Effects of Suction and Freestream Velocity on a Hydromagnetic Stagnation-Point Flow and Heat Transport in a Newtonian Fluid Toward a Stretching Sheet

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Forced flow of an electrically conducting Newtonian fluid due to an exponentially stretching sheet is studied numerically. Free stream velocity is present and so is suction at the sheet. The governing coupled, nonlinear, partial differential equations of flow and heat transfer are converted into coupled, nonlinear, ordinary differential equations by similarity transformation and are solved numerically using shooting method, and curve fitting on the data is done by differential transform method together with Padé approximation. Prescribed exponential order surface temperature (PEST) and prescribed exponential order surface heat flux are considered for investigation of heat transfer related quantities. The influence of Chandrasekhar number, suction/injection parameter, and freestream parameter on heat transport is presented and discussed. Coefficient of friction and heat transport is also evaluated in the study. The results are of interest in extrusions and such other processes. [DOI: 10.1115/1.4033460]

Keywords: exponential stretching, freestream, hydromagnetic flow, heat transfer, shooting method, differential transform

1 Introduction

The flow past a stretching sheet has several important engineering applications, viz., polymer processing unit of a chemical engineering plant, metal working process in metallurgy, hot rolling, wire drawing, glass fiber, and drawing of plastic films. Sakiadis [1–3] initiated the theoretical study of these applications by considering the boundary layer flow over a continuous solid surface moving with constant speed. Crane [4] studied the steady two-dimensional boundary layer flow caused by the stretching sheet. Thereafter, various aspects of the above boundary layer problem on continuous moving surface were considered by many researchers (for example, Grubka and Bobba [5], Chen and Char [6], Siddheshwar and Mahabaleshwar [7], and references therein).

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Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. During this process of drawing, the strips are sometimes stretched. The properties of the final product depend on the rate of cooling.

Sanjayanand and Khan [8] analyzed the effects of various physical parameters such as local viscoelastic parameter, Prandtl number, local Reynolds number, local Eckert number, and Schmidt number on momentum, heat, and mass transfers in a viscoelastic boundary layer fluid flow over an exponentially stretching continuous sheet. The effect of radiation on the boundary layer flow and heat transfer of a viscous fluid over an exponentially stretching sheet is studied by Sajid and Hayat [9]. The homotopy method is employed to determine the convergent series expansions of velocity and temperature. Khan et al. [10] presented a combination of variational iterative method and Padé approximants to investigate the two-dimensional exponential stretching sheet problem. Similarity transformation is used to convert the nonlinear partial differential equations corresponding to the momentum and heat equations into nonlinear ordinary differential equations. Numerical solutions of these equations are obtained by shooting method. Mukhopadhyay [11,12] analyzed the magnetohydrodynamic (MHD) boundary layer flow and heat transfer toward an exponentially stretching sheet embedded in a thermally stratified medium in the presence of a magnetic field and subject to suction. A steady two-dimensional boundary layer flow of a viscous incompressible radiating fluid over an exponentially stretching sheet, in the presence of transverse magnetic field, is studied by Reddy and Reddy [13]. Recently, many researchers (e.g., Jat and Gopi Chand [14] and Wong et al. [15]) studied the steady two-dimensional laminar flow of a viscous incompressible fluid over an exponentially stretching/shrinking permeable sheet with viscous dissipation and radioactive heat flux.

Sparse literature is available on stretching sheet problems, stretching exponentially, in a fluid that is flowing with uniform velocity (called generally as uniform freestream) (see Refs. [16–18] and references therein). Bhattacharya and Vajravelu [19] investigated the stagnation-point flow and heat transfer over an exponentially shrinking sheet.

In the present work, we study the boundary layer flow behavior and heat transfer of a Newtonian fluid past an exponentially stretching sheet in the presence of an external magnetic field, suction (injection), and a freestream. Cooling of perforated films is an important problem as we know, and hence, the rate of cooling is an important issue in these problems. Horizontal freestream (bath) and transverse suction/injection as mechanisms for faster cooling can be of prime importance in such problems. Using boundary layer approximation and a similarity transformation in exponential form, the governing mathematical equations are transformed into coupled, nonlinear ordinary differential equations, which are then solved numerically by shooting method.

