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Bénard–Marangoni ferroconvection with magnetic field dependent viscosity

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ABSTRACT

The effect of magnetic field dependent viscosity on the onset of Bénard–Marangoni ferroconvection in a horizontal layer of ferrofluid is investigated theoretically. The lower boundary is taken to be rigid with fixed temperature, while the upper free boundary at which temperature-dependent surface tension effect is considered is non-deformable and subject to a general thermal condition. The Rayleigh–Ritz method with Chebyshev polynomials of the second kind as trial functions is employed to extract the critical stability parameters numerically. The results show that the onset of ferroconvection is delayed with an increase in the magnetic field dependent viscosity parameter (Λ) and Biot number (Bi) but opposite is the case with an increase in the value of magnetic Rayleigh number (R_m) and nonlinearity of magnetization (M_3). Further, increase in R_m, M_3, and decrease in Λ and Bi is to decrease the size of the convection cells.

1. Introduction

A typical ferromagnetic fluid contains single domain nanoparticles of magnetic material (iron, cobalt or magnetite) stably suspended in a liquid carrier with low electrical conductivity. Each particle is encapsulated by a monolayer of surfactant in order to prevent particle coalescence due to magnetic attraction. The average size of magnetic nanoparticles is about 10 nm. Magnetic colloids have magnetic susceptibility which is thousands times larger than that of natural materials. Such fluids became the subject of a special branch of magnetohydrodynamics termed as ferrohydrodynamics (Rosensweig [1]) and found applications in various areas of science, technology and nanotechnology (Bashtovoy et al. [2], [3]).

The magnetization of ferromagnetic fluids depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is similar to buoyancy driven convection. Buoyancy driven convection in a layer of ferrofluid heated uniformly from below in the presence of a uniform magnetic field has been studied extensively over the years. In the first theoretical study (Finlayson [4]), which dealt with convection in a horizontal layer of magnetic fluid subject to a vertical temperature gradient and placed in a transverse uniform magnetic field, the concentration of magnetic particles was assumed to be constant. Therefore only thermo-gravitational and thermomagnetic mechanisms of convection were considered. The discussed theory predicted a destabilizing influence of the magnetic field and extensively continued over the years (Lalas and Carmi [5]; Shliomis [6]; Gotoh and Yamada [7]; Stiles and Kagan [8]; Kaloni and Lou [9]). The non-linear stability analysis for a magnetized ferrofluid layer heated from below for stress-free boundaries has been performed by Sunil and Mahjan [10]. A variety of velocity and temperature boundary conditions on the onset of ferroconvection in an initially quiescent ferrofluid layer has been considered by Nanjundappa and Shivakumara [11]. Recently, thermal convection of ferrofluids in the presence of a uniform vertical magnetic field with the boundary temperatures modulated sinusoidally about some reference values has been discussed by Singh and Bajaj [12], while Belyaev and Smorodin [13] have studied the effect of an alternating uniform magnetic field on the onset of convection in a horizontal layer of a ferrofluid within the framework of a quasi-stationary approach.

It is a well established fact that convection can also be induced by surface-tension forces provided it is a function of temperature. In view of the fact that heat transfer is greatly enhanced due to convection, the magnetic convection problems offer new possibilities for new applications in cooling with motors, loud speakers, transmission lines, and other equipment where magnetic field is already present. If the ferrofluid layer has an upper surface open to atmosphere then the instability is due to

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the combined effects of the buoyancy as well as temperature-dependent surface tension forces, known as Bénard–Marangoni ferroconvection. A limited number of studies have addressed the effect of surface tension forces on ferroconvection in a horizontal ferrofluid layer. Linear and non-linear stability of combined buoyancy-surface tension effects in a ferrofluid layer heated from below is considered by Qin and Kaloni [14]. The coupling between Marangoni and Rosensweig instabilities by considering two semi-infinite incompressible and immiscible viscous fluids of infinite lateral extent in which one of them is ferromagnetic and the other is a usual Newtonian liquid is studied by Weilepp and Brand [15]. Shivakumara et al. [16] have investigated the effect of different forms of basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces with an idea of understanding control of ferroconvection. The Rayleigh–Bénard–Marangoni instability in a ferrofluid layer in the presence of weak vertical magnetic field normal to the boundaries has been discussed by Hennenberg et al. [17]. The onset of Marangoni ferroconvection with different initial temperature gradients is analyzed by Shivakumara and Nanjundappa [18].

Thermal convection in ferromagnetic fluids is gaining much importance due to its astounding physical properties. One such property is viscosity of the ferromagnetic fluid. The viscosity of the ferrofluid is predicted by dimensional analysis to be a function of the ratio of hydrodynamic stress to magnetic stress (Rosenswieg et al. [19]). The effect of a homogeneous magnetic field on the viscosity of a fluid with solid particles possessing intrinsic magnetic moments has been investigated by Shliomis [20]. The effect of magnetic field dependent (MFD) viscosity on the onset of ferroconvection in a rotating ferrofluid layer is discussed by Vaidyanathan et al. [21], with or without dust particles by Sunil et al. [22] and the non-linear stability analysis has also been performed by Sunil et al. [23]. Recently, Nanjundappa et al. [24] have investigated the effect of MFD viscosity on the onset of convection in a ferromagnetic fluid layer in the presence of a vertical magnetic field by considering the bounding surfaces are either rigid-ferromagnetic or stress- free with constant heat flux conditions.

The intent of the present paper is to study coupled Bénard–Marangoni ferroconvection in a ferrofluid layer in the presence of a uniform vertical magnetic field with magnetic field dependent viscosity. The lower boundary is rigid with fixed temperature, while the upper non-deformable free boundary is subjected to temperature dependent surface tension forces and a general thermal boundary condition on the perturbation temperature is imposed. The study helps in understanding control of ferroconvection by magnetic field dependent viscosity, which is useful in many heat transfer related problems particularly in materials science processing. The resulting eigenvalue problem is solved numerically by employing the Rayleigh–Ritz method with Chebyshev polynomials of the second kind as trial functions.

The paper is organized as under. Section 2 is devoted to the formulation of the problem. The method of solution is discussed in Section 3. In Section 4, the numerical results are discussed and some important conclusions follow in Section 5.

