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Effect of radiative transfer on the onset of convection in a porous medium

D. VORTMEYER
Institute B for Thermodynamics, Technical University of Munich, Arcisstr. 21, 8000 Munich 2, Federal Republic of Germany

and

N. RUDRAIAH and T. P. SASIKUMAR
UGC-DSA Centre in Fluid Mechanics, Department of Mathematics, Central College, Bangalore University, Bangalore 560001, India

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Abstract—The effect of radiative transfer on thermal convection of a thin fluid-saturated densely packed porous layer bounded by stress-free radiating horizontal planes heated from below is studied using linear theory. The coefficients of absorption, emission and scattering are computed from packed bed properties using a two-flux model. The Milne–Eddington approximation is employed to determine approximate solutions valid for optically thin (transparent) and optically thick (opaque) gray media which absorb and emit thermal radiation. The effect of radiation parameters on the cell size and on the onset of convection are studied in detail using the Galerkin approximation. It is shown that the effect of radiation is to inhibit the onset of convection in a porous medium. The physical explanation for this is given, taking into account the increase in thermal conductivity due to the combined effects of the porosity of the medium and radiation.

1. INTRODUCTION

THE STUDY of heat and mass transfer by convection through fluid saturated porous media is an area of rapid growth in contemporary heat and mass transfer research because of its importance in many branches of science and engineering [1–5]. Heat transfer by conduction and convection in a porous medium is based on a set of partial differential equations, and has been extensively studied for more than four decades [1–5], with applications, for example, to geothermal reservoirs [1] and thermal insulation engineering [5]. But heat transfer by combined conduction, convection and radiation in an absorbing, emitting and scattering porous medium has not been given much attention, in spite of its applications in the design of furnaces and cooling towers, the processing of glass and other semi-transparent materials and the storage of solar energy.

Determination of radiation flux in a fluid-saturated porous medium involving absorption, emission and scattering requires a solution of coupled momentum and energy equations expressing the intensity of radiation at each location of the medium. Mathematically, this problem is quite formidable because it requires the solution of non-linear integrodifferential equations. Because of the complexity of the problem, many authors adopted an approximate form of solutions.

In ordinary viscous flow, Schuster [6] introduced the two-flux model which reduces the non-linear coupled integral equation into a differential equation for which a closed-form solution has been found [7]. In the case of a fluid-saturated porous medium, the literature on this is very sparse. References [8, 9] studied the one-dimensional heat transfer by conduction and radiation. Leung and Edwards [10] later studied the effect of combined conduction, convection and radiation on heat transfer in a one-dimensional semi-infinite isotropic homogeneous absorbing, emitting and scattering porous medium when the flow is steady, using an analytical technique. Sharma and Singh [11] have studied the effect of radiation on the stability of convective flow through porous media and obtained a sufficient condition for stability in shear flows. To our knowledge, the condition for the onset of convection in a radiating fluid saturated porous medium has not been given any attention, and the study of it is the main object of this paper. Assuming the boundary and the media to be gray, we obtain analytical solutions for flow through porous media using a two-flux model for radiative flux. We concentrate mainly on the thermal instability of a fluid-saturated horizontal porous layer, bounded by free radiating surfaces, with the assumption of very low and very high absorption coefficients.

2. FORMULATION OF THE PROBLEM

We consider a densely packed radiating fluid-saturated horizontal porous layer confined between two parallel infinite stress-free impermeable iso-

