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# New results on edge rotation distance graphs

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**Abstract**

The concept of edge rotations and distance between graphs was introduced by Gary Chartrand et al. A graph $G$ can be transformed into a graph $H$ by an edge rotation if $G$ contains distinct vertices $u$,$v$ and $w$ such that $uv \in E(G)$, $uw \notin E(G)$ and $H \cong G - uv + uw$. In this paper we consider rotations on some snake related graphs followed by some general results.

**Keywords**: Edge rotation, edge rotation distance graphs, $r$-distance graph, triangular snake, double triangular snake, alternating double triangular snake, quadrilateral snake, double quadrilateral snake, alternating double quadrilateral snake.
**AMS Subject Classification(2010):** 05C12.

## 1 Introduction

Unless mentioned otherwise, for terminology and notation the reader may refer Buckley and Harary [2] and Chartrand and Zhang [5], new ones will be introduced as and when found necessary.

In this paper, by a graph $G$, we mean a simple, undirected, connected graph without self-loops. The *order* and *size* are respectively the number of vertices denoted by $n$ and the number of edges denoted by $m$.

The *distance* $d(u, v)$ between any two vertices $u$ and $v$, of $G$, is the length of a shortest path between $u$ and $v$. The *eccentricity* $e(u)$ of a vertex $u$ is the distance to a farthest vertex from $u$. The maximum and the minimum eccentricity amongst the vertices of $G$ are respectively called the *diameter* $diam(G)$ and *radius* $rad(G)$. If $d(u, v) = e(u)$, $(v \neq u)$ then we say that $v$ is an *eccentric vertex* of $u$.

The distance between isomorphism classes of graphs was introduced by Zelinka in [14] which was also studied for trees by Zelinka in [15].'*Edge Rotations*' or the concept of rotation between edges of the graphs and the distance between such graphs was introduced by Chartrand et al. [3] which were based on [14] and [15]. A graph $G$ can be transformed into a graph $H$ by an edge rotation given by $H \cong G - uv + uw$ where $u$, $v$ and $w$ are distinct vertices of $G$ such that

$uv \in E(G)$ and $uw \notin E(G)$. Later, Zelinka [16] gave a comparison of various distances for the isomorphism classes of graphs and trees, which was based on the concept of edge rotations.

Zelinka studied various aspects by using the concept of distance between graphs and edge rotations in [17], [18] and [19].

The rotation distance between graphs $G$ and $H$ is denoted by $d_r(G,\ H)$, if there exists a sequence of graphs $G_1,\ G_2,\ \ldots,\ G_{k-1}$ such that $G_1$ is obtained by an edge rotation on $G$, and for each $1 \leq i \leq k$, $G_{i+1}$ is obtained by an edge rotation on $G_i$, with $H$ obtained from $G_{k-1}$ by one edge rotation. In this case we denote the rotation distance from $G$ to $H$ as $d_r(G,\ H)$ and it is equal to $k$.

**Definition 1.1.** [3] Let $S = \{\ G_1,\ G_2,\ \ldots,\ G_k\ \}$ be a set of graphs all of the same order and the same size. Then the rotation distance graph $D(S)$ of $S$ has $S$ as its vertex set and vertices (graphs) $G_i$ and $G_j$ are adjacent if $d_r(G_i,G_j) = 1$, where $d_r(G_i,\ G_j)$ is the rotation distance between $G_i$ and $G_j$.
A graph G is an edge rotation distance graph(ERDG) (or $r$ - *distance graph*) if G $\cong$ D(S) for some set S of graphs.

In 1990, Chartrand et al. [4] showed that the cycles, the complete bipartite graphs $K_{3,3}$, and $K_{2,p}$ $(p \geq 1)$ are edge rotation distance graphs. In 1997, Jarrett [10] gave a proof using different technique and showed complete graphs, trees, wheel $(W_{1,n})$ and the complete bipartite graph $K_{m,n}$ $(3 \leq m \leq n)$ are edge rotation distance graphs. In [8], Huilgol et al. showed that the generalized Petersen graph, $G_p(n, 1)$, the generalized star, $K_{(1,n)}$ are edge rotation distance graphs.