2. Mathematical formulation

We consider a Boussinesq ferrofluid layer of thickness d with no lateral boundaries and a uniform magnetic field H_0 acting normal to the boundaries. The lower and the upper boundaries are maintained at constant but different temperatures T_0 and $T_1(<T_0)$, respectively. A Cartesian co-ordinate system (x, y, z) is used with the origin at the lower boundary and the z-axis vertically upward. Gravity acts in the negative z-direction, $\vec{g}=-g\hat{k}$, where $\hat{k}$ is the unit vector in the z-direction. The layer is bounded below by a rigid surface while the free surface which is subjected to temperature dependent surface tension forces is assumed to be flat and non-deformable. The surface tension σ is assumed to vary linearly with temperature in the form

$$\sigma=\sigma_0-\sigma_T(T-T_0) \tag{1}$$

where σ_0 is the unperturbed value and $-\sigma_T$ is the rate of change of surface tension with temperature. The fluid density ρ is assumed to vary linearly with temperature in the form

$$\rho=\rho_0[1-\alpha_t(T-T_0)] \tag{2}$$

where α_t is the thermal expansion coefficient and ρ_0 is the density at $T=T_0$.

In the study of ferroconvection, we have to solve the Maxwell equations simultaneously with the balance of mass, linear momentum and energy. Since the fluid is assumed to be electrically not conducting, the Maxwell equations reduce to

$$\nabla\cdot\vec{B}=0 \tag{3}$$

$$\nabla\times\vec{H}=0 \tag{4}$$

where $\vec{B}$ is the magnetic induction and $\vec{H}$ the intensity of magnetic field. In view of Eq. (4), we can express the magnetic field by a scalar potential

$$\vec{\mathrm{H}}=\nabla\varphi \tag{5}$$

Further $\vec{B}$, $\vec{M}$ and $\vec{H}$ are related by

$$\vec{B}=\mu_0(\vec{M}+\vec{H}) \tag{6}$$

where $\vec{M}$ is the magnetization and μ_0 the magnetic permeability of vacuum.

Following Finlayson [4], we assume that the magnetization is aligned with the magnetic field, but allow dependence on the magnitude of magnetic field as well as on the temperature in the form,

$$\vec{M}=[M_0+\chi(H-H_0)-K(T-T_0)](\vec{H}/H) \tag{7}$$

where $M_0=M(H_0,T_0)$, $H=\left|\vec{H}\right|$, $M=\left|\vec{M}\right|$, $\chi=(\partial M/\partial H)_{H_0,T_0}$ is the magnetic susceptibility and $K=-(\partial M/\partial T)_{H_0,T_0}$ is the pyromagnetic coefficient.

The momentum equation is

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\vec{q}\right]=-\nabla p+\rho\vec{g}+\mu_0(\vec{M}\cdot\nabla)\vec{H}+2\nabla\cdot\left[\eta\underset{\sim}{D}\right] \tag{8}$$

where $\vec{q}=(u,v,w)$ is the velocity, p the pressure, t the time and $\underset{\sim}{D}=[\nabla\vec{q}+(\nabla\vec{q})^T]/2$ the rate of strain tensor. The fluid is assumed to be incompressible having variable viscosity. Experimentally, it has been demonstrated that the magnetic viscosity has got exponential variation, with respect to magnetic field (Rosenwieg et al. [19]). As a first approximation, for small field variation, linear variation of magnetic viscosity has been used in the form $\eta=\eta_0(1+\vec{\delta}\cdot\vec{B})$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity and is considered to be isotropic (Vaidyanathan et al. [21]), η_0 is taken as viscosity of the fluid when the applied magnetic field is absent.

Neglecting viscous dissipation, the energy equation is [4]

$$\left[\rho_0C_{V,H}-\mu_0\vec{H}\cdot\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}+\mu_0T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt}=k_t\nabla^2T \tag{9}$$

where, $C_{V,H}$ is the specific heat capacity at constant volume and magnetic field per unit mass, and k_t the thermal conductivity.

The continuity equation is

$$\nabla\cdot\vec{q}=0 \tag{10}$$

We follow the stability analysis as outlined in the work of Finlayson [4]. The basic state is quiescent and is given by

$$\vec{q}=0,\quad p=p_b(z),\quad T_b=T_0-\beta z\left(\beta=\frac{\Delta T}{d}\right)$$
$$\vec{H}_b=\left[H_0-\frac{K\beta z}{1+\chi}\right]\hat{k},\quad \vec{M}_b=\left[M_0+\frac{K\beta z}{1+\chi}\right]\hat{k} \tag{11}$$

To study the stability of the system, we perturb all the variables in the form

$$\vec{q}=\vec{q}\,',\quad p=p_b(z)+p',\quad \eta=\eta_b(z)+\eta',\quad T=T_b(z)+T'$$
$$\vec{H}=\vec{H}_b(z)+\vec{H}\,',\quad \vec{M}=\vec{M}_b(z)+\vec{M}\,' \tag{12}$$

where, $\vec{q}\,', p', \eta', T', \vec{H}\,'$ and $\vec{M}\,'$ are perturbed variables and are assumed to be small.

Substituting Eq. (12) into Eq. (3), using Eqs.(6) and (7), and assuming that $K\beta d \ll (1+\chi)H_0$ as propounded by Finlayson [4], we obtain (after dropping the primes)

$$H_x+M_x=(1+M_0/H_0)H_x,$$
$$H_y+M_y=(1+M_0/H_0)H_y,$$
$$H_z+M_z=(1+\chi)H_z-KT \tag{13}$$

where (H_x, H_y, H_z) and (M_x, M_y, M_z) are (x, y, z) components of perturbed magnetic field and magnetization, respectively.

