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RADIATION AND NON-DARCY EFFECTS ON CONVECTION IN POROUS MEDIA

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RADIATION AND NON-DARCY EFFECTS ON CONVECTION IN POROUS MEDIA

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Combined conduction, convection and radiation heat transfer in a gray fluid-saturated sparsely packed porous medium is examined analytically for marginal convection using linear stability analysis. The effects of boundary and inertia which were absent in the usual Darcy model are considered. The Milne–Eddington approximation is employed to determine the solutions valid for transparent and opaque media which absorbs and emits thermal radiation. It is shown that the nature of the bounding surfaces and radiation significantly influence the critical Rayleigh and wave numbers. The mechanism for suppressing or augmenting convection is discussed in detail. The results obtained using Galerkin technique are compared with the existing results of Darcy model and of non-radiating systems and agreement is found.
KEYWORDS   Radiation   Non-Darcy   Convection   Porous media.

INTRODUCTION

It is known that although radiative heat transfer is neglected in homogeneous reactors, the situation is different in packed bed reactors (Vortmeyer (1972)). A large amount of heat is emitted by the surface of the solid particle at high temperature, since the emitted energy is proportional to T^4. Therefore, the temperature gradients in the reaction zones of exothermic reactions lead to radiative fluxes as they produce conductive fluxes. For radiation, the magnitude of the fluxes depends mainly on temperature level and surface emissivity. These fluxes contribute much to heat and mass transfer in porous media. This has important applications in variety of areas, including collection and storage of solar energy, processing of glasses and other semi-transparent materials, geothermal energy and fibrous and foam insulations. Most of the work on this has been concerned with problems of combined conduction and radiation in a porous medium (Vortmeyer (1972), Fernades and Francis (1982) and Ho and Ozisik (1988)). However, heat transfer by combined conduction, convection and radiation is more likely to occur in many of the practical problems mentioned above. Inspite of these applications this problem has not been given much attention.

Recently, Vortmeyer *et al.* (1988) have studied this heat transfer problem using Darcy's law which neglects the boundary and inertia effects on fluid flow and heat transfer. The Darcy model is valid in a densely packed porous medium made up of uniform spherical particles, for which the porosity and permeability are small.

To achieve effective insulation and high efficiency in many practical problems sited above it is advantageous to use special type of solid materials having high permeability and porosity. In such sparsely packed porous media the boundary and inertia effects can not be neglected. Beavers and Sparrow (1969) have shown that though inertia effects are not important in low-porosity porous media they are significant in high-porosity media. The inertia effects also become important at high Darcy–Reynolds number. Further, the distortion of velocity at the boundaries gives rise to the usual viscous force, which was first postulated by Brinkman (1947). In the absence of radiation, Vafai and Tien (1981), Rudraiah (1984) and Gill and Minkowycz (1988) have studied convection, incorporating the effects of boundary and inertia. These effects are also significant on heat transfer by combined conduction, convection and radiation because of their importance in engineering applications where high temperature and higher designing accuracy are required to improve the power systems. The results of earlier work (Vortmeyer *et al.* (1988)) are not of much use here, because they neglect the non-Darcy effects. These non-Darcy effects are considered in this paper using Darcy–Forchheimer–Brinkman model (Gill and Minkowycz (1988)).

To bridge the gap between the mathematical complexity of the exact radiative transfer theory and the need of concise engineering formulae, the two-flux model (Vortmeyer *et al.* (1980)) is employed to estimate the radiative flux. The complete set of conservative equations in this case are extremely difficult to solve. Therefore, these equations are solved analytically adopting Milne–Eddington approximation considering transparent and opaque media. The medium is assumed to be gray and black and has constant properties. We ignore the radiative stress in the momentum equation, because its magnitude is negligible when compared to the more significant molecular stress (Arpaci and Guzum (1973)). Hence the radiation contribution comes only through the energy equation. We note that the principle of exchange of stability is valid in the present problem as in the viscous flow (Arpaci and Guzum (1973)). Therefore we study only the condition for the onset of marginal convection using Galerkin technique. The critical Rayleigh and wave numbers are obtained, which are functions of absorptive, radiative intensity and porous parameters. To show that a single-term Galerkin expansion gives reasonable results, the results obtained by this method are compared with those obtained using Darcy law and viscous flow and some important conclusions are drawn.

