Comparison of toroidal viscosity with neoclassical theory

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Toroidal rotation profiles are measured with charge exchange spectroscopy for a plasma heated with tangential neutral beam injection (NBI) in the Compact Helical System (CHS) heliotron/torsatron device [Ida *et al.*, Phys. Rev. Lett. **67**, 58 (1991)] to estimate the toroidal viscosity. The toroidal viscosity derived from the toroidal rotation velocity shows good agreement with the neoclassical toroidal viscosity plus the perpendicular viscosity ($\mu_\perp = 2\ \mathrm{m^2/s}$). © *1997 American Institute of Physics.* [S1070-664X(97)01101-4]

I. INTRODUCTION

In heliotron/torsatron devices, because of the nonaxis symmetry of the magnetic field, the toroidal viscosity causes the damping of the toroidal rotation velocity. The toroidal rotation profiles are measured using charge exchange spectroscopy[1] and the magnitude of the toroidal viscosity has been compared with the preliminary neoclassical calculations in the Compact Helical System (CHS).[2] Strictly speaking, toroidal viscosity (parallel viscosity in the toroidal direction), not the parallel viscosity (viscosity parallel to the magnetic field line), is evaluated in this paper. (In the original paper, the magnitude of toroidal viscosity was assumed to be identical to that of the parallel viscosity.)

The transit frequency for a particle in the magnetic field ripple is roughly $\omega_i = v_{\mathrm{th}}(Mn + m\iota)/R$, where v_{th} is the thermal velocity of ions, M is the toroidal period number in the helical components of the field, n is the toroidal mode number in one field period, m is the poloidal mode number, ι is the rotational transform, and R is the major radius. For a tokamak, where $n=0$ and $m=1$, with $q=1/\iota$, the transit frequency of the toroidal effect is $\omega_{ti} = v_{\mathrm{th}}/(qR)$, where q is the safety factor. For helical devices with $n=1$ and $M \gg m\iota$, the transit frequency in the helical ripple is approximately $\omega_{hi} = (v_{\mathrm{th}} M/R)$. In the original paper, one dominant helical mode in CHS, where $M=8$ and $(m,n)=(2,1)$, was assumed in the calculation of the transit frequency, and ω_{ti} in the Shaing paper[3] is replaced by ω_{hi}, with everything else the same. On the other hand, in this paper all helical components of the magnetic field in the CHS are included in the calculation of ∇B. In the original paper, the magnitude of ∇B was characterized by the magnetic field modulation strength, γ, defined as $\gamma^2 = \langle (\hat{\mathbf{n}} \cdot \nabla \mathbf{B})^2 / B^2 \rangle$, where $\hat{\mathbf{n}}$ is the normalized vector in the direction of magnetic field and $\langle\ \rangle$ is a flux-surface-average operator. The magnetic field modulation strength γ is approximated to be $(\epsilon_h M/R)$ near the plasma edge, where ϵ_h is the ripple of the (2,1) component and the single helicity assumption is a good approximation except for the plasma center. However, γ near the plasma center should be estimated by including other Fourier spectral components, mainly the (0,1) and (1,1) components, because $\epsilon_h = 0$ at the plasma center. Toroidal viscosity for a three-dimensional (3-D) magnetic structure in a stellarator has been published.[4,5] In this paper, neoclassical toroidal viscosity[6,7] is calculated more precisely using the transit frequency, including all magnetic field Fourier spectral components, and then the neoclassical toroidal viscosity is compared with the measurements.

CHS is a heliotron/torsatron device (poloidal period number $L=2$, and toroidal period number $M=8$) with a major radius, R_{ax} of 95 cm and an average minor radius a of 20 cm (the major radius of the vacuum magnetic axis and the average minor radius in the standard configuration). The magnetic field ripple on the magnetic axis on the inboard side ($R_{\mathrm{ax}} = 90-95$ cm) is negligible, however, it increases sharply for $R_{\mathrm{ax}} > 95$ cm and reaches 8% at $R_{\mathrm{ax}} = 101.6$ cm. Therefore the magnetic field ripple at the plasma center can be modified from zero to 8% by shifting the magnetic axis R_{ax} from 89.9 to 101.6 cm and the damping of toroidal rotation due to transit time magnetic pumping (TTMP) can be studied.

