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On Tests for Detecting More Nbu-ness Property of Life Distributions

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Abstract: A simple test procedure is proposed for testing the null hypothesis that two life distributions are equal against the alternative that one is more new better than used than the other. The proposed test is based on a sub sample of size two from each sample. The properties of the test procedure such as asymptotic normality, consistency etc., are established. The performance of the test procedure is evaluated by finding Pitman asymptotic relative efficiencies(ARE) for different distributions in comparison with the test procedure due to Hollander and Proschan(1986). A simple two-sample test statistic for testing the null hypothesis of equality of two life distributions against the alternative that one life distribution possesses more of New better than used property than does another life distribution at a specified age. The newly proposed test is based on a U-statistic whose kernel is based on minima of a sub sample of size two. The asymptotic normality of the proposed test is established. The Pitman ARE of the proposed test relative to the test proposed by Lim, Kim and Park(2005) are computed by considering the alternative distributions with location and scale parameters. The Pitman AREs show that the performance of the tests proposed here is better as compared to its competitors.

Key words: NBU, NBU-t_0, positive ageing, pitman ARE, Two-sample test, U-statistic

INTRODUCTION

In reliability and survival analysis, the study of life time of units whose distribution possesses positive ageing plays vital role. The classes of life distribution possessing positive ageing were discussed by Bryson and Siddiqui(1969) and then in a book by Barlow and Proschan (1981). The problem of testing exponentiality against monotone failure rate were considered by Proschan and Pyke(1967), Barlow and Proschan (1969) among others. An important class of life distribution possessing positive ageing is the class of new better than used and its extensions. The NBU class of distributions plays an important role in maintenance policy of a system, which is a subclass of NBU-t_0 class of distributions. The NBU-ness or NBU-t_0 property of a life distribution is of interest in reliability theory. Testing exponentiality against the new better than used alternatives are discussed by Hollander and Proschan (1972), Koul (1977), Kumazawa (1983), Ahmad (1994, 2004) among others. Many authors proposed a number of tests for detecting such a property ; for example, Hollander and Proschan (1972) and Hollander et al.(1986a). Later, Ebrahimi and Habibullah(1990) extended the results of Hollander et al.(1986a) to propose a notion of new better than used of order k and developed a class of tests for detecting NBU-t_0 property. Recently Pandit(2004,2005) and Pandit and Anuradha(2007) proposed test procedures for detecting NBU-t_0 alternatives.

Hollander, Park and Proschan (1986b) developed a test procedure for testing the null hypothesis that two life distributions F and G are equal versus the alternative hypothesis that F is 'more NBU than' G. Pandit and Gudaganavar (2007) proposed test procedures for detecting more NBU and NBU-t_0 alternatives. Recently, Lim, Kim and Park(2005) developed a class of test procedures for testing the null hypothesis that two life distributions F and G are equal against the alternative that F is 'more NBU-t_0 than' G.

In material and methods section we propose two new test procedures one for testing equality of two distributions against the alternative that one distribution possesses more NBU property than another distribution and another for testing equality of two distributions against the alternative that one distribution possesses more NBU property than another distribution at specified age.

MATERIALS AND METHODS

A life is represented by a non-negative random variable X with distribution function F and survival function $\bar{F}$ =1 - F. A distribution F is said to be new better than (NBU) used if $\bar{F}(x+t) \le \bar{F}(x)\bar{F}(t)$, for all x, t $\ge$ 0. Hollander, Park and Proschan (1986) introduced the larger class of life distributions called new better than used of age t_o defined as follows.

Definition 1: The distribution F is said to be new better than used of age to, (NBU-to), if

$$\bar{F}(x+t_0) \le \bar{F}(x)\bar{F}(t_0) \quad \text{for all } x^30. \qquad (1.1)$$

The concept of new better than used of specified age can be interpreted as 'a used item of age t_0 has stochastically less remaining life than does a new item.

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The NBU-t_0 class is a larger class of life distributions than NBU and the boundary members of NBU-t_0 class include not only exponential distribution but also other life distributions, such as one truncated at t_0 or one with periodic failure rate of period t_0.

