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Ignition Condition for p-^{11}B Reactor with LHD type Magnetic Field Configuration

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Abstract

Proton-Boron fusion reactor (p-^{11}B reactor) might be able to be achieved by the combination of the LHD type magnetic field configuration and the ion cyclotron range of frequency (ICRF) heating scheme of protons. The LHD magnetic field has the excellent ability for the high energy particle confinements. This characteristic is studied by numerical computations of high energetic particles in the LHD magnetic field where the strong ICRF field is applied. It is shown by the Langevin equation analysis that the steady state distribution function of ICRF heated proton becomes to the quasilinear plateau distribution function (QPDF). The fusion reaction rate of p-^{11}B is calculated for QPDF protons and studied the ignition condition. It is found that the ignition condition becomes possible to be satisfied if effective temperature of proton is of the order of 300 keV and $n\tau \gtrsim 10^{22}$ s/m^3.

Keywords:

LHD, ICRF heating, chaotic orbit, chaotic field line, p-^{11}B reactor

1. Introduction

Energy resources, whose consumption keeps increasing year by year, should be transfer to more clean ones. Proton-Boron fusion reactor (p-^{11}B reactor)[1,2] probably offers the ultimate energy resource. The fuels (Proton and Boron) are ubiquitous on the earth and fast neutrons are not generated. Main reaction

$$\mathrm{p} + {}^{11}\mathrm{B} \rightarrow 3\alpha + \mathrm{Q}_\alpha,\ \mathrm{Q}_\alpha \simeq 8.7\ \mathrm{MeV} \tag{1}$$

is neutron-free. The side reaction

$$\mathrm{p} + {}^{11}\mathrm{B} \rightarrow \mathrm{n} + {}^{11}\mathrm{C} - 2.8\ \mathrm{MeV} \tag{2}$$

is an endothermal reaction and produces only slow neutrons when energy of proton exceeds about 3 MeV or more.

A detailed study of the p-^{11}B reactor has been done [3]. It has been, however, pointed out that the p-^{11}B reactor could not satisfy the ignition condition since the bremsstrahlung power loss due to the large atomic number of boron ($Z_B = 5$) is too large.

But, the progress of LHD experimental and theoretical studies has brought a new prospect for the p-^{11}B reactor. Experiments and computational studies are establishing the high performance for production and confinement of high energetic particles in LHD configurations. The ion cyclotron range of frequency (ICRF) heating of LHD plasma is very promising [4-7]. High β plasma confinement is progressive in LHD experiments. Theoretical studies show the potentiality of high beta plasma confinement by the combination of the line-tying effect of diverter field lines and high magnetic shear structure in peripheral regions [8].

In Sec. **2** we briefly summarize the ICRF heating process in LHD. The steady state distribution function of the ICRF-heated proton in LHD is evolved in Sec. **3**. Evaluation of ignition condition is summarized in Sec. **4**. Summary and discussions are in Sec. **5**.

2. ICRF heating process in LHD

The quasilinear theory on the evolution of the plasma distribution function under ICRF heating has been developed by Stix [9]. The average change in energy per ICRF resonance zone transit, $\Delta W_\perp$ is given by,

$$\Delta W_\perp = \frac{M_p}{2}\,\frac{e^2}{M_p^2}\,E_{RF}^2\,\frac{2\pi}{\frac{\partial B}{\partial \ell}v_\parallel}\,\frac{M_p}{e}\,, \tag{3}$$

where M_p is the mass of a proton, e the elementary electric charge, E_{RF} the RF electric intensity, B the magnetic field intensity, ℓ the length along the magnetic field line and $v_\parallel$ the particle speed parallel to the magnetic field line, respectively.

