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Effects of non-uniform temperature gradient and magnetic field on the onset of convection in fluids with suspended particles under microgravity conditions

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The effects of a non-uniform temperature gradient and magnetic field on the onset of convection driven by surface tension in a horizontal layer of Boussinesq fluid with suspended particles confined between an upper free / adiabatic boundary and a lower rigid / isothermal boundary have been considered. A linear stability analysis is performed. The microrotation is assumed to vanish at the boundaries. The Galerkin technique is used to obtain the eigenvalues. The influence of various parameters on the onset of convection has been analysed. Six different non-uniform temperature profiles are considered and their comparative influence on onset is discussed. It is observed that the electrically conducting fluid layer with suspended particles heated from below is more stable compared to the classical electrically conducting fluid without suspended particles. The critical wave number is found to be insensitive to the changes in the parameters but sensitive to the changes in the Chandrasekhar number. The problem has possible applications in microgravity space situations.

Recently there has been great interest in the theory and modelling of materials processing in the microgravity environment. The development of convection and corresponding heat transfer are examples of the physical phenomena to be encountered in these types of problems. Among the effects to be considered here are those of surface tension, crystalline anisotropy, non-equilibrium solidification and convection in the melt. These are relevant to the growth of large single crystals, the manufacture of semi-conductor devices and metallurgical processing. The results of space exploration, particularly the mechanism of prevention of buoyancy driven convection, are useful in understanding the physical processes involved in manufacturing these materials. Even though the microgravity environment in space is known to reduce the convection driven by buoyancy force, Marangoni convection will be generated due to the variation of surface tension with temperature. Simulating the microgravity environment in the laboratory to prevent altogether the buoyancy driven convection is difficult.

The Rayleigh-Benard situation in Eringen's[1-6] micropolar fluids has been investigated by many authors[7-14]. The main results from all these studies is that for heating from below stationary convection is the preferred mode. But it is a well known fact that the onset of convection in Benard's experiments is produced not simply by buoyancy force but primarily by variation of surface tension with the temperature. The latter effect is generally referred to as Marangoni instability. Pearson[15] was the first person to make an analytical study of this effect. According to Pearson's[15] theory for a critical value of the Marangoni number, the layer displays a short-wave pattern of stationary cellular convection. The effect of a uniform vertical magnetic field on the thermocapillary instability of a Newtonian layer of electrically conducting fluid (Marangoni magneto-convection) was first considered by Nield[16] and later by Rudraiah *et al.*[17,18]. Subsequently, Maekawa and Tanasawa[19] considered the same problem with inclined magnetic field. All the above works are for a non-deformable surface. Sarma[20], Kaddame and Lebon[21] and Wilson[22,23] studied Marangoni magneto-convection considering a deformable free surface. Recently, Milandinova and Slavtchev[24] made a weak non-linear analysis of Marangoni magneto-convection.

The objective of this paper is to suggest additional mechanisms of controlling convection using suspended particles, applied magnetic field and non-uniform basic temperature gradients. The micropolar fluid description is used in the paper for the fluid with suspended particles. The single term Galerkin expansion technique has been utilized to obtain the critical Marangoni number.

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Mathematical Formulation and Solution

Consider an infinite horizontal layer of a Boussinesquian electrically conducting micropolar fluid layer of depth 'h' permeated by an externally applied magnetic field H_o normal to the fluid. A cartesian coordinate system is taken with the origin in the lower boundary and z-axis vertically upwards. The x-axis is along the lower plate. Let ΔT be the temperature difference between lower and upper boundaries of the fluid. The interface at the upper boundary has a temperature dependent surface tension $\sigma(T)$. Expanding $\sigma(T)$ by Taylor series about T_o, we get

$$\sigma(T)=\sigma(T_0)+\frac{(T-T_0)}{1!}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}T}\right)_{T_0}+\frac{(T-T_0)^2}{2!}\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}T^2}\right)_{T_0}+------.$$

Since T-T_o is quite small in our analysis, we may write

$$\sigma(T)=\sigma_o-\sigma_1(T-T_0) \qquad \ldots (1)$$

where $\sigma_1 = -\left(\frac{\mathrm{d}\sigma}{\mathrm{d}T}\right)_{T_0}$ and $\sigma_o=\sigma(T_o)$.