We develop a simple and efficient test for testing the null hypothesis that two life distributions F and G are equal against the alternative that F is 'more NBU than' G in the proposed two-sample more NBU test subsection. The proposed test is based on U-statistics whose kernels are based on the sub samples drawn from the two samples. We present a new test procedure for testing against the alternative that F is 'more NBU at specified age than' G, in the proposed two-sample more NBU-t_0 test subsection.

The Proposed two-sample more NBU test: Let X_1, $X_2,\ldots,X_m$ and Y_1, $Y_2,\ldots,Y_n$ denote two random samples from continuous life distributions F and G, respectively. We want to develop test statistic for testing the null hypothesis

H_0: F = G (the common distribution is not specified)
versus the alternative hypothesis

H_1: F is 'more NBU' than G.

Consider the parameter,

$$\Lambda(F,G)=\Lambda(F)-\Lambda(G),$$

where

$$\Delta(F)=\int_0^\infty\int_0^\infty \bar{F}(x).\bar{F}(y)dxdy-\int_0^\infty\int_0^\infty \bar{F}(x+y)dxdy \quad \text{and}$$

$$\Delta(F)=\int_0^\infty\int_0^\infty \bar{F}(x).\bar{F}(y)dxdy-\int_0^\infty\int_0^\infty \bar{F}(x+y)dxdy$$

.

Here, $\Delta(F)$ and $\Delta(G)$ can be considered as the measure of degree of the NBU-ness. Ahmed (2004) test used this measure as basis for their test statistic. If F (G) belongs to NBU, then $\Delta(F)>0$ ($\Delta(G)>0$) and $\Delta(F,G)$ can be taken as a measure by which F is 'more NBU-t_0' than G. Under H_0, γ_{to} (F,G) and it is strictly greater than zero under H_1.

An unbiased estimator for $\Delta(F,G)$, which is a U-statistic is defined as

$$U_{m,n}=U_m-U_n$$

where

$$U_m=\frac{[m(m-1)]^{-1}\sum_{i\neq j}\sum X_iX_j-[2m]^{-1}\sum_{i=1}^{m}X_i^2}{\bar{X}^2}$$

and

$$U_n=\frac{[n(n-1)]^{-1}\sum_{i\neq j}\sum Y_iY_j-[2n]^{-1}\sum_{i=1}^{n}Y_i^2}{\bar{Y}^2}$$

Here $\bar{X}$ are $\bar{Y}$ the sample means of x-sample and y-sample respectively. The asymptotic normality of the test $U_{m,n}$ is presented in the following theorem.

Theorem: The asymptotic distribution of $\sqrt{N}\left[U_{m,n}-\Lambda(F,G)\right]$ is normal with mean zero and variance given by

$$\sigma^2(U_{m,n})=\sigma_1^2+\sigma_2^2$$

where $$\sigma_1^2=\frac{Var\left(2\mu X_1-\frac{X_1^2}{2}-\frac{\mu_1^{(2)}}{2}\right)}{\lambda}$$

and $$\sigma_2^2=\frac{Var\left(2\mu_2 Y_1-\frac{Y_1^2}{2}-\frac{\mu_2^{(2)}}{2}\right)}{1-\lambda}$$

where $\mu_1=E(X_1),\ \mu_2=E(Y_1)$

and $\mu_1^{(2)}=E(X_1^2),\ \mu_1^{(2)}=E(X_1^2)$

Proof: Proof follows from Hoeffding(1948)).

A consistent estimator, $\hat{\sigma}^2(U_{m,n})$ for $\sigma^2(U_{m,n})$ is obtained by replacing $\bar{F}(x)$ by $\bar{F}_m(x)$, the empirical survival function of F and $\bar{G}(y)$, by $\bar{G}_n(y)$ the empirical survival function of G. Hence, approximate α-level test rejects H_0 in favour of H_1 $\frac{\sqrt{N}U_{m,n}}{\hat{\sigma}^2(U_{m,n})}>Z_\alpha$, if, where Z_α is the upper α - percentile point of standard normal distribution. Since, $\Delta(F,G)>0$ under H_1 and the asymptotic normality of $U_{m,n}$, the test based on $U_{m,n}$ is consistent against the alternative F is 'more NBU than' G.

The Proposed two-sample more NBU-t_0 test: Let X_1, $X_2,\ldots,X_m$ and Y_1, $Y_2,\ldots,Y_n$ denote two random samples from continuous life distributions F and G, respectively. We want to develop test statistic for testing the null hypothesis

H_0: F = G (the common distribution is not specified)
versus the alternative hypothesis

H_1: F is 'more NBU-t_0' than G,

based on the two random samples.