II. TOROIDAL AND PERPENDICULAR VISCOSITIES

As is well known in axisymmetric systems, the direction of the flow to be damped by the parallel viscosity is determined by the symmetry, i.e., the flow in the direction without symmetry (poloidal flow) is damped. The plasma flow velocity $\mathbf{V}_a$ is governed by the momentum equation

$$m_a n_a \frac{\partial \mathbf{V}_a}{\partial t} = e_a n_a (\mathbf{E} + \mathbf{V}_a \times \mathbf{B}) - \nabla p_a - \nabla \cdot \Pi_a + \mathbf{F}_a + \mathbf{F}_a^f, \tag{1}$$

where m_a and n_a are the mass and the density of species a ($a=i$ for ion and $a=e$ for electrons), respectively, p_a and Π_a are pressure and stress tensor, respectively, $\mathbf{E}$ and $\mathbf{B}$ are the electric field and magnetic field, respectively, $\mathbf{F}_a$ and $\mathbf{F}_a^f$ are the forces due to collisions between ion and electrons and plasma and fast ions, respectively. By summing ion and electron equations and neglecting electron viscosities, the ion flow velocity, $\mathbf{V}_i$, is given by

$$m_i n_i \frac{\partial \mathbf{V}_i}{\partial t} = \mathbf{j} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_i + \mathbf{F}_i^f + \mathbf{F}_e^f, \tag{2}$$

where $\mathbf{j}$ and p are the current density and the total pressure in the plasma, respectively. When the neutral beam is injected in the tangential direction, the force driven by neutral beam injection, $\mathbf{F}^{\mathrm{nbi}}$, mainly contributes to $\mathbf{F}_i^f + \mathbf{F}_e^f$ In the steady state, the toroidal force driven by neutral beam injection, $\mathbf{F}^{\mathrm{nbi}}$, is balanced with viscosities in the toroidal direction,

 1070-664X/97/4(2)/310/5/$10.00

$$\langle \mathbf{B}_\phi \cdot \mathbf{F}^{\mathrm{nbi}} \rangle = \langle \mathbf{B}_\phi \cdot \nabla \cdot \Pi_i \rangle, \tag{3}$$

where $\mathbf{B}_\phi$ is the toroidal magnetic field and $\langle\rangle$ is the flux average. As we discuss later, the measured damping of plasma flow cannot be explained by the neoclassical toroidal viscous force alone. With respect to the viscous forces, we consider two viscosities: one is the viscosity coupled to a velocity gradient in the direction parallel to the toroidal velocity and the other is the viscosity associated with a velocity gradient in the direction perpendicular to the velocity (radial direction). These viscosities are called toroidal (nearly parallel) and perpendicular viscosity. There are off-diagonal terms of the transport matrix in these viscosities and the plasma rotations are not independent of temperature or density gradients (which are considered to be part of the internal forces). Taking the standard diagonal terms of the transport matrix, the viscous forces can be expressed by the perpendicular and toroidal viscosity coefficients, $\mu_\perp, \mu_\parallel$;

$$\langle \mathbf{B}_\phi \cdot \nabla \cdot \Pi_i \rangle = B_\phi n_i m_i \mu_\parallel V_\phi - B_\phi n_i m_i \mu_\perp \nabla^2 V_\phi. \tag{4}$$

