

# Helicon waves and efficient plasma production

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Helicon waves generated by radio-frequency (rf) waves are experimentally demonstrated to have the characteristics of Landau damping, as predicted theoretically, and fully ionized plasmas are realized by this efficient coupling of rf powers to plasmas. Excited waves are identified as a helicon wave by measuring wavelengths in the plasma along the magnetic field and comparing with the dispersion relation. Good agreement is found between experimental and theoretical results.

## I. INTRODUCTION

The plasma production by radio-frequency (rf) waves using either inductive or capacitive coupling methods has been well known to realize appreciable high density and temperature plasmas easily. Rather than simply relying on the large oscillatory electric fields of the antenna to accelerate electrons to ionizing energies, resonant excitation methods have been found to be more effective.<sup>1</sup> Recent experiments by Boswell *et al.*<sup>2-4</sup> have shown that fully ionized plasmas whose density is higher than  $1 \times 10^{12} \text{ cm}^{-3}$  can be produced with a special antenna to excite a helicon wave in a chamber of 10 cm in diameter and 120 cm in length with an rf power of 180 W at 8.8 MHz and a confining magnetic field of 0.75 kG. It has been shown that wave properties are consistent with those expected of helicon waves. Shoji<sup>5</sup> has obtained almost the same results with a different antenna from that used by Boswell *et al.*<sup>2-4</sup> In a theoretical analysis, Chen<sup>6,7</sup> has suggested that the rate of energy absorption by wave damping may be due to Landau damping of the helicon wave that has an electric field component parallel to the magnetic field. This can interact directly with electrons, and hence, provides an efficient means for transferring energy to electrons that subsequently suffer inelastic collisions in the column. Landau damping has been observed and is the main subject of the present paper.

In the present paper detailed measurements are performed of the helicon wave and the efficient plasma production. Spatial variations of damping waves are measured by interferometry to obtain damping rates and wavelengths in the plasma along the magnetic field. The observed wave is consequently confirmed to obey the dispersion relation of the helicon wave, and its damping rate has the characteristics of Landau damping although the collisional damping is not negligible in our experiments. Most plasma properties observed experimentally are consistent with those predicted theoretically.

In Sec. II, we describe a theoretical background for the helicon wave. After describing in Sec. III the experimental apparatus, the observations are presented with possible interpretations in the two subsections of Sec. IV. The conclusion is given in Sec. V.

## II. THEORETICAL BACKGROUND

Helicon waves are known to be right-handed circularly polarized electromagnetic waves, propagating along the magnetic field.<sup>8-10</sup> The frequency  $\omega$  is in the range of  $\omega_{ci} \ll \omega_1 \ll \omega \ll \omega_{ce} \ll \omega_{pe}$ , where  $\omega_{ci}$  and  $\omega_{ce}$  are the ion and electron cyclotron frequencies,  $\omega_1$  is the lower hybrid frequency, and  $\omega_{pe}$  is the electron plasma frequency. Using a cylindrical geometry  $(r, \theta, z)$  and assuming the magnetic field  $\mathbf{B}_0$  to be in the  $z$  direction, the helicon wave in a cylindrical plasma is described by the equations<sup>8-10</sup>

$$\begin{aligned} \nabla \times \mathbf{E}_1 &= -\frac{\partial \mathbf{B}_1}{\partial t}, \\ \nabla \times \mathbf{B}_1 &= \mu_0 \mathbf{j}_1, \\ \mathbf{E}_1 &= \mathbf{j}_1 \times \mathbf{B}_0 / en_e, \end{aligned} \quad (1)$$

where  $\mathbf{E}_1$ ,  $\mathbf{B}_1$ , and  $\mathbf{j}_1$  are small perturbations of the electric field, the magnetic field, and the plasma current, respectively, and  $n_e$  is the time-averaged plasma density. We assume perturbations of the form  $\exp i(m\theta + kz - \omega t)$ . After some algebraic manipulations, the eigenmode equation for  $B_{1z}$ , the axial component of  $\mathbf{B}_1$ , is obtained:<sup>7</sup>