MATHEMATICAL FORMULATION

Consider an incompressible Boussinesq fluid-saturated high porosity porous layer of finite depth 'h', confined between two infinite horizontal isothermal and radiating surfaces heated from below and cooled from above. The lower plate is kept on the xy-plane and z-axis is vertically upwards. For this physical configuration, the basic conservative equations with the radiative contribution in

the energy equation are

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\partial \vec{q}/\partial t + C_b \,|\vec{q}|\, \vec{q}/\sqrt{k} = -\nabla p/\rho_s + \rho\vec{g}/\rho_s - v\vec{q}/k + v\nabla^2\vec{q} \tag{2}$$

$$\partial T/\partial t + (\vec{q} \cdot \nabla)T = K\nabla^2 T + H \tag{3}$$

$$\rho = \rho_s[1 - \alpha(T - T_s)] \tag{4}$$

The quantities are defined in the nomenclature. Here $C_b/\sqrt{k} = 1.75(1 - \phi)/\phi^3 d$, $k = d^2\phi^3/175(1 - \phi)^2$ (Rohsenow and Hartnett (1973)).

BASIC STATE

The basic system is quiescent with heat transfer by conduction and radiation given by

$$0 = H_b + Kd^2T_b/dz^2. \tag{5}$$

If F is the z-component of the radiative heat flux, then

$$H_b = -dF/dz. \tag{6}$$

We may now write (5), using (6), in the integrated form

$$F - K\beta = C \tag{7}$$

where C is the constant of integration. Here, the basic temperature gradient β is related to the radiative flux, F, which depends on the radiative intensity, I.

The computation of I for gray emitting packed bed is based on the fact that the particles emit, absorb and reflect heat. In addition, radiative energy may penetrate the bed through the void volume. This can be analysed by considering two-flux model, as shown in Figure 1, for which
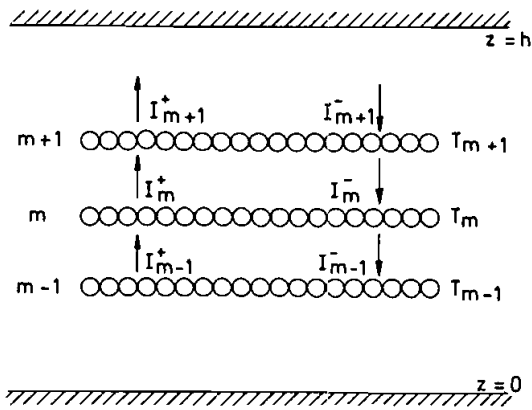
$$I(s) = I^+ - I^- \tag{8}$$

FIGURE 1   Two flux step model

$$I_m^+ = NI_{m-1}^+ + \epsilon(1 - N)\sigma T_m^4 + I_m^-(1 - N)(1 - \epsilon)$$

$$I_m^- = NI_{m+1}^- + \epsilon(1 - N)\sigma T_{m+1}^4 + I_m^+(1 - N)(1 - \epsilon)$$

where I^+ and I^- are the net forward and reverse respectively. Since the average diameter of the particles is small compared to the width of the bed, these two fluxes are related to each other through the set of linear ordinary differential equations

$$dI^+/ds = -K_a I^+ + K_b \epsilon \sigma T^4/\pi - K_g I^-$$
$$dI^-/ds = K_a I^- - K_b \epsilon \sigma T^4/\pi + K_g I^+ \tag{9}$$

where σ is the Stefan's constant, the total absorption coefficient K_a, emission coefficient K_b and scattering coefficient K_g are given by (Vortmeyer (1980))

$$K_a = [2(1 + N)^2 + (1 - N^2)(1 - \epsilon)^2(1 - N)]/[(1 + N)^2 - (1 - N)^2(1 - \epsilon)^2](1 + N)d \tag{10}$$

$$K_b = 2[(1 + N)^2 - (1 - N^2)(1 - \epsilon)]\epsilon(1 - N)/[(1 + N)^2 - (1 - N)^2(1 - \epsilon)^2](1 + N)d \tag{11}$$

$$K_g = 2[(1 + N)^2 + (1 - N^2)](1 - \epsilon)(1 - N)/[(1 + N)^2 - (1 - N)(1 - \epsilon)^2](1 + N)d. \tag{12}$$

