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Lict edge semientire graph of a planar graph

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Abstract

In this paper, we introduce the concept of the Lict edge semientire graph of a planar graph. We present characterizations of graphs whose lict edge semientire graphs are planar, outerplanar and Maximal outerplanar, crossing number one. Further, we establish a characterization of graphs whose lict edge semi entire graphs are Eularian and Hamiltonian.

Keywords and phrases : *Pathos, pathoslength, edge semientire graph, outerplanar, crossing number.*

1. Introduction

The lict graph[4] $n(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of cutvertices of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent or the corresponding members of G are incident.

The edge semientire graph[3] $e_e(G)$ of a plane graph G is the graph whose vertices can be put in one to one correspondence with the edges and regions of G in such a way that two vertices of $e_e(G)$ are adjacent if and only if the corresponding elements of G are adjacent. The line graph $L(G)$ of a graph G is the graph whose vertex set coincides with the edge set of G and in which two vertices are adjacent if and only if the corresponding edges are adjacent in G.

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A regionvertex is a vertex in lict edge semientire graph corresponding to the regions of G.

We now define the lict edge semientire graph $n_e(G)$ of a planar graph G whose vertex set is the union of the set of edges, set of vertices and set of regions of G in which two vertices are adjacent if and only if the corresponding edges are adjacent, edges are incident to the cutvertex and edges are lies on the region. In Figure 1, a graph G and its lict edge semientire graph $n_e(G)$ of a planar graph are shown.

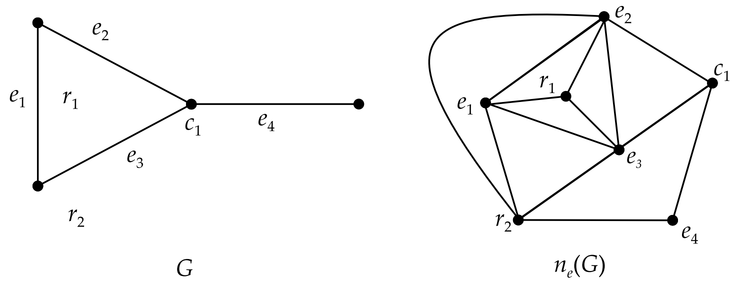

Figure 1

All the undefined terms may be referred to Harrary[2]. All graphs considered here are finite, undirected and without loops or multiple edges.

We need the following theorem for the proof of our further results.

Theorem 1. *If G is a (p, q) graph whose vertices have degree d_i then $L(G)$ has q vertices and q_L edges where $q_L = -q + \frac{1}{2}\sum d_i^2$.*

Theorem 2. *The line graph $L(G)$ of a graph G has crossing number one if and only if G is planar and 1 or 2 holds:*

(1) *The maximum degree $\Delta(G)$ is 4 and there is unique non cutvertex of degree.*

(2) *The maximum degree $\Delta(G)$ is 5, every vertex of degree 4 is a cutvertex, there is a unique vertex of degree 5 and has almost 3 edges in any block.*

Theorem 3. *A connected graph G is isomorphic to its line graph if and only if it is a cycle.*

Theorem 4. *The lict graph $n(G)$ is planar if and only if $\deg v \leq 3$.*

Theorem 5. *The edge semi entire graph $e_e(G)$ is planar if and only if*

(1) *G is a tree*

(2) $\deg v \leq 3$ *for every vertex v of G.*

Theorem 6. *Let G be a plane graph. A necessary and sufficient condition for $e_e(G)$ to be eulerian is that each of the following holds:*

(1) *Each edge of G is adjacent to even number of elements*
(2) *Each region of G has even number of elements adjacent to it.*

Theorem 7. *If G is a hamiltonian plane graph, then $e_e(G)$ is also hamiltonian.*

Lict edge semientire graph $n_e(G)$ of a planar graph G

We start with a few preliminary results.

Remark 1. For any graph G, $L(G) \subset n(G) \subset n_e(G)$ and $e_e(G) \subset n_e(G)$.

Remark 2. For any graph G, $n_e(G)$ is nonseparable.