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r}{K^2 - k^2} \frac{\partial B_{1z}}{\partial r} \right) \\ + \left[ 1 - \frac{m^2}{r^2} \frac{1}{K^2 - k^2} + \frac{m}{r} \frac{\partial}{\partial r} \left( \frac{1/K}{K^2 - k^2} \right) \right] B_{1z} = 0, \end{aligned} \quad (2)$$

where  $K = \omega n_e \mu_0 / k B_0$ . The solution of Eq. (2) is found to be an  $m$ th order Bessel function of the first kind  $J_m(\rho)$  if the uniform plasma is contained in a cylinder of radius  $a$ . We define a new variable  $\rho$  such that  $\rho^2 = (K^2 - k^2)a^2$ . Although the dispersion relation is determined by the boundary conditions, the same dispersion relation is obtained both in the plasma bounded by an insulator and in the plasma contained in a conducting cylinder:

$$k/K = -mJ_m(\rho)/\rho J'_m(\rho), \quad (3)$$

where  $J'_m(\rho)$  is a derivative of  $J_m(\rho)$ . When  $k$  is smaller than  $A/a$  where  $A$  is defined as the argument which yields the first zero of  $J_1(\rho)$ , that is, 3.83, the dispersion relation for  $m = 0$  and 1 modes is approximately given by<sup>6,7</sup>

$$\frac{\omega}{k} = \frac{A}{ae\mu_0} \frac{B_0}{n_e}. \quad (4)$$

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We note that, up to here,  $K$ ,  $k$ , and  $\rho$  are all real.

Since the  $B_1$  and  $j_1$  lines of the helicon wave are more or less helical, an antenna of the helical shape, that is, a helical antenna is considered to be able to excite the helicon wave by the rf at the frequency of  $\omega_{ci} \ll \omega \ll \omega_{ce}$ .<sup>5</sup> For example, an antenna consisting of two leading wires twisted helically can excite an  $m = 1$  mode.

The Landau damping rate is given by<sup>6</sup>

$$\frac{\text{Im}(k)}{\text{Re}(k)} = 2\sqrt{\pi} \frac{c^2}{\omega_{pe}^2} \frac{A^2}{a^2} \zeta^3 \exp(-\zeta^2), \quad (5)$$

where  $\zeta = \omega/kv_{th}$ ,  $v_{th} = \sqrt{2k_B T_e/m_e}$ ,  $T_e$  is the electron temperature, and  $m_e$  is the electron mass. Equation (5) is such a steep function of  $\zeta$  that a small change in  $T_e$  or  $k$  can appreciably increase the damping. Introducing the definition of  $\zeta$  and inserting Eq. (4) into Eq. (5), we may write Eq. (5) as

$$\frac{\text{Im}(k)}{\text{Re}(k)} = 2\sqrt{\pi} \frac{v_{th}}{\omega_{ce}} \frac{A}{a} \zeta^4 \exp(-\zeta^2). \quad (6)$$

Thus it is clear that the damping rate maximizes when  $\zeta^4 \exp(-\zeta^2)$  is at its maximum value of 0.54, which occurs for  $\zeta = \sqrt{2}$ . Figure 1 shows the dependence of  $\text{Im}(k)/\text{Re}(k)$  on the wavelength  $\lambda$  for  $T_e = 3$  and 10 eV at  $B_0 = 1$  kG, together with the  $\lambda$  dependence of  $n_e$ . The damping rates maximize surely at  $\zeta (\lambda \omega / 2\pi v_{th}) = \sqrt{2}$ , and  $n_e$  increases monotonically with a decrease in  $\lambda$ . Equation (5) also says that the damping rate is inversely proportional to  $n_e$ , that is,  $B_0$ , when  $\zeta$  is kept at  $\sqrt{2}$ . On the other hand, the damping rate due to the resistivity is given by<sup>6</sup>

$$\frac{\text{Im}(k)}{\text{Re}(k)} = \frac{\nu_{ei}}{\omega} \frac{c^2}{\omega_{pe}^2} \frac{A^2}{a^2}, \quad (7)$$

