

Finite Element Development for Analysis of Smart Structures

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ABSTRACT: In the present study, a numerical solution based on the finite element method has been developed to analyze the deformation, electric potentials of a piezoelectric smart structure subjected to external mechanical or electrical loadings. The formulation of the finite element for static analysis has been presented based on isoparametric formulation. The element considered in the present study is eight noded hexahedral elements. A computer code based on the above formulation has been developed using MATLAB software to solve the three dimensional structures integrated with piezoelements. The experiments have been conducted on the piezoelectric smart structures consisting carbon epoxy beam and the results obtained were used for validating the present finite element code developed and found to have good agreement

Keywords: Smart structures, Piezoelectric, Finite Element and MATLAB

1. INTRODUCTION

Piezoelectric materials are widely used in smart structure application due to their high bandwidth, high output force, and compact size. The effective use of such material on different application can be done if their behavior is well understood [1-3]. The Finite element method can be used so that number of iteration can be done to understand and optimize the process. The effective numerical and experimental methods are needed to evaluate behavior and applications of piezoelectric structures subjected to electromechanical loading [4-7]. In the present work formulation of a finite element has been done. The implementation of the formulation has been done using MATLAB software. The developed code is validated using number of problems involving different materials such as isotropic, orthotropic and piezoelectric materials and a combination of these materials. One such example is piezo electric actuation of carbon epoxy beam has been discussed in detail in this paper. The present solution are compared with result obtained by experimental approaches

2. CONSTITUTIVE EQUATIONS

The basic constitutive equations for the linear theory of piezoelectric are as follows [8]

$$\begin{aligned} \{\sigma\} &= [C] \{\varepsilon\} - [d] \{E\} \\ \{D\} &= [d]^T \{\varepsilon\} + [b] \{E\} \end{aligned} \quad (1)$$

where $\{\sigma\} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\}^T$ is the stress vector, $\{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{23}, \varepsilon_{13}, \varepsilon_{12}\}^T$ Strain tensor, $\{E\} = \{E_1, E_2$

& $E_3\}$ the electric field, $\{D\} = \{D_1, D_2, \& D_3\}$, the electric displacement or Electric flux density vector, $[C]$ the elasticity constants matrix, $[b]$ the dielectric constants matrix, $[d]$ the piezoelectric coupling coefficients matrix or Piezoelectric constants.

3. FINITE ELEMENT MODELING OF PIEZOELECTRIC MATERIAL USING 3D HEXAHEDRAL ELEMENT

In the finite element formulation [9-10], the displacements u, v, w and the potentials are approximated as functions of the nodal displacements u_n and nodal potential ϕ_n where n is node number of the element and the nodal shape functions N_i such that

$$\begin{aligned} \{u\} &= [N_u] \{u_n\} \\ \{\phi\} &= [N_\phi] \{\phi_n\} \end{aligned} \quad (2)$$

The electric field vector of an element is represented as follows

$$\{E\} = \langle E_x, E_y, E_z \rangle^T \quad (3)$$

where

$$E_i = -\phi_{,i} \quad (4)$$

Consequently

$$\begin{aligned} \{\varepsilon\} &= [B_u] \{u_n\} \\ \{E\} &= -[B_\phi] \{\phi_n\} \end{aligned} \quad (5)$$

The matrix $[B_u]$ and $[B_\phi]$ contains the derivatives of the shape functions for the displacements and potentials which is written as follows

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$$[B_u] = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \\ 0 & \partial_z & \partial_y \\ \partial_z & 0 & \partial_x \\ \partial_y & \partial_x & 0 \end{bmatrix} [N_u] \quad (6)$$

$$[B_\phi] = \begin{bmatrix} \partial_x \\ \partial_y \\ \partial_z \end{bmatrix} [N_u] \quad (7)$$

The external virtual work done by the external mechanical and electrical forces is

$$\begin{aligned} \delta W_{mech}^e &= \delta u_n \{F\} \\ \delta W_{elect}^e &= \delta \phi_n \{Q\} \end{aligned} \quad (8)$$