Remark 3. If a graph G is K_2 then, $n_e(G)$ is complete.

Remark 4. If G itself is a block then $n_e(G) = e_e(G)$.

In the following theorem, we obtain the number of vertices and edges in lict edge semientire graph.

Theorem 8. *For any (p, q) graph G whose vertices have degree d_i, cutvertices c, regions r and l_i be the number of edges to which cutvertex c_i belongs and e_k be the number of edges in which the region r_k lies, the lict edge semientire graph $n_e(G)$ has $(q + c + r)$ vertices and $\sum \left[\frac{d_i^2}{2} + l_i\right] + e_k$ edges.*

Proof. By definition of $n_e(G)$, the number of vertices is the union of edges, cutvertices and regions of G. Hence $n_e(G)$ has $(q + c + r)$ vertices. Further the number of edges in $n(G)$ is the sum of number of edges incident with cutvertices in G, the sum of number of edges in $L(G)$. Since the number of edges in $L(G)$ has $-q + \sum \frac{d_i^2}{2}$, hence the number of edges in $n_e(G)$ is the sum of edges in $n(G)$ and the number of edges bounded by the regions e_k. In addition, the total number of edges that lie on the region $\sum r_i$ of G is q. Hence

$$E[n_e(G)] = -q + \sum \left[\frac{d_i^2}{2} + l_i\right] + q + e_k$$
$$= \sum \left[\frac{d_i^2}{2} + l_i\right] + e_k\,.$$

Planar lict edge semientire graph

In this section, we obtain the condition for planarity of lict edge semientire graph.

Theorem 9. *The lict edge semientire graph $n_e(G)$ is planar if and only if* $\deg v \leq 3$, *for every vertex v of G.*

Proof. Suppose $n_e(G)$ is planar. Assume $\deg v \geq 4$. If there exists a vertex v of degree 4, then by definition, $L(G)$ is planar which contains $\langle K_4 \rangle$ as an induced subgraph. So the lict graph $n(G)$ contains $\langle K_5 \rangle$ as an induced subgraph, which is nonplanar. Clearly, $n_e(G)$ is also nonplanar, a contradiction.

Conversely suppose $\deg v \leq 3$. By the Theorem 5, $e_e(G)$ is planar and by the Theorem 4, $n(G)$ is also planar. Clearly, $n_e(G)$ is planar. □

We now present a characterization of graphs whose lict edge semientire graph is outerplanar and maximum outerplanar.

Theorem 10. *The lict edge semientire graph $n_e(G)$ is outer planar if and only if G is a path.*

Proof. Suppose $n_e(G)$ is outerplanar. Assume that G has a vertex v of degree 3. The edges incident to v and the cutvertex v form $\langle K_4 \rangle$ as a subgraph in $n_e(G)$. Hence $n_e(G)$ is non outerplanar, a contradiction.

Conversely, suppose T is a path P_t of length $t \geq 1$. For $t = 1$, the result is obvious. For $t > 1$, the graph $n(G)$ has $(t-1)$ blocks which are K_3. Since G has exactly one region it follows that the region vertex, corresponding to this region, together with the above blocks form $(t-1)$ number of induced subgraphs which are all $\langle K_4 - x \rangle$ in $n_e(G)$. Hence $n_e(G)$ is outerplanar. □

Theorem 11. *The lict edge semientire graph $n_e(G)$ is maximum outerplanar if and only if G is a path.*

Proof. Proof follows from the Theorem 10. □

In the next theorem, we characterize lict edge semientire graph in terms of crossing number one.

Theorem 12. *The lict semientire graph $n_e(G)$ has a crossing number one if and only if the following conditions hold:*

(1) $\deg v \leq 3$, *for every vertex v of G, and*

(2) *G has unique vertex of degree* 4, *which is not a cutvertex.*

Proof. Suppose $n_e(G)$ has crossing number one. Then it is nonplanar. By Theorem 9, $\deg v \geq 4$ for every vertex v of G. We now consider the following cases:

Case 1. Assume G has a vertex u of degree 5. If u is not a cutvertex, then by Theorem 4, the regionvertex is adjacent to edges of G. Clearly $C[n_e(G)] > 1$, a contradiction. If u is a cutvertex then the edges incident to this vertex together with the cutvertex form $\langle K_6 \rangle$ as a subgraph in $n(G)$, the regionvertex is adjacent to atleast one vertex of $\langle K_6 \rangle$ in $n_e(G)$. This gives $C[n_e(G)] > 1$, a contradiction.

