View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by ePrints@Bangalore University

International Communications in Heat and Mass Transfer 38 (2011) 906–910

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Effects of thermal nonequilibrium and non-uniform temperature gradients on the onset of convection in a heterogeneous porous medium ☆

I.S. Shivakumara [a,1], Jinho Lee [a,*], K. Vajravelu [a,2], A.L. Mamatha [a,b]

[a] School of Mechanical Engineering, Yonsei University, Seoul 120–749, South Korea
[b] Smt. Rukmini Shedthi Memorial National Government First Grade College, Department of Mathematics, Barkur, 576 210, Udupi District, India

ARTICLE INFO

Available online 30 April 2011

Keywords:
Heterogeneous porous medium
Thermal nonequilibrium
Convection
Basic temperature gradient

ABSTRACT

The simultaneous effect of local thermal nonequilibrium (LTNE), vertical heterogeneity of permeability, and non-uniform basic temperature gradient on the criterion for the onset of Darcy–Benard convection is studied. The eigenvalue problem is solved numerically using the Galerkin method. The interaction of various types of permeability heterogeneity and non-uniform basic temperature gradient functions on the stability characteristics of the system is analyzed. It is observed that the linear variation (about the mean) of the permeability and the basic temperature gradient with depth has no added effect on the criterion for the onset of convection. However, the concurrent variation in heterogeneous permeability and non-uniform basic temperature gradient functions has more stabilizing effect on the system, while opposite is the trend when the effect of non-uniform basic temperature gradient alone is present.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

Buoyancy-driven convection in a layer of fluid (in a saturated porous medium) heated uniformly from below has been studied extensively by several researchers over the years because of its natural occurrence and also its relevance in various applications such as biomedical engineering, drying processes, thermal insulation, radioactive waste management, transpiration cooling, geophysical systems, and contaminant transport in groundwater, ceramic processing, solid-matrix compact heat exchangers and many others. Copious literature is available on this as well as related topics and it is well documented in the literature (Ingham and Pop [1], Vafai [2], Nield and Bejan [3], Vadasz [4]).

The effect of heterogeneity in either permeability or thermal conductivity or both on thermal convective instability in a layer of fluid in a porous medium is of importance since there can be dramatic effects in the case of heterogeneity (Braester and Vadasz [5], Simmons et al. [6] and Prasad and Simmons [7]). The effects of hydrodynamic and thermal heterogeneity, for the case of variation in both the horizontal and vertical directions, on the onset of convection in a horizontal layer of saturated porous medium uniformly heated from below, are studied analytically for the case of weak heterogeneity by Nield and Kuznetsov [8]. A discussion on the effect of heterogeneity on the onset of convection induced by a vertical density gradient in a saturated porous medium has been made by Nield and Simmons [9]. Whereas, the combined effects of vertical and horizontal heterogeneity on the onset of transient convection in a porous medium are investigated by Nield and Kuznetsov [10]. Recently, Nield and Kuznetsov [11] have studied the effect of vertical heterogeneity on the onset of convection in a horizontal layer of fluid in a saturated porous medium, uniformly heated from below but with a non-uniform basic temperature gradient resulting from transient heating or otherwise. All the above studies are based on local thermal equilibrium (LTE) model.

However, in many practical applications involving hyper-porous materials and also media in which there is a large temperature difference between the fluid and the solid phases, it has been realized that the assumption of LTE model is inadequate for proper understanding of the heat transfer problems. In such circumstances, the local thermal non-equilibrium (LTNE) effects are to be taken into consideration. Therefore, the recent trend in the study of thermal convective instability problems in porous media is to account for LTNE effects by considering a two-field model for energy equation each representing the fluid and solid phases separately. Under certain circumstances, investigations have been carried out in the recent past to know LTNE effects on forced and free convection in fluid saturated porous media.

The LTNE effects on forced convection flows in a porous medium have been covered exhaustively in the excellent reviews by Vafai and Amiri [12] and Kuznetsov [13]. Banu and Rees [14] have studied the criterion for the onset of convection in a Darcy porous medium using LTNE model. The effect of LTNE on the onset of convection in a porous

☆ Communicated by H. Yoshida and S. Maruyama.
* Corresponding author.
E-mail addresses: shivakumarais@gmail.com (I.S. Shivakumara), jinholee@yonsei.ac.kr (J. Lee), vajravel@mail.ucf.edu (K. Vajravelu).
[1] Permanent address: UGC-CAS in Fluid Mechanics, Department of Mathematics, Bangalore University, Bangalore-560 001, India.
[2] Permanent address: Department of Mathematics, University of Central Florida, Orlando, FL 32816, USA.