Where $\{F\}$, $\{Q\}$ are external mechanical force and electrical charge vectors. Consequently, mechanical and electrical equilibrium equations can be written as follows

$$\int \langle \delta u_n \rangle [B_u]^T ([c][B_u]\{u_n\} + [d][B_\phi]\{\phi_n\}) dV = \langle \delta u_n \rangle \{F\} \quad (9)$$

and

$$\int \langle \delta \phi_n \rangle [B_\phi]^T ([d]^T [B_u] \{u_n\} - [b][B_\phi]\{\phi_n\}) dV = \langle \delta \phi_n \rangle \{Q\} \quad (10)$$

or

$$\begin{aligned} (\int [B_u]^T [c] [B_u] dV) \{u_n\} + (\int [B_u]^T [d] [B_\phi] dV) \{\phi_n\} &= \{F\} \\ (\int [B_\phi]^T [d]^T [B_u] dV) \{u_n\} + (-\int [B_\phi]^T [b] [B_\phi] dV) \{\phi_n\} &= \{Q\} \end{aligned} \quad (11)$$

The stiffness matrices are defined as follows

$$\begin{aligned} [K_{uu}] &= \int_v [B_u]^T [c] B_u dV \\ [K_{\phi\phi}] &= -\int_v [B_\phi]^T [b] B_\phi dV \\ [K_{\phi u}] &= \int_v [B_\phi]^T [d]^T B_u dV \\ [K_{u\phi}] &= [K_{\phi u}]^T \end{aligned} \quad (12)$$

$$\begin{aligned} [K_{uu}]\{u_n\} + [K_{u\phi}]\{\phi_n\} &= \{F\} \\ [K_{\phi u}]\{u_n\} + [K_{\phi\phi}]\{\phi_n\} &= \{Q\} \end{aligned}$$

$$\begin{bmatrix} [K_{uu}] & [K_{u\phi}] \\ [K_{\phi u}] & [K_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} \{u_n\} \\ \{\phi_n\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \quad (13)$$

$$[K] \{u\} = \{F\} \quad (14)$$

The above equations are included in formulation of element to add the capability for analyses of piezoelectric smart structures and the same is coded using MATLAB software.

4. EXPERIMENTS CONDUCTED USING PIEZOELECTRIC MATERIAL

For the purpose of validating the result of developed finite element code experiment was carried out. In this section a brief description of instrumentation used during the experimentation with smart materials is given. Figure 1 shows the different instrumentation and a smart beam used in current program [11-13]

- Function Generator: Used for generation of signal
- Piezo Sensing System : Used for sensing the voltage
- Active Band Pass Filter : Used for filtering the waves
- Piezo Actuation System :Used for generation of low current and high voltage source for driving piezo-actuator
- Dimmerstat with power transformer: Used to obtain variable DC supply
- Linear variable differential transformer: Used for measuring the displacement.



Figure 1: The Smart Structure and the Instrumentation used

In this experiment the base beam is made of carbon/epoxy composite. A piezoelectric patch is mounted on this beam and subjected to electric field. The geometry details and composite beam with PZT is shown in Figure 2 and Figure 3. This case study involves the combination of orthotropic material such as composite and piezoelectric materials.

4.1 Specimen Preparation

In the present work fabrication of carbon /epoxy composite beam was carried out for experimental studies involving piezoelectric materials. The resin LY 556 is heated at 60° C and harder HT 972 are mixed and acetone

is added to the mixture of resin and hardeners to achieve required consistency. The carbon fibers in fabric form will be used as reinforcement with 60% volume fraction. The impregnations of the fabric are carried out on a heated plate at 60° C and the entrapped air was removed with help of rollers. The curing is done for 3 hrs under 120° C and pressure of 7 bars and a post curing for 1 hr at 180° C. Then it is cut to required shape using diamond-cutting equipment. The composite is a cross ply lamination and the fibers orientation angles in the alternative layers are 0°/90°/0°/90°.