In neoclassical theory, the toroidal viscosity is determined by the toroidal flow velocity, v_ϕ (see the Appendix), the thermal velocity $(2eT_i/m_i)^{1/2}$, and the geometric factor of the magnetic field. The toroidal viscosity coefficient $\mu_\parallel$, can be expressed as

$$\mu_\parallel = 2\frac{\sqrt{2eT_i/m_i}J}{\lambda_{\mathrm{PL}\phi} R B_\phi}, \tag{5}$$

where $2\pi J$ is poloidal current outside the flux surface and n_i, m_i, and T_i are ion density, mass, and temperature, respectively. In CHS, ions are in the plateau collisionality regime and $\lambda_{\mathrm{PL}\phi}$ is defined as

$$\frac{1}{\lambda_{\mathrm{PL}\phi}} \equiv \left(\frac{\sqrt{\pi}(J + \iota I)}{2\langle B^2 \rangle} \right) \times \left\langle \left(\frac{\mathbf{B}_\phi \cdot \nabla B}{B} \right) \sum_{mn} \frac{1}{|m\iota + Mn|} \left(\frac{1}{B} \hat{\mathbf{n}} \cdot \nabla B \right)_{mn} \times \exp^{i(m\theta + Mn\phi)} \right\rangle, \tag{6}$$

where m and Mn are the poloidal and toroidal period number of the magnetic field spectrum. By replacing the terms of the Fourier series with $1/(m\iota + Mn)(\hat{\mathbf{n}} \cdot \nabla B/B)$, making the approximation of single helicity ($n=1$, $m=2$, then $M \gg m\iota$), ignoring the difference between parallel and toroidal viscosity ($B_\phi \approx B$ and $(\mathbf{B}_\phi \cdot \nabla B)/B \approx \hat{\mathbf{n}} \cdot \nabla B$), and taking the large aspect ratio limit ($J \approx B_\phi R$) with no net current ($I=0$), the formula (6) is approximated to be $(\frac{1}{2}) \times (\pi^{1/2} R \gamma^2)/M$. Then the viscosity coefficient is simplified as $\mu_\parallel \approx \pi^{1/2}\gamma^2(R/M)(2eT_i/m_i)^{1/2}$. This formula is identical to that in the original paper, which was derived from the Shaing formula[3] by replacing ω_{ti} to $\omega_{hi} = (v_{\mathrm{th}} M/R)$.

Figure 1 shows the magnetic field spectrum contributing to the damping of the toroidal velocity ($n \neq 0$) for the various magnetic axis, R_{ax}. Near the plasma edge ($\rho > 0.5$) the main contribution to the modulation of magnetic field is the helical component of helical coils of $(m,n) = (2,1)$, regardless of the position of the magnetic axis. However, the (2,1) mode vanishes at the plasma center and the (0,1) and (1,1) modes mainly contribute modulation of the magnetic field and damping of the central toroidal rotation velocity. These modes increase as the magnetic axis is shifted outward. The toroidal viscosity coefficients, $\mu_\parallel$, are closely related with the three-dimensional magnetic field structure. In the assumption of single helicity in the calculation of the transit frequency, the toroidal viscosity coefficient is simplified to $\mu_\parallel \approx (2\pi^{1/2}\gamma^2 eT_i)/(m_i \omega_{hi}) = \pi^{1/2}\gamma^2(R/M)(2eT_i/m_i)^{1/2}$ with the magnetic field modulation strength, γ. In order to extend the magnetic field modulation strength, γ, applicable even in the three-dimensional calculation, the parameter $\gamma_{\text{3-D}}$ is defined with the toroidal viscosity coefficient $\mu_\parallel$ as

