

ADMS, Volume 1, Issue 1, 2012, Pages 72-82
© 2012 Advancement and Development in Technology International

TEST PROCEDURES FOR DETECTING ‘MORE NBU-NESS’ PROPERTY OF LIFE DISTRIBUTIONS

PARAMESHWAR V. PANDIT¹ and C. M. MATH²

¹Department of Statistics
Bangalore University
Bangalore-560056
India

²Department of Statistics
KLEs G. H. College
Haveri, Karnataka
India
e-mail: panditpv12@gmail.com
e-mail: panditpv@rediffmail.com
e-mail: chandrashekar.math@gmail.com

Abstract

Testing the hypothesis of no ageing against positive ageing has been considered by many authors in the literature. However, very few tests procedures for detecting whether a life distribution possesses ‘more positive ageing’ than the other distribution are developed. Hollander, Park and Proschan (1986) proposed a test procedure to detect ‘More NBU-ness’ property of life distributions, Pandit and Gudaganavar (2009) developed a procedure which is an improvement over the test due to Hollander, Park and Proschan (1986). In this paper, a test is developed to decide whether one life distribution possesses more ‘new better than used’ (NBU) property than does another life distribution. The asymptotic performance of the test

2010 Mathematics Subject Classification: 62G10, 62N05.

Keywords and phrases: More NBU property, U-statistics, Two-sample test, Asymptotic

Received November 04, 2011

procedure is evaluated in terms of Pitman asymptotic relative efficiency, It is found that new test performs better than the tests in the literature.

1. Introduction

In the literature, more attention is given to testing the hypothesis of no ageing against positive ageing. Testing against NBU have been discussed by Hollander and Proschan (1972), Koul (1977), Kumazawa (1983), Deshpande & Kochar (1983), Ahmad (1994, 2004) among others. Recently, Pandit and Anuradha (2007 a, b, c) have proposed new test procedures based on linear (and/convex) combination of two statistics, In this paper, we propose a new class of test statistics for the problem of testing exponentiality against NBU class of alternatives. However, very few tests procedures for detecting whether a life distribution possesses ‘more positive ageing’ than the other distribution are developed, Hollander, Park and Proschan (1986) proposed a test procedure to detect ‘More NBU-ness’ property of life distributions. Pandit and Gudaganavar (2009, 2010) developed two procedures which are the improvements over the test due to Hollander, Park and Proschan (1986) based two different measures. In this paper, a test is developed to decide whether one life distribution possesses more ‘new better than used’ (NBU) property than does another life distribution. The NBU property is given in the following definition.

Definition: A life distribution F is new better than used if

$$\bar{F}(x+y) \leq \bar{F}(x)\bar{F}(y), \quad x, y \geq 0 \quad (1)$$

where $\bar{F} = 1 - F$. The dual concept of new worse than used (NWU) is defined by reversing the inequality in (1).

Inequality (1) may be interpreted as stating that a used item of any age had stochastically smaller residual life length than does a new item. We refer Barlow and Proschan (1981) for discussion of the NBU class and its basic role in the study of maintenance policies. However, the situations where tests for detecting degree of ‘NBU-ness’ is useful, can be found in Hollander, Park and Proschan (1986).

In section 2, we propose a test for two sample problem and present the asymptotic distribution of the two sample statistic. The asymptotic relative efficiency the proposed two sample test relative to the existing test is considered in section 3. In section 4, we present the remarks and conclusions.

2. The Proposed Two-sample More NBU Test

Let X_1, X_2, \dots, X_m and Y_1, Y_2, \dots, Y_n denote two random samples from continuous life distributions F and G , respectively. We want to develop test statistic for testing the null hypothesis

$$H_0 : F = G \text{ (the common distribution is not specified)}$$

versus the alternative hypothesis

$$H_1 : F \text{ is 'more NBU' than } G.$$

Consider the parameter, for an integer $m > 1$

$$\gamma(F, G) = \gamma(F) - \gamma(G),$$

$$\text{where } \gamma(F) = \int_0^{\infty} \bar{F}^k\left(\frac{x}{k}\right) dF(x) \text{ and } \gamma(G) = \int_0^{\infty} \bar{G}^k\left(\frac{y}{k}\right) dG(y).$$

Here, $\gamma(F)$ and $\gamma(G)$ can be considered as the measure of degree of the NBU-ness. Ahmed (2004) test used this measure as basis for their test statistic. If $F(G)$ belongs to NBU, then $\gamma(F) > \frac{1}{2} \left(\gamma(G) > \frac{1}{2} \right)$ and $\gamma(F, G)$ can be taken as a measure by which F is 'more NBU' than G . Under H_0 , $\gamma(F, G) = 0$ and it is strictly greater than zero under H_1 .

