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# Effect of temperature-dependent viscosity on the onset of Bénard–Marangoni ferroconvection in a ferrofluid saturated porous layer

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**Abstract** The criterion for the onset of Bénard–Marangoni ferroconvection in an initially quiescent magnetized ferrofluid saturated horizontal Brinkman porous layer is investigated in the presence of a uniform vertical magnetic field. The viscosity is considered to be varying exponentially with temperature. The lower rigid boundary and the upper free boundary at which the surface tension effects are accounted for are assumed to be perfectly insulated to temperature perturbations. The eigenvalue problem is solved numerically using the Galerkin technique and analytically by regular perturbation technique with wave number $a$ as a perturbation parameter. It is observed that the analytical and numerical results are very well comparable. The characteristics of stability of the system are strongly dependent on the viscosity parameter $B$. The effect of $B$ on the onset of Bénard–Marangoni ferroconvection in a porous layer is dual in nature depending on the choices of the physical parameters, and a sublayer starts to form at higher values of $B$. The nonlinearity of fluid magnetization $M_3$ is found to have no influence on the onset of ferroconvection, whereas an increase in the value of the magnetic number $M_1$ and the Darcy number $Da$ is to advance the onset of Bénard–Marangoni ferroconvection in a porous layer.

## 1 Introduction

Ferrofluids are synthesized in the laboratory, and the idiosyncrasy of these fluids is the combination of normal liquid behavior with a magnetic control of their flow and properties. Since advective transport in a ferrofluid can be readily controlled by using an external magnetic field, these fluids have promising applications in heat-transfer-related problems. The growing importance of microscale heat exchangers in micro-electromechanical system (MEMS) and nano-electromechanical system (NEMS) devices has initiated a great deal of research that

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addresses heat transfer in miniaturized configurations. Using ferrofluids in these applications and manipulating the flow by external magnetic fields can be a viable alternative to enhance convection in these devices. In addition, many terrestrial electronic cooling devices rely on free convection. However, the corresponding space (or hypo-gravity, i.e., less than normal gravity) applications require innovative methods to sustain convection. Ferromagnetic convection is a substitute for gravity-induced free convection. Realizing the importance of ferrofluids in many heat transfer applications, thermal convection in a layer of ferrofluid has been studied extensively and is well documented in the literature [1–6]. Its counterpart in a layer of ferrofluid saturated porous medium has also received due attention of researchers in the recent past [7–11].

On the other hand, if the surface of a ferrofluid layer is free and open to the atmosphere, then convection can also be induced by temperature-dependent surface tension forces at the free surface known as Marangoni ferroconvection. In view of the fact that heat transfer is significantly enhanced due to convection, Marangoni ferroconvection offers new possibilities for application in cooling of motors in space, loudspeakers, transmission lines and other equipments in micro-gravity environment where a magnetic field is already present. In most of the cases, the combined effect of buoyancy and surface tension forces on convective instability in a ferrofluid layer also becomes important. Realizing these aspects, a limited number of studies have addressed the effect of surface tension forces on ferroconvection in a horizontal ferrofluid layer. Linear and nonlinear stability of combined buoyancy–surface tension effects in a ferrofluid layer heated from below has been analyzed by Qin and Kaloni [12]. Odenbach [13] has carried out experiments to investigate the onset and the flow profile of thermomagnetic convection in a cylindrical fluid layer under microgravity conditions. The linear stability analysis of a layer of magnetic fluid with deformable free surface which is heated uniformly from below and subject to a vertical magnetic field has been studied by Weilepp and Brand [14], considering the temperature dependence of the surface tension and buoyancy. Odenbach [15] has shown that microgravity experiments can provide unique experimental conditions allowing the investigation of magnetic effects in ferrofluids covered by gravitational action in normal terrestrial examinations. The coupling between Marangoni and Rosensweig instabilities by considering two semi-infinite incompressible and immiscible viscous fluids of infinite lateral extent in which one of them is ferromagnetic and the other is a usual Newtonian liquid has been addressed by Weilepp and Brand [16]. The effect of different forms of basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces has been discussed by Shivakumara et al. [17], while Hennenberg et al. [18] have considered Rayleigh–Bénard–Marangoni instability in a ferrofluid layer in the presence of a weak vertical magnetic field normal to the boundaries. Shivakumara and Nanjundappa [19] have analyzed the onset of Marangoni ferroconvection with different initial temperature gradients with the object of understanding control of convection. Nanjundappa et al. [20] have investigated theoretically the effect of magnetic-field-dependent viscosity on the onset of Bénard–Marangoni ferroconvection in a horizontal layer of ferrofluid. Nanjundappa et al. [21] have studied the effect of internal heat generation on the onset of Bénard–Marangoni convection in a horizontal ferrofluid layer heated from below in the presence of a uniform vertical magnetic field. Recently, Nanjundappa et al. [22] have investigated the onset of penetrative Benard–Marangoni convection in a horizontal ferromagnetic fluid layer in the presence of a uniform vertical magnetic field via an internal heating model.