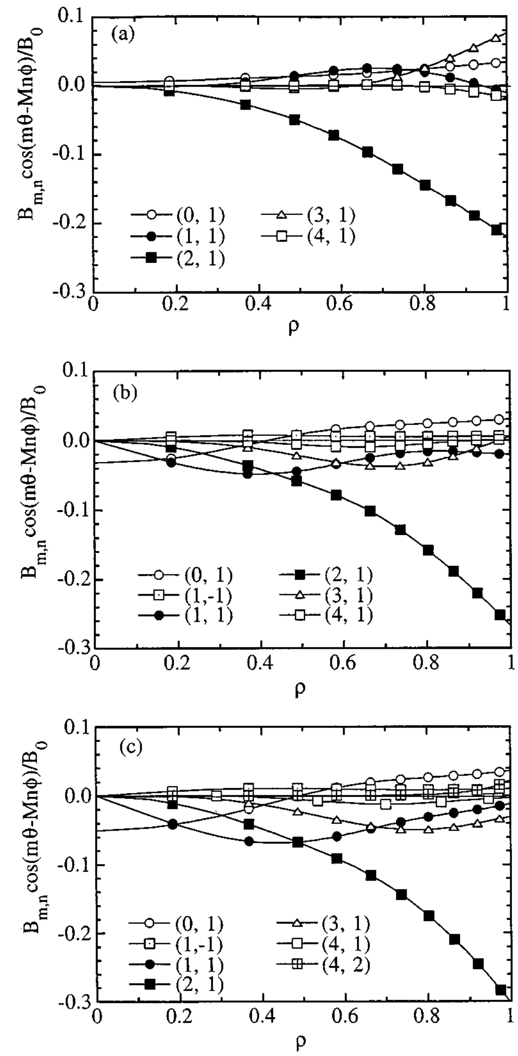

FIG. 1. Radial profile of the magnetic field spectrum in the CHS torsatron/heliotron for the vacuum magnetic axis of $R_{\mathrm{ax}} = 89.9$, 94.9, and 97.4 cm.

$$\gamma_{\text{3-D}} \equiv \sqrt{\frac{\mu_\parallel}{\sqrt{2\pi e T_i/m_i}(R/M)}}. \tag{7}$$

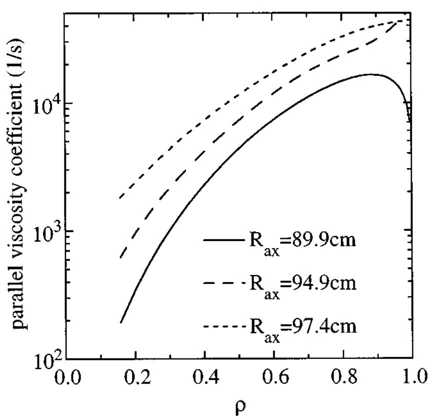

FIG. 2. Radial profile of the toroidal viscosity coefficient in the CHS torsatron/heliotron for the vacuum magnetic axis of $R_{\mathrm{ax}}=89.9$, 94.9, and 97.4 cm.

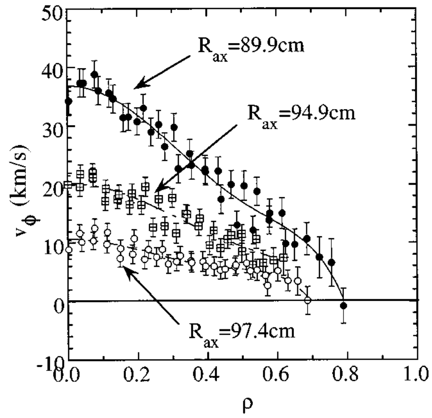

FIG. 3. Radial profile of the toroidal rotation velocity measured in the CHS torsatron/heliotron for a vacuum magnetic axis of $R_{\mathrm{ax}}=89.9$, 94.9, and 97.4 cm.

The 3-D magnetic field strength, $\gamma_{\text{3-D}}$, no longer depends on the thermal velocity of ions, but mainly depends on the magnetic field structure.

Figure 2 shows the neoclassical toroidal viscosity coefficient profiles calculated from the magnetic structure, including the finite β effect in the plateau regime in the CHS. The toroidal viscosity coefficient increases very rapidly toward the plasma edge, which gives strong damping of the toroidal rotation velocity, regardless of the position of the vacuum magnetic axis, R_{ax}. As the magnetic axis is shifted outward, the toroidal viscosity coefficient increases even near the plasma center. The increase of the toroidal viscosity coefficient by shifting the plasma from 89.9 cm, where there is negligible ripple, to 97.4 cm, where the helical ripple is more than 2%, is one order of magnitude near the plasma center.

