Solitary radial electric field structure in tokamak plasmas

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The solitary structure solution of the radial electric field E_r in the tokamak plasmas is obtained. It is shown to be stable under an external power supply, like a biased electrode at the edge. The radial gradient is governed by the ion viscosity and the nonlinearlity of the perpendicular conductivity. The radial structure of E_r and reduction of turbulent transport, which belong to key issues of the high confinement mode (H-Mode) [F. Wagner *et al.*, Phys. Rev. Lett. **49**, 1408 (1982)], are self-consistently determined. A bifurcation from a radially-uniform one to a solitary one occurs at a certain applied voltage, and a hysteresis is associated. © *1998 American Institute of Physics.* [S1070-664X(98)01312-3]

The finding of the high-confinement mode (H-mode) in tokamak plasmas[1] is one of the first experimental demonstrations of the structural transition in confined plasmas, which are in the far-nonequilibrium state. An electric field bifurcation has been proposed for the mechanism of the H-mode transition,[2] and the important role of the structure of the radial electric field E_r on the plasma confinement is now widely recognized (see review, e.g., Refs. 3, 4.) Related to the electric field, the impact on the micro turbulence has been investigated most intensively.[5–7] The interface between the regions with different electric polarity was discussed,[3,8] and spatio-temporal evolution of E_r in the case of improved confinements has attracted attention, e.g., Refs. 9, 10. The experiment has been done by use of the biased electrode near the plasma periphery,[11] to study the turbulent suppression and the nonlinear relation between the radial current and E_r. The data constitutes a basis to understand the plasma nonlinearity that induces the electric field bifurcation. Several attempts at analysis have been done,[12] but the physics mechanism that determines the gradient of E_r is left unresolved there.

In this article, we study the spatial structure of the radial electric field in the presence of the radial current across the magnetic field. It is found that there exists a solution of the solitary structure of E_r. The gradient and its impacts on the turbulence suppression are self-consistently determined. The ion viscosity, coupled with the nonlinearity in the perpendicular conductivity, governs the gradient of the radial electric field. It is shown that the bifurcation of E_r takes place from a radially-homogeneous one to the solitary structure at a threshold voltage imposed on the electrode.

The charge conservation relation combined with the Poisson's relation governs E_r as $(\partial/\partial t)E_r = -(1/\epsilon_0\epsilon_\perp) \times(J_r^{NET} - J_{ext})$, where J_r^{NET} is the net radial current in the plasma, J_{ext} is the current that is driven into the electrode by the external circuit, ϵ_0 is the vacuum susceptibility, and $\epsilon_\perp$ is a dielectric constant of the magnetized plasma. (The effect of ion polarization current is kept in $\epsilon_\perp$, because we are interested in a time change that is much slower than the ion cyclotron frequency.) The radial current is composed of two components, $J_r^{NET} = J_r - \epsilon_0\epsilon_\perp \nabla\cdot\mu_i\nabla E_r$. The first term J_r is the ''local current,'' which is determined by the radial electric field at the same radial location. The second is caused by the shear viscosity of ions, μ_i, and includes the diffusion operator.[3] Note that the evaluation of $\epsilon_\perp$, is performed by solving the Newton equation for the toroidal plasmas (see, e.g., Refs. 3, 12–14). The equation of E_r in a stationary state is a nonlinear diffusion equation, as

$$\nabla\cdot\mu_i\nabla E_r - \frac{1}{\epsilon_0\epsilon_\perp}(J_r - J_{ext}) = 0. \quad (1)$$

The local current J_r and E_r is related through the perpendicular conductivity, $J_r = \sigma(E_r)E_r$. In this article, we study the case that the neoclassical current[12] is dominant in J_r. This is because we are interested in the situation that the plasma is away from the condition that may allow spontaneous bifurcations, for which the contribution of other mechanisms to J_r is known to be important.[3] (Associated with this simplification, the diamagnetic velocity and neoclassical poloidal velocity are neglected.) We are interested in the very steep gradient of E_r. Compared to the structure of E_r, the other plasma parameters are slowly varying in space, so that the other plasma parameters are treated as constant for simplicity.

