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Stability of Heterogeneous Flows to Nonaxisymmetric Disturbances

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With 2 Figures

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1. Introduction

The stability of heterogeneous fluids has been extensively investigated by Taylor (1931), Synge (1933), Miles (1961), Howard and Gupta (1962) and others. These results have also been extended to include the stability of cylindrical masses of fluid but mostly for axisymmetric disturbances. When the fluid is homogeneous and incompressible and is having a 'solid body' rotation the stability for nonaxisymmetric disturbances has been investigated by Howard and Gupta (1962), Ludwieg (1961) and Pedley (1968). In the case of a homogeneous fluid, in addition to the solid body rotation, the presence of even a small axial shear makes the system unstable as has been shown by Ludwieg (1961) for a narrow gap and by Pedley (1968) without this restriction. In this paper we consider the stability for non-axisymmetrical disturbances of a cylindrical mass of heterogeneous fluid, with an exponential variation of density in the radial direction and having both axial and azimuthal velocities. Assuming the dependence of the radial perturbation velocity on r to be of the form $u = \gamma^{1-m} H(r)$, we discuss two cases $m = {}^1/_2$ and 1. The general stability criteria, for both cases, have been derived. In the second case ($m = 1$), as an illustration the general stability criterion is applied to the Poiseuille type flow and a bound for instability is obtained. The growth rate of the most rapidly growing disturbances is also determined.

2. Formulation of the Problem

We have for the basic velocity $[0, V(r), W(r)]$ in r, θ, z directions, the fluid being contained between two infinitely long coaxial concentric cylinders of radii r_1 and r_0, $(r_1 \leqq r \leqq r_0)$. The usual process of perturbation and linearization of the momentum, continuity and incompressibility equations leads to the following equations:

$$\varrho_0 \left[\frac{\partial u'}{\partial t} + \frac{V}{r}\frac{\partial u'}{\partial \theta} + W\frac{\partial u'}{\partial z} - 2\frac{V v'}{r}\right] = -\frac{\partial p'}{\partial r} - \varrho' g, \tag{1}$$

$$\varrho_0 \left[\frac{\partial v'}{\partial t} + u'\frac{dV}{dr} + \frac{V}{r}\frac{\partial v'}{\partial \theta} + W\frac{\partial v'}{\partial z} + \frac{u' V}{r}\right] = -\frac{1}{r}\frac{\partial p'}{\partial \theta}, \tag{2}$$

$$\varrho_0 \left[\frac{\partial w'}{\partial t} + u' \frac{dW}{dr} + \frac{V}{r} \frac{\partial w'}{\partial \theta} + W \frac{\partial w'}{\partial z} \right] = - \frac{\partial p'}{\partial z}, \tag{3}$$

$$\frac{\partial u'}{\partial r} + \frac{u'}{r} + \frac{1}{r} \frac{\partial v'}{\partial \theta} + \frac{\partial w'}{\partial z} = 0, \tag{4}$$

$$\frac{\partial \varrho'}{\partial t} + u' \frac{d\varrho_0}{dr} + \frac{V}{r} \frac{\partial \varrho'}{\partial \theta} + W \frac{\partial \varrho'}{\partial z} = 0, \tag{5}$$

where ϱ_0 is the density distribution of the basic flow, u', v', w' are the perturbed velocities, p' the perturbed pressure, ϱ' the perturbed density. In the above equations it must be noted that the perturbed density ϱ' is assumed to be negligibly small, except when it is multiplied with the acceleration due to gravity g, which is the usual Boussinesq approximation. Further we assume the dependence of the perturbed quantities on the dependent variables r, θ, z and t in the form:

$$x'(r, \theta, z, t) = x(r) e^{i(\sigma t + n\theta + kz)}, \tag{6}$$

where $x(r)$ is the corresponding function of r only and σ may be complex. Substituting the form (6) in equations (1) to (5) and eliminating all variables except u we obtain:

$$\gamma^2 D[SD^* u] - \left\{ \gamma^2 + \gamma r D \left[S \left(\frac{D\gamma}{r} + \frac{2nV}{r^3} \right) \right] - \frac{2kVS}{r^2} [krD_* V - nDW] - \beta g(r) \right\} u = 0 \tag{7}$$

with the boundary conditions

$$u(r_1) = u(r_0) = 0, \tag{8}$$

where

$$\gamma = \sigma + \frac{nV(r)}{r} + kW(r), \qquad S = \frac{r^2}{n^2 + k^2 r^2}, \qquad \beta = - \frac{D\varrho_0}{\varrho_0},$$

$$D_* = \frac{d}{dr} + \frac{1}{r}, \qquad D = \frac{d}{dr}.$$

The motion is unstable if equations (7) and (8) possess a non-trivial solution and the imaginary part of σ (and hence γ) is negative for every pair of wave-numbers (n, k).