III. TOROIDAL ROTATION VELOCITY IN CHS

Figure 3 shows toroidal rotation velocity profiles as a result of a major radius scan ($R_{\mathrm{ax}}=89.8$, 94.9, 97.4 cm), which is controlled by the vertical field strength in the CHS heliotron/torsatron. The plasma is produced initially by electron cyclotron heating (ECH) in hydrogen gas and sustained with tangential neutral beam injection (NBI) (absorbed power of 0.5 MW in the direction parallel to the helical current). The line-averaged density reaches about $2\times10^{13}\mathrm{cm}^{-3}$ after NBI.

The parameter dependence of the viscosity coefficient is studied by changing the field ripple to check whether it is neoclassical or not. The damping of toroidal rotation velocity due to charge exchange loss can be neglected, except at the plasma periphery. Here we introduce the effective viscosity coefficient μ_{eff}(1/s) as an indication of how strong the damping of the central velocity is by toroidal and perpendicular viscosities $n_i m_i \mu_\parallel v_\phi$, and $-n_i m_i \mu_\perp \nabla^2 v_\phi$ in the plasma, where $\mu_\parallel$(1/s) and $\mu_\perp$(m^2/s) are the toroidal and perpendicular viscosity coefficients, respectively. The effective viscosity coefficient μ_{eff} is defined as μ_{eff}^{-1} $=v_\phi(0)m_i n_i(0)/F_\phi^{\mathrm{nbi}}(0)$, where $F_\phi^{\mathrm{nbi}}(0)$ is the toroidal force due to neutral beam injection. The toroidal force due to injected fast ions has been estimated for each plasma axis using an analytical model,[8] where the fast ion birth distribution and the shine-through are calculated. Since the deposition of the fast ion energy declines due to orbit loss and charge exchange loss, their effects have been involved in the model to match the result of the 3-D Monte Carlo code HELIOS.[9] The major source of error in the effective viscosity is the toroidal force, while the error bar of toroidal rotation velocity itself is 5%–20%, as seen in Fig. 3. One of the main errors in evaluating $\gamma_{\text{3-D}}$ in the measurements is due to the uncertainty in the description of finite β equilibria. The uncertainties of the evaluated $\gamma_{\text{3-D}}$ are 5%–20%. Since the central toroidal rotation velocity is determined by the magnetic field strength $\gamma_{\text{3-D}}$ in the central region (not the central $\gamma_{\text{3-D}}$ value alone), because of the diffusion process [see Eq. (8)], the $\gamma_{\text{3-D}}$ values near the center ($\rho=0.2$) are taken as a reference.

The central toroidal velocity $v_\phi(0)$ is determined by solving the diffusion equations of

$$\frac{F_\phi^{\mathrm{nbi}}(r)}{m_i n_i(r)}=\mu_\parallel(r)V_\phi(r)-\mu_\perp\nabla^2 V_\phi(r), \tag{8}$$

with the perpendicular viscosity coefficient constant in space and boundary conditions of $v_\phi(a)=0$, $\partial v_\phi/\partial r(0)=0$. If there is no perpendicular viscosity this effective viscosity coefficient is equal to the toroidal viscosity coefficient, $\mu_{\mathrm{eff}}=\mu_\parallel(0)$. Since the effective viscosity, μ_{eff}, is defined with the central toroidal rotation velocity and the central torque due to neutral beam injection, μ_{eff}^{-1} represents the magnitude of central toroidal velocity and does not represent the local value of perpendicular viscosity plus toroidal viscosity.

Figure 4 shows the inverse of the effective viscosity coefficient as a function of the modulation parameter of the magnetic field, $\gamma_{\text{3-D}}$. The effective toroidal viscosity coefficient μ_{eff} shows the $\gamma_{\text{3-D}}^2$ dependence, as predicted by neo-