The form of $\sigma(T)$ in Eq. (1) was used by Pearson[15].
The governing equations for the problem are

$$\nabla.\vec{q}=0 \qquad \ldots (2)$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right]=-\nabla P+(2\zeta+\eta)\nabla^2\vec{q}+\zeta\nabla\times\vec{\omega}+\mu_m(\vec{H}.\nabla)\vec{H} \qquad \ldots (3)$$

$$\rho_0 I\left[\frac{\partial\vec{\omega}}{\partial t}+(\vec{q}.\nabla)\,\vec{\omega}\right]=(\lambda'+\eta')\nabla(\nabla.\vec{\omega})+\eta'\nabla^2\vec{\omega},+\zeta(\nabla\times\vec{q}-2\vec{\omega}) \qquad \ldots (4)$$

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}}+\left(\vec{q}-\frac{\beta}{\rho_0 C_v}\nabla\times\vec{\omega}\right).\nabla\mathrm{T}=\chi\nabla^2\mathrm{T}, \qquad \ldots (5)$$

$$\frac{\partial\vec{H}}{\partial t}+(\vec{q}.\nabla)\vec{H}=(\vec{H}.\nabla)\,\vec{q}+\gamma_m\nabla^2\vec{H}, \qquad \ldots (6)$$

$$\nabla.\vec{H}=0, \qquad \ldots (7)$$

where $\vec{q}$ is the velocity, $\vec{\omega}$ is the spin, T is the temperature, $\vec{H}$ is the magnetic field, $P=p+\frac{\mu_m}{2}H_0^2$ is the hydromagnetic pressure, ρ_0 is the density of the fluid at a reference temperature T=T_0, ζ is the coupling viscosity coefficient or vortex viscosity, η is the shear kinematic viscosity coefficient, I is the moment of inertia, λ' and η' are the bulk and shear spin viscosity coefficient, β is the micropolar heat conduction coefficient, C_v is the specific heat, χ is the thermal conductivity, α is the coefficient of thermal expansion and $\gamma_m=1/\mu_m\sigma_m$ is the magnetic viscosity (σ_m : electrical conductivity and μ_m : magnetic permeability).

The Eqs. (2) – (7) are solved subject to containment conditions appropriate for a rigid and thermally perfect conducting wall on the underside and a free surface on the upper side. This free surface is adjacent to a non-conducting medium and subject to a constant heat flux (i.e. adiabatic). Further, the no-spin boundary condition is assumed for micro-rotation. Since the shear stress for a non-classical fluid with suspended particles is no different from that of classical fluids, the boundary conditions for flat free boundaries used by Nield[16] in respect of Newtonian fluids are appropriate for micropolar fluids also.

In the quiescent state the velocity $\vec{q}$, the spin $\vec{\omega}$, the temperature T and the magnetic field $\vec{H}$ have the following solution:

$$\vec{q}=0, \quad \vec{\omega}=0, \quad \vec{H}=H_0\hat{k}, \quad -\frac{h}{\Delta\mathrm{T}}\frac{d\mathrm{T}_o}{dz}=f\ (z) \qquad \ldots (8)$$

where $f\ (z)$ is a non-dimensional basic temperature gradient satisfying the condition

$$\int_0^1 \mathrm{f}\ (\mathrm{z})\ dz=1. \qquad \ldots (9)$$

The various non-uniform basic temperature gradients considered in this paper are presented in Table 1.

Table 1—Various non-uniform basic temperature gradients

Model	Non-uniform temperature gradient	$f(z)$
1	Linear	1
2	Heating from below	$\begin{cases}\varepsilon^{-1} & 0\le z<\varepsilon\\ 0 & \varepsilon<z\le 1\end{cases}$
3	Cooling from above	$\begin{cases}0 & 0\le z<1-\varepsilon\\ \varepsilon^{-1} & 1-\varepsilon<z\le 1\end{cases}$
4	Step function	$\delta(z-\varepsilon)$
5	Inverted parabolic	$2(1-z)$
6	Parabolic	$2z$

This type of basic temperature gradients arise due to sudden heating or cooling, radiation, through flow, etc. (see Lebon and Cloot[25]).

