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Self-sustained annihilation of magnetic islands in helical plasmas

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The evolution of the magnetic island which is induced by the resonant deformation by external currents in helical systems (such as the large helical device (LHD) [A. Iiyoshi, Phys. Plasmas **2**, 2349 (1995)]) is analyzed. The defect of the bootstrap current, caused by the magnetic island, has a parity which reduces the size of the magnetic island, if the bootstrap current enhances the vacuum rotational transform. The width of magnetic island can be suppressed to the level of ion banana width if the pressure gradient exceeds a threshold value. This island annihilation is self-sustained. That is, the annihilation continues, for fixed beta value, until the external drive for island generation exceeds a threshold. The effects of the reversal of the direction of the bootstrap current and of the sign of radial electric field are also investigated. The possibility of the neoclassical tearing mode in the LHD-like plasma is discussed. © *2005 American Institute of Physics*.
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I. INTRODUCTION

The problem of abrupt transitions has attracted attention in the research on the magnetic confinement of plasma.[1] Examples include the change of confinement modes (e.g., the improved confinement modes), the topological change of magnetic structure, the change of radiation loss patterns, and others. One of the characteristic examples of the transition appears as a magnetohydrodynamic (MHD) instability named the tearing mode.[2,3] Analysis was made for a case of externally driven magnetic island.[4,5] A nonlinear instability was found, which is called the neoclassical tearing mode (NTM) (Refs. 6–9). This is a subcritically excited tearing mode under the influence of the pressure gradient. It has been shown that the subcritical NTM is triggered by the stochastic turbulence noise.[10,11] Another phenomenon, which is also attributed to the onset of subcritical excitation of the magnetic island, is the Snake event.[12,13] In this case, a dense plasma is sustained, after the pellet injection, in a localized helical tube on low-order rational surfaces.

The magnetic island formation in toroidal helical plasmas has also been subject to intensive studies. The classical problem is the destruction of the magnetic surface at the rational surfaces.[14] The influence of the vertical field and Pfirsch-Schlüter (PS) current on the island formation has been discussed. (For instance, see Refs. 15–22.) The island size can depend on the plasma pressure. Recently, experiments on the large helical device (LHD) (Ref. 23) have been performed, where global magnetic islands are induced by the external helical resonant current.[24–26] Under certain conditions, the externally induced island is annihilated. The annihilation process was found to be persistent, in a certain finite range of variance of external control parameters. This observation is not explained by the effect of the vertical field or Pfirsch-Schlüter current, and suggests that there is a self-sustaining process that maintains island annihilation.

In this paper, the theory of the neoclassical tearing mode is extended to the case of the LHD-like plasmas, where the external resonant current exists and the magnetic island is induced. It is shown that the island width is suppressed to the level of the ion banana width, if the plasma beta value exceeds a threshold value. The island width does not grow, in the finite-beta plasmas in LHD-like systems, until the magnitude of the external drive exceeds the threshold value. The island annihilation occurs not at a particular value but in a region with finite range of controlling parameters. That is, a self-sustained state where the magnetic island is annihilated is explained by this model. A test of this theoretical model is also proposed. The direction of the bootstrap current can be reversed in the helical systems.[27] In such cases, the plasma pressure gradient makes the island width larger. The neoclassical tearing mode possibly occurs even in the absence of the external drive of the island. The influence of the radial electric field is also investigated. This gives a new insight for the evolution of magnetic islands in toroidal plasmas.

II. MODEL

An evolution of the magnetic island in the presence of bootstrap current has been discussed in the theory of neoclassical tearing mode. As the first step in studying the island annihilation in helical plasmas, we choose a simple model in a deterministic picture, neglecting the stochastic noise by background turbulence. The global parameters such as the mean pressure profile are treated as prescribed parameters. We do not consider the three-dimensional dependence of the perturbation. The influences of the helical magnetic field are represented in the average magnetic-surface quantities, such as the pressure gradient, rotational transform, magnetic shear, and bootstrap current density. These quantities are treated as the magnetic-surface quantities and considered here as the given parameters. By this simplification, the mesoscale islands, which might be induced by the combination of the excited global magnetic island and the helical asymmetry, are not considered here. Only the single global island (excited externally) in the average magnetic structure is analyzed here.