where  $\nu_{ei}$  is the electron-ion collision frequency. A comparison between the damping rates represented by Eqs. (6) and (7) indicates that Landau damping is dominant in the range of  $n_e \leq 7.2 \times 10^{12} \text{ cm}^{-3}$ , while the collisional damping is larger than Landau damping in the region of  $n_e \geq 7.2 \times 10^{12} \text{ cm}^{-3}$ ;  $a = 2.5$  cm,  $T_e = 3$  eV, and  $\omega/2\pi = 7$  MHz are used. The damping rate due to the resistivity  $\text{Im}(k)/\text{Re}(k)$  depends little on  $n_e$ , as known from Eq. (7), and is regarded as constant within 20% in the range of  $10^{12} \text{ cm}^{-3} \leq n_e \leq 10^{14} \text{ cm}^{-3}$ .

### III. EXPERIMENTAL ARRANGEMENT

A schematic of the experimental apparatus is shown in Fig. 2. The argon plasma produced with a helical antenna is confined in a uniform magnetic field  $B_0$  of up to 3 kG. The antenna is located outside the plasma, that is, on the Pyrex tube of 5 cm in diameter and 50 cm in length, which is connected to the end of the stainless-steel vacuum chamber with a length of 170 cm and an inner radius of 23 cm. The stainless-steel vacuum chamber is electrically grounded. The base pressure is kept less than  $2 \times 10^{-7}$  Torr, and the gas pressure  $p$  is in the range of  $2 \times 10^{-4} \text{ Torr} \leq p \leq 3 \times 10^{-3} \text{ Torr}$  during the experiments. The gas inlet is located at the opposite end of the Pyrex tube to the stainless-steel vacuum chamber.

The antenna consists of two copper ribbons of 2.5 cm in width, which are wound around the Pyrex tube and have half

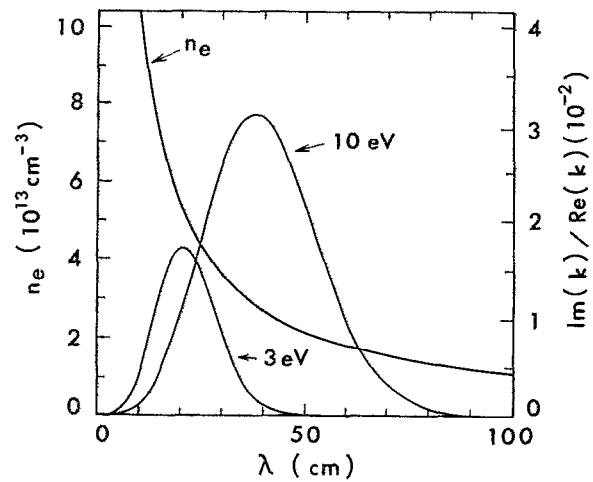


FIG. 1. Plasma density  $n_e$ , and Landau damping rates  $\text{Im}(k)/\text{Re}(k)$  for  $T_e = 3$  and 10 eV, calculated using Eqs. (4) and (5) at  $B_0 = 1$  kG,  $a = 2.5$  cm, and  $\omega/2\pi = 7$  MHz.

of a winding, in order to excite an  $m = 1$  mode, as shown in Fig. 2(b).<sup>5</sup> The length of the antenna is chosen to be 25 cm. The rf power at 7 MHz,  $P_{rf}$ , is supplied from an oscillator-amplifier system and is varied up to 2 kW. The power  $P_{rf}$  is delivered to the plasma via a coaxial cable and is matched into the antenna with two high voltage variable capacitors arranged in a  $\pi$  network.<sup>3</sup> To minimize damage to the rf circuit, the rf supply is pulsed at 83.3 Hz with a 16.7% duty