First, we study the case that the ion viscosity μ_i is constant. The dependence of the conductivity on E_r is symbolically written as $\sigma(E_r) \equiv \sigma(0)f(X)$, where E_r is normalized as $X = e\rho_p E_r/T$ (ρ_p: ion poloidal gyroradius, T: ion temperature). The neoclassical theory has given $f(X) \simeq \exp(-X^2)$ in the collisionless limit and $f(X) \simeq 1/(\nu_*^2 + X^2)$ in a collisional case.[12] The essential thing is that $f(X)$ satisfies the relations $f(0)=1$ and $Xf(X)\to 0$ as $|X|\to\infty$. For this analytic study, we consider the radially-thin shell structure, and introduce the normalization in space and current density as $x=(r-r_0)/l$ and $I=(e\rho_p/T\sigma(0))J_{ext}$, where $l=\sqrt{\mu_i/\sigma(0)}$. (The radius r_0 is chosen at the middle between two electrodes.) Then Eq. (1) is rewritten as

$$\frac{\partial^2}{\partial x^2}X-f(X)X+I=0. \tag{2}$$

The solitary solution of the electric field, which has cylindrical symmetry, is searched for. The solution is much more localized than the distance between the two magnetic surfaces, on which the electrodes are located. The boundary condition is chosen as $\partial X/\partial x\rightarrow 0$ (i.e., $X\rightarrow X_1$) at $|x|\rightarrow\infty$. We choose $x=0$ at the surface of the symmetry.

The stationary solution is obtained. Equation (2) has a trivial solution, which is constant in space, as $X=X_1$, where X_1 is the solution of the equation $f(X_1)X_1=I$. [The equation $f(X)X=I$ has two solutions, X_1 and X_2. X_1 is chosen by the condition $|X_1|<|X_2|$.] Besides this trivial solution, there is a nontrivial solution with the solitary radial electric field. Equation (2) is integrated as $2^{-1}(dX/dx)^2=\int_{X_1}^{X}Xf(X)dX-IX+\mathrm{const}\equiv F(X)$. Function $F(X)$ takes the minimum at $X=X_1$ and the maximum at $X=X_2$, respectively, and is a decreasing function of X in the region of $X>X_2$. The integral constant is chosen as $F(X_1)=0$, to satisfy the boundary condition at $|x|\rightarrow\infty$. The solution $X(x)$ is given as

$$x=\int^{X}\{2F(X)\}^{-1/2}dX. \tag{3}$$

This solution gives the solitary structure of the radial electric field.

The solution is studied near the critical current, $I\simeq I_*$, where the local current $Xf(X)$ takes the maximum with respect to X at $X=X_*$. Expanding $F(X)$ in the vicinity of $I\simeq I_*$ as $F(X)=C\{(X_*-X_1)(X-X_1)^2-(X-X_1)^3/3\}+\cdots$, we have the solution as

$$X(x)=X_*+3\alpha^2-3\alpha^2\left(\frac{e^{\alpha Cx}-1}{e^{\alpha Cx}+1}\right)^2, \tag{4}$$

where $\alpha\equiv C^{-1/4}(I_*-I)^{1/4}$ and $C=(-1/2)(\partial^2/\partial X^2)\times[Xf(X)]_{X=X_*}$. The peak height scales as $(I_*-I)^{1/2}$ and the width scales like $(I_*-I)^{-1/4}$.

To study the voltage–current relation quantitatively, let us take a model from $f(X)=1-X^2/3X_*^2(|X|<\sqrt{3}X_*)$ and $f(X)=0(|X|>\sqrt{3}X_*)$. This model keeps an essential feature of the conductivity, i.e., f gradually becomes smaller if $|X|$ is small, and $f\ll 1$ holds in the large $|X|$ limit. This form of f provides an exact analytic solution for the solitary radial electric field structure. Figure 1 illustrates the solitary solution in the case of $X_1/X_*=0.6$. By performing the integral $V=\int_{-d/2}^{d/2}X(x)dx$, the voltage difference between the electrodes is calculated. (d is a distance between the electrode.) In the asymptotic limit $y_1 d\gg 1$, one has explicit relations: $V=4\sqrt{6}[\pi/2-\arctan(y_1^{-1}(\sqrt{1-X_1^2/3X_*^2}+\sqrt{2}X_1/\sqrt{3}X_*))]X_*+X_1 d$ for the case of $X_*/\sqrt{3}<X_1<X_*$, and $V=(4\sqrt{2}/3)C_m^{3/2}X_*^3I^{-2}+2\sqrt{2C_m}(\sqrt{3}X_*-X_1)X_*I^{-1}+4\sqrt{6}\times[\pi/2-\arctan(C_2y_1/2+X_1/\sqrt{3}X_*y_1)]X_*+X_1 d$ for $X_1<X_*/\sqrt{3}$. Here, coefficients are given as $y_1=\sqrt{1-X_1^2X_*^{-2}}$, $C_2=(c_2+\sqrt{c_3})(1-X_1/\sqrt{3}X_*)^{-1}$, $C_m=c_3(\sqrt{3}-X_1/X_*)^2/8$, $c_2\equiv 2(1+X_1/\sqrt{3}X_*)(1-2X_1/\sqrt{3}X_*)(1-X_1^2X_*^{-2})^{-1}$, and $c_3\equiv 2(1+X_1/\sqrt{3}X_*)(1-\sqrt{3}X_1/X_*)(1-X_1^2X_*^{-2})^{-1}$. Figure 2 illustrates the $V-I$ curve in the case of $d=20$. The voltage difference V is rewritten as $V=V_{peak}+X_1 d$, where V_{peak} is due to the deviation of the solitary solution from the constant one. For the trivial solution, $X=X_1$, the voltage difference is given by $V=X_1 d$.

