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# Ferromagnetic Convection in a Rotating Ferrofluid Saturated Porous Layer

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**Abstract** The effect of Coriolis force on the onset of ferromagnetic convection in a rotating horizontal ferrofluid saturated porous layer in the presence of a uniform vertical magnetic field is studied. The boundaries are considered to be either stress free or rigid. The modified Brinkman–Forchheimer-extended Darcy equation with fluid viscosity different from effective viscosity is used to characterize the fluid motion. The condition for the occurrence of direct and Hopf bifurcations is obtained analytically in the case of free boundaries, while for rigid boundaries the eigenvalue problem has been solved numerically using the Galerkin method. Contrary to their stabilizing effect in the absence of rotation, increasing the ratio of viscosities, $\Lambda$, and decreasing the Darcy number $Da$ show a partial destabilizing effect on the onset of stationary ferromagnetic convection in the presence of rotation, and some important observations are made on the stability characteristics of the system. Moreover, the similarities and differences between free–free and rigid–rigid boundaries in the presence of buoyancy and magnetic forces together or in isolation are emphasized in triggering the onset of ferromagnetic convection in a rotating ferrofluid saturated porous layer. For smaller Taylor number domain, the stress-free boundaries are found to be always more unstable than in the

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case of rigid boundaries. However, this trend is reversed at higher Taylor number domain because the stability of the stress-free case is increased more quickly than the rigid case.

**Keywords** Rotation · Ferroconvection · Porous medium · Coriolis force · Viscosity ratio

**List of Symbols**

| | |
|---|---|
| $A = (\rho_0 C)_m/(\rho_0 C_p)_{\mathrm{f}}$ | Ratio of heat capacities |
| $a = \sqrt{\ell^2 + m^2}$ | Overall horizontal wave number |
| $\vec{B}$ | Magnetic induction |
| $C$ | Specific heat |
| $C_{V,H}$ | Specific heat at constant volume and magnetic field |
| $c_F$ | Dimensionless form drag constant |
| $d$ | Thickness of the porous layer |
| $D = \mathrm{d}/\mathrm{d}z$ | Differential operator |
| $Da = k/d^2$ | Darcy number |
| $\vec{g}$ | Acceleration due to gravity |
| $\vec{H}$ | Magnetic field intensity |
| $H_0$ | Imposed uniform vertical magnetic field |
| $\hat{k}$ | Unit vector in z-direction |
| $k$ | Permeability of the porous medium |
| $k_1$ | Thermal conductivity |
| $K = -(\partial M/\partial T)_{H_0,\,T_0}$ | Pyromagnetic co-efficient |
| $\ell, m$ | Wave numbers in the $x$ and $y$ directions |
| $\vec{M}$ | Magnetization |
| $M_0 = M(H_0, T_0)$ | Constant mean value of magnetization |
| $M_1 = \mu_0 K^2 \beta/(1+\chi)\alpha_{\mathrm{t}}\rho_0 g$ | Magnetic number |
| $M_3 = (1 + M_0/H_0)/(1+\chi)$ | Nonlinearity of magnetization parameter |
| $N = RM_1 = \mu_0 K^2 \beta^2 d^4/(1+\chi)\mu\kappa A$ | Magnetic Rayleigh number |
| $p$ | Pressure |
| $\mathrm{Pr} = \nu/\kappa$ | Prandtl number |
| $\vec{q} = (u, v, w)$ | Velocity vector |
| $R = \alpha_{\mathrm{t}} g \beta d^4/\nu\kappa$ | Rayleigh number |
| $t$ | Time |
| $T$ | Temperature |
| $T_0$ | Temperature of the lower boundary |
| $T_1$ | Temperature of the upper boundary |
| $Ta = 4\Omega^2 d^4/\nu^2\varepsilon^2$ | Taylor number |
| $W$ | Amplitude of vertical component of perturbed velocity |
| $(x, y, z)$ | Cartesian co-ordinates |
| $Z$ | Amplitude of vertical component of vorticity |

**Greek symbols**

| | |
|---|---|
| $\alpha_{\mathrm{t}}$ | Thermal expansion coefficient |
| $\beta = \Delta T/d$ | Temperature gradient |

| | |
|---|---|
| $\chi = (\partial M/\partial H)_{H_0}, T_0$ | Magnetic susceptibility |
| $\Delta T (= T_0 - T_1)$ | Constant temperature difference between the boundaries |
| $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ | Laplacian operator |
| $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ | Horizontal Laplacian operator |
| $\varepsilon$ | Porosity of the porous medium |
| $\kappa = k_1/(\rho_0 C)_2$ | Effective thermal diffusivity |
| $\vec{\Omega} = \Omega\hat{k}$ | Constant angular velocity |
| $\omega$ | Growth rate |
| $\Lambda = \tilde{\mu}_f/\mu_f$ | Ratio of viscosities |
| $\mu_f$ | Dynamic viscosity |
| $\tilde{\mu}_f$ | Effective viscosity |
| $\mu_0$ | Free space magnetic permeability of vacuum |
| $v = \mu_f/\rho_0$ | Kinematic viscosity |
| $\varphi$ | Magnetic potential |
| $\Phi$ | Amplitude of perturbed magnetic potential |
| $\rho$ | Fluid density |
| $\rho_0$ | Reference density |
| $\Theta$ | Amplitude of perturbed temperature |

**Subscripts**

| | |
|---|---|
| b | Basic state |
| f | Fluid |
| s | Solid |

## 1 Introduction

Thermogravitational convection in a layer of magnetized ferrofluids in the presence of a uniform magnetic field, known as ferromagnetic convection, is analogous to classical Benard convection and has been investigated extensively because of promising potential in heat transfer applications. An extensive literature pertaining to this field and also the important applications of these fluids in many practical problems are given in the books by Rosensweig (1985), Berkovsky et al. (1993), Hergt et al. (1998). Ganguly et al. (2004) have given an overview of prior research on heat transfer in ferrofluid flows and also discussed the heat transfer augmentation due to the thermomagnetic convection. In his review article, Odenbach (2004) has focused on recent developments in the field of rheological investigations of ferrofluids and their importance for the general treatment of ferrofluids.

Ferromagnetic convection in a porous medium has also been investigated owing to its importance in controlled emplacement of liquids or treatment of chemicals, and emplacement of geophysically imageable liquids into particular zones for subsequent imaging etc. Rosensweig et al. (1978) have studied experimentally the penetration of ferrofluids in the Hele-Shaw cell. The stability of the magnetic fluid penetration through a porous medium in high uniform magnetic field oblique to the interface is studied by Zhan and Rosensweig (1980). Thermal convective instability in a layer of ferrofluid saturating a porous medium in the presence of a vertical magnetic field is studied by Vaidyanathan et al. (1991). Their analysis is limited to free–free boundaries and to the case of effective viscosity equal to fluid viscosity. Qin and Chadam (1995) have carried out the nonlinear stability analysis of ferroconvection in a porous layer by including the inertial effects to accommodate high velocity. Sekar et al (1996) have investigated ferroconvection in an anisotropic porous medium for stress-free boundaries by considering anisotropy only in the permeability of the porous medium. The laboratory scale experimental results of the behavior of ferrofluids in porous media consisting of sands and sediments are presented in detail by Borglin et al. (2000). Shivakumara et al. (2008, 2009a) have investigated in detail the onset of thermomagnetic convection in a ferrofluid saturated porous medium for various types of velocity and temperature boundary conditions. Recently, Nanjundappa et al. (2010) have discussed the buoyancy-driven convection in a ferromagnetic fluid saturated porous medium.

The study of fluids in rotation is in itself an interesting topic for research. Ferrofluids are known to exhibit peculiar characteristics when they are set to rotation and hence investigating the effects of rotation on thermal convective instability is scientifically and technologically important. Das Gupta and Gupta (1979) have studied thermal convective instability in a rotating layer of ferrofluid heated uniformly from below. Venkatasubramanian and Kaloni (1994) have discussed the effect of rotation on thermo-convective instability of a horizontal layer of ferrofluid confined between stress-free, rigid-paramagnetic and rigid-ferromagnetic boundaries. Effect of rotation on ferrothermohaline convection is studied by Sekar et al. (2000). Thermal convection in a rotating layer of a magnetic fluid is discussed by Auernhammer and Brand (2000). The weakly nonlinear instability of a rotating ferromagnetic fluid layer heated from below is discussed by Kaloni and Lou (2004). Shivakumara and Nanjundappa (2006a) have studied the effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection. Effect of magnetic field dependent viscosity and rotation on ferroconvection in the presence of dust particles has been investigated by Sunil et al. (2006). Shivakumara and Nanjundappa (2006b) have investigated the effect of rotation on the onset of coupled Benard–Marangoni ferroconvection in a horizontal ferrofluid layer.

The corresponding problem of ferromagnetic convection in a rotating porous medium is discussed by Sekar et al. (1993) and Vaidyanathan et al. (2002). In the latter article, the effect of magnetic field dependent viscosity is also taken into consideration. Subsequently, many researchers have extended these studies to include various additional effects. The effect of rotation on ferromagnetic fluid heated and soluted from below saturating a porous medium is studied by Sunil and Sharma (2004). The onset of centrifugal convection in a magnetic fluid saturated porous medium under zero gravity condition is investigated by Saravanan and Yamaguchi (2005). Sunil and Amit Mahajan (2008) have performed nonlinear stability analysis for rotating magnetized ferrofluid heated from below saturating a porous medium for free boundaries. Saravanan (2009) has investigated the influence of magnetic field on the onset of convection induced by centrifugal acceleration in a magnetic fluid saturated porous medium.