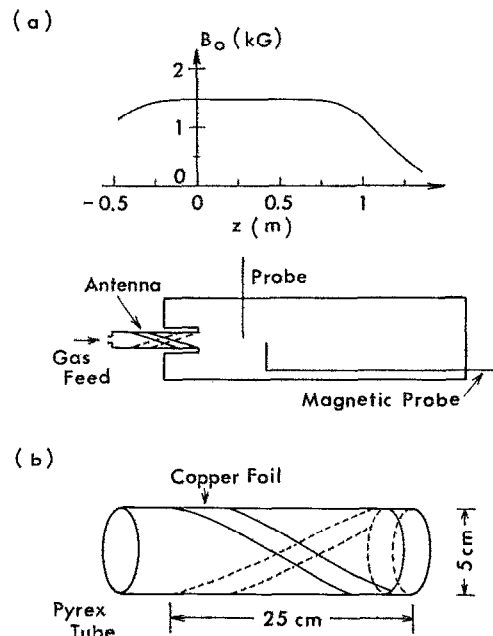


FIG. 2. Schematic of experimental apparatus. (a) Magnetic configuration  $B_0$  and the vacuum vessel. (b) The exciting helical antenna with half of a winding.

cycle. The plasma density  $n_e$  is measured at  $t = 1.5$  msec after the oscillator is turned on at  $t = 0$  msec with a boxcar integrator with a gate width of 0.1 msec, while  $T_e$  is obtained in the afterglow plasma because  $T_e$  of 4–6 eV at  $t = 1.5$  msec is considered to be affected by the rf wave.

Radial profiles of plasma parameters are measured at  $z = 25$  cm, and axial profiles are obtained in the range of  $25 \text{ cm} < z < 75 \text{ cm}$ . Here, the origin of the  $z$  axis is located at the end of the Pyrex tube, which is connected to the stainless-steel vacuum chamber, as shown in Fig. 2(a). Measurements of  $n_e$  and  $T_e$  are performed with a Langmuir probe calibrated against a microwave interferometer; absolute measurements of the line-integrated electron density are made with an 8 mm microwave interferometer. The wavelength of the helicon wave is measured by interferometry with magnetic probes, which are located inside the stainless-steel vacuum chamber, and are movable along the  $z$  axis. The magnetic probes consist of single layer solenoidal coils of 4 mm in diameter and 6 mm in length with 50 windings.

#### IV. EXPERIMENTAL RESULTS AND INTERPRETATION

##### A. Production of fully ionized plasmas

Figure 3 shows the radial profiles of  $n_e$  and  $T_e$ , measured at  $p = 8 \times 10^{-4}$  Torr,  $P_{rf} = 0.7$  kW, and  $B_0 = 0.7$  kG. It is clear that  $T_e$  is constant across the plasma, and that  $n_e$  is almost uniform in the region inside the Pyrex tube, that is, in the region of  $|r| \leq a = 2.5$  cm although the  $n_e$  profile depends on  $B_0$  in practice, as will be described later. The electron temperature  $T_e$  is also found to always be 3–4 eV, and to depend little on such parameters as  $B_0$ ,  $p$ , and  $P_{rf}$ .

The  $p$  dependence of  $n_e$  is shown in Fig. 4, obtained at  $B_0 = 1$  kG and  $P_{rf} = 1.5$  kW. The plasma density  $n_e$  is apparently proportional to  $p$  in the region of  $p \leq 1 \times 10^{-3}$  Torr, and saturates in the high- $p$  region. Since the density  $n_0$  of neutral argon atoms at  $p = 1 \times 10^{-3}$  Torr is  $\sim 2.6 \times 10^{13} \text{ cm}^{-3}$  at the room temperature of 300 K, it is confirmed that argon atoms are completely ionized in the range of

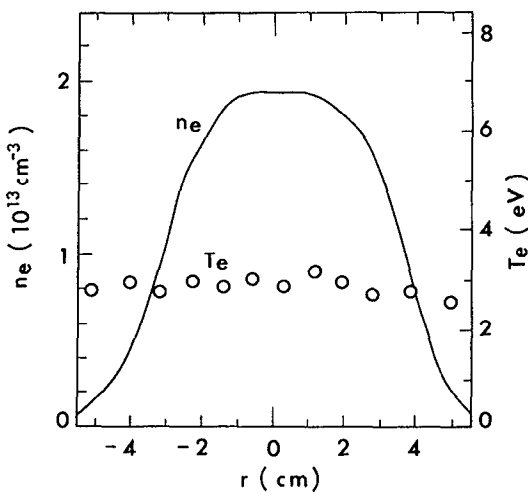


FIG. 3. Radial profiles of  $n_e$  and  $T_e$ , measured at  $p = 8 \times 10^{-4}$  Torr,  $P_{rf} = 0.7$  kW, and  $B_0 = 0.7$  kG.