Coefficients K_a, K_b and K_g depend on the transmittance number N which is a function of the emissivity ϵ and porosity ϕ. We see that $(K_a - K_g)$ will be the true absorption coefficient. In this case the scattering coefficient K_g is zero and absorption coefficient equals emission coefficient, and is given by

$$K_a = K_b = 2(1 - N)/(1 + N)d. \tag{13}$$

Then the radiative heat transfer equations (9) simplify to

$$dI^+/ds = K_a[\sigma T^4/\pi - I^+]$$
$$dI^-/ds = -K_a[\sigma T^4/\pi - I^-]. \tag{14}$$

Combining these, using (8) and defining the black-body intensity as

$$B = \sigma T^4/\pi \tag{15}$$

we get

$$dI/ds = K_a[B - I]. \tag{16}$$

This equation of transfer is analogous to the one given by Kourganoff (1952) in the case of pure viscous flow. Also the radiative heating rate of the fluid-saturated porous medium is

$$H = -\int (dI(s)/ds) \, dw \tag{17}$$

where w is the element of solid angle and the integral is taken over an angle 4π.

In the basic state all the quantities are functions of z only and hence the equation of transfer (16) takes the form

$$\mu_3 \, dI/ds = K_a[B - I]. \tag{18}$$

Here K_a is given by (13) and μ_3 is the direction cosine of the vector s in the z-direction. Assuming Milne–Eddington approximation, we can obtain the differential equation for F, using the radiative equation (18) in the form (Vortmeyer *et al.* (1988))

$$d^2F/dz^2 - A^2F = -A^2XC/(1 + X) \tag{19}$$

where $X = 4\pi Q/3K_aK$ and $Q = (4\sigma/\pi)(T_b + T')^3$ which is assumed to be constant. Solving (7) and (19) using the radiative boundary conditions

$$\begin{aligned} dF/dz &= -2K_ahF &&\text{at } z = 1/2 \\ dF/dz &= 2K_ahF &&\text{at } z = -1/2 \end{aligned} \tag{20}$$

we get

$$f = \beta/\bar{\beta} = L\cosh(Az) + M. \tag{21}$$

Here

$$L = X/[(2X + \sqrt{(3 + 3X)}/2)\sinh(A/2) + \cosh(A/2)]$$

and

$$M = L[\sqrt{(3 + 3X)}\sinh(A/2)/2 + \cosh(A/2)]/X.$$

The quantities $X = 4\pi Q/3K_aK$ and $A = K_ah\sqrt{3(1 + X)}$ are the radiative parameters which determine the radiative intensity and absorptivity.

LINEAR STABILITY ANALYSIS

On the basic state discussed in the above section we superimpose a small perturbation of the form

$$\vec{q} = \vec{q}', \qquad T = T_b + T', \qquad p = p_b + p', \qquad H = H_b + H' \tag{22}$$

where the primes denote the perturbed quantities. Substituting these into (1)–(4), linearizing, assuming the principle of exchange of stability is valid, eliminating the pressure and expressing the resulting equations in terms of the z-component of velocity, W, we obtain the perturbation equations in the form

$$\nabla^4W' - \nabla^2W'/k = -\alpha g\nabla_1^2T'/v \tag{23}$$

$$W'\beta = K\nabla^2T' + H'. \tag{24}$$

The contribution of radiative heating rate to the energy equation relates to the temperature through a differential equation under the two approximations, transparent and opaque, based on the optical thickness, K_ah compared to the wave length, a (Vortmeyer *et al.* (1988)). For the derivation of this relation we combine (17) and (18) to get

$$H' = -4\pi K_aB + K_a \int I(s)\, dw. \tag{25}$$

For a transparent medium $(K_a h \ll a)$, the second term in (25) can be neglected compared to the first term. Hence in the linear analysis using (15) we get