In this paper we consider the edge rotations on ladder graph, triangular snake, quadrilateral snake, double triangular snake, double quadrilateral snake, alternate triangular snake, alternate quadrilateral snake. A triangular snake is a connected graph in which all blocks are triangles and the block cut point graph is a path[12]. Since these graphs contain cycles as subgraphs, to generate them we use the method used by Jarrett [10] with slight modifications to prove all of the above specified graphs are Edge Rotation Distance Graphs(ERDG). Here the number of vertices and edges are denoted by $n'$ and $m'$, in order to avoid confusion.

**Definition 1.2.** [13] A triangular snake $T_n$ is obtained from a path $u_1,\ u_2,\ u_3,\ \ldots,\ u_n$ by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$ for $1 \leq i \leq n-1$.

**Definition 1.3.** [13] A double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path.

**Definition 1.4.** [13] An alternate triangular snake $AT_n$ is obtained from a path $u_1,\ u_2,\ u_3,\ \ldots,$ $u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to a new vertex $v_i$.

**Definition 1.5.** [13] An alternate double triangular snake $ADT_n$ consists of two alternate triangular snakes that have a common path.

**Definition 1.6.** [13] A quadrilateral snake $Q_n$ is obtained from a path $u_1$, $u_2$, $u_3$, …, $u_n$ by joining $u_i$ and $u_{i+1}$ to new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$.

**Definition 1.7.** [13] An alternate quadrilateral snake $AQ_n$ is obtained from a path $u_1$, $u_2$, $u_3$, …, $u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$.

**Definition 1.8.** [13] An alternate double quadrilateral snake $A(D(Q_n))$ consists of two alternate quadrilateral snakes that have a common path.

**Definition 1.9.** [13] A polygonal chain $G_{m,n}$ is a connected graph all of whose m blocks are polygons on n sides.

**Definition 1.10.** A ladder, $L_n$ is defined as the cartesian product of a path and $K_2$, that is, $L_n = P_n \times K_2$.

## 2 Edge Rotations on Snakes

We use the method by Jarrett [10] with some modifications to prove the following snake related theorems.

**Theorem 2.1.** Every triangular snake is an ERDG.

**Proof:** We first generate a $T_2$. Since the same pattern is repeated we just change the labeling and thus generate a $T_n$. Since a $T_2$ is nothing but a triangle, the construction is as follows.

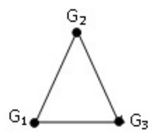


Figure 1: The triangular snake graph - $T_2$.

Let $S = \{G_1, G_2, G_3\}$. Consider the graphs $G_1$, $G_2$ and $G_3$ as shown in Figure 2. We first show that the $d_r(G_1, G_2) = d_r(G_2, G_3) = d_r(G_3, G_1) = 1$.
We see that the edge $xu_2$ is rotated to $xu_6$, thus resulting in one rotation between the graphs $G_1$ and $G_2$. Similarly, we observe the edge $yu_4$ rotated to $yu_2$ between the graphs $G_2$ and $G_3$ to show one rotation. Also, the edge $yu_2$ in $G_3$ is rotated to $yu_4$ in $G_1$. Thus, the rotation distance between each of these graphs is equal to one resulting in a $T_2$. Thus $D(S) \cong T_2$.
To generate a $T_3$ , we use graph(vertex), i.e., $G_3$, by just changing the labels of the vertices.

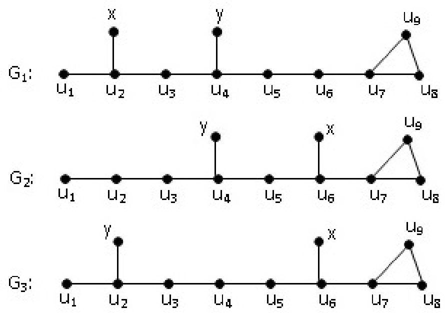


Figure 2: Rotations on a triangular snake graph.