Defining

$$u = \gamma^{1-m} H(r) \tag{9}$$

equation (7) can be written, after some simplification, as

$$D[S\gamma^{2(1-m)} D_* H] - \gamma^{2(1-m)} \left[1 + \frac{2nr}{\gamma} D \left(\frac{SV}{r^3} \right) + \frac{mr}{\gamma} D \left(\frac{SD\gamma}{r} \right) + \frac{S}{\gamma^2} \left\{ m(1-m)(D\gamma)^2 - \frac{2kV}{r^2} (krD_* V - nDW) \right\} + \frac{gD\varrho_0}{\gamma^2 \varrho_0} \right] H = 0 \tag{10}$$

with

$$H(r_1) = H(r_0) = 0. \tag{11}$$

3. Stability Analysis

To discuss stability, we shall now consider two types of flow depending on the values of m.

Type 1: $m = {}^1/_2$: In the case of homogeneous fluids this situation was considered by HOWARD and GUPTA (1962) and they showed that a necessary condition for instability is:

$$k^2 \Phi - 2kn \frac{V}{r^2} DW - \frac{1}{4} (D\gamma)^2 < 0$$

for some r in the range (r_1, r_0). However, this problem is extended in this section to include heterogeneous fluids with density stratification $\beta = -\frac{1}{\varrho_0} D\varrho_0 > 0$.

When $m = {}^1/_2$, equation (10) becomes

$$D[S\gamma D_* H] - \left[\gamma + 2nrD\left(\frac{SV}{r^3}\right) + \frac{r}{2} D\left(\frac{SD\gamma}{r}\right) + \frac{S}{\gamma}\left\{\left(\frac{D\gamma}{2}\right)^2 - \frac{2kV}{r^2}(krD_* V - nDW)\right\} + \frac{gD\varrho_0}{\gamma\varrho_0}\right] H = 0. \tag{12}$$

Multiplying equation (12) by $r\bar{H}$ where $\bar{H}$ is the complex conjugate of H and integrating term by term with respect to r between the limits r_1 and r_0, and setting the imaginary part of the resulting equation equal to zero, we obtain:

$$\sigma_i \left\{\int_{r_1}^{r_0} (Sr\,|D_* H|^2 + r|H|^2)\,dr + \int_{r_1}^{r_0} S\left[-\left(\frac{D\gamma}{2}\right)^2 + k^2\Phi - 2\frac{knV}{r^2} DW - \frac{gD\varrho_0}{S\varrho_0}\right] r\left|\frac{H}{\gamma}\right|^2 dr\right\} = 0 \tag{13}$$

where

$$\Phi = \frac{1}{r^3} D(V^2 r^2). \tag{14}$$

$\sigma_i = 0$ implies neutral stability since the amplitude of perturbations would then not change with time. However, $\sigma_i < 0$ is a necessary condition for instability for the perturbations would then grow with time. Hence, a necessary condition for instability is that in the second integral of equation (13) the integrand should be negative. In other words, a sufficient condition for the flow to be stable is that the integrand of the second integral should be greater than, or equal to zero. That is,

$$-\left(\frac{D\gamma}{2}\right)^2 + k^2\Phi - \frac{2knV}{r^2} DW - \frac{gD\varrho_0}{S\varrho_0} \geqq 0 \tag{15}$$

since $S \geqq 0$.

Case 1:

If the flow is homogeneous ($D\varrho_0 = 0$) and the disturbances are axisymmetric ($n = 0$) the condition for stability, from condition (15), is:

$$\Phi \geqq \left(\frac{DW}{2}\right)^2 \tag{16}$$

which, in the absence of axial shear, ($DW = 0$), yields the Rayleigh's criterion for stability that the square of the angular velocity should nowhere decrease as we go along a radius.

