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Onset of Bénard–Marangoni ferroconvection with a convective surface boundary condition: The effects of cubic temperature profile and MFD viscosity ☆

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ABSTRACT

The combined effects of basic cubic temperature profiles and magnetic field dependent (MFD) viscosity on the onset of Bénard-Marangoni convection in a ferrofluid layer are studied. The lower boundary is rigid-isothermal, while the upper free boundary open to the atmosphere is flat and subject to a general thermal boundary condition. The Galerkin technique is employed to extract the critical stability parameters numerically. The results indicate that the basic cubic temperature profiles have a profound influence on the stability characteristics of the system and can be effectively used to either suppress or augment the onset of Bénard–Marangoni ferroconvection. Besides, increasing the magnetic Rayleigh number and the nonlinearity of magnetization hastens, while an increase in the Biot number and MFD viscosity parameter delays the onset of Bénard–Marangoni ferroconvection. The existing results in the literature are obtained as particular cases from the present study.

1. Introduction

Ferrofluids are stable colloidal suspensions of magnetic nano-particles dispersed in a carrier liquid. In the absence of an external magnetic field the magnetic moments of the particles are randomly orientated and there is no net macroscopic magnetization. In an external magnetic field, however, the magnetic moments of particles easily orient and a large (induced) magnetization prevails. There are two additional features in ferrofluids not found in ordinary fluids, the Kelvin force and the body couple [1]. In addition, in an external magnetic field, a ferrofluid exhibits additional rheological properties such as a field-dependent viscosity, special adhesion properties, and a non-Newtonian behavior, among others [2]. The theory of thermal convective instability in a ferrofluid layer began with Finlayson [3] and extensively continued over the years ([4–6]). Recently, Shivakumara et al. [7] have investigated the onset of thermogravitational convection in a horizontal ferrofluid layer with viscosity depending exponentially on temperature.

On the other hand, if the surface of a ferrofluid layer is free and open to the atmosphere then convection can also be induced by temperature dependent surface tension forces at the free surface known as Marangoni ferroconvection. In view of the fact that heat transfer is greatly enhanced due to convection, Marangoni ferroconvection offers new possibilities for application in cooling of motors in space, loudspeakers, transmission lines and other equipments in micro-gravity environment where a magnetic field is already present. In most of the cases, the combined effect of buoyancy and surface tension forces on convective instability in a ferrofluid layer becomes important. Realizing these aspects, a limited number of studies have addressed the effect of surface tension forces on ferroconvection in a horizontal ferrofluid layer. Linear and nonlinear stability of combined buoyancy–surface tension effects in a ferrofluid layer heated from below has been analyzed by Qin and Kaloni [8]. The linear stability analysis of a layer of a magnetic fluid with a deformable free surface which is heated uniformly from below and subject to a vertical magnetic field has been analyzed considering the temperature dependence of the surface tension and buoyancy by Weilepp and Brand [9]. Odenbach [10] has investigated experimentally the stability of a free surface of a magnetic fluid subjected to a magnetic field parallel to the fluid surface under strongly reduced gravity. The Bénard–Marangoni problem of ferrofluids has been studied for different situations in Refs. [11–13]. Idris and Hashim [14] have investigated the instability of Bénard–Marangoni convection in a horizontal layer of ferrofluid under the influence of a linear feedback control and cubic temperature profile. Nanjundappa et al. [15] have studied the effect of internal heat generation on the onset of Bénard–Marangoni convection in a horizontal ferrofluid layer heated from below in the presence of a uniform vertical magnetic field. Recently, the effect of temperature dependent viscosity on Marangoni ferroconvection is considered by Nanjundappa et al. [16]. In the contemporary heat transfer research involving ferrofluids it is imperative to understand the mechanisms for control (suppression or augmentation) of Bénard–Marangoni ferroconvection which is

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useful in materials science processing under microgravity conditions. One of the effective mechanisms is to maintain a non-uniform basic temperature gradient across the ferrofluid layer. The main objective of the present study is to consider this aspect and analyze different forms of basic cubic temperature profiles on the onset of Bénard–Marangoni convection in a horizontal ferrofluid layer in the presence of a uniform vertical magnetic field. In investigating the problem, the effect of magnetic field dependent viscosity is also considered as the viscosity of the ferrofluid varies significantly with magnetic field. The lower boundary of the ferrofluid layer is rigid-isothermal, while the upper boundary is free, non-deformable and subject to a general thermal boundary condition. Moreover, at the upper free boundary the surface tension effects due to temperature are considered. The resulting eigenvalue problem is solved numerically by employing the Galerkin technique with Chebyshev polynomials as trial functions.