That is we perform the rotations in the reverse directions, viz., $G_3$ to $G_2$, $G_2$ to $G_1$ and then finally $G_3$ to $G_1$, thus forming one more $C_3$.
In a similar way a $T_n$ is generated using $2n-1$ graphs. Hence, a $T_n$ is an edge rotation distance graph. ∎

**Theorem 2.2.** Every double triangular snake is an ERDG.

**Proof:** We first prove $D(T_2)$ is an ERDG by fixing the value of $n' = 4$. We generate a cycle of length 4 and then show that the rotation distance between the first and third vertex is one thus forming a $D(T_2)$. Let $S = \{G_1, G_2, G_3, G_4\}$. Consider the graphs $G_1$, $G_2$, $G_3$ and $G_4$ as shown in Figure 4.

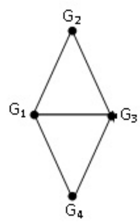


Figure 3: A double triangular snake - $D(T_2)$.

We first show that the $d_r(G_1, G_2) = d_r(G_2, G_3) = d_r(G_3, G_4) = d_r(G_4, G_1) = 1$. Also, we show that $d_r(G_1, G_3) = 1$.
We see that the edge $xu_2$ is rotated to $xu_6$, thus resulting in one rotation between the graphs $G_1$ and $G_2$. Similarly, we observe the edge $yu_4$ rotated to $yu_8$ between the graphs $G_2$ and $G_3$ to show one rotation. The edge $xu_6$ in $G_3$ is rotated to $xu_2$ in $G_4$ and $yu_8$ in $G_4$ is rotated to $yu_4$ in $G_1$ to show one rotation between $G_4$ and $G_1$. Also, $u_1u_3$ is rotated to $u_1u_4$ between the graphs $G_1$ and $G_3$ to show one rotation between them. In order to equalize the size between the remaining graphs $G_2$ and $G_4$ we add an edge $u_2u_4$. Thus, the rotation distance between

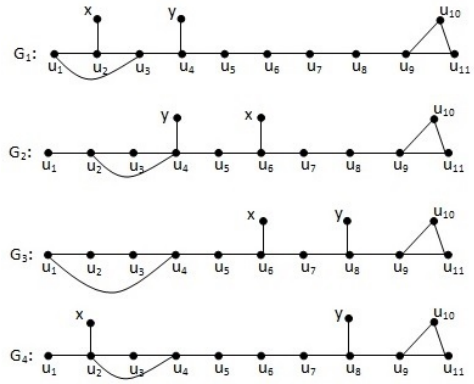


Figure 4: Rotations on a double triangular snake.

each of these graphs is equal to one resulting in a $D(T_2)$. Thus $D(S) \cong D(T_2)$.
In a similar way a $D(T_n)$ is generated. Hence, a $D(T_n)$ is an edge rotation distance graph. ■

**Theorem 2.3.** Every alternating double triangular snake is an ERDG.

**Proof:** The alternating double triangular snake is denoted by $ADT_n$. Here we will generate an $ADT_2$ followed by a path. To generate this through edge rotations we shall first generate a $C_4$ and then from the third vertex( i.e., $G_3$) we use edge rotation between graphs $G_3$ and a new graph $G_1$ and thus generate a path of length 1. Let $S = \{G_1, G_2, G_3, G_4, G_1\}$. Consider the graphs $G_1$, $G_2$, $G_3$ and $G_4$ as shown in Figure 6.

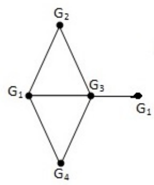


Figure 5: An alternate double triangular snake - $ADT_3$.

Now we show that the $d_r(G_1, G_2) = d_r(G_2, G_3) = d_r(G_3, G_4) = d_r(G_4, G_1) = 1$. Also, we show $d_r(G_1, G_3) = 1$. To generate a path of length one from the graph $G_3$ we consider one more new graph $G_1$ and thus show the rotation distance between $G_3$ and $G_1$ is one.
We see that the edge $xu_2$ is rotated to $xu_6$, thus resulting in one rotation between the graphs $G_1$ and $G_2$. Similarly, we observe the edge $yu_4$ rotated to $yu_8$ between the graphs $G_2$ and $G_3$ to show one rotation. The edge $xu_6$ in $G_3$ is rotated to $xu_2$ in $G_4$ and $yu_8$ in $G_4$ is rotated to $yu_4$ in $G_1$ to show one rotation between $G_4$ and $G_1$. Also, $u_1u_3$ is rotated to $u_1u_4$ between the graphs $G_1$ and $G_3$ to show one rotation between them. In order to equalize the size between the remaining graphs $G_2$ and $G_4$ we add an edge $u_2u_4$. Thus, the rotation distance between