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If we substitute typical values of the ICRF experiments of LHD as

$$E_{RF} = 20\ \text{kV/m},\ B = 2.75\ \text{T},\ \frac{1}{B}\ \frac{\partial B}{\partial \ell} = 0.3\ \text{m}, \qquad (4)$$

we get for 2 keV protons

$$\Delta W_{\perp} \simeq 3.48\ \text{keV}, \qquad (5)$$

where $v_{\|}$ is the thermal velocity of 2 keV proton and assumed to be constant. In this case, the standard statistical theory predicts that protons are necessary the total $(10^3/\Delta W_{\perp})^2 \simeq 8.2 \times 10^4$ events (almost equal to 4.6 s for 2 keV protons in LHD) for their average energy to reach to 1 MeV (= 10^3 keV) by the ICRF heating. In the computer simulations of the ICRF heating in LHD (total particle number = 726), however, we are observing average energies reaching to 0.7 MeV by the heating during 12.4 ms. In addition, the ICRF experiments of LHD have been observed a smooth high energy proton tail reaching to 500 keV [6].

These results imply that a rapid heating process — "runaway ion heating process" — works in the ICRF heating of LHD. That is, the accelerated ions are accelerated preferentially further more in the ICRF heating of LHD. The rapid heating process is caused with the trapped and near trapped particles whose magnetic mirror turning points coincide with the ion cyclotron resonance pints. As they dwell for a considerably longer time near resonance, particles within this class gain much more energy than the quasilinear average change in eq. (3)[10]. For the realization of rapid heating process, non-particle-loss is necessary in the course of the transition from passing particle to helical trapped mirror particle. In addition, a complete confinement of a helical trapped particle is essential factor for rapid heating process. In the following, we show that the LHD magnetic field configuration satisfies these two requirements, the runaway ion heating process is possible and the steady state distribution function do not become the Maxwellian.

In LHD, the magnetic field is produced by the continuous helical coils (ℓ/m = 2/10). There are 3 types of particles, the passing, reflecting, and chaotic orbit particles according as the initial pitch angles of particles. Passing particles and reflecting particles can form the closed drift surface and be confined completely in collisionless case. Reflecting particles, which are trapped in the helical mirror region and are possible to circulate around the magnetic axis, have shorter periodic lengths compared to passing particles. Then adiabaticity of reflecting particles becomes stronger than that of passing particles. This fact realizes the good confinement of reflecting particles and the high performance of ICRF heating in LHD. Reflecting particles can be confined even in the outside the last close flux surface (LCFS).

Chaotic orbits particles, whose pitch angles are in the range of transition regions between passing particles and reflecting particles, have the finite but controllable lifetime. Lifetimes of chaotic orbit particles become long (short) by increasing (decreasing) magnetic field strength or by decreasing (increasing) the particle energies. LHD has no loss-cone [11]. These nature are shown by Poincarè plot in ($v_{\|}$, $v_{\perp}$) plain of the ICRF heating process (Fig. 1). Dense plots expanding in $v_{\perp}$ direction are accumulated in small $v_{\perp}$ region. This means that the energy changing in each transitions of the ICRF resonance region is relatively small and its direction is almost perpendicular to the magnetic field. On the other hands, ICRF-heated particles with large $v_{\perp}$ show sparse plots expanding in reflection region. This means that the energy changing in each transitions of the ICRF resonance region is large. The ICRF-heated particles can pass through the chaotic orbit region without the degradation effect. High energy ions can be heated further more by preferentially absorbing the RF energy. Thus, we consider that the runaway ion heating process occurs in ICRF heating of LHD plasma.