0735-1933/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2011.04.023

layer has been studied using a non-Darcian model for stress-free boundaries by Malashetty et al. [15]. Straughan [16] has considered a problem of thermal convection in a fluid-saturated porous layer using a global nonlinear stability analysis with a LTNE model. Postelnicu [17] has studied the onset of Darcy–Brinkman convection using LTNE model for rigid isothermal boundaries. Shivakumara et al. [18,19] have analyzed the effects of temperature dependent viscosity and quadratic density as well as boundary effects, while Lee et al. [20] have investigated the effect of various forms of basic non-uniform temperature gradients on the onset of thermal convection in a porous layer using LTNE model. All these studies are limited to the case of constant permeability.

In geophysical and engineering applications the porous domain is frequently heterogeneous in permeability and also the possibility of existing non-uniform temperature gradient due to differential heating with depth is common. Therefore, the influence of heterogeneity in permeability and non-uniform basic temperature gradient on natural convection in a layer of porous medium is of practical interest. To the best of our knowledge, the simultaneous effect of vertical heterogeneity in the permeability (existing due to series of horizontal layers in each of which the permeability is uniform) and non-uniform basic temperature gradient on the onset of convection in a Newtonian fluid saturated horizontal porous layer heated from below using a LTNE model has not received any attention in the literature despite its importance in geophysical and engineering applications. The intent of the present work is to develop the formalism required to determine the criterion for the onset of convection and the analysis presented is quite general. In the present study, the assumptions put forth by Nield and Kuznetsov [11] are being used and the resulting eigenvalue problem is solved numerically using the Galerkin method for various interactions of vertical heterogeneity in permeability and non-uniform basic temperature gradient functions and the results are presented graphically. Furthermore, the effects of LTNE model over the classical LTE model on the onset of convection are highlighted.

2. Mathematical formulation

The physical configuration is as shown in Fig. 1. It consists of an incompressible Newtonian viscous fluid saturated heterogeneous horizontal porous layer of characteristic thickness d confined by rigid boundaries and heated from below. The lower surface is held at constant temperature T_L, while the upper surface is at T_U ($<T_L$). A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and the z-axis directed vertically upward in the direction of the gravitational field. The Oberbeck–Boussinesq approximation on the density and LTNE with two-field model for temperatures are used.

The basic equations governing the flow, following Nield and Bejan [3], are:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

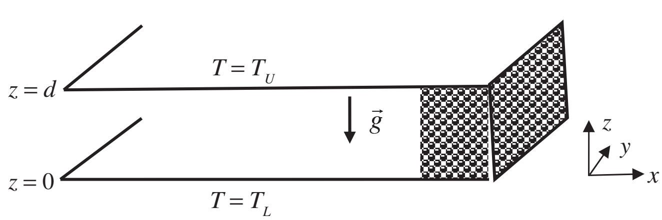

Fig. 1. Physical configuration.

$$\rho_0 c_a \frac{\partial \vec{q}}{\partial t} = -\nabla p + \rho_f \vec{g} - \frac{\mu}{K(z)} \vec{q} \tag{2}$$

$$\varepsilon (\rho_0 c)_f \frac{\partial T_f}{\partial t} + (\rho_0 c)_f \left(\vec{q} \cdot \nabla\right) T_f = \varepsilon k_f \nabla^2 T_f + h\left(T_s - T_f\right) \tag{3}$$

$$(1-\varepsilon)(\rho_0 c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon) k_s \nabla^2 T_s - h\left(T_s - T_f\right) \tag{4}$$

$$\rho_f = \rho_0\left[1-\beta\left(T_f - T_L\right)\right] \tag{5}$$

where, $\vec{q}$ the velocity vector, p the excessive pressure over the reference hydrostatic value, ρ_f the fluid density, $K(z)$ the variable permeability of the porous medium, c_a the acceleration coefficient, ε the porosity of the porous medium, μ the fluid viscosity, T_f the temperature of the fluid phase, T_s the temperature of the solid phase, c the specific heat, k_f the thermal conductivity of the fluid, k_s the thermal conductivity of the solid, β the thermal expansion coefficient of the fluid and h is the inter-phase heat transfer coefficient which depends on the nature of the porous matrix and the saturating fluid. The vertical heterogeneity in the porous medium $K(z)$ may be visualized as one can divide the field into horizontal layers, within each of which one can employ arithmetic mean permeability with the mean taken over that particular layer [9]. In this way a continuously varying permeability is discretized in terms of the vertical position coordinate. The time derivative term is included in the momentum equation to look eventually for the occurrence of oscillatory convection.