In the non-porous domains it has been shown that, in contrast to the convection problems in non-rotating systems, viscosity has a destabilizing effect on stationary convection in a rotating system at high rotation rates (Chandrasekhar 1961). Probing for such a possibility in ferrofluid-saturated rotating porous domains is not only warranted but also would be more interesting. Therefore, the objectives of this study are of three fold. Firstly, revisit the problem of ferromagnetic convection in a rotating sparsely packed horizontal porous layer for the case of stress free boundaries and unveil some interesting observations which have been overlooked by the previous investigators in analyzing the problem. Secondly, investigate the problem numerically for realistic rigid boundary conditions, which has not been given any attention in the literature. Thirdly, unfold the similarities and differences in the results between the two types of velocity boundary conditions as well as when the buoyancy and magnetic forces are acting together and also in isolation. To achieve the above objectives,

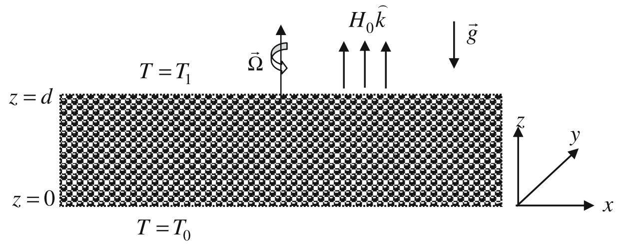


**Fig. 1** Physical configuration

we have employed the Brinkman–Forchheimer-extended Darcy model with fluid viscosity different from effective viscosity to describe the flow in the porous medium and also to know the influence of viscosity on the criterion for the onset of ferromagnetic convection in a rotating porous layer. A comparative study has also been conducted to analyze the relative effects of the boundaries on the stability characteristics of the system. The results are presented for three different cases namely, (i) when the buoyancy and magnetic forces are simultaneously present, (ii) when only the magnetic forces are present, and (iii) when only the buoyancy forces are present. It is observed that there is a qualitative agreement between the results of free–free and rigid–rigid boundaries. In particular, it is shown that increasing the permeability of the porous medium and also the effective viscosity hasten the onset of stationary ferromagnetic convection in the presence of rotation; a result of contrast noticed and documented in the absence of rotation. Although the stress-free boundaries is found to be always unstable than that of rigid boundaries for small Taylor number domain, they show more stabilizing effect on the system than the rigid boundaries at higher Taylor number domain. Our results are shown to agree well with the earlier published ones in the corresponding limits.

## 2 Formulation of the Problem

We consider an initially quiescent ferrofluid saturated horizontal porous layer of depth $d$ in the presence of a uniform applied magnetic field $H_0$ acting in the vertical direction. The lower and upper boundaries of the porous layer are maintained at constant temperatures $T_0$ and $T_1(<T_0)$ respectively, and thus, a constant temperature difference $\Delta T(= T_0 - T_1)$ is maintained between the boundaries. A Cartesian coordinate system $(x, y, z)$ is used with the $z$ axis normal to the porous layer. The entire system is rotating with constant angular velocity $\vec{\Omega} = \Omega\hat{k}$, where $\hat{k}$ is the unit vector in the vertical direction (see Fig. 1). It is assumed that the rotation does not disrupt the isotropy of the porous medium. The flow in the porous medium is described by the Brinkman–Forchheimer-extended Darcy equation with fluid viscosity different from effective viscosity, and the Boussinesq approximation on the density is made.

The governing equations for the flow of an incompressible ferrofluid in a layer of rotating porous medium are:

The continuity equation

$$\nabla \cdot \vec{q} = 0. \tag{1}$$

The momentum equation

$$\rho_0 \left[ \frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q} \right] = -\nabla \mathrm{p} + \rho_0 \left[1 - \alpha_{\mathrm{t}}(T - T_0)\right]\vec{g} - \frac{\mu_{\mathrm{f}}}{k}\vec{q} + \tilde{\mu}_{\mathrm{f}}\nabla^2\vec{q} + \nabla \cdot (\vec{H}\vec{B})$$

$$+\frac{\rho_0 c_F}{\sqrt{k}}\left|\vec{q}\right|\vec{q}+2\frac{\rho_0}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right)+\frac{\rho_0}{2}\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|\right). \tag{2}$$

The energy equation

$$A\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)\,T=k_1\nabla^2 T. \tag{3}$$

The Maxwell equations in the magnetostatic limit are:

$$\nabla\cdot\vec{B}=0$$
$$\nabla\times\vec{H}=0 \quad \text{or} \quad \vec{H}=\nabla\varphi. \tag{4a,b}$$

Further, $\vec{B}$, $\vec{M}$, and $\vec{H}$ are related by

$$\vec{B}=\mu_0\left(\vec{M}+\vec{H}\right). \tag{5}$$

The quantities appeared in the above equations are defined in the nomenclature. We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of magnetic field as well as the temperature in the form:

$$\vec{M}=\frac{\vec{H}}{H}M\,(H,T). \tag{6}$$

The magnetic equation of state is linearized about $H_0$ and $T_0$ to become

$$M=M_0+\chi\,(H-H_0)-K\,(T-T_0) \tag{7}$$

where $\chi=(\partial M/\partial H)_{H_0,\,T_0}$ is the magnetic susceptibility $K=-(\partial M/\partial T)_{H_0,\,T_0}$ is the pyro-magnetic co-efficient, $M_0=M(H_0,T_0)$, $H=\left|\vec{H}\right|$ and $M=\left|\vec{M}\right|$.

The basic state is assumed to be quiescent, and the standard linear stability analysis procedure as outlined in the studies of Venkatasubramanian and Kaloni (1994) as well as Shivakumara et al. (2009a) is followed to obtain the stability equations in the dimensionless form as follows:

$$\left[\Lambda\left(D^2-a^2\right)-Da^{-1}-\omega\right]\left(D^2-a^2\right)W=-a^2R\left[M_1D\Phi-(1+M_1)\,\Theta\right]+Ta^{1/2}DZ \tag{8}$$

$$\left(D^2-a^2-Pr\omega\right)\Theta=-W \tag{9}$$

$$\left(D^2-a^2M_3\right)\Phi-D\Theta=0 \tag{10}$$

$$\left[\Lambda\left(D^2-a^2\right)-Da^{-1}-\omega\right]Z=-Ta^{1/2}DW. \tag{11}$$

Here, $D=\mathrm{d}/\mathrm{d}z$ is the differential operator, $a=\sqrt{\ell^2+m^2}$ is the overall horizontal wave number, $R=\alpha_{\mathrm{t}}g\beta d^4/\nu\kappa$ is the thermal Rayleigh number, $Ta=4\Omega^2d^4/\nu^2\varepsilon^2$ is the Taylor number, $Pr=\nu A\varepsilon/\kappa$ is the modified Prandtl number, $\Lambda=\tilde{\mu}_{\mathrm{f}}/\mu_{\mathrm{f}}$ is the ratio of viscosities, $M_1=\mu_0K^2\beta/(1+\chi)\alpha_t\rho_0 g$ is the magnetic number, $N=RM_1=\mu_0K^2\beta^2d^4/(1+\chi)\mu\kappa$ is the magnetic Rayleigh number, $M_3=(1+M_0/H_0)/(1+\chi)$ is the measure of nonlinearity of magnetization and $Da=k/d^2$ is the Darcy number.

The constant-temperature ferromagnetic boundaries are considered to be either free or rigid. Thus, the boundary conditions are:

$$W=0=D^2W,\quad \Theta=0,\quad D\Phi=0=DZ \tag{12a}$$

on the free boundary, and

$$W = 0 = DW, \quad \Theta = 0, \quad \Phi = 0 = Z \tag{12b}$$

on the rigid boundary.

## 3 Method of Solution

Equations 8–11 together with the chosen boundary conditions, Eq. 12a or Eq. 12b, constitute an eigenvalue problem. Two types of velocity boundary conditions are considered for discussion, namely,

(i) Both boundaries free, and
(ii) Both boundaries rigid.

### 3.1 Both Boundaries Free

For this case, the eigenvalue problem can be solved exactly. We assume the solution for $W$, $\Theta$, $\Phi$, and $Z$, satisfying the respective boundary conditions in the form:

$$W = A_0 \sin \pi z, \quad \Theta = B_0 \sin \pi z, \quad \Phi = -(C_0/\pi) \cos \pi z, \quad Z = -(E_0/\pi) \cos \pi z \tag{13}$$

where $A_0,\ B_0,\ C_0$, and $E_0$ are constants.

