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Loss of Exchange Symmetry in Multiqubit States under Ising Chain Evolution *

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Keeping in view of importance of exchange symmetry aspects in studies on spin squeezing of multiqubit states, we show that the one-dimensional Ising Hamiltonian with nearest neighbor interactions does not retain the exchange symmetry of initially symmetric multiqubit states. Specifically we show that among 4-qubit states obeying exchange symmetry, all states except W class (and their linear combination) lose their symmetry under time evolution with Ising Hamiltonian. Attributing the loss of symmetry of the initially symmetric states to rotational asymmetry of the one-dimensional Ising Hamiltonian with more than 3 qubits, we indicate that all N-qubit states ($N \geq 5$) obeying permutation symmetry lose their symmetry after time evolution with Ising Hamiltonian.

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Multiqubit states that are symmetric under the interchange of particles form an important class among quantum states due to their experimental significance and mathematical elegance.[1−5] They are the quantum states obeying exchange symmetry and the N-qubit symmetric states belong to the $(N + 1)$-dimensional subspace of the 2^N-dimensional Hilbert space, the subspace being the maximal multiplicity space of the collective angular momentum. In general, multi-atom systems that are symmetric under permutation of the particles allow for an elegant description in terms of the collective variables of the system.

Atomic spin squeezed states[4,6−13] are quantum correlated states with reduced fluctuations in one of the collective spin components and they have possible applications in atomic interferometers and high precision atomic clocks. Spin squeezing, in the original sense, is defined for multiqubit states belonging to the symmetric subspace of the collective angular momentum space.[6] In fact Kitagawa and Ueda[6] have defined a parameter that quantifies the spin-squeezing in symmetric multiqubit states. If $J_i = \frac{1}{2}\sum_{\alpha=1}^{N}\sigma_{\alpha i}$, $i = x, y, z$ denotes the components of the collective angular momentum operator of an N qubit system, the spin squeezing parameter ξ is defined as[6]

$$\xi^2 = 2(\Delta \boldsymbol{J}_\perp)^2/J, \quad J = N/2, \tag{1}$$

where the subscript $\perp$ refers to an axis perpendicular to the mean spin direction $\boldsymbol{n}$ in which the minimal value of the variance $(\Delta J_\perp)^2$ is obtained. The system is said to be spin squeezed when the parameter ξ is less than 1. The relationship between spin squeezing and quantum entanglement in symmetric multiqubit systems has been an interesting area of study[14−16] and it has been shown that for a two-qubit symmetric state, spin squeezing is equivalent to its bipartite entanglement.[14] An extension of this result to symmetric multiqubit systems shows that the presence of spin squeezing essentially reflects pairwise entanglement.[16] Even though spin squeezing serves only as a sufficient condition for pairwise entanglement in arbitrary symmetric multiqubit systems, for a special class of symmetric multiqubit systems it was shown that spin squeezing is a necessary and sufficient condition for pairwise entanglement.[16]

With the observation that the detection of spin squeezing forms a useful diagnostic tool in the early stages of the construction of a quantum computer,[10] the spin squeezing produced in several models of interacting spins has been studied.[10,16] Ising type Hamiltonian with nearest neighbor interactions[10] is one of the interaction models considered in these studies. The one-dimensional Ising type Hamiltonian with N spins and a constant coupling between any two nearest neighbors[10] is given by

$$\mathcal{H} = \frac{\hbar\chi}{4}\sum_{\alpha=1}^{N}\sigma_{\alpha x}\sigma_{\alpha+1 x}, \tag{2}$$

where we identify the $(N + 1)$th spin with the first one in the chain. Here $\sigma_{\alpha x}$ and $\sigma_{\alpha+1x}$ are the Pauli spin matrices for the spin at sites α and $\alpha + 1$, respectively, and χ is a constant characterizing the coupling strength between any two nearest neighbors in the chain. It is not difficult to see that $J_x = \frac{1}{2}\sum_{\alpha=1}^{N}\sigma_{\alpha x}$, the x component of the collective angular momentum operator is a constant of motion as it commutes with the Hamiltonian $\mathcal{H}$ of the system. The Ising type Hamiltonian arises in proposals for quantum computation with atoms in optical lattices.[17,18] In these proposals, the atom interacts with nearest neighbors and it has been shown that this interaction produces spin squeezing.[10] Spin squeezed states are routinely produced in several laboratories as they are quite experimentalist-friendly and in addition to the practical applications of spin squeezing such as atomic clocks, they provide a demonstration of the entangling capabilities of the system. Thus studies on spin

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squeezed states and the interaction models that produce spin squeezing form a prominent area of study.

