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Throughflow and Quadratic Drag Effects on Thermal Convection in a Rotating Porous Layer

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Abstract A linear stability analysis is implemented to study thermal convective instability in a horizontal fluid-saturated rotating porous layer with throughflow in the vertical direction. The modified Forchheimer-extended Darcy equation that includes the time-derivative and Coriolis terms is employed as a momentum equation. The criterion for the occurrence of direct and Hopf bifurcations is obtained using the Galerkin method. It is shown that if a Hopf bifurcation is possible it always occurs at a lower value of the Darcy–Rayleigh number than the direct bifurcation. Increase in the throughflow strength and inertia parameter is to decrease the domain of Prandtl number up to which Hopf bifurcation is limited but opposite is the trend with increasing Taylor number. The effect of rotation is found to be stabilizing the system, in general. However, in the presence of both rotation and Forchheimer drag a small amount of vertical throughflow as well as inertia parameter show some destabilizing effect on the onset of direct bifurcation; a result of contrast noticed when they are acting in isolation. The existing results in the literature are obtained as limiting cases from the present study.

Keywords Thermal convection · Porous medium · Throughflow · Rotation · Direct and Hopf bifurcations

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List of Symbols

a	Horizontal wave number, $\sqrt{l^2+m^2}$
C_d	Form drag constant
d	Depth of the porous layer
D	Differential operator, $\mathrm{d}/\mathrm{d}z$
Da	Darcy number, K/d^2
$\vec{g}$	Gravitational acceleration
G	Inertia parameter, $\rho_0 C_\mathrm{d}\kappa/\mu d$
$\hat{\mathrm{k}}$	Unit vector in the vertical z-direction
K	Permeability of the porous medium
l, m	Wave numbers in x-, y-directions
M	Ratio of heat capacities
p	Pressure
Pe	Scaled Peclet number, $Q/2\pi$
Pr	Prandtl number, $\mu/\rho_0\kappa$
$\vec{q}$	Velocity vector (u, v, w)
Q	Peclet number, $w_0 d/\kappa$
R	Darcy–Rayleigh number, $\alpha_t\rho_0 g(T_0-T_1)Kd/\mu\kappa$
R_t	Scaled Darcy–Rayleigh number, R/π^2
T	Temperature
T_0	Temperature at the lower boundary
T_1	Temperature at the upper boundary
t	Time
$W(z)$	Amplitude of perturbed vertical velocity
w_0	Constant vertical throughflow velocity
x, y, z	Space coordinates

Greek symbols

α	Scaled square of the wave number, a^2/π^2
α_t	Thermal expansion coefficient
β	Scaled Vadasz number, χ/π^2
χ	Modified Darcy–Prandtl number or the Vadasz number, $Pr\phi M/Da$
ϕ	Porosity
μ	Dynamic viscosity
κ	Thermal diffusivity
$\vec{\zeta}$	Vorticity vector, $\nabla\times\vec{q}$
$\vec{\Omega}$	Constant angular velocity, $\Omega\hat{k}$
$\Im$	Porous media-related Taylor number, $(2\Omega\rho_0 K/\mu\phi)^2$
$\Theta(z)$	Amplitude of perturbed temperature
τ	Scaled time, $Pr\phi/Da$
ρ	Density
ρ_0	Reference density at $T=T_0$
σ	Growth rate
ω	Frequency

Subscripts/superscripts

b Basic state
c Critical
d Direct bifurcation
H Hopf bifurcation

1 Introduction

Thermal convection in a rotating fluid-saturated porous layer heated from below has been the subject of experimental and theoretical interest because of its natural occurrence in geophysical and oceanic flows. The study has also motivated researchers because of its practical applications in engineering. Some of the important areas of applications in engineering include the food processing, chemical process, solidification and centrifugal casting of metals, rotating machinery, and so on. The stability characteristics of a fluid-saturated porous layer have attracted researchers since long time and a comprehensive review concerning the various aspects of flow through porous media may be found in the books by Bear (1988), Ingham and Pop (1998), Nield and Bejan (2006), Vafai (2000, 2005), and Vadasz (2008).

The problem of thermal convective instability in a rotating porous layer when the basic state is quiescent has been studied by many authors. Friedrich (1983) has investigated both linear and nonlinear thermal convection in a rotating porous layer. Patil and Vaidyanathan (1983) have studied thermal convection in a rotating fluid-saturated porous layer under the influence of variable viscosity using the Brinkman model. Palm and Tyvand (1984) have used the Darcy model to study the linear stability problem of thermal convection in a rotating porous layer and they have shown that their results are equivalent to those of non-rotating anisotropic porous medium case. Using the Brinkman model, Qin and Kaloni (1995) have studied the nonlinear stability of a rotating porous layer by including the convective inertia term in the Brinkman model. Vadasz (1998) has used both linear and weak nonlinear theories to study the effect of Coriolis force on gravity-driven convection in a rotating porous layer heated from below by employing the modified Darcy model. An excellent review of research on thermal convection in a rotating porous medium is given by Vafai (2000). A nonlinear stability analysis for thermal convection in a rotating porous layer has been performed by Straughan (2001). The problem of onset of thermal convection in a rotating porous medium bounded between rigid boundaries has been considered by Desaive et al. (2002). Shivakumara et al. (2009) have investigated linear and weakly nonlinear thermal convection in a rotating porous layer and they have shown that decrease in the permeability and increase in the effective viscosity of the fluid have a destabilizing effect on the onset of stationary convection at high rotation rates. Recently, Falsaperla et al. (2010) have considered the problem of thermal convection in a rotating horizontal layer of porous medium with Newton–Robin type of temperature boundary conditions.

However, the in situ processing of energy resources such as coal, oil shale, or geothermal energy and many practical problems cited above often involves the throughflow in the porous medium and in such problems it is pertinent to consider the rotational effects for a better understanding of the problem. The importance of buoyancy-driven convection in such systems may become significant when precise processing is required. Moreover, the throughflow effect in such circumstances may be of interest because of the possibility of controlling the convective instability by adjusting the throughflow in addition to the Coriolis force due to rotation.

