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# Magnetic Field and Force of Helical Coils for Force Free Helical Reactor (FFHR)

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## Abstract

The electromagnetic force on a helical coil becomes smaller by decreasing the coil pitch parameter which is the angle of the coil to the toroidal direction. This makes it possible to enlarge the central toroidal field or to simplify the supporting structures of the coil. The plasma minor radius, however, becomes smaller with the pitch parameter, and a higher field is necessary to attain the same plasma performance. Another important item in a helical reactor is the distance between the helical coil and the plasma to gain enough space for blankets. In order to reduce the mass of the coil supports, a lower aspect ratio is advantageous, and an optimum value of the pitch parameter will exist around 1.2 and 1.0 for the helical systems of the pole numbers of 2 and 3, respectively.

## Keywords:

electromagnetic force, magnetic field, helical coil, force-free, superconducting magnet, fusion reactor

## 1. Introduction

The main feature of the Force Free Helical Reactor (FFHR) [1] is relatively small electromagnetic force on helical coils, which makes it possible to enlarge the central toroidal field or to simplify the supporting structures. Another feature is relatively wide space for blankets between the helical coil and the plasma by reducing the plasma minor radius [2]. In order to attain the 'force free' condition, the external transverse magnetic field in the helical coil should be small. That condition is attained by decreasing the angle of the coil to the toroidal direction, which is called a pitch parameter. The plasma minor radius, however, changes with the pitch parameter, and it diminishes at an excessively smaller pitch parameter.

The transverse magnetic field is important for properties of a superconductor, especially for the critical current and the stability margin [3]. The external transverse field becomes small in the 'force free' condition, and the self field becomes dominant. The self field is proportional to the square root of the coil current and the current density. Since the current is determined from the basic parameters of the helical system, the current density is important for design of the coil and the structure to gain sufficient space for the blankets.

This paper intends to clarify the features of the electromagnetic force on the helical coil and the maximum magnetic field in the coil for various pitch parameters, and an optimum pitch parameter is discussed from the viewpoint of the engineering of superconducting magnets.

## 2. Method of Estimation of 'Force Free'

The coordinate of the helical coil is shown in Fig. 1, where $R_0$ and $a_c$ are the major radius and the minor radius of the helical coil, and $W$ and $H$ are width and height of the coil in the cross-section. The current $I$ of

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the helical coil is given by

$$I = \frac{2\pi R_0 B_0}{\mu_0 m} \tag{1}$$

where $B_0$, $m$ and $\mu_0$ are the central toroidal field, the pitch number, and the permeability in vacuum. The current is proportional to the toroidal field and the major radius of the reactor. On the other hand, the current density $j$ of the helical coil is given by

$$j = \frac{I}{W \cdot H}. \tag{2}$$

In the case of similar shape with the constant magnetic field, the current density is in inverse proportion to the size. Consequently, the coil size becomes relatively smaller in the larger reactor with the constant magnetic field and the constant current density. Since the current density is very important for the design of superconducting magnets, it was fixed to 25 or 50 MA/m$^2$ in this study. The major parameters of $B_0$ and $R_0$ were set 10 T and 10 m, which are alterable with the constant ratio of $B_0/R_0$.

The magnetic field in the coil is separated into the self field and the external field. The self field is defined as the field in the coil induced by the coil current itself, and it does not produce motional force. The external field is induced by the loop current of all the coils. The 'force free' condition is attained in the case that the helical coil is aligned with the external field, that is, the average perpendicular component of the external field is zero. In this condition, the self field becomes dominant in the transverse field. The maximum self field in a straight round conductor is expressed as

$$\left(B_{\text{self}}\right)_{\max}^{\text{round}} = \mu_0 \frac{I}{2\pi r} \mu_0 \sqrt{\frac{I \times j}{4\pi}}, \tag{3}$$

where $r$ is a radius of the conductor in the cross-section.