2. Mathematical formulation

We consider a Boussinesq ferrofluid layer of thickness d in the presence of a uniform vertical magnetic field H_0. The lower and upper boundaries are maintained at constant but different temperatures T_0 and $T_1 (< T_0)$ respectively. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and the z-axis is directed vertically upward. The gravitational force $(0, 0, -g)$ acts in the negative z-direction. Experimentally, it has been demonstrated that the viscosity η of ferrofluids varies significantly with respect to the magnetic field (Rosensweig et al. [17]). As a first approximation, for small field variation, the linear variation of viscosity with magnetic field has been considered in the form $\eta = \eta_0\left(1 + \vec{\delta} \cdot \vec{B}\right)$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity and is considered to be isotropic (Vaidyanathan et al. [18]), and η_0 is the viscosity of the fluid in the absence of magnetic field. Several investigators in the past have followed this assumption in the study of Marangoni convection and the free surface is assumed to be non-deformable (zero capillary number). At the upper free surface, the surface tension σ is assumed to vary linearly with temperature in the form

$$\sigma = \sigma_0 - \sigma_T(T - T_0), \tag{1}$$

where, σ_0 is the unperturbed value and $-\sigma_T$ is the rate of change of surface tension with temperature T.

The governing equations under the Oberbeck–Boussinesq approximation are given by the following:

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\nabla\right)\vec{q}\right] = -\nabla p + \rho\vec{g} + \nabla \cdot \left[\eta\left(\nabla\vec{q} + \nabla\vec{q}^{T}\right)\right] + \mu_0\left(\vec{M} \cdot \nabla\right)\vec{H} \tag{3}$$

$$\left[\rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt} + \mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H} \cdot \frac{D\vec{H}}{Dt} = k_t \nabla^2 T \tag{4}$$

$$\rho = \rho_0[1 - \alpha_t(T - T_a)]. \tag{5}$$

Maxwell's equations in the magnetostatic limit:

$$\nabla \cdot \vec{B} = 0, \nabla \times \vec{H} = 0, \vec{B} = \mu_0\left(\vec{M} + \vec{H}\right) \tag{6a, b, c}$$

$$\vec{M} = [M_0 + \chi(H - H_0) - K(T - T_0)]\left(\frac{\vec{H}}{H}\right). \tag{7}$$

Here, $\vec{q}$ is the velocity, p the pressure, ρ the fluid density, $\vec{M}$ the magnetization, $\vec{H}$ the magnetic field intensity, $\vec{B}$ the magnetic flux density, μ_0 the magnetic permeability of vacuum, k_t the thermal conductivity, $C_{V,H}$ the specific heat at constant volume and magnetic field, ρ_0 the reference density, α_t the thermal expansion coefficient, $T_a = (T_0 + T_1)/2$ the average temperature, $M_0 = M(H_0, T_0)$, $\chi = (\partial M/\partial H)_{H_0, T_0}$ the magnetic susceptibility, $K = -(\partial M/\partial T)_{H_0, T_0}$ the pyromagnetic coefficient, $H = \left|\vec{H}\right|$ and $M = \left|\vec{M}\right|$.

The basic state is quiescent and given by

$$\vec{q} = 0, p = p_b(z), \eta = \eta_b(z), -\frac{dT_b}{dz} = f(z), \vec{H}_b = \left[H_0 - \frac{K\beta z}{1 + \chi}\right]\hat{k}, \quad \vec{M}_b = \left[M_0 + \frac{K\beta z}{1 + \chi}\right]\hat{k}, \tag{8a}$$

where, $\beta = (T_l - T_u)/d$ is the temperature gradient, $\hat{k}$ is the unit vector in the vertical direction and the subscript b denotes the basic state. Following [14,19], the basic state cubic temperature profile is taken in the form

$$T_b = T_1 - a_1(z - d) - a_2(z - d)^2 - a_3(z - d)^3, \tag{8b}$$

where, a_1, a_2 and a_3 are constants. The principle of exchange of stability is assumed to be valid as there are no physical mechanisms to set up oscillatory motions. Following the standard linear stability analysis procedure, the stability equations in dimensionless form can then be shown to be (for details see [3,13,14]):

$$(1 + \Lambda)\left(D^2 - a^2\right)^2 W = Raa^2\Theta - RaM_1a^2 f(z)(D\Theta - \Theta) \tag{9}$$