NOMENCLATURE

a	dimensionless wave number, $(l^2+m^2)^{1/2}$	R	Lapwood–Rayleigh number, $\alpha g\bar{\beta}kh^2/K\nu$
a_{ct}	critical wave number in transparent approximation	R_{ct}	critical R in the transparent approximation
a_{co}	critical wave number in opaque approximation	R_{co}	critical R in the opaque approximation
A^2	$3K_a^2h^2(1+X)$	s	heat content per unit volume
d	porous particle diameter	T	temperature
D	derivative, d/dz	x, y, z	horizontal and vertical space coordinates
f	non-dimensional temperature gradient	X	non-dimensional radiative parameter, $4\pi Q/3K_aKs$.
F_z	z-component of the radiative flux		
g	gravitational acceleration		
h	vertical length scale	Greek symbols	
H	rate of radiative heating per unit volume	α	coefficient of expansion
k	permeability	β	basic temperature gradient, dT_b/dz
K	effective thermal diffusivity	$\bar{\beta}$	mean value of β throughout the medium
K_a	defined by equation (8)	ε	emissivity
K_c	effective thermal conductivity	ν	kinematic viscosity
K_b	defined by equation (9)	ρ	density
K_g	defined by equation (10)	σ	Stefan's constant
l, m	horizontal wave numbers in the x- and y-directions	ϕ	porosity.
N	transmittance number	Operators	
p	pressure	$\langle\cdots\rangle$	$\int(\cdots)\,dz$
$\mathbf{q}$	Darcy velocity vector, (U, V, W)	∇^2	$\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$
Q	assumed as constant, $(4\sigma/\pi)T^3$	∇_1^2	$\partial^2/\partial x^2+\partial^2/\partial y^2$.

thermal surfaces of thickness h, heated from below and cooled from above. We use a Cartesian coordinate system (x, y, z) with the x–y plane on the lower plate and the z-axis vertically upwards. Neglecting the contribution of the radiative stress to the momentum equation, the Boussinesq formulation of the conservation of momentum, energy, mass and state equations are

$$\nabla\cdot\mathbf{q} = 0 \tag{1}$$

$$(1/\phi)(\partial\mathbf{q}/\partial t)+(1/\phi^2)(\mathbf{q}\cdot\nabla)\mathbf{q} = -(1/\rho_0)\nabla p + (\rho/\rho_0)\mathbf{g}-(\nu/k)\mathbf{q} \tag{2}$$

$$(\partial T/\partial t)+(\mathbf{q}\cdot\nabla)T = K\nabla^2T+H/s \tag{3}$$

$$\rho = \rho_0[1-\alpha(T-T_0)]. \tag{4}$$

The quantities are defined in the Nomenclature. Equation (2) is the modified Darcy equation, known as the Lapwood–Darcy equation [4]. Here the dependent variables are all volumetric-averaged quantities. We will try to find the effect of radiative flux on the departure of the following basic state.

3. BASIC STATE

The basic state in which the energy transfer is by conduction and radiation is quiescent. Then the total heat flux, I_T, in a packed bed, caused by a temperature difference, consists of two parts. One is the flux, $I_L(s)$, depending on the effective conductivity of the heat (i.e. conduction) and the other is the radiative flux, $I(s)$, where s is the heat content per volume. Since we assume the basic state to be steady and one-dimensional, the total heat flux I_T is a constant over the length of the bed. That is

$$I_T = I_L(s)+I(s) = \text{const.} \tag{5}$$

$I_L(s)$ may be computed, following Yagi *et al.* [12], by making use of the effective thermal conductivity K_c of the packed bed.

The computation of radiation flux, $I(s)$, for a gray emitting packed bed is based on the fact that the particles emit, absorb and reflect heat. In addition, radiative energy may penetrate the bed through the void volume.

In this paper, we consider only an idealized two-flux model consisting of identical spheres for which

$$I(s) = I^+ - I^- \tag{6}$$

where I^+ and I^- are the net forward and reverse fluxes, respectively.

If the bed is densely packed and the average diameter of the particles is small compared to the width of the bed, these two fluxes are related to each other through the set of non-linear ordinary differential equations

$$\mathrm{d}I^+/\mathrm{d}s = -K_\mathrm{a}I^+ + K_\mathrm{b}\varepsilon(\sigma/\pi)T^4 - K_\mathrm{g}I^-$$
$$\mathrm{d}I^-/\mathrm{d}s = K_\mathrm{a}I^- - K_\mathrm{b}\varepsilon(\sigma/\pi)T^4 + K_\mathrm{g}I^+ \quad (7)$$

where σ is Stefan's constant, and the total absorption coefficient K_a, emission coefficient K_b and scattering coefficient K_g are given by [9]