Without considering rotational effect, several investigators have investigated the problem of convective instability in a porous medium with throughflow and copious literature is available on this topic. It is recognized that the throughflow is stabilizing if the bounding surfaces of the porous layer are of the same type but destabilizing in one particular direction if the boundaries are not of the same type (Homsy and Sherwood 1976; Jones and Persichetti 1986; Nield 1987; Shivakumara 1999; Shivakumara and Sureshkumar 2007 and references therein). However, it has been shown that the throughflow is destabilizing even if the boundaries are of the same type if the porous layer is heated internally (Khalili and Shivakumara 1998) and also in the presence of an additional diffusing component (Shivakumara and Khalili 2001; Shivakumara and Nanjundappa 2006). Besides, similar result is shown to be true even in dealing with the effect of throughflow on the stability of an Oldroyd-B binary fluid-saturated porous layer (Shivakumara and Sureshkumar 2008). Hill (2007) has performed linear and nonlinear stability analyses of vertical throughflow in a fluid-saturated porous layer, while Hill et al. (2007) have extended the problem to account for penetrative convection by considering density is quadratic in temperature. The nonlinear dynamics of a saline boundary layer formed by throughflow near the surface of a porous medium is considered by Pieters and Schuttlelaars (2008). Brevdo and Ruderman (2009a,b) have analyzed convective instability in a porous medium with inclined temperature gradient and vertical throughflow, while the nature of unstable three-dimensional localized disturbances on the problem is studied by Brevdo (2009). Recently, Barletta et al. (2010) have considered the combined effect of viscous dissipation and vertical throughflow on the onset of convective rolls in a horizontal porous layer.

Nonetheless, the simultaneous effect of throughflow and Coriolis force due to rotation on the onset of thermal convection in a rotating porous layer has not been given any attention despite its relevance and occurrence in many practical problems cited above. Moreover, in the presence of strong throughflow the inertial effects become significant and the Darcy law which holds for small velocities doesn't take care of these effects. Under the circumstances, the use of a non-Darcian model is warranted for a better understanding of the problem. For many naturally occurring porous media, the appropriate inertia term in the momentum equation is $|\vec{q}|\,\vec{q}$; a drag term quadratic in the velocity $\vec{q}$ (Nield and Joseph 1985). The aim of the present study is to investigate the combined effect of throughflow and quadratic drag on the stability characteristics of a rotating porous layer heated from below using the modified Forchheimer-extended Darcy model and explore the possibility for any of the unusual effect of throughflow under the influence of rotation as observed previously in other situations such as internal heating and double diffusive systems. Because of the presence of throughflow, the eigenvalue problem turns out to be one with variable coefficient. A Galerkin-type weighted residuals method is used to obtain the condition for the occurrence of direct and Hopf bifurcations. The results are exhibited graphically and the combined effect of throughflow, quadratic drag, and rotation on the onset of convection is discussed in detail.

2 Problem Formulation

We consider a Newtonian fluid-saturated horizontal porous layer of thickness d with vertical throughflow of constant velocity w_0 and subject to rotation with a constant angular velocity $\vec{\Omega} = \Omega\hat{k}$, where $\hat{k}$ is the unit vector in the vertical z-direction, as presented in Fig. 1. The top and bottom boundaries are maintained at different constant temperatures which are high at the bottom (T_0) and low at the top (T_1). A Cartesian co-ordinate system (x, y, z) is used

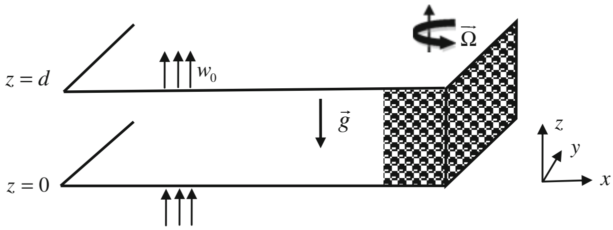

Fig. 1 Physical configuration

with the origin at the bottom of the porous layer and the z-axis vertically upward. Gravity, $\vec{g} = -g\hat{k}$ acts in the negative z-direction.

The continuity, momentum, and energy equations are, respectively,

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho_0 \left[\frac{1}{\phi} \frac{\partial \vec{q}}{\partial t} + \frac{C_{\mathrm{d}}}{K} |\vec{q}|\, \vec{q} + \frac{2}{\phi} \left(\Omega \hat{k} \times \vec{q} \right) \right] = -\nabla p + \rho_0 \left\{ 1 - \alpha_t \left(T - T_0 \right) \right\} \vec{g} - \frac{\mu}{K} \vec{q} \tag{2}$$

$$M \frac{\partial T}{\partial t} + (\vec{q}.\nabla)\, T = \kappa \nabla^2 T. \tag{3}$$

In the above equations, $\vec{q} = (u, v, w)$ denotes the filtration velocity vector, p the pressure, T the temperature, K the permeability of the porous medium, μ the dynamic viscosity, ϕ the porosity, α_t the thermal expansion coefficient, M the ratio of heat capacities, ρ_0 the reference density, κ the thermal diffusivity, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ the Laplacian operator, and the coefficient, C_{d}, called the form drag constant, is independent of the viscosity and the other properties of the fluid but dependent on the geometry of the medium (Georgiadis and Catton 1986). It may be noted that the Boussinesq approximation has been applied and the time-derivative term is included in the Forchheimer-extended Darcy model to account for the convection to set in as oscillatory as well (Vadasz 1998). Let us non-dimensionalize the quantities by choosing the characteristic length as d, initially time as d^2/κ, filtration velocity as κ/d, pressure as $\mu\kappa/K$ and temperature as $(T_0 - T_1)$. Then, the non-dimensional equations, written in the same notation as their dimensional counterparts, are:

$$\nabla \cdot \vec{q} = 0 \tag{4}$$

$$\frac{\partial \vec{q}}{\partial \tau} + G\, |\vec{q}|\, \vec{q} + \sqrt{\Im} \left(\hat{k} \times \vec{q} \right) + \vec{q} = -\nabla p + RT\hat{k} \tag{5}$$

$$\chi \frac{\partial T}{\partial \tau} + (\vec{q}.\nabla)\, T = \nabla^2 T. \tag{6}$$