$$\left(D^2 - a^2\right)\Theta = -(1 - M_2)f(z)W \tag{10}$$

$$\left(D^2 - a^2 M_3\right)\Phi = D\Theta, \tag{11}$$

where, $D = d/dz$ is the differential operator, $a = \sqrt{l^2 + m^2}$ the overall horizontal wave number, W the amplitude of vertical component of perturbed velocity, Θ the amplitude of perturbed temperature, Φ the amplitude of perturbed magnetic potential, $Ra = \alpha_t g\beta d^2/\kappa\nu$ the thermal Rayleigh number, $M_1 = \mu_0 K^2\beta/(1 + \chi)\alpha_t\rho_0 g$ the magnetic number, $M_2 = \mu_0 T_0 K^2/\rho_0 C_0(1 + \chi)$ the non-dimensional parameter, $M_3 = (1 + M_0/H_0)/(1 + \chi)$ the measure of nonlinearity of fluid magnetization parameter and $\Lambda = \delta\mu_0(M_0 + H_0)$ the MFD viscosity parameter and the non-dimensional temperature gradient $f(z)$ is given by

$$f(z) = a_1^* + 2a_2^*(z - 1) + 3a_3^*(z - 1)^2. \tag{12}$$

The typical value of M_2 for magnetic fluids with different carrier liquids turns out to be of the order of 10^{-6} ([3]) and hence its effect is neglected as compared to unity. Three types of basic temperature gradients are considered for discussion as mentioned below ([14,19]):

Reference steady-state temperature gradient	$f(z)$	a_1^*	a_2^*	a_3^*
Linear	1	1	0	0
Cubic 1	$3\ (z-1)^2$	0	0	1
Cubic 2	$0.66 + 1.02\ (z-1)^2$	0.66	0	0.34

The boundary conditions are:

$$W = DW = \Theta = \Phi = 0 \text{ at } z = 0 \tag{13a}$$

$$W = (1 + \Lambda)D^2W + a^2 Ma\Theta = D\Theta + Bi\Theta = \Phi = 0 \text{ at } z = 1, \tag{13b}$$

where, $Ma = \sigma_T\Delta Td/\mu\kappa$ is the Marangoni number $Bi = hd/k_t$ is the Biot number. The case $Bi = 0$ and $Bi \to \infty$ respectively correspond to constant heat flux and isothermal conditions at the upper boundary.

3. Method of solution

Eqs. (9)–(11) together with the boundary conditions constitute an eigenvalue problem with R_t or Ma as an eigenvalue. The eigenvalue problem is solved numerically using the Galerkin method with wave number as a perturbation parameter. Accordingly, the variables are written in a series of basis functions as

$$W(z) = \sum_{i=1}^{n} A_i W_i(z), \Theta(z) = \sum_{i=1}^{n} B_i\, \Theta_i(z), \Phi(z) = \sum_{i=1}^{n} C_i\, \Phi_i(z), \qquad (14)$$

where, A_i, B_i, C_i are unknown constants to be determined. The basis functions $W_i(z)$, $\Theta_i(z)$, $\Phi_i(z)$ are generally chosen such that they satisfy the corresponding boundary conditions but not the differential equations. For rigid ferromagnetic boundaries, they are chosen as

$$W_i = \left(z^4 - 2z^3 + z^2\right) T^*_{i-1}, \Theta_i = \left(z - z^2/2\right) T^*_{i-1}, \Phi_i = \left(z - z^2\right) T^*_{i-1}, \quad (15)$$

where, T_i^* ' s are the modified Chebyshev polynomials. The above trial functions satisfy all the boundary conditions except, $(1 + \Lambda) D^2 W + a^2 Ma \Theta = 0 = D\Theta + Bi\Theta$ at z = 1 but the residual from this is included as the residual from Eqs. (9)–(11). Multiplying momentum Eq. (9) by $W_i(z)$, energy Eq. (10) by $\Theta_i(z)$, magnetic potential Eq. (11) by $\Phi_i(z)$, performing the integration by parts with respect to z from z = 0 to z = 1 and using the boundary conditions, we obtain a system of linear homogeneous algebraic equations inA_i, B_i, C_i. A non-trivial solution exists if and only if the characteristic determinant is equal to zero. This leads to a relation

$$f\left(Ra, Ma, \Lambda, Bi, M_1, M_3, a_1^* a_2^*, a_3^*, a\right) = 0. \qquad (16)$$

The critical values of Ra_c and Ma_c are determined numerically with respect to wave numbera_c.