Let us non-dimensionalize the variables by setting

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \vec{q}^{\,*} = \frac{\vec{q}}{\left(\varepsilon\kappa_f / d\right)}, t^* = \frac{t}{\left(d^2/\kappa_f\right)}, p^* = \frac{p}{\left(\mu\kappa_f\varepsilon / K_0\right)}$$
$$T_f^* = \frac{T_f - T_U}{T_L - T_U}, T_s^* = \frac{T_s - T_U}{T_L - T_U}, \tag{6}$$

where K_0 is the mean value of $K(z)$ and $\kappa_f = k_f/(\rho_0 c)_f$ is the thermal diffusivity of the fluid. Using the non-dimensional quantities in Eq. (6), Eqs. (1)–(5) can be written (after dropping the asterisks) as

$$\nabla \cdot \vec{q} = 0 \tag{7}$$

$$\gamma_a \frac{\partial \vec{q}}{\partial t} = -\nabla p + R_D\left[T_f - \frac{1}{\beta(T_L - T_U)}\right]\hat{k} - \Gamma(z)\vec{q} \tag{8}$$

$$\frac{\partial T_f}{\partial t} + \left(\vec{q} \cdot \nabla\right) T_f = \nabla^2 T_f + H\left(T_s - T_f\right) \tag{9}$$

$$\alpha \frac{\partial T_s}{\partial t} = \nabla^2 T_s - \gamma H\left(T_s - T_f\right). \tag{10}$$

Here, $R_D = \rho_0 \beta g (T_L - T_U) K_0 d / \varepsilon \mu_f \kappa_f$ is the Darcy–Rayleigh number, $H = hd^2/\varepsilon k_f$ is the scaled inter-phase heat transfer coefficient, $\gamma_a = c_a K_0 \kappa_f / \nu d^2$ is the acceleration parameter, $\alpha = \kappa_f/\kappa_s$ is the ratio of diffusivities, $\gamma = \varepsilon k_f/(1-\varepsilon) k_s$ is the porosity modified conductivity ratio and $\Gamma(z) = K_0/K(z)$ is the permeability heterogeneity function.

The basic state is quiescent and is given by

$$\vec{q}_b = 0, \quad \nabla p_b = R_D\left[T_{fb} - \frac{1}{\beta(T_L - T_U)}\right]\hat{k} \tag{11a, b}$$

$$\frac{\partial T_{fb}}{\partial t} = \nabla^2 T_{fb}, \quad \alpha \frac{\partial T_{sb}}{\partial t} = \nabla^2 T_{sb}. \tag{11c, d}$$

As propounded by Nield and Kuznetsov [11], a simplification in the form of a quasi-static approximation is introduced which consists of freezing the basic temperature distributions at a given instant of time. This simplification is justified so long as the disturbances are growing faster than the basic profile is evolving [3]. Besides, it is assumed that the solid and fluid phases have identical temperatures at the bounding surfaces of the porous layer. Under the circumstances, the basic state temperature distribution for fluid and solid phases admit a solution of the form

$$-\frac{d}{\Delta T}\frac{dT_{fb}}{dz} = f(z), -\frac{d}{\Delta T}\frac{dT_{sb}}{dz} = g(z). \tag{11e, f}$$

Here, $f(z)$ and $g(z)$ are the basic non-uniform temperature gradients for fluid and solid phases respectively such that $\int_0^1 f(z)dz = 1 = \int_0^1 g(z)dz$.