Case 2:

If the flow is heterogeneous ($D\varrho_0 \neq 0$) and the disturbances are axisymmetric ($n = 0$) the condition for stability, from condition (15), is:

$$\frac{\Phi}{(DW)^2} + J(r) \geqq \frac{1}{4} \tag{17}$$

where $J(r) \equiv -\dfrac{g D\varrho_0}{\varrho_0 (DW)^2}$ is the Richardson number.

Case 3:

If the flow is heterogeneous ($D\varrho_0 \neq 0$), the disturbances are non-axisymmetric ($n \neq 0$) and the azimuthal velocity $V = 0$, then, a sufficient condition for stability is:

$$J(r) \geqq \frac{Sk^2}{4}$$

or

$$J\left(\frac{r}{r_0}\right) \geqq \frac{\dfrac{r^2}{r_0^2}}{4\left(\dfrac{r^2}{r_0^2} + \alpha^2\right)} \tag{18}$$

where $\alpha = \dfrac{n}{k r_0}$ is the ratio of the azimuthal to axial wave numbers. The relation (18) is an interesting one in that it gives the effect of non-axisymmetric disturbances on the stability. When the disturbances are axisymmetric ($\alpha = 0$) the sufficient condition for stability is reduced to the well known result:

$$J\left(\frac{r}{r_0}\right) \geqq \frac{1}{4}. \tag{19}$$

The variations of $J\left(\dfrac{r}{r_0}\right)$ with $\dfrac{r}{r_0}$ for different α's are plotted in figure (1) in which each curve represents a sort of stability curve for a particular α. The region above and including the points on the curve represents the region of stability and the region below the curve represents a possible unstable domain.

Case 4:

For a rigid body rotation $V = \Omega r$ a sufficient condition for stability, from condition (15), is:

$$4k^2\Omega^2 - \frac{2kn\Omega}{r} DW - \frac{g}{s}\frac{D\varrho_0}{\varrho_0} - k^2\left(\frac{DW}{2}\right)^2 \geqq 0. \tag{20}$$

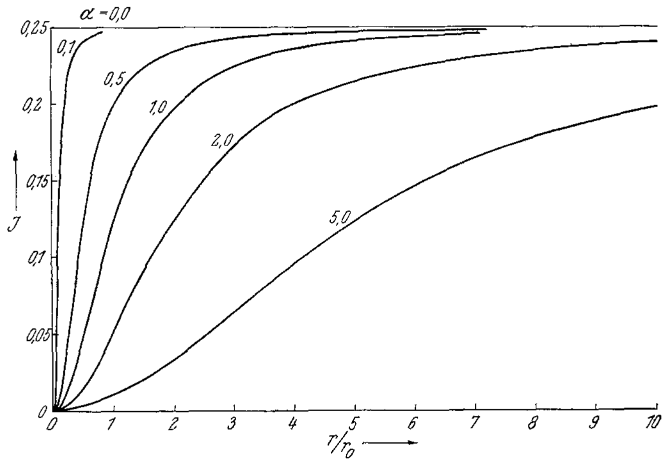

Fig. 1. Stability curves: J versus r/r_0 for various $\alpha = n/kr_0$

In the case of axisymmetric disturbances this becomes

$$4\Omega^2 - g\frac{1}{\varrho_0}D\varrho_0 - \frac{1}{4}(DW)^2 \geqq 0.$$

Thus a sufficient condition for stability is:

$$\frac{(DW)^2}{4} + \frac{gD\varrho_0}{\varrho_0} \leqq 4\Omega^2. \tag{21}$$

When there is no stratification the flow is stable if

$$|DW| \leqq 4\Omega \tag{22}$$

everywhere in the fluid.