At this juncture, an illustration of the fact that spin-squeezing criterion given by Kitagawa and Ueda[6] is applicable only for symmetric multiqubit states may be in order. As any two uncorrelated non-squeezed systems cannot possess any spin-squeezing amongst themselves, one should not expect to obtain positive results for spin squeezing in such states. However, one can obtain such a result when the Kitagawa–Ueda spin-squeezing criterion is applied to some separable non-symmetric states of the form $|\psi\rangle = |\psi_1\rangle \otimes \psi_2\rangle$ $(|\psi_1\rangle \neq |\psi_2\rangle)$. For instance, the spin squeezing parameter ξ of the state $|\psi\rangle = \binom{\sqrt{3}/2}{1/2} \otimes \binom{-\sqrt{3}/2}{1/2}$ is found to be less than 1 implying that a separable state is spin-squeezed. On evaluation of the parameter ξ for any symmetric separable state $|\psi\rangle = |\psi_1\rangle \otimes |\psi_1\rangle$, one obtains the expected result that $\xi > 1$ thus confirming the applicability of the Kitagawa–Ueda spin squeezing criterion to symmetric multiqubit states. Another spin squeezing criterion, applicable to arbitrary multiqubit states, was given by Wineland *et al.*[7] in the context of Ramsey spectroscopy. The squeezing parameter ξ_w defined by Wineland *et al.*[7] is given by

$$\xi_w^2 = \frac{N(\Delta \boldsymbol{J}_\perp)^2}{|\langle \boldsymbol{J} \cdot \boldsymbol{n}\rangle|^2}, \tag{3}$$

where the symbols have the same meaning as that in Eq. (1). One can see that ξ_w reduces to ξ when $|\langle \boldsymbol{J} \cdot \boldsymbol{n}\rangle| = J = N/2$. The Wineland criterion[7] is used by several authors[10,16] to detect spin squeezing in multiqubit states interacting through Hamiltonian models such as one-dimensional Ising chains. Spin squeezed states satisfying the criterion $\xi_w < 1$ are shown to have reduced frequency noise and thus are useful in spectroscopic studies.[7]

We wish to point out here that caution has to be exercised while analyzing the spin squeezing nature of a quantum state so as to relate it with the pairwise entanglement properties of that state. In fact, the quantum entanglement between any two qubits of a multiqubit system can be the same for an arbitrary choice of qubits only when the state is symmetric under interchange of qubits. Thus, while examining the connection between pairwise entanglement and spin-squeezing of a symmetric multiqubit system, interacting through a particular Hamiltonian model, one has to ascertain whether the exchange symmetry of the state is affected by the interacting Hamiltonian or not. As any relationship between spin squeezing and pairwise entanglement in non-symmetric states is bound to give invalid results, permutation symmetry of a multiqubit state has to be ascertained before relating the spin squeezing nature to its pairwise entanglement. The main motivation of the present work is to show that permutation symmetry of an initially symmetric multiqubit state cannot be taken for granted while considering its time evolution with different interaction models. As exchange symmetry aspects are not given due consideration in studies on spin squeezed states generated through time evolution of an initially symmetric multiqubit state with Ising type interaction models,[19] we feel that an exploration of this aspect in an explicit manner is important. We carry out one such study in this article. In particular, we show that none of the N qubit ($N \geq 4$) symmetric states, except 4-qubit W states and their linear combinations, are likely to retain their exchange symmetry under evolution with Ising type Hamiltonian, one of the important interaction models considered for spin squeezing studies.[10]

An Ising chain with two qubits ($N = 2$) is given by

$$\mathcal{H} = \frac{\hbar\chi}{4}(\sigma_{1x}\sigma_{2x} + \sigma_{2x}\sigma_{1x}). \tag{4}$$