$$K_\mathrm{a} = 2[(1+N)^2+(1-N^2)(1-\varepsilon)^2](1-N)/[(1+N)^2-(1-N)^2(1-\varepsilon)^2](1+N)d \quad (8)$$

$$K_\mathrm{b} = 2[(1+N)^2-(1-N^2)(1-\varepsilon)]\varepsilon(1-N)/[(1+N)^2-(1-N)^2(1-\varepsilon)^2](1+N)d \quad (9)$$

$$K_\mathrm{g} = 2[(1+N)^2+(1-N^2)](1-\varepsilon)(1-N)/[(1+N)^2-(1-N)(1-\varepsilon)^2](1+N)d. \quad (10)$$

Coefficients K_a, K_b and K_g depend on the transmittance number N, which is a function of the emissivity ε and porosity ϕ. We see that $(K_\mathrm{a}-K_\mathrm{g})$ will be the true absorption coefficient. In this paper we consider only the black-body radiation, for which $\varepsilon = 1$. In this case the scattering coefficient K_g is zero; the absorption coefficient equals the emission coefficient, and is given by

$$K_\mathrm{a} = K_\mathrm{b} = 2(1-N)/(1+N)d. \quad (11)$$

The radiative heat transfer equation (7) then simplifies to

$$\mathrm{d}I^+/\mathrm{d}s = K_\mathrm{a}[(\sigma/\pi)T^4 - I^+]$$
$$\mathrm{d}I^-/\mathrm{d}s = -K_\mathrm{a}[(\sigma/\pi)T^4 - I^-]. \quad (12)$$

Combining these, using equation (6) and defining the black-body intensity as

$$B = (\sigma/\pi)T^4 \quad (13)$$

we get

$$\mathrm{d}I(s)/\mathrm{d}s = K_\mathrm{a}[B - I(s)]. \quad (14)$$

This equation of transfer is analogous to the one given by Kourganoff [13] in the case of pure viscous flow. Also, the radiative heating rate of the fluid-saturated porous medium is

$$H = -\int [\mathrm{d}I(s)/\mathrm{d}s]\,\mathrm{d}\omega \quad (15)$$

where ω is the element of the solid angle and the integral is taken over the solid angle of 4π.

In the basic state, all the quantities are functions of z only and the equation of transfer, equation (14), takes the form

$$\mu_3(\mathrm{d}I/\mathrm{d}s) = K_\mathrm{a}[B - I] \quad (16)$$

where K_a is given by equation (11) and μ_3 is the directional cosine of vector s in the z-direction.

The energy equation for the basic state is

$$0 = (H_\mathrm{b}/s) + K(\mathrm{d}^2T_\mathrm{b}/\mathrm{d}z^2). \quad (17)$$

If F_z is the z-component of this heat flux, then

$$H_\mathrm{b} = -(\mathrm{d}F_z/\mathrm{d}z) \quad (18)$$

and we may write equation (17) in the integrated form

$$F_z - Ks\beta = C \quad (19)$$

where C is the constant of integration.

Assuming the Milne–Eddington approximation, we can obtain the differential equation for F_z, using the radiative transfer equation, equation (16), in the form [7]

$$(\mathrm{d}^2F_z/\mathrm{d}z^{*2}) - A^2F_z = -A^2XC/(1+X). \quad (20)$$

Here $z^* = z/h - 1/2$; in the subsequent analysis we will write z for z^*. Solving equations (19) and (20), using boundary conditions

$$\mathrm{d}F_z/\mathrm{d}z = -2K_\mathrm{a}hF_z \quad \text{at } z = \tfrac{1}{2}$$
$$\mathrm{d}F_z/\mathrm{d}z = 2K_\mathrm{a}hF_z \quad \text{at } z = -\tfrac{1}{2} \quad (21)$$

we get

$$f = \beta/\bar{\beta} = L\cosh(Az) + M \quad (22)$$

where

$$L = X[[2X/A + \tfrac{1}{2}(3+3X)^{1/2}]\sinh(A/2) + \cosh(A/2)]^{-1}$$

$$M = L[\tfrac{1}{2}(3+3X)^{1/2}\sinh(A/2) + \cosh(A/2)]/X$$

and $\bar{\beta}$ is the mean value of β throughout the medium in which there is marginal stability.