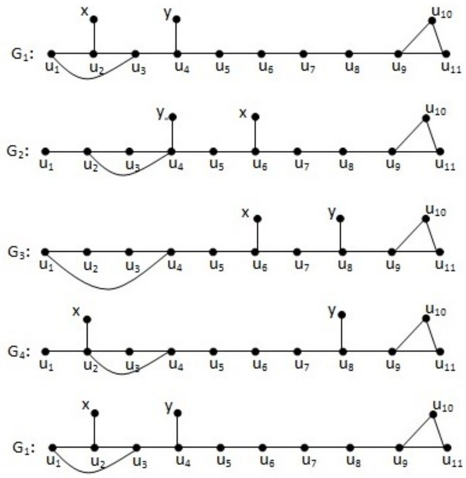


Figure 6: Rotations on an alternate double triangular snake.

each of these graphs is equal to one resulting in a $D(T_2)$. Thus $D(S) \cong D(T_2)$.
Now, to form $ADT_2$ (a path of length 1, from graph $G_3$), we consider the graph $G_1$ once again and show the rotation distance between $G_3$ and $G_1$ is one. The edge $u_1u_4$ in $G_3$ is rotated to $u_1u_3$ in the fifth graph ($G_1$) to form an $ADT_2$.
Thus, in a similar way an $AD(T_n)$ is generated. Hence, an $AD(T_n)$ is an edge rotation distance graph. ∎

**Theorem 2.4.** Every quadrilateral snake is an ERDG.

**Proof:** A quadrilateral snake is denoted by $Q_n$. If $n = 2$, then it is a $C_4$. Since the same pattern is repeated, we generate a $Q_2$, and thus by changing the order of labeling we generate a $Q_n$.

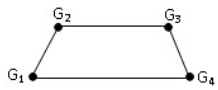


Figure 7: A quadrilateral snake.

Let $S = \{G_1, G_2, G_3, G_4\}$. Consider the graphs $G_1$, $G_2$, $G_3$ and $G_4$ as shown in Figure 8.

We first show that the $d_r(G_1, G_2) = d_r(G_2, G_3) = d_r(G_3, G_4) = d_r(G_4, G_1) = 1$. Here in the above set of graphs, to show the rotation between the graph $G_1$ and $G_2$, the edge $xu_2$ is rotated to edge $xu_6$. The edge $yu_4$ is rotated to $yu_8$ between the graphs $G_2$ and $G_3$. The edge $xu_6$ is rotated to $xu_2$ between the graphs $G_3$ and $G_4$. The edge $yu_8$ is rotated to $yu_4$ between the

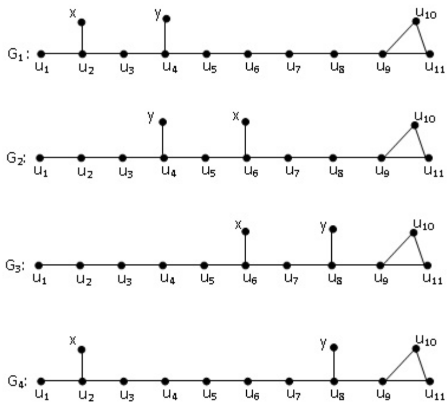


Figure 8: Rotations on a quadrilateral snake.

graphs $G_4$ and $G_1$ to show one rotation. Thus, the rotation distance between each of these graphs is equal to one resulting in a $Q_2$. Thus $D(S) \cong Q_2$.
Similarly we show that $Q_n$ is an edge rotation distance graph. ∎

**Theorem 2.5.** Every double quadrilateral snake is an ERDG.

**Proof:** We generate a cycle of length 6 and then show that the rotation distance between the first and fourth vertex is one and thus forming a $D(Q_2)$.