Substituting Eq.(12) into Eq. (8) and linearizing, we obtain in components (after neglecting the primes)

$$\rho_0\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\eta_0\left[1+\mu_0\delta(M_0+H_0)\right]\nabla^2u+\mu_0(M_0+H_0)\frac{\partial H_x}{\partial z} \tag{14}$$

$$\rho_0\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\eta_0\left[1+\mu_0\delta(M_0+H_0)\right]\nabla^2v+\mu_0(M_0+H_0)\frac{\partial H_y}{\partial z} \tag{15}$$

$$\rho_0\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\rho_0\alpha_t gT+\eta_0\left[1+\mu_0\delta(M_0+H_0)\right]\nabla^2w+\mu_0(M_0+H_0)\frac{\partial H_z}{\partial z}-\mu_0K\beta H_z+\frac{\mu_0K^2\beta T}{1+\chi} \tag{16}$$

Differentiating Eqs. (14) and (15) partially with respect to x and y, respectively, and adding, we obtain

$$\nabla_1^2p=-\rho_0\alpha_t g\frac{\partial T}{\partial z}+\mu_0(M_0+H_0)\frac{\partial}{\partial z}(\nabla\cdot\vec{H})-\mu_0K\beta\frac{\partial H_z}{\partial z}+\frac{\mu_0K^2\beta}{1+\chi}\frac{\partial T}{\partial z} \tag{17}$$

where $\nabla_1^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the horizontal Laplacian operator. Eliminating the pressure term from Eq. (16), using Eq. (17), we obtain

$$\left[\rho_0\frac{\partial}{\partial t}-\eta_0\{1+\delta\mu_0(M_0+H_0)\}\nabla^2\right]\nabla^2w=-\rho_0\alpha_t g\nabla_1^2T+\mu_0K\beta\frac{\partial}{\partial z}(\nabla_1^2\phi)+\frac{\mu_0K^2\beta}{1+\chi}(\nabla_1^2T) \tag{18}$$

where $\nabla^2=\nabla_1^2+\partial^2/\partial z^2$ is the Laplacian operator.

As before, substituting Eq. (12) into Eq. (9) and linearizing, we obtain (after neglecting primes)

$$\rho_0C_0\frac{\partial T}{\partial t}-\mu_0KT_0\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial z}\right)=\left(\rho_0C_0-\frac{\mu_0K^2T_0}{1+\chi}\right)w\beta+k_t\nabla^2T \tag{19}$$

where $\rho_0C_0=\rho_0C_{V,H}+\mu_0KH_0$.

Finally, Eqs. (3) and (4), after using Eqs. (12) and (13), yield (after neglecting primes)

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\varphi+(1+\chi)\frac{\partial^2\varphi}{\partial z^2}-K\frac{\partial T}{\partial z}=0. \tag{20}$$

Since the principle of exchange of stability is valid, we assume the normal mode solution in the form

$$\{w,T,\varphi\}=\{W,\Theta,\Phi\}(z)e^{i(lx+my)} \tag{21}$$

where l and m are wave numbers in the x and y directions respectively. Substituting Eq. (21) in Eqs. (18)–(20) and non-dimensionalizing the quantities in the form

$$(x^*,y^*,z^*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right),\quad W^*=\frac{d}{\nu}W,\quad t^*=\frac{\nu}{d^2}t,$$
$$\Theta^*=\frac{\kappa}{\beta\nu d}\Theta,\quad \Phi^*=\frac{(1+\chi)\kappa}{K\beta\nu d^2}\Phi \tag{22}$$

we get

$$(1+\Lambda)(D^2-a^2)^2W=(Ra+R_m)a^2\Theta-a^2R_mD\Phi \tag{23}$$

$$(D^2-a^2)\Theta=-\mathrm{W} \tag{24}$$

$$(D^2-a^2M_3)\Phi-D\Theta=0 \tag{25}$$

where $D=d/dz$ is the differential operator, $a=\sqrt{l^2+m^2}$ is the overall horizontal wave number, $Ra=\alpha_t g\beta d^4/\kappa\nu$ the thermal Rayleigh number, $R_m=RaM_1=\mu_0K^2\beta^2d^4/(1+\chi)\kappa\mu$ the magnetic Rayleigh number, $\Lambda=\delta\mu_0(M_0+H_0)$ the non-dimensional magnetic field dependent viscosity parameter, $M_1=\mu_0K^2\beta/(1+\chi)\alpha_t\rho_0 g$ the magnetic number, $M_3=(1+M_0/H_0)/(1+\chi)$ the measure of non-linearity of magnetization parameter, $M_2=\mu_0T_0K^2/\rho_0C_0(1+\chi)$ the non-dimensional parameter and its value for different carrier liquids turns out to be of the order of $10^{-6}$and hence its effect is neglected as compared to unity.

The above equations are to be solved subject to appropriate boundary conditions. The boundary conditions considered are

$$W=DW=\Theta=\Phi=0 \text{ at } z=0 \tag{26}$$

$$W=(1+\Lambda)D^2W+Ma\,a^2\Theta=D\Theta+Bi\Theta=D\Phi=0 \text{ at } z=1 \tag{27}$$

where $Ma=\sigma_T\,\Delta T\,d/\mu\kappa$ the Marangoni number and $Bi=hd/k_t$ is the Biot number. The case $Bi=0$ and $Bi\to\infty$,respectively, correspond to constant heat flux and isothermal conditions at the upper boundary.

3. Method of solution

Eqs. (23)–(25) together with the boundary conditions (26) and (27) constitute a Sturm–Liouville problem with the Marangoni number Ma or the Rayleigh number Ra, as an eigenvalue while keeping other physical parameters fixed. To solve the resulting eigenvalue problem, Rayleigh–Ritz's method is used. Accordingly, the variables are written in a series of basis functions as

$$W=\sum_{i=1}^{n}A_iW_i(z),\quad \Theta(z)=\sum_{i=1}^{n}C_i\Theta_i(z) \text{ and } \Phi(z)=\sum_{i=1}^{n}D_i\Phi_i(z) \tag{28}$$

where the trial functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions, and A_i, C_i and D_i are constants. Substituting Eq.(28) into Eqs.(23)–(25), multiplying the resulting momentum Eq. (18) by $W_j(z)$, energy Eq. (19) by $\Theta_j(z)$ and magnetic potential Eq. (20) by $\Phi_j(z)$; performing the integration by parts with respect to z between z=0 and 1 and using the boundary conditions (26) and (27), we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}C_i + E_{ji}D_i = 0 \quad (29)$$

$$F_{ji}A_i + G_{ji}C_i = 0 \quad (30)$$

$$H_{ji}C_i + I_{ji}D_i = 0 \quad (31)$$

The coefficients $C_{ji} - I_{ji}$ involve the inner products of the basis functions and are given by

$$C_{ji} = (1+\Lambda)\left[\langle D^2W_jD^2W_i\rangle + 2a^2\langle DW_jDW_i\rangle + a^4\langle W_jW_i\rangle\right]$$

$$D_{ji} = -a^2(Ra+R_m)\langle \Theta_jW_i\rangle + a^2MaDW_j(1)\Theta_i(1)$$

$$E_{ji} = a^2R_m\langle W_jD\Phi_i\rangle$$

$$F_{ji} = -\langle \Theta_jW_i\rangle$$

$$G_{ji} = \langle D\Theta_jD\Theta_i\rangle + a^2 < \Theta_j\Theta_i > + Bi\Theta_j(1)\Theta_i(1)$$

$$H_{ji} = \langle \Phi_jD\Theta_i\rangle$$

$$I_{ji} = \langle D\Phi_jD\Phi_i\rangle + a^2M_3\langle \Phi_j\Phi_i\rangle$$

where the inner product is defined as $\langle\cdots\rangle = \int_0^1(\cdots)\,dz$. The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} \\ F_{ji} & G_{ji} & 0 \\ 0 & H_{ji} & I_{ji} \end{vmatrix} = 0 \quad (32)$$