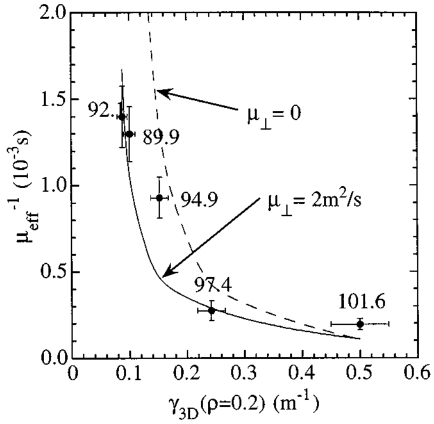

FIG. 4. Inverse of the effective viscosity coefficient as a function of magnetic field modulation strength $\gamma_{3\text{-D}}$ with the prediction of neoclassical toroidal viscosity, $n_i m_i \mu_\parallel v_\phi$, and neoclassical toroidal viscosity plus anomalous perpendicular viscosity, $n_i m_i \mu_\parallel v_\phi - n_i m_i \mu_\perp \nabla^2 v_\phi (\mu_\perp = 2\ \mathrm{m^2/s})$, in the CHS torsatron/heliotron.

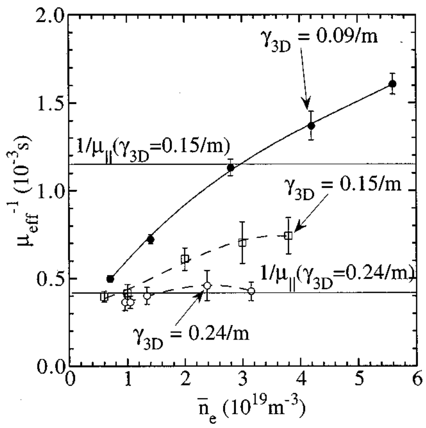

FIG. 5. Inverse of the effective viscosity coefficient as a function of line-averaged density near the plasma center ($\rho = 0.2$) for various magnetic field modulation, $\gamma_{3\text{-D}}$. The two straight lines are the inverse of the neoclassical toroidal viscosities for $\gamma_{3\text{-D}} = 0.15\ \mathrm{m}^{-1}$ and $0.24\ \mathrm{m}^{-1}$. The inverse of neoclassical toroidal viscosities for $\gamma_{3\text{-D}} = 0.09\ \mathrm{m}^{-1}$ is off the scale.

classical theory in the region, where the neoclassical toroidal viscosity becomes dominant, $\gamma_{3\text{-D}} > 0.2$. When the modulation of B decreases below 0.2, the neoclassical toroidal viscosity becomes small and anomalous perpendicular viscosity becomes dominant. The anomalous perpendicular viscosity coefficient, $\mu_\perp$, to fit the measured data is $2\ \mathrm{m^2/s}$. The coefficient of perpendicular viscosity of $2\ \mathrm{m^2/s}$ has a similar magnitude to that of effective thermal diffusivity ($\chi_{\text{eff}} = 5-6\ \mathrm{m^2/s}$).[10] Since the toroidal viscosity increases sharply toward the plasma edge, the toroidal velocity away from the plasma center ($\rho > 0.6$) is determined mainly by toroidal viscosity. In fact, the toroidal velocity becomes almost zero at $\rho = 0.6$ for $R_{\text{ax}} = 97.4$ cm and $\rho = 0.8$ for $R_{\text{ax}} = 89.9$ cm, where the toroidal viscosity coefficient exceeds 10^4 1/s. The measured velocity away from the plasma center shows good agreement with the neoclassical prediction.

IV. DISCUSSION

Since the plasma is in the plateau regime, the neoclassical toroidal viscosity coefficient is independent of collisionality (electron density or ion temperature), except for v_{th} in formula (5). In order to check, the effective viscosity coefficient is measured at various densities. In this density scan, ion temperature was more or less constant. As seen in Fig. 5, the effective viscosity coefficient shows at most only weak dependence on the electron density, when the modulation of B is large ($\gamma = 0.24\ \mathrm{m}^{-1}$) and neoclassical toroidal viscosity is dominant. However, when the modulation of B is small ($\gamma = 0.09\ \mathrm{m}^{-1}$) and neoclassical toroidal viscosity is negligible, the effective viscosity coefficient has a strong dependence on electron density. This density dependence is a feature of anomalous transport, because strong density dependence is also observed in the energy transport, which is governed by anomalous transport. In fact, the global energy confinement shows strong density dependence in CHS,[11] as predicted by LHD scaling[12] ($\tau_E = 0.17 P_{\text{abs}}^{-0.58} n_e^{-0.69} B^{0.84} a^2 R^{0.75}$, where τ_E is global confinement time in s, P_{abs}, n_e, B, a, R are absorbed power in MW, line-averaged electron density in $10^{20}\ \mathrm{m}^{-3}$, magnetic field strength in T, and minor and major radius in m, respectively).