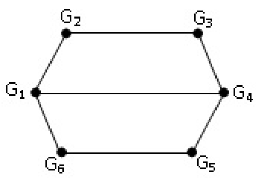


Figure 9: A double quadrilateral snake $D(Q_2)$.

Let $S = \{G_1, G_2, G_3, G_4, G_5, G_6\}$. Consider the graphs $G_1$, $G_2$, $G_3$, $G_4$, $G_5$ and $G_6$ as shown in Figure 10. We first show that the $d_r(G_1, G_2) = d_r(G_2, G_3) = d_r(G_3, G_4) = d_r(G_4, G_5) = d_r(G_5, G_6) = d_r(G_6, G_1) = 1$. Here in the above set of graphs, to show the rotation between the graph $G_1$ and $G_2$, the edge $xu_2$ is rotated to edge $xu_6$. The edge $yu_4$ is rotated to $yu_8$ between the graphs $G_2$ and $G_3$. The edge $xu_6$ is rotated to $xu_{10}$ between the graphs $G_3$ and $G_4$. The edge $yu_8$ is rotated to $yu_{12}$ between the graphs $G_4$ and $G_5$ to show one rotation. Similarly, the edge $xu_{10}$ is rotated to $xu_2$ between the graphs $G_5$ and $G_6$. And the edge $yu_{12}$ is rotated to $yu_4$ between the graphs $G_6$ and $G_1$ to show one rotation.

To show the rotation distance between $G_1$ and $G_4$ is one we add the edge $u_1$ $u_3$ to $G_1$ and the edge $u_1$ $u_4$ to $G_4$. Since the size of the graphs $G_1$ and $G_4$ changes we add an extra edge

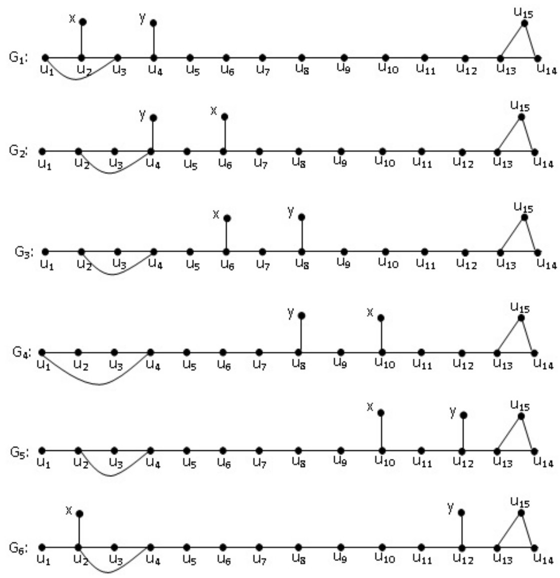

Figure 10: Rotations on a double quadrilateral snake.

$u_2u_4$ to the remaining graphs namely $G_2$, $G_3$, $G_5$ and $G_6$.
Thus, the rotation distance between each of these graphs is equal to one resulting in a $D(Q_2)$. Thus $D(S) \cong D(Q_2)$. Extending the construction we get a $D(Q_n)$ by considering $5n - 4$ graphs. ■

**Theorem 2.6.** Every alternating double quadrilateral snake is an ERDG.

**Proof:** To prove this theorem, we use the proof of Theorem 2.5, with a slight change. We first generate a $DQ_2$ and then show that the rotation distance between the fourth graph ($G_4$,(vertex)) and the new graph(one again $G_1$ to be considered) is one, thus forming a path of length 1.

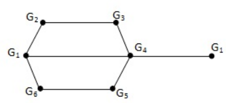

Figure 11: An alternating double quadrilateral snake.