We now perturb the basic solution and write

$$\vec{q} = \vec{q}\,', p = p_b + p', T_f = T_{fb} + T_f', T_s = T_{sb} + T_s'. \tag{12}$$

Eq. (12) is substituted into Eqs. (7)–(10) and linearized to get

$$\nabla\cdot\vec{q}\,' = 0 \tag{13}$$

$$\gamma_a \frac{\partial \vec{q}\,'}{\partial t} = -\nabla p' + R_D T_f' \hat{k} - \Gamma(z)\vec{q}\,' \tag{14}$$

$$\frac{\partial T_f'}{\partial t} + \left(\vec{q}\,'\cdot\nabla\right) T_{fb} = \nabla^2 T_f' + H\left(T_s' - T_f'\right) \tag{15}$$

$$\alpha \frac{\partial T_s'}{\partial t} = \nabla^2 T_s' - \gamma H\left(T_s' - T_f'\right). \tag{16}$$

Eliminating the pressure term from the momentum equation by taking curl twice and retaining the vertical component of the resulting equation and assuming the normal mode solution in the form (after noting the principle of exchange of stability holds [11] as there are no physical mechanisms to make the system oscillatory),

$$\left(w', T_f', T_s'\right) = [W(z), \Theta(z), \Phi(z)] \exp(ilx + imy) \tag{17}$$

leads to the following equations:

$$\Gamma(z)\left(D^2 - a^2\right)W + D\Gamma(z)DW = -a^2 R_D \Theta \tag{18}$$

$$\left(D^2 - a^2 - H\right)\Theta = -f(z)W - H\Phi \tag{19}$$

$$\left(D^2 - a^2 - \gamma H\right)\Phi = -\gamma H\Theta. \tag{20}$$

Here, $D = d/dz$ is the differential operator. The functions $f(z)$ and $\Gamma(z)$are chosen in the following form:

$$f(z) = 1 + \alpha_1\left(z - \frac{1}{2}\right) + \alpha_2\left(z^2 - \frac{1}{3}\right), \; \Gamma(z) = 1 + \beta_1\left(z - \frac{1}{2}\right) + \beta_2\left(z^2 - \frac{1}{3}\right) \tag{21a, b}$$

where α_1, α_2, β_1 and β_2 are constants. It is noted that the above quadratic functions have unit mean. For the homogeneous porous medium and uniform temperature gradient case, $\alpha_1 = 0 = \alpha_2$ and $\beta_1 = 0 = \beta_2$.

The boundaries are impermeable with constant temperatures and hence the boundary conditions are:

$$W = 0, \Theta = 0 = \Phi. \tag{22}$$

3. Numerical solution

Eqs. (18)–(20) together with the boundary conditions Eq. (22) constitute an eigenvalue problem with R_D as the eigenvalue. The resulting eigenvalue problem is solved numerically using the Galerkin technique. In this method, the test (weighted) functions are same as the base (trial) functions. Thus, $W(z)$, $\Theta(z)$ and $\Phi(z)$ are expanded in the series form

$$W(z) = \sum_{i=1}^{n} A_i W_i(z), \Theta(z) = \sum_{i=1}^{n} B_i \Theta_i(z), \Phi(z) = \sum_{i=1}^{n} C_i \Phi_i(z) \tag{23}$$

where A_i, B_i and C_i are unknown coefficients. The base functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ are assumed in the following form:

$$W_i = z(1-z)T_{i-1}^*, \quad \Theta_i = z(z-1)T_{i-1}^* = \Phi_i \tag{24}$$

where T_i^*s are the modified Chebyshev polynomials, such that W_i, Θ_i and Φ_i satisfy the corresponding boundary conditions. Multiplying Eq. (18) by $W_j(z)$, Eq. (19) by $\Theta_j(z)$ and Eq. (20) by $\Phi_j(z)$; performing the integration by parts with respect to z between z = 0 and 1, and using the boundary conditions, we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}B_i = 0 \tag{25}$$

$$E_{ji}A_i + F_{ji}B_i + G_{ji}C_i = 0 \tag{26}$$

$$H_{ji}B_i + I_{ji}C_i = 0. \tag{27}$$

The coefficients C_{ji}–I_{ji} involve the inner products of the base functions and are given by

$$\begin{aligned}
C_{ji} &= \langle\left(1 + \alpha_1(z - 1/2) + \alpha_2\left(z^2 - 1/3\right)\right)DW_j DW_i\rangle \\
&\quad + \langle\left(1 + \alpha_1(z - 1/2) + \alpha_2\left(z^2 - 1/3\right)\right)W_j W_i\rangle \\
&\quad - \langle(\alpha_1 + 2\alpha_2 z)W_j DW_i\rangle \\
D_{ji} &= -a^2 R_D \langle W_j \Theta_i\rangle, E_{ji} = -\langle\left(1 + \beta_1(z - 1/2) + \beta_2\left(z^2 - 1/3\right)\right)\Theta_j W_i\rangle \\
F_{ji} &= \langle D\Theta_j D\Theta_i\rangle + \left(a^2 + H\right)\langle\Theta_j\Theta_i\rangle, G_{ji} = -H\langle\Theta_j\Phi_i\rangle \\
H_{ji} &= -\gamma H\langle\Phi_j\Theta_i\rangle, I_{ji} = \langle D\Phi_j D\Phi_i\rangle + \left(a^2 + \gamma H\right)\langle\Phi_j\Phi_i\rangle
\end{aligned} \tag{28}$$

where the inner product is defined as $\langle\cdots\rangle = \int_0^1 (\cdots)dz$.