The states spanning the three-dimensional symmetric subspace are the so called *triplet states* given by

$$\begin{aligned} |1,1\rangle &= |00\rangle = |0\rangle \otimes |0\rangle \equiv \text{spin-up state},\\ |1,0\rangle &= |\psi\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle}{\sqrt{2}},\\ |1,-1\rangle &= |11\rangle = |1\rangle \otimes |1\rangle \equiv \text{spin-down state}. \end{aligned}$$

Here the symbol '$\otimes$' stands for Kronecker product and the symbol '$|j,m\rangle$' stand for the angular momentum states.

As σ_{1x}, σ_{2x} correspond to spin flip operation on 1st and 2nd qubit respectively, it is easy to see

$$\mathcal{H}|00\rangle \propto |11\rangle, \quad \mathcal{H}|11\rangle \propto |00\rangle, \quad \mathcal{H}|\psi\rangle \propto |\psi\rangle$$

Hence repeated application of $\mathcal{H}$ on these states results in the same states. Thus the action of the time-evolution operator $\boldsymbol{U} = \exp(-i\mathcal{H}t/\hbar)$ on basis states of the symmetric subspace results in their linear combination, ensuring the symmetry of the 2-qubit Dicke states under Ising chain evolution.

Considering the Ising chain with three qubits ($N = 3$), we have

$$\mathcal{H} = \frac{\hbar\chi}{4}(\sigma_{1x}\sigma_{2x} + \sigma_{2x}\sigma_{3x} + \sigma_{3x}\sigma_{1x}). \tag{5}$$

The set of all symmetric 3-qubit states is spanned by the four basis states (3-qubit Dicke states)

$$\begin{aligned} |\psi_1\rangle &= |3/2, 3/2\rangle = |000\rangle,\\ |\psi_2\rangle &= |3/2, 1/2\rangle = \frac{|001\rangle + |010\rangle + |100\rangle}{\sqrt{3}},\\ |\psi_3\rangle &= |3/2, -1/2\rangle = \frac{|110\rangle + |101\rangle + |011\rangle}{\sqrt{3}},\\ |\psi_4\rangle &= |3/2, -3/2\rangle = |111\rangle. \end{aligned} \tag{6}$$

Our main task here is to check whether an arbitrary symmetric 3-qubit state retains its exchange symmetry under Ising chain evolution. For this, we need to examine whether the basis states of the symmetric subspace of three qubits remain symmetric after interaction with the 1D Ising chain.

Though the exchange symmetry of each of the 3-qubit Dicke states after interaction with the 1D Ising chain can be checked by inspection as is done for the 2-qubit case, it is easier to examine whether the evolved states $|\psi'_\alpha\rangle = \boldsymbol{U}|\psi_\alpha\rangle$ (α = 1, 2, 3, 4), $\boldsymbol{U} = \exp(-i\mathcal{H}t/\hbar)$ being the unitary operator corresponding to 3-qubit Ising chain Hamiltonian, remain in the symmetric subspace. It is a simple matter to notice that the evolved states $|\psi'_\alpha\rangle = \boldsymbol{U}|\psi_\alpha\rangle$ (α = 1, 2, 3, 4) remain in the symmetric subspace iff $|\psi'_\alpha\rangle$ is expressible as $|\psi'_\alpha\rangle = \sum_m c^*_m|3/2,m\rangle$ such that $\sum_m|c_m|^2 = 1$. In fact, $c_m = \langle\psi'_\alpha|3/2,m\rangle$ and we need to evaluate the set of coefficients $c_m = \langle\psi'_\alpha|3/2,m\rangle$, $m = -3/2, -1/2, 1/2, 3/2$ for each α (α = 1, 2, 3, 4). The coefficients c_m for each of the states $|\psi_\alpha\rangle$ are given in Table 1 and it is evident that the symmetry of the 3-qubit Dicke states is unhampered by the Ising chain interaction.