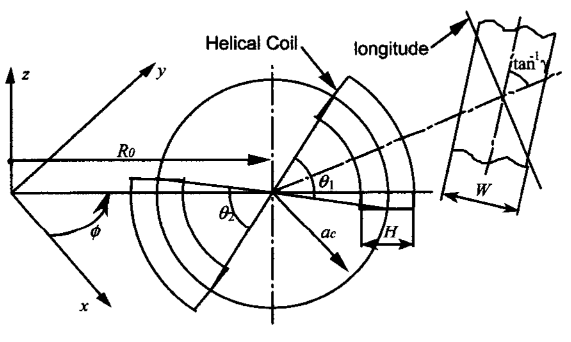


Fig. 1 A coordinate of helical coil.

The maximum field is determined by the current and the current density. In the case of a straight conductor with rectangular shape, the maximum field appears at the middle of the shorter side, which is written as

$$\left(B_{\text{self}}\right)_{\max}^{rec} = \mu_0 \frac{\sqrt{I \times j}}{4\pi\sqrt{x}} \left\{ \ln\left(1 + 4x^2\right) + 4x \tan^{-1}\frac{1}{2x} \right\} < \left(B_{\text{self}}\right)_{\max}^{\text{round}} \tag{4}$$

where $x \equiv \frac{H}{W}$ $(0 < x \leq 1)$.

In the design of helical reactors, the supporting structures for the helical coils are very important. The electromagnetic force on the helical coil is separated into the minor-radius and overturning components, and the former is dominant in a standard operation. The minor-radius hoop force per unit length $f_a$ is given by

$$f_a = B_\perp \cdot I \propto \frac{B_\perp}{B_0} \frac{R_0 B_0^2}{m}, \tag{5}$$

where $B_\perp$ is an average transverse field in the coil. The hoop force is mainly supported by the tension of the structure. The total minor-radius hoop force $F$ is given by

$$F = m \cdot \frac{\int f_a \cdot dL}{2\pi} = \langle f_a \rangle \cdot m \cdot R_0 \sqrt{1 + \gamma^2} \propto \alpha \cdot R_0^2 B_0^2 \tag{6}$$

where, $\gamma = \frac{m}{\ell} \frac{a_c}{R_0}$, $\alpha \equiv \frac{\langle B_\perp \rangle}{B_0} \cdot \sqrt{1 + \gamma^2}$,

and $\langle\ \rangle$ means the average. The normalized force per unit length $\langle f_a \rangle/(B_0\, I)$ is equal to the ratio of the external transverse field to the $B_0$, and it represents a degree of 'force free'. The non-dimensional factor $\alpha$ will be the index of the necessary weight of the coil supports.

## 3. Magnetic Field and Force of Helical Coil

### 3.1 Method of Calculation

Because of the complicated shape of helical coils, a numerical method is necessary to calculate electromagnetic field and force. We modified the magnetic field calculation code for helical coils with a rectangular cross section [4]. In the code, a helical coil is approximated as a set of many short body-current-elements. The uniform vertical field is applied to make the vertical field zero at the major radius. Accuracy of the calculations depends mainly on the size of the divided elements. In these calculations, the number of body-current-elements was set more than 7,200 per coil to suppress the modeling error within 1 %. The magnetic field was calculated at 20 × 10 points in the cross-sections by the poloidal pitch of 22.5 degree. The maximum transverse field $(B_\perp)_{max}$ was estimated to be the maximum in the calculated values $(B_\perp)_{cal}$ plus the self field by one of the 20 × 10 segments. It is expressed as

$$\left(B_\perp\right)_{max} = Max\left\{\left(B_\perp\right)_{cal}\right\} + \mu_0\sqrt{\frac{I \times j}{4\pi \cdot n}}, \quad (7)$$

where $n$ is the number of the segments. The electromagnetic force was calculated by being summed up in each cross section and being multiplied by the current.