2 Mathematical Formulation

We consider a steady, two-dimensional boundary layer flow due to a stretching sheet of an incompressible, weakly electrically conducting Newtonian fluid. The liquid is at rest and the motion is effected by pulling the sheet at both ends with equal force parallel to the sheet and with speed u, which varies exponentially with the distance from the slit (see Fig. 1). The boundary layer equations governing the flow and heat transfer, assuming the viscous dissipation to be negligible, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\mu_m^2\sigma H^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\left(\frac{\mu}{\rho C_p}\right)\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

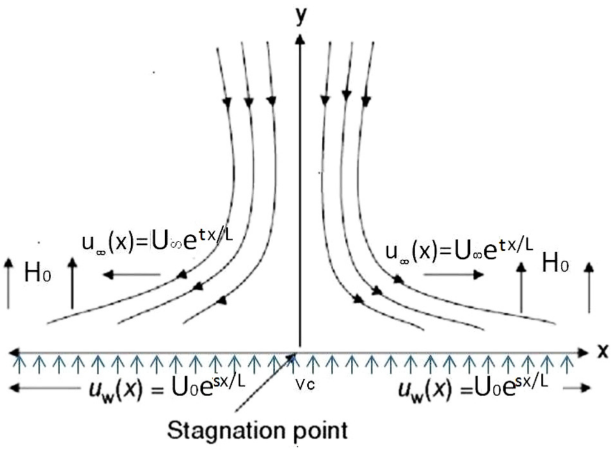

Fig. 1 Schematic of the stretching sheet

The last term in Eq. (2) arises from the Lorentz force in the case of a weakly electrically conducting liquid. We employ the following boundary conditions on velocity:

$$\left.\begin{aligned} u = u_w(x) = U_0 e^{s\frac{x}{L}}, v = v_w \quad &\text{at} \quad y = 0 \\ u \to U_\infty e^{t\frac{x}{L}} \quad &\text{as} \quad y \to \infty \end{aligned}\right\} \tag{4}$$

We consider general nonisothermal and variable heat flux boundary conditions to solve Eq. (3). We take up the thermal boundary conditions and the energy equation in Sec. 4. In Sec. 3 we consider the solution of the momentum equation (2) subject to the boundary conditions (4).

3 Local Similarity Transformation of the Momentum Equation

We now make Eqs. (1), (2), and (4) dimensionless using the following definition:

$$\left.\begin{aligned} (X, Y) = \frac{(x, y\sqrt{\text{Re}})}{L}, (U, V, V_w) = \frac{(u, v\sqrt{\text{Re}}, v_w\sqrt{\text{Re}})}{U_0} \\ \text{Re} = \frac{U_0 L}{\nu}, P = \frac{p}{\rho U_0^2} \end{aligned}\right\} \tag{5}$$

and also introduce the stream function $\psi(X, Y)$ as follows:

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \tag{6}$$

to get the following equations:

$$\frac{\partial \psi}{\partial Y}\frac{\partial^2 \psi}{\partial X \partial Y} - \frac{\partial \psi}{\partial X}\frac{\partial^2 \psi}{\partial Y^2} = \lambda_e^2 t e^{2tX} + \frac{\partial^3 \psi}{\partial Y^3} - Q\left(\frac{\partial \psi}{\partial Y} - \lambda_e e^{tX}\right) \tag{7}$$

$$\left.\begin{aligned} \frac{\partial \psi}{\partial Y} = e^{sX}, \frac{\partial \psi}{\partial X} = -V_w \quad &\text{at} \quad Y = 0 \\ \frac{\partial \psi}{\partial Y} = \lambda_e e^{tX} \quad &\text{as} \quad Y \to \infty \end{aligned}\right\} \tag{8}$$

where $Q = (\mu_m^2 \sigma H^2 L)/(\rho U_0)$ is the Chandrasekhar number and $\lambda_e = U_\infty/U_0$ (freestream parameter).

We now outline the procedure of obtaining a local similarity solution for the exponential stretching sheet problem by using the transformation

$$\psi(X, \eta) = A_e e^{mX} f(\eta) \quad \text{and} \quad \eta = B_e e^{nX} Y \tag{9}$$

where $A_e, B_e, m,$ and n are to be determined. We note that a similarity transformation is not possible to use in this problem.