The characteristic equation formed from Eqs. (25)–(27) for the existence of non-trivial solution is solved numerically, for different values of γ and H as well as for different forms of $f(z)$ and $\Gamma(z)$. The Newton–Raphson method is used to obtain the Rayleigh number R_D as a function of wave number a when all the parameters and functions are fixed: the bisection method is built-in to locate the critical stability parameters (R_{Dc},a_c) to the desired degree of accuracy. It is observed that the series expansions in Eq. (23) converge (when we consider the first six terms).

4. Results and discussion

The effects of LTNE, the vertical heterogeneity of permeability and a non-uniform basic temperature gradient resulting from transient heating are investigated on the criterion for the onset of convection in a layer of Newtonian fluid saturated Darcy porous medium. Various models of $f(z)$ and $\Gamma(z)$ as shown in Table 1 are considered in analyzing respectively the interaction of non-uniform basic temperature gradient and the vertical heterogeneity of permeability on the onset of convection. The resulting eigenvalue problem for different models is solved numerically by the Galerkin method. At this juncture, it is instructive to look at the critical Darcy–Rayleigh number (R_{Dc})

Table 1
Various forms of basic temperature gradients $f(z)$ and the vertical heterogeneity of permeability $\Gamma(z)$.

Models	α_1	α_2	β_1	β_2	Nature of $f(z)$	Nature of $\Gamma(z)$
$M1$	0	0	0	0	Uniform	Homogeneous
$M2$	1	0	1	0	Linear variation in z	Linear variation in z
$M3$	0	1	0	1	Quadratic variation in z only	Quadratic variation in z only
$M4$	1	1	1	1	General quadratic variation in z	General quadratic variation in z
$M5$	1	1	0	0	General quadratic variation in z	Homogeneous
$M6$	0	0	1	1	Uniform	General quadratic variation in z

and the corresponding wave number (a_c) for various levels of the Galerkin approximation to know the process of convergence and also the accuracy of the results. Table 2 shows the numerically computed values of R_{Dc} and the corresponding a_c. From the tabulated values, it is clear that the results converge for six terms in the Galerkin expansion. Also, the critical stability parameters (R_{Dc}, a_c) for model $M1$ are found to be in close agreement with those of Banu and Rees [14] obtained analytically.

The neutral stability curves in the (R_D, a) plane for different models $M1$–$M6$ are presented in Fig. 2(a) for two values of $\gamma(=1, 5)$ when $H=100$, and for two values of $H(=10, 100)$ when $\gamma=0.5$ in Fig. 2(b). The region below each curve corresponds to stable state. From these figures it is obvious that the neutral curves exhibit single minimum. The Darcy–Rayleigh numbers turn out to be the same for models $M1$ and $M2$ as well as $M3$ and $M4$ as observed in the case of LTE model. It is thus observed that the linear variation in $f(z)$ and $\Gamma(z)$ with depth has no additional influence on the stability characteristics of the system. However, the Darcy–Rayleigh numbers are different for models $M5$ and $M6$. The effect of increasing γ is to reduce the Darcy–Rayleigh number and to decrease the region of stability, while opposite is the trend with increasing H.