### 3.2 Normalized Force and Maximum Field

A high ratio of width to height of the helical coil is useful to reduce the maximum transverse field and to enlarge the blanket space, but it will bring problems for maintenance ports. The optimum value should be determined by the total design. The ratio of 2 was selected in this study as a moderate value. The normalized electromagnetic force per unit length and the index of the total hoop force are shown in Figs. 2 and 3, respectively. The normalized force becomes lower with the smaller pitch parameter, and the 'force free' condition is attained at the pitch parameter of 0.6 to 0.85. The curves are shifted in the positive y-direction with higher current density or lower aspect ratios. The 'force free' condition seems to depend on the aspect ratio, but it depends mainly on the ratio of area occupied by the coils in the torus surface. The tendency of the total hoop force versus the pitch parameter is similar to the normalized force per unit length.

The maximum transverse field in the helical coil is shown in Fig. 4. It is almost proportional to the square root of the current density. Besides, it becomes gradually higher at the smaller pitch parameter in spite of the lower external transverse field. The main reason is the increase of the current with the decrease of the pitch number. The field in the coil of the pole number of

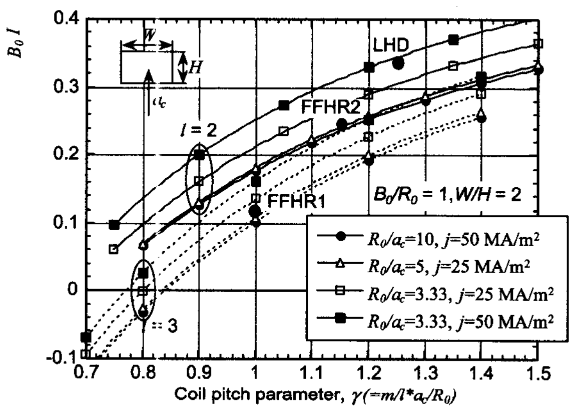


Fig. 2 The force per unit length of the helical coil normalized with $B_0$ $I$.

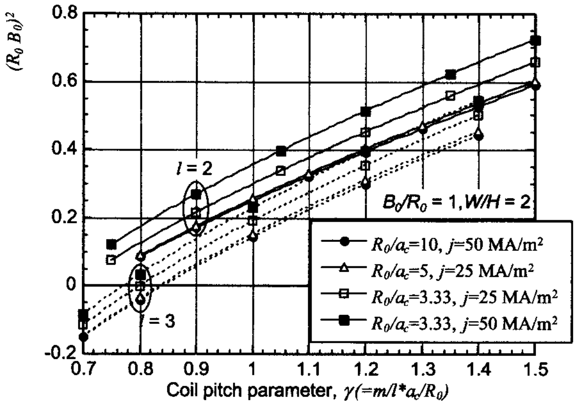


Fig. 3 The total hoop force of the helical coil normalized with $(R_0\ B_0)^2$.

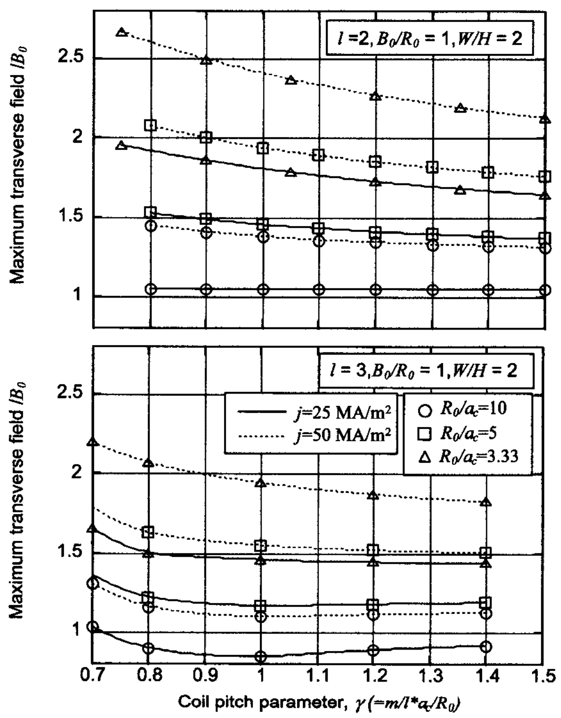


Fig. 4 The transverse magnetic field in the helical coil for the pole number of 2 and 3.