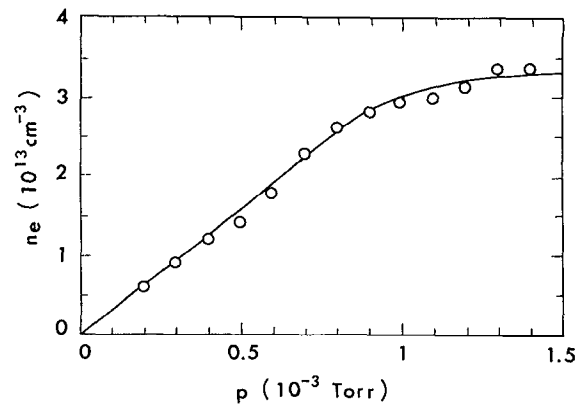


FIG. 4. Dependence of  $n_e$  on  $p$  at  $B_0 = 1$  kG and  $P_{rf} = 1.5$  kW.

$p \leq 1 \times 10^{-3}$  Torr. The reason why  $n_e$  is a little larger than  $n_0$  in the range of  $p \leq 1 \times 10^{-3}$  Torr, is attributed to the fact that  $p$  is measured at the position close to the vacuum pump rather than the gas inlet. In the high- $p$  region, it is considered that there is not enough rf power for all neutral atoms to be ionized. In this case, the discharge color is found to be light pink arising from the many lines of neutral argon in the red part of the spectrum. On the other hand, the color of the central volume is dark blue due to emission of Ar II lines when the fully ionized plasma is achieved.

Figure 5 shows the dependence of  $n_e$  on  $P_{rf}$  for various  $B_0$ 's. The gas pressure  $p$  is kept at  $8 \times 10^{-4}$  Torr in the rest of this paper except for the experiments to obtain Fig. 9. It is found that the large rf power is required to some extent in order to realize the fully ionized plasma, and  $B_0$  affects the plasma production seriously in the manner that the larger  $B_0$  makes it easier to achieve 100% ionization. In our system, it is difficult for the matching between the plasma and rf circuit

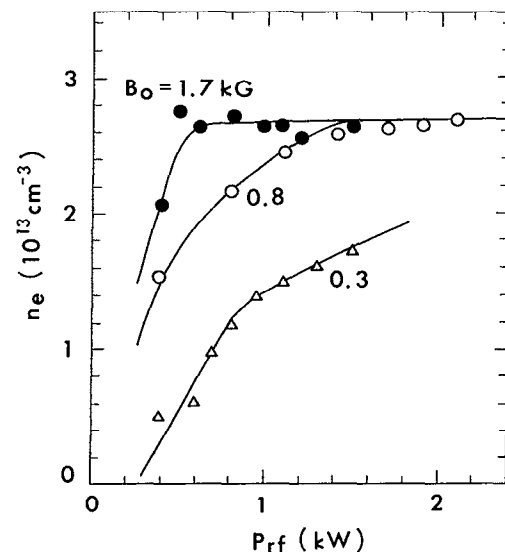


FIG. 5. Dependence of  $n_e$  on  $P_{rf}$  for various  $B_0$ 's.