We now suppose that the initial state is slightly disturbed. The linearized equations of motion allow the solution of a disturbance in the form

$$[W, \Omega_z, T, H_z] = [W(z), G(z), T(z), H_z(z)] \exp[i(lx + my)], \quad \dots (10)$$

where l and m are the horizontal component of the wave number $\vec{a}$. We use this expression in the linearized version of the basic equations and non-dimensionalise the resulting equations using the following definitions:

$$(x^*, y^*, z^*) = \frac{(x, y, z)}{h}, \ \vec{q}^* = \frac{\vec{q}'}{\chi/h}, \ \vec{\omega}^* = \frac{\vec{\omega}'}{\chi/h^2},$$
$$T^* = \frac{T'}{\Delta T}, \quad \vec{H}^* = \frac{\vec{H}'}{H_0} \quad \dots (11)$$

We assume the principle of exchange of stability to be valid and hence deal with only stationary convection which is governed by the following equations after using Eq. (11):

$$(1 + N_1)(D^2 - a^2)^2 W + N_1 (D^2 - a^2) G + Q\frac{\Pr}{Pm}(D^2 - a^2) DH_z = 0, \quad \dots (12)$$

$$N_1 (D^2 - a^2) W - N_3 (D^2 - a^2) G + 2N_1 G = 0, \quad \dots (13)$$

$$(D^2 - a^2) T + f(z)(W - N_5 G) = 0, \quad \dots (14)$$

$$(D^2 - a^2) H_z + \frac{Pm}{\Pr} DW = 0, \quad \dots (15)$$

where $D \equiv \frac{\mathrm{d}}{\mathrm{d}z}$, $a^2 = l^2 + m^2$,

$N_1 = \frac{\zeta}{\zeta + \eta}$ (Coupling Parameter),

$N_3 = \frac{\eta'}{(\zeta + \eta) h^2}$ (Couple Stress Parameter),

$N_5 = \frac{\beta}{\rho_0 C_v h^2}$ (Micropolar Heat Conduction Parameter),

$\Pr = \frac{\zeta + \eta}{\chi}$ (Prandtl number),

$Pm = \frac{\zeta + \eta}{\gamma_m}$ (Magnetic Prandtl number),

$R = \frac{\alpha g \Delta T h^3 \rho_0}{(\zeta + \eta)\chi}$ (Rayleigh number) and

$Q = \frac{\mu_m H_0^2 h^2}{(\zeta + \eta)\gamma_m}$ (Chandrasekhar number).

In writing Eqs. (12)-(15) the asterisks have been omitted.

Eliminating H_z between Eqs. (12) and (15), we get

$$(1 + N_1)(D^2 - a^2)^2 W + N_1 (D^2 - a^2) G - QD^2 W = 0. \quad \dots (16)$$

Eqs. (13), (14) and (16) are solved subject to the following boundary conditions (see Nield[16] and Rudraiah and Siddheshwar[26]):

$$\left.\begin{array}{ll} W = DW = T = G = 0 & at\ z = 0 \\ W = D^2 W + a^2 MT = DT = G = 0 & at\ z = 1 \end{array}\right\}, \quad \dots (17)$$

where $M = \frac{\sigma_T \Delta T h}{\mu\chi}$ is the Marangoni number. Eq. (17) indicates the use of rigid, isothermal lower boundary and upper, free, thermally insulating boundary (with respect to the perturbation). The condition on G is the spin-vanishing boundary condition.

We now use the single-term Galerkin expansion technique to find the critical eigenvalue. Multiplying Eq.(16) by W, Eq.(13) by G and Eq.(14) by T, integrating the resulting equations by parts with respect to z from 0 to 1, using the boundary conditions (17) and using $W = \mathrm{A}\ W_1$, $G = \mathrm{B}\ G_1$, $T = \mathrm{C}\ T_1$ in which A, B and C are constants and W_1, G_1 and T_1 are trial functions, yield the following eigenvalue equation:

$$M = \frac{\left[\left\langle (DT_1)^2 \right\rangle + a^2 \left\langle T_1^2 \right\rangle\right]\left(C_1 C_2 + N_1^2 C_3^2\right)}{(1+N_1)a^2\, DW_1(1) T_1(1)\, C_4}, \quad \ldots (18)$$

where

$$C_1 = N_3 \left\langle (DG_1)^2 \right\rangle + \left(N_3 a^2 + 2N_1\right)\left\langle G_1^2 \right\rangle,$$

$$C_2 = -(1+N_1)\left[\left\langle \left(D^2 W_1\right)^2 \right\rangle + 2a^2 \left\langle (DW_1)^2 \right\rangle + a^4 \left\langle W_1^2 \right\rangle\right] - Q\left\langle (DW_1)^2 \right\rangle,$$

$$C_3 = \left\langle (DG_1)(DW_1) \right\rangle + a^2 \left\langle W_1 G_1 \right\rangle \text{ and}$$

$$C_4 = \left\langle f(z) W_1 T_1 \right\rangle C_1 - N_5 N_1 \left\langle f(z) T_1 G_1 \right\rangle C_3$$