Starting with an Ising chain with 4 qubits, we evaluate the corresponding unitary time-evolution operator and it is given by

$$\boldsymbol{U} = \exp\Big(-\frac{i\mathcal{H}t}{\hbar}\Big) = \boldsymbol{I} + \boldsymbol{A}^2(\cos(\chi t)-1) - i\boldsymbol{A}\sin(\chi t), \tag{7}$$

where

$$\mathcal{H} = \frac{\hbar\chi}{4}(\sigma_{1x}\sigma_{2x}+\sigma_{2x}\sigma_{3x}+\sigma_{3x}\sigma_{4x}+\sigma_{4x}\sigma_{1x}),\ \boldsymbol{A} = \frac{\mathcal{H}}{\hbar\chi}.$$

The basis states $|j,m\rangle$ where $j = 2$ and $m = -2, -1, 0, 1, 2$ of the symmetric subspace are given by

$$\begin{aligned}
|\phi_1\rangle &= |2,2\rangle = |0000\rangle,\\
|\phi_2\rangle &= |2,1\rangle = \frac{1}{2}[|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle],\\
|\phi_3\rangle &= |2,0\rangle = \frac{1}{2}[|0011\rangle + |1100\rangle + |0101\rangle\\
&\qquad + |1010\rangle + |0110\rangle + |1001\rangle],\\
|\phi_4\rangle &= |2,-1\rangle = \frac{1}{2}[|1110\rangle + |1101\rangle + |1011\rangle + |0111\rangle],\\
|\phi_5\rangle &= |2,-2\rangle = |1111\rangle.
\end{aligned} \tag{8}$$

The time evolved states $|\phi'_\alpha\rangle = \boldsymbol{U}|\phi_\alpha\rangle$ (α = 1, 2, 3, 4, 5) remain in the symmetric subspace iff $|\phi'_\alpha\rangle$ is expressible as $|\phi'_\alpha\rangle = \sum_m c^*_m|2,m\rangle$ such that $\sum_m|c_m|^2 = 1$. We have evaluated the set of coefficients $c_m = \langle\phi'_\alpha|2,m\rangle$, $m = -2, -1, 0, 1, 2$ for each α (α = 1, 2, 3, 4, 5) and these coefficients are given explicitly in Table 2.

It is readily seen from Table 2 that though all the states $|\phi_\alpha\rangle$ (α = 1 to 5) are initially symmetric (at time $t = 0$, $\sum_m|c_m|^2 = 1$ for all α), their time-evolved counterparts $|\phi'_\alpha\rangle$ are not restricted to the symmetric subspace. After time evolution, $|\phi_2\rangle = |2,1\rangle$ and $|\phi_4\rangle = |2,-1\rangle$ the so-called W states, are the only two that remain in the symmetric subspace and hence are symmetric under the interchange of particles. We thus conclude that not all 4-qubit symmetric states retain their exchange symmetry after Ising chain interaction. Only a subclass of symmetric states, of the form $a|2,1\rangle + b|2,-1\rangle$ where a, b are any two complex numbers can retain their exchange symmetry under Ising chain evolution.

It is important to notice here that though the 4-qubit W states retain their exchange symmetry under Ising chain evolution, N qubit W states ($N \geq 5$) do not possess this property of symmetry retention. For instance, if $\mathcal{H}$ denotes the 5-qubit Ising chain Hamiltonian with nearest-neighbor interactions, one can see that the coefficients c_m, $m = 5/2, 3/2, \cdots, -5/2$ for the time evolved W state $\Psi_w = \exp(-i\mathcal{H}t/\hbar)|5/2,3/2\rangle$ are given by

$$\begin{aligned}
c_{\frac{5}{2}} &= c_{\frac{1}{2}} = c_{-\frac{3}{2}} = 0,\\
c_{\frac{3}{2}} &= \frac{1}{40}\Big(15 + 12\cos\frac{\chi t}{2} + 13\cos\chi t + 8i\sin\frac{\chi t}{2}\\
&\qquad + 12i\sin(\chi t)\Big),\\
c_{-\frac{1}{2}} &= \frac{1}{20\sqrt{2}}\Big(-6 - 6\cos\frac{\chi t}{2} + 12\cos\chi t\\
&\qquad - 2i\sin\frac{\chi t}{2} + 13i\sin(\chi t)\Big),\\
c_{-\frac{5}{2}} &= \frac{1}{8\sqrt{5}}\Big(-3 + 3\cos\chi t - 4i\sin\frac{\chi t}{2} + 2i\sin(\chi t)\Big).
\end{aligned} \tag{9}$$

Table 1. Demonstration of symmetry retention in 3-qubit states under Ising chain evolution.