Using the transformation (9) in Eqs. (7) and (8), we get the following boundary valve problem:

$$\begin{aligned} f''' + \frac{A_e}{B_e} m e^{(m-n)X} f f'' - \frac{A_e}{B_e}(m+n) e^{(m-n)X} (f')^2 \\ - \frac{Q e^{-2nX}}{B_e^2} f' + \frac{Q \lambda_e e^{tX}}{A_e B_e^3 e^{(m+3n)X}} + \frac{\lambda_e^2 t e^{2tX}}{A_e B_e^3 e^{(m+3n)X}} = 0 \end{aligned} \tag{10}$$

$$\left.\begin{aligned} f(0) = \frac{-V_w}{A_e m e^{mX}}, f'(0) = \frac{e^{sX}}{A_e B_e e^{(m+n)X}} \quad &\text{at} \quad \eta = 0 \\ f'(\infty) = \frac{\lambda_e e^{tX}}{A_e B_e e^{(m+n)X}} \quad &\text{as} \quad \eta \to \infty \end{aligned}\right\} \tag{11}$$

Equations (10) and (11) dictate the choice $A_e = B_e = m = n = 1$ and $s = t = 2$. With this choice, Eqs. (10) and (11) reduce to

$$f''' + f f'' - 2(f')^2 - Q_x f' + (Q_x \lambda_e + 2\lambda_e^2) = 0 \tag{12}$$

$$\left.\begin{aligned} f(0) = -V_{wx}, f'(0) = 1 \quad &\text{at} \quad \eta = 0 \\ f'(\infty) = \lambda_e \quad &\text{as} \quad \eta \to \infty \end{aligned}\right\} \tag{13}$$

where $Q_x = Q/e^{2X}$ (local Chandrasekhar number) and $V_{wx} = V_w/e^X$ (local suction/injection parameter).

4 The Energy Equation

Using Eqs. (5) and (6) in Eq. (3), we get the following nondimensionalized heat equation:

$$\frac{\partial \psi}{\partial Y}\frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X}\frac{\partial \theta}{\partial Y} = \frac{1}{\text{Pr}}\frac{\partial^2 \theta}{\partial Y^2} + \frac{\nu U_0 \text{Re}}{C_p L (T_w - T_\infty)}\left(\frac{\partial^2 \psi}{\partial Y^2}\right)^2 \tag{14}$$

where

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{15}$$

In order to solve Eq. (14), we consider the following nonisothermal and variable heat flux boundary conditions for exponential stretching sheet problem.

4.1 PEST.

The boundary conditions in this case are

$$\left.\begin{aligned} T = T_w = T_\infty + (T_w - T_\infty) e^{\left(\frac{x}{L}\right)} \quad &\text{at} \quad y = 0 \\ T \to T_\infty \quad &\text{as} \quad y \to \infty \end{aligned}\right\} \tag{16}$$

In terms of dimensionless temperature θ, we can write

$$\left.\begin{aligned} \theta = e^X \quad &\text{at} \quad Y = 0 \\ \theta = 0 \quad &\text{as} \quad Y \to \infty \end{aligned}\right\} \tag{17}$$

Equations (14) and (17) read as

$$\Theta_e'' + \text{Pr} f \Theta_e' - \text{Pr} f' \Theta_e + E_{\text{PEST}} \text{Pr} (f'')^2 = 0 \tag{18}$$

$$\left.\begin{aligned} \Theta_e = 1 \quad &\text{at} \quad \eta = 0 \\ \Theta_e = 0 \quad &\text{as} \quad \eta \to \infty \end{aligned}\right\} \tag{19}$$

where $\Theta_e(\eta) = e^{-X}\theta$ and $E_{\text{PEST}} = (\nu U_0 \text{Re} e^{3X})/[C_p L (T_w - T_\infty)]$ is the local Eckert number of PEST case.