In the above equations, $R = \alpha_t \rho_0 g (T_0 - T_1) K d/\mu\kappa$ is the Darcy–Rayleigh number, $\Im = (2\Omega\rho_0 K/\mu\phi)^2$ is the porous media-related Taylor number, $G = \rho_0 C_{\mathrm{d}} \kappa/\mu d$ is the inertia parameter, $\chi = Pr\phi M/Da$ is the modified Darcy–Prandtl number and also known as the Vadasz number in the literature, $Pr = \mu/\kappa\rho_0$ is the Prandtl number, $Da = K/d^2$ is the Darcy number, and $\tau = (Pr\phi/Da)t$.

The basic state is not quiescent and the solution to the basic state is given by

$$\begin{aligned}\vec{q}_{\mathrm{b}}(z) &= Q\hat{k}\\ p_{\mathrm{b}}(z) &= p_0 - R\frac{\left(e^{Qz}-Qz\right)}{Q\left(e^{Q}-1\right)} - (G|Q|+1)\,Qz\\ T_{\mathrm{b}}(z) &= -\frac{\left(e^{Qz}-Qz\right)}{\left(e^{Q}-1\right)}\end{aligned}\tag{7}$$

where $Q = w_0 d/\kappa$ is the Peclet number and the subscript "b" denotes the basic state.

Assume small perturbations around the basic solution in the form

$$\vec{q} = \vec{q}_{\mathrm{b}} + \vec{q}\,', \quad p = p_{\mathrm{b}} + p', \quad T = T_{\mathrm{b}} + T'. \tag{8}$$

Substituting Eq. 8 in Eq. 5, linearizing and eliminating the pressure term by operating curl, we obtain

$$\frac{\partial\vec{\zeta}}{\partial\tau} + (G|Q|+1)\,\vec{\zeta} - \sqrt{\Im}\frac{\partial\vec{q}}{\partial z} = R\left(\frac{\partial T}{\partial y}\hat{i} - \frac{\partial T}{\partial x}\hat{j}\right) \tag{9}$$

where $\vec{\zeta} = \nabla\times\vec{q}$ is the vorticity vector. Again taking curl on Eq. 9 and retaining the vertical component of the resulting equation, we get

$$\left(\frac{\partial}{\partial\tau} + G|Q| + 1\right)\nabla^2 w + \sqrt{\Im}\frac{\partial\zeta_z}{\partial z} = R\nabla_{\mathrm{h}}^2 T \tag{10}$$

where ζ_z is the vertical component of the vorticity vector and $\nabla_{\mathrm{h}}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator. Eliminating ζ_z from Eq. 10, using Eq. 9, we obtain

$$\left(\frac{\partial}{\partial\tau} + G|Q| + 1\right)^2\nabla^2 w + \Im\frac{\partial^2 w}{\partial z^2} = R\left(\frac{\partial}{\partial\tau} + G|Q| + 1\right)\nabla_{\mathrm{h}}^2 T. \tag{11}$$

Substituting Eq. 8 in Eq. 6 and linearizing, we obtain

$$\left(\chi\frac{\partial}{\partial\tau} + Q\frac{\partial}{\partial z} - \nabla^2\right)T = f(z)\,w \tag{12}$$

where

$$f(z) = \frac{Qe^{Qz}}{\left(e^{Q}-1\right)} \tag{13}$$

is the basic temperature gradient. As $Q \to 0$, we note that $f(z) \to 1$. The presence of vertical throughflow is thus to deviate the basic temperature gradient to nonlinear one with respect to porous layer height and it has a profound effect on the stability of the system. Assume a normal mode expansion of the dependent variables in the form

$$\{w, T\} = \{W(z), \Theta(z)\}\exp\left[i(\ell x + my) + \sigma\tau\right] \tag{14}$$

where ℓ and m are wave numbers in the x and y directions, respectively, and σ is the growth rate. Substituting Eq. 14 in Eqs. 11 and 12, we obtain

$$(\sigma + G|Q| + 1)^2\left(D^2 - a^2\right)W + \Im D^2 W + a^2 R\,(\sigma + G|Q| + 1)\,\Theta = 0 \tag{15}$$

$$\left(D^2 - QD - a^2 - \chi\sigma\right)\Theta + f(z)\,W = 0 \tag{16}$$

where $D = \mathrm{d}/\mathrm{d}z$ is the differential operator and $a = \sqrt{l^2 + m^2}$ is the overall horizontal wave number.

The boundaries are impermeable to velocity perturbations and are perfect conductors of heat. Then, the boundary conditions are:

$$W = \Theta = 0 \quad \text{at } z = 0, 1. \tag{17}$$

3 Solution Procedure

The eigenvalue problem defined by Eqs. 15–17 is a two-point boundary value problem with variable coefficient. A Galerkin-type weighted residuals method is employed to obtain an approximate solution to the eigenvalue problem as this has the advantage of dealing with a large parameter space with minimum mathematical computations. Multiplying Eq. 15 by $AW(z)$, Eq. 16 by $B\Theta(z)$, integrating across the porous layer and eliminating the constants A and B, we obtain an expression for R in the form

$$R = \frac{\left[(\sigma + G\,|Q| + 1)^2 \left\langle (DW)^2 + a^2 W^2 \right\rangle + \Im \left\langle (DW)^2 \right\rangle\right] \times \left[\left\langle (D\Theta)^2 + Q\Theta D\Theta + \left(a^2 + \chi\sigma\right)\Theta^2 \right\rangle\right]}{a^2 (\sigma + G\,|Q| + 1) \langle W\Theta \rangle \langle -f(z)\, W\Theta \rangle} \tag{18}$$