4. Numerical results and discussion

It may be noted that Eq. (16) leads to the characteristic equation giving the Marangoni number Ma or the thermal Rayleigh number Ra as a function of the wave number a, the parameters R_m, Bi, M_1, M_3, Λ and for different forms of basic temperature profiles (i.e., linear, cubic 1 and cubic 2 temperature profiles). Computations reveal that the convergence in finding the critical eigenvalue (Ma_c or Ra_c) crucially depends on the value of MFD viscosity parameter Λ and various forms of basic temperature profiles. The Galerkin method is used to find the eigenvalues numerically as this technique is found to be more convenient to tackle different forms of basic temperature profiles. The results presented here are for $i = j = 6$ the order at which the convergence is achieved, in general. To validate the numerical solution procedure used, a new magnetic parameter S, independent of the temperature gradient, was introduced in the form $R_m = Ra^2\, S$, where $S = \mu_0 K^2 \kappa \nu / (1 + \chi)\rho_0 g^2 \alpha^2 d^4$. The critical thermal Rayleigh number (Ra_c) and the critical magnetic Rayleigh number (R_{mc}) obtained for different values of Marangoni number Ma and for linear temperature profile (i.e., $a_1^* = 1$, $a_2^* = 0$ and $a_3^* = 0$) are exhibited in Table 1 when $S = 10^{-4}$ and $\Lambda = 0$ along with the results of Qin and Kaloni [8]. We note that there is a close agreement between the results of present analysis and those obtained by Qin and Kaloni [8] using different numerical approach (see Table 1). The values of magnetic parameters chosen are based on the physical parameters for a commercially available magnetic fluid EMG 905 produced by Ferrofluidics [20]; density ρ[kg/m^3] = 1.24 × 10^3, kinematic viscosity (27 °C) ν[m^2/s] = 12 × 10^{-6}, thermal diffusivity κ[m^2/s] = 8 × 10^{-8}, heat capacity c_p[J/kgK] = 1.47 × 10^3, coefficient of thermal expansion α_t[1/K] = 8.6 × 10^{-4}, susceptibility at low field $\chi = 1.9$, pyromagnetic coefficient at $H = 50$ kA/m[A/km] = 110 and mean particle diameter [nm] = 10.2. For such fluids the magnetic parameters have the following order of magnitude $M_1 \sim 10^{-4} - 10$ and $M_3 \geq 1$.

Table 1
Comparison of critical values of Ra_c and R_{mc} for different values of Ma and Bi for $S = 10^{-4}$, $M_3 = 1$ and $\Lambda = 0$ with linear temperature profile $f(z) = 1$.

Bi	Ma	Qin and Kaloni [8]		Present analysis	
		Ra_c	R_{mc}	Ra_c	R_{mc}
0	0	652.87	42.624	649.84	42.229
	20	493.23	24.426	491.07	24.115
	40	335.98	11.255	335.38	11.248
	60	171.44	2.939	171.90	2.955
	70	85.67	0.734	85.74	0.735
	79.61	0.00	0.00	0.00	0.00
10	0	892.06	79.577	903.55	81.640
	150	628.88	39.298	638.19	40.729
	250	418.23	17.492	427.29	18.258
	300	301.89	9.114	302.73	9.405
	350	176.10	3.101	177.74	3.158
	413.4	0.00	0.00	0.00	0.00