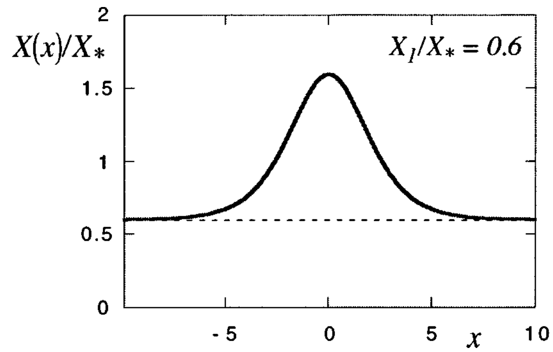

FIG. 1. Solitary structure of the radial electric field. Model form $f(X)$ is taken as $f(X)=1-X^2/3X_*^2(|X|<\sqrt{3}X_*)$ and $f(X)=0(|X|>\sqrt{3}X_*)$. The parameter is $X_1/X_*=0.6$ ($I/I_*=0.792$). The dotted line shows the trivial solution $X=X_1$.

The solitary structure is characterized by the peak value of the radial electric field $X(0)$ and the radial width Δ. In the small I limit, asymptotic forms hold generally as $X(0)\propto X_*^2I^{-1}$, $\Delta\propto X_*/I$, and $V_{peak}\simeq X_*^3I^{-2}$. In the case of $I\simeq I_*$, it is explicitly calculated as $V_{peak}=12C^{-5/4}(I_*-I)^{1/4}+\cdots$.

The bifurcation is described by the voltage–current relation. The $V-I$ curve is a multi-valued function, as is shown in Fig. 2. For a fixed value of current, two solutions of V are given. For a fixed value of V, one, or three solutions of I are available. In the experimental condition, the external circuits are often composed of the power supply of V_{ext} and the internal resistance. Then the applied voltage between the electrode V and the current density I is constrained as $V=V_{ext}-\hat{r}_iI$ (the coefficient $\hat{r}_i$ is proportional to the internal resistance), as is shown by the solid (or dashed) lines in Fig. 2. The cross-points of the $V-I$ curve and the constraints $V=V_{ext}-\hat{r}_iI$ give the solutions. In the cases of high and low

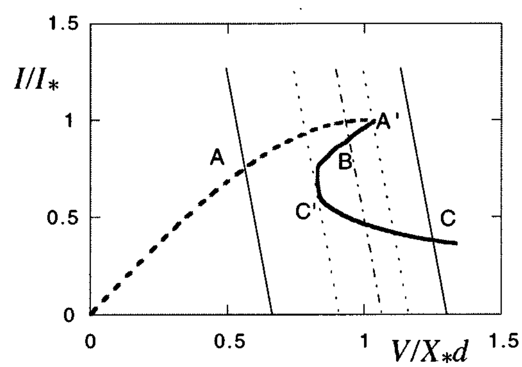

FIG. 2. The relation between the voltage V and the current I for the solitary structure of E_r (thick solid line) and that of the homogeneous E_r (thick dashed line). A constraint by the external circuit, $V=V_{ext}-\hat{r}_iI$, as is shown by the thin lines. Bifurcation to the solitary structure takes place at A', and the back transition occurs at C'.

V_{ext} (thin solid lines), solutions are given by A or C and are stable. Bifurcation from the constant one to the solitary structure takes place at A', and the back transition occurs at C'. When three roots are given (a thin dashed–dotted line), the second solution B is unstable. We see a hysteresis of the electric field structure as a function of the voltage in the power supply. Depending on the characteristics of the external circuit, this system also shows the limit cycle oscillation. The details will be reported in a separate article.