4. LINEAR STABILITY ANALYSIS

In this section we study the linear stability problem subjected to infinitesimal disturbances. We determine the critical value of the Rayleigh number, at which convection sets in, in the limiting cases of transparent and opaque gray media when the horizontal boundaries are idealized as planar stress-free surfaces. Using the usual process of linearization, eliminating the pressure and assuming that the principle of exchange of stability is valid, we get the z-component of the momentum equation in the form

$$\nabla^2W' = (\alpha gk/\nu)\nabla_1^2T' \quad (23)$$

and the energy equation

$$W'\beta = K\nabla^2T' + H'/s. \quad (24)$$

Here primes denote perturbed quantities, which are assumed to be small compared to the basic state quantities. Eliminating T' between equations (23) and (24), using h as the length scale and assuming a solution of the form (some function of z) $\exp[\mathrm{i}(lx+my)]$ with

$$\mathrm{D} = (\mathrm{d}/\mathrm{d}z), \quad \nabla_1^2W = -a^2W,$$
$$\nabla^2\mathrm{W} = (\mathrm{D}^2 - a^2)W \quad (25)$$

we get

$$-a^2\beta W/h^2 = (K\nu/k\alpha gh^4)(\mathrm{D}^2-a^2)^2W + \nabla_1^2(H/s) \quad (26)$$

where, for simplicity, the primes are omitted. The boundary conditions are

$$W = T = 0 \quad \text{at } z = \pm\tfrac{1}{2}. \tag{27a}$$

Since the temperature is assumed to be constant over the boundaries, these conditions also require that

$$D^2 W = 0 \quad \text{at } z = \pm\tfrac{1}{2}. \tag{27b}$$

The analytical solution of equation (26) is very difficult because of $\nabla_1^2(H/s)$, which depends on the complicated integral over the layer. However, we note that analytical solutions of equation (26) are possible for two simple approximate forms for $\nabla_1^2(H/s)$, one valid when the fluid-saturated medium is optically thin (i.e. the transparent approximation), and the other when it is optically thick (i.e. the opaque approximation). For the transparent approximation, where $K_a^2 h^2 \ll a^2$, we obtain from the radiative transfer equation, equation (16), following Goody [7], the result

$$\nabla_1^2 H = -4\pi Q K_a \nabla_1^2 T \tag{28a}$$

and this, using equation (23) becomes

$$\nabla_1^2 H = -(4\pi Q v K_a/\alpha g k)\nabla^2 W. \tag{28b}$$

Similarly, for the optically thick approximation where $K_a^2 h^2 \gg a^2$, we obtain

$$\nabla_1^2 H = (4\pi Q/3K_a)\nabla^2(\nabla_1^2 T) \tag{29a}$$

which, using equation (23), becomes

$$\nabla_1^2 H = (4\pi Q v/3K_a k\alpha g)\nabla^4 W. \tag{29b}$$

Equations (28b) and (29b) differ from those describing ordinary viscous flow [7] in the order of the differential equation and in the nature of the relative absorption coefficient.

5. GALERKIN METHOD

In this section we obtain the critical Rayleigh number for transparent and opaque approximations.

5.1. *Transparent approximation*

In this case, the momentum and energy equations (23) and (26), respectively, using equations (25) and (28), become

$$(D^2 - a^2)W = -(\alpha g k/v)a^2 T \tag{30}$$

$$(h^2/K)\beta f W = (D^2 - a^2)T - 3XK_a^2 h^2 T. \tag{31}$$

It is interesting to note that in the process of deriving equations (30) and (31), no radiative boundary conditions have been used—hence they are equally valid if the boundaries are black bodies or mirrors. The nature of the boundaries affects only β, the basic temperature gradient.