It has also been realized the possibility and importance of Marangoni convection in porous media. Several studies have been undertaken in the past to understand such an instability problem in an ordinary viscous fluid saturating a porous medium. Patberg et al. [23] have studied Marangoni effects in packed distillation columns. It is observed that large differences in refreshing of the liquid on wetted particles can be produced by the Marangoni effect. White and Perroux [24] have examined experimentally that bulk liquid convection can be produced in porous media by macroscopic gradients in surface tension. In their seminal paper, Hennenberg et al. [25] have discussed in detail the Marangoni convection in a liquid-saturated porous matrix. Shivakumara et al. [26] have investigated coupled Darcy–Bénard–Marangoni convection in a liquid-saturated porous layer.

The earlier studies are mainly concerned with either Bénard–Marangoni convection in an ordinary viscous fluid saturated porous layer or ferroconvection in a magnetized ferrofluid saturated porous medium. Nonetheless, certain observed convective motions in many heat transfer applications in ferrofluid saturated porous media cannot be attributed to the buoyancy mechanism alone. Therefore, probing convective instability problems in a sparsely packed porous medium saturated with magnetized ferrofluids involving both buoyancy and surface tension forces will be of fundamental and practical importance. Moreover, the majority of ferrofluids are either water based or oil based, and naturally, the viscosity of these fluids varies with temperature. The viscosity of water is far more sensitive to temperature variations, and oils are known to have viscosity decreasing exponentially with temperature rather than linearly. Several investigators have considered the variation in viscosity with temperature in analyzing thermal convective instability in a fluid layer [27–29] as well as in a layer of fluid saturated porous medium [30–32], but the studies are limited to ordinary viscous fluids.

To the best of our knowledge, due attention has not been given to investigate the variation in viscosity due to temperature on ferroconvection despite its relevance and importance in many heat transfer applications. Stiles and Kagan [33] have investigated thermal convective instability in a ferrofluid layer heated from below by considering a linear variation in viscosity with temperature. Shivakumara et al. [34] have investigated the onset of thermogravitational convection in a horizontal ferrofluid layer with viscosity depending exponentially on temperature. Recently, Najundappa et al. [35] have studied the effect of temperature-dependent viscosity on the onset of Marangoni–Bénard ferroconvection under microgravity conditions in a horizontal ferrofluid layer in the presence of a uniform vertical magnetic field. However, variation in viscosity attributable to temperature changes on coupled buoyancy and surface tension driven convective instability in a ferrofluid saturated porous medium has not received any attention in the literature.

The intent of the present study is to analyze the influence of viscosity varying exponentially with temperature on the onset of coupled Bénard–Marangoni convection in a ferrofluid saturated Brinkman porous layer in the presence of a uniform vertical magnetic field. In investigating the problem, the lower rigid and upper free boundary at which the temperature-dependent surface tension forces are accounted for are considered to be perfectly insulated to temperature perturbations. The resulting eigenvalue problem is solved numerically using the Galerkin technique. Besides, an analytical formula is obtained for the critical Rayleigh/Marangoni number by regular perturbation technique with wave number $a$ as a perturbation parameter. The results are presented graphically for various values of physical parameters in the presence of buoyancy and/or surface tension forces.