Our statistic is motivated by considering the concept of new better than used of specified age t_0.

Consider the parameter,

$$\gamma_{t_0}(F,G)=\gamma_{t_0}(F)-\gamma_{t_0}(G),$$

where

$$\gamma_{t_0}(F) = \mu_1 \bar{F}(t_0) - \int_0^\infty \bar{F}(x+t_0)dx$$

and $$\gamma_{t_0}(G) = \mu_2(t_0)\bar{G}(x) - \int_0^\infty \bar{G}(x+t_0)dx$$.

Here $\gamma_{t0}(F)$ can $\gamma_{t0}(G)$ be considered as the measure of degree of the NBU-ness at age t_0. Ahmed (2004) test used this measure as basis for their test statistic. If F (G) belongs to NBU-t_0 class, then $\gamma_{t0}(F) > 0$ ($\gamma_{t0}(G) > 0$) and $\gamma_{t0}(F,G)$ can be taken as a measure by which F is 'more NBU-t_0' than G. $\gamma_{t0}(F,G)=0$ Under H_0, and it is strictly greater than zero under H_1.

An unbiased estimator for $\gamma_{t0}(F,G)$, which is a U-statistic is defined as

$$V_{t0} = V_{t0}^1 - V_{t0}^2$$

where

$$V_{t0}^1 = \frac{[m(m-1)]^{-1}\sum_{i\neq j}\sum\left[X_i - X_j + t_0\right]I\left(X_j - t_0\right)}{S_m^2}$$

and $$V_{t0}^2 = \frac{[n(n-1)]^{-1}\sum_{i\neq j}\sum\left[Y_i - Y_j + t_0\right]I\left(Y_j - t_0\right)}{S_n^2}$$

Here

$$S_m^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_i - \bar{X}\right)^2 \text{ and } S_n^2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(Y_j - \bar{Y}\right)^2 .$$

The asymptotic normality of the test V_{t0} is presented in the following theorem.

Theorem: The asymptotic distribution of $\sqrt{N}\left[V_{t_0} - \gamma(F,G)\right]$ is normal with mean zero and variance given by

$$\sigma^2\left(V_{t_0}\right) = \frac{\eta_{t_0}(F)}{\lambda} + \frac{\eta_{t_0}(G)}{1-\lambda}$$

where $\eta_{t_0}(F) = \bar{F}(t_0)F(t_0)$

and $\eta_{t_0}(G) = \bar{G}(t_0)G(t_0)$.

Proof: Proof follows from Hoeffding(1948)).

A consistent estimator, $\hat{\sigma}^2\left(V_{t_0}\right)$ for $\sigma^2\left(V_{t_0}\right)$ is obtained by replacing $\bar{F}(t_0)$ by $\bar{F}_m(t_0)$, the empirical survival function of F and $\bar{G}(t_0)$ by $\bar{G}_m(t_0)$, the empirical survival function of G. Hence, approximate a-level test rejects H_0 in favour of H_1, if $\frac{\sqrt{N}V_{t_0}}{\hat{\sigma}^2\left(V_{t_0}\right)} > Z_\alpha$, where is the upper - percentile point of standard normal distribution. Since, $\gamma(F,G) > 0$ under H_1 and the asymptotic normality of V_{t_0}, the test based on V_{t_0} is consistent against the alternative F is 'more NBU-t_0 than' G.

RESULTS AND DISCUSSIONS

The performance of the test procedure, $U_{m,n}$ developed for testing whether F is 'more NBU' than G is studied in subsection "Asymptotic Relative Efficiency of $U_{m,n}$" and that of V_{t_0} developed for testing whether F is 'more NBU at specified age' than G is presented in subsection "Asymptotic Relative Efficiency of V_{t_0}."