where $\langle \cdots \rangle$ denotes the integration with respect to z. We choose the trial functions satisfying the respective boundary conditions in the form $W = \sin(\pi z) = \Theta$ and substitute in the above equation to get (after performing the integration)

$$R_t = \frac{\left[(\sigma + 2\pi G\,|Pe| + 1)^2 (1 + \alpha) + \Im\right] (1 + \alpha + \beta\sigma) \left(Pe^2 + 1\right)}{\alpha\, (\sigma + 2\pi G\,|Pe| + 1)} \tag{19}$$

where $R_t = R/\pi^2, \alpha = a^2/\pi^2, \beta = \chi/\pi^2$, and $Pe = Q/2\pi$ are the scaled quantities. To investigate the stability of the system, now we set the real part of σ equal to zero and let $\sigma = i\omega$ in the above equation. After clearing the complex quantity from the denominator, Eq. 19 yields

$$R_t = \frac{(1+\alpha)\left[\delta(1+\alpha) - \beta\omega^2\right]\left(Pe^2+1\right)}{\alpha} + \Im \frac{\left[\delta(1+\alpha) + \beta\omega^2\right]\left(Pe^2+1\right)}{\alpha\left(\omega^2+\delta^2\right)} + i\omega N \tag{20}$$

where

$$N = \frac{\left(Pe^2+1\right)}{\alpha}\left[(1+\alpha)(1+\alpha+\beta\delta) + \Im \frac{(\beta\delta - 1 - \alpha)}{\left(\omega^2+\delta^2\right)}\right] \tag{21}$$

with $\delta = 2\pi G\,|Pe| + 1$. Since R_t is a physical quantity, it must be real and from Eq. 20 it implies either $\omega = 0$ or $N = 0$. Accordingly, we obtain the condition for the occurrence of direct or Hopf bifurcation.

3.1 Direct Bifurcation ($\omega = 0$)

Direct bifurcation occurs at $R_t = R_t^{\mathrm{d}}$, where

$$R_t^{\mathrm{d}} = \frac{\delta(1+\alpha)^2\left(Pe^2+1\right)}{\alpha} + \Im \frac{(1+\alpha)\left(Pe^2+1\right)}{\delta\alpha}. \tag{22}$$

We note that R_t^{d} attains its critical value R_{tc}^{d} at $\alpha = \alpha_c^{\mathrm{d}}$, where

$$\alpha_c^{\mathrm{d}} = \sqrt{1 + \frac{\Im}{(2\pi G\,|Pe| + 1)^2}} \tag{23}$$

and the corresponding R_{tc}^{d} is given by

$$R_{tc}^{\mathrm{d}} = (2\pi G\,|Pe| + 1)\left(Pe^2 + 1\right)\left[1 + \sqrt{1 + \frac{\Im}{(2\pi G\,|Pe| + 1)^2}}\right]^2. \tag{24}$$

Equation 23 reveals that increasing the Taylor number is to increase the wave number and opposite is the trend with increasing Pe. From Eq. 24, it is evident that R_{tc}^{d} is an even function of Pe and hence the direction of throughflow does not have any influence on the stability of the system. In the absence of throughflow ($Pe = 0$), we note that

$$R_{tc}^{\mathrm{d}} = \left[1 + \sqrt{1 + \Im}\right]^2, \quad \alpha_c^{\mathrm{d}} = \sqrt{1 + \Im}. \tag{25}$$

This result is identical to the critical values presented by Vadasz (1998). In addition, if $\Im = 0$ then we have $R_{tc}^{\mathrm{d}} = 4$ and $\alpha_c^{\mathrm{d}} = 1$ which are the known exact values.

3.2 Hopf Bifurcation ($\omega \neq 0$, $N = 0$)

The condition $N = 0$ provides the solution for the frequency ω of the oscillations in the form

$$\omega^2 = \frac{(1 + \alpha - \beta\delta)}{(1 + \alpha)\,(1 + \alpha + \beta\delta)}\Im - \delta^2. \tag{26a}$$

From the above equation, we note that the presence of rotation is the cause for the occurrence of Hopf bifurcation and in its absence ($\Im = 0$) it is seen that $\omega^2 < 0$ and hence Hopf bifurcation is not possible. A necessary condition for the existence of Hopf bifurcation can be obtained by imposing the condition $\omega^2 > 0$, and this inequality gives

$$\delta^2\,(1 + \alpha)^2 + \left(\delta^3\beta - \Im\right)(1 + \alpha) + \beta\delta\Im < 0 \tag{26b}$$

which in turn yields the following condition to allow a range of positive values of α that accommodate Hopf bifurcation

$$0 < \beta < \left(3 - 2\sqrt{2}\right)\frac{\Im}{(2\pi G\,|Pe| + 1)^3}. \tag{26c}$$

From the above condition, it is seen that the presence of throughflow as well as inertia effect is to decrease the range of β values and when $Pe = 0$ it coincides with the result of Vadasz (1998). Utilizing the above condition, the corresponding range of values of α which is consistent with Hopf bifurcation is

$$\frac{\left(\Im - \beta\delta^3\right) - \sqrt{\left(\Im^2 - 6\beta\delta^3\Im + \beta^2\delta^6\right)}}{2\delta^2} < (1 + \alpha) < \frac{\left(\Im - \beta\delta^3\right) + \sqrt{\left(\Im^2 - 6\beta\delta^3\Im + \beta^2\delta^6\right)}}{2\delta^2}. \tag{27}$$

Eliminating ω^2 from Eq. 20 using Eq. 26a and noting $N = 0$, it is observed that Hopf bifurcation occurs at $R_t = R_t^{\mathrm{H}}$, where

$$R_t^{\mathrm{H}} = \frac{2\delta\left(Pe^2+1\right)}{\alpha}\left[(1+\alpha)(1+\alpha+\beta\delta) + \Im\frac{\beta^2}{(1+\alpha+\beta\delta)}\right]. \tag{28}$$