Finally, the influence of the radial electric field inhomogeneity on the ion viscosity is investigated. The shear viscosity of ions has two origins: one is the collisional transport, μ_c, and the other is the turbulent transport, μ_N. The turbulent transport could depend on the electric field gradient, and the ratio $|\omega_{El}/\gamma_{dec}|$ is the key parameter, where $\omega_{E1}=(dE_r/dr)B^{-1}$ and γ_{dec} is the nonlinear decorrelation rate of the fluctuations that cause the turbulent transport.[5–7] Analytic formulas have been derived as $\mu_N=\mu_N(0)(1+\omega_{E1}^2/\gamma_{dec}^2)^{-1}$ (when $|\omega_{E1}/\gamma_{dec}|$ is small) and $\mu_N \propto \mu_N(0)|\omega_{E1}/\gamma_{dec}|^{-\nu}$ (when $|\omega_{E1}/\gamma_{dec}|$ is large, $\nu<1$). We chose, as an interpolation formula, $\mu_N=\mu_N(0)(1+(2/\nu)\times(\omega_{E1}/\gamma_{dec})^2)^{-\nu/2}$. The explicit form of the coefficient γ_{dec} is given in, e.g., Ref. 3. Introducing normalized coefficients as $H_1\equiv(eT/\gamma_{dec}Bl\rho_p)^2$, $\mu_{i0}\equiv\mu_i(X\to0)=\mu_N(X\to0)+\mu_c$, and $\eta\equiv\mu_N(X\to0)/\mu_{i0}$, we rewrite as $\mu_i=\mu_{i0}\{1-\eta+\eta(1+(2/\nu)H_1(dX/dx)^2)^{-\nu/2}\}$. Length l is defined as $l=\sqrt{\mu_{i0}/\sigma(0)}$. Equation (2) is integrated as

$$\frac{\eta\nu}{4H_1}\left\{\frac{1}{1-\nu/2}+\frac{1-\nu}{1-\nu/2}\left(1+\frac{2H_1}{\nu}\left(\frac{dX}{dx}\right)^2\right)^{1-\nu/2} -2\left(1+\frac{2H_1}{\nu}\left(\frac{dX}{dx}\right)^2\right)^{-\nu/2}\right\}+\frac{1}{2}(1-\eta)\left(\frac{dX}{dx}\right)^2=F(X). \tag{5}$$

Equation (5) provides a self-consistent solution for E_r and turbulence suppression. The peak value of $X, X(0)$, is not modified, because it is determined by the relation $F(X(0))=0$. The solution also has the same asymptotic form at $|x|>\Delta$. The coupling with the suppression of the turbulent transport makes the solitary structure of E_r more peaked, but does not change the qualitative nature. If the coefficient H_1 is small, $(2/\nu)H_1X_*^2\ll1$, the solution $X(x)$ is unaltered from Eq. (3), and the maximum suppression factor is given as $\mu_N/\mu_N(0)\simeq(1+H_1X_*^2)^{-1}$. In an intermediate range, $1\ll(2/\nu)H_1X_*^2\ll((1-\nu)/(1-\nu/2))^{2/\nu}\eta^{2/\nu}(1-\eta)^{1-2/\nu}$, the maximum of the gradient is estimated as $X'\simeq(X_*^2(1-\nu/2)/(1-\nu))^{1/(2-\nu)}(2H_1/\nu)^{\nu/(4-2\nu)}$, and the maximum suppression factor is given as $\mu_N/\mu_N(0)\simeq(2H_1X_*^2(1-\nu/2)/\nu(1-\nu))^{-\nu/(2-\nu)}$. In the case of large coefficient H_1, $((1-\nu)/(1-\nu/2))^{2/\nu}\eta^{2/\nu}(1-\eta)^{1-2/\nu}\ll(2/\nu)H_1X_*^2$, the left hand side of Eq. (5) is approximated as $2^{-1}(1-\eta)(dX/x)^2$: The maximum of the gradient is approximately given as $X'\simeq(1-\eta)^{-1/2}X_*$. The maximum suppression factor is given as $\mu_N/\mu_N(0)\simeq(1-\eta)^{-2/\nu}(2H_1X_*^2/\nu)^{-\nu/2}$, satisfying the relation $\mu_N<(1-\nu/2)(1-\nu)^{-1}\mu_c$. The anomalous transport coefficient is reduced to the level of collisional one and the momentum transport barrier is locally formed.

In summary, the solitary-ring structure of the radial electric field in the tokamak plasmas is obtained. The stable solitary structure is sustained by the external steady power supply. The radial structure and the suppression of the turbulent transport are self-consistently obtained. Comparisons of the theoretical results (e.g., bifurcation condition and radial shape of E_r) with experimental observations allow us to evaluate fundamental parameters, such as of the nonlinear J_r–E_r relation, ion viscosity, or decorrelation rate of fluctuations.

In this article, several simplifications are made due to analytic transparency. The pressure-driven radial current in the limit of $E_r=0$ and the neutral particle effects are neglected. In addition, the turbulent transport can affect J_r, and J_r (anomalous) depends on E_r as well as E_r' (see Ref. 3 and references therein). The influence of such an effect is considered in relation with improved confinement.[10] The E_r' term could appear in Eq. (2), and a radially-moving solitary structure of E_r is allowed. [If we have a term like $\alpha X'$ in Eq. (2), then the solution is given as $X(x-\alpha\tau)$ where $\tau=t/t_N$ and $t_N=\epsilon_0\epsilon_\perp/\sigma(0)$. It has a velocity α.] The solution of Eq. (2) also includes the one in which multiple solitary structures are confined between the electrodes. These corrections and variations will be reported in a separate article.

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