Substituting Eq. 13 into Eqs. 8–11, we obtain the following matrix equation:

$$\begin{pmatrix} \delta^2\left(\Lambda\delta^2 + Da^{-1} + \omega\right) & -(1+M_1)\,a^2 R & M_1 a^2 R & -\sqrt{Ta} \\ 1 & -\left(\delta^2 + Pr\omega\right) & 0 & 0 \\ 0 & \pi^2 & -\left(\pi^2 + a^2 M_3\right) & 0 \\ \pi^2\sqrt{Ta} & 0 & 0 & \left(\Lambda\delta^2 + Da^{-1} + \omega\right) \end{pmatrix} \begin{pmatrix} A_0 \\ B_0 \\ C_0 \\ E_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

where, $\delta^2 = \pi^2 + a^2$. A nontrivial solution to the above matrix equation occurs, provided

$$R = \frac{\left(\delta^2 + Pr\omega\right)\left(\pi^2 + M_3 a^2\right)\left[\pi^2 Ta + \delta^2\left(Da^{-1} + \delta^2\Lambda + \omega\right)^2\right]}{a^2\left\{\pi^2 + M_3\left(1+M_1\right)a^2\right\}\left(Da^{-1} + \delta^2\Lambda + \omega\right)}. \tag{15}$$

To examine the stability of the system, the real part of $\omega$ is set to zero and take $\omega = i\omega_i$ in Eq. 15. After clearing the complex quantities from the denominator, Eq. 15 yields

$$R = \frac{\left(\pi^2 + a^2 M_3\right)}{a^2\left\{\pi^2 + M_3\left(1+M_1\right)a^2\right\}\left(\left(Da^{-1}\right)^2 + 2Da^{-1}\Gamma^2\Lambda + \delta^4\Lambda^2 + \omega_i^2\right)}\left(\Delta_1 + i\omega_i\Delta_2\right) \tag{16}$$

where,

$$\begin{aligned} \Delta_1 = {} & \left(Da^{-1}\right)^3\delta^4 + \delta^2\left(\delta^4\Lambda^2 + \omega_i^2\right)\left(\delta^4\Lambda - Pr\omega_i^2\right) + \pi^2 Ta\left(\delta^4\Lambda + Pr\omega_i^2\right) + Da^{-1}\pi^2 Ta\delta^2 \\ & + \left(Da^{-1}\right)^2\left(3\delta^6\Lambda - Pr\delta^2\omega_i^2\right) + 3Da^{-1}\delta^8\Lambda^2 + Da^{-1}\delta^4\omega_i^2 - 2Da^{-1}\delta^4 Pr\Lambda\omega_i^2 \\ \Delta_2 = {} & \left(Da^{-1}\right)^3 Pr\delta^2 + \left(Da^{-1}\right)^2\delta^4\left(1 + 3Pr\Lambda\right) + Da^{-1}\pi^2 PrTa + 2Da^{-1}\delta^6\Lambda \\ & + 3Da^{-1}\delta^6 Pr\Lambda^2 + Da^{-1}\delta^2 Pr\omega_i^2 + \delta^2\pi^2 Ta\left(Pr\Lambda - 1\right) + \delta^4\left(1 + Pr\Lambda\right)\left(\delta^4\Lambda^2 + \omega_i^2\right) \end{aligned}$$

Since the Rayleigh number $R$ is a physical quantity, it must be real. Hence, from Eq. 16 it implies either $\omega_i = 0$ or $\Delta_2 = 0$ $(\omega_i \neq 0)$, and accordingly the condition for direct and Hopf bifurcation is obtained.

*3.1.1 Direct bifurcation* $(\omega_i = 0)$

The direct bifurcation occurs at $R = R^{\mathrm{d}}$, where

$$R^{\mathrm{d}} = \frac{\delta^2\left(\pi^2 + M_3 a^2\right)\left(\pi^2 Ta + \delta^2\left(Da^{-1} + \delta^2\Lambda\right)^2\right)}{a^2\left(\pi^2 + M_3\left(1 + M_1\right)a^2\right)\left(Da^{-1} + \delta^2\Lambda\right)}. \tag{17}$$

When $M_1 = 0$, (i.e., ordinary viscous fluid case) and for a non-porous medium domain $(Da^{-1} = 0,\ \Lambda = 1)$, Eq. 17 coincides, respectively, with that of Shivakumara et al. (2009b), and Venkatasubramanian and Kaloni (1994).

For very large $M_1$, we obtain the results for the magnetic mechanism operating in the absence of buoyancy effects. The corresponding magnetic Rayleigh number $N$ can be expressed as follows:

$$N = R^{\mathrm{d}} M_1 = \frac{\left(\pi^2 + M_3 a^2\right)\left\{\pi^2 Ta\delta^2 + \delta^4\left(Da^{-1} + \Lambda\delta^2\right)^2\right\}}{a^4 M_3\left(Da^{-1} + \Lambda\delta^2\right)}. \tag{18}$$

The critical values of $R^{\mathrm{d}}$ and $N$ (i.e., $R_{\mathrm{c}}^{\mathrm{d}}$ and $N_{\mathrm{c}}$) with respect to the wave number $a$, are obtained numerically for various values of physical parameters.

*3.1.2 Hopf bifurcation* $(\omega_i \neq 0)$

The Hopf bifurcation (oscillatory onset) corresponds to $\Delta_2 = 0$ $(\omega_i \neq 0)$ in Eq. 16, and this gives a dispersion relation which can be expressed in the form:

$$\omega_i^2 = -\delta^4 Pr^2\left(Da^{-1}/\delta^2 + \Lambda\right)^2 + \frac{\left\{1 - Pr\left(Da^{-1}/\delta^2 + \Lambda\right)\right\} Pr^2\pi^2 Ta}{\delta^2\left\{1 + Pr\left(Da^{-1}/\delta^2 + \Lambda\right)\right\}}. \tag{19}$$

Since $\omega_i^2 > 0$, from Eq. 19 it is clear that the necessary conditions for the occurrence of Hopf bifurcation are

$$Pr < \frac{\delta^2}{\delta^2\Lambda + Da^{-1}},\ Ta > \frac{\delta^2\left(Da^{-1} + \Lambda\delta^2\right)^2\left\{\delta^2 + Pr\left(Da^{-1} + \Lambda\delta^2\right)\right\}}{\pi^2\left\{\delta^2 - Pr\left(Da^{-1} + \Lambda\delta^2\right)\right\}}. \tag{20}$$

The above conditions are independent of magnetic parameters and the same as those for the case of ordinary viscous fluid saturated porous medium (Shivakumara et al. 2009). For a non-porous domain case (i.e., $Da^{-1} = 0$ and $\Lambda = 1$), the above conditions reduce to

$$0 < Pr < 1, \quad Ta > \frac{\delta^6\left(1 + Pr\right)}{\pi^2\left(1 - Pr\right)}. \tag{21}$$

These conditions are the same as those of Chandrasekhar (1961) and Venkatasubramanian and Kaloni (1994).

The Hopf bifurcation occurs at $R = R^{\mathrm{H}}$, where

$$R^{\mathrm{H}} = \frac{\left(\pi^2 + a^2 M_3\right)\Delta_1}{a^2\left\{\pi^2 + M_3\left(1 + M_1\right)a^2\right\}\left[\left(Da^{-1}\right)^2 + 2Da^{-1}\delta^2\Lambda + \delta^4\Lambda^2 + \omega_i^2\right]} \tag{22}$$

where $\omega_i^2$ is given by Eq. 19. From Eq. 20, it is observed that the onset of oscillatory ferromagnetic convection in rotating porous media is possible only when the value of the Prandtl number is less than unity and the Taylor number exceeds a threshold. Since the Prandtl number is greater than unity for ferrofluids (whether they are water based or any other organic liquid based), oscillatory convection is not a preferred mode of instability in a ferrofluid saturated rotating porous layer. Under the circumstances, the study is restricted to stationary convection only.

### 3.2 Both Boundaries Rigid

For the boundary conditions considered, it is not possible to obtain an analytical solution as in the case of free–free boundaries, and we have to resort to numerical methods. The Galerkin method is employed to obtain the critical stability parameters for the onset of stationary ferromagnetic convection. Accordingly, the variables are written in series of basis functions as

$$W = \sum_{i=1}^{n} A_i W_i(z), \quad \Theta(z) = \sum_{i=1}^{n} B_i \Theta_i(z), \quad \Phi(z) = \sum_{i=1}^{n} C_i \Phi_i(z),$$
$$\mathrm{Z}(z) = \sum_{i=1}^{n} D_i \mathrm{Z}_i(z) \tag{23}$$

where the trial functions $W_i(z)$, $\Theta_i(z)$, $\Phi_i(z)$ and $\mathrm{Z}_i(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions, and $A_i$, $B_i$, $C_i$ and $D_i$ are constants. Substituting Eq. 23 into Eqs. 8–11 (with $\omega = 0$), multiplying the resulting momentum equation by $W_j(z)$, energy equation by $\Theta_j(z)$, magnetic potential equation by $\Phi_j(z)$, and the vorticity equation by $\mathrm{Z}_j(z)$; performing the integration by parts with respect to $z$ between $z = 0$ and $z = 1$ and using the boundary conditions (12b), we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji} A_i + D_{ji} B_i + E_{ji} C_i + F_{ji} D_i = 0 \tag{24}$$
$$G_{ji} A_i + H_{ji} B_i = 0 \tag{25}$$
$$I_{ji} B_i + J_{ji} C_i = 0 \tag{26}$$
$$K_{ji} A_i + L_{ji} D_i = 0. \tag{27}$$