We observe that the direction of throughflow is not altering R_t^{H}. It is interesting to note that, using Eqs. 22 and 28, Eq. 26a can be written as

$$\omega^2 = \frac{\alpha\delta}{\left(Pe^2+1\right)(1+\alpha+2\beta\delta)}\left(R_t^{\mathrm{d}} - R_t^{\mathrm{H}}\right). \tag{29}$$

From the above equation, it is evident that if a Hopf bifurcation is possible it always occurs at a lower value of the Darcy–Rayleigh number than the direct bifurcation. It is observed that R_t^{H} attains its critical value R_{tc}^{H} at $\alpha = \alpha_{\mathrm{c}}^{\mathrm{H}}$, where $\alpha_{\mathrm{c}}^{\mathrm{H}}$ satisfies the equation

$$\left(\alpha_{\mathrm{c}}^{\mathrm{H}}\right)^4 + a_1\left(\alpha_{\mathrm{c}}^{\mathrm{H}}\right)^3 + a_2\left(\alpha_{\mathrm{c}}^{\mathrm{H}}\right)^2 + a_3\left(\alpha_{\mathrm{c}}^{\mathrm{H}}\right) + a_4 = 0. \tag{30a}$$

Here,

$$\begin{aligned} a_1 &= 2(1+\beta\delta) \\ a_2 &= \beta\delta(1+\beta\delta) \\ a_3 &= -2\left\{(1+\beta\delta)^2 + \beta^2\Im\right\} \\ a_4 &= -\beta^2(1+\beta\delta)\Im - (1+\beta\delta)^3. \end{aligned} \tag{30b}$$

It may be noted that in the absence of throughflow ($Pe = 0$), Eqs. 28 and 29 reduce to the expression obtained by Vadasz (1998).

4 Results and Discussion

The effect of throughflow and quadratic drag on the onset of thermal convection in an infinite horizontal fluid-saturated porous layer in the presence of rotation is investigated analytically using the modified Forchheimer-extended Darcy model. The presence of throughflow is to alter the basic temperature gradient from linear to nonlinear with respect to porous layer height. The onset thresholds for both direct and Hopf bifurcations are derived by employing a Galerkin-type weighted residuals method. It is shown that if a Hopf bifurcation is possible it always occurs at a lower value of the Darcy–Rayleigh number than the direct bifurcation.

The neutral stability curves in the (R_t, a) plane are presented in Figs. 2 and 3 for different values of physical parameters. Figure 2a and b exhibits the effect of Vadasz number (modified Darcy–Prandtl number) β for two values of $|Pe| = 0.1$ and 0.5, respectively, when $\Im = 50$ and $G = 0.5$. We note that Hopf bifurcation Rayleigh number increases with increasing β indicating its effect is to delay the onset of Hopf bifurcation and $\beta = 0$ provides a lower limit for all the neutral stability curves. The bifurcation points from where Hopf bifurcation solutions branch off from the direct bifurcation curve are clearly identified. The neutral stability curves associated with $\Im = 100$ and $G = 0.5$ are shown in Fig. 3a and b, respectively, for $|Pe| = 0.1$ and 0.5, in order to know the effect of Taylor number on the neutral stability curves. A closer inspection of the figures reveals that the effect of increasing Taylor number has a stabilizing effect on the system and decreasing β is to shift Hopf bifurcation neutral curves toward a lower wave number region. Besides, increasing the throughflow strength is

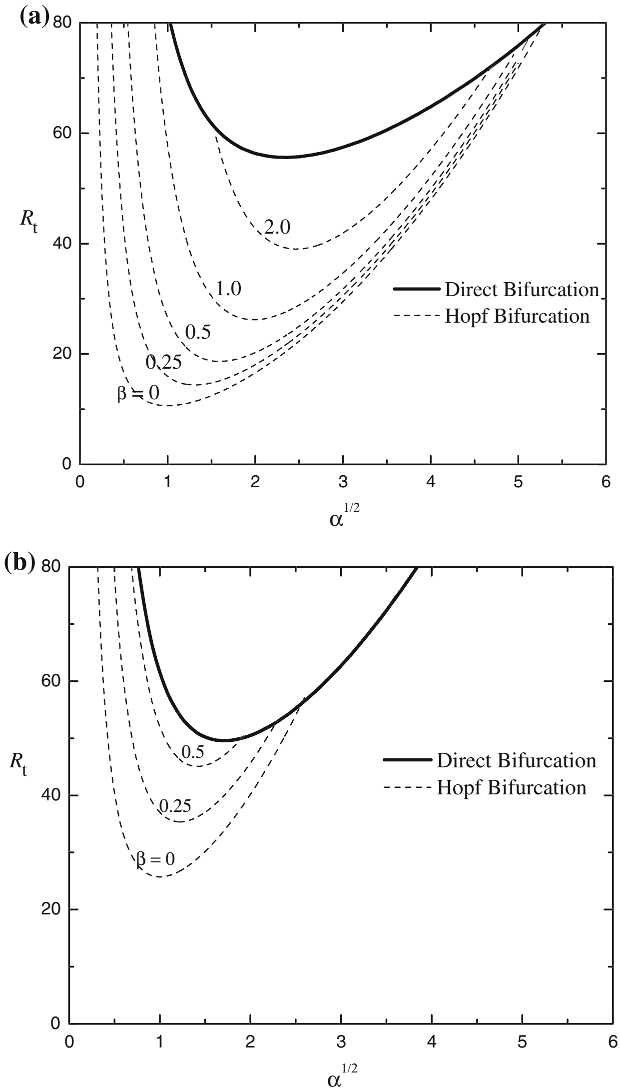

Fig. 2 Neutral stability curves for different values of β for **a** $|Pe| = 0.1$ and **b** $|Pe| = 0.5$ when $\Im = 50$ and $G = 0.5$

to decrease the domain of Vadasz number (modified Darcy–Prandtl number) up to which it can support Hopf bifurcation but opposite is the case with increasing Taylor number.