In the "Rutherford regime" of the island evolution, which is relevant to cases where the island is large enough to be observed experimentally, the equation that describes the evolution of the resonant magnetic island is given as

$$\frac{\partial}{\partial t}A + \eta \Lambda A = 0, \tag{1}$$

where $A \equiv \tilde{A}_* q^2 R / B r_s^3 q'$ is the normalized amplitude of the (m,n)-Fourier component of helical vector potential perturbation $\tilde{A}_*$ at the mode rational surface $r=r_s$, η is the inverse of resistive diffusion time $\eta = \eta_\parallel \mu_0^{-1} r_s^{-2} \tau_{\mathrm{Ap}} = R_M^{-1}$, where $\eta_\parallel$ stands for a parallel resistivity and R_M is the Lundquist number (magnetic Reynolds number), and $-\Lambda A$ is the (m,n)-Fourier component of helical current on the rational surface $r=r_s$. (See, e.g., Refs. 28 and 29 for the explanation of mechanisms.) Here, the time is normalized to poloidal Alfvén transit time, $\tau_{\mathrm{Ap}} = qR/\nu_A$ (ν_A, Alfvén velocity) and the length to r_s.

An explicit form of the growth rate is given, within the neoclassical transport theory, by

$$-\Lambda A = 2\Delta' A^{1/2} - \beta_{\mathrm{bs}}\left(a_{\mathrm{bs}}\frac{A}{W_1 + A} + \frac{\rho_b^2}{r_s^2}\frac{L_q}{L_p} a_{\mathrm{pc}} \frac{A}{W_2^2 + A^2}\right), \tag{2}$$

where the first, second, and third terms of the right-hand side (RHS) stand for the effects of the current density gradient (including the effects of the external current that generates the resonant magnetic island), the bootstrap current, and the ion polarization current, respectively.

The first term in the RHS of Eq. (2) stands for the effect of the external resonant current, such as the effects of the external magnetic field coil or the resonant component of the PS current. An induction of the magnetic island by the external coil and resonant PS current has been analyzed in detail,[16–22] and the contribution of these effects are parametrized by the magnetic island width at the stationary state δ. According to Ref. 5, we write

$$\Delta' = \left(\frac{\delta^2}{r_s^2}\frac{1}{A} - 1\right)|\Delta'_0|, \tag{3}$$

where the coefficient Δ'_0 represents the stabilizing influence in the absence of the external current. (We study here the case that the linear tearing mode is stable, $\Delta'_0 < 0$. The island width δ is not normalized, in order to explicitly show the parameter dependence.) The second and third terms in the RHS of Eq. (2) represent the effects by the bootstrap current and ion-polarization current, respectively. (See Ref. 11 and references therein for details.) The coefficient β_{bs} indicates the plasma beta value