The variation of R_{Dc} and a_c as a function of $\log_{10} H$ is shown in Fig. 3(a) and (b) respectively for different values of γ for the models $M1$–$M6$. Fig. 3(a) demonstrates that, R_{Dc} is independent of γ for smaller values of H and observed that it remains almost independent of H for $\gamma \geq 10$, for a particular model considered. This is because, for very small values of H and higher values of γ there is no significant transfer of heat between the fluid and solid phases, and hence the condition for the onset of convection is not affected by the properties of the solid phase. This corresponds to classical LTE case. For other values of γ, however, R_{Dc} varies with γ as well as H but remains independent of H at higher values of H. This may be attributed to the fact that, at higher values of H, the condition for the onset of convection is based on the mean properties of the medium and hence the critical Rayleigh number varies with γ. The figures also indicate that for moderate and large values of H, the critical Darcy–Rayleigh number decreases with increasing values of γ. This is because an increase in the value of γ leads to a significant transfer of heat through both the solid and fluid phases. Further inspection of the figures reveal that the system is more stabilizing for models $M3$ and $M4$ and least stable for model $M5$. In other words, the presence of heterogeneity in permeability is to delay the onset of convection when compared to homogeneous porous medium case. Moreover, the critical Darcy–Rayleigh numbers for model $M5$ are lower than those of model $M6$ indicating that the effect of non-uniform basic temperature gradient is to hasten the onset of convection.

Fig. 3(b) indicates that, a_c remains unaffected and also takes the same value in the small-H as well as the large-H limits (LTE case), while at intermediate values of H (LTNE case) it attains a maximum value for different values of γ. Moreover, at intermediate values of H, we note that an increase in the value of γ is to decrease the critical wave number and hence its effect is to increase the size of convection cells for all six models considered. A closer inspection at the figure

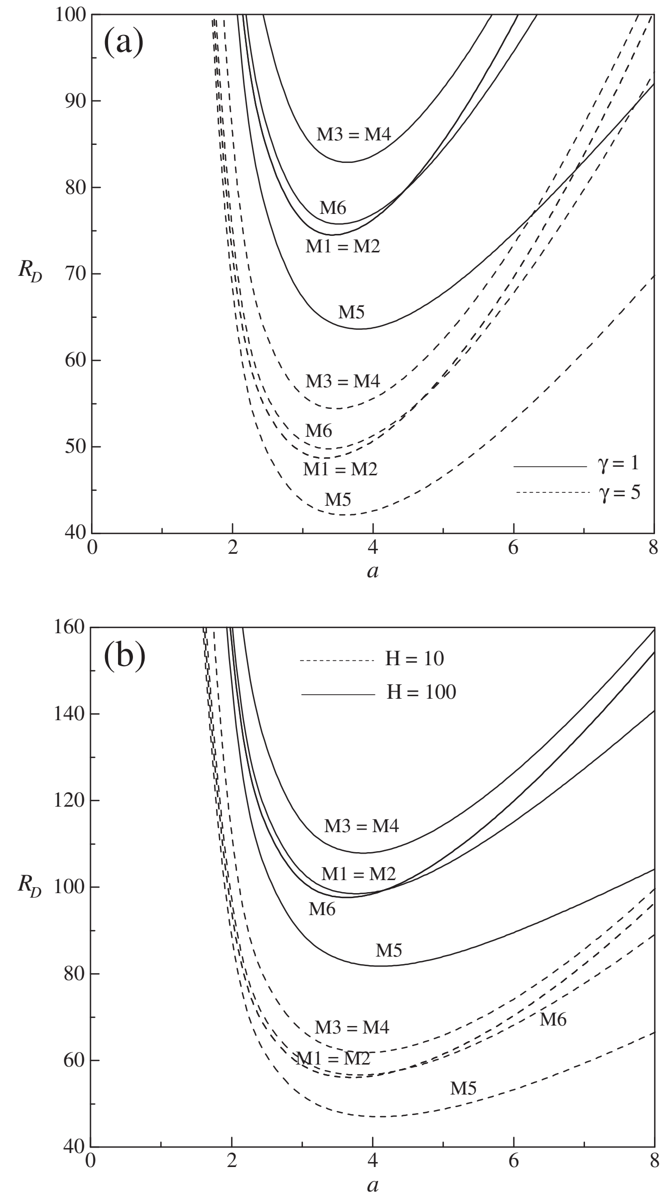

Fig. 2. Neutral curves for different forms of $f(z)$ and $\Gamma(z)$ when (a) $H=100$ (b) $\gamma=0.5$.

Table 2
Comparison of results for different orders of Galerkin approximations for $H=100$.