In Eq.(18), $\langle - - \rangle$ denotes integration with respect to z between z=0 and z=1.

M (z, W_1, G_1, T_1) in Eq. (18) is a functional and Euler – Lagrange equations for the extremisation of M are Eqs (12)-(15).

We select the trial functions

$$W_1 = z^2 (1 - z^2),\ G_1 = z(1 - z),\ T_1 = z(2 - z) \quad \ldots (19)$$

such that they satisfy all the boundary conditions (17) except the one given by $D^2W + a^2MT = 0$ at $z = 1$, but the residual from this is included in the residual from the differential equations. Substituting Eqs (19) in (18) and performing the integration, we can calculate the critical Marangoni number M_c, which attains its minimum at a_c^2 .

Results and Discussion

The effects of non-uniform basic temperature gradient and magnetic field on the onset of Marangoni convection in an electrically conducting micropolar fluid have been studied. Six non-uniform temperature profiles are chosen for study. It is observed that for the critical Marangoni number, M_c, the following inequality holds for the six models under question

$$M_{c4} < M_{c3} < M_{c6} < M_{c2} < M_{c1} < M_{c5},$$

i.e., the step function is the most destabilising basic temperature distribution and inverted parabolic is the most stabilising basic temperature distribution. In the case of piecewise linear and step function profiles, the critical Marangoni number M_c depends on the thermal depth, ε, in addition to depending on the parameters of the problem. In the case of piecewise linear profile heating from below, cooling from above and step function profiles the minimum value of M_c is attained at $\varepsilon = 0.93$, $\varepsilon = 0.43$ and $\varepsilon = 0.74$ respectively.

Before embarking on a discussion of the results let us make some comments on the parameters N_1, N_3 and N_5 arising due to the suspended particles. Assuming the Clausius-Duhem inequality Eringen[4] presented certain thermodynamic restrictions, which lead to non-negativeness of N_1, N_3 and N_5. For $\zeta = 0$ ($N_1 = 0$) it is clear that equation (12) for W becomes independent of G, i.e. it is uncoupled. As $\zeta \to \infty$, we see that $N_1 \to 1$ and $N_3 \to 0$. This is the Stokesian description of suspension. Thus, it is obvious that couple stress comes into play only at small values of N_3. This supports the contention that $N_1 \in [0,1]$ and that N_3 is small positive real number. Coming to N_5 it has to be finite because the increasing of concentration has to practically stop somewhere and hence N_5 has to be a positive, finite real number.

The typical order of magnitudes of N_1, N_3 and N_5 mentioned above apply to fluid systems encountered in materials processing under microgravity in space. With the above background and with the motive

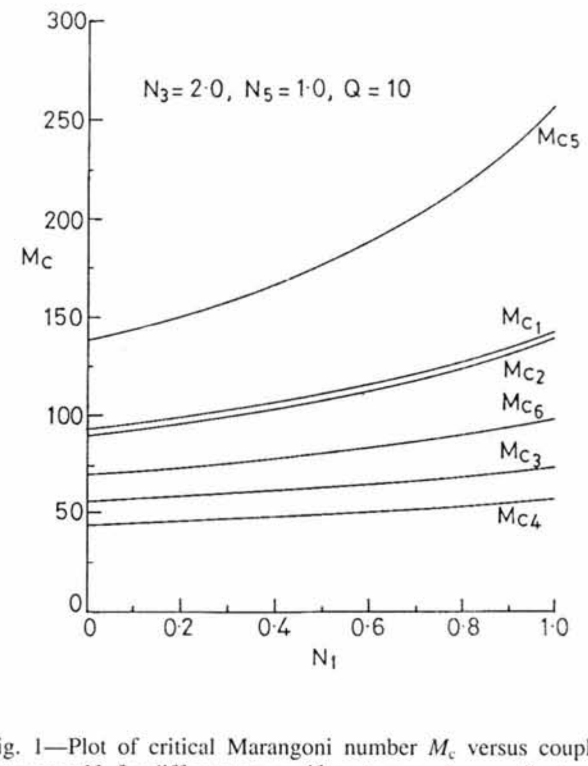

Fig. 1—Plot of critical Marangoni number M_c versus coupling parameter N_1 for different non-uniform temperature gradients.

