Quantum effects on the entanglement fidelity in elastic scatterings in strongly coupled semiclassical plasmas

Young-Dae Jung[1,2,a)] and Daiji Kato[1]
[1]*National Institute for Fusion Science, Toki, Gifu, 509-5292, Japan*
[2]*Department of Applied Physics, Hanyang University, Ansan, Kyunggi-Do 426-791, South Korea*

(Received 23 July 2008; accepted 18 September 2008; published online 15 October 2008)

The quantum effects on the entanglement fidelity in elastic electron-ion scatterings are investigated in strongly coupled semiclassical plasmas. The screened pseudopotential model and partial wave analysis are employed to obtain the entanglement fidelity in strongly coupled semiclassical plasmas as a function of the thermal de Broglie wavelength, Debye length, and projectile energy. It is shown that the quantum effect significantly enhances the entanglement fidelity in strongly coupled semiclassical plasmas. It is also found that the entanglement fidelity increases with increasing the projectile energy. In addition, it is shown that the plasma screening effect increases the entanglement fidelity slightly. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2997334]

Electron-ion scattering in plasmas[1–6] has been of great interest since this process is widely used to diagnose various plasmas. The plasma identified by the Debye–Hückel screened potential has been known as the ideal plasma since the average energy of interaction between particles is small compared to the average kinetic energy of a particle. Recently, the significance of investigation of various physical properties of strongly coupled plasmas such as the interiors of astrophysical compact objects and inertial confinement fusion plasmas has increased remarkably.[3–6] It is quite obvious that the physical properties of matter existing under such strongly coupled plasmas differ drastically from those of weakly coupled classical plasmas. In these strongly coupled semiclassical plasmas, the interaction potential would not be characterized by the ordinary Debye–Hückel model because of nonideal particle interactions due to collective and quantum effects.[3,7] Recently, the entanglement fidelity in the scattering process has received noteworthy attention since it is known that the quantum correlation plays an important role in understanding the quantum measurements and information processing.[8] However, the deportment of the entanglement fidelity in elastic electron-ion scatterings in strongly coupled semiclassical plasmas has not been specifically investigated as yet. Thus, in this paper we investigate the quantum effects on the entanglement fidelity in low-energy elastic electron-ion scattering in strongly coupled semiclassical plasmas. The screened pseudopotential model,[3] taking into account the quantum and plasma screening effects, is employed to represent electron-ion interactions in strongly coupled semiclassical plasmas. In addition, the partial wave method[9] is applied to investigate the entanglement fidelity in strongly coupled semiclassical plasmas as a function of the thermal de Broglie wavelength, Debye length, and projectile energy.

The stationary-state Schrödinger equation[10] for the potential $V(\mathbf{r})$ is represented as

$$(\nabla^2 + k^2)\psi_k(\mathbf{r}) = \frac{2\mu}{\hbar^2} V(\mathbf{r})\psi_k(\mathbf{r}), \tag{1}$$

where $\psi_k(\mathbf{r})$ is the solution of the scattered wave function, $k(=\sqrt{2\mu E/\hbar^2})$ is the wave number, μ is the reduced mass of the collision system, and E is the kinetic energy of the projectile. Here the final state wave function $\psi_k(\mathbf{r})$ is represented by means of the partial wave expansion[9] in the following form:

$$\psi_k(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \sum_{l=0}^{\infty} i^l (2l+1) D_l^k R_l^k(r) P_l(\cos\theta), \tag{2}$$

where D_l^k is the expansion coefficient, $R_l^k(r)$ is the solution of the radial wave equation, $P_l(\cos\theta)$ is the Legendre polynomial, and l is the angular momentum quantum number. For a spherically symmetric potential $V(r)$, it has been known that the expansion coefficient D_l^k and the radial wave equation for $R_l^k(r)$ are, respectively, represented by[8]

$$D_l^k = \left[1 + i\frac{2\mu k}{\hbar^2}\int_0^\infty dr\, r^2 j_l(kr) V(r) R_l^k(r)\right]^{-1}, \tag{3}$$

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2}V(r) + k^2\right] R_l^k(r) = 0, \tag{4}$$

where $j_l(kr)$ is the spherical Bessel function. The solution of the radial wave equation $R_l^k(r)$ is represented by the spherical Bessel function $j_l(kr)$ and spherical Neumann function $n_l(kr)$:

$$R_l^k(r) = j_l(kr) + (2\mu k/\hbar^2)\left[n_l(kr)\int_0^r dr\, r^2 j_l(kr) V(r) R_l^k(r) + j_l(kr)\int_r^\infty dr\, r^2 n_l(kr) V(r) R_l^k(r)\right]. \tag{5}$$