γ	Model	Approximations							
		$i=j=1$		$i=j=2$		$i=j=5$		$i=j=6$	
		R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c
0.5	$M1$	96.347	3.488	96.347	3.488	95.314	3.463	95.314	3.463
	$M2$	96.347	3.488	95.702	3.473	95.314	3.463	95.314	3.463
	$M3$	105.803	3.706	105.544	3.700	104.504	3.674	104.504	3.674
	$M4$	105.803	3.706	104.637	3.680	105.713	3.701	105.712	3.701
	$M5$	100.765	3.706	82.106	3.817	80.525	3.944	80.539	3.943
	$M6$	101.165	3.489	97.281	3.614	96.538	3.588	96.538	3.588
1	$M1$	73.221	3.294	73.221	3.294	72.340	3.271	72.340	3.271
	$M2$	73.221	3.294	72.699	3.280	72.340	3.271	72.340	3.271
	$M3$	80.898	3.479	80.687	3.474	79.789	3.452	79.789	3.452
	$M4$	80.898	3.479	79.947	3.457	80.774	3.474	80.774	3.474
	$M5$	77.046	3.479	63.212	3.550	62.286	3.627	62.296	3.627
	$M6$	76.882	3.294	74.470	3.378	73.800	3.355	73.799	3.355

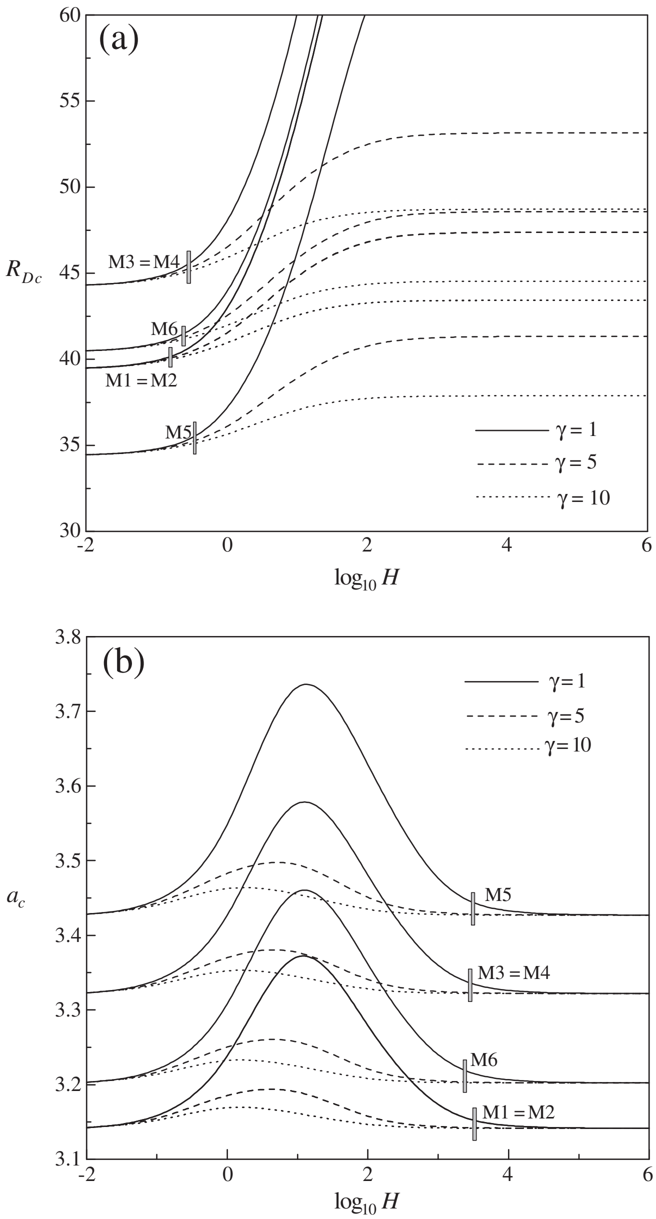

Fig. 3. Variation of (a) R_{Dc} and (b) a_c with $\log_{10} H$ for different values of γ.

reveals that the critical wave number is higher for model $M5$ and the least (as well as same) for models $M1$ and $M2$.

5. Conclusions

From the foregoing study, it is observed that the linear variation (about the mean) of the permeability and basic temperature gradient with depth has no added effect on the criterion for the onset of convection. However, the system is more stabilizing when both heterogeneous permeability and non-uniform basic temperature gradient functions vary simultaneously with depth, and destabilizing the most when the effect of non-uniform basic temperature gradient alone is present. Thus the effect of heterogeneous permeability is to delay the onset of convection and opposite is the trend in the presence of non-uniform basic temperature gradient. The porosity modified conductivity ratio γ has no effect on the onset of convection in the small-H limit, while for other values of H increase in the value of γ is to hasten the onset of convection. The LTNE model exhibits more stabilizing effect on the system than LTE model. The critical wave number for different values of γ in the small-H and large-H limits coincide but attain a maximum value at the intermediate values of H, and in that case increasing γ is to decrease the critical wave number. Moreover, the critical wave number is the least for the case of homogeneous porous medium and uniform temperature gradient, and higher if only the basic temperature gradient function varies quadratically.