## 2 Mathematical formulation

We consider a horizontal layer of Brinkman porous medium of thickness $d$ saturated with an electrically non-conducting Boussinesq magnetized ferrofluid with an imposed spatially uniform magnetic field $H_o$ in the vertical direction as shown in Fig. 1. The lower and upper boundaries are maintained at constant but different temperatures $T_l$ and $T_u(<T_l)$, respectively. A Cartesian coordinate system $(x, y, z)$ is used with the origin at the bottom, and $z$-axis is directed vertically upward. Gravity acts in the negative $z$-direction, $\vec{g} = -g\hat{k}$, where $\hat{k}$ is the unit vector in the $z$-direction. For most of the fluids, the capillary number is very small. Several investigators in the past have followed this assumption in the study of Marangoni convection, and the free surface is assumed to be non-deformable (zero capillary number). At the upper free surface, the surface tension $\sigma$ is assumed to vary linearly with temperature in the form

$$\sigma = \sigma_0 - \sigma_T (T - T_0) \tag{1}$$

where $\sigma_0$ is the unperturbed value and $-\sigma_T$ is the rate of change of surface tension with temperature $T$, whereas the viscosity $\eta$ of the ferrofluid is assumed to vary exponentially with temperature in the form

$$\eta = \eta_0 \exp\left[-\gamma \left(T - T_r\right)\right] \tag{2}$$

where $\eta_0$ is the reference value at the reference temperature $T_r$ and $\gamma$ is a positive constant.

The governing mathematical equations used are as follows:

The continuity equation is

$$\nabla \cdot \vec{q} = 0. \tag{3}$$

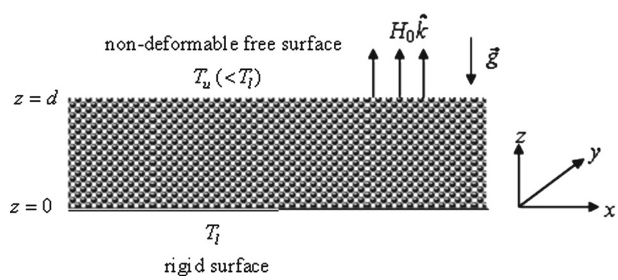



**Fig. 1** Physical configuration

The momentum equation for an incompressible fluid with variable viscosity is

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}\cdot\nabla)\vec{q}\right]=-\nabla p+\rho\vec{g}+\nabla\cdot\left[\frac{\eta}{\varepsilon}\left(\nabla\vec{q}+\nabla\vec{q}^T\right)\right]+\mu_0\left(\vec{M}\cdot\nabla\right)\vec{H}-\frac{\eta}{k}\vec{q}. \quad (4)$$

The temperature equation for an incompressible fluid which obeys the modified Fourier's law as given by Finlayson [2] is

$$\varepsilon\left[\rho_0 C_{V,H}-\mu_0\vec{H}\cdot\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}+(1-\varepsilon)(\rho_0 C)_S\frac{\partial T}{\partial t}+\mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt}=k_1\nabla^2 T. \quad (5)$$

The density equation of state for a Boussinesq magnetic fluid is

$$\rho=\rho_0\ \left[1-\alpha_t\left(T-T_a\right)\right]. \quad (6)$$

Maxwell's equations for non-conducting fluids are

$$\nabla\cdot\vec{B}=0,\nabla\times\vec{H}=0,\vec{B}=\mu_0\left(\vec{M}+\vec{H}\right). \quad (7.1\text{–}3)$$