Asymptotic Relative Efficiency of $U_{m,n}$: We study the asymptotic relative efficiency of $U_{m,n}$, relative to the $T_{m,n}$ test of Hollander, Park and Proschan(1986b) for the two pairs of distributions ($F_{i,q}$,G). Here, we assume that G is an exponential distribution with mean one. We consider $F_{1,\theta}$ as Weibull distribution and $F_{2,\theta}$ as Linear failure rate distribution as defined below:

1. Weibull Distribution:

$$\bar{F}_{1,\theta}(x) = \exp\left(-x^\theta\right), \theta > 0, x \geq \theta.$$

2. Linear Failure Rate Distribution

$$\bar{F}_{2,\theta}(x) = \exp\left[-\left(x + \theta\frac{x^2}{2}\right)\right], x > 0, \theta \geq 0.$$

The Pitman asymptotic efficacy is

$$Eff\left(U_{m,n}, F_{i,\theta}, G\right) = \sigma_0^{-2}\left(U_{m,n}\right)\left\{\frac{d}{d\theta}\gamma\left(F_{i,\theta}, G\right)\right\}_{\theta\to\theta_0}$$

The ARE's of the proposed tests with respect to the $T_{m,n}$ test of Hollander, Park and Proschan(1986b) for Weibull distribution with parameter θ, $F_{1,q}$ and Linear failure rate distribution are respectively 1.8432 and 2.6923.

Next, The Pitman efficacies of two sample test based on $U_{m,n}$ proposed in this chapter is determined by specifying a common distribution with parameter q in the null hypothesis and by considering sequence of alternatives ($F_{\theta\phi N}$, F_θ) where, $\phi_N = 1 + \frac{a}{\sqrt{N}}$, a>0 be a constant. The sequence of distribution considered here are ($F_{1,\theta\phi N}$, $F_{1,\theta}$), ($F_{2,\theta\phi N}$, F_θ) and ($F_{3,\theta\phi N}$, F_θ), where $F_{1,\theta}$, $F_{2,\theta}$ and $F_{3,\theta}$ are given by

1) Weibull Distribution

$$\bar{F}_{1,\theta}(x) = \exp\left(-x^\theta\right), x > 0$$

and

$$\bar{F}_{1,\theta\phi N}(x) = \exp\left\{-x^{\theta\phi N}\right\}, x > 0$$

2) Linear Failure Rate Distribution

$$\bar{F}_{2,\theta}(x) = \exp\left[-\left(x + \theta\frac{x^2}{2}\right)\right], x > 0, \theta \geq 0$$

and

$$\bar{F}_{2,\theta\phi N}(x) = \exp\left[-\left(x + \theta\phi N\frac{x^2}{2}\right)\right], x > 0, \theta \geq 0$$

3) Makeham Distribution

$$\bar{F}_{3,\theta}(x) = \exp\left\{-\left[x + \theta\left(x + e^{-x} - 1\right)\right]\right\}, \theta > 0, x \geq 0$$

and

$$\bar{F}_{3,\theta\phi N}(x) = \exp\left\{-\left[x + \theta\phi_N\left(x + e^{-x} - 1\right)\right]\right\},$$
$$\theta > 0, x \geq 0$$

It is to be noted that as $N \to \infty$, the sequence of alternatives converges to the null hypothesis. The efficacy of the $U_{m,n}$ test is given by

$$eff(V) = \frac{\left\{\Delta'\left(F_{\theta\phi_N}, F_\theta\right)\right\}_{\phi-1}}{\sigma_{H_0}(V)}$$

where σ_{H_0} ($U_{m,n}$) is null asymptotic standard deviation of $U_{m,n}$ and We present the asymptotic relative efficiencies(ARE) of $U_{m,n}$ relative to $T_{m,n}$ in Table 1.

$$\Delta'\left(F_{\theta\phi_N}, F_\theta\right)/\phi-1 = \left[\frac{d}{d\phi}\Delta\left(F_{\theta\phi_N}, F_\theta\right)\right]_{\phi-1}$$

Asymptotic Relative Efficiency of V_{t_0} : We study the asymptotic relative efficiency of V_{t_0} , relative to the $T_{t_0}^k$ test of Lim,Kim and Park (2005) for the three pairs of distributions ($F_{i,q}$,G). Here, we assume that G is an exponential distribution with mean one. Here we consider the following distributions for F which are given as follows:

1. Exponential Distribution with location parameter:

$$\bar{F}_{1,\theta}(x) = \exp\{-[x - \theta]\}, \theta > 0, x \geq \theta.$$

2. Linear Failure Rate Distribution

$$\bar{F}_{2,\theta}(x) = \exp\left[-\left(x + \theta\frac{x^2}{2}\right)\right], x > 0, \theta \geq 0.$$