Very recently, it has been shown that the entanglement

a)Permanent address: Department of Applied Physics, Hanyang University, Ansan, Kyunggi-Do 426-791, South Korea. Electronic mail: ydjung@hanyang.ac.kr.

fidelity for elastic scatterings can be represented by the absolute square of the scattered wave function for a given interaction potential; i.e., $f_k \propto |\int d^3\mathbf{r}\psi_k(\mathbf{r})|^2$.[8] Since the partial s-wave ($l=0$) gives the main contribution in low collision energies, the entanglement fidelity for low-energy elastic scatterings can be written as

$$f_k \propto \frac{|\int_0^\infty dr\, r^2 R_{0,k}(r)|^2}{1 + \left|\frac{2\mu k}{\hbar^2}\int_0^\infty dr\, r^2 V(r) R_{0,k}(r)\right|^2}. \quad (6)$$

In strongly coupled semiclassical plasmas,[3] the ranges of the number density and temperature are known to be around $10^{20}-10^{24}$ cm^{-3} and 10^3-10^7 K, respectively. Recently, the precise form of the screened effective pseudopotential[3] of the particle interactions in strongly coupled semiclassical plasmas, taking into account both the quantum mechanical and plasma screening effects, was obtained on the basis of the dielectric response function analysis. Using the screened effective pseudopotential model,[3] the electron-ion (Ze) interaction potential V^{SCSP} in strongly coupled semiclassical plasmas (SCSPs) is given by

$$V^{\mathrm{SCSP}}(r,\lambda,\Lambda) = -\frac{Ze^2}{\sqrt{1-4\lambda^2/\Lambda^2}}\left[\frac{e^{-A(\lambda,\Lambda)r}}{r} - \frac{e^{-B(\lambda,\Lambda)r}}{r}\right], \quad (7)$$

where $\lambda(=\hbar/\sqrt{2\pi\mu k_B T})$ is the thermal de Broglie wavelength, k_B is the Boltzmann constant, T is the plasma temperature, Λ is the Debye length, $A^2 \equiv (1-\sqrt{1-4\lambda^2/\Lambda^2})/(2\lambda^2)$, and $B^2 \equiv (1+\sqrt{1-4\lambda^2/\Lambda^2})/(2\lambda^2)$. The screened effective potential V^{SCSP} is required to be valid in the domain $2\lambda < \Lambda$;[3] i.e., the region of weakly degenerate plasmas. For strongly coupled plasmas, the range of the coupling parameter is $\Gamma[=(Ze)^2/(ak_BT)] > 1$, where a is the average distance between particles, the density parameter is $r_s(=a/a_0) > 1$, and the degeneracy parameter is $\theta(=k_BT/E_F) > 1$, $a_0[=\hbar^2/(me^2)]$ is the first Bohr radius of the hydrogen atom, m is the rest mass of the electron, and E_F is the Fermi energy of electrons.[3] The entanglement fidelity for low-energy elastic electron-ion scatterings in strongly coupled semiclassical plasmas is represented by

$$f_k^{\mathrm{SCSP}} \propto \frac{\left|\int_0^\infty dr\, r^2 \frac{\sin(kr)}{kr}\right|^2}{1 + \left|\frac{2\mu k}{\hbar^2}\int_0^\infty dr\, r^2 V^{\mathrm{SCSP}}(r,\lambda,\Lambda)\frac{\sin(kr)}{kr}\right|^2}. \quad (8)$$

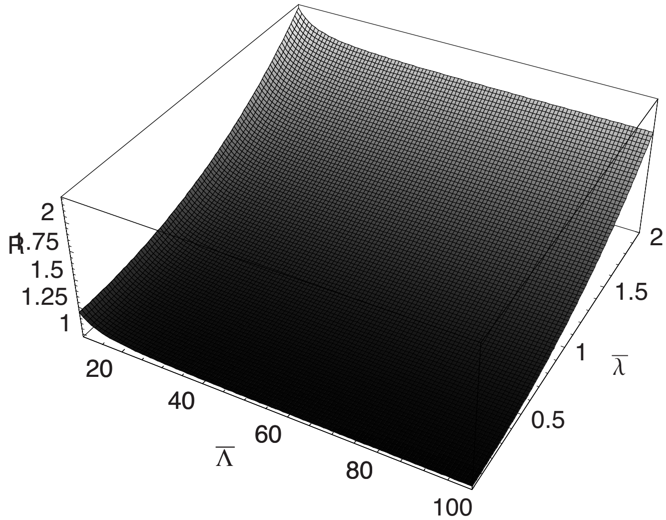

FIG. 1. The surface plot of the ratio $R(\bar{E},\bar{\lambda},\bar{\Lambda})$ of the entanglement fidelity for the elastic electron-ion scattering in strongly coupled semiclassical plasmas V^{SCSP} to that for the Coulomb potential V^{Coul} as a function of the scaled Debye length $(\bar{\Lambda})$ and scaled thermal de Broglie wavelength $(\bar{\lambda})$ when $\bar{E}=0.1$.