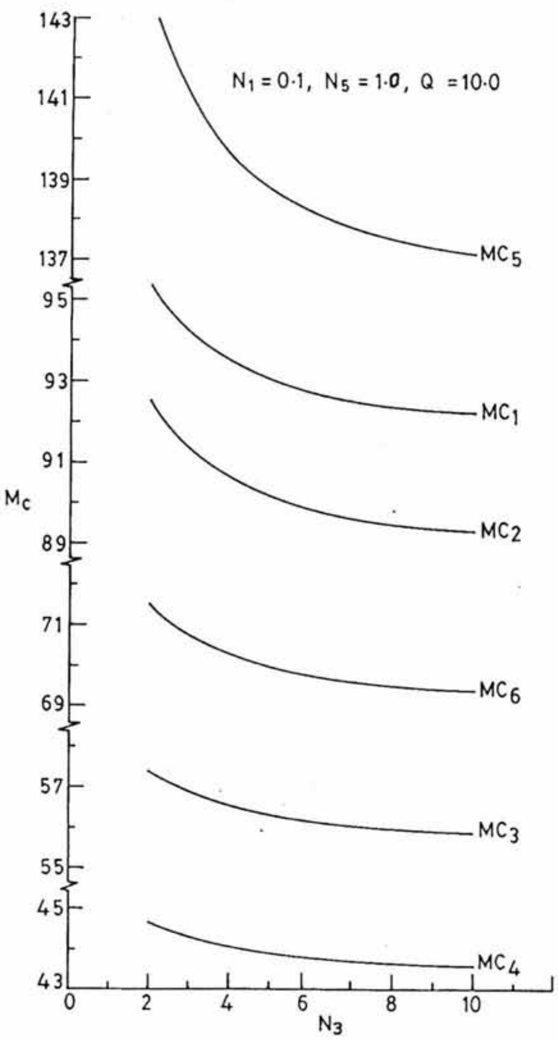

Fig. 2—Plot of M_c versus couple stress parameter N_3 for different non-uniform temperature gradients.

