J. Plasma Physics (2009), *vol.* 75, *part* 1, *pp.* 111–115.

doi:10.1017/S0022377808007514 Printed in the United Kingdom

Reflection of an electromagnetic pulse from a relativistically moving plasma

MILOŠ M. ŠKORIĆ[1], BOŽIDAR B. STANIĆ[2], LJUPČO HADŽIEVSKI[2] and LJUBOMIR NIKOLIĆ[3]

[1]National Institute for Fusion Science, Toki-shi 509-5292, Japan
[2]Vinča Institute of Nuclear Sciences, POB 522, Belgrade 11001, Serbia
[3]ECERF, University of Alberta, Edmonton AB, Canada, T6G 2V4
(skoric.milos@nifs.ac.jp)

(Received 22 February 2008 and in final form 30 June 2008, first published online 22 August 2008)

Abstract. The reflection of an obliquely incident electromagnetic pulse from a moving plasma half-space is studied. Using the Lorentz transformations, covariance of Maxwell's equations and the principle of phase invariance to transform between the rest frame and the moving frame, calculations can be conveniently performed in the moving frame. An analytical formula for the linear reflected waveform as a function of the incident angle shows temporal compression and pulse amplification at relativistic velocities of relevance for the generation of ultra-short laser optical pulses.

1. Introduction

An interaction between time-dependent electromagnetic (EM) pulses with a dispersive medium and plasma, the so-called transient EM wave phenomenon, has been a classical problem in applied electrodynamics for over three decades. For example, the transient reflection and transmission of an obliquely incident EM pulse at the steady (non-moving) plasma–vacuum interface has been solved analytically in a closed form by Chabris and Bolle [1] and Stanić and Škorić [2, 3]. A growing interest in nonlinear relativistic plasmas [4] in the last decade has been followed by an upsurge of activity in laser optical and ultra-fast plasma phenomena in a relativistic regime. In particular, a generation of ultra-short (attosecond range) light and relativistic particle bunches is of importance in various applications (see [5, 6] and references therein). This gave a special motivation to revisit the problem of the transient EM pulse interaction with a relativistic plasma [7]. Here, we consider a general problem of a linear reflection of a time-dependent EM pulse from a plasma half-space moving at the relativistic velocity. A similar study that inconsistently applied the relativistic electrodynamics had obtained the incorrect result [8].

2. Formulation

A time-dependent EM plane wave pulse is incident at the moving cold plasma–vacuum interface. The incident angle is θ_{i} and the plane of incidence is Oxz, as shown in Fig. 1. The incident electric field (S-polarization) of the EM pulse in the

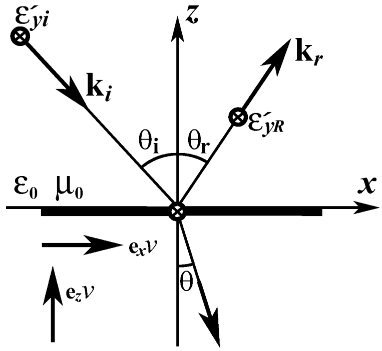

Figure 1. Geometry of the problem.

time domain, by inverse Fourier transformation, is

$$E_{y\mathrm{i}} = (1/2\pi)\int_{-\infty}^{+\infty} E_0 \exp[j(\omega_\mathrm{i} t - \mathbf{k}_\mathrm{i}\mathbf{r})]\, d\omega_\mathrm{i}$$

$$\equiv E_0\delta(t - (x/c)\sin\theta_\mathrm{i} + (z/c)\cos\theta_\mathrm{i}), \tag{2.1}$$

where $\delta(t)$ is the Dirac function and ω_i and $\mathbf{k}_\mathrm{i}$ are the angular frequency and the wavenumber vector in the observer's rest frame K, respectively. The uniform plasma half-space is moving with the velocity $\mathbf{v}$, where we have two special cases: (i) $\mathbf{v} = \mathbf{e}_x v$ and (ii) $\mathbf{v} = \mathbf{e}_z v$. The rest frame of the moving plasma is K'.