$$\nabla_1^2 H' = -4\pi Q K_a \nabla_1^2 T'. \tag{26a}$$

For opaque medium $(K_a h \gg a)$, H in (25) can be expanded in a power series in terms of K_a^{-1}. The resulting series after successive integration with respect to w using (15) leads to

$$\nabla_1^2 H' = (4\pi Q/3 K_a)\nabla^2(\nabla_1^2 T'). \tag{26b}$$

On substituting (26a) and (26b) into (24) we get

$$\nabla_1^2(W'\beta) = K\nabla^2(\nabla_1^2 T') - 4\pi Q K_a \nabla_1^2 T' \tag{27a}$$

for transparent medium and

$$\nabla_1^2(W'\beta) = K\nabla^2(\nabla_1^2 T') - (4\pi Q/3 K_a)\nabla^2(\nabla_1^2 T') \tag{27b}$$

for opaque medium.

The solution of the momentum and the energy equations are assumed to be of the form

$$\begin{bmatrix} W'(x, y, z) \\ T'(x, y, z) \end{bmatrix} = \begin{bmatrix} W(z) \\ T(z) \end{bmatrix} \exp(i(1x + my)) \tag{28}$$

where $W(z)$ and $T(z)$ are the amplitude of the perturbed quantities. Substituting (28) into (23), (27a) and (27b), non-dimensionalizing using the transformation

$$W \to KW/h, \qquad D \to D/h, \qquad a \to a/h, \qquad T \to \bar{\beta}hT \tag{29}$$

we obtain

$$(D^2 - a^2)^2 W - P(D^2 - a^2)W = Ra^2 T \tag{30}$$

$$-Wf = (D^2 - a^2 - A^2 X/(1 + X))T \tag{31a}$$

for transparent medium and

$$-Wf = (1 + X)(D^2 - a^2)T \tag{31b}$$

for opaque medium. Here, $R = -\alpha g \bar{\beta} h^4/Kv$ is the Rayleigh number and $P = h^2/k$ is the porous parameter.

The exact solutions of (31a) and (31b) are difficult due to the presence of f. Therefore to get an eigenvalue relation for the present problem we use the Galerkin method. In this method, we multiply (30) by W, (31a) and (31b) by T, integrate with respect to z from $-1/2$ to $1/2$ and obtain the equations

$$\langle (D^2 W)^2 + (2a^2 + P)(DW)^2 + (a^4 + Pa^2)W^2 \rangle = Ra^2 \langle WT \rangle \tag{32}$$

$$\langle WTf \rangle = \langle (DT)^2 + (a^2 + A^2 X/(1 + X))T^2 \rangle \tag{33a}$$

$$\langle WTf \rangle = (1 + X)\langle (DT)^2 + a^2 T^2 \rangle. \tag{33b}$$

We substitute $W = CW_1$, $T = ET_1$ and eliminate the constants C and E from the resulting equations and for simplicity neglecting the suffix unity, we get

$$R_t = \langle (D^2 W)^2 + (2a^2 + P)(DW)^2 + (a^4 + Pa^2)W^2 \rangle$$
$$\times \langle (DT)^2 + (a^2 + A^2 X/(1 + X))T^2 \rangle / a^2 \langle WT \rangle \langle WTf \rangle \tag{34a}$$

in the transparent approximation and

$$R_o = (1 + X)\langle (D^2W)^2 + (2a^2 + P)(DW)^2 + (a^4 + Pa^2)W^2 \rangle$$
$$\times \langle (DT)^2 + a^2T^2 \rangle /a^2 \langle WT \rangle \langle WTf \rangle \tag{34b}$$

in the opaque approximation.

CRITICAL RAYLEIGH NUMBER

Since the dependence of the values of the critical Rayleigh number on the boundaries are crucial, we now discuss the problem considering different boundary conditions. We may consider the three cases: (i) both boundaries free, (ii) lower boundary rigid and upper boundary free and (iii) both boundaries rigid. In each case we compute R_t and R_o and the results will be discussed in the final section.