Figure 4a and b, respectively, shows the variation of R_{tc}^{d} and R_{tc}^{H} (respective critical Darcy–Rayleigh number obtained with respect to wave number) as a function of $|Pe|$ for different values of $\Im$ and for two values of $G = 0$ (absence of inertial effect) and 0.5 when $\beta = 2$. The results for $G = 0$ is shown by dotted lines in these figures. Since the inertial effect is toggled with vertical throughflow, the variation of R_{tc}^{d} and R_{tc}^{H} as a function of inertia parameter G is shown in Fig. 5a and b, respectively, for different values of $\Im$ when $|Pe| = 0.7$ and $\beta = 1$. From the figures, we observe the following:

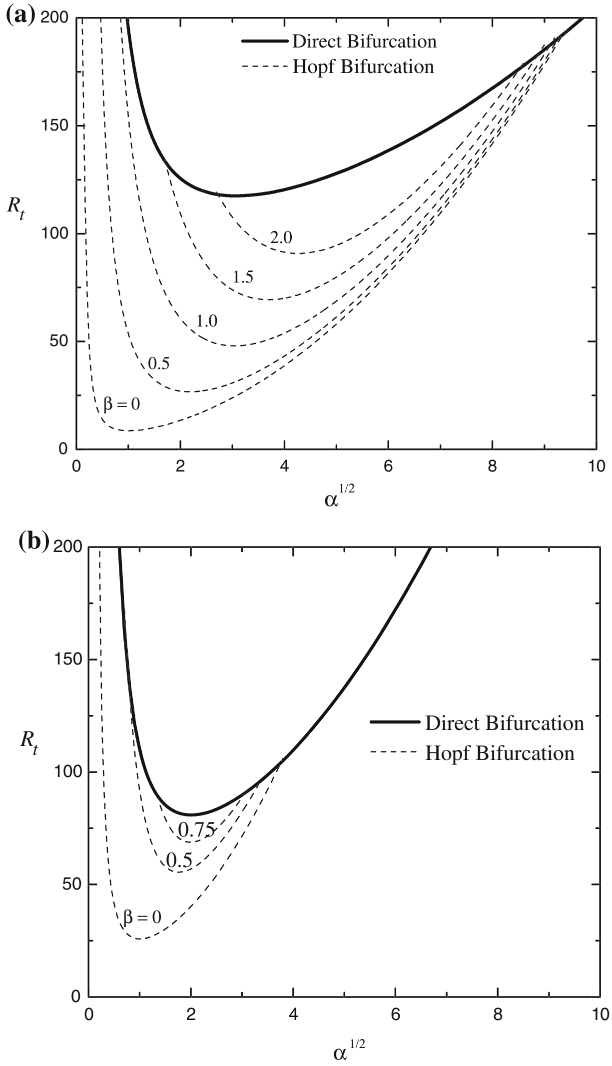

Fig. 3 Neutral stability curves for different values of β for **a** $|Pe| = 0.1$ and **b** $|Pe| = 0.5$ when $\Im = 100$ and $G = 0.5$

(i) In the absence of rotation, the effect of throughflow is always stabilizing.

(ii) In the presence of rotation but in the absence of Forchheimer drag ($G = 0$), the effect of throughflow is then also stabilizing.

(iii) In contrast to the above situations, it is interesting and important to note that a small amount of throughflow shows some destabilizing effect initially if the rotation and inertia effects are simultaneously present ($\Im \neq 0, G \neq 0$) and the destabilization manifests itself as a minimum in the (R_{tc}^{d}, $|Pe|$)-plane but throughflow again makes the system more stable with further increasing in its strength. Similar is the trend with increasing G when $\Im \neq 0$ and $|Pe| \neq 0$.

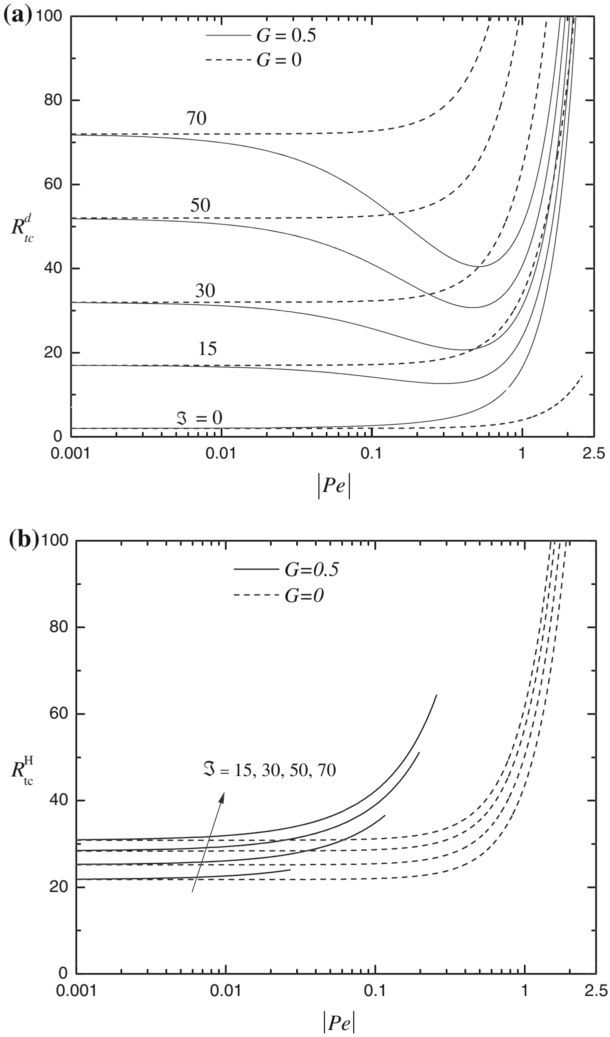

Fig. 4 Variation of **a** $R^{\mathrm{d}}_{t\mathrm{c}}$ and **b** $R^{\mathrm{H}}_{t\mathrm{c}}$ as a function of $|Pe|$ for $G = 0.5$ and $\beta = 2$

(iv) Increasing G is to increase $R^{\mathrm{d}}_{t\mathrm{c}}$ in the absence of rotation but opposite is the trend in the presence of rotation.