The differences between the parallel viscosity in the original paper and the toroidal viscosity in this paper are discussed. The approximations made in the original paper were a replacement of the Fourier series with $(1/M) \times (\hat{\mathbf{n}} \cdot \nabla B / B)$ and the replacement of toroidal viscosity with parallel viscosity ($B_\phi \approx B$) and $(\mathbf{B}_\phi \cdot \nabla B)/B \approx \hat{\mathbf{n}} \cdot \nabla B$). Since $1/M$ is the maximum value of $1/(Mn + m\iota)$, where $n = -1$, 1, 2 and $m = 0$, 1, 2, 3, 4, $(1/M)(\hat{\mathbf{n}} \cdot \nabla B / B)$ values should be larger than the sum of the Fourier series in formula (6). The toroidal effect, ϵ($1/\epsilon = 5$ at the plasma edge), is comparable to the helical ripple ϵ_h in a low aspect ratio heliotron/torsatron device like the CHS. Therefore the magnetic field modulation strength estimated along the magnetic field $\hat{\mathbf{n}} \cdot \nabla B$, should be larger than that in the toroidal direction, $(\mathbf{B}_\phi \cdot \nabla B)/B$. Then the parallel viscosity was overestimated in the original paper, especially near the plasma edge because of these effects. In order to match the toroidal rotation profiles measured, an artificial multiplicative factor of 0.3 for the parallel viscosity ($0.3\mu_\parallel$) was taken to evaluate the perpendicular viscosity to be $3.5\ \mathrm{m^2/s}$. This artificial multiplicative factor causes the underestimation in toroidal viscosity near the plasma center, where the effects described above are smaller. Therefore a larger perpendicular viscosity was needed to explain the toroidal rotation velocity near the plasma center.

The toroidal viscosity coefficient derived from the toroidal rotation velocity shows good agreement with neoclassical toroidal viscosity coefficient calculated with the a three-dimensional magnetic structure, when the perpendicular viscosity with the coefficient, $\mu_\perp$, of $2\ \mathrm{m^2/s}$ is added to the neoclassical viscosities.

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APPENDIX: THE TOROIDAL VISCOSITY OF IONS IN THE PLATEAU REGIME

In nonaxisymmetric systems, both the direction and the magnitude of the damping should be specified. Therefore the parallel viscosity in the toroidal direction (toroidal viscosity) neglecting the heat flux can be expressed as

$$\langle \mathbf{B}_\phi \cdot \nabla \cdot \Pi_a \rangle = \mu_a \langle \mathbf{V}_a \cdot \nabla \theta_a^* \rangle, \tag{A1}$$

where μ_a is the viscosity coefficient mainly determining the magnitude of the damping,

$$\mu_a = \left(\frac{2 n_a m_a v_{\mathrm{th}}}{\lambda_a} \right), \tag{A2}$$

where $n_a, m_a v_{\mathrm{th}} [=(2eT_a/m_a)^{1/2}]$ are the density, mass, and thermal velocity of the particle species a, and the explicit expression of the coefficient λ_a for ions in the plateau regime will be given later. The parameter θ_a^* in (A1) is an angle-like variable mainly determining the direction of the damping due to parallel viscosity:

$$\theta_a^* \equiv (I + \langle G_\phi \rangle_a)\theta + (J - \iota \langle G_\phi \rangle_a)\phi, \tag{A3}$$