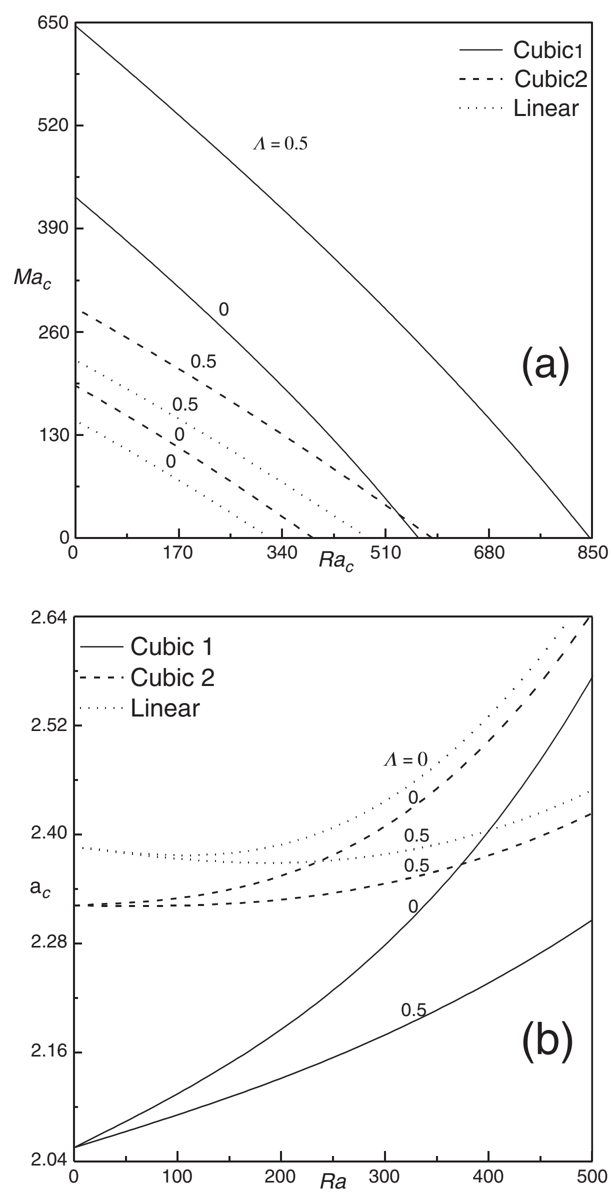

Fig. 1. (a). Locus of Ma_c and Ra_c for different values of Λ for $Bi = 2$, $M_1 = 2$ and $M_3 = 1$.(b). Variation of a_c verses Ra for different values of Λ for $Bi = 2$, $M_1 = 2$ and $M_3 = 1$.

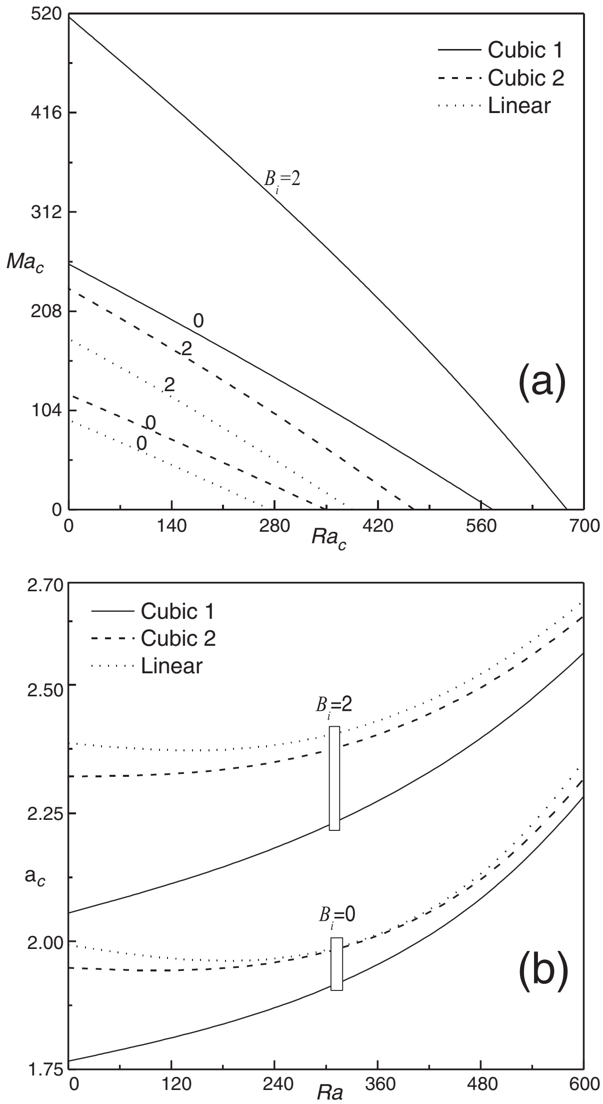

Fig. 2. (a). Locus of Ma_c and Ra_c for different values of Bi for $\Lambda = 0.2$, $M_1 = 2$ and $M_3 = 1$.(b). Variation of a_c verses Ra for different values of Bi for $\Lambda = 0.2$, $M_1 = 2$ and $M_3 = 1$.