State	$c_1 = \langle\psi'_\alpha\vert 3/2,3/2\rangle$	$c_2 = \langle\psi'_\alpha\vert 3/2,1/2\rangle$	$c_3 = \langle\psi'_\alpha\vert 3/2,-1/2\rangle$	$c_4 = \langle\psi'_\alpha\vert 3/2,-3/2\rangle$	$\sum_m\vert c_m\vert^2$
$\vert\psi_1\rangle$	$\frac{e^{3i\chi t/4}}{4}(1+3e^{-i\chi t})$	0	$\frac{i\sqrt{3}}{2}e^{i\chi t/4}\sin\frac{\chi t}{2}$	0	1
$\vert\psi_2\rangle$	0	$\frac{e^{3i\chi t/4}}{4}(3+e^{-i\chi t})$	0	$\frac{i\sqrt{3}}{2}e^{i\chi t/4}\sin\frac{\chi t}{2}$	1
$\vert\psi_3\rangle$	$\frac{i\sqrt{3}}{2}e^{i\chi t/4}\sin\frac{\chi t}{2}$	0	$\frac{e^{3i\chi t/4}}{4}(3+e^{-i\chi t})$	0	1
$\vert\psi_4\rangle$	0	$\frac{i\sqrt{3}}{2}e^{i\chi t/4}\sin\frac{\chi t}{2}$	0	$\frac{e^{3i\chi t/4}}{4}(1+3e^{-i\chi t})$	1

Table 2. Demonstration of loss of exchange symmetry in 4-qubit states under Ising chain evolution.

State	$c_1 = \langle\phi'_\alpha\vert 2,2\rangle$	$c_2 = \langle\phi'_\alpha\vert 2,1\rangle$	$c_3 = \langle\phi'_\alpha\vert 2,0\rangle$	$c_4 = \langle\phi'_\alpha\vert 2,-1\rangle$	$c_5 = \langle\phi'_\alpha\vert 2,-2\rangle$	$\sum_m\vert c_m\vert^2$
$\vert\phi_1\rangle$	$\frac{1}{4}(3+\cos\chi t)$	0	$\frac{-1+\cos(\chi t)+2i\sin\chi t}{2\sqrt{6}}$	0	$\frac{1}{4}(-1+\cos\chi t)$	$\frac{1}{6}(5+\cos(\chi t))$
$\vert\phi_2\rangle$	0	$\frac{1+e^{i\chi t}}{2}$	0	$\frac{-1+e^{i\chi t}}{2}$	0	1
$\vert\phi_3\rangle$	$\frac{-1+\cos\chi t+2i\sin\chi t}{2\sqrt{6}}$	0	$\frac{1+5\cos(\chi t)+4i\sin\chi t}{6}$	0	$\frac{-1+\cos(\chi t)+2i\sin\chi t}{2\sqrt{6}}$	$\frac{1}{9}(8+\cos\chi t)$
$\vert\phi_4\rangle$	0	$\frac{-1+e^{i\chi t}}{2}$	0	$\frac{1+e^{i\chi t}}{2}$	0	1
$\vert\phi_5\rangle$	$\frac{1}{4}(-1+\cos\chi t)$	0	$\frac{-1+\cos(\chi t)+2i\sin\chi t}{2\sqrt{6}}$	0	$\frac{1}{4}(3+\cos\chi t)$	$\frac{1}{6}(5+\cos(\chi t))$