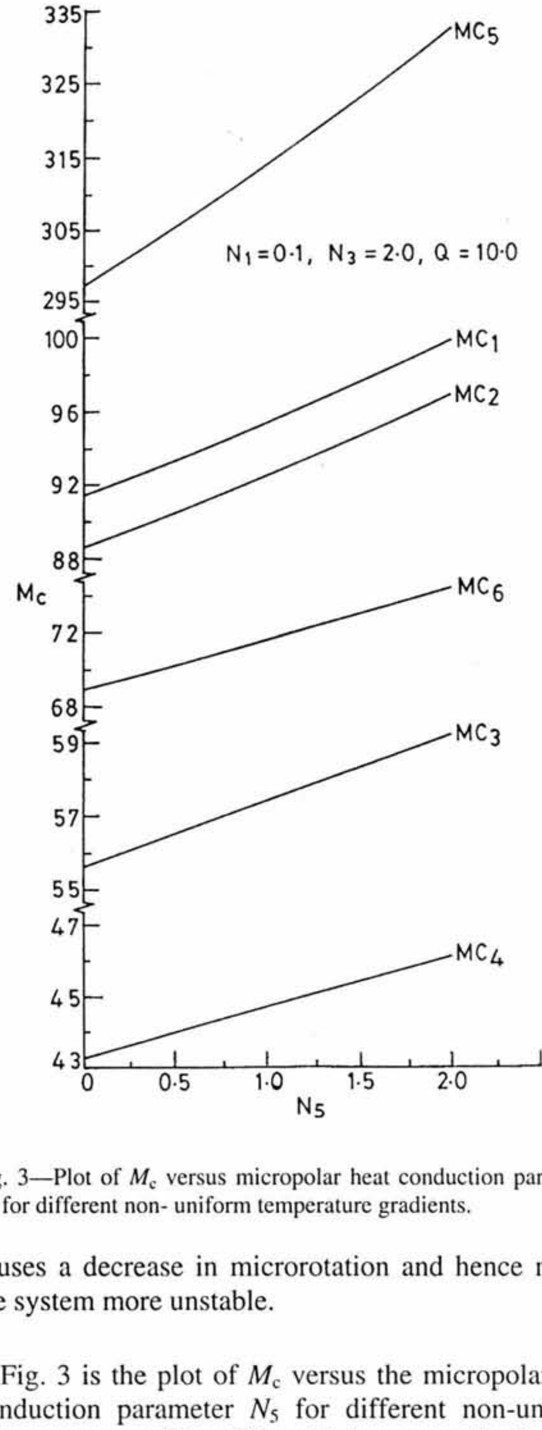

Fig. 3—Plot of M_c versus micropolar heat conduction parameter N_5 for different non- uniform temperature gradients.

specified in the introduction we now discuss the results presented by the Figs 1-6.

Fig. 1 is the plot of M_c versus the coupling parameter N_1 for different non-uniform temperature gradients. Clearly M_c increases with N_1. Increase in N_1 indicates the increase in the concentration of microelements. These elements consume the greater part of the energy of the system in developing the gyrational velocities of the fluid and as a result the onset of convection is delayed. Therefore, the increase in N_1 is to stabilise the system.

Fig. 2 is the plot of M_c versus the couple stress parameter N_3 for different non-uniform temperature profiles. Clearly M_c decreases with the increase in N_3 and ultimately levels off to the Newtonian value. Increase in N_3, decreases the couple stress of the fluid which causes a decrease in microrotation and hence makes the system more unstable.

Fig. 3 is the plot of M_c versus the micropolar heat conduction parameter N_5 for different non-uniform temperature profiles. When N_5 increases, the heat induced into the fluid due to these microelements also increases, thus reducing the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of instability. This result can also be anticipated because Eq. (5) clearly shows that the effect of the suspended particles is to deduct from the velocity. Thus, increase in N_5 is to stabilise the system.

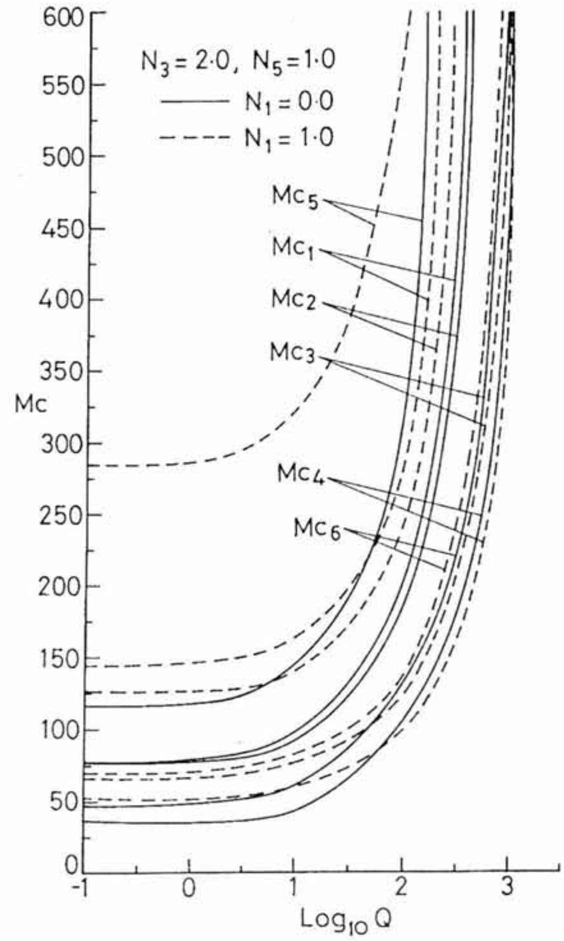

Fig. 4—Plot of M_c versus Chandrasekhar number Q for different non-uniform temperature gradients and N_1.