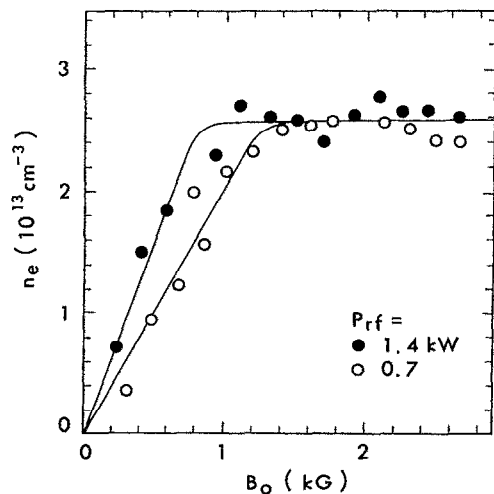


FIG. 6. Density  $n_e$  as a function of  $B_0$  for  $P_{rf} = 0.7$  and  $1.4$  kW.

to be adjusted at  $P_{rf}$ 's below  $\sim 0.4$  kW, so that we cannot produce the plasma in this  $P_{rf}$  range.

The  $B_0$  dependence of  $n_e$  is shown in Fig. 6 for  $P_{rf} = 0.7$  and  $1.4$  kW. The electron density  $n_e$  is roughly proportional to  $B_0$  in the region of  $B_0 \leq 1$  kG, although the slope of the curve depends a little on  $P_{rf}$ . This agrees qualitatively with the relation between  $n_e$  and  $B_0$ , predicted in the dispersion relation of the helicon wave. For quantitative agreement, the measurement of the wavelength is, of course, required, as known from Eq. (4). In the range of  $B_0 \gtrsim 1$  kG,  $n_e$  is constant independently of  $B_0$  since the fully ionized plasma is accomplished. The radial  $n_e$  profile is studied by changing  $B_0$  at  $P_{rf} = 1.4$  kW, and the result is shown in Fig. 7. When  $B_0$  is smaller than  $\sim 0.45$  kG, the  $n_e$  profile around  $r = 0$  cm has a concave shape and the distance between two ridges is almost equal to the inner diameter of the Pyrex tube. By increasing  $B_0$ , the  $n_e$  profile is varied drastically and becomes peaked at  $r = 0$  cm, as shown in Fig. 7. This dependence of the radial  $n_e$  profile on  $B_0$  may be explained by

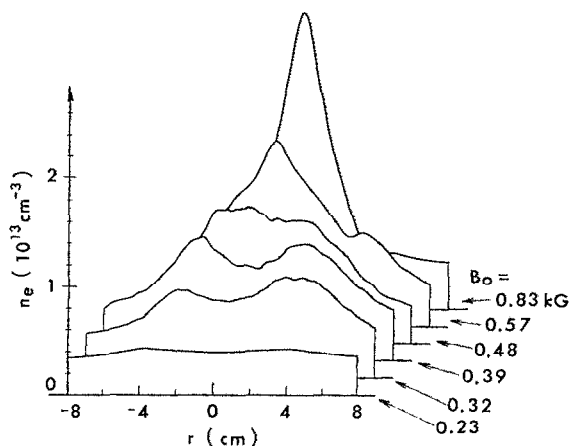


FIG. 7. Dependence of radial  $n_e$  profile on  $B_0$  at  $P_{rf} = 1.4$  kW.

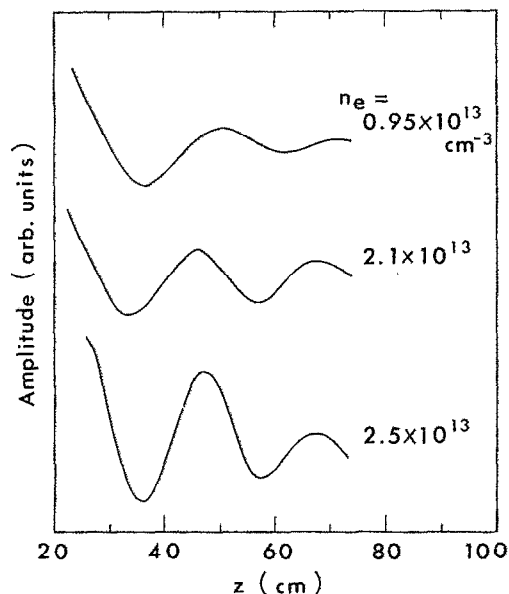


FIG. 8. Tracings of spatial variations of damped waves;  $B_0/n_e$ 's are  $4.2 \times 10^{-14}$  kG cm<sup>3</sup>,  $3.5 \times 10^{-14}$  kG cm<sup>3</sup>, and  $3.2 \times 10^{-14}$  kG cm<sup>3</sup> from the top.