Case 5:

If instead of rigid body rotation, we consider a potential vortex flow where $V = \frac{\Gamma}{r}$, we have $\Phi = 0$ and the motion is stable if

$$-\left(\frac{kDW}{2} - \frac{n\Gamma}{r^3}\right)^2 - \frac{2kn\Gamma}{r^3}DW - \frac{g}{S}\frac{1}{\varrho_0}D\varrho_0 \geqq 0. \tag{23}$$

In the case of axisymmetric disturbances this becomes

$$-\left(\frac{DW}{2}\right)^2 - \frac{g}{\varrho_0}D\varrho_0 \geqq 0.$$

Thus a sufficient condition for stability is:

$$-\frac{g}{\varrho_0}D\varrho_0 \geqq \left(\frac{DW}{2}\right)^2 \tag{24}$$

which leads to a condition identical to (19).

Type 2: $m = 1$: In the case of a homogeneous fluid this situation was considered by PEDLEY (1968) who proved that when $\varepsilon \ll 1$, where ε is defined below, the flow is unstable with respect to non-axisymmetric disturbances.

In this section, following the analysis of PEDLEY (1968), we consider the stability of heterogeneous fluid for non axisymmetric disturbances. This problem is a natural extension of Pedley's problem of homogeneous fluid to heterogeneous fluid with density stratification $\beta = -\frac{1}{\varrho_0} D\varrho_0 > 0$.

We now consider equation (7) which is equation (10) with $m = 1$ according to the transformation (9). The following transformations are used in this equation:

$$\varkappa = \frac{r}{r_0}, \quad V(r) = \Omega r_0 \varkappa \Phi(\varkappa), \quad W(\varkappa) = W_0 \Psi(\varkappa),$$
$$y = \frac{ru}{r_0 W_0}, \quad \alpha = k r_0 > 0, \quad \varepsilon = \frac{W_0}{2\Omega r_0}, \quad \omega = \frac{\gamma}{2\Omega} = \frac{\sigma}{2\Omega} + \frac{n}{2}\Phi(\varkappa) + \varepsilon\alpha\Psi(\varkappa) \tag{25}$$

where $\Phi(\varkappa)$ and $\Psi(\varkappa)$ are of order one and they are such that $\Phi(1) = 1$ and $\Psi(1) = 0$. For a solid body rotation $\Phi(\varkappa) = 1$.

Equation (7), using equation (25) and after rearrangement and simplification becomes

$$\frac{d}{d\varkappa}\left(\frac{\varkappa y'}{n^2 + \alpha^2\varkappa^2}\right) + \left\{-\frac{1}{\varkappa} - \frac{1}{\omega}\frac{d}{d\varkappa}\left(\frac{\varkappa W' + n\varphi}{n^2 + \alpha^2\varkappa^2}\right)\right.$$
$$\left. + \frac{\alpha\varphi}{\omega^2(n^2 + \alpha^2\varkappa^2)}\left(\frac{\alpha}{2}\frac{d}{d\varkappa}\varkappa^2\varphi - \varepsilon n\Psi'\right) + \frac{G\varepsilon^2\beta}{\omega^2\varkappa}\right\} y = 0 \tag{26}$$

where

$$G\varepsilon^2 = \frac{g r_0}{4\Omega^2 r_0^2}$$

and the primes denote the differentiation with respect to $\varkappa$. The boundary conditions are $y(\varkappa_1) = y(1) = 0$ where $\varkappa_1 = \frac{r_1}{r_0}$. Equation (26) as it is, is intractable, a situation which calls for some simplifying assumptions. Hence we consider the destabilising influence of a small axial shear superimposed on the rotation so that ε defined in (25) is small and we therefore restrict our attention to small values of ε. We set $\alpha = \delta\varepsilon n$ where $\delta = O(1)$ and $\delta n > 0$ since $\alpha > 0$. Further, we assume that

$$\Phi(\varkappa) = 1 + \varepsilon^2 h(\varkappa) \tag{27}$$

where h is of order one and $h(1) = 0$. ω defined in (25) is now a constant within order ε^2

$$\omega = \frac{\sigma}{2\Omega} + \frac{n}{2} + n\varepsilon^2\left(\frac{h}{2} + \delta\Psi\right)$$
$$= \omega_0^2 + O(\varepsilon^2)$$

say, so that, if $O(\varepsilon^2)$ is neglected with respect to unity, equation (26) reduces to:

$$\frac{1}{n^2}\frac{d}{d\varkappa}(\varkappa y') + \left\{-\frac{1}{\varkappa} - \frac{\varepsilon^2}{n\omega_0}\left(\frac{3h'}{2} + \frac{\varkappa h''}{2} + \delta\Psi' + \delta\varkappa\Psi'' - 2\delta^2\varkappa\right) + \frac{\varepsilon^2\delta\varkappa}{\omega_0^{\,2}}\left[\left(\delta - \frac{\Psi'}{\varkappa}\right) + \frac{G\beta}{\delta\varkappa^2}\right]\right\} y = 0 \tag{28}$$

where $G = \frac{g r_0}{(W_0)^2}$ is the reciprocal of Froude number.