3 is always lower than 2 because of the lower current per coil.

## 4. Discussion

The total hoop force becomes larger with the coil minor radius but the increase is small as shown in Fig. 5. According to the scaling law [5], the energy confinement time is proportional to more than the second power of the plasma minor radius. Consequently, the sum of coil supports should become less at lower plasma aspect ratios. On the other hand the plasma minor radius also depends on the pitch parameter. A sample of the dependence in the case of a higher aspect ratio of 50 is shown in Fig. 6. In the case of the pole number of 2, the radius becomes smaller sharply at the smaller pitch parameter to diminish at lower than 0.9. In the case of the pole number of 3, the largest value appears around 1.0. The necessary thickness of the blanket is estimated to be almost 1 m, and the average plasma minor radius will be larger than 1.5 m even for a high field reactor [2]. The adequate ratio of the plasma minor radius to the coil radius will be higher than 0.5 in order to generate a sufficient plasma radius for an economical size. Therefore, the optimum values of the pitch parameter will be around 1.2 and 1.0 for the pole numbers of 2 and 3, respectively.

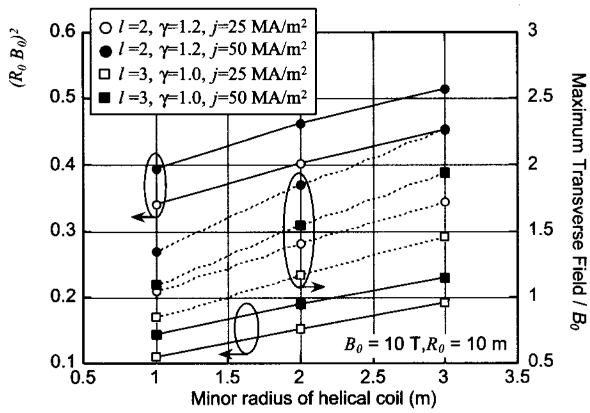


Fig. 5 The total hoop force and the maximum transverse magnetic field in the helical coil versus the major radius of the coil.

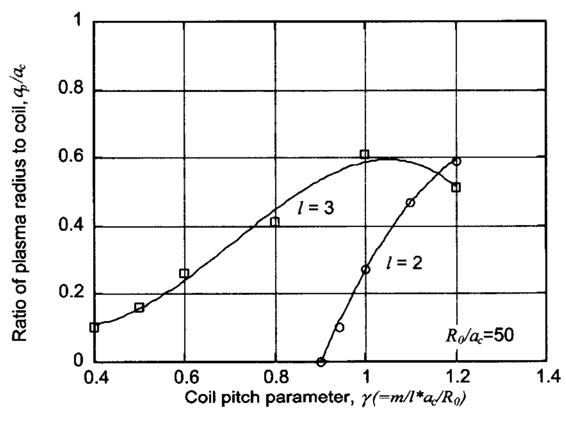


Fig. 6 The plasma radius for the pole number of 2 and 3 in the case of the aspect ratio of 50.

From the viewpoint of design of superconducting magnets, the lower maximum field is desirable. The pole number of 3 is superior to 2 in this aspect, as long as the plasma confinement is comparable.

## 5. Summary

The motional force on the helical coils is intrinsically reduced by the inclined shape, and it becomes smaller at the smaller pitch parameter or at the higher ratio of area occupied by the coils. On the other hand the plasma minor radius changes with the pitch parameter. In view of compatibility of the plasma minor radius and the blanket space, the optimum pitch parameters will be around 1.2 and 1.0 for the pole numbers of 2 and 3, respectively. Considering the strong effect of the plasma radius on the confinement time and the small change of the total hoop force with the aspect ratio, the lower aspect ratio will reduce the necessary mass of the coil supports. Although the maximum transverse field becomes higher at the lower aspect ratio, it can be reduced by lowering the current density. From the viewpoint of magnets engineering, the pole number of 3 is advantageous because of smaller total force and the lower maximum field, as long as the plasma confinement is comparable.

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