Both Boundaries Free

The boundary conditions are

$$W = D^2W = T = 0 \qquad \text{at } z = \pm 1/2. \tag{35}$$

In this case we select the trial functions

$$W = (z^2 - 1/4)(z^2 - 5/4), \qquad T = 1/4 - z^2 \tag{36}$$

satisfying the boundary conditions (35). Substituting (36) in (34a) and (34b), using (21) and performing integration, we get

$$R_t = G[10 + a^2 + A^2X/(1 + X)]/J \tag{37a}$$

and

$$R_o = (1 + X)G(10 + a^2)/J \tag{37b}$$

where $G = 28[3024 + 306(2a^2 + P) + 31a^2(a^2 + P)]$ and $J = 51a^2[17M + (420L/A^7)[(4A^4 - 96A^2 - 1440)\sinh(A/2) + (720A - A^3)\cosh(A/2)]]$.

Lower Boundary Rigid and Upper Boundary Free

The boundary conditions are

$$W = DW = T = 0 \qquad \text{at } z = -1/2$$
$$W = D^2W = T = 0 \qquad \text{at } z = 1/2. \tag{38}$$

The suitable trial functions satisfying the boundary conditions (38) are

$$W = (z + 1/2)^2(1 - 2z)(1 - z), \qquad T = 1/4 - z^2. \tag{39}$$

Substituting (39) into (34a) and (34b), using (21) and performing integration

we get

$$R_t = U[10 + a^2 + A^2X/(1 + X)]/V \tag{40a}$$

and

$$R_0 = (1 + X)U(10 + a^2)/V \tag{40b}$$

where

$$U = 28[4536 + 216(2a^2 + P) + 19(a^4 + Pa^2)]$$

and

$$V = 39a^2[13M + (840L/A^7)[A^4 - 132A^2 - 1440)\sinh(A/2)$$
$$+ (6a^3 + 720A)\cosh(A/2)]].$$

Both Boundaries Rigid

The boundary conditions are

$$W = DW = T = 0 \qquad \text{at } z = \pm 1/2 \tag{41}$$

we choose the trial functions

$$W = (z^2 - 1/4)^2, \qquad T = 1/4 - z^2 \tag{42}$$

satisfying the boundary conditions (41). Substituting (42) into (34a) and (34b), using (21) and performing integration, we get

$$R_t = Y[10 + a^2 + A^2X/(1 + X)]/Z \tag{43a}$$

and

$$R_0 = (1 + X)Y(10 + a^2)/Z \tag{43b}$$

where

$$Y = 28[504 + 12(2a^2 + P) + a^2(P + a^2)]$$

and

$$Z = 27a^2[M + (168OL/A^7)[(A^3 + 60A)\cosh((A/2) - (A^2 + 10)\sinh(A/2)]].$$

R_t and R_0 attains their minimum values R_{tc} and R_{oc}, the critical Rayleigh numbers for transparent and opaque approximations respectively, at the corresponding critical wave numbers a_{tc} and a_{oc}. By assigning values to the physical parameters and minimizing R with respect to a, both R_c and a_c are determined for the above two approximations. The effect of different parameters on the onset of instability and on the cell size are computed both in transparent and opaque approximations, for all the three boundary combinations discussed above. The results are depicted in Figures 2-4 and are discussed in the next section.

RESULTS AND CONCLUSIONS

The effects of radiation, the porous parameter and the boundary combination on the linear stability of a radiating fluid-saturated porous layer heated from below
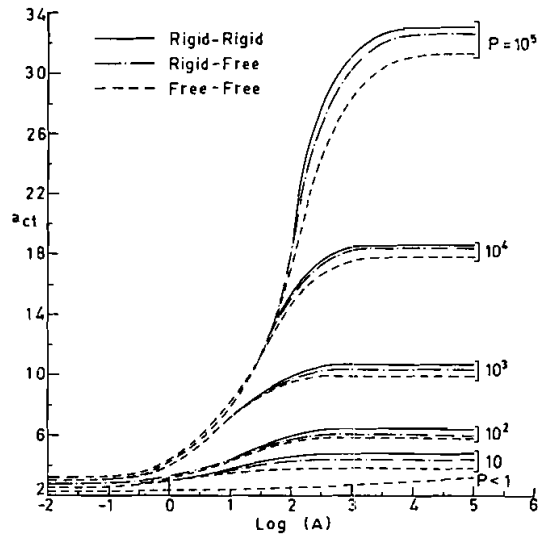
FIGURE 2 a_{ct}Vs. log(A) for $X = 10^6$ and different P.