4.2 Prescribed Exponential Order Heat Flux (PEHF).

In this case, the boundary conditions assumed on T are

$$\left.\begin{aligned} -k\frac{\partial T}{\partial y} = T_1 e^{\left(\frac{3x}{2L}\right)} \quad &\text{at} \quad y = 0 \\ T \to T_\infty \quad &\text{as} \quad y \to \infty \end{aligned}\right\} \tag{20}$$

Following the procedure of Sec. 1 on Eq. (14), we get

$$\Phi_e'' + \Pr f\Phi_e' - \frac{\Pr}{2}f'\Phi_e + E_{\mathrm{PEHF}}\Pr(f'')^2 = 0 \tag{21}$$

$$\left.\begin{array}{lll}\Phi_e' = -1 & \text{at} & \eta = 0 \\ \Phi_e = 0 & \text{as} & \eta \to \infty\end{array}\right\} \tag{22}$$

where

$$\Phi_e(\eta) = be^{\frac{-X}{2}}\theta, \quad b = \frac{k(T_w - T_\infty)\sqrt{\mathrm{Re}}}{T_1 L}$$

and $E_{\mathrm{PEHF}} = \{2\nu U_0 \mathrm{Re}e^{[(7X)/2]}b\}/[C_pL(T_w - T_\infty)]$ is the local Eckert number of PEHF case. The PEST and PEHF boundary conditions are essentially nonisothermal and variable heat flux boundary conditions, respectively. In view of the fact that "∞" in this problem is around 6, the x/L values are not very large. This essentially means that varying the sheet temperature or the imposed flux does not have the large variations. Such boundary conditions support a local similarity solution. Due to the adoption of such conditions and the inherently nonlinear equations, the problem cannot have an analytical solution.

The two boundary-value problems arising in the problem are solved using shooting method. Curve fitting for the obtained data is done using differential transform method-Padé method [20]. In general, for most parameters' values, {3,7} are a good Padé approximant for temperature. The accuracy chosen for the solution in the problem is 10^{-6}.

The local skin-friction coefficient C_f is given by

$$\left.\begin{array}{l} C_f = \dfrac{\tau_w}{\rho(u_w)^2} \\ \text{where} \quad \tau_w = \mu\left(\dfrac{\partial u}{\partial y}\right)_{y=0}\end{array}\right\} \tag{23}$$

Solving Eq. (23), we get

$$C_f = \frac{1}{e^X\sqrt{\mathrm{Re}}}f''(0) \tag{24}$$

The rate of heat transfer in terms of the Nusselt number, Nu_x, at the wall is given by

$$\mathrm{Nu}_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{25}$$

where $q_w = -k(\partial T/\partial y)_{y=0}$.

Equation (25) can now be rewritten as

$$\mathrm{Nu}_x = \begin{cases} -\sqrt{\mathrm{Re}}Xe^X\Theta_e'(0) & \text{in PEST} \\ \dfrac{-\sqrt{\mathrm{Re}}X}{b}e^{\frac{X}{2}}\Phi_e'(0) & \text{in PEHF}\end{cases} \tag{26}$$

5 Results and Discussion

The boundary-value problems involving semi-infinite intervals arising in the exponentially stretching sheet problems (ESSPs) are solved for a convergent solution by using a combination of the shooting and differential transform methods. Table 1 shows the concurrence of our results with that of Mustafa et al. [21] in the absence of freestream. The results on the skin friction and local Nusselt number are presented through Tables 2 and 3. We have separate notation for temperatures in the stretching sheet problem. The terms Θ_e and Φ_e represent the scaled temperatures for the PEST and PEHF cases, respectively, of ESSP. At this point, we note that the effect of magnetic field is to diminish the magnitude of boundary layer velocity, and the freestream helps in the cooling of the stretching sheet. Thus, the desired rate of cooling of the sheet can be achieved by a proper combination of the strength of the magnetic field and the freestream velocity. This might possibly help scientists in having a desired property in the stretching film and is thus a useful piece of information in an extrusion process.

From Table 2, it is clear that the following results are true:

(1) Effect of suction/injection parameter is to decrease the skin-friction coefficient.
(2) The skin-friction coefficient decreases with increase in λ_e.
(3) The skin-friction coefficient is directly proportional to Q.