The quantum effects on the entanglement fidelity for elastic electron-ion scatterings in strongly coupled semiclassical plasmas can then be obtained by the ratio (R) of the entanglement fidelity for the screened pseudopotential V^{SCSP} [Eq. (7)] to that for the pure Coulomb potential ($V^{\mathrm{Coul}}=-Ze^2/r$),

$$R(k,\lambda,\Lambda) = \frac{f_k^{\mathrm{SCSP}}(k,\lambda,\Lambda)}{f_k^{\mathrm{Coul}}(k)} = \frac{1 + \left|-\frac{2Ze^2\mu k}{\hbar^2}\int_0^\infty dr\, r\frac{\sin(kr)}{kr}\right|^2}{1 + \left|-\frac{2Ze^2\mu k}{\hbar^2\sqrt{1-4\lambda^2/\Lambda^2}}\int_0^\infty dr\, r^2\left[\frac{e^{-A(\lambda,\Lambda)r}}{r} - \frac{e^{-B(\lambda,\Lambda)r}}{r}\right]\frac{\sin(kr)}{kr}\right|^2}. \quad (9)$$

After some mathematical manipulations with the scaled parameters $Aa_Z=[1-\sqrt{1-4\bar{\lambda}^2/\bar{\Lambda}^2}]^{1/2}/(\sqrt{2}\bar{\lambda})$ and $Ba_Z=[1+\sqrt{1-4\bar{\lambda}^2/\bar{\Lambda}^2}]^{1/2}/(\sqrt{2}\bar{\lambda})$, the fidelity ratio $R(\bar{E},\bar{\lambda},\bar{\Lambda})$ for the elastic electron-ion scattering including the quantum and plasma screening effects in strongly coupled semiclassical plasmas is obtained as the following analytic form:

$$R(\bar{E},\bar{\lambda},\bar{\Lambda}) = \frac{\bar{E}+4}{\bar{E} + \left[\frac{8\bar{E}\bar{\lambda}^2}{(2\bar{E}\bar{\lambda}^2+1)^2 + 4\bar{\lambda}^2/\bar{\Lambda}^2 - 1}\right]^2}, \quad (10)$$

where $a_Z(=a_0/Z)$ is the first Bohr radius of hydrogenic ion with nuclear charge Z, $\bar{\lambda}(\equiv\lambda/a_Z)$ is the scaled thermal de

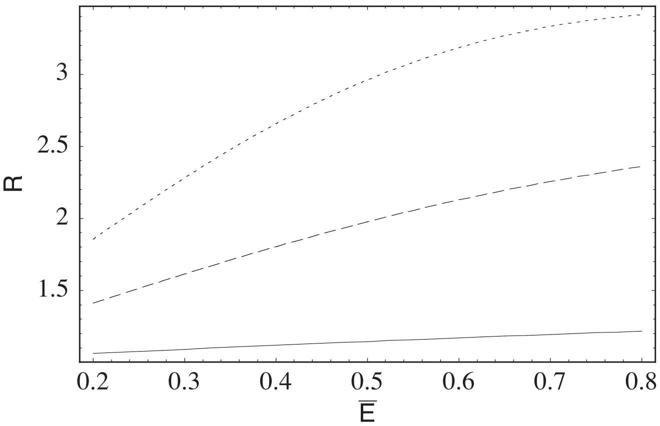

FIG. 2. The fidelity ratio $R(\bar{E},\bar{\lambda},\bar{\Lambda})$ as a function of the scaled projectile energy $(\bar{E})$ when $Z=6$. The solid line represents the case of $\Gamma=0.1$, $r_s=54.8$, and $\theta=5895.3$. The dashed line represents the case of $\Gamma=1.2$, $r_s=189.7$, and $\theta=1700.6$. The dotted line represents the case of $\Gamma=2.9$, $r_s=294.9$, and $\theta=1093.9$.