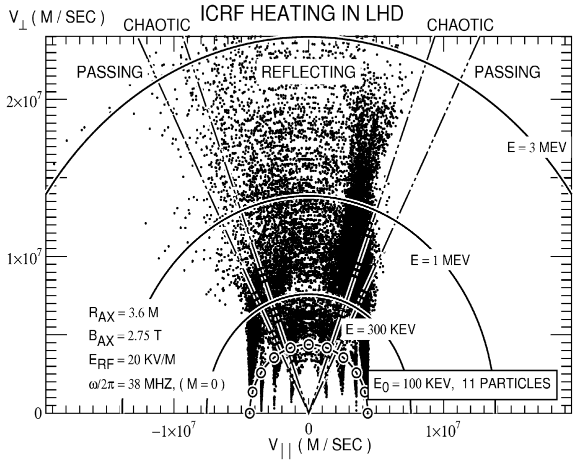

Fig. 1 ICRF heating process in LHD. Poincarè plots are shown in ($v_{\|}$, $v_{\perp}$) at the poloidal section of $\phi = \pi/2$, that is the starting poloidal section of particles (initial conditions are shown by open circles : total 11 particles). The chaotic orbit region has no degradation effect for heating process in LHD. Particles are lost to the vacuum vessel wall along the diverter field lines with $v_{\|} < 0$ when $E \gtrsim 1$ MeV.

3. Steady state distribution function of ICRF-heated proton in LHD

The simplest model, which determines the steady state distribution function of runaway ion heating process, can be assumed to be built up by the Langevin equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = (-\mu' + \alpha' v^2 - \beta' v^4)v + R(t), \qquad (6)$$

where v is the ion velocity, μ' the coefficient for friction force mainly due to electron drag, α' the coefficient for the runaway ion heating process and β' the coefficient for a termination process of ion heating, respectively. $R(t)$ is a random force due to resonant cyclotron heating process. The Fokker-Planck equation corresponding to this Langevin equation is given by [12]

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v}\left\{\left(-g + D\frac{\partial}{\partial v}\right)f\right\}, \tag{7}$$

where f is the distribution function of ion,

$$g(v) = (-\mu' + \alpha' v^2 - \beta' v^4)v \tag{8}$$

and D is defined by the auto-correlation of random force $R(t)$,

$$\langle R(t)R(t')\rangle = D\delta(t - t'). \tag{9}$$

The steady state solution of the Fokker-Planck equation (7) is given by [12]

$$f(v) = C'\exp\left(\frac{1}{D}\int \mathrm{d}v g(v)\right), \tag{10}$$

where C' is a constant for normalization. This steady state distribution function can be rewritten as a function of energy E:

$$f(E) = n_p\, C\left(\frac{M_p}{2\pi T}\right)^{3/2} \times \exp\left[-\left\{\frac{E}{T} - \frac{1}{\alpha}\left(\frac{E}{T}\right)^2 + \frac{1}{\beta}\left(\frac{E}{T}\right)^3\right\}\right],$$

$$E = \frac{1}{2}M_p \boldsymbol{v}^2, \tag{11}$$

where

$$\frac{\mu'}{D} = \frac{M_p}{T}, \frac{\alpha'}{D} = \left(\frac{M_p}{T}\right)^2 \frac{1}{\alpha},$$

$$\frac{\beta'}{D} = \frac{3}{4}\left(\frac{M_p}{T}\right)^3 \frac{1}{\beta}, \tag{12}$$

n_p and T are the density and temperature of the proton. If the population inverted distribution function is formed, the particle energy transfers to the RF energy. Then, the steady state distribution function will be settled down to the formation of the plateau, even under the runaway ion heating process. The condition of the plateau formation

$$\left.\frac{\mathrm{d}}{\mathrm{d}E}f(E)\right|_{E=E_{plateau}} = \left.\frac{\mathrm{d}^2}{\mathrm{d}E^2}f(E)\right|_{E=E_{plateau}} = 0, \tag{13}$$

reduces to

$$\beta = 3\alpha^2, \frac{E_{plateau}}{T} = \alpha. \tag{14}$$

Then the parameter α determines the energy at the plateau. We expect that the ICRF heating of LHD can sustain the high energy proton to the optimal energy level ($E_{plateau} = T$). Thus, it is assumed that $\alpha = 1$ and $\beta = 3$. We call this distribution function as the quasilinear plateau distribution function (QPDF). Explicit form of QPDF for proton is given by