taking account of the confinement of the plasma.<sup>6</sup> Basically, the heating and ionization will be localized near the walls, and so is the energy deposition. This causes the concave  $n_e$  profile with the ridges at the inner-wall position of the Pyrex tube. By increasing  $B_0$ , the energy confinement, which is determined by the end plate sheaths and the intensity of  $\mathbf{B}_0$ , may be better on the axis, leading to the appearance of a dense core.

## B. Dispersion relation and damping of the helicon wave

The phase and amplitude of the helicon wave as a function of axial position  $z$  are obtained by interferometry with the magnetic probe. Tracings of the spatial variations of the damped waves are shown in Fig. 8 at  $B_0 = 0.4 - 0.8$  kG and  $P_{rf} = 1.3 - 1.6$  kW. An important point in the figure is that the wavelength is about half of the antenna length, and is varied depending on  $n_e$ , strictly speaking, on  $B_0/n_e$  as expected from Eq. (4). This suggests that the wavelength is not determined by the antenna length, and is varied automatically to provide the best coupling or to satisfy the dispersion relation. We cannot obtain the dispersion relation directly since the frequency of the oscillator is fixed, but the relation between  $n_e$  and  $kB_0$  can be obtained, and compared with the theoretical prediction. Figure 9 shows the dependence of  $n_e$  on  $kB_0$  under various conditions of  $B_0$ ,  $P_{rf}$ , and  $p$ . The experimental results represented by open circles are obtained in the low- $B_0$  region, where  $n_e$  does not saturate with the variation of  $B_0$  and the relatively flat  $n_e$  profile is observed inside the Pyrex tube. The solid line represents  $n_e$  given by Eq. (3);  $a = 2.5$  cm and  $\omega/2\pi = 7$  MHz are used. It is evident that  $n_e$  is proportional to  $kB_0$ , and there is relatively good agreement between the experimental and theoretical results, indicating that the observed wave obeys the dispersion relation of the helicon wave.

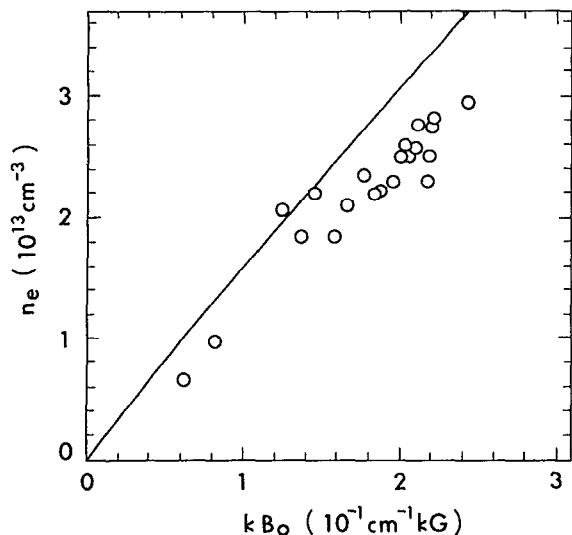


FIG. 9. Dependence of  $n_e$  on  $kB_0$  under various conditions of  $B_0$ ,  $P_{rf}$ , and  $p$ .

The most important point which is noticed in studying the relation between  $n_e$  and  $k$ , is that  $\zeta$  approaches  $\sqrt{2}$  by increasing  $n_e$  and keeping  $B_0$  constant, and that  $n_e$  has a maximum at  $\zeta \sim \sqrt{2}$  where the Landau damping rate maximizes. In fact, the lowest tracing in Fig. 8 yields  $\zeta = 1.49 \sim \sqrt{2}$  at  $T_e = 3$  eV and  $\omega/2\pi = 7$  MHz. This suggests that the damping of the helicon wave can be explained by Landau damping in our experiments. The variation of  $B_0$  makes it possible to change  $n_e$  with keeping  $\zeta$  at  $\sim \sqrt{2}$ .