To obtain the required equations in the Galerkin method we multiply equation (30) by W and equation (31) by T, integrate with respect to z from $-1/2$ to $1/2$ and obtain the equations

$$\langle (DW)^2 + a^2 W^2 \rangle = (\alpha g k a^2/v)\langle WT \rangle \tag{32}$$

and

$$(h^2\beta/K)\langle WTf \rangle = -\langle (DT)^2 + GT^2 \rangle \tag{33}$$

where $G = a^2 + 3XK_a^2 h^2$. We substitute $W = EW_1$ and $T = FT_1$ into equations (32) and (33) to obtain

$$E\langle (DW_1)^2 + a^2 W_1^2 \rangle = F(\alpha g k a^2/v)\langle W_1 T_1 \rangle \tag{34}$$

$$E(h^2\beta/K)\langle W_1 T_1 f \rangle = -F\langle (DT_1)^2 + GT_1^2 \rangle \tag{35}$$

where E and F are constants and W_1 and T_1 are the trial functions. Substituting F from equation (35) into equation (34), removing the common factor E and omitting, for simplicity, subscript one on the dependent variables, we get

$$R = [\langle (DW)^2 + a^2 W^2 \rangle \langle (DT)^2 + GT^2 \rangle] / [a^2 \langle WT \rangle \langle WTf \rangle]. \tag{36}$$

The critical Rayleigh number in this case is determined for isothermal free boundaries using equations (27a) and (27b). The trial functions satisfying these boundary conditions are

$$W = \sin[n\pi(z + \tfrac{1}{2})], \quad T = \sin[n\pi(z + \tfrac{1}{2})]. \tag{37}$$

Substituting equations (37) into equation (36), using equation (22) and performing the integration, we get

$$R = [(n\pi)^2 + a^2][(n\pi)^2 + a^2 + 3K_a^2 h^2 X] / a^2[M + 8n^2\pi^2 L \sinh(A/2)/A(A^2 + 4n^2\pi^2)]. \tag{38}$$

R attains its minimum value, R_{ct}, at the critical wave number

$$a_{ct} = [\pi^4 + A^2 X\pi^2/(1 + X)]^{1/4} \tag{39}$$

with $n = 1$. The critical wave number a_{ct} determines the width of the cell in the least stable mode of motion. For large values of X, $a_{ct} = (\pi^4 + a^2\pi^2)^{1/4}$, which is a function of A only. Therefore, the radiative parameter X affects the cell size only for moderate values of X.

The critical Rayleigh number from equation (38) and using equation (39) is

$$R_{ct} = \pi^2[1 + \sqrt{(1 + (A^2 X/(1 + X)\pi^2))}]^2 / [M + 8\pi^2 L \sinh(A/2)/A(A^2 + 4\pi^2)]. \tag{40}$$

For $X \to 0$ or $A \to 0$, which represents no radiation, equation (39) gives $a_{ct} = \pi$ and equation (40) gives $R_{ct} = 4\pi^2$; these expressions coincide with those given by Lapwood [14]. The results are discussed in the final section.

5.2. *Opaque approximation*

In this case, the momentum equation remains the same as equation (30), but the energy equation takes the form

$$\beta W f = (K/h^2)(1 + X)(D^2 - a^2)T. \tag{41}$$

Using the procedure explained in Section 5.1, we get

$$R = (1+X)\langle(\mathrm{D}W)^2+a^2W^2\rangle\langle(\mathrm{D}T)^2+a^2T^2\rangle/ a^2\langle WT\rangle\langle WTf\rangle. \quad (42)$$

We select the same trial function as that given by equations (37). Then equation (42) with $n = 1$ becomes

$$R = (1+X)(\pi^2+a^2)^2/ a^2[M+8\pi^2 L \sinh (A/2)/A(A^2+4\pi^2)]. \quad (43)$$

The critical wave number is

$$a_{co} = \pi \quad (44)$$

and the corresponding critical Rayleigh number is

$$R_{co} = 4\pi^2(1+X)/ [M+8\pi^2 L \sinh (A/2)/A(A^2+4\pi^2)]. \quad (45)$$

The results are discussed in the next section. Here we note that a_{co} has the same value as that given by Lapwood [14].