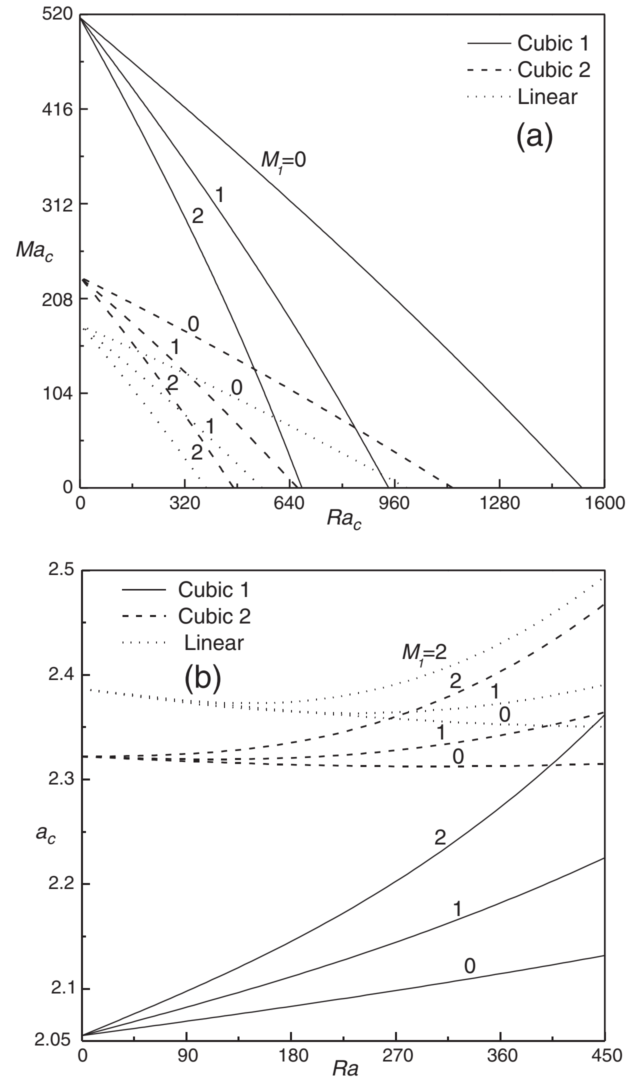

Fig. 3. (a). Locus of Ma_c and Ra_c for different values of M_1 for $\Lambda = 0.2$, $Bi = 2$ and $M_3 = 1$.(b). Variation of a_c verses Ra for different values of M_1 for $\Lambda = 0.2$, $Bi = 2$ and $M_3 = 1$.

We now look into the solution of the complete problem, which involves the effect of all the parameters Ra, Ma, R_m, Bi, Λ,M_1 and M_3 on the criterion for the onset of ferroconvection. The salient characteristics of these parameters are exhibited graphically in Figs. 1–7. These figures exhibit a tight coupling between the buoyancy, magnetic and surface tension forces. Fig. 1(a) shows the locus of the critical Marangoni number Ma_c and the Rayleigh number Ra_c for different values of MFD viscosity parameter Λ for $Bi = 2$, $M_1 = 2$ and $M_3 = 1$ for different forms of basic temperature profiles. It is seen that the curves are slightly convex and an increase in the Rayleigh number has a destabilizing effect on the system. That is, buoyancy and surface tension forces complement with each other. Also, an increase in the MFD viscosity parameter has a stabilizing effect on the system. Moreover, $(Ma_c \text{ or } Ra_c)_{\text{linear}} < (Ma_c \text{ or } Ra_c)_{\text{cubic 2}} < (Ma_c \text{ or } Ra_c)_{\text{cubic 1}}$ suggesting that the cubic 1 temperature profile is more stabilizing than cubic 2 and the linear temperature profile is the least stable. Thus, it is possible to control Bénard–Marangoni ferroconvection effectively by the choice of different forms of basic temperature profiles. The variation in a_c as a function of Ra is elucidated in Fig. 1(b) for different forms of basic temperature profile with different values of Λ. It may be noted that the critical wave number a_c increases monotonically with increasing Ra. Moreover, an increase in the value of Λ is to decrease a_c and thus its effect is to reduce the size of convection cells and also $(a_c)_{\text{cubic 1}} < (a_c)_{\text{cubic 2}} < (a_c)_{\text{linear}}$.

The plots in Fig. 2(a) represent the locus of Ma_c and Ra_c for different values of heat transfer coefficient Bi (i.e., Biot number) when $\Lambda = 0.2$, $M_3 = 1$ and $M_1 = 2$ for three different forms of basic temperature profiles. From the figure it is evident that an increase in the value of Bi is to increase Ra_c as well as Ma_c and thus its effect is to delay the onset of Bénard–Marangoni ferroconvection. This may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable. Fig. 2(b) represents the corresponding critical wave number a_c and it indicates that increase in the value of Bi and Ra is to increase a_c and thus their effect is to reduce the size of convection cells.

Fig. 3(a) presents the locus of Ra_c and Ma_c for different forms of basic temperature profiles and for various values of magnetic number M_1 when $\Lambda = 0.2$, $M_3 = 1$ and $Bi = 2$. The results for $M_1 = 0$ correspond to ordinary viscous fluid and it is observed that higher heating is required to have instability in this case. Besides, the curves of different M_1 become closer as the value of M_1 increases. Thus the combined effect of surface tension, magnetic and buoyancy forces is to reinforce together