An unbiased estimator for $\gamma(F, G)$, which is defined as

$$T_{m, n}^k = T_m - T_n,$$

where T_m and T_n are U -statistics with kernels of degree $(k + 1)$ which are defined as

$$h_1(x_1, x_2, \dots, x_{k+1}) = \frac{1}{k+1} \sum_{i=1}^{k+1} I(\min(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_{k+1}) > mx_i)$$

and

$$h_2(y_1, y_2, \dots, y_{k+1}) = \frac{1}{k+1} \sum_{j=1}^{k+1} I(\min(y_1, \dots, y_{j-1}, y_{j+1}, \dots, y_{k+1}) > my_j)$$

respectively. Here $I(\cdot)$ is an indicator function.

Asymptotic normality of the test $T_{m,n}^k$. In this subsection, we study the asymptotic distribution of $T_{m,n}^k$. For that define

$$\xi_1(F) = E[\psi_1(X_1)]^2 - [\gamma(F)]^2, \text{ where}$$

$$\psi_1(x_1) = E\{h_1(x, X_2, \dots, X_{k+1})\}$$

$$= \frac{1}{k+1} \left\{ KP \left(\min(x, X_2, \dots, X_k) > \frac{X_{k+1}}{k} \right) + P \left(\min(X_2, \dots, X_{k+1}) > \frac{x}{k} \right) \right\}$$

$$= \frac{1}{k+1} \{k.A + B\}.$$

Here,

$$\begin{aligned} A &= P \left(\min(x, X_2, \dots, X_k) > \frac{X_{k+1}}{k} \right) \\ &= \int_0^x \left[\bar{F}^{k-1} \left(\frac{u}{k} \right) - \bar{F}^{k-1}(x) \right] dF(u) + \bar{F}^{k-1}(x) \cdot F(kx) \end{aligned}$$

and

$$\begin{aligned} B &= P \left(\min(X_2, \dots, X_{k+1}) > \frac{x}{k} \right) \\ &= \bar{F}^k \left(\frac{x}{k} \right). \end{aligned}$$

Next, $\xi_1(G)$ is defined as

$$\begin{aligned}\xi_1(G) &= E[\Psi_1^*(Y_1)]^2 - [\gamma(G)]^2, \text{ where} \\ \Psi_1^*(y_1) &= E\{h_2(y, Y_2, \dots, Y_{k+1})\} \\ &= \frac{1}{k+1} \left\{ kP\left(\text{Min}(y, Y_2, \dots, Y_k) > \frac{Y_{k+1}}{k}\right) + P\left(\text{Min}(Y_2, \dots, Y_{k+1}) > \frac{y}{k}\right) \right\} \\ &= \frac{1}{k+1} \{k \cdot A_1 + B_1\}\end{aligned}$$

Here,

$$\begin{aligned}A_1 &= P(\text{Min}(y, Y_2, \dots, Y_k) > Y_{k+1}) \\ &= \int_0^y \left[\bar{F}^{k-1}\left(\frac{u}{k}\right) - \bar{F}^{k-1}(y) \right] dF(u) + \bar{F}^{k-1}(y) \cdot F(ky)\end{aligned}$$

and

$$\begin{aligned}B_1 &= P\left(\text{Min}(X_2, \dots, X_{k+1}) > \frac{y}{k}\right) \\ &= \bar{F}^k\left(\frac{y}{k}\right).\end{aligned}$$

The asymptotic normality of the test $T_{m,n}^k$ is presented in the following theorem.

Theorem 2.1. The asymptotic distribution of $\sqrt{N}[T_{m,n}^k - \gamma(F, G)]$ is normal with mean zero and variance given by $\sigma^2(T_{m,n}^k) = \sigma_1^2 + \sigma_2^2$, where $\sigma_1^2 = \frac{(k+1)^2 \xi_1(F)}{\lambda}$ and $\sigma_2^2 = \frac{(k+1)^2 \xi_1(G)}{1-\lambda}$, where $\xi_1(F)$ and $\xi_1(G)$ is as defined above.

Under $H_0 : F = G = F_0$, then $\xi_1(F) = \xi_1(G) = \xi_1(F_0)$ and if F_0 is exponential distribution function then,

$$(k + 1)^2 \xi_1(F_0) = \left[\frac{1}{3} + \frac{k^2}{2k + 1} - \frac{(k + 1)^2}{4} + \frac{k^4}{k(4k - 1)} \right]$$

Proof: Proof follows from Hoeffding (1948).