The coefficients $C_{ji} - I_{ji}$ involve the inner products of the basis functions and are given by

$$C_{ji} = \Lambda < D^2 W_j D^2 W_i > +(2a^2\Lambda + Da^{-1}) < DW_j DW_i > +a^2(a^2\Lambda + Da^{-1}) < W_j W_i >$$
$$D_{ji} = -a^2 R(1 + M_1) < W_j \Theta_i >, E_{ji} = a^2 R M_1 < W_j D\Phi_i >, F_{ji} = -\sqrt{Ta} < W_j D\mathrm{Z}_i >$$
$$G_{ji} = - < \Theta_j W_i >, H_{ji} = < D\Theta_j D\Theta_i > +a^2 < \Theta_j \Theta_i >,$$
$$I_{ji} = - < D\Phi_j \Theta_i >, J_{ji} = < D\Phi_j D\Phi_i > +a^2 M_3 < \Phi_j \Phi_i >, K_{ji} = -\sqrt{Ta} < \mathrm{Z}_j DW_i >$$
$$L_{ji} = \Lambda[< D\mathrm{Z}_j D\mathrm{Z}_i > +a^2 < \mathrm{Z}_j \mathrm{Z}_i >] + Da^{-1} < \mathrm{Z}_j \mathrm{Z}_i > \tag{28}$$

where the inner product is defined as

$$< \cdots > = \int_0^1 (\cdots) \mathrm{d}z.$$

The above set of homogeneous algebraic equations can have a nontrivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} & F_{ji} \\ G_{ji} & H_{ji} & 0 & 0 \\ 0 & I_{ji} & J_{ji} & 0 \\ K_{ji} & 0 & 0 & L_{ji} \end{vmatrix} = 0 \tag{29}$$

The eigenvalue has to be extracted from the above characteristic equation. For this, we select the trial functions as

$$\begin{aligned} W_i &= (z^4 - 2z^3 + z^2)T^*_{i-1}, \quad \Theta_i = z(z-1)T^*_{i-1}, \\ \Phi_i &= (z^3 - 3z^2 + 2z)T^*_{i-1}, \quad \mathrm{Z}_i = (z^3 - 3z^2 + 2z)T^*_{i-1} \end{aligned} \tag{30}$$

where $T_i^*$s are the modified Chebyshev polynomials, such that $W_i$, $\Theta_i$, $\Phi_i$, and $\mathrm{Z}_i$ satisfy the corresponding boundary conditions. The characteristic Eq. 29 is solved numerically for different values of physical parameters involved therein using the Newton–Raphson method to obtain the thermal Rayleigh number/magnetic Rayleigh number as the case may be as a function of wave number $a$, and the bisection method is built-in to locate the critical stability parameters ($R_c^d$, $a_c$) or ($N_c$, $a_c$) to the desired degree of accuracy. The critical stability parameters computed numerically as explained above, are found to converge by considering six terms in the series expansion of the basis functions given by Eq. 23.

## 4 Results and Discussion

The effect of Coriolis force on thermal convective instability in a rotating sparsely packed ferrofluid saturated horizontal porous layer is investigated for two types of velocity boundary conditions namely, (i) both boundaries free, and (ii) both boundaries rigid. Since the basic state is quiescent, it is found that inertia has no effect on the onset of ferromagnetic convection in a rotating ferrofluid-saturated porous layer. The results for the above two types of velocity boundary conditions are discussed below.

### 4.1 Free–Free Boundaries

For this particular case in this study, the condition for the occurrence of direct (stationary onset) and Hopf bifurcations (oscillatory onset) is obtained analytically. It has been established that the oscillatory convection occurs only when the Prandtl number is less than unity and the Taylor number exceeds a threshold value. Since the Prandtl number is greater than unity for ferrofluids, the discussion is limited to the stationary onset. The results have been analyzed when buoyancy and magnetic forces are acting simultaneously and also in isolation. The similarities and differences between these mechanisms on the onset of stationary ferromagnetic convection in a rotating porous layer are examined. We emphasize here only those aspects which have been overlooked by the previous investigators.

*Case (i): Simultaneous presence of buoyancy and magnetic forces*

In the above case of this study, the effects of both buoyancy and magnetic forces on the stability of the system are considered and the thermal Rayleigh number $R$ turns out to be the eigenvalue. From Eq. 17, we note that $\partial R^d/\partial Ta > 0$, $\partial R^d/\partial M_1 < 0$ and $\partial R^d/\partial M_3 < 0$. Hence, the effect of increasing $Ta$ has stabilizing, while increasing $M_1$ and $M_3$ have destabilizing effect on the system. However, we note that

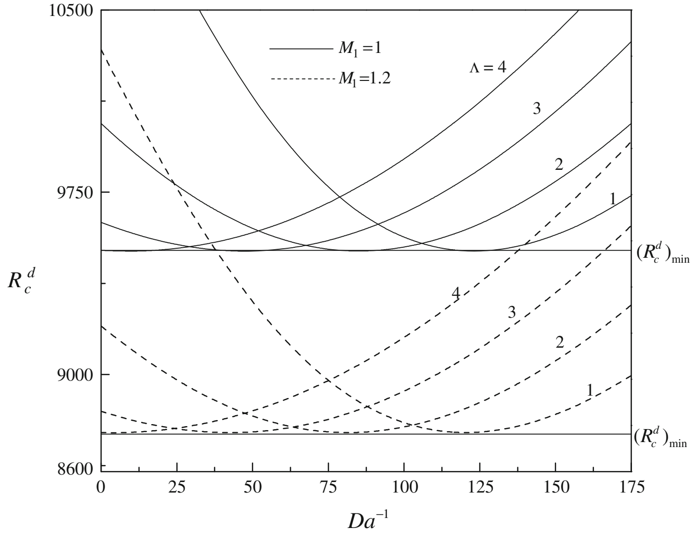


**Fig. 2** Variation of $(R_c^d)_{\min}$ as a function of $Da^{-1}$ for different values of $\Lambda$ and $M_1$ when $M_3 = 1$ and $Ta = 10^5$

$$\frac{\partial R^d}{\partial Da^{-1}} = \frac{\left(a^2 M_3 + \pi^2\right)\delta^2}{a^2\left\{a^2(1+M_1)M_3+\pi^2\right\}}\left[2\delta^2 - \frac{\pi^2 Ta + \delta^2\left(Da^{-1}+\delta^2\Lambda\right)^2}{\left(Da^{-1}+\delta^2\Lambda\right)^2}\right] \quad (31)$$

$$\frac{\partial R^d}{\partial \Lambda} = \frac{\left(a^2 M_3 + \pi^2\right)\delta^4}{a^2\left\{a^2(1+M_1)M_3+\pi^2\right\}}\left[2\delta^2 - \frac{\pi^2 Ta + \delta^2\left(Da^{-1}+\delta^2\Lambda\right)^2}{\left(Da^{-1}+\delta^2\Lambda\right)^2}\right]. \quad (32)$$

When $Ta = 0$, from the above equations, it is observed that $R^d$ is an increasing function of $Da^{-1}$ and $\Lambda$ indicating their effect is to stabilize the fluid motion against ferromagnetic convection. When $Ta \neq 0$, however, it is seen that the right-hand side of Eqs. 31 and 32 may be either negative or positive depending on the choices of parametric values. That is to say that an increase in the value of $Da^{-1}$ and $\Lambda$ might lead to an instability of a rotating ferrofluid saturated porous layer. This aspect has been analyzed in detail and the critical Rayleigh numbers ($R_c^d$) obtained with respect to the wave number ($a$) for various values of physical parameters are presented graphically in Figs. 2, 3, 4, 5.

Figures 2 and 3 show the destabilization due to $Da^{-1}$ on the steady onset for various values of $\Lambda$and for two values of $M_1(= 1, 1.2)$ and $M_3(= 1, 2)$, respectively, when the value of $Ta$ is fixed at $10^5$. From these figures, it is observed that the destabilization due to $Da^{-1}$ manifests itself as minimum in the $R_c^d$ versus $Da^{-1}$ curve. The range of $Da^{-1}$ up to which the system becomes destabilized decreases with an increase in the value of $\Lambda$. This may be due to a delicate balance between Coriolis and Darcy frictional forces, while elsewhere a strong "two-dimensionality" prevails, being provided at lower values of $Da^{-1}$ by Coriolis forces, and at higher values of $Da^{-1}$ by frictional forces. This phenomenon is similar to the one observed by Chandrasekhar (1961) in the study of thermal instability in a rotating electrically conducting fluid layer in the presence of vertical magnetic field, where it is observed that rotation/magnetic field destabilizes the system although their individual effect is to make the system more stable. Moreover, it is found that $R_c^d$ attains its minimum value with $Da^{-1}$, denoted by $(R_c^d)_{\min}$, at $Da^{-1} = Da_{\min}^{-1}$, where