$$\beta_{\mathrm{bs}} = \frac{2\varepsilon^{1/2} L_q}{L_p}\beta_p, \tag{4}$$

a_{bs} indicates the ratio between the bootstrap current and poloidal beta value, such as $J_{\mathrm{bs}} = \mu_0^{-1}\varepsilon^{1/2} a_{\mathrm{bs}} \beta_p L_p^{-1} B_p$, and a_{pc} denotes the sign of the effect of ion-polarization current, $a_{\mathrm{pc}} = \omega(\omega - \omega_{*\mathrm{pi}})\omega_{*\mathrm{pi}}^{-2}$, where ω is the frequency of the island deformation which is observed on the frame, which is moving with the $E \times B$ drift velocity.[30] As was clarified in Refs. 8 and 30, the essential elements in determining the coefficient a_{pc} is the relative velocity of the propagation of the mode and ion diamagnetic motion. Of course, as is discussed in Ref. 31, the solution of the kinetic equation by employing the viscosity in the presence of magnetic island provides a numerical factor $\omega_{*\mathrm{pi}}$ in the expression of a_{pc}. Such an advanced modeling is useful if one extends the present model to quantitative comparison with the experiments. We do not calculate the coefficient a_{bs} in this paper, but treat a_{bs} as a given parameter. Influences of the helical geometry are included in this coefficient, and its explicit calculation is left to the literature.[32] Other important parameters are as follows: ρ_b is the banana width; L_q and L_p are the gradient scale lengths of safety factor and pressure, respectively. In the following, we take $W_1 = \rho_b^2 r_s^{-2}$, noting the fact that the turbulent transport coefficient is reduced in the magnetic island of LHD.[33] The ion-polarization current is chosen as stabilizing in Eq. (2). This point is discussed in Sec. IV.

III. EXTERNALLY INDUCED MAGNETIC ISLAND

A. Finite-beta effect

From Eq. (2), one sees the effect of the bootstrap current (and the ion polarization drift effect) on the growth of the magnetic island. When $\beta_{\mathrm{bs}} > 0$, the second term in the RHS of Eq. (2) is stabilizing. When $\beta_{\mathrm{bs}} < 0$ holds, it is destabilizing.

The sign of the coefficient β_{bs} is determined by two factors. The first one is the ratio L_q/L_p. The pressure profile is usually a decreasing function of radius, so that the ratio L_q/L_p is positive for the configuration such as LHD (in which the safety factor q is decreasing to the edge) and negative for normal operation of tokamaks (in which the safety factor is increasing to the edge). The other is the coefficient a_{bs}. In normal conditions of tokamaks or the standard configuration of LHD, the bootstrap current is in the direction such that it enhances the background rotational transform. (In this case $a_{\mathrm{bs}} > 0$.) The coefficient a_{bs} can change the sign in helical systems by a particular choice of the Fourier component of magnetic fields.[27,32] Considering these two factors, one sees that, in the standard LHD operation condition, the second term has stabilizing influence on the externally driven magnetic island.

B. Width of saturated island

The saturation width of the island is evaluated from the relation $\Lambda A = 0$. From Eq. (2), one sees that the second term in the RHS is effective for $A > \rho_b^2 r_s^{-2}$, i.e., the island width is larger than ρ_b. The effect of the bootstrap current is the dominant stabilization effect, and the ion-polarization drift may be effective when the island width approaches to ρ_b. We study in this section the standard cases where $a_{\mathrm{bs}} > 0$ and $a_{\mathrm{pc}} > 0$

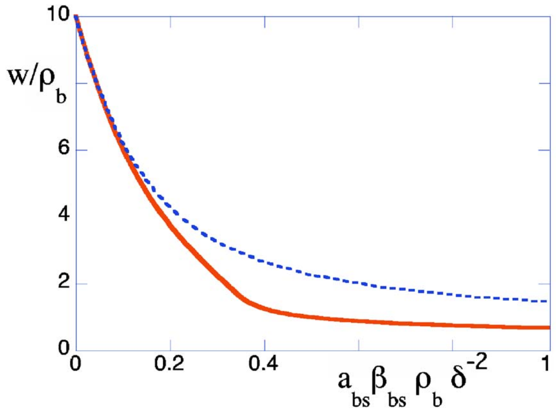

FIG. 1. The width of the magnetic island as a function of the plasma beta value. The parameter δ is fixed, $\delta/\rho_b=10$. The dashed line indicates the suppression by the bootstrap current effect and the solid line includes the ion polarization drift. We simply choose $2|\Delta'_0|=1$, $L_q/L_p=3$, and $a_{\rm pc}=a_{\rm bs}$.