The approximate α -level test rejects H_0 in favour of H_1 , if $\frac{\sqrt{N}T_{m,n}^k}{\sigma^2(T_{m,n}^k)} > z_\alpha$, Where z_α is the upper α -percentile point of standard normal distribution. Since, $\gamma(F, G) > 0$ under H_1 and from the asymptotic normality of $T_{m,n}^k$, the test based on $T_{m,n}^k$ is consistent against the alternative F is 'more NBU than' G .

3. Asymptotic Relative Efficiency

We study the asymptotic relative efficiency of $T_{m,n}^k$, relative to the $V_{k,n}$ test of Hollander, Park and Proschan (1986) for the two pairs of distributions $(F_{i,\theta}, G)$. Here, we assume that G is an exponential distribution with mean one. We denote $F_{1,\theta}$ as Weibull distribution and $F_{2,\theta}$ as Makeham distribution are as defined below:

1. Weibull Distribution:

$$\bar{F}_{1,\theta}(x) = \exp \{-x^\theta\}, \theta > 0, x \geq \theta.$$

2. Makeham Distribution

$$\bar{F}_{2,\theta}(x) = \exp[-x + \theta(x + e^{-x} - 1)], \quad x > 0, \quad \theta \geq 0.$$

The ARE's of the proposed tests $T_{m,n}^k$ with respect to the test of Hollander, Park and Proschan (1986) for various distributions are presented in the following table 1.

Table 1Asymptotic Relative Efficiency $T_{m, n}^k$ relative to $V_{m, n}$

k	Weibull	Makeham
2	5.0606	6.5845
3	2.9786	3.7409
4	1.9366	2.7853
5	1.4897	2.2914
6	1.2363	1.9853

Next, we compute the efficacy of the two sample test based on $T_{m, n}^k$ by specifying the common null distribution in the null hypothesis as F_θ with $\theta \geq 1$ and considering sequence of alternatives $(F_{\theta\phi_N}, F_\theta)$, where $\phi = 1 + \frac{\alpha}{\sqrt{N}}$, α being arbitrary positive constant. Note that as $N \rightarrow \infty$, the sequence of alternatives converges to the null hypothesis, The efficacy of the $T_{m, n}^k$ test is given by

$$Eff(T_{m, n}^k) = \frac{[\gamma(F_{\theta\phi_N}, F_\theta)]^2}{\sigma_0^2(T_{m, n}^k)},$$

where $\sigma_0^2(T_{m, n}^k)$ is null asymptotic variance of $\sqrt{N}T_{m, n}^k$, and

$$\gamma(F, G) = \left[\frac{d\gamma(F_{\theta\phi_N}, F_\theta)}{d\phi} \right]_{\phi=1}.$$

The sequence of alternatives considered here are $(F_{1, \theta\phi_N}, F_{1, \theta})$ and $(F_{2, \theta\phi_N}, F_{2, \theta})$ whose functional forms are as given in below:

1. Weibull distribution,

$$\bar{F}_{1, \theta\phi}(x) = \exp(-x^{\theta\phi}), \quad x > 0, \quad \theta \geq 1$$

and

$$\bar{F}_{1, \theta}(x) = \exp(-x^\theta), \quad x > 0, \quad \theta \geq 1.$$

2. Makeham Distirbution

$$\bar{F}_{2, \theta\phi}(x) = \exp[-x + \theta\phi(x + e^{-x} - 1)], \quad x > 0, \quad \theta \geq 0$$

and

$$\bar{F}_{2, \theta}(x) = \exp[-x + \theta(x + e^{-x} - 1)], \quad x > 0, \quad \theta \geq 0.$$

The Asymptotic relative efficiencies of the proposed test $T_{m, n}^k$ relative to the test due to Hollander, Park and Proschan (1986) $V_{k, n}$ for $(F_{1, \theta\phi_N}, F_{1, \theta})$ and $(F_{2, \theta\phi_N}, F_{2, \theta})$ are presented in table 2 and 3 respectively.