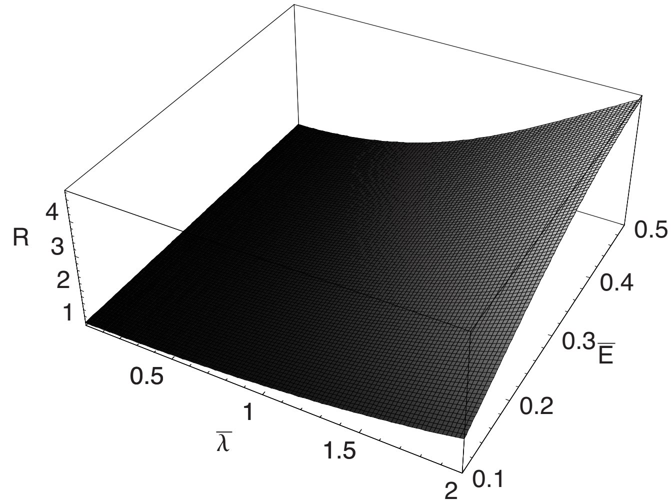

FIG. 3. The surface plot of the fidelity ratio $R(\bar{E},\bar{\lambda},\bar{\Lambda})$ as a function of the scaled thermal de Broglie wavelength $(\bar{\lambda})$ and scaled projectile energy $(\bar{E})$ when $\bar{\Lambda}=100$

Broglie wavelength, $\bar{\Lambda}(\equiv\Lambda/a_Z)$ is the scaled Debye length, $\bar{E}[\equiv E/(Z^2Ry)]$ is the scaled projectile energy, and $\mathrm{Ry}[=me^4/(2\hbar^2)\cong 13.6\ \mathrm{eV}]$ is the Rydberg constant.

Figure 1 shows the surface plot of the ratio of the entanglement fidelity for the elastic scattering in strongly coupled semiclassical plasmas to that for the Coulomb potential as a function of the scaled Debye length $(\bar{\Lambda})$ and scaled thermal de Broglie wavelength $(\bar{\lambda})$. From this figure, it is shown that the quantum effect significantly enhances the entanglement fidelity in strongly coupled semiclassical plasmas. It is also found that the plasma screening effect slightly enhances the entanglement fidelity. In addition, the quantum effects are found to be more significant and important than the plasma screening effects on the entanglement fidelity for the elastic electron-ion scattering in strongly coupled semiclassical plasmas. Figure 2 represents the fidelity ratio as a function of the scaled projectile energy $(\bar{E})$ for various values of the thermal de Broglie wavelength $(\bar{\lambda})$. Figure 3 show the surface plot of the fidelity ratio as a function of the scaled thermal de Broglie wavelength $(\bar{\lambda})$ and scaled projectile energy $(\bar{E})$. As we have seen in those figures, the fidelity ratio is found to be increased with an increase of the projectile energy. It is also interesting to note that the quantum mechanical effects on the entanglement fidelity are found to be increased with increasing the projectile energy. These results would provide useful information on the quantum and plasma screening effects on the entanglement fidelity for the elastic electron-ion scatterings in strongly coupled semiclassical plasmas.

One of the authors (Y.-D.J.) gratefully acknowledges the Director-General Professor O. Motojima, Director Professor M. Sato, Director Professor Y. Hirooka, and Professor I. Murakami for warm hospitality while visiting the National Institute for Fusion Science (NIFS) in Japan as a long-term visiting professor. The authors are also grateful to NIFS for supporting the research. This work was done while Y.-D.J. visited NIFS.

[1] T.-Y. Wu, *Kinetic Equations of Gases and Plasmas* (Addison-Wesley, Reading, 1997), p. 159.
[2] V. P. Shevelko, *Atoms and Their Spectroscopic Properties* (Springer, Berlin, 1997), p. 153.
[3] T. S. Ramazanov and K. N. Dzhumagulova, Phys. Plasmas **9**, 3758 (2002).
[4] T. Ramazanov, K. Galiyev, K. N. Dzhumagulova, G. Röpke, and R. Redmer, Contrib. Plasma Phys. **43**, 39 (2003).
[5] T. S. Ramazanov and K. N. Turekhanova, Phys. Plasmas **12**, 102502 (2005).
[6] T. S. Ramazanov, K. N. Dzhumagulova, and Y. A. Omarbakiyeva, Phys. Plasmas **12**, 092702 (2005).
[7] T. S. Ramazanov and S. K. Kodanova, Phys. Plasmas **8**, 5049 (2001).
[8] K. Mishima, M. Hayashi, and S. H. Lin, Phys. Lett. A **333**, 371 (2004).
[9] A. G. Sitenko, *Lectures in Scattering Theory* (Pergamon, Oxford, 1971), p. 51.
[10] C. J. Joachain, *Quantum Collision Theory* (North-Holland, Amsterdam, 1975), p. 112.