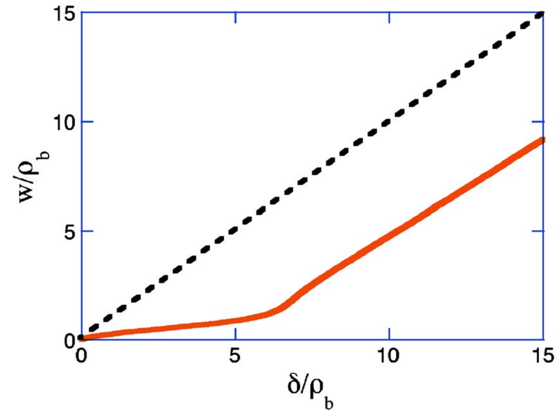

FIG. 2. The width of the island as a function of the intrinsic island width δ for a fixed value of the plasma pressure, $a_{\rm bs}\beta_{\rm bs}=0.3$ and $r_s/\rho_b=50$. (Other parameters are the same as in Fig. 1.)

hold. The marginal condition gives the island width w, $w/r_s=\sqrt{A}$, at the stationary state. In the limit of $A \gg \rho_b^2 r_s^{-2}$, one has

$$\frac{w}{r_s}=-\frac{a_{\rm bs}\beta_{\rm bs}}{4|\Delta'_0|}+\sqrt{\frac{a_{\rm bs}^2\beta_{\rm bs}^2}{16|\Delta'_0|^2}+\frac{\delta^2}{r_s^2}}. \tag{5}$$

In a small beta limit, one has

$$\frac{w}{r_s}\simeq\frac{\delta}{r_s}-\frac{a_{\rm bs}\beta_{\rm bs}}{4|\Delta'_0|}. \tag{6}$$

In the limit of large $\beta_{\rm bs}$, Eq. (5) gives

$$\frac{w}{r_s}\sim\frac{2|\Delta'_0|}{a_{\rm bs}\beta_{\rm bs}}\frac{\delta^2}{r_s^2}, \tag{7}$$

showing a slow reduction with respect to the plasma pressure. When the island width approaches to ρ_b, the stabilization by the ion-polarization current is effective. The island width is chopped off to the level of ρ_b if the condition

$$\beta_{\rm bs}\frac{\rho_b}{r_s}\sqrt{a_{\rm bs}a_{\rm pc}\frac{L_q}{L_p}}\geq\frac{|\Delta'_0|}{2}\frac{\delta^2}{r_s^2} \tag{8}$$

is satisfied. The critical beta for annihilation of the induced magnetic island is given as

$$\beta_p\geq\beta_{\rm p*}=\frac{|\Delta'_0|}{4\varepsilon^{1/2}}\sqrt{\frac{L_p^3}{a_{\rm bs}a_{\rm pc}L_q^3}}\frac{\delta^2}{\rho_b r_s}. \tag{9}$$

Figure 1 illustrates the island width as a function of the plasma beta value. The dashed line is the case where only the first term in the parenthesis of Eq. (2) is taken into account. The reduction of the island is shown. The solid line shows the case where the effect of the ion-polarization current is included. A sharper reduction when the island width approaches to ρ_b is demonstrated. The analytic estimate is found to be relevant.

The response of the island width as a function of the external drive is illustrated in Fig. 2. In this figure, the saturation island width is shown as a function of δ for fixed plasma beta value. Below the threshold,

$$\frac{\delta}{r_s}\leq\frac{\delta_c}{r_s}=2(\varepsilon a_{\rm bs}a_{\rm pc})^{1/4}\left(\frac{L_q}{L_p}\right)^{3/4}\left(\frac{1}{|\Delta'_0|}\frac{\rho_b}{r_s}\beta_{\rm p}\right)^{1/2}, \tag{10}$$

the induced island width is suppressed to the level of the banana width of ions. When the width of the induced island exceeds this threshold, $\delta\geq\delta_c$, the island starts to grow. In a large δ limit, $\delta\gg\delta_c$, the island width approaches to δ, $w\to\delta$.