(v) In the case of a Hopf bifurcation, the variation of $R^{\mathrm{H}}_{t\mathrm{c}}$ with $|Pe|$ and G represents that throughflow and inertia parameter are always stabilizing in the presence of rotation. The curves in the respective figures end at a point where no more critical $R^{\mathrm{H}}_{t\mathrm{c}}$ which are consistent with $\omega_{\mathrm{c}}^2 > 0$ exist.

(vi) Increase in the value of G is to decrease the range of $|Pe|$ up to which it can support the occurrence of a Hopf bifurcation for different values of $\Im$.

The dual behavior of throughflow on the stability characteristics of the system under the simultaneous presence of rotation and Forchheimer drag may be viewed as follows. In general, the effect of throughflow is to confine significant thermal gradients to a thermal

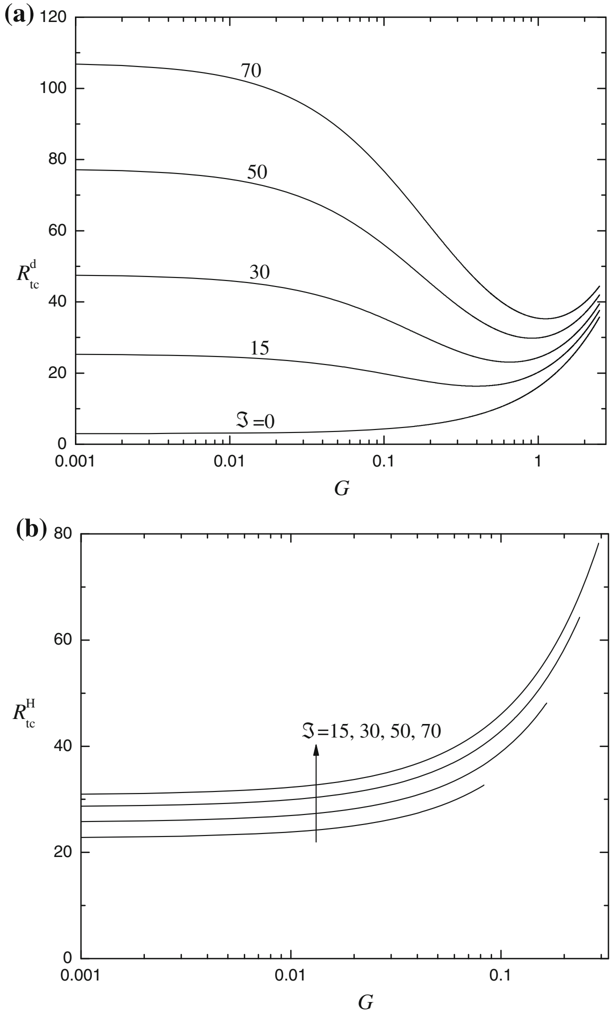

Fig. 5 Variation of **a** R_{tc}^{d} and **b** R_{tc}^{H} as a function of G for $|Pe| = 0.7$ and $\beta = 1$

boundary layer at the boundary toward which the throughflow is directed. As a result of this, the effective length scale becomes smaller than the porous layer thickness and, therefore, the Darcy–Rayleigh number, which is proportional to the length scale, will be much less than the actual critical Darcy–Rayleigh number and hence throughflow is stabilizing irrespective of inertial effect due to throughflow. In the presence of rotation, however, the purely stabilizing effect provided by the throughflow and inertial effect changes because a viscous boundary layer appears and there will be toggling between these mechanisms and the temperature gradients will not be restricted to a smaller length scale. But once the throughflow strength increases, it confines again the thermal gradients to a much smaller length scale and hence it makes the system more stable as observed in the absence of rotation. Thus, we note that higher throughflow strength is required to make the system stabilizing with increasing Taylor number.

Table 1 Variation of critical Darcy–Rayleigh and Peclet numbers for different values of $\Im$

$\Im$	$G = 0.1$		$G = 0.5$	
	R_{tc}^{d}	$\lvert Pe_{\mathrm{c}}\rvert$	R_{tc}^{d}	$\lvert Pe_{\mathrm{c}}\rvert$
5	10.341	0.105	9.535	0.181
15	23.051	0.163	18.977	0.296
30	40.278	0.192	30.683	0.363
50	62.067	0.210	44.718	0.409
60	72.700	0.216	51.382	0.426
70	83.212	0.220	57.881	0.439

Table 2 Variation of critical Darcy–Rayleigh number and inertial parameter for different values of $\Im$

$\Im$	$\lvert Pe\rvert = 0.1$		$\lvert Pe\rvert = 0.5$	
	R_{tc}^{d}	G_{c}	R_{tc}^{d}	G_{c}
5	9.0337	1.9672	11.1803	0.3934
15	15.6469	4.5725	19.3649	0.9145
30	22.128	7.1257	27.3861	1.4252
50	28.5671	9.6624	35.3553	1.9325
70	33.8011	11.7243	41.833	2.3449

The values of Peclet number $|Pe|$ and inertia parameter G at which R_{tc}^{d} takes the least value are denoted by $|Pe_{\mathrm{c}}|$ and G_{c}, respectively. The values of $|Pe_{\mathrm{c}}|$ (with $G = 0.1$ and 0.5) and G_{c} (with $|Pe| = 0.1$ and 0.5) computed for different values of Taylor number are presented in Tables 1 and 2, respectively. From the tables, it is seen that increasing $\Im$ is to increase the value of $|Pe_{\mathrm{c}}|$ and G_{c}. That means the range of $|Pe|$ and G up to which the system becomes destabilizing increases with an increase in the Taylor number.