Assuming that $|\omega_0^{\,2}| = O(\varepsilon^2)$ (PEDLEY 1968) the second term in the curly bracket becomes small compared with the remaining ones and we are then left with a Sturm-Liouville problem. Multiplying equation (28) by $\bar{y}$, the complex conjugate of y, integrating over $(\varkappa_1, 1)$ and using the boundary conditions $y(\varkappa_1) = y(1) = 0$, we get

$$\int_{\varkappa_1}^{1}\left[\varkappa\,|y'|^2 + \frac{n^2}{\varkappa}\,|y|^2\right] d\varkappa = \frac{n\varepsilon^2}{\omega_0^{\,2}}\int_{\varkappa_1}^{1}\left[\delta\left(\delta - \frac{\Psi'}{\varkappa}\right) + \frac{G\beta}{\varkappa^2}\right]\varkappa\,|y|^2. \tag{29}$$

From this it follows that the characteristic values for $\omega_0^{\,2}$ are all real and some of them are negative, implying instability, if

$$\left(\delta - \frac{\Psi'}{\varkappa}\right) + \frac{G\beta}{\varkappa^2} < 0 \tag{30}$$

which is a necessary and sufficient condition for the flow to be unstable for non-axisymmetric disturbances. In the absence of axial shear, $(\Psi' = 0)$, the integrand of the integral on the right hand side of (29) is always positive and hence $\omega_0^{\,2}$ are real and positive. There fore in the absence of axial shear the motion is always stable even in heterogeneous fluid. If $\Psi' \neq 0$ the flow will be unstable if

$$\Psi' > \delta\varkappa + \frac{G\beta}{\delta\varkappa} \tag{31}$$

for positive δ.

4. Poiseuille Flow

The general theory discussed above is applied to Poiseuille flow in a rotating pipe. Here

$$\Psi(\varkappa) = 1 - \varkappa^2, \qquad \beta = \beta_0\varkappa^2. \tag{32}$$

This flow, from the condition (30), using (32), is unstable if

$$\delta^2 + 2\delta + G\beta_0 < 0. \tag{33}$$

Since $G\beta_0$ is always positive, it follows that condition (33) will be satisfied only when δ is negative. In other words, for instability, in the case of Poiseuille flow, δ has to be negative and hence n is negative. The condition for stability from equation (33), now becomes

$$\frac{G\beta_0 + \delta^2}{\delta} > -2. \tag{34}$$

Since the maximum value of the magnitude of the left hand side of inequality (34) is zero, it follows that the bound for instability is

$$0 > \frac{G\beta_0 + \delta^2}{\delta} > -2 \tag{35}$$

for negative δ. This gives a bound both on $G\beta_0$ and δ. The stability curve is drawn for $G\beta_0$ against δ in figure 2. In the region within the curve shown the flow is unstable and on and outside the curve it is stable.

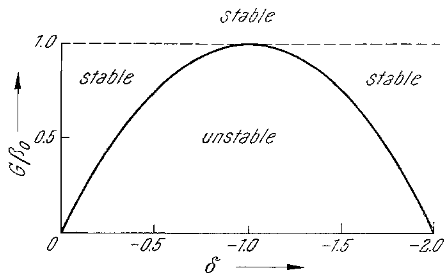

Fig. 2. Stability curve: $G\beta_0$ versus δ

We may calculate the growth rate of unstable disturbances by actually solving equation (28). Equation (28) using (32) becomes

$$\varkappa^2 y'' + \varkappa y' + [\lambda^2 \varkappa^2 - n^2] y = 0 \tag{36}$$

where

$$\lambda^2 = \frac{\varepsilon^2 n^2}{\omega_0^2} [\delta(\delta + 2) + G\beta_0].$$

The solution of equation (36), satisfying the condition $y(\varkappa) = O(\varkappa)$ as $\varkappa \to 0$ is

$$y = T_m(\lambda \varkappa) \tag{37}$$

where $m = -n$, and T_m is the Bessel function of the first kind with order m.