The eigenvalue has to be extracted from the characteristic Eq.(32). We select the trial functions as

$$W_i = z^2(1-z)T^*_{i-1}, \quad \Theta_i = z(1-z/2)T^*_{i-1} \text{ and } \Phi_i = z^2(1-2z/3)T^*_{i-1} \quad (33)$$

where T^*_is are the Chebyshev polynomials of the second kind, such that W_i, Θ_i and Φ_i satisfy the corresponding boundary conditions except, $(1+\Lambda)D^2W + Ma\ a^2\Theta = 0 = D\Theta + Bi\Theta$ at $z=1$ but the residuals from the equations are included as residuals from the differential equations.

4. Numerical results and discussion

It may be noted that Eq.(32) leads to the characteristic equation giving the Marangoni number Ma or the Rayleigh number Ra as a function of the wavenumber a, the parameters R_m, B_i, M_3 and Λ. The inner products involved in the equation are evaluated analytically in order to avoid errors in the numerical integration. Computations reveal that the convergence in finding Ma_c or Ra_c crucially depends on the value of MFD viscosity parameter Λ. The results presented here are for $i=j=6$ the order at which the convergence is achieved, in general. In order to validate the numerical solution procedure used, first the critical values (Ra_c, Ma_c, a_c) obtained from the present study under the limiting conditions are compared with the previously published results of Davis [25] in Table 1. The results tabulated in Table 1 for different values of heat transfer coefficient Bi (i.e. Biot number) are for $\Lambda=0$ and $R_m=0$ (i.e., classical Bénard–Marangoni convection for ordinary viscous fluid). From the table it is evident that there is an excellent agreement between the results of the present study and the previously published ones. This verifies the applicability and accuracy of the method used in solving the problem.

Table 1
Comparison of Ma_c for different values of Ra and Bi with $Rm=0$ and $\Lambda=0$.

Bi	Ra	**Davis** [25] Ma_c	**Present study** Ma_c
0	0	79.61	79.608
	100	68.43	68.484
	200	57.12	57.116
	300	45.49	45.491
	400	33.59	33.589
	500	21.39	21.387
	600	8.857	8.857
	669	0.000	0.000
10	0	413.4	413.444
	100	378.7	378.741
	300	305.0	304.980
	500	225.1	225.116
	700	138.6	138.634
	900	44.73	44.730
	989.49	0.000	0.000

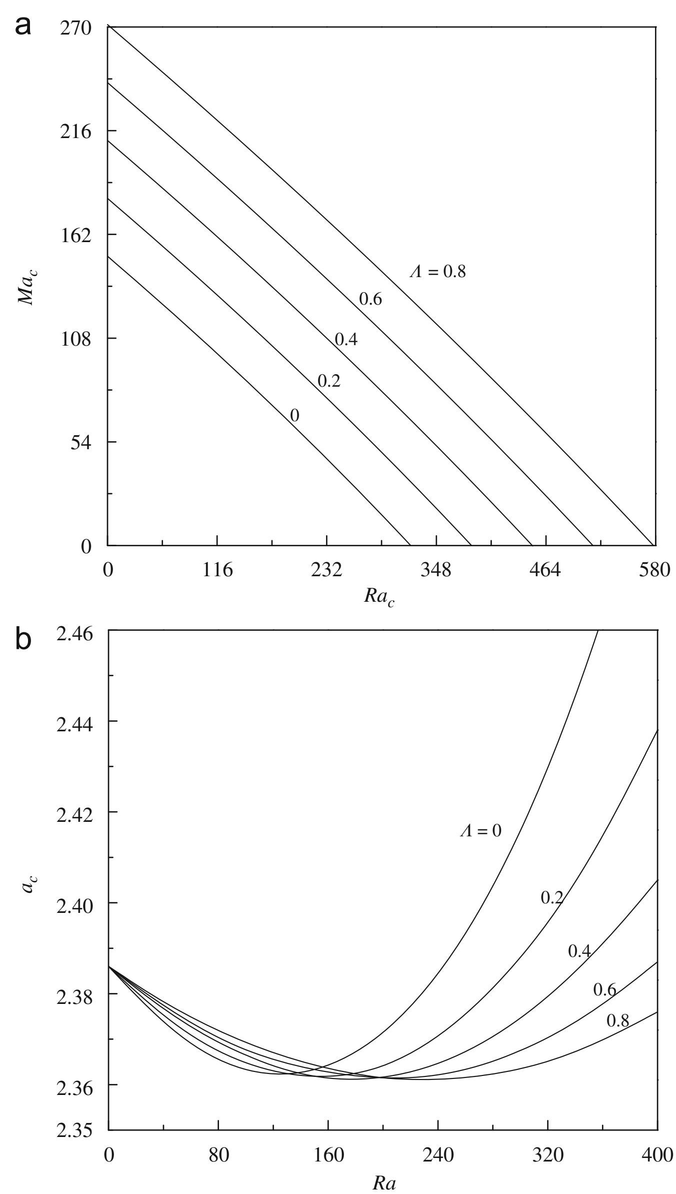

Fig. 1. (a) Locus of critical Marangoni number Ma_c and Rayleigh number Ra_c for different values of Λ for $Bi=2$, $M_1=2$ and $M_3=1$. (b) Critical wave number a_c as a function of Ra for different values of Λ for $Bi=2$, $M_1=2$ and $M_3=1$.