3. Analytical theory

Making use of the Lorentz transformations, the covariance of Maxwell's equations and the principle of phase invariance, to transform between the rest (laboratory) frame and the moving frame (see, e.g., [9, Ch. 7]); the incident electric field in the moving frame K', can be represented as

$$E'_{y\mathrm{i}} = (1/2\pi)\int_{-\infty}^{+\infty} \gamma(1 - \mathbf{k}_\mathrm{i}\mathbf{v}/\omega_\mathrm{i})E_0 \exp[j(\omega_\mathrm{i} t - \mathbf{k}_\mathrm{i}\mathbf{r})]\, d\omega_\mathrm{i}$$

$$= (1/2\pi)\int_{-\infty}^{+\infty} \gamma(1 + \mathbf{k}'_\mathrm{i}\mathbf{v}/\omega'_\mathrm{i})E'_0 \exp[j(\omega'_\mathrm{i} t' - \mathbf{k}'_\mathrm{i}\mathbf{r}')]\, d\omega'_\mathrm{i}, \tag{3.1}$$

with physical quantities with the 'prime' superscript corresponding to the moving frame K', and where

$$\omega'_\mathrm{i} = \gamma(1 - \mathbf{k}_\mathrm{i}\mathbf{v}/\omega_\mathrm{i})\omega_\mathrm{i}, \quad \gamma = (1 - v^2/c^2)^{-1/2} = (1 - \beta^2)^{-1/2}, \tag{3.2}$$

$$\mathbf{k}'_\mathrm{i} = \mathbf{k}_\mathrm{i} - \gamma\omega_\mathrm{i}\mathbf{v}/c^2 + (\gamma - 1)(\mathbf{k}_\mathrm{i}\mathbf{v})\mathbf{v}/v^2, \quad \text{and} \tag{3.3}$$

$$E'_0 = \gamma(1 - \mathbf{k}_\mathrm{i}\mathbf{v}/\omega_\mathrm{i})E_0, \quad E'_{01} = \gamma(1 + \mathbf{k}'_\mathrm{i}\mathbf{v}/\omega'_\mathrm{i})E'_0. \tag{3.4}$$

An analogous procedure is followed to reduce the oblique incidence case to a normal incidence in laser plasmas with clear savings in computational expense [5, 10].

With the $\exp(j\omega'_\mathrm{i}t')$ time-dependence suppressed, the incident electric field in the frequency domain in the moving frame is given by

$$\mathscr{E}'_{y\mathrm{i}} = E'_{01}\exp(-j\mathbf{k}'_\mathrm{i}\mathbf{r}'), \tag{3.5}$$

and the frequency domain expression for the reflected field (see [2, 3, 9]) is simply

$$\mathscr{E}'_{y\mathrm{R}} = \frac{1-N'}{1+N'} E'_{01} \exp(-j\mathbf{k}'_{\mathrm{r}}\mathbf{r}'), \tag{3.6}$$

where the well-known index of refraction for cold plasma at rest in K' is

$$N' = |1 - (\omega'_{\mathrm{p}}/\omega' \cos\theta'_{\mathrm{i}})^2|^{1/2}, \quad \omega'_{\mathrm{i}} = \omega'_{\mathrm{r}} = \omega'_{\mathrm{t}} \equiv \omega'.$$

The vacuum dispersion relation $\omega^{(\prime)}_{\mathrm{i,r}} = k^{(\prime)}_{\mathrm{i,r}} c$ is valid in the K and K' frames.

The time domain expression for the reflected field (in the K' frame) is

$$E'_{y\mathrm{R}} = (1/2\pi) \int_{-\infty}^{+\infty} \mathscr{E}'_{y\mathrm{R}} \exp(j\omega' t')\, d\omega'. \tag{3.7}$$

Using again the Lorentz transformations, the covariance of Maxwell's equations and the principle of phase invariance to transform back from the moving frame K' to the laboratory frame K, the time domain reflected field becomes

$$\begin{aligned} E_{y\mathrm{R}} &= (1/2\pi) \int_{-\infty}^{+\infty} \gamma(1 + \mathbf{k}'_{\mathrm{r}}\mathbf{v}/\omega'_{\mathrm{i}}) \mathscr{E}'_{y\mathrm{R}} \exp(j\omega' t')\, d\omega' \\ &= \gamma(1 + \mathbf{k}'_{\mathrm{r}}\mathbf{v}/\omega'_{\mathrm{i}}) E'_{y\mathrm{R}}. \end{aligned} \tag{3.8}$$