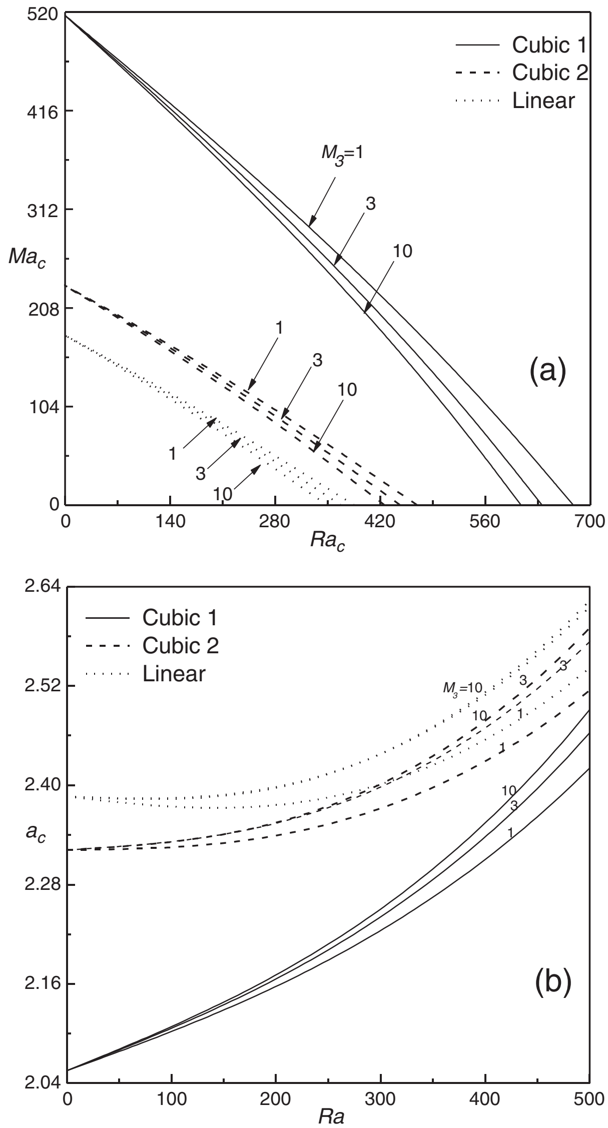

Fig. 4. (a). Locus of Ma_c and Ra_c for different values of M_3 for $\Lambda = 0.2$, $Bi = 2$ and $M_1 = 2$.(b). Variation of a_c verses Ra for different values of M_3 for $\Lambda = 0.2$, $Bi = 2$ and $M_1 = 2$.

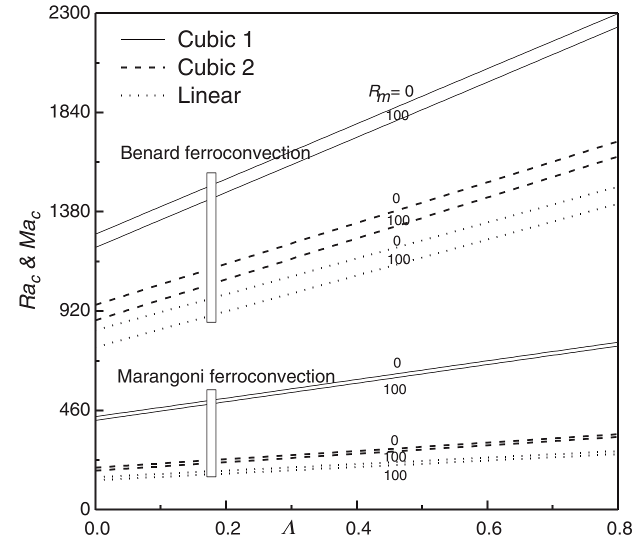

Fig. 6. Variation of Ma_c and Ra_c verses Λ with two values of R_m for $Bi = 2$ and $M_3 = 1$.

and to hasten the onset of Bénard–Marangoni ferroconvection compared to their effect in isolation. Although the critical wave number a_c remains invariant for different values of M_1 at lower values of Ra, it increases with further increase in the value of Ra (see Fig. 3(b)). Further inspection of Fig. 3(b) shows that the cubic 1 basic temperature profile is more stabilizing when compared to the other two profiles. The deviation in the critical wave number amongst different values of M_1, with increasing M_1 as well as Ra, is to increase the critical wave number a_c and hence to reduce the size of the convection cells.

The measure of non-linearity of fluid magnetization, denoted through the parameter M_3, on the onset of ferroconvection in a ferrofluid layer is depicted in Fig. 4(a). The curves of Ma_c as a function of Ra_c are shown in Fig. 4(a) for different values of M_3 are for $\Lambda = 0.2$, $Bi = 2$ and $M_1 = 2$. It can be seen that an increase in M_3 is to decrease Ra_c and Ma_c but only marginally and thus it has a destabilizing effect on the system. This may be due to the fact that the application of magnetic field makes the ferrofluid to acquire larger magnetization which in turn interacts with the imposed magnetic field and releases more energy to drive the flow faster. Hence, the system becomes unstable with a smaller temperature gradient as the value of M_3 increases. Alternatively, a