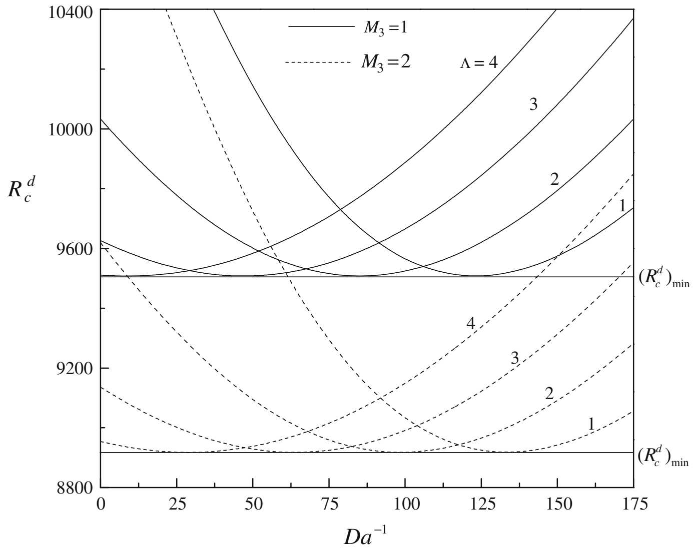


**Fig. 3** Variation of $(R_c^d)_{min}$ as a function of $Da^{-1}$ for different values of $\Lambda$ and $M_3$ when $M_1 = 1$ and $Ta = 10^5$

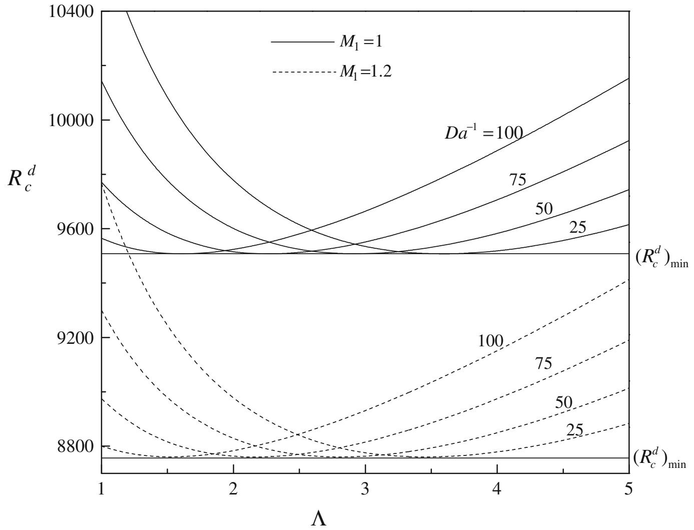


**Fig. 4** Variation of $(R_c^d)_{min}$ as a function of $\Lambda$ for different values of $Da^{-1}$ and $M_1$ when $M_3 = 1$ and $Ta = 10^5$

$$Da_{\min}^{-1} = \frac{\pi\sqrt{Ta}}{\sqrt{\pi^2 + a_c^2}} - \Lambda\left(\pi^2 + a_c^2\right). \tag{33}$$

It is evident that $Da_{\min}^{-1}$ decreases with an increase in the value of $\Lambda$ but increases with an increase in the value of $Ta$. Also, we note that $Da_{\min}^{-1}$ is not dependent on magnetic parameters explicitly. From the figures, it is also observed that increasing $M_1$ (see Fig. 2) and $M_3$

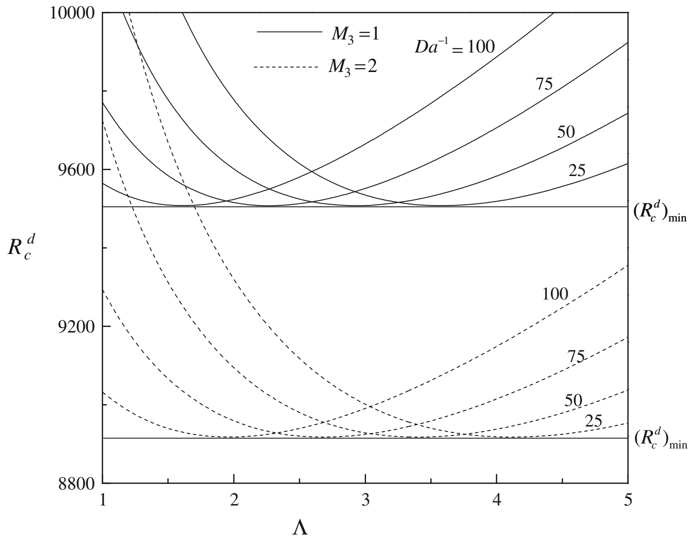


**Fig. 5** Variation of $(R_c^d)_{min}$ as a function of $\Lambda$ for different values of $Da^{-1}$ and $M_3$ when $M_1 = 1$ and $Ta = 10^5$

(see Fig. 3) is to decrease the critical Rayleigh number, and hence, their effect is to hasten the onset of ferromagnetic convection. Nevertheless, the destabilization due to increase in the nonlinearity of the fluid magnetization $M_3$ is only marginal. This may be attributed to the fact that a higher value of $M_3$ would arise either due to a larger pyromagnetic coefficient or larger temperature gradient. Both these factors are conducive for generating a larger gradient in the Kelvin body force field, possibly promoting the instability.

A similar type of behavior is observed by varying $\Lambda$ and the results are presented in Figs. 4 and 5. The dual role of viscosity ratio $\Lambda$ on the onset of stationary ferromagnetic convection in a rotating ferrofluid saturated porous layer is evident from these figures, where we note that $R_c^d$ passes through a minimum with an increase in the value of $\Lambda$. In this case, $R_c^d$ attains its minimum value with $\Lambda$ (i.e., $(R_c^d)_{min}$) at $\Lambda = \Lambda_{min}$ where

$$\Lambda_{min} = \frac{\pi\sqrt{Ta}}{(\pi^2 + a_c^2)\sqrt{(\pi^2 + a_c^2)}} - \frac{Da^{-1}}{(\pi^2 + a_c^2)}. \tag{34}$$

From the above equation, it is noted that $\Lambda_{min}$ decreases with an increase in the value of $Da^{-1}$, while it increases with increasing $Ta$, and it is also independent of magnetic parameters explicitly.

*Case (ii): Magnetic forces alone present*

In this case, only the magnetic forces contribute to the stability of the system and the magnetic Rayleigh number $N$ $(= RM_1)$turns out to be the eigenvalue. The variation of $N_c$ (i.e., critical value of $N$ with respect to the wave number) as a function of $Da^{-1}$ and $\Lambda$ is shown in Figs. 6 and 7, respectively for two values of $M_3 = 1, 2$. From the figures, it is observed that initially both $Da^{-1}$ and $\Lambda$ shows a partial destabilizing effect on ferromagnetic convection depending on the strength of rotation as observed in the previous case. Moreover, there is a coupling between the values of $Da^{-1}$ and $\Lambda$ in destabilizing the system with respect to $\Lambda$ and $Da^{-1}$. Also, the coupling between $\Lambda$ and $Da_{min}^{-1}$ or $Da^{-1}$ and $\Lambda_{min}$ is such that the

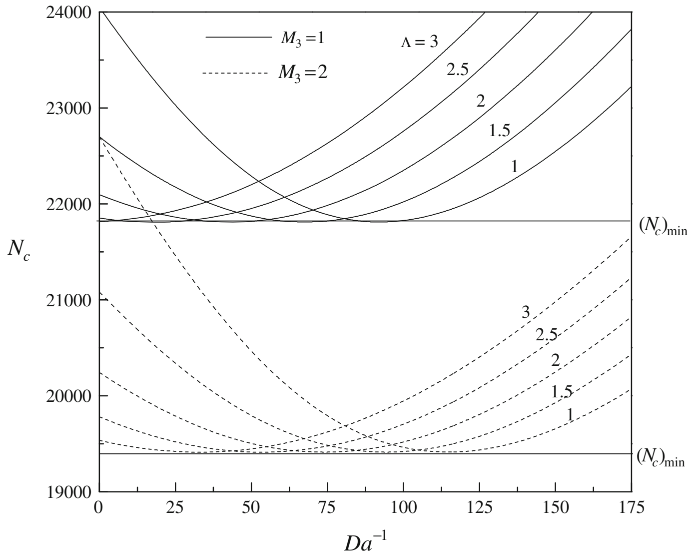


**Fig. 6** Variation of $(N_c)_{min}$ as a function of $Da^{-1}$ for different values of Λ and $M_3$ when $Ta = 10^5$

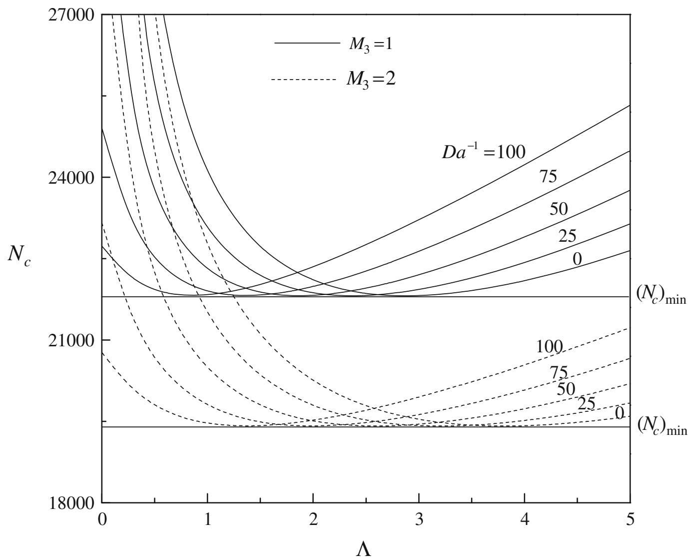


**Fig. 7** Variation of $(N_c)_{min}$ as a function of Λ for different values of $Da^{-1}$ and $M_3$ when $Ta = 10^5$

$(N_c)_{min}$, the minimum value of $N_c$ with respect to $Da^{-1}$ or Λ as the case may be, is the same for a fixed value of Taylor number.