The expression for $E'_{y\mathrm{R}}$ found by the standard method of contour integration (see, e.g., [3, 11]) appears in the following form:

$$E'_{y\mathrm{R}} = -(2E'_0/\tau') J_2(a'\tau') U(\tau'), \tag{3.9}$$

where

$$\tau' = t' - \mathbf{k}'_{\mathrm{r}}\mathbf{r}'/\omega', \quad a' = \omega'_{\mathrm{p}}/\cos\theta'_{\mathrm{i}}, \tag{3.10}$$

and $U(\tau')$ is the Heaviside unit step function, while $J_2(x)$ is the Bessel's function of the first kind of second order. We note that (3.9) is the Green's function solution, while a linear solution to another incident pulse profile is found by a straightforward convolution integration.

Further, we shall discuss two special cases of the moving plasma half-space: (i) $\mathbf{v} = \mathbf{e}_x v$ (parallel to the interface) and (ii) $\mathbf{v} = \mathbf{e}_y v$ (normal to the interface). By making use of the Lorentz transformations, the following results are obtained.

(i) For $\mathbf{v} = \mathbf{e}_x v$ we have

$$E_{y\mathrm{R}} = -(2E_0/\tau) J_2(a\tau) U(\tau), \tag{3.11}$$

where

$$\tau = t - \mathbf{k}_{\mathrm{r}}\mathbf{r}/\omega_{\mathrm{r}} = t - (x/c)\sin\theta_{\mathrm{i}} - (z/c)\cos\theta_{\mathrm{i}}, \tag{3.12}$$

and

$$a = \omega_{\mathrm{p}}/\cos\theta_{\mathrm{i}},$$

where ω_{p} is the standard (rest frame) electron plasma frequency.

The reflected field is identical to the case of a non-moving plasma half-space [1–3]; as expected, the normally incident wave does not 'observe' the plasma motion across the surface, in the x-direction.

(ii) In the case $\mathbf{v} = \mathbf{e}_z v$, the reflected field is

$$E_{y\mathrm{R}} = -(2E_0\alpha_0/\xi) J_2(\alpha_1\xi) U(\xi), \tag{3.13}$$

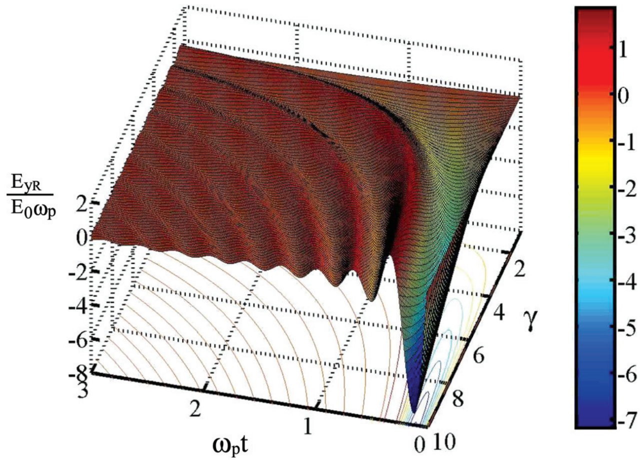

Figure 2. Reflected EM field in time as a function of the plasma velocity γ.

where

$$\alpha_0 = \gamma^2(1 + 2\beta\cos\theta_{\rm i} + \beta^2), \quad \alpha_1 = |\omega_{\rm p}/\gamma(\beta + \cos\theta_{\rm i})|, \tag{3.14}$$

and

$$\xi = \alpha_0 t - (x/c)\sin\theta_{\rm i} - (z/c)\gamma^2|(1+\beta^2)\cos\theta_{\rm i} + 2\beta|. \tag{3.15}$$

4. Results and discussion

It is now apparent that the plasma motion strongly modifies both the amplitude and the oscillatory phase of the reflected field (3.13), with, at the same time, a departure from the classical Snell's law ($\theta_{\rm i} \neq \theta_{\rm r}$), where the reflected wave angle equals the incident angle. More precisely, from (3.15) one readily calculates

$$\tan\theta_{\rm r} = \sin\theta_{\rm i}/\gamma^2|(1+\beta^2)\cos\theta_{\rm i} + 2\beta|,$$

which, for large $\beta > 0$, predicts $\theta_{\rm r} < \theta_{\rm i}$, i.e. the reflection angle closer to normal incidence. We note that earlier authors [8], by using the known steady-state ω-domain reflection coefficient for a moving plasma, to find the time-dependent solution, erroneously performed inverse Fourier transform over the incident $\omega_{\rm i}$, instead of integrating over the reflected frequency $\omega_{\rm r}$ (Doppler shifted) which would yield a correct result identical to (3.13).