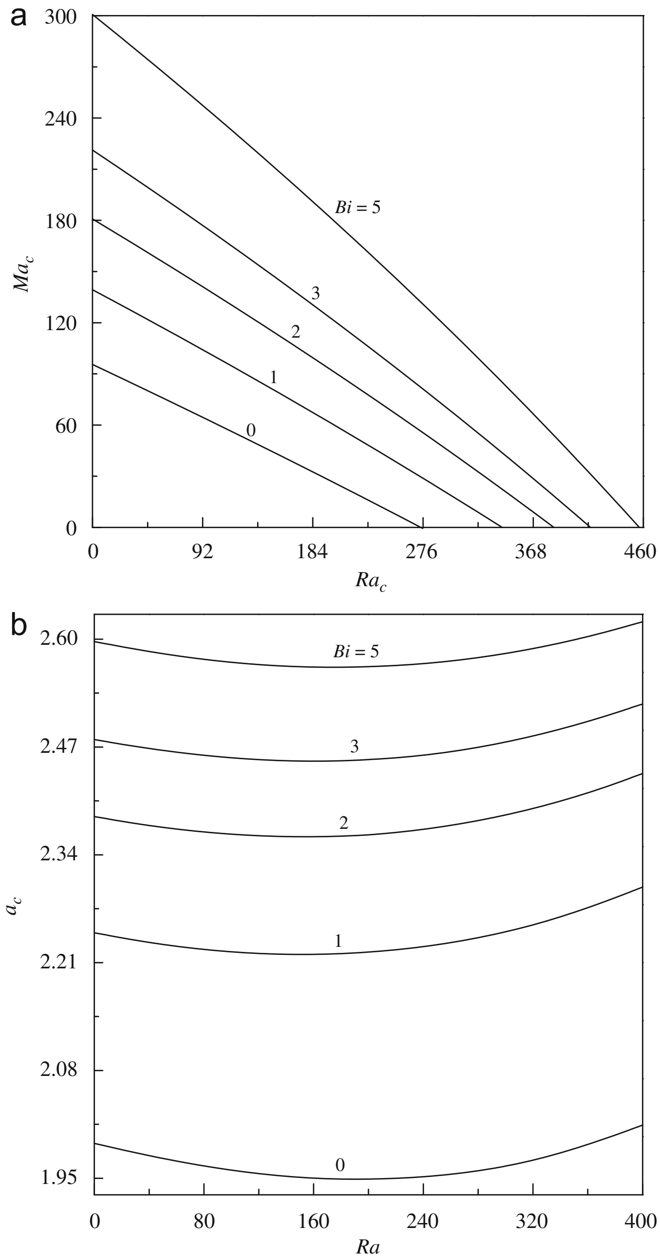

Fig. 2. (a) Locus of critical Marangoni number Ma_c and Rayleigh number Ra_c for different values of Bi for $\Lambda=0.2$, $M_1=2$ and $M_3=1$. (b) Critical wave number a_c as a function of Ra for different values of Bi for $\Lambda=0.2$, $M_1=2$ and $M_3=1$.

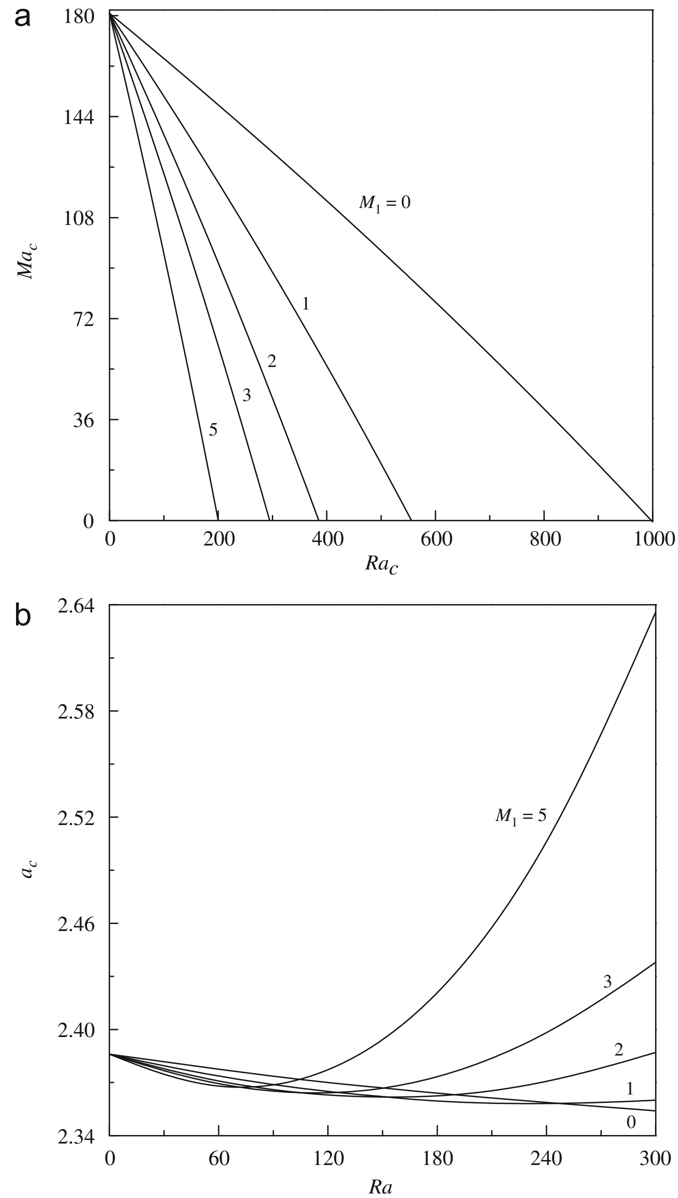

Fig. 3. (a) Locus of critical Marangoni number Ma_c and Rayleigh number Ra_c for different values of M_1 for $\Lambda=0.2$, $Bi=2$ and $M_3=1$. (b) Critical wave number a_c as a function of Ra for different values of M_1 for $\Lambda=0.2$, $Bi=2$ and $M_3=1$.