The value of  $\text{Im}(k)/\text{Re}(k)$  can be also obtained from the tracing of the spatial variation of the damped wave. Figure 10 shows the dependence of  $\text{Im}(k)/\text{Re}(k)$  on  $n_e$ , obtained by changing  $B_0$ . The wave number  $k$  is kept constant to satisfy  $\zeta = \sqrt{2}$ , as mentioned above. The experimental results are plotted by open circles. There is a tendency for  $\text{Im}(k)/\text{Re}(k)$  to decrease with an increase in  $n_e$ . This is characteristic of Landau damping, and cannot be explained by the collisional damping since  $\text{Im}(k)/\text{Re}(k)$  due to the resistivity depends little on  $n_e$ , as known from Eq. (7). However, for example, at  $n_e = 2.5 \times 10^{13} \text{ cm}^{-3}$ , the Landau damping rate is about one third of the collisional damping rate, and is about one-fourth of the measured damping rate. Landau damping is larger than the collisional damping in the range of  $n_e \lesssim 7.2 \times 10^{12} \text{ cm}^{-3}$ . Taking account of these results, both Landau damping and collisional damping are considered to occur at the same time in our  $n_e$  range. The solid line shown in Fig. 10 represents the damping rate predicted theoretically, which consists of the Landau damping rate plus the collisional damping rate. Good agreement is found between the experimental and theoretical results.

## V. CONCLUSIONS

Experiments reported here demonstrate the excitation of a helicon wave in a cylindrical magnetoplasma and the production of a fully ionized plasma. An axial wavelength

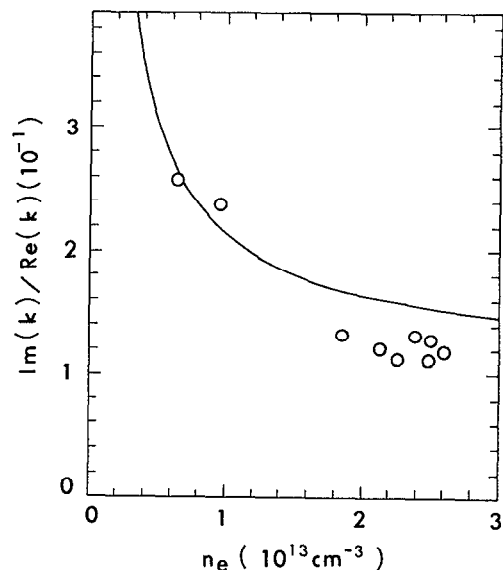


FIG. 10. Dependence of  $\text{Im}(k)/\text{Re}(k)$  on  $n_e$ , obtained by changing  $B_0$  at  $P_{rf} = 1.5$  kW.

measured in the vacuum chamber indicates that the axial wavelength is not determined by the antenna length, but is automatically varied depending on the density and the magnetic field to satisfy the dispersion relation of the helicon wave. As the magnetic field is increased, the density increases according to the dispersion relation of the helicon wave, and the central core of the plasma becomes fully ionized at an appreciable rf power with the optical emissions being typical of Ar II. It is also shown that the density has a maximum at  $\zeta (= \omega/kv_{th}) \sim \sqrt{2}$  where the Landau damping rate maximizes, and that there is a tendency for  $\text{Im}(k)/\text{Re}(k)$  to decrease with an increase in the density. These are characteristic of Landau damping. However, both Landau damping and collisional damping are considered to occur at the same time in our density range, since the collisional damping rate is larger than the Landau damping rate at densities above  $7.2 \times 10^{12} \text{ cm}^{-3}$ . There is good agreement between the experimental and theoretical damping rates when the theoretical damping rate consists of both the Landau damping rate and the collisional damping rate. Landau damping rather than the collisional damping is considered to play an important role on the plasma production, because the resonant mechanism can accelerate electrons to ionizing energies more efficiently.

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