Table 1 Comparison of values of $-f''(0)$ with those of Mustafa et al. [21] when λ_e=0

Ref. [21]	Present value
1.281810	1.281835

Table 2 Values of $-f''(0)$ for different parameters

Parameters	$-f''(0)$
V_{wx}	$Q_x = 0$ and $\lambda_e = 0.2$
−1	1.655993
0	1.195144
1	0.867681
λ_e	$Q_x = 0$ and $V_{wx} = 0$
0.1	1.253608
0.2	1.195144
0.4	1.005649
Q_x	$\lambda_e = 0.2$ and $V_{wx} = 1$
0	0.867681
1	1.098032
2	1.297733

Table 3 Values of $-\Theta_e'(0)$ and $\Phi_e(0)$ for different parameters

Parameters	$-\Theta_e'(0)$(PEST)	$\Phi_e(0)$(PEHF)
$E_{\mathrm{PEST}}/E_{\mathrm{PEHF}}$	$Pr = 1, \lambda_e = 0.1$, $V_{wx} = 1$ and $Q_x = 0$	
0.5	0.348653	3.084491
1	0.178513	3.419940
1.5	0.008379	3.755404
Pr	$E_{\mathrm{PEST}}/E_{\mathrm{PEHF}} = 0.5, \lambda_e = 0.1$, $V_{wx} = 1$ and $Q_x = 1$	
0.5	0.284127	4.526484
1	0.348653	3.75839
1.5	0.376686	3.509042
λ_e	$E_{\mathrm{PEST}}/E_{\mathrm{PEHF}} = 0.5, \mathrm{Pr} = 2$, $V_{wx} = 1$ and $Q_x = 1$	
0.1	0.389775	3.400175
0.4	0.584437	2.567514
1	0.898777	1.665495
V_{wx}	$E_{\mathrm{PEST}}/E_{\mathrm{PEHF}} = 0.5, \mathrm{Pr} = 2$, $\lambda_e = 0.1$ and $Q_x = 1$	
–1	2.037869	0.688278
0	0.910172	1.338882
1	0.389775	3.400175
Q_x	$E_{\mathrm{PEST}}/E_{\mathrm{PEHF}} = 0.5, \mathrm{Pr} = 2$, $\lambda_e = 0.1$ $V_{wx} = 1$	
0	0.535806	2.802307
1	0.389775	3.400175
2	0.264183	3.964549

The following results are extracted from Table 3:

(1) It is observed that $-[\Theta_e'(0)]$ decreases as E_{PEST} increases, and $\Phi_e(0)$ has the opposite behavior to that of $-[\Theta_e'(0)]$.
(2) In regard to the influence of Pr, we observe that $-[\Theta_e'(0)]$ increases with increase in Pr, and $\Phi_e(0)$ decreases with increase in Pr.
(3) The effect of λ_e on the temperature profiles is similar to that of Pr.
(4) Increase in suction/injection parameter decreases $-[\Theta_e'(0)]$ and increases $\Phi_e(0)$.
(5) $-[\Theta_e'(0)]$ decreases with increase in Q, and $\Phi_e(0)$ increases with increase in Q.

6 Conclusion

(1) PEHF is better suited for faster cooling of the stretching sheet than the PEST boundary condition. This is because the boundary layer temperature is higher in PEST case compared to PEHF case.
(2) The dissipation effect on local Nusselt number is less pronounced compared to that of other parameters.
(3) Freestream takes away heat, and hence, we observe that cooling is faster in the presence of freestream.
(4) A combination of the magnitude of the freestream velocity and the strength of the applied magnetic field cools the extrusion at a required rate, and thus can be used to impart a desired property to the extrudate.

Nomenclature

C = specific heat
C_f = skin-friction coefficient
H = uniform applied magnetic field
L = reference length
Nu_x = Nusselt number
p = pressure
Pr = Prandtl number
Re = local Reynolds number
T = temperature
u = dimensional horizontal velocity component
U = nondimensional horizontal velocity component
v = dimensional vertical velocity component
V = nondimensional vertical velocity component
x = dimensional horizontal Cartesian coordinate
X = dimensionless horizontal Cartesian coordinate
y = dimensional vertical Cartesian coordinate
Y = dimensionless vertical Cartesian coordinate

Greek Symbols

α = thermal diffusivity
η = similarity variable
θ = dimensionless temperature of PEST
Θ = scaled temperature of PEHF
μ = dynamic viscosity
μ_m = magnetic permeability
ν = kinematic coefficient of viscosity
ρ = density
σ = electrical conductivity
τ = shear stress
ϕ = dimensionless temperature of PEHF in exponential stretching sheet problem
Φ = scaled temperature of PEHF
ψ = stream function

Subscripts

e = value corresponding to exponential stretching
p = value at constant pressure
w = value at wall
0 = reference value
$'$ = differentiation with respect to η
∞ = value at infinity

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