6. RESULTS AND DISCUSSION

The effect of radiation on the onset of convection in an absorbing and emitting gray porous medium is studied using linear theory. The Galerkin procedure is used to determine the critical Rayleigh number for different values of the radiative parameters A and X. Although A and X are related to each other through the relation $A^2 = 3K_a^2h^2(1+X)$, they are chosen in such a way that they determine the values of K_a valid for different approximations.

The critical wave number, a_{ct}, a function of A and X, is computed from equation (39) for different values of the radiative parameter A in the case of transparent approximation, and the results are depicted in Fig. 1. In this figure we have also drawn the results for $X \to 0$, which correspond to non-radiating systems, and we note that the values coincide with those given by Lapwood [14]. It is clear that the increase in A increases a_{ct}, and hence we conclude that the effect of radiation is to contract the cells. Equation (44) shows that radiation has no effect on the cell size in the case of the opaque approximation.

The critical Rayleigh number given by equations (40) and (45) for transparent and opaque approximations, respectively, are computed for different values of A and X, and the results are shown in Fig. 2. The full lines starting from the left-hand side are for the transparent approximation, computed using equation (40), and that starting from the right-hand side is for the opaque approximation computed using equation (45). The dashed lines are over the range where neither approximation is valid. We see that there is no radiative effect on the convective motion when A takes values less than unity; the curves coincide with $X = 0$ in Fig. 2. The critical Rayleigh number in this case is the same as the one given by Lapwood [14]. We also note that for smaller values of A the curves converge to a point and hence become independent of X. For $A > 1$, however, the critical Rayleigh number increases and hence stabilizes the system. The following are the reasons for this stabilization.

(1) Radiation contracts the cells, resulting in the increase in Rayleigh number to cause stabilization.

(2) We know that the effective thermal conductivity K_c in the porous medium is given by [15]

$$K_c+\phi[(K_s-K_f)/K_f^{1/3}]K_c^{1/3}-K_s = 0$$

where subscripts f and s denote the quantities for fluid and solid, respectively. This is higher than that in the ordinary viscous flow owing to the contribution from the solid particles. The increase in thermal conductivity also promotes stability.

(3) The Rayleigh number is defined in terms of the temperature difference between the plates, as in the case of the usual Rayleigh–Lapwood convection [14]. The radiation influences the Rayleigh number through the radiative heat flux $\langle WTf\rangle$ (see equations (36) and (42)). The effect of radiation is to make the heat flux $\langle WT\rangle$ minimum and hence to increase the critical Rayleigh number.

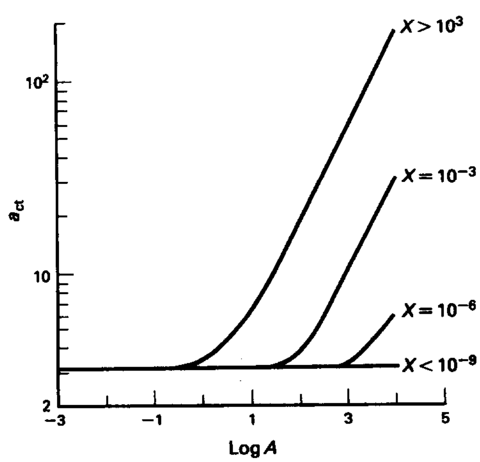

FIG. 1. Critical wave number as a function of A for transparent medium.

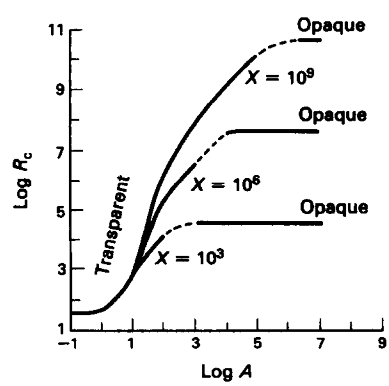

FIG. 2. Critical Lapwood–Rayleigh number as a function of A for different values of X.