with the poloidal (toroidal) angle $\theta(\phi)$ in the Boozer coordinate system, the poloidal (toroidal) current outside (inside) of the flux surface $2\pi J(2\pi I)$, the rotational transform ι, and the geometric factor $\langle G_\phi \rangle_a$. The geometric factor $\langle G_\phi \rangle_a$ depends on the collisionality of the species a. The geometric factor in the plateau regime, $\langle G_\phi \rangle^{\mathrm{PL}}$, is given by

$$\langle G_\phi \rangle^{\mathrm{PL}} \equiv \frac{\left\langle \left(\frac{\mathbf{B}_\phi \cdot \nabla B}{B} \right) \right\rangle \sum_{mn} \frac{1}{|m\iota + Mn|} \left[\frac{1}{2B} \left(\frac{\partial}{\partial \theta} + \iota \frac{\partial}{\partial \phi} \right) (B^2 \langle g_2 \rangle - \langle B^2 \rangle g_2) \right]_{mn} \exp^{i(m\theta + Mn\phi)}}{\left\langle \left(\frac{\mathbf{B}_\phi \cdot \nabla B}{B} \right) \right\rangle \sum_{mn} \frac{1}{|m\iota + Mn|} \left[\left(\frac{\partial}{\partial \theta} + \iota \frac{\partial}{\partial \phi} \right) \right]_{mn} \exp^{i(m\theta + Mn\phi)}}, \tag{A4}$$

where m and Mn are the poloidal and toroidal period number of the magnetic field spectrum and g_2 is the solution of the following differential equation:

$$\mathbf{B} \cdot \nabla \left(\frac{g_2}{B^2} \right) = \mathbf{B} \times \nabla \psi \cdot \nabla \left(\frac{1}{B^2} \right), \tag{A5}$$

with the condition $g_2(B = B_{\max}) = 0$.

In CHS plasmas, there is no net toroidal current $(I = 0)$ and only poloidal current $(J \approx B_\phi R)$. The first term of θ_a^* can be neglected in the plasma core $(r < 0.5)$, because the toroidal velocity is dominant in these plasmas, with the tangential neutral beam injected in the toroidal direction. For instance, the normalized geometric factor for ions, $\langle G_\phi \rangle_i / (J/\iota) = 0.076$ and 0.137, $\iota(v_\phi/R)/(v_\theta/r) = 0.3$ and 0.2 for $\rho = 0.3$ and 0.5, respectively, in the plasma with $R_{\mathrm{ax}} = 94.9$ cm. Therefore the J_ϕ term is dominant in (A3) for most of the plasma region in the CHS. This is because the toroidal torque and therefore the toroidal damping force is larger than poloidal damping force in this experiment, where the torque from the neutral beam is almost parallel to the toroidal direction. By neglecting the other terms, $\langle G_\phi \rangle_i \theta$ and $\iota \langle G_\phi \rangle_i \phi$ in (A3), the toroidal viscosity becomes a function of the toroidal velocity alone and there is no coupling of the poloidal velocity, $\langle \mathbf{V}_a \nabla \theta_a^* \rangle \approx -J V_\phi / R$.

Thus the toroidal viscosity of ions in the plateau regime driven from (A1) is simplified as

$$\langle \mathbf{B}_\phi \cdot \nabla \cdot \Pi_i \rangle \approx \left(\frac{2 n_i m_i \sqrt{2eT_i/m_i} V_\phi J}{\lambda_{\mathrm{PL}\phi} R} \right), \tag{A6}$$

$$\frac{1}{\lambda_{\mathrm{PL}\phi}} \equiv \left(\frac{\sqrt{\pi}(J + \tau I)}{2\langle B^2 \rangle} \right) \times \left\langle \left(\frac{\mathbf{B}_\phi \cdot \nabla \mathbf{B}}{B} \right) \times \sum_{mn} \frac{1}{|m\iota + Mn|} \left(\frac{1}{B} \hat{\mathbf{n}} \cdot \nabla \mathbf{B} \right)_{mn} \exp^{i(m\theta + Mn\phi)} \right\rangle, \tag{A7}$$

where, n_i, m_i, and T_i are ion density, mass, and temperature, respectively. The toroidal viscosity is given by the toroidal flow velocity.

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