$$f_p(E) = n_p \times C\left(\frac{M_p}{2\pi T}\right)^{3/2}$$

$$\times \exp\left[-\left\{\frac{E}{T} - \left(\frac{E}{T}\right)^2 + \frac{1}{3}\left(\frac{E}{T}\right)^3\right\}\right], \tag{15}$$

where $C = 1/1.9242060\cdots$. Effective temperature T_{eff} for the QPDF is defined by

$$n_p\frac{3}{2}T_{p\_eff} = \int \mathrm{d}^3\boldsymbol{v}\frac{M_p(v_x^2 + v_y^2 + v_z^2)}{2}f(\boldsymbol{v}), \tag{16}$$

and this relation is reduced to

$$T_{p\_eff} = T \times 0.959735333668\cdots. \tag{17}$$

4. Ignition condition for p-^{11}B reactor

We have calculated the p-^{11}B fusion reaction rate $\langle\sigma v\rangle$ of protons with QPDF (eq. (11)) assuming $\beta = 3$.

$$\langle\sigma v\rangle = \int \mathrm{d}^3\boldsymbol{v}_b \int \mathrm{d}^3\boldsymbol{v}_p \sigma\left(\frac{1}{2}M_p(\boldsymbol{v}_p - \boldsymbol{v}_b)^2\right)$$

$$|\boldsymbol{v}_p - \boldsymbol{v}_b| f_p(\boldsymbol{v}_p) f_b(\boldsymbol{v}_b), \tag{18}$$

where σ is the p-^{11}B fusion cross section and $f_b(\boldsymbol{v}_b)$ is the distribution function for boron assuming Maxwellian distribution with the temperature T_b.

The most fundamental condition necessary for a fusion reactor to keep burning is given by the following,

$$\eta \equiv \frac{P_\alpha}{P_B} > 1, \tag{19}$$

where $P_\alpha = n_p n_b \langle\sigma v\rangle Q_\alpha$ is the heating power density by α particles, and P_B is the power density of bremsstrahlung loss due to electrons. If eq. (19) is not satisfied, it is impossible to keep burning in the p-^{11}B reactor of which confinement performance is dramatically improved. In eq. (19), the electron temperature is assumed to be determined by the power balance between the losses due to bremsstrahlung and heating due to electron drug of protons. Since the high energetic tail is decreased very much in QPDF protons, it is assumed that the distribution function of the electron is also QPDF of effective temperature T_{e_eff}. In this case, T_{e_eff} is proportional to $T_{p\_eff}^{1/2}$ and electron drug time τ_E becomes almost twice compared with the case of Maxwellian distribution function.

The density ratio $\delta \equiv n_b/n_p$ is one of an important factor for the sustainment of p-^{11}B burning. If δ is too large, bremsstrahlung loss exceeds the fusion power density. If we assume that $n_e = (1 + 5\delta)n_p$ and $T_b = T_p/20$, δ optimized to maximize the value of η approximately becomes $0.1418588\cdots$. Then, the ignition condition given by eq. (19) for the p-^{11}B reactor is reduced to

$$\eta = \frac{P_\alpha}{P_B} = \frac{1.58 \times 10^{22}\langle\sigma v\rangle}{(T_{p\_eff}/e)^{1/4}} > 1, \tag{20}$$

since P_B is proportional to $T_{e\_eff}^{1/2}$. Figure 2(a) shows η calculated by eq. (20). It can be seen from Fig. 2(a) that η for the p-^{11}B reactor can exceed unity around $T_{p\_eff} \simeq 300$ keV.