The reflected waveforms for $E_{y\rm R}$, as functions of time and the plasma velocity $v(\mathbf{v} = \mathbf{e}_z v)$ for normal incidence ($\theta_{\rm i} = 0$), are plotted in Fig. 2. For $\beta = v/c < -0.5$, the reflected field is not given, since the amplitude becomes small; obviously, for $\beta \to -1, E_{y\rm R} \to 0$. The time delays in terms of the inverse plasma frequency of the maximum positive and negative reflected amplitude, as a function of plasma velocity β, deduced from Fig. 2 are shown in Fig. 3. Large compression and amplification of the reflected pulse (by a factor of $\sim 2\gamma$) at highly relativistic plasma motion reveals a remarkable feature and some potential of this linear mechanism for ultra-short (attosecond) pulse generation by low-intensity high-repetition-rate femtosecond laser pulse scattering by counter-propagating relativistic electron beams [5, 12]. For example, a short green laser light pulse ($\lambda \sim 0.5\ \mu$m) reflected

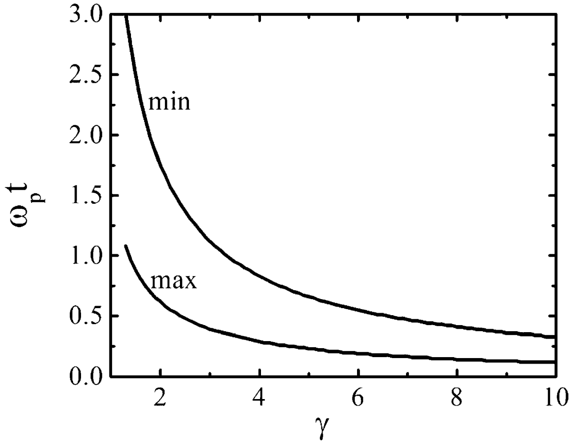

Figure 3. Time period of the first and second peaks in the reflected wave versus the plasma velocity.

from 5 MeV electrons ($\gamma \sim 10$) at critical density gives a main reflected pulse width of around 60 attoseconds. Another important point is that the reflected pulse width is basically determined by the relativistically upshifted electron plasma frequency which can be high in laser–solid density plasmas.

Acknowledgement

This work was supported in part by the Ministry of Science of Serbia, Project No. 141034.

References

[1] Chabris, D. B. and Bolle, D. M. 1971 *Radio Sci.* **6**, 1143.

[2] Stanić, B. V. and Škorić, M. M. 1973 *J. Phys. D: Appl. Phys.* **7**, L137.

[3] Stanić, B. V. and Škorić, M. M. 1973 *Electron. Lett.* **9**, 583.

[4] Shukla, P. D., Rao, N. N., Yu, M. Y. and Tsintsadze, N. L. 1986 *Phys. Rep.* **138**, 1.

[5] Mourou, G. A., Tajima, T. and Bulanov, S. V. 2006 *Rev. Mod. Phys.* **78**, 309.

[6] Naumova, N. M., Nees, J. M. and Mourou, G. M. 2005 *Phys. Plasmas* **12**, 056707.

[7] Stanić, B. V. and Škorić, M. M. 1974 *Proceedings of 7th SPIG Conf. (Rovinj, Yugoslavia)*, (ed. V. Vujnović) Zagreb, Institute of Physics, p. 221.

[8] Ratan, I. and Gautama, G. D. 1973 *J. Phys. D: Appl. Phys.* **6**, L85.

[9] Papas, C. H. 1965 *Theory of Electromagnetic Wave Propagation*. New York: McGraw-Hill.

[10] Bourdier, A. 1983 *Phys. Fluids* **7**, 1804.

[11] Gröbner, W. and Hofreiter, N. 1950 *Integraltafel. Teil II–Bestimtte Integrale*. Berlin: Springer, p. 144.

[12] Nikolić, Lj., Škorić, M. M., Ishiguro, S., Sakagami, H., Vidal, F. and Johnston, T. W. 2008 *J. Phys: Conf. Series* **112**, 022086.