We now look into the solution of the complete problem, which involves the effect of all the parameters R_a, R_m, Bi, Λ, M_1 and M_3 on the criterion for the onset of convection. The salient characteristics of these parameters are exhibited graphically in Figs. 1–7 and also in Table 2. These figures exhibit a tight coupling between the buoyancy, magnetization and surface tension forces. Fig. 1(a) shows the locus of the critical Marangoni number Ma_c and the Rayleigh number Ra_c for different values of MFD viscosity parameter Λ for $Bi=2$, $M_1=2$ and $M_3=1$. From the figure, it is obvious that there is a strong coupling between the critical Rayleigh and the Marangoni numbers, and an increase in the Rayleigh number has a destabilizing effect on the system. Thus, when the buoyancy force is predominant, the surface tension force becomes negligible and vice-versa. From Fig. 1(a), it is seen that the critical Rayleigh and Marangoni numbers increase with an increase in the MFD viscosity parameter and thus it has a stabilizing effect on the system. That is, the effect of increasing Λ is to delay the onset of Bénard–Marangoni ferroconvection. The variation in a_c as a function of Ra is elucidated in Fig. 1(b) for different values of Λ with $Bi=2$, $M_1=2$ and $M_3=1$. It may be noted that the curves of different Λ cross over each other with an increase in the value of Ra. That is, an increase in the value of Λ increases marginally the critical wave number a_c up to some value of Ra, depending on the value of Λ, and an opposite trend prevails with further increase in the value of Ra.

The plots in Fig. 2(a) represents the locus of critical Marangoni number Ma_c and Rayleigh number Ra_c for different values of the heat transfer coefficient Bi for $\Lambda=0.2$, $M_3=1$ and $M_1=2$. The critical Rayleigh and Marangoni numbers increase with an increase in Bi and thus its effect is to delay the onset of Bénard–Marangoni ferroconvection. This may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top free surface and hence makes the system

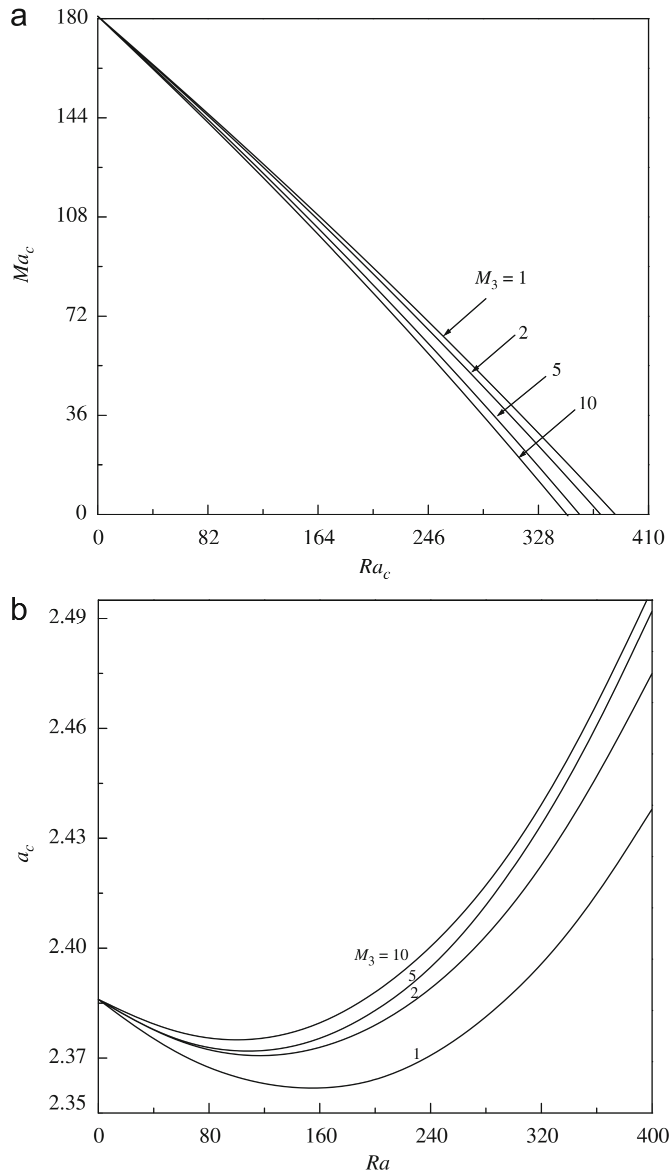

Fig. 4. (a) Locus of critical Marangoni number Ma_c and Rayleigh number Ra_c for different values of M_3 for Λ=0.2, Bi=2 and M_1=2. (b) Critical wave number a_c as a function of Ra for different values of M_3 for Λ=0.2, Bi=2 and M_1=2.

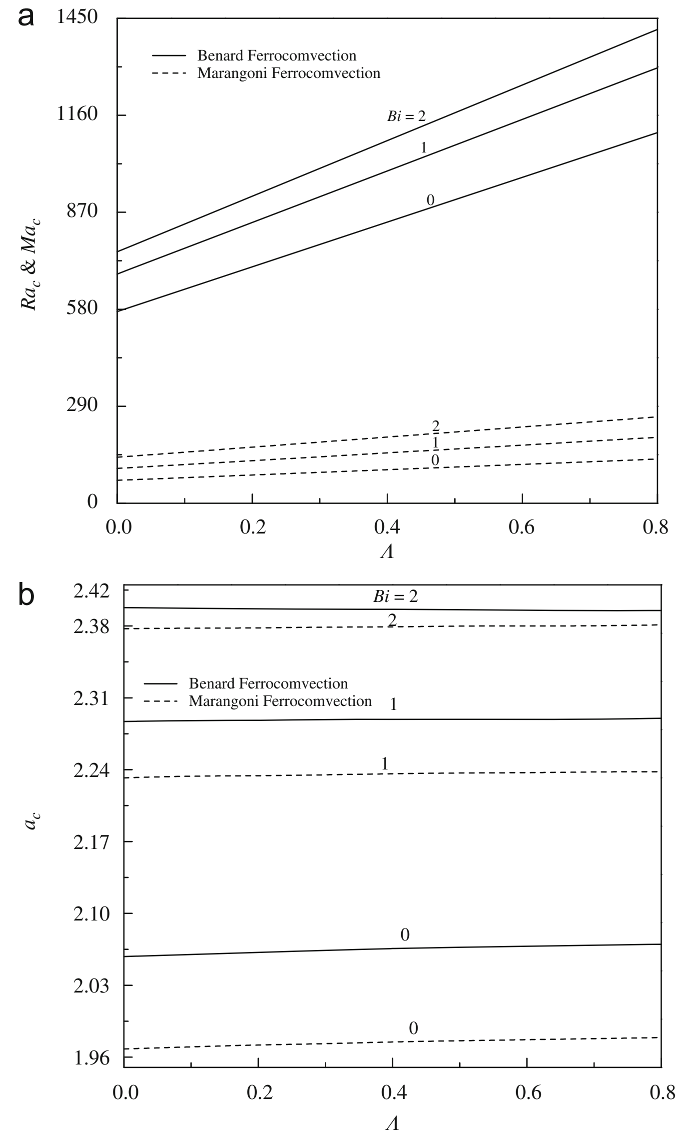

Fig. 5. (a) Critical values of Ma_c and Ra_c as a function of Λ for different values of Bi for R_m=100 and M_3=1. (b) Critical wave number a_c as a function of Λ for different values of Bi for R_m=100 and M_3=1.

more stable. Fig. 2(b) represents the corresponding critical wave number and it indicates that increase in the value of Bi is to increase a_c and thus its effect is to reduce the size of convection cells. It is also seen that the critical wave number passes through a minimum with increasing Ra.