### 4.2 Rigid–Rigid Boundaries

For the case considered, the resulting eigenvalue problem is solved numerically using the Galerkin technique. The convergence is achieved by using a sixth-order Galerkin expansion

**Table 1** Comparison of critical Rayleigh number $R_c^d$ and the corresponding wave number $a_c$ for different values of $Ta$ when $M_1 = 0 = Da^{-1}$ and $\Lambda = 1$

| Chandrasekhar (1961) | | | Present study | |
|---|---|---|---|---|
| $Ta$ | $R_c^d$ | $a_c$ | $R_c^d$ | $a_c$ |
| 10 | 1713.0 | 3.10 | 1712.67 | 3.12 |
| 100 | 1756.6 | 3.15 | 1756.35 | 3.16 |
| 500 | 1940.5 | 3.30 | 1940.20 | 3.32 |
| 1000 | 2151.7 | 3.50 | 2151.34 | 3.48 |
| 2000 | 2530.5 | 3.75 | 2530.13 | 3.75 |
| 5000 | 3469.2 | 4.25 | 3468.44 | 4.26 |
| 10000 | 4713.1 | 4.80 | 4712.06 | 4.79 |

**Table 2** Comparison of critical Rayleigh number $R_c^d$ and the corresponding wave number $a_c$ for different values of $\Lambda$ with $Da^{-1} = 1$ when $M_1 = 0 = Ta$

| $\Lambda$ | Guo and Kaloni (1995) | | Present analysis | |
|---|---|---|---|---|
| | $R_c^d$ | $a_c$ | $R_c^d$ | $a_c$ |
| 0.1 | 215.1 | 3.149 | 215.06 | 3.149 |
| 0.5 | 898.3 | 3.124 | 898.31 | 3.123 |
| 0.8 | 1410.6 | 3.121 | 1410.65 | 3.121 |
| 0.9 | 1581.4 | 3.121 | 1581.43 | 3.120 |
| 1.0 | 1752.2 | 3.114 | 1752.21 | 3.120 |
| 1.2 | 2093.7 | 3.119 | 2093.77 | 3.119 |

of the trial functions. To verify the accuracy of the numerical method employed, first, the test computations were carried out to compare the critical thermal Rayleigh number and the corresponding wave number under the limiting cases of classical rotating ordinary viscous fluid layer (i.e., $M_1 = 0 = Da^{-1}$ and $\Lambda = 1$) for different values of Taylor number $Ta$ and ordinary viscous fluid saturated porous layer (i.e., $M_1 = 0 = Ta$) for different values of $Da^{-1}$ and $\Lambda$. The critical stability parameters $R_{+}c^{d}$ and $a_c$ so computed for the above cases are compared with those of Chandrasekhar (1961) obtained using variational procedure with second-order approximation and Guo and Kaloni (1995) obtained using compound matrix method in Tables 1 and 2, respectively. The comparisons show excellent agreement on the numerical results.

To unfurl the salient characteristics of permeability and viscosity on the stability of the system, the variations of $R_c^d$ and $N_c$ are shown in Figs. 8, 9, 10, 11, 12 13 as a function of $Da^{-1}$ and $\Lambda$. We note that $R_c^d$ passes through a minimum with increasing $\Lambda$ for different values of $Da^{-1}$ shown in Figs. 8 and 9 for two values of $M_1$ (= 1 and 1.2 with $M_3 = 1$) and $M_3$(= 1 and 2 with $M_1 = 1$) when $Ta = 10^6$. For the above combination of values, Figs. 10 and 11 show the same behavior with increasing $Da^{-1}$ as well, for different values of $\Lambda$. Thus, increasing the viscosity of the fluid and decreasing the permeability of the porous medium show partial destabilizing effects on the system; a contrasting result is observed when compared to the nonrotating porous layer case. The above dual behavior of $Da^{-1}$ and $\Lambda$ on the stability of the system is found to be true even in the absence of buoyancy forces (i.e., when the magnetic forces alone are present) and the same is evident from Figs. 12 and 13. From these two figures, it is seen that $N_c$ passes through a minimum with increasing $Da^{-1}$ and $\Lambda$, and also note that the system is more stabilizing when the magnetic forces alone are present.

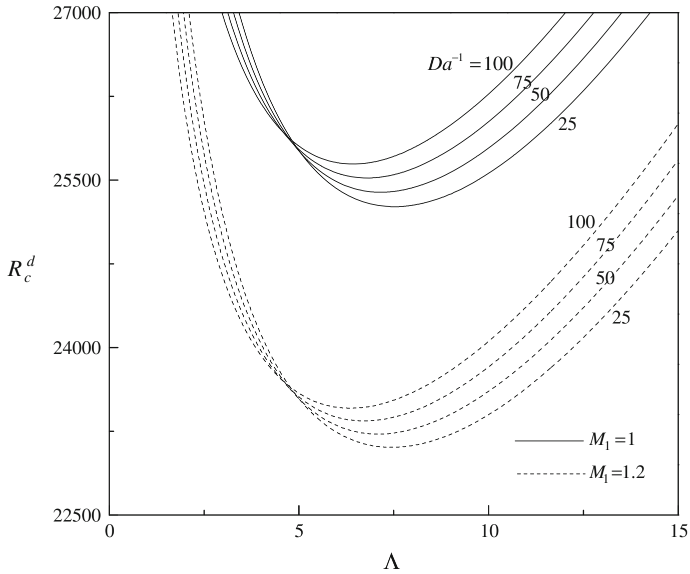


**Fig. 8** Variation of $R_c^d$ as a function of $\Lambda$ for different values of $Da^{-1}$ and $M_1$ when $M_3 = 1$ and $Ta = 10^6$ for rigid–rigid boundaries

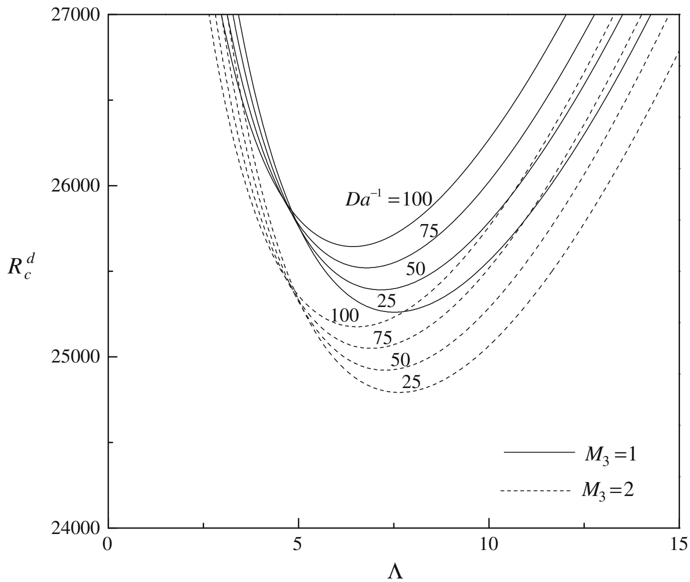


**Fig. 9** Variation of $R_c^d$ as a function of $\Lambda$ for different values of $Da^{-1}$ and $M_3$ when $M_1 = 1$ and $Ta = 10^6$ for rigid–rigid boundaries

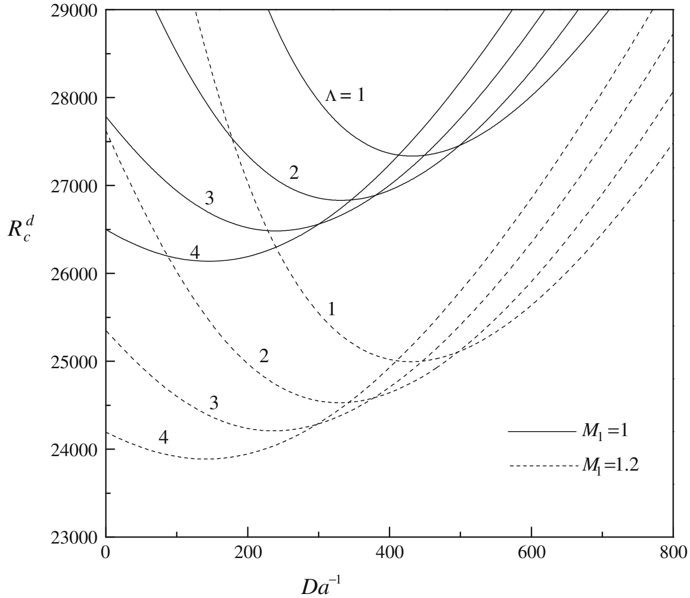


**Fig. 10** Variation of $R_c^d$ as a function of $Da^{-1}$ for different values of $\Lambda$ and $M_3$ when $M_1 = 1$ and $Ta = 10^6$ for rigid–rigid boundaries