The variation of R_{tc}^{H} and the corresponding critical wave number α_{c} as a function of β for various values of Taylor number is elucidated in Fig. 6a and b, respectively, for two values of $|Pe| = 0.1$ and 0.5 by fixing the value of $G = 0.5$. Figure 6a shows that increasing $|Pe|$ and $\Im$ is to increase the critical Hopf bifurcation Rayleigh number and hence their effect is to delay the onset of the same. In the figure, the curves end at a point where no more critical R_{tc}^{H} which are consistent with $\omega_{\mathrm{c}}^2 > 0$ exist. Furthermore, it is seen that the R_{tc}^{H} curves end at lower values of β for $|Pe| = 0.5$ when compared to $|Pe| = 0.1$, but the curves definitely end at higher values of β with increasing Taylor number. The critical wave number of a Hopf bifurcation increases with increasing β and $\Im$ but decreases slightly with increasing $|Pe|$ (see Fig. 6b).

Figure 7 shows the variation of critical frequency ω_{c}^2 as a function of β. It is observed that ω_{c}^2 decays as β increases for different values of $\Im$ and $|Pe|$ considered and also higher values of frequency correspond to small β. Besides, ω_{c}^2 increases with increasing Taylor number but decreases with increasing throughflow strength.

5 Conclusions

From the above study, it is observed that the presence of rotation introduces Hopf bifurcation and its effect is to stabilize the system, in general. A necessary condition for the existence of a Hopf bifurcation is found and it is shown that if a Hopf bifurcation is possible it always

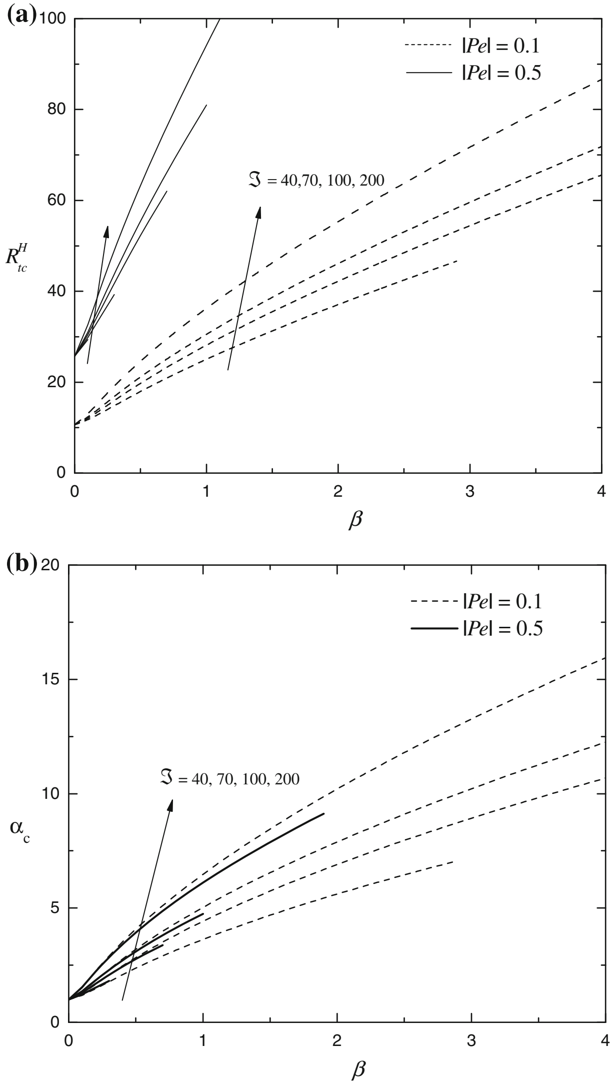

Fig. 6 Variation of **a** R_{tc}^{H} and **b** α_{c} as a function of β for different values of $\Im$ and for two values of $|Pe|$ with $G = 0.5$

occurs at a lower value of the Darcy–Rayleigh number than the direct bifurcation. Moreover, increasing the vertical throughflow-dependent Peclet number and also inertia parameter is to decrease the range of Prandtl number up to which Hopf bifurcation is limited, while increase in the Taylor number is to increase this range. Throughflow stabilizes the system always in the absence of rotation as well as in the presence of rotation when there is no effect of quadratic drag. To the contrary, throughflow exhibits a dual behavior on the stability of the system in the presence of rotation and Forchheimer drag. That is, a small amount of throughflow destabilizes the system in the case of direct bifurcation and the range of Peclet number up to which the system gets destabilized increases with increasing Taylor number and also

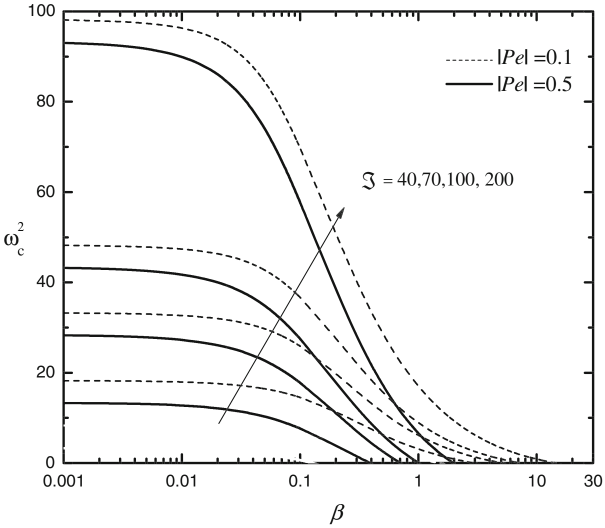

Fig. 7 Variation of ω_c^2 as a function of β for different values of $\Im$ and for two values of $|Pe|$ with $G = 0.5$

inertia parameter. Although increase in the inertia parameter is to stabilize the system in the absence of rotation, it shows a dual behavior on the stability of the system in the presence of rotation with throughflow. However, in the case of a Hopf bifurcation throughflow is always stabilizing in the presence of rotation and Forchheimer drag as well. The effect of increasing Vadasz number is to delay the onset of a Hopf bifurcation. Increasing the Taylor number is to increase the critical wave number and frequency of oscillations and an opposite trend is noticed with increasing Peclet number.

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