Fig. 3(a) presents the locus of the critical values of Ra_c and Ma_c for various values of magnetic number M_1 for Λ=0.02, M_3=1 and Bi=2. The curve of M_1=0 corresponds to the case when only the buoyancy force is in effect and it lies above all other curves of different $M_1(\neq 0)$. This indicates that increasing M_1 is to make the system more unstable due to increase in the destabilizing magnetic force. Besides, the curves of different M_1 become closer as the value of M_1 increases. Although the critical wave number a_c remains invariant for different values of M_1 at lower values of Ra it increases with further increase in the value of Ra (see Fig. 3(b)). Further, the deviation in the critical wave number amongst different values of M_1 increases with increasing M_1 as well as Ra.

Fig. 4(a) presents the critical Marangoni number Ma_c as a function of critical Rayleigh number Ra_c for several values of nonlinearity of magnetization parameter M_3 for Λ=0.2, Bi=2 and M_1=2. It can be seen that an increase in M_3 is to decrease Ra_c and Ma_c but only marginally and thus it has a destabilizing effect on the stability of the system. This may be due to the fact that the application of magnetic field makes the ferrofluid to acquire larger magnetization which in turn interacts with the imposed magnetic field and releases more energy to drive the flow faster. Hence, the system becomes unstable with a smaller temperature gradient as the value of M_3 increases. Alternatively, a higher value of M_3 would arise either due to a larger pyromagnetic coefficient or larger temperature gradient. Both these factors are conducive for generating a larger gradient in the Kelvin body force field, possibly promoting the instability.

The variation of critical wave number a_c as a function of Rayleigh number Ra is shown in Fig. 4(b) for different values of M_3. From the figure, we note that an increase in M_3 is to increase a_c and hence its effect is to decrease the dimension of convection cells. From the figure it is also seen that the critical wave number

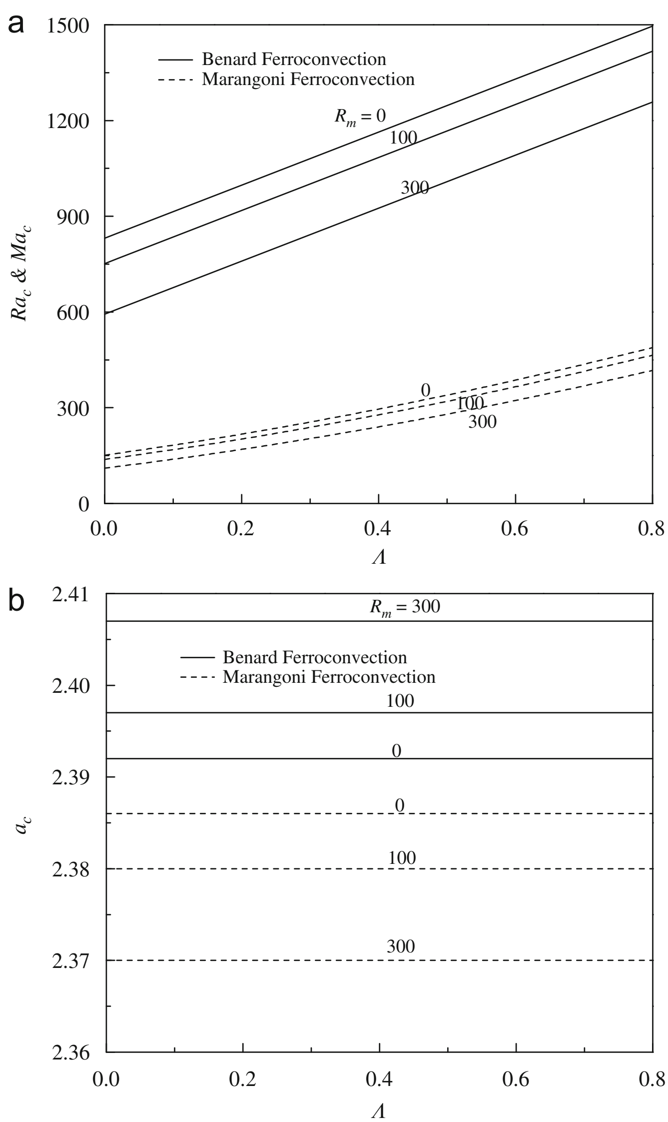

Fig. 6. (a) Critical values of Ma_c and Ra_c as a function of Λ for different values of Rm for Bi=2 and M_3=1. (b) Critical wave number a_c as a function of Λ for different values of R_m for Bi=2 and M_3=1.