The piezoelectric actuator is bonded on to composite beam after cleaning the surface and marking the position. The leads of the pig tail provided with the piezoelectric materials are connected to the instruments by using another terminal and the specimen is clamped to conduct the experiment.

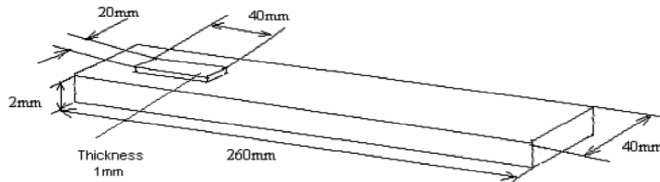


Figure 2: Geometric Details



Figure 3: Composite Beam with PZT Actuator

4.2 Experimental Procedure

The block diagram of the experiment is similar to one shown in the Figure 4.

The procedure for conducting the experiment is as follows:

- The composite beam with piezoelectric material is clamped tightly at one side as cantilever.

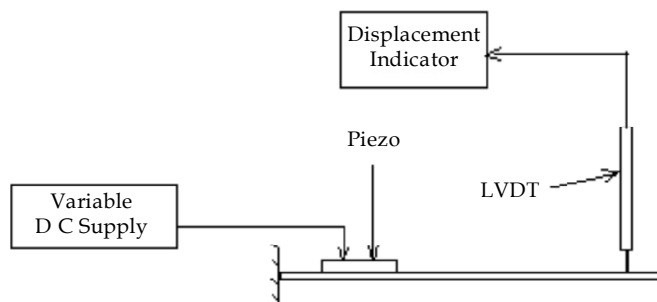


Figure 4: The Block Diagram of the Experiment for Smart Beam

- The variable DC supply (Dimmerstat with power transformer, variable DC supply 0-200V and 1A) is applied to PZT on the beam
- The supplied voltage to the PZT on beam causes it to deflect.
- The deflection thus obtained at tip the beam is measured using linear variable differential transformer

5. FINITE ELEMENT ANALYSIS

The finite element model of the smart beam with same configuration as that of the beam used in experiment is considered for the purpose of comparison. The finite element model of the composite beam is shown in the Figure 5. The convergence studies are carried out by increasing the number of elements in the finite element mesh. The results of finite element model consisting of 260 elements and 501 nodes and the experimental results are given in the Table 1. The results of present code are in good agreement with experimental results.

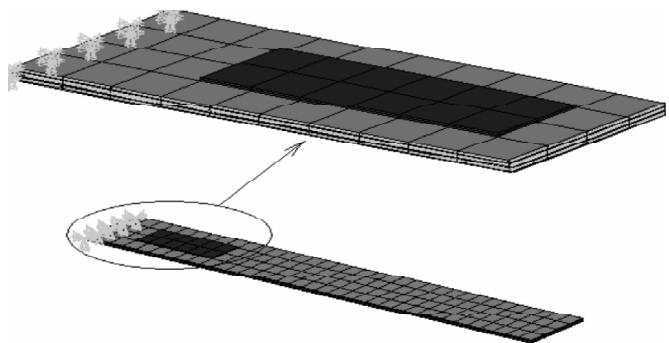


Figure 5: Finite Element Model

Table 1
Comparison of Deflection of Smart Carbon Composite Beam

Sl. No.	Voltage applied (Volts)	Deflection (μ m)	
		Experiment	Present code
1	50	16	15.7
2	100	31	30.6
3	150	46	45.3
4	200	61	60.4

6. CONCLUSIONS

The finite element code for analysis of smart structures is developed using MATLAB programming language. Also experiment was carried out on the smart carbon epoxy beam with piezoelectric material attached and related instruments are discussed. Simple experiments are devised to obtain static behaviour of the smart structure. These experimental results were also useful to validate the present code. The developed code is validated by comparing the results of experiments

carried out by using piezoelectric material in combination with orthotropic materials such as composite. The finite element code developed for structure analysis and electromechanical analysis of the smart composite structure is found to have good agreement with the experimental results.

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