The comparison of the results between transparent and opaque approximations reveals that although the former approximation affects the cell size, the critical Rayleigh number in the latter case is much higher than that of the former case—that is, increase in optical thickess delays the instability. Therefore, for material processing in the laboratory, the results of the opaque approximation are more suitable than those of the transparent approximation.

To assess the validity of the results obtained from the Galerkin approximation, the results obtained here for $f(z) = 1$ in the absence of radiation are compared with those of Lapwood [14], and good agreement is found. We conclude that the Galerkin expansion procedure used in this paper gives good results with minimum mathematics.

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EFFET DU TRANSFERT RADIATIF SUR L'APPARITION DE LA CONVECTION DANS UN MILIEU POREUX

Résumé—On étudie par la théorie linéaire l'effet du transfert radiatif sur la convection thermique dans une mince couche poreuse saturée de fluide, limitée par des plans horizontaux radiants, sans contrainte, et chauffée par le bas. Les coefficients d'absorption, d'émission et de diffusion sont calculés à partir des propriétés de lit fixe en utilisant un modèle à deux flux. L'approximation de Milne–Eddington est utilisée pour déterminer des solutions approchées valables pour des milieux gris optiquement minces (transparents) ou épais (opaques) qui absorbent et émettent un rayonnement thermique. L'effet des paramètres de rayonnement sur la dimension de la cellule et sur l'apparition de la convection est étudié en détail avec l'approximation de Galerkine. On montre que l'effet du rayonnement est d'inhiber la convection dans un milieu poreux. L'explication physique est donnée en tenant compte de l'accroissement de la conductivité thermique dû aux effets combinés de la porosité du milieu et du rayonnement.

EINFLUSS DER STRAHLUNG AUF DAS EINSETZEN DER KONVEKTION IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Der Einfluß der Wärmestrahlung auf die thermische Konvektion in einer dünnen, flüssigkeitsgesättigten, dicht gepackten porösen Schicht, die von zwei kräftefreien wärmeabstrahlenden horizontalen Platten begrenzt und von unten beheizt wird, wird mit der linearen Theorie untersucht. Die Absorptions-, Emissions- und Streuungskoeffizienten werden aus den Stoffeigenschaften der porösen Schüttung mit einem Zwei-Strom-Modell berechnet. Die Milne–Eddington-Näherung wird zur Bestimmung von Näherungslösungen verwendet, die für strahlungsdurchlässige (transparente) und für strahlungsundurchlässige (opake) graue Stoffe gültig sind. Der Einfluß der Strahlungsparameter auf die Größe der Konvektionszellen und auf das Einsetzen der Konvektion wird detailliert mit dem Galerkin-Näherungsverfahren untersucht. Es zeigt sich, daß Strahlung das Einsetzen der Konvektion in einem porösen Medium hemmt. Dies wird physikalisch dadurch erklärt, daß die Wärmeleitfähigkeit durch die überlagerten Einflüsse von Porosität und Strahlung zunimmt.

ВЛИЯНИЕ РАДИАЦИОННОГО ТЕПЛОПЕРЕНОСА НА ВОЗНИКНОВЕНИЕ КОНВЕКЦИИ В ПОРИСТОЙ СРЕДЕ

Аннотация—На основе линейной теории исследуется влияние радиационного теплопереноса на тепловую конвекцию в тонком насыщенном жидкостью плотном пористом слое, ограниченном ненапряженными излучающими горизонтальными плоскостями, нагреваемыми снизу. Коэффициенты поглощения, излучения и рассеяния рассчитываются на основе свойств плотного слоя с использованием двухпотоковой модели. Для определения приближенных решений, применимых в случае оптически тонких (прозрачных) или толстых (непрозрачных) серых сред, поглощающих и испускающмх тепловое излучение используется приближение Милна–Эддингтона. С помощью аппроксимации Галеркина детально изучается влияние параметров радиации на размеры ячеек и возникновение конвекции. Показано, что радиационный теплоперенос препятствует возникновению конвекции в пористой среде. Физически это объясняется фактом увеличения удельной теплопроводности пористой среды благодаря радиационному теплопереносу.