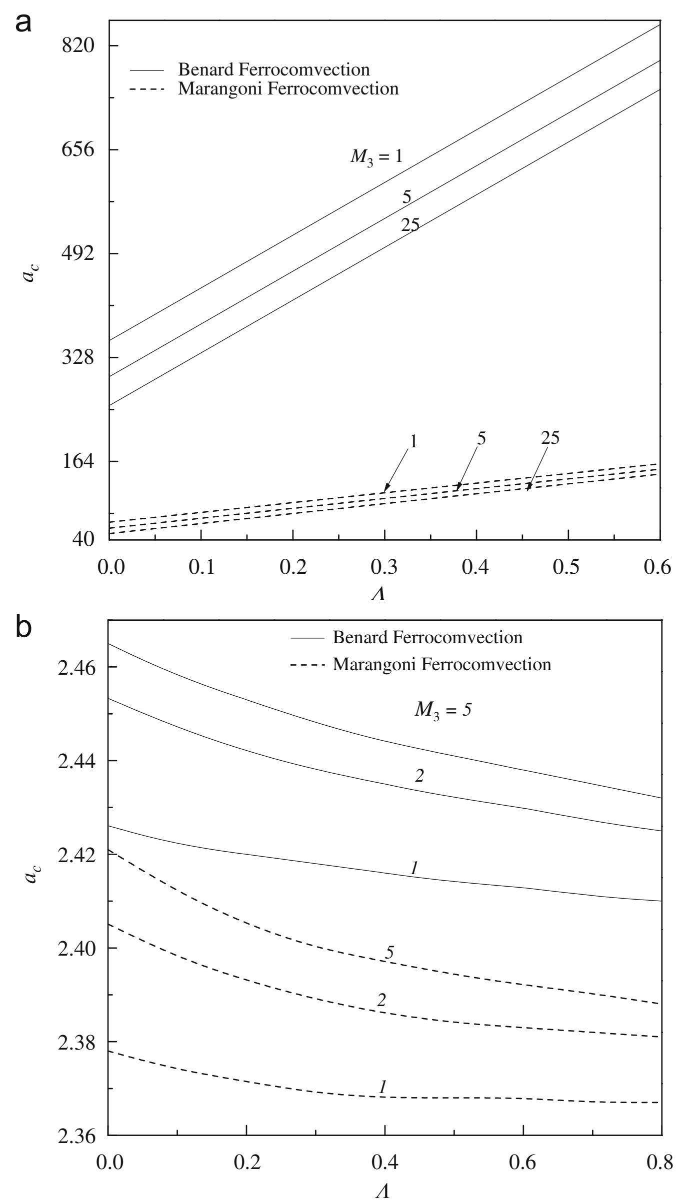

Fig. 7. (a) Critical values of Ma_c and Ra_c as a function of Λ for different values of M_3 for Bi=2 and R_m=600. (b) Critical wave number a_c as a function of Λ for different values of M_3 for Bi=2 and R_m=600.

decreases initially with increasing Ra but eventually increases with further increase in the value of Ra.

Figs. 5–7 show the critical values of Ma_c (pure Marangoni ferroconvection) and Ra_c (pure Bénard ferroconvection) as well as corresponding a_c for different values of Bi, R_m and M_3, respectively, as a function of MFD viscosity parameter Λ. From the figures, it is seen that $Ma_c < Ra_c$ and the effect of increasing Λ is to delay the onset of Bénard/Marangoni ferroconvection. Further, increase in Bi (Fig. 5(a)) and decrease in R_m (Fig. 6(a)) and M_3 (Fig. 7(a)) is to increase the critical Rayleigh/Marangoni number and hence has a stabilizing effect on the system. Moreover, increase in Bi (Fig. 5(b)), R_m (Fig. 6) band M_3(Fig. 7(b)) is to decrease the width of convection cells. The critical wave numbers a_c for Bénard ferroconvection are always found to be higher than those of pure Marangoni ferroconvection (see Figs. 5–7(b)). Further inspection of these figures reveals that the variation in a_c with Λ is insignificant but for different values of M_3 it decreases monotonically with Λ.

The tight coupling between buoyancy, surface tension and magnetic forces is exhibited quantitatively by tabulating the values of triplets (Ra_c, Ma_c, R_{mc}) for different values of Λ and M_3 with Bi=2 in Table 2. It is observed that increase in one of these decreases the other and vice-versa. As M_3 increases, R_{mc} decreases and the results reduce to that of classical Bénard–Marangoni problem for ordinary viscous fluids as $M_3 \to \infty$. That is, R_{mc}=R_{ac} as $M_3 \to \infty$.

5. Conclusions

The effect of MFD viscosity on the criterion for the onset of coupled Bénard–Marangoni convection in a ferrofluid layer is investigated since the viscosity of the magnetic fluid varies with an applied magnetic field. The lower rigid surface of the ferrofluid layer is heated from below, while a general thermal condition is used at the upper free surface subjected to a surface tension decreasing with temperature. The resulting eigenvalue problem is solved numerically by employing the Rayleigh–Ritz technique with either Rayleigh number (Ra) or Marangoni number (Ma) as

Table 2
The critical instability parameters Ra_c and R_{mc} for different values of Λ and Ma when Bi=2.

Λ	Ma_c	Ra_c (R_m=0)	R_{mc} M_3=1 (Ra=0)	R_{mc} M_3=10 (Ra=0)	R_{mc} M_3=25 (Ra=0)	R_{mc} $M\to\infty$ (Ra=0)
0	0	831.27	1046.7	894.635	861.211	831.27
	50	579.525	722.888	624.305	600.728	579.525
	100	304.572	375.905	328.012	315.659	304.572
	150	4.27148	5.211	4.59223	4.42246	4.27148
	150.679	0.0	0.0	0.0	0.0	0.0
0.2	0	997.524	1256.036	1073.562	1033.453	997.524
	50	747.670	934.177	805.379	774.991	747.670
	100	478.679	592.955	515.647	496.178	478.679
	150	189.353	232.373	203.788	196.168	189.353
	180.815	0.0	0.0	0.0	0.0	0.0
0.5	0	1246.91	1570.05	1341.95	1291.82	1246.91
	50	998.934	1250.15	1075.91	1035.37	998.934
	100	735.787	914.677	792.686	762.729	735.787
	150	456.859	563.857	492.019	473.489	456.859
	200	161.094	197.308	173.321	166.861	161.094
	226.019	0.0	0.0	0.0	0.0	0.0

the eigenvalue. The effect of magnetic field dependent viscosity measured through the parameter Λ on the physical parameters of importance Ra as well as Ma is analyzed in detail.

The following conclusions can be drawn from the present study

(i) The effect of increase in the value of magnetic field dependent viscosity parameter Λ is to increase the value of critical stability parameters Ra_c or Ma_c and hence its effect is to delay the onset of Bénard–Marangoni ferroconvection.
(ii) Increase in the value of Biot number Bi is to delay the onset of Bénard–Marangoni ferroconvection, while increase in the value of magnetic Rayleigh number R_m and nonlinearity of fluid magnetization parameter M_3 is to advance the onset of Bénard–Marangoni ferroconvection.
(iii) The buoyancy and surface tension forces complement with each other and it is always found that $Ma_c < Ra_c$; a result in accordance with ordinary viscous fluids.
(iv) The effect of increase in Bi and Λ as well as decrease in M_1 and M_3 values is to decrease the dimension of the convection cells.
(v) As $M_3 \to \infty$, the results reduce to that of the Bénard–Marangoni problem for ordinary viscous fluids.

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