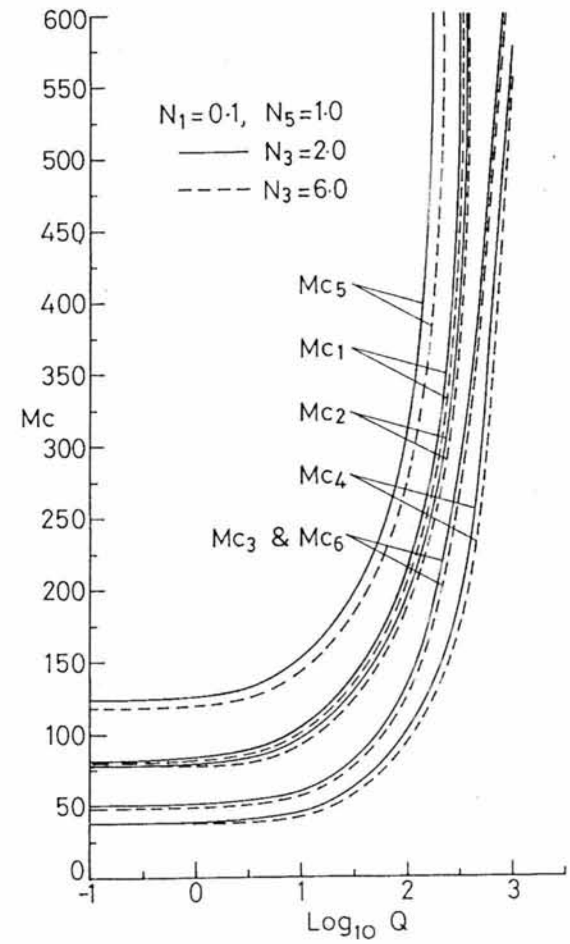

Fig. 5—Plot of M_c versus Q for different non-uniform temperature gradients and N_3.

Table 2—The values of critical wave number a_c^2 for various values of N_1, N_3, N_5 and Q for $f(z) = 1$

$Q = 0.0$				$Q = 10.0$				$Q = 1000.0$			
N_1	N_3	N_5	a_c^2	N_1	N_3	N_5	a_c^2	N_1	N_3	N_5	a_c^2
0.1	2.0	1.0	5.91	0.1	2.0	1.0	6.47	0.1	2.0	1.0	9.89
0.5	2.0	1.0	5.89	0.5	2.0	1.0	6.31	0.5	2.0	1.0	9.02
1.0	6.0	0.5	5.78	1.0	6.0	0.5	6.08	1.0	6.0	0.5	8.19
0.1	10.0	1.0	5.91	0.1	10.0	1.0	6.47	0.1	10.0	1.0	9.89
0.1	2.0	1.5	5.91	0.1	2.0	1.5	6.47	0.1	2.0	1.5	9.89
			5.91				6.47				9.89
			5.91				6.47				9.89
			5.91				6.47				9.89
			5.91				6.47				9.89

Fig. 4 is the plot of M_c versus Chandrasekhar num ber Q for different non-uniform temperature gradients and two values of N_1. It is observed that as Q increases M_c also increases. It is also observed that as N_1 increases, M_c also increases for small values of Q. However, for very large values of Q, the critical M_c is less than the Newtonian value. This result may possibly suggest a value of Q upto which the present theoretical study applies. Thus, the increase in the concentration of suspended particles is to stabilise the system along with the magnetic field.

Fig. 5 is the plot of M_c versus Q for different non-uniform temperature gradients and two values of N_3. The increase in Q increases M_c thus reiterating an earlier observation. From the figure, we see that the