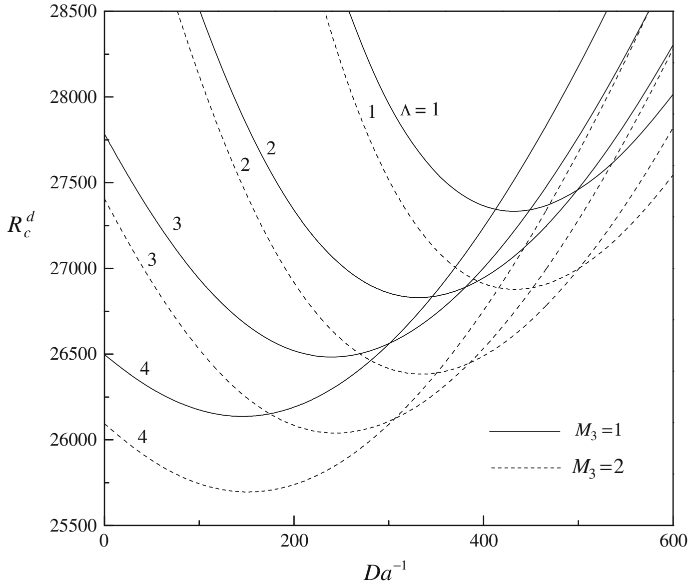


**Fig. 11** Variation of $R_c^d$ as a function of $Da^{-1}$ for different values of $\Lambda$ and $M_3$ when $M_1 = 1$ and $Ta = 10^6$ for rigid–rigid boundaries

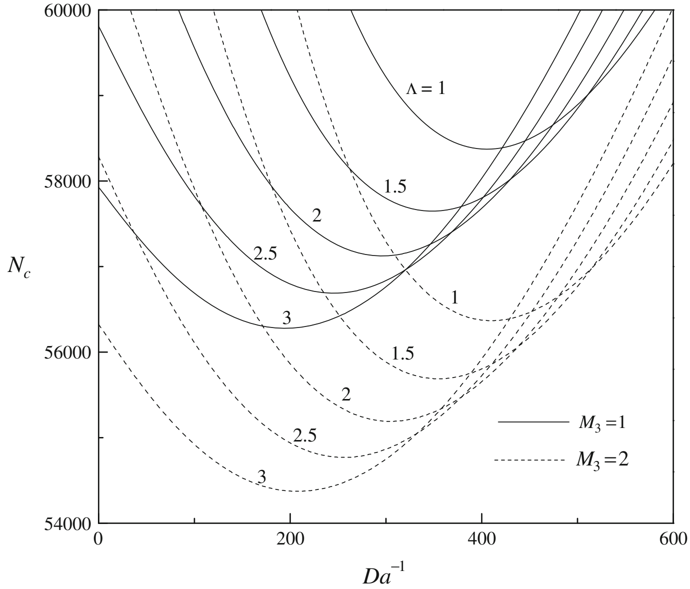


**Fig. 12** Variation of $N_c$ as a function of $Da^{-1}$ for different values of $\Lambda$ and $M_3$ when $Ta = 10^6$ for rigid–rigid boundaries

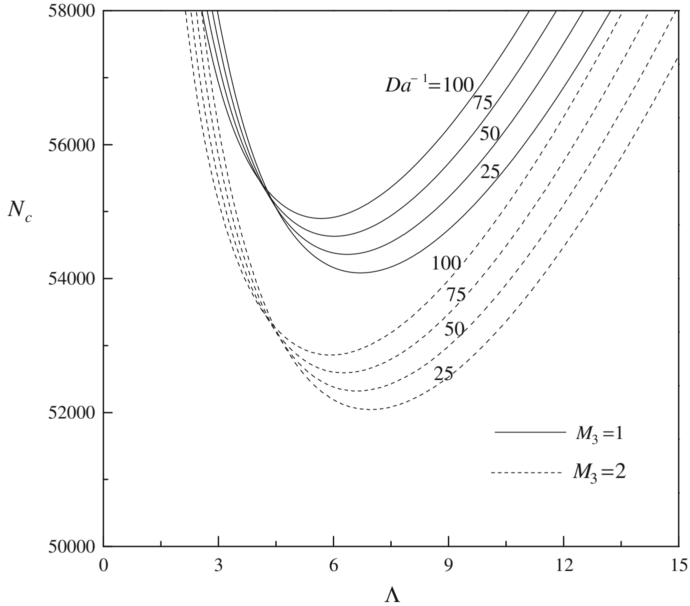


**Fig. 13** Variation of $N_c$ as a function of $\Lambda$ for different values of $Da^{-1}$ and $M_3$ when $Ta = 10^6$ for rigid–rigid boundaries

**Table 3** Values of $(R_c^d)_{min}$, $(N_c)_{min}$ and $Da_m^{-1}$ for various vales of $\Lambda$ when $M_1 = 1$, $M_3 = 1$ and $Ta = 10^6$

| Types of boundaries | $\Lambda$ | Simultaneous presence of buoyancy and magnetic forces | | Buoyancy forces alone present | | Magnetic forces alone present | |
|---|---|---|---|---|---|---|---|
| | | $(R_c^d)_{min}$ | $Da_{min}^{-1}$ | $(R_c^d)_{min}$ | $Da_{min}^{-1}$ | $(N_c)_{min}$ | $Da_{min}^{-1}$ |
| Free–free boundaries | 1.0 | 30065.00 | 471.59 | 51284.00 | 547.741 | 68966.00 | 397.866 |
| | 1.5 | 30065.00 | 452.587 | 51284.00 | 532.937 | 68966.00 | 373.192 |
| | 2.0 | 30065.00 | 433.584 | 51284.00 | 518.133 | 68966.00 | 348.518 |
| | 2.5 | 30065.00 | 414.581 | 51284.00 | 503.328 | 68966.00 | 323.844 |
| Rigid–rigid boundaries | 1.0 | 27329.50 | 431.597 | 51263.80 | 455.214 | 58316.80 | 403.632 |
| | 1.5 | 27037.80 | 379.004 | 50779.10 | 406.291 | 57596.90 | 346.590 |
| | 2.0 | 26825.90 | 331.268 | 50422.10 | 361.87 | 57078.50 | 294.696 |
| | 2.5 | 26647.90 | 285.488 | 50118.90 | 379.379 | 56646.30 | 244.645 |

**Table 4** Values of $(R_c^d)_{min}$, $(N_c)_{min}$ and $Da_m^{-1}$ for various vales of $\Lambda$ when $M_1 = 1$, $M_3 = 1$ and $Ta = 2 \times 10^6$

| Types of boundaries | $\Lambda$ | Simultaneous presence of buoyancy and magnetic forces | | Buoyancy forces alone present | | Magnetic forces alone present | |
|---|---|---|---|---|---|---|---|
| | | $(R_c^d)_{min}$ | $Da_{min}^{-1}$ | $(R_c^d)_{min}$ | $Da_{min}^{-1}$ | $(N_c)_{min}$ | $Da_{min}^{-1}$ |
| Free–free boundaries | 1.0 | 42518.40 | 682.671 | 72526.50 | 786.888 | 97532.60 | 583.108 |
| | 1.5 | 42518.40 | 663.668 | 72526.50 | 772.083 | 97532.60 | 558.433 |
| | 2.0 | 42518.40 | 644.666 | 72526.50 | 757.279 | 97532.60 | 533.759 |
| | 2.5 | 42518.40 | 625.663 | 72526.50 | 742.475 | 97532.60 | 509.085 |
| Rigid–rigid boundaries | 1.0 | 38985.90 | 659.405 | 73052.40 | 689.332 | 83305.30 | 623.956 |
| | 1.5 | 38590.40 | 600.798 | 72400.00 | 634.871 | 82325.70 | 560.444 |
| | 2.0 | 38297.50 | 548.122 | 71913.40 | 585.872 | 81602.70 | 503.316 |
| | 2.5 | 38067.70 | 499.291 | 71527.90 | 540.422 | 81038.90 | 450.289 |

### 4.3 Similarities and Differences Between the Results of Free–Free and Rigid–Rigid Boundaries

The results obtained for free–free and rigid–rigid boundaries when the buoyancy and magnetic forces are acting together or in isolation are viewed quantitatively in Tables 3, 4 and 5, 6 with the perspective of understanding the effect of boundaries on the onset of ferromagnetic convection in a rotating porous layer. The tables display the numerically computed values