For this we need to show the rotation distance between the graph $G_4$ and $G_1$ (considered once again) is one. Since we have already added an edge $u_1u_4$ to the graph $G_4$, we add an edge $u_1u_3$ to $G_1$, and thus by performing this rotation shows the distance between them is one, and forming a path of required length.
Thus, the above mentioned procedure generates an $AD(Q_2)$. Since, this pattern is repeated the basic number of graphs to generate such a snake is $6 + 1$. To generate $AD(Q_n)$, the number of graphs required is $6 * (n/2)$, for $n$, even and $6 * \lfloor (n/2) \rfloor + 1$, for $n$, odd. ■

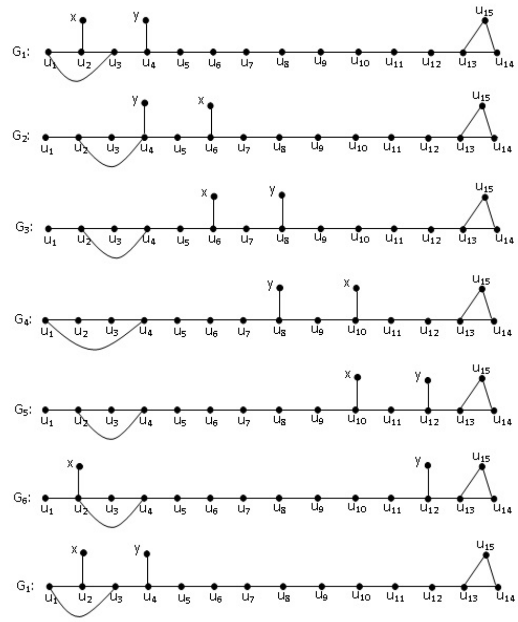


Figure 12: Rotations on an alternating double quadrilateral snake.

**Theorem 2.7.** A ladder graph, $L_n$ is an ERDG.

**Proof:** Let $P$: $u_1$, $u_2$, …, $u_{n+2}$ be a path and $G$ be a graph obtained by adding two new vertices $u_{n+3}$, $u_{n+4}$ and three new edges $u_{n+2}u_{n+3}$, $u_{n+3}u_{n+4}$, $u_{n+4}u_{n+2}$. Then, for $i$ = 1, 2, …, $n-1$, define $G_i$ to be a graph obtained from $G$ by adding one new vertex $x$ adjacent only to $u_i$. We also define $G_n$ as the graph obtained from $G$ by adding one new vertex $x$ adjacent only to $u_1$. For all $n$, we add a new edge $u_1$ $u_3$ for all $G_n$. For $n = 3$, the graphs $G_1$, $G_2$ and $G_3$ are shown in Figure 14.

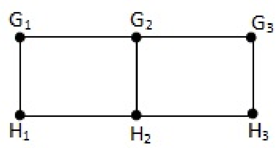


Figure 13: A ladder graph.

Every graph $G_i$ has exactly one vertex of degree one, and an edge rotation changes the degrees of exactly two vertices, $d_r(G_i, G_j) > 1$. On the other hand for $i = 1, 2, 3,$ …, $n-2$, $G_{i+1} \cong G_i - xu_i + xu_{i+1}$ and consequently $d_r(G_i, G_{i+1}) = 1$ and $d_r(G_n, G_{n-1}) = 1$, since $G_n \cong G_{n-1} - xu_{n-1} + xu_1$; thus $D(\{G_1, G_2, \ldots, G_n\}) \cong P_n$.
Similarly we generate one more path from the set of graphs $\{H_1, H_2, \ldots, H_n\}$.

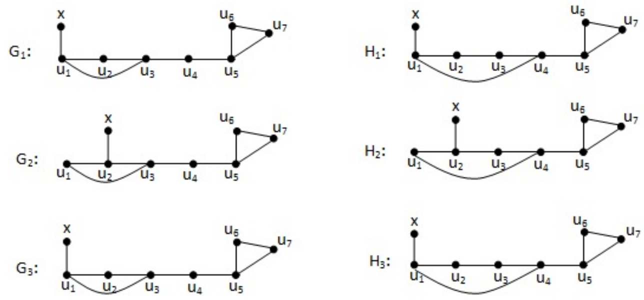


Figure 14: Rotations on a ladder graph.

Now we show that the rotation distance between each of these graphs $G_i$ and $H_i$ is one. Since $G_i \cong H_i$ - $u_1u_3$ + $u_1u_4$, $d_r(G_i, H_i) = 1$.
Hence, $D(\{G_1, G_2, \ldots, G_n, H_1, H_2, \ldots, H_n\}) \cong L_n$. ∎

**Remark 2.8.** A polygonal chain $G_{m,n}$ is an ERDG.

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