Acknowledgments

One of the authors (ISS) wishes to thank the BK 21 Program of the School of Mechanical Engineering, Yonsei University, Seoul, Korea for inviting him as a visiting Professor and also the Bangalore University for sanctioning sabbatical leave. The author (KV) thanks the University of Central Florida for the sabbatical leave.

References

[1] D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, Pergamon, Oxford, 1998.
[2] K. Vafai (Ed.), Handbook of Porous Media, Marcel Dekker, New York, 2000.
[3] D.A. Nield, A. Bejan, Convection in Porous Media, 3 rd edn. Springer-Verlag, New York, 2006.
[4] P. Vadasz (Ed.), Emerging Topics in Heat and Mass Transfer in Porous Media, Springer, New York, 2008.
[5] C. Braester, P. Vadasz, The effect of a weak heterogeneity of a porous medium on natural convection, J. Fluid Mech. 254 (1993) 345–362.
[6] C.T. Simmons, T.R. Fenstemaker, J.M. Sharp, Variable-density flow and solute transport in heterogeneous porous media: approaches, resolutions and future challenges, J. Contam. Hydrol. 52 (2001) 245–275.
[7] A. Prasad, C.T. Simmons, Unstable density-driven flow in heterogeneous porous media: a stochastic study of the Elder [1967b] 'short heater' problem, Water Resour. Res. 39 (2003) #1007.
[8] D.A. Nield, A.V. Kuznetsov, The effects of combined horizontal and vertical heterogeneity on the onset of convection in a porous medium, Int. J. Heat Mass Trans. 50 (2007) 3329–3339.
[9] D.A. Nield, C.T. Simmons, A discussion on the effect of heterogeneity on the onset of convection in a porous medium, Transp. Porous Med 68 (2007) 413–421.
[10] D.A. Nield, A.V. Kuznetsov, The effects of combined horizontal and vertical heterogeneity on the onset of convection in a porous medium, Int. J. Heat Mass Trans. 50 (2007) 1909–1915.
[11] D.A. Nield, A.V. Kuznetsov, The onset of convection in a heterogeneous porous medium with transient temperature profile, Transp. Porous Med. (2010), doi:10.1007/s11242-010-9586-8.
[12] K. Vafai, A. Amiri, Non-Darcian effects in combined forced convective flows, in: D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media Oxford; Pergamon, 1998, pp. 313–329.
[13] A.V. Kuznetsov, Thermal non-equilibrium forced convection in porous media, in: D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, Pergamon, Oxford, 1998, pp. 103–130.
[14] N. Banu, D.A.S. Rees, Onset of Darcy–Benard convection using a thermal non-equilibrium model, Int. J. Heat Mass Trans. 45 (2002) 2221–2228.
[15] M.S. Malashetty, I.S. Shivakumara, Kulkarni Sridhar, The onset of Lapwood–Brinkman convection using a thermal non-equilibrium model, Int. J. Heat Mass Trans. 48 (2005) 1155–1163.
[16] B. Straughan, Global nonlinear stability in porous convection with a thermal non-equilibrium model, Proc. R. Soc. A 462 (2006) 409–418.
[17] A. Postelnicu, The onset of a Darcy–Brinkman convection using a thermal nonequilibrium model. Part II, Int. J. Thermal Sci. 47 (2008) 1587–1594.
[18] I.S. Shivakumara, A.L. Mamatha, M. Ravisha, Effects of variable viscosity and density maximum on the onset of Darcy–Benard convection using a thermal non-equilibrium model, J. Porous Media 13 (7) (2010) 613–622.
[19] I.S. Shivakumara, A.L. Mamatha, M. Ravisha, Boundary and thermal non-equilibrium effects on the onset of Darcy–Brinkman convection in a porous layer, J. Eng. Math. 45 (2010) 213–226.
[20] Jinho Lee, I. S. Shivakumara and A. L. Mamatha, Effect of non-uniform temperature gradients on thermo-gravitational convection in a porous layer using a thermal non-equilibrium model. J. Porous Media in press.