**Table 5** Values of $(R_c^d)_{min}$, $(N_c)_{min}$ and $\Lambda_{min}$ for various vales of $Da^{-1}$when $M_1 = 1, M_3 = 1$ and $Ta = 10^6$

| Types of boundaries | $Da^{-1}$ | Simultaneous presence of buoyancy and magnetic forces | | Buoyancy forces alone present | | Magnetic forces alone present | |
|---|---|---|---|---|---|---|---|
| | | $(R_c^d)_{min}$ | $\Lambda_{min}$ | $(R_c^d)_{min}$ | $\Lambda_{min}$ | $(N_c)_{min}$ | $\Lambda_{min}$ |
| Free–free boundaries | 10 | 30065.00 | 13.145 | 51284.00 | 19.162 | 68966.00 | 8.860 |
| | 20 | 30065.00 | 12.882 | 51284.00 | 18.824 | 68966.00 | 8.657 |
| | 30 | 30065.00 | 12.619 | 51284.00 | 18.486 | 68966.00 | 8.455 |
| | 50 | 30065.00 | 12.093 | 51284.00 | 17.811 | 68966.00 | 8.049 |
| | 100 | 30065.00 | 10.777 | 51284.00 | 16.122 | 68966.00 | 7.036 |
| Rigid–rigid boundaries | 10 | 25177.10 | 7.727 | 47068.50 | 8.539 | 53866.50 | 6.839 |
| | 20 | 25230.10 | 7.581 | 47169.10 | 8.387 | 53978.30 | 6.703 |
| | 30 | 25282.70 | 7.436 | 47269.00 | 8.235 | 54089.30 | 6.566 |
| | 50 | 25386.70 | 7.144 | 47466.60 | 7.929 | 54398.40 | 6.293 |
| | 100 | 25639.00 | 6.410 | 47947.30 | 7.160 | 54839.80 | 5.609 |

**Table 6** Values of $(R_c^d)_{min}$, $(N_c)_{min}$ and $\Lambda_{min}$ for various vales of $Da^{-1}$ when $M_1 = 1, M_3 = 1$ and $Ta = 2 \times 10^6$

| Types of boundaries | $Da^{-1}$ | Simultaneous presence of buoyancy and magnetic forces | | Buoyancy forces alone present | | Magnetic forces alone present | |
|---|---|---|---|---|---|---|---|
| | | $(R_c^d)_{min}$ | $\Lambda_{min}$ | $(R_c^d)_{min}$ | $\Lambda_{min}$ | $(N_c)_{min}$ | $\Lambda_{min}$ |
| Free–free boundaries | 10 | 42518.40 | 18.699 | 72526.50 | 27.238 | 97532.60 | 12.614 |
| | 20 | 42518.40 | 18.436 | 72526.50 | 26.901 | 97532.60 | 12.411 |
| | 30 | 42518.40 | 18.173 | 72526.50 | 26.563 | 97532.60 | 12.208 |
| | 50 | 42518.40 | 17.647 | 72526.50 | 25.887 | 97532.60 | 11.803 |
| | 100 | 42518.40 | 16.331 | 72526.50 | 24.199 | 97532.60 | 10.790 |
| Rigid–rigid boundaries | 10 | 35583.70 | 10.987 | 66523.10 | 12.139 | 76132.10 | 9.728 |
| | 20 | 35636.90 | 10.842 | 66624.00 | 11.987 | 76244.40 | 9.592 |
| | 30 | 35689.80 | 10.697 | 66724.40 | 11.835 | 76356.00 | 9.455 |
| | 50 | 35794.80 | 10.405 | 66923.70 | 11.530 | 76577.30 | 9.183 |
| | 100 | 36051.90 | 9.674 | 67412.80 | 10.764 | 77119.10 | 8.500 |

of $(R_c^d)_{min}$ and $(N_c)_{min}$ along with $Da_{min}^{-1}$ for different values of $\Lambda$ as well as $\Lambda_{min}$ for different values of $Da^{-1}$ for two values of Taylor number $Ta = 10^6$ and $2 \times 10^6$ when $M_1 = 1, M_3 = 1$. The results obtained for realistic rigid–rigid boundaries mimic qualitatively those observed in the case of free–free boundaries. We note that the principal effect of increasing the Taylor number is to make the system more stable. In contrast to the results noticed at lower Taylor number domain, the stress-free boundaries is found to be more stable than that of rigid boundaries at higher values of Taylor number because the stability of the stress-free case is increased more quickly than the rigid case. In the case of free–free boundaries, interestingly it is noted that there is a tight coupling between the values of $\Lambda$ and $Da^{-1}_{\min}$ as well as $Da^{-1}$ and $\Lambda_{\min}$ in destabilizing the system such that $(R^{\mathrm{d}}_{\mathrm{c}})_{\min}$ and $(N_{\mathrm{c}})_{\min}$ remain unaltered for a fixed value of $Ta$. However, **i**n the case of rigid–rigid boundaries, there is no such coupling is noticed between the values of $Da^{-1}_{\min}$ and $\Lambda$ as well as $\Lambda_{\min}$ and $Da^{-1}$ such that $(R^{\mathrm{d}}_{\mathrm{c}})_{\min}$ as well as $(N_{\mathrm{c}})_{\min}$ remain the same. Further, increasing $\Lambda(Da^{-1})$ is to decrease $Da^{-1}_{\min}(\Lambda_{\min})$, while increase in $Ta$ is to increase these minimum values. Nonetheless, the values of $Da^{-1}_{\min}$ and $\Lambda_{\min}$ for a fixed value of Taylor number are not the same when the buoyancy and magnetic forces are acting together or in isolation. This is because the critical wave numbers are different for these three cases considered. Irrespective of the boundaries, it is noted that

$$\left[(N_{\mathrm{c}})_{\min}\right]_{\text{magnetic}} > \left[\left(R^{\mathrm{d}}_{\mathrm{c}}\right)_{\min}\right]_{\text{buoyancy}} > \left[\left(R^{\mathrm{d}}_{\mathrm{c}}\right)_{\min}\right]_{\text{buoyancy}+\text{magnetic}}$$

for a fixed value of Taylor number. This indicates that the onset of convection is delayed the most when the magnetic forces alone are present. In other words, the combined effect of buoyancy and magnetic forces is to reinforce together and to hasten the onset of ferromagnetic convection in a rotating porous layer. In addition,

$$\left[Da^{-1}_{\min}\right]_{\text{buoyancy}} > \left[Da^{-1}_{\min}\right]_{\text{buoyancy}+\text{magnetic}} > \left[Da^{-1}_{\min}\right]_{\text{magnetic}}$$

for all values of $\Lambda$ considered, and

$$[\Lambda_{\min}]_{\text{buoyancy}} > [\Lambda_{\min}]_{\text{buoyancy}+\text{magnetic}} > [\Lambda_{\min}]_{\text{magnetic}}$$

for all values of $Da^{-1}$ considered. It is also observed that (not displayed here) the critical wave number is higher when the onset of convection is due to magnetic forces alone as compared to instability due to the simultaneous presence of buoyancy and magnetic forces.

## 5 Conclusions

The linear stability theory is used to investigate the criterion for the onset of ferromagnetic convection in a rotating sparsely packed ferrofluid saturated porous layer heated from below in the presence of a uniform vertical magnetic field. Some interesting observations, which were overlooked by the previous investigators, have been unveiled in the case of stress-free boundaries, and the problem has been investigated newly for realistic rigid boundary conditions. The resulting eigenvalue problem is solved exactly for stress-free boundaries and numerically using the Galerkin method in the case of rigid boundaries. Contrary to their usual stabilizing effect in the absence of rotation, it has been shown that increasing $Da^{-1}$ and $\Lambda$ exhibits destabilizing effect on the onset of stationary convection depending on the strength of rotation. This result is true whether the buoyancy and magnetic forces are acting together or in isolation and also irrespective of the velocity boundary conditions. The double minimum of thermal Rayleigh number $R^{\mathrm{d}}$ or magnetic Rayleigh number $N$ with respect to $a$ and $Da^{-1}$ or $a$ and $\Lambda$, denoted by $\left(R^{\mathrm{d}}_{\mathrm{c}}\right)_{\min}$ or $(N_{\mathrm{c}})_{\min}$, is computed numerically to assess the behavior of $Da^{-1}$ and $\Lambda$ on the stability characteristics of the system. In the case of free boundaries, it is identified that there is a coupling between $Da^{-1}_{\min}$ and $\Lambda$ or $\Lambda_{\min}$ and $Da^{-1}$ such that $\left(R^{\mathrm{d}}_{\mathrm{c}}\right)_{\min}$ and also $(N_{\mathrm{c}})_{\min}$ remain the same for a fixed value of Taylor number.

However, such a coupling is not noticed in the case of rigid boundaries and observed that $\left(R_{\mathrm{c}}^{\mathrm{d}}\right)_{\min}$and $(N_{\mathrm{c}})_{\min}$ decrease with increasing $\Lambda$and also decreasing $Da^{-1}$. In general, it is seen that the system is more stabilizing when the magnetic forces alone are present and the effect of increasing Taylor number is to delay the onset of ferromagnetic convection in a porous medium. The stress-free boundaries are found to be less stable than rigid boundaries only at small Taylor number domain. However, free boundaries offer more stability than rigid boundaries at high Taylor number domain. Increasing the value of magnetic number $M_1$and the nonlinearity of fluid magnetization parameter $M_3$ is to hasten the onset of ferromagnetic convection in a rotating porous layer. The effect of increasing $\Lambda$, $M_3$ and $Da^{-1}$ as well as decreasing $M_1$ and $Ta$ is to decrease the dimension of convection cells. The critical wave number is higher when the onset of ferromagnetic convection in a rotating porous layer is only due to magnetic forces as compared to the simultaneous presence of buoyancy and magnetic forces.

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