has been investigated. The problem is solved analytically using a single term Galerkin procedure. The critical wave number a_c and the critical Rayleigh number R_c are computed in both transparent and opaque approximations for all the three boundary combinations and are depicted in Figures 2–4 and the following conclusions have been drawn.

In Figure 2, a_{tc} is drawn against A for different values of P for transparent approximation ($A < 10^5$). The effect of A is visible only when $X > 10^3$ and a_{tc} is independent of X. From this figure it is clear that both A and P have significant

FIGURE 3   Log(R_c) Vs. Log(A) for $X = 10^6$ and different P. The lines at the left and right part are for transparent and opaque approximations respectively. The dotted lines in the middle part represent the interpolation of the two approximations where neither of them holds.
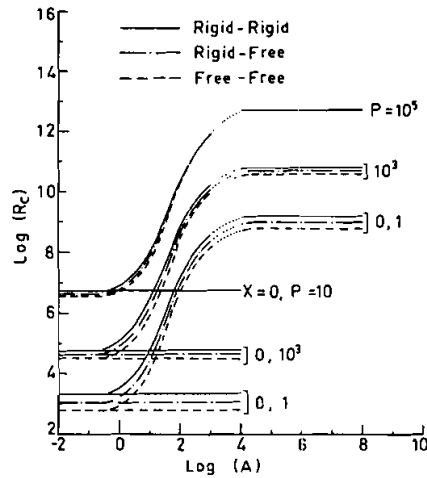
FIGURE 4   Log(R_c)Vs. Log(A) for $P = 10^2$ and different X. The lines at the left and right part are for transparent and opaque approximations respectively. The dotted lines in the middle part represent the interpolation of the two approximations where neither of them holds.
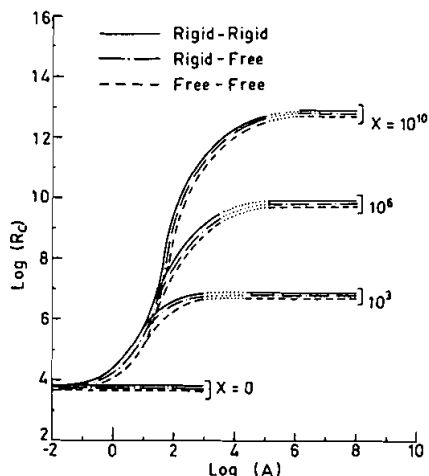
influence in contracting the cells only when $1.0 < A < 10^5$. But when $A < 1.0$ the contraction of cells is only due to P and radiation has no influence on the cell size. Boundary effects influence the cell size only for values of A in the range $10^2 < A < 10^5$. In particular, in the rigid-free case the cell gets contracted more than that in the free-free case and it is much more contracted in the rigid-rigid case (see Figure 2). It is found, from the computed results, that in opaque approximation the radiative parameters have no effect on the cell size. Thus the wave number, a_{oc}, in this approximation coincides with that in a non-radiating system.

In Figures 3 and 4, the critical Rayleigh numbers, R_c, is drawn against A for different values of P and X. We see that the critical Rayleigh number increases with A and X. Hence the effect of radiation is to inhibit the onset of convection both in transparent and opaque approximations. The extent of inhibition depends on the values of A, X and P. From Figure 3 it is clear that for small values of A the critical Rayleigh number is independent of X. As A increases R_c increases in transparent approximation but it is independent of A in opaque approximation. The porous parameter also inhibits the onset of convection. The porous parameter and radiative intensity significantly effect the onset of convection for values of A in the range $10^2 < A < 10^5$, i.e., only in large absorptive transparent medium.