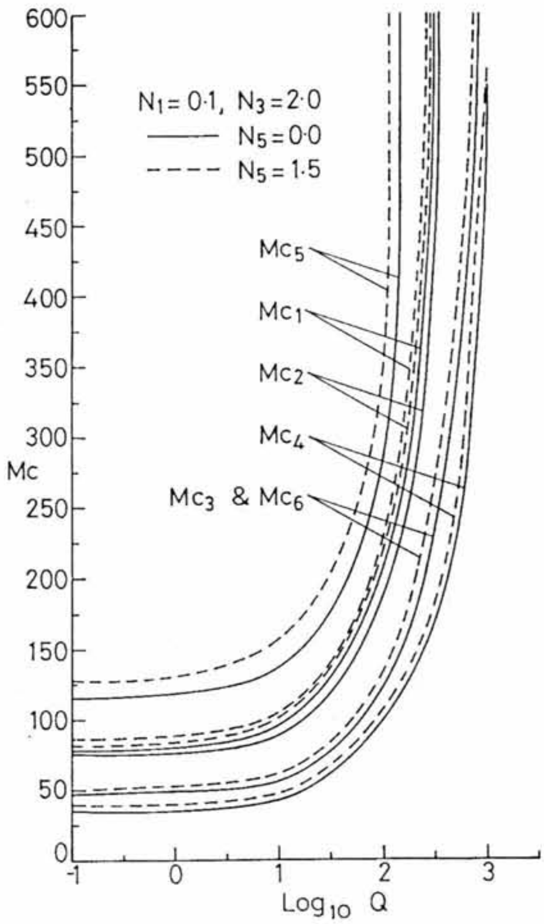

Fig. 6—Plot of M_c versus Q for different non-uniform temperature gradients and N_5.

effect of N_3 on the system is very small compared to the effects of the other micropolar parameters.

Fig. 6 is the plot of M_c versus Q for different non-uniform temperature gradients and two values of N_5. In the figure the contribution of micropolar heat conduction is clearly brought out. We observe that micropolar heat conduction leads to delayed convection, a result which as we noted earlier is only to be anticipated.

It has also been found that the critical wave number is, in general, insensitive to the changes in the micropolar parameters but is influenced by the magnetic field. A strong magnetic field succeeds in inducing only the coupling number N_1 into influencing a_c^2. These are shown in Table 2.

The above results indicate that the externally applied magnetic field is an effective means of controlling Marangoni convection in electrically conducting micropolar fluids. The results suggest that Marangoni convection in Newtonian fluids may be delayed by adding micron sized electrically inert suspended particles. Further, by creating conditions for an appropriate basic temperature gradient we can also make an *a priori* decision on advancing or delaying convection. In the limit $N_1 \to 0$, we recover the results of Rudraiah *et al* [17] from the present study and those of Rudraiah and Siddheshwar[26] in the limit $Q \to 0$.

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References

1 Eringen A C, *Int J Eng Sci*, 2 (1964) 205.
2 Eringen A C, *Int J Eng Sci*, 7 (1969) 115.
3 Eringen A C, *Int J Eng Sci*, 18 (1980) 5.
4 Eringen A C, *J Math Anal Appl*, 38 (1972) 480.
5 Eringen A C, *Int J Eng Sci*, 28 (1990) 133.
6 Eringen A C, *Int J Eng Sci*, 29 (1991) 1515.
7 Datta A B & Sastry V U K, *Int J Eng Sci*, 14 (1976) 631.
8 Bhattacharya S P & Jena S K, *Int J Eng Sci*, 23 (1984) 13.
9 Payne L E & Straughan B, *Int J Eng Sci*, 27 (1989) 827.
10 Siddheshwar P G & Pranesh S, *Int J Eng Sci*, 36 (1998) 1173.
11 Siddheshwar P G & Pranesh S, *Int J Eng Sci*, 36 (1998) 1183.
12 Siddheshwar P G & Pranesh S, *JMMM*, 192 (1999) 159.
13 Siddheshwar P G & Pranesh S, *JMMM*, 219 (2000) L153.
14 Siddheshwar P G & Pranesh S, *Acta Mech*, (2001) (In Press).
15 Pearson J R A, *J. Fluid Mech*, 4 (1958) 489.
16 Nield D A, *ZAMM*, 17 (1966) 131.
17 Rudraiah N, Ramachandramurthy V & Chandna O P, *Int J Heat Mass Transfer*, 28 (8) (1985) 1621.
18 Rudraiah N, Chandna O P & Garg M R, *Ind J Technol* , 24 (1986) 285.
19 Maekawa T & Tanasawa I, *Int J Heat Mass Transfer*, 31 (1988) 285.
20 Sarma G S R, *PCH Physico Chemical Hydrodynamics*, 6 (1985) 283.
21 Kaddame A & Lebon G, *Microgravity Q*, 3 (1993) 1.
22 Wilson S K, *Q J Mech Appl Math*, 46 (1993) 211.
23 Wilson S K, *Phys Fluids A*, 6 (1994) 3591.
24 Milandinova S P & Slavtchev S G, *ZAMM*, 78 (1998) 345.
25 Lebon G & Cloot A, *J. Non-Equilib. Thermody*, 6 (1981) 15.
26 Rudraiah N & Siddheshwar P G, *Int J Aerospace Technol, France*, 4 (2000) 517.