One might be interested in a parameter dependence of the critical size δ_c. In the collisionless regime, $a_{\rm bs}$ is weakly dependent on the plasma parameter, so that the relation

$$\delta_c\propto\sqrt{\beta_p\rho_b}\propto n^{1/2}T^{3/4} \tag{11}$$

holds. In the plateau regime, $\nu>v_{\rm th}/qR$ (ν, collision frequency; $v_{\rm th}$, thermal velocity), one has a reduction of the bootstrap current, which may be modeled as $a_{\rm bs}\propto v_{\rm th}/\nu qR\propto n^{-1}T^2$. Therefore, the critical width has a dependence

$$\delta_c\propto\sqrt{\beta_p\rho_b v_{\rm th}/\nu qR}\propto T^{7/4}, \tag{12}$$

and is independent of the density.

The result that the island width is suppressed in the region Eq. (10) [or Eq. (12)] indicates that this is a self-sustained healing state against the external generation of the magnetic island. The self-sustained healing requires, by definition, that the island annihilation (below the level of the ion gyroradius) not only happens at particular values of global parameters but also continues to hold even if global parameters change in a certain range. The annihilation of the magnetic island by the finite plasma pressure is an example of the self-sustained healing of the islands in helical plasmas.

It is noted that the excitation of the magnetic island, shown in Fig. 2, has the feature of the supercritical excitation. The island width w is a monotonic increasing function of δ, and no bifurcation happens in this case. No hysteresis is predicted in Fig. 2.

C. Variations

The case of an opposite bootstrap current ($a_{\rm bs}<0$) is also studied. In this case, the defect of the bootstrap current has a destabilizing effect. Figure 3 shows the island width as

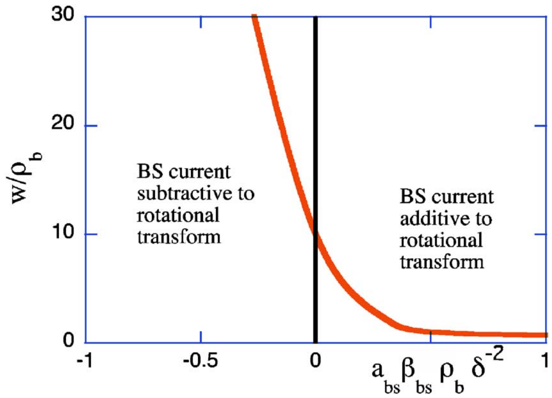

FIG. 3. The island width for positive and negative values of $\beta_{\rm bs}$. (Other parameters are the same as in Fig. 1.)

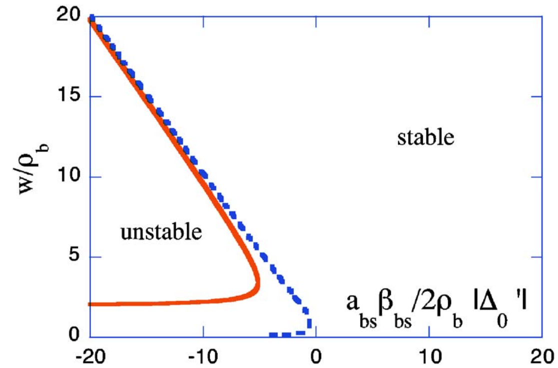

FIG. 4. Magnetic island width as a function of the plasma pressure gradient when the bootstrap current is in the counterdirection. We take $\delta=0$ in this case. The solid line shows that the ion-polarization current has a stabilizing effect, $a_{\rm pc}=|a_{\rm bs}|$. The dashed line indicates the case that it is destabilizing for $a_{\rm bs}<0$ ($a_{\rm pc}=a_{\rm bs}$). (Other parameters are the same as in Fig. 1.)

a function of $\beta_{\rm bs}$ for a fixed value of δ. In the configuration where the q value is decreasing to the surface, the magnetic island width is enhanced by the plasma pressure if the bootstrap current is in the counterdirection (reducing the vacuum rotational transform).