The power loss due to the heat conduction and convection flow (P_L) is also important for the p-^{11}B reactor. P_L is characterized by the energy confinement time τ as

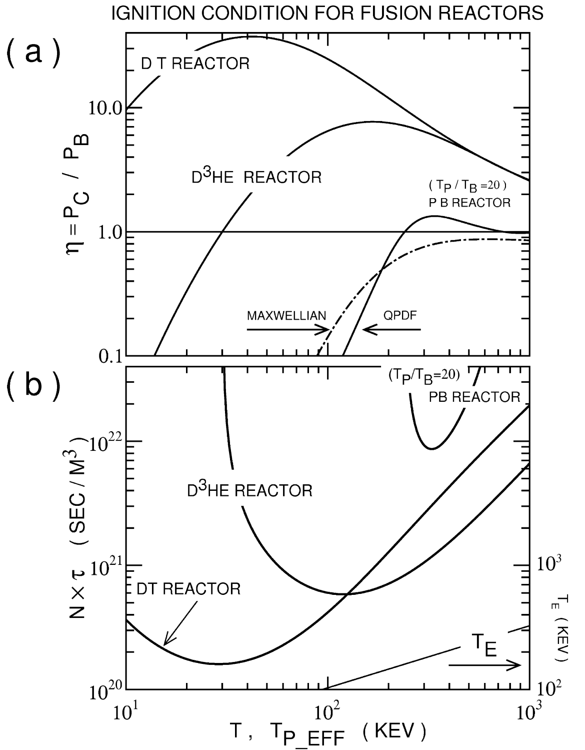

Fig. 2 Ignition condition for p-^{11}B reactor. (a) : η shows the ratio of fusion power density P_c and the radiation power density P_B by bremsstrahlung. Dot-dashed curve shows that the ignition condition ($\eta > 1$) cannot be satisfied by Maxwellian proton. (b) : $n \times \tau$ values necessary to keep the ignition condition (τ : energy confinement time of core plasma). For the comparison, ignition conditions of the DT and D^3 He reactors are shown, also.

$$P_L = \left\{ \frac{3}{2} n_p T_{p\_eff} + \frac{3}{2} n_b T_b + \frac{3}{2} n_e T_{e\_eff} \right\} \frac{1}{\tau}. \qquad (21)$$

If P_L is considered, the ignition condition necessary for a fusion reactor to keep burning can be written as

$$\frac{P_\alpha}{P_B + P_L} > 1. \qquad (22)$$

In this case, $n_e\tau$ value for the p-^{11}B reactor is shown in Fig. 2(b). This result indicates that the ignition condition (eq. (22)) becomes possible when T_{e_eff} is of the order of 300 keV and $n_e\tau \gtrsim 10^{22}$ s/m^3.

5. Summary and discussions

We show that the LHD magnetic field has excellent ability for the high energy particles confinement by the numerical computation of the ICRF-heated particles and that the steady state distribution function of the ICRF-heated protons becomes to QPDF given by eq. (15). Based on these results, the possibility of the p-^{11}B reactor using the LHD type magnetic field and the ICRF heating scheme are studied. It is found that the ignition condition becomes possible to be satisfied in the case of T_{p_eff} ~ 300 keV and $n_e\tau \gtrsim 10^{22}$ s/m^3.

To prevent the dilution of fuels and to reduce the radiation loss, the α ash exhaust is very important for fusion reactor. The combinations of the LHD magnetic field and ICRF heating scheme could realize the active α ash exhaust. An electric power necessary for exhausting the α ash and holding the QPDF-distribution function of protons is a circulating power of the fusion power plant, which is desirable to be in small power level. Since the fusion-produced α particles are in population inversion, the excitation of the microinstability can be expected. Because the ratio of cyclotron frequency between the α particle and the proton is 1 : 2, there is a possibility that holding of the proton QPDF-distribution and a rapid exhaust of the α ash are achieved simultaneously by this microinstability. In this case, the circulating power of the fusion power plant can be greatly decreased. N.J. Fisch has pointed out the possibility of α power channeling using ion-Bernstein waves in tokamak [13]. The detailed analysis of the microinstability in p-^{11}B reactor is an interesting problem in the future.

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