The boundary condition $y(1) = 0$ shows that the eigenvalues for λ are given by the equation

$$T_m(\lambda) = 0. \tag{38}$$

For given values of m and β_0 in the unstable range the largest value of $(-\omega_0^2)$ (i.e., the largest growth value) is given by the smallest value of λ satisfying equation (38), that is, by the first zero $j_{m,1}$ of the Bessel function. For all values of m this first zero is greater than m, and for large values of m it is given asymptotically by (Watson 1944)

$$j_{m,1} \approx m + 1.86 m^{1/3} + O(m^{-1/3}). \tag{39}$$

Thus, the corresponding value of ω_0^2 is given by

$$\begin{aligned} \omega_0^2 &= \frac{\varepsilon^2 m^2}{j_{m,1}^2} [\delta(\delta + 2) + G\beta_0] \\ &\approx \varepsilon^2 [\delta(\delta + 2) + G\beta_0] [1 + O(m^{-1/3})]. \end{aligned} \tag{40}$$

The maximum value of $-[\delta(\delta + 2) + G\beta_0]$ occurs for $\delta = -1$, so the most rapidly growing disturbance is given by $\delta = -1$ and $m \to \infty$. Thus

$$\omega_0^2 = \varepsilon^2 (G\beta_0 - 1). \tag{41}$$

Since

$$\omega_0^2 = \left(\frac{\sigma}{2\Omega} + \frac{n}{2}\right)^2 = \frac{(\sigma_r + n\Omega)^2}{4\Omega^2} - \frac{\sigma_i^2}{4\Omega^2} + \frac{2i\sigma_i(\sigma_r + n\Omega)}{4r^2}$$

it follows that, for unstable motion, $\sigma_r = -n\Omega$ and

$$\sigma_i^2 = 4\varepsilon^2\Omega^2(1 - GB_0)$$

or

$$\sigma_i = \pm 2\varepsilon\Omega\sqrt{1 - G\beta_0} \tag{42}$$

where $G\beta_0 < 1$; otherwise σ_i becomes imaginary. Thus

$$\sigma = \sigma_r + i\sigma_i = -n\Omega \pm 2i\varepsilon\Omega\sqrt{1 - G\beta_0}. \tag{43}$$

Thus the growth rate of the most rapidly growing disturbance is $2\varepsilon\Omega\sqrt{1 - G\beta_0}$ which in the case of zero stratification ($\beta_0 = 0$) reduces to $2\varepsilon\Omega$.

5. Conclusions

We have discussed the stability, for non-axisymmetric disturbances, of the flow in the annulus of two coaxial concentric cylinders taking the density stratification $-\frac{1}{\varrho_0}\frac{d\varrho_0}{dr} = \beta$ to be positive. Both the axial and azimuthal velocities are assumed to be present. Depending on the value of m in equation (9) two types of flow are considered. When $m = 1/2$, we found that the motion is always stable if

$$-\left(\frac{D\gamma}{2}\right)^2 + k^2\Phi - \frac{2knV}{r^2} DW + \frac{\beta g}{S} \geqq 0.$$

This reduces to the well known results when the disturbances are axisymmetric. In the case $m = 1$, when the axial velocity is small compared to the azimuthal velocity, a necessary and sufficient condition for instability is

$$\delta\left(\delta - \Psi' \frac{1}{\varkappa}\right) + \frac{G\beta}{\varkappa^2} < 0.$$

As an illustration this analysis is applied to a Poiseuille type of flow and bounds for instability are given by

$$O > \frac{G\beta_0 + \delta^2}{\delta} > -2.$$

Further, the growth rate of the most rapidly growing disturbance is found to be $2\varepsilon\Omega\sqrt{1 - G\beta_0}$. The analysis shows that for zero density stratification the flow is always unstable whereas the flow can be made stable for $G\beta_0 \geqq 1$ where G is the reciprocal of Froude number and β_0 a measure of density stratification.

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