The other issue is the role of the ion-polarization current. It was shown that the coefficient $a_{\rm pc}$ (positive if stabilizing) is proportional to the sign of $\omega(\omega-\omega_{*\,\rm pi})$. The island which is driven by the external drive is stationary in the laboratory frame. The radial electric field has two branches in LHD-like plasmas.[34] When the radial electric field in the LHD plasma is in the so-called electron root $E_r>0$, the plasma motion is in the direction of the ion diamagnetic drift. The relative motion of the island in the plasma frame is in the direction of the electron diamagnetic drift, i.e., $\omega(\omega-\omega_{*\,\rm pi})>0$. The term with $a_{\rm pc}$ is stabilizing. In contrast, if the radial electric field is that of the ion root, $E_r<0$, there is a possibility $\omega(\omega-\omega_{*\,\rm pi})<0$. Such a situation holds if the radial electric field is negative and the $E\times B$ velocity is smaller than the ion-pressure gradient drift velocity. Therefore, depending on the branch of the radial electric field, the ion-polarization current effect can be destabilizing. (Note that the radial electric field is subject to the bifurcation owing to the presence of magnetic islands.[31,35] In this analysis the radial electric field is treated as a parameter.)

D. Possibility of neoclassical tearing instability

The destabilization effect in the case of $a_{\rm bs}<0$ suggests the neoclassical tearing instability in the helical systems. Thus, the tearing instability can take place for the configuration with $a_{\rm bs}<0$ in the LHD-like plasmas.

Putting $\delta=0$ into Eq. (2), the balance between the stabilizing force associated with the magnetic configuration and the destabilization owing to the defect of bootstrap current on the resonance magnetic surface can be analyzed. In the small A limit, the leading term in the destabilizing force depends linearly on A, while the stabilizing term has a dependence on $A^{1/2}$. The latter dominates the former in the small A limit. This suggests that the perturbation can be unstable through subcritical excitation. The threshold amplitude $w_{\rm th}$ exists and the perturbation becomes unstable if the amplitude exceeds this threshold, $A^{1/2}>w_{\rm th}/r_s$. When the polarization current effect is stabilizing, the threshold amplitude is estimated as $w_{\rm th}\sim O(\rho_b)$ as was discussed for tokamaks. If it is also destabilizing, $a_{\rm pc}=a_{\rm bs}$, the threshold size of the island is evaluated in the large $\beta_{\rm bs}$ limit as

$$\frac{w_{\rm th}}{r_s}\simeq\frac{2|\Delta_0'|}{|a_{\rm bs}|\beta_{\rm bs}(1+L_q/L_p)}\left(\frac{\rho_b^2}{r_s^2}\right). \tag{13}$$

The threshold amplitude for the onset of NTM becomes much smaller in this case. When the island width exceeds this threshold, the island grows and reaches the finite-amplitude stationary state. In the large amplitude limit, the saturation amplitude is given as

$$\frac{w}{r_s}=\frac{|a_{\rm bs}|}{2|\Delta'_0|}\beta_{\rm bs}. \tag{14}$$

The island width increases as the plasma pressure increases, when $a_{\rm bs}<0$ holds. Figure 4 illustrates the marginal stability condition $\Lambda=0$ in the case of $\delta=0$.