It can be readily seen that

$$\sum_m |c_m|^2 = \frac{1}{400}\Big(257 + 130\cos\frac{\chi t}{2} + 11\cos(\chi t) + 2\cos\frac{3\chi t}{2}\Big), \tag{10}$$

clearly indicating that the state $\Psi_w = \exp\left(-\frac{i\mathbf{H}t}{\hbar}\right)|\frac{5}{2},\frac{3}{2}\rangle$ does not belong to the 6-dimensional symmetric subspace at times $t>0$. It is not difficult to see that for the time evolved obverse W-state $\Psi_{\bar{w}} = \exp\left(-\frac{i\mathcal{H}t}{\hbar}\right)|\frac{5}{2},-\frac{3}{2}\rangle$, the coefficients c_m are the same (except for order) as for the state Ψ_w thus resulting in the same conclusion of loss of symmetry at times $t>0$. In fact, not just W states but *all N qubit symmetric states* ($N\geq 5$), the states belonging to the $(N+1)$-dimensional symmetric subspace, are likely to lose their exchange symmetry on interaction with an Ising chain with the nearest neighbor interactions. This can be checked in an analogous manner by evaluating the coefficients c_m, $m=\frac{N}{2},\frac{N}{2}-1,\cdots,-\frac{N}{2}$ and by observing that $\sum_m |c_m|^2 \neq 1$ for all N qubit Dicke states. This implies that none of the time-evolved N qubit Dicke states ($N\geq 5$) are expressible in terms of the N qubit Dicke states themselves (which are basis states of the $(N+1)$-dimensional symmetric subspace) and hence do not exhibit exchange symmetry. Alternatively, one can check the loss of symmetry through inspection (like how we illustrated the retention of symmetry in 2-qubit case). The loss of symmetry is evident in just the action of the N-qubit Ising chain Hamiltonian on the corresponding Dicke states. Repeated applications of the Hamiltonian does not initiate symmetry any further and the action of unitary time evolution operator corresponding to N-qubit Ising spin chain results in non-symmetric states.

The retention/loss of exchange symmetry in initially symmetric states on time evolution with the Ising chain interaction can be attributed to the existence/non-existence of rotational symmetry in the Ising chain Hamilotonian. In fact, although the Ising chain Hamiltonians corresponding to 2 and 3 qubits exhibit rotational symmetry as they commute with $\boldsymbol{J}^2$ (the squared collective angular momentum operator), the Hamiltonians corresponding to $N>3$ do not commute with it and hence are rotationally asymmetric. One needs to notice here the status of 4-qubit W states with reference to evolution with the Ising chain; in spite of the rotational asymmetry of 4-qubit Ising chain Hamiltonian, the W-states retain their exchange symmetry on evolution with the Ising chain. This privilege is not preserved by the W states with more than 4 qubits as we have indicated above.

A discussion on the symmetry aspect for other interactions of physical interest may be in order, at this stage. In particular, on observing that the spin chain modelled by isotropic Heisenberg Hamiltonian possesses rotational symmetry for all N, one can expect that the exchange symmetry of multiqubit symmetric states is to be unhampered on time evolution with isotropic Heisenberg Hamiltonian. An anisotropy along either x, y or z direction will spoil the rotational symmetry and thereby multiqubit states interacting with an anisotropic Heisenberg Hamiltonian are unlikely to retain their exchange symmetry. An examination of the rotational symmetry of the interacting Hamiltonian is therefore quite useful when considering their effect on the exchange symmetry of multiqubit states.

In summary, we have shown that symmetric N-qubit states ($N\geq 5$) lose their exchange symmetry after interaction with a spin chain modelled by 1D Ising Hamiltonian with nearest neighbor interaction. Specifically we have shown that 2 and 3 qubit symmetric states retain their exchange symmetry under Ising chain evolution but all 4-qubit symmetric states, except the states of W type, lose their symmetry under the same interaction. The rotational symmetry/asymmetry of the interacting Hamiltonian is seen to be the main reason for retention/loss of exchange symmetry in multiqubit states. We emphasize here that permutation symmetry aspects are important either in studying the collective behavior such as spin-squeezing or in relating the spin-squeezing behavior of a symmetric N-qubit state with its pairwise entanglement properties. However, exchange symmetry properties are assumed to hold good under 1D Ising chain, when such a study is reported in Ref. [19]. Through this work, we hope to initiate proper clarifications on the retention of exchange symmetry in future investigations on spin squeezing or in any study where exchange symmetry matters.

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