Table 2. ARE of $T_{m, n}^k$ w. r. t. $V_{k, n}$ for $(F_{1, \theta\phi_N}, F_{1, \theta})$

$K \rightarrow \theta$	2	3	4	5
2	0.3188	0.5551	0.6858	0.7869
3	1.2605	2.1411	2.6342	2.9176
4	1.1974	1.9058	2.2426	2.4202
5	2.3262	3.4804	3.9999	4.2744
6	2.3622	3.3915	3.8592	4.1114
7	2.8256	3.9605	4.4896	4.7787
8	3.5982	4.9780	5.6351	5.9965
9	4.5118	6.1991	7.0140	7.4633
10	4.9283	6.7474	7.6330	8.1219

Table 3. ARE of $T_{m, n}^k$ w. r. t. $V_{k, n}$ for $(F_{2, \theta\phi_N}, F_{2, \theta})$

00	2	3	4	5
2	2.4453	2.3308	2.0656	1.8253
3	2.2943	2.0697	1.7835	1.5503
4	2.1544	1.8613	1.5701	1.3474
5	2.0308	1.6934	1.4042	1.1927
6	1.9211	1.5546	1.2706	1.0689
7	1.8243	1.4384	1.1614	0.9712
8	1.7372	1.3386	1.0691	0.8885
9	1.6590	1.2525	0.9912	0.8191
10	1.5877	1.1770	0.9236	0.7597

4.4. Some remarks.

(1) The Asymptotic relative efficiencies of the proposed test with respect to the test due to Hollander, Park and Proschan (1986) is computed for two pairs of distributions (F_θ, G) with G is exponential with mean one and F_θ as Weibull and Makeham distributions.

(2) It is observed that the proposed test performs better for the alternatives considered F_θ is either Weibull or Makeham distributions when G is exponential.

(3) The asymptotic relative efficiencies of the test proposed with respect to Hollander, Park and Proschan (1986) is computed for three pairs of distributions $(F_{1, \theta\phi_N}, F_{1, \theta})$ and $(F_{2, \theta\phi_N}, F_{2, \theta})$ with F_1, F_2 as Weibull, Makeham distributions respectively.

(4) It is observed that the proposed test performs better than the test due to Hollander, Park and Proschan (1986) when the underlying distribution is Weibull or Makeham.

(5) The test due to Hollander, Park and Proschan (1986) performs better for Linear failure rate distributions.

(6) The optimum value of m to use the proposed two sample test is 2.

(7) Hence, if the data under consideration is exactly NBU, the new test proposed would be a better choice.

References

- (1) I. A. Ahmad, A class of statistics useful in testing increasing failure rate average and new better than used life distributions. *J. of Stat. Plann. and Infer.*, 41, (1994), 141-149.
- (2) I. A. Ahmad, A simple and more efficient new approach to life testing. *Comm. Statist, Theor., and Meth.*, 9 (33) (2004), 2199-2215.
- (3) J. V. Deshpande and S. C. Kochar, A linear combination of two U-statistics for testing new better than used, *Commn. Statist. Theor. Meth.*, 2 (12) (1983), 153-159.
- (4) W. Hoeffding, A class of statistics with asymptotically normal distribution, *Ann. Math. Statist.*, 19 (1948), 293-325.
- (5) R. M. Hollander and F. Proschan, Testing whether New is Better than used, *Ann. Math. Stat.*, 43 (1972), 1136-1146.
- (6) M. Hollander, D. H. Park and F. Proschan, Testing whether F is "more NBU" than G , *Microelectron, Reliab.*, 26 (1986), 39-44.
- (7) H. L. Koul, A test for new is better than used, *Comm. Statist. Theo. Meth.*, A6 (1977), 363-373.
- (8) H. L. Koul, A class of tests for testing new is better than used, *Canad. J. Statistics*, 6 (2) (1978), 249-271.
- (9) Y. Kumazawa, Testing for new is better than used, *Comm. Statist. Theo. Meth.*, 12 (1983), 311-321.
- (10) P. V. Pandit and M. P. Anuradha, A note on linear combination of two U-statistics for testing new better than used, *Journal of Prob. and Stat. Sci.*, 2 (5) (2007), 209-215.
- (11) P. V. Pandit and M. P. Anuradha, Convex combination of two U-statistics for testing new better than used, *Model Assisted Stat. and Appl.*, 2 (2007b), 1-9.
- (12) P. V. Pandit and M. P. Anuradha, Linear combination of two U-statistics for testing new better than used, *J. of Comb., Inform. and System Sci.*, 32 (2007c), 17-28.
- (13) P. V. Pandit and N. V. Gudaganavar, On Tests for Detecting More Nbu-ness Property of Life Distributions, *Research Journal of Mathematics and Statistics*, 1 (1) (2009), 9-13.
- (14) P. V. Pandit and N. V. Gudaganavar, On Detecting new better than used property of life distributions, *Arya Bhatta Journal of Mathematics and Informatics*, 2 (2010), 1-8.