According to Eq. (21), $\beta/\bar{\beta} \rightarrow 1$ if either A or X tend to zero independently. If A and X are both greater than unity, there is a boundary layer in which the variation of temperature is exponential and which tend to a discontinuity as $A \rightarrow \infty$. If $X \gg A^2$, $\beta/\bar{\beta}$ become a function of A only; Figure 5 shows a number of profiles for this limiting case.

These figures also reveal that the opaque medium with rigid-rigid boundaries is the most stable situation while less absorptive transparent medium with free-free
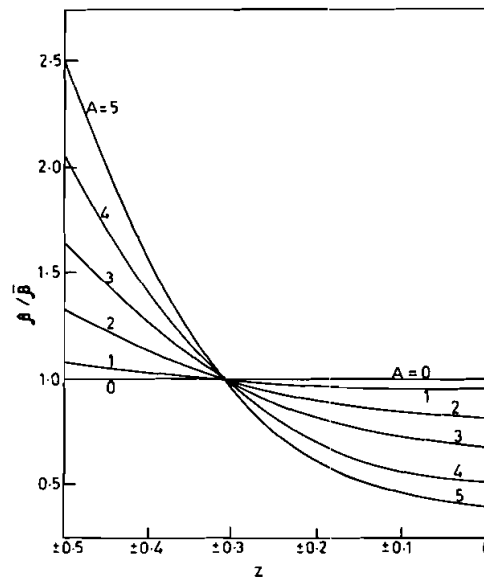
FIGURE 5 $\beta/\bar{\beta}$ as a function of Z for the limiting case $X \gg A^2$.

boundary is the most unstable one. Thus, radiation may be effectively used in the control of convection. We conclude that for large values of the porous parameter $(P > 10^5)$ the results obtained in the present paper are comparable with those obtained using Darcy model. Finally we note that although the non-linear inertia effect is significant in a high porosity porous media it has no effect on the onset of convection, since the base state is quiescent and the analysis is linear.

ACKNOWLEDGEMENTS

NOMENCLATURE

a	dimensionless wave number, $\sqrt{1^2 + m^2}$
A	radiative parameter determining the absorption, $3K_a h\sqrt{(1 + X)}$
B	black body intensity, $\sigma T^4/\pi$
C_b	drag coefficient
d	porous particle diameter
D	derivative, d/dz

f	non-dimensional temperature gradient, $\beta/\bar{\beta}$
F	z-component of the radiative flux
g	gravitational acceleration
h	vertical length scale, medium thickness
H	rate of radiative heating per unit volume
I	radiative intensity
k	permeability
K	effective thermal diffusivity
K_a	absorption coefficient, defined by (10)
K_b	emission coefficient, defined by (11)
K_g	scattering coefficient, defined by (12)
l, m	horizontal wave numbers in the x- and y-direction
N	transmittance number
p	pressure
P	porous parameter, h^2/k
$\vec{q}$	Darcy velocity vector, (U, V, W)
Q	assumed as constant, $(4\sigma/\pi)(T_b + T')^3$
R	Rayleigh number, $-\propto g\bar{\beta}h^4/Kv$
s	heat content per unit volume
t	time
T	temperature
x, y, z	horizontal and vertical space co-ordinates
X	nondimensional parameter determining the radiative heating, $4\pi Q/3K_a K$

Greek Symbols

α	coefficient of expansion
β	basic temperature gradient, dT_b/dz
$\bar{\beta}$	mean value of β throughout the medium
ϵ	emissivity
v	kinematic viscosity
ρ	density
σ	Stefan's constant
ϕ	porosity

Operators

$|\vec{q}|$	$\sqrt{U^2 + V^2 + W^2}$
$\langle \cdots \rangle$	$\int_{1/2}^{-1/2} (\cdots)\,dz$

∇^2 $\qquad$ $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$

∇_1^2 $\qquad$ $\partial^2/\partial x^2 + \partial^2/\partial y^2$

Subscripts

b	basic state
c	critical
o	opaque medium
t	transparent medium
s	reference state
oc	critical value in opaque medium
tc	critical value in transparent medium

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