
www.journalofgraphlabeling.com
Iournal of craph Labeling A Graph Theoretical Research Journal


Special Issue honouring Dr. KM. Kathiresan Volume 2 Issue 22016

## Honorary Editors

KM Kathiresan, India Sin-Min Lee, USA Harris Kwong, USA Joseph A Gallian, USA W C Shiu, China

## Managing Editors

Tao-Ming Wang, Taiwan M Salman A N, Indonesia

Founder • Editor-in-Chief
G Marimuthu, India

# SQUARE SUM LABELING OF DISJOINT UNION OF GRAPHS 

## AUTHORS INFO

Medha Itagi Huilgol* ${ }^{*}$ and V.Sriram,
Department of Mathematics, Bangalore University, Bengaluru, India.
E.Mail : medha@bub.ernet.in
vs140580@gmail.com
*Corresponding Author

## ARTICLE INFO

J. Graph Label. 2(2) (2016), 103-106.

Article History :
AMS MSC: 05C78.
Received
: 08.12.2015
Received in revised form : 05.01.2016
Accepted : 10.01.2016
Available online $\quad: 20.01 .2016$

Keywords: Square sum labeling, square sum graph.

JOURNAL INFO

© 2015 Journal of Graph Labeling.
All rights reserved.


#### Abstract

A $(p, q)$ graph $G$ is said to be square sum, if there exists a bijection $f: V(G) \rightarrow$ $\{0,1,2, \ldots, p-1\}$ such that the induced function $f^{*}: E(G) \rightarrow N$ defined by $f^{*}(u v)=$ $(f(u))^{2}+(f(v))^{2}$, for every $u v \in E(G)$ is injective. In this paper we prove that if $G_{1}$ and $G_{2}$ are square sum, then $G_{1} \cup G_{2} \cup G_{3}$ is also square sum, where $G_{3}$ is a set of isolated vertices.


## 1 Introduction

All graphs considered here are finite, undirected, and simple. For terminologies and notations not defined in this paper, the reader is referred to [1]. Labeling of a graph $G$ is an assignment of integers either to the vertices or edges or both subject to certain conditions. A dynamic survey to know about the numerous graph labeling methods is regularly updated by J.A. Gallian [4]. Acharya and Germina [2] defined a square sum labeling of a $(p, q)$-graph $G$ as follows.

A $(p, q)$-graph $G$ is said to be square sum, if there exists a bijection $f: V(G) \rightarrow$ $\{0,1,2, \ldots, p-1\}$ such that the induced function $f^{*}: E(G) \rightarrow N$ defined by $f^{*}(u v)=$ $(f(u))^{2}+(f(v))^{2}$, for every $u v \in E(G)$ is injective.

Several results on square sum labeling can be found in [3], [5], [6], [7].

## 2 Results

Theorem 2.1. Let $G_{1}$ and $G_{2}$ be square sum graphs then $G_{1} \cup G_{2} \cup G_{3}$ is square sum, with $\left|G_{3}\right|=\mathcal{L}$, isolates.

Proof. Let $G_{1}$ be a $\left(p_{1}, q_{1}\right)$-graph and $G_{2}$ be a $\left(p_{2}, q_{2}\right)$-graph. Let $V\left(G_{1}\right)$ be labeled as $\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{p_{1}}\right\}$ and let the edge set of $G_{1}$ be labeled as $\left\{e_{1}, e_{2}, e_{3}, \ldots, e_{q_{1}}\right\}$. Let $V\left(G_{2}\right)$ be labeled as $\left\{u_{1}, u_{2}, u_{3}, \ldots, u_{p_{2}}\right\}$ and let the edge set of $G_{1}$ be labeled as $\left\{e_{1}^{\prime}, e_{2}^{\prime}, e_{3}^{\prime}, \ldots, e_{q_{2}}^{\prime}\right\}$. Without loss of generality let $p_{1} \geq p_{2}$. Since $G_{1}$ is square sum there exists a bijection $f: V\left(G_{1}\right) \rightarrow\left\{0,1,2, \ldots, p_{1}-1\right\}$ such that the induced function $f^{*}: E\left(G_{1}\right) \rightarrow N$ defined by $f^{*}\left(v_{i} v_{j}\right)=\left(f\left(v_{i}\right)\right)^{2}+\left(f\left(v_{j}\right)\right)^{2}$, for every $v_{i} v_{j} \in E\left(G_{1}\right)$ and $i \neq j$ is injective. Also since $G_{2}$ is square sum there exists a bijection, say $g$, $g: V\left(G_{2}\right) \rightarrow\left\{0,1,2, \ldots, p_{2}-1\right\}$ such that the induced function $g^{*}: E\left(G_{2}\right) \rightarrow N$ defined by $g^{*}\left(u_{m} u_{n}\right)=\left(g\left(u_{m}\right)\right)^{2}+\left(g\left(u_{n}\right)\right)^{2}$, for every $u_{m} u_{n} \in E\left(G_{2}\right)$ and $m \neq n$ is injective.