Case 2. Assume G has atleast two vertices of degree 4. Suppose v_1 and v_2 are two noncutvertices of degree 4. Then $L(G)$ has atleast two crossings, by Theorem 2, $C[n_e(G)] > 1$, a contradiction. Suppose v_1 and v_2 are two cutvertices of degree 4. Then cutvertices v_1 and v_2 together with their corresponding four incident edges form two $\langle K_6 \rangle$ as subgraphs in $n(G)$ and hence in $n_e(G)$. Hence $C[n_e(G)] > 1$, a contradiction.

Conversely, suppose G holds both the conditions of the Theorem. Let v_1 be the noncutvertex of degree 4. Then by Theorems 2 and 3, $n(G)$ has crossing number one and hence $n_e(G)$ has crossing number one. □

Theorem 13. *The lict edge semientire graph $n_e(G)$ is eulerian if and only if G is a cycle C_n, n is even.*

Proof. The cycle C_n does not contains a cutvertex. By the Remark 4, $n_e(G) = e_e(G)$. Also by Theorem 6, $e_e(G)$ is eulerian and hence $n_e(G)$ is eulerian.

Theorem 14. *The lict edge semientire graph $n_e(G)$ is hamiltonian if and only if $G \neq K_2$.*

Proof. Suppose $n_e(G)$ is hamiltonian. Assume that G is K_2. Then this edge is incident with regionvertex w to form K_2, which is nonhamiltonian, a contradiction.

Conversely, suppose $G \neq K_2$, we now consider the following cases:

Case 1. If G is a path and has exactly one regionvertex. Let $V[n(G)] = (e_1, e_2, \ldots e_n) \cup (c_1, c_2, \ldots c_{n-2})$, where $(c_1, c_2, \ldots c_{n-2})$ are cutvertices of G. Each block is a triangle and each block consist as vertices $B_1 = (e_1, c_1, e_2), B_2 = (e_2, c_2, e_3), \ldots B_n = (e_{n-1}, c_{n-2}, e_n)$. Also in $n_e(G)$, the regionvertex w is adjacent to $(e_1, e_2, \ldots e_n)$. Hence

$V[n_e(G)] = (e_1, e_2, \ldots e_n) \cup (c_1, c_2, \ldots c_{n-2}) \cup w$ form a cycle $we_1c_1e_2c_2e_3 \ldots e_{n-1}c_{n-1}c_nw$ containing all the vertices of $n_e(G)$. Clearly $n_e(G)$ is hamiltonian.

Case 2. If G is a tree and has exactly one regionvertex. Let $[n(G)] = (e_1, e_2, \ldots e_n) \cup (c_1, c_2, \ldots c_j)$, where $(c_1, c_2, \ldots c_j)$ are the cutvertices of G. Clearly, each block is K_3 if degree of the cutvertex is two and is K_4 if degree of the cutvertex is three. In $n_e(G)$, the regionvertex w is adjacent to $(c_1, c_2, \ldots c_j)$. By Remark 2, $n_e(G)$ is nonseparable. Clearly, the vertices $(e_1, e_2, \ldots e_n) \cup (c_1, c_2, \ldots c_j) \cup w$ form $we_1c_1e_2c_2e_3e_4 \ldots c_je_nw$ containing all the vertices of $n_e(G)$. Hence $n_e(G)$ is hamiltonian.

Case 3. If G is hamiltonian graph, then by Theorem 7, $e_e(G)$ is hamiltonian. Hence $n_e(G)$ is hamiltonian.

Case 4. If G is the graph other than above types of graphs, then by Remark 2, $n_e(G)$ is nonseparable, hence it is hamiltonian.

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