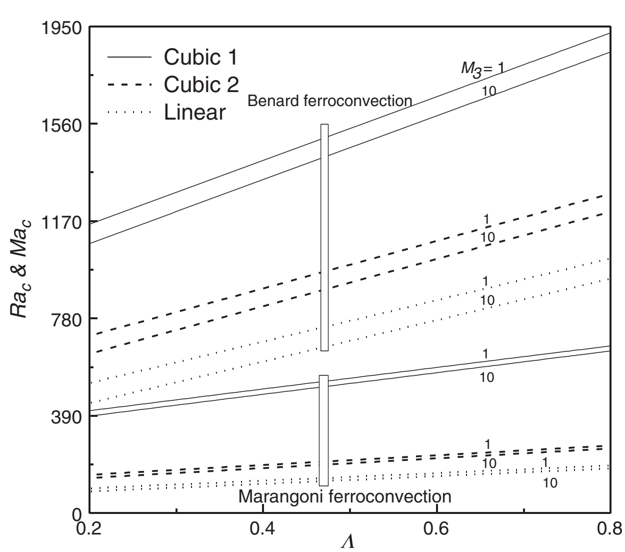

Fig. 5. Variation of Ma_c and Ra_c verses Λ with two values of M_3 for $R_m = 100$ and $Bi = 2$.

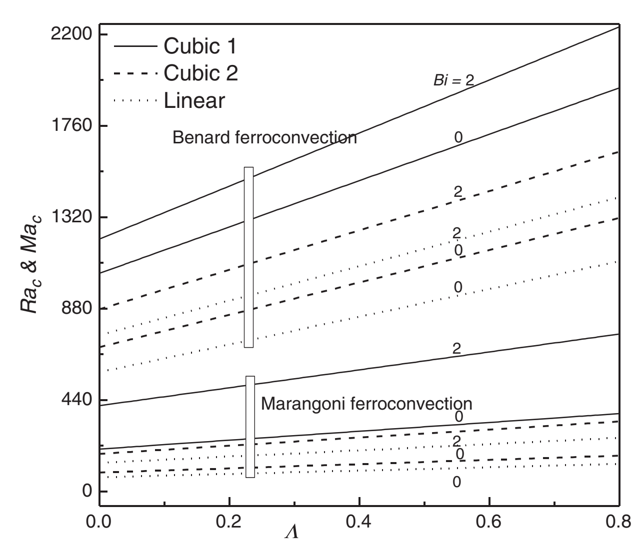

Fig. 7. Variation of Ma_c and Ra_c verses Λ with two values of Bi for $M_3 = 1$ and $R_m = 100$.

higher value of M_3 would arise either due to a larger pyromagnetic coefficient or larger temperature gradient. Both these factors are conducive for generating a larger gradient in the Kelvin body force field, possibly promoting the instability. The variation of a_c as a function of Ra is shown in Fig. 4(b) for different values of M_3. From the figure, we note that increasing M_3 and Ra is to increase a_c and hence to decrease the dimension of convection cells.

Figs. 5–7 show the isolation presence of surface tension and buoyancy forces on the onset of ferroconvection. These figures illustrate the variation of Ma_c (pure Marangoni ferroconvection, $Ra = 0$) and Ra_c (pure Bénard ferroconvection, $Ma = 0$) as a function of Λ for different values of physical parameters M_3, R_m and Bi for three types of basic temperature profiles. From the figures, it is seen that the effect of increasing Λ is the delay of the onset of Bénard/Marangoni ferroconvection and also an increase in M_3 (Fig. 5), R_m (Fig. 6) and a decrease in Bi (Fig. 7) which decreases the critical Rayleigh/Marangoni number and hastens the onset of ferroconvection. Further, $(Ma_c)_{\text{linear}<\text{cubic 2}<\text{cubic 1}} < (Ra_c)_{\text{linear}<\text{cubic 2}<\text{cubic 1}}$ as observed in ordinary viscous fluids.

5. Conclusions

The buoyancy, surface tension and magnetic forces reinforce each other to hasten the onset of ferroconvection. It is demonstrated that the onset of Bénard–Marangoni ferroconvection can be controlled effectively by the proper choice of basic temperature profiles. The cubic 1 basic temperature profile delays, while linear profile hastens the onset of Bénard–Marangoni ferroconvection. Besides, an increase in the value of Λ, Bi and a decrease in R_m and M_3 delays the onset of Bénard–Marangoni ferroconvection. The dimension of convection cells decreases with increasing Bi, and Λ as well as decrease in M_1 and M_3. The critical wave numbers for the cubic 1 basic temperature profile are lower than those of the cubic 2 and linear temperature profiles. The critical Marangoni numbers are lower than the critical Rayleigh numbers for all temperature profiles considered.

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