E. Implication to experiments

Before closing, the relevance of this model to the experimental observation is discussed. First, this model provides a self-sustaining annihilation of the externally driven magnetic island. The annihilation holds within a certain finite range of variation of the global parameters. Such a self-sustaining mechanism can be seen in the case where either the external coil current is modified or the island annihilation by the resonant PS current works. For the latter case, one may consider the case that the resonant PS current tends to reduce the island width as the plasma pressure increases. When the island width becomes smaller than the critical size obtained here, the island is annihilated, and this annihilation occurs in a finite range of plasma pressure, not at a particular value of pressure (where the exact cancellation between the effects of coil current and PS current occurs). Second, this model predicts that the reduction of the island width is controlled by the factor $\beta_{\rm p}\rho_{\rm b}$ and the collisionality. That is, the island is reduced both by the plasma pressure gradient and by the higher temperature. The dependencies, Eqs. (11) and (12), are consistent with experimental observations.[25] Third, this model is consistent with the onset of the island by the pellet injection.[26] When the pellet is injected, the plasma temperature decreases on magnetic surfaces in the island, while the change of the pressure is small in a fast time. The increment of the collisionality causes the reduction of the bootstrap current at the position of O point, and thus eliminates the defect of the bootstrap current which has a stabilizing force on the magnetic island. When the temperature recovers after the pellet injection, the island is suppressed. The fourth issue is the time scale of the appearance of the magnetic island. Consider the case that the initial condition is chosen as the annihilated state and that the stabilization term disappears at $t=0$. In such a case, the dependence of Δ' on A (i.e., $\Delta' \simeq \delta^2 A^{-1}$) accelerates the growth and gives $w \sim (\eta t)^{1/3}$ (Ref. 5). This may explain a fast reappearance of the externally driven magnetic island after the pellet injection. Fifth, the test in the case of reversing the direction of the bootstrap current is discussed. Sixth, the role of ion-polarization drift could be tested. The sign of $E \times B$ velocity, outside the island, can be changed in LHD.[34] The plasmas with the radial electric field in the electron-root have a much higher threshold for the onset of the magnetic island. This provides an experimental test for the stabilizing effects of the ion-polarization current. The quantitative comparison, together with the examination of the proposed test, will enrich the understanding of the island formation in toroidal plasmas.

IV. SUMMARY AND DISCUSSION

In this paper, we studied the excitation of the magnetic island in helical systems by the external resonant deformation. The theory of the neoclassical tearing mode was extended to the case of the LHD-like plasmas. A simple model theory was developed, where only a single resonant Fourier component was taken. A self-sustaining mechanism was proposed for the island annihilation in helical systems. The island width is suppressed to the level of the ion banana width, if the plasma beta value exceeds a certain threshold value. The island width does not grow, in the finite-beta plasmas in LHD-like systems, until the size of the externally driven magnetic island exceeds the threshold value. Above this threshold, the island width grows when the externally driven island grows. This is one possible explanation for the island annihilation which was observed on the LHD. A test of this theoretical model was also proposed. The direction of the bootstrap current can be reversed in the helical systems. In such cases, the plasma pressure gradient makes the island width larger. The neoclassical tearing mode possibly occurs even in the absence of the external drive of the island. Under such circumstances, the threshold for the seed island for the onset of NTM depends on whether the radial electric field is in the electron root or in the ion root. This gives a new insight for the evolution of the magnetic islands in toroidal plasmas.

It should be noticed that the response of the magnetic island in the regime where the island width is close to the banana width may be subject to modification by an elaborated theory of neoclassical processes. We note that the role of ripple-trapped ions in the polarization current is not included. A quantitative analysis in a region of small magnetic islands is left for future work. Another issue is the role of background turbulence on the island growth: In some cases, the accelerated island formation was predicted theoretically.[36,37] In the case of subcritical excitation (i.e., $a_{bs} < 0$), the excitation by the turbulent noise is essential.[38,39,40,41] The island may have an impact on the radial electric field and influence the transport.[42,43] In addition, the mesoscale islands which could be excited by the external field possibly influence the island evolution.[44] Such effects can influence on the evolution of islands. There remains much work in connection with the physics of magnetic island in toroidal plasmas.

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