3. NBU-t_0 Distribution:

$$\bar{F}_{1,\theta}(x) = \exp\left[-x + \frac{\theta x^2}{2t_0}\right] I\left(0 \leq x < t_0\right) +$$
$$\exp\left[-x + \frac{\theta t_0}{2}\right] I\left(x \geq t_0\right), 0 \leq \theta \leq \frac{2}{3}$$

The Pitman asymptotic efficacy is

$$Eff\left(V_{t_0}, F_{i,\theta}, G\right) = \sigma_0^{-2}\left(V_{t_0}\right)\left\{\frac{d}{d\theta}\gamma\left(F_{i,\theta}, G\right)\right\}_{\theta \to \theta_0}$$

The ARE's of the proposed tests with respect to the test proposed by Lim, Kim and Park(2005) for exponential distribution with location parameter, $F_{1,q}$ for values of 0.1, 0.3, 0.5 are respectively 3.8109, 6.6444 and 21.4096. The table1 gives the asymptotic relative efficiency (ARE) of V_{t_0} relative to the statistic due to Lim,Kim and Park(2005), $T_{t_0}^k$ for different pairs of distributions ($F_{i,q}$,G) .

From Table 2, it is evident that the newly proposed test is better linear failure rate distribution and the performance of the newly proposed test is better than NBU-t_0 distribution for t_0, the specified age is above 0.5. It is expected that a test should be better for NBU –t_0 distribution when the unit has survived for a fixed period of time.

Table 1: ARE of $U_{m,n}$ w.r.t. $T_{m,n}$

θ	($F_{1,\theta\phi N}$, $F_{1,\theta}$)	($F_{2,\theta\phi N}$, $F_{2,\theta}$)	($F_{3,\theta\phi N}$, $F_{3,\theta}$)
2	0.4393	0.3396	0.2941
3	2.5386	4.4836	0.5190
4	1.1465	5.7197	0.6620
5	2.6801	6.8133	0.7655
6	1.9453	7.8100	0.8457
7	2.2138	8.7193	0.9058
8	2.9478	9.5681	0.9541
9	3.90961	0.3634	0.9964
10	3.51951	1.1166	1.0311

Table 2: ARE of V_{t_0} w.r.t. $T_{t_0}^k$

	T_0					
Distributions	0.1	0.3	0.5	0.7	0.9	1.0
$F_{2,q}$	3.7462	3.3088	2.9507	2.6575	2.4175	2.3143
$F_{3,q}$	0.0445	0.5258	2.1156	7.0143	16.1347	20.1794

CONCLUSIONS

From the above discussion, we conclude as:

(1) The asymptotic relative efficiency of the proposed test, $U_{m,n}$ with respect to the test due to Hollander, Park and Proschan (1986b) is computed for two pairs of distributions (F_q,G) with G as exponential with mean one and F_q as Weibull and Linear failure rate distributions.

(2) It is observed that the proposed test, $U_{m,n}$, performs better for the alternatives considered F_q is either weibull or linear failure rate distribution when it is the comparison with exponential distribution.

(3) We have studied asymptotic performance of the newly proposed test, $U_{m,n}$, when the common null distribution is Weibull, Makeham and Linear failure rate distribution.

(4) It is observed from table 1, that the performance of the new test, $U_{m,n}$, is better than the test due to Hollander, Park and Proschan (1986b) $T_{m,n}$ when the common null distribution is Weibull, Makeham and Linear failure rate distribution.

(5) Hence if the data under consideration is exactly NBU, the new test proposed would be a better choice.

(6) The Asymptotic relative efficiencies of the proposed test, , with respect to the test due to Lim,Kim and Park(2005) is computed for three pairs of distributions (F_q,G) with G is exponential with meanone and F_q as exponential with location parameter q, Makeham distribution with parameter q and NBU-t_0 distribution.

(7) It is observed that the proposed test, , performs better for the alternatives considered F_q is either exponential or Makeham for all the ages.

(8) Hence, if the data under consideration is exactly NBU-t_0 , the new test would be a better choice when the age t_0 is greater than 0.3.

(9) Similar studies about the Pitman ARE comparisons have been made by taking common distributions as Weibull, Makeham and NBU-t_0 distributions and found that new test is better choice.

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