Here, $\vec{q}=(u,v,w)$ is the seepage velocity vector, $p$ is the pressure, $\rho$ is the fluid density, $\vec{B}$ is the magnetic induction, $\vec{M}$ is the magnetization, $\vec{H}$ is the magnetic-field intensity, $\vec{B}$ is the magnetic flux density, $\mu_0$ is the magnetic permeability of vacuum, $k$ is the permeability of the porous medium, $\varepsilon$ is the porosity of the porous medium, $k_t$ is the thermal conductivity, $C_{V,H}$ is the specific heat at constant volume and magnetic field, $\rho_0$ is the reference density, $\alpha_t$ is the thermal expansion coefficient, $T_a=(T_l+T_u)/2$ is the average temperature, $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$ is the Laplacian operator, and the subscript $s$ represents the solid. In view of Eq. (7.2), $\vec{H}$ can be expressed as

$$\vec{H}=\nabla\varphi \quad (8)$$

where $\varphi$ is the magnetic potential.

Since the magnetization depends on the magnitude of magnetic field and temperature (see Finlayson [2]), we have

$$\vec{M}=\frac{\vec{H}}{H}\left[M_0\ +\ \chi\left(H-H_0\right)\ -\ K\left(T-T_a\right)\right] \quad (9)$$

where $M_0=M\left(H_0,T_a\right)$ is the saturation magnetization, $\chi=(\partial M/\partial H)_{H_0,T_a}$ is the magnetic susceptibility, $K=-(\partial M/\partial T)_{H_0,T_a}$ is the pyromagnetic co-efficient, $H=\left|\vec{H}\right|$ and $M=\left|\vec{M}\right|$.

It is clear that there exists the following solution for the quiescent basic state:

$$\vec{q}_b=0, \quad (10.1)$$

$$p_b(z)=p_0-\rho_0 gz-\frac{1}{2}\rho_0\alpha_t g\beta z\left(z-d\right)-\frac{\mu_0 M_0 K\beta}{1+\chi}z-\frac{\mu_0 K^2\beta^2}{2\left(1+\chi\right)^2}z\left(z-d\right), \quad (10.2)$$

$$T_b(z)=T_a-\beta\left(z-\frac{d}{2}\right), \quad (10.3)$$

$$\vec{H}_b(z)=\left[H_0-\frac{K\beta}{1+\chi}\left(z-\frac{d}{2}\right)\right]\hat{k}, \quad (10.4)$$

$$\vec{M}_b(z)=\left[M_0+\frac{K\beta}{1+\chi}\left(z-\frac{d}{2}\right)\right]\hat{k} \quad (10.5)$$

where $\beta=(T_l-T_u)/d$ is the temperature gradient, and the subscript $b$ denotes the basic state. To investigate the conditions under which the quiescent solution is stable against small disturbances, we consider a perturbed state such that

$$\vec{q}=\vec{q}',\ p=p_b(z)+p',\ \eta=\eta_b(z)+\eta',\ T=T_b(z)+T',$$
$$\vec{H}=\vec{H}_b(z)+\vec{H}',\ \vec{M}=\vec{M}_b(z)+\vec{M}' \quad (11)$$

where $\vec{q}'$, $p'$, $T'$, $\vec{H}'$ and $\vec{M}'$ are perturbed variables and are assumed to be small. Then, we note that

$$\eta = \eta_0 \exp\left[\gamma\beta(z - d/2) + \gamma(T_r - T_a) - \gamma T'\right]. \tag{12}$$

Substituting Eq. (11) into Eq. (7.3) and using Eqs. (7.1, 2), we obtain (after dropping the primes)

$$H_x + M_x = (1 + M_0/H_0)\, H_x,\ H_y + M_y = (1 + M_0/H_0)\, H_y,\ H_z + M_z = (1 + \chi)\ H_z - KT. \tag{13}$$

Again substituting Eq. (11) into momentum Eq. (4), linearizing, eliminating the pressure term by operating curl twice and using Eq. (13), the $z$-component of the resulting equation can be obtained as (after dropping the primes)

$$\begin{aligned} \frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 w\right) &= \frac{\eta(z)}{\varepsilon}\nabla^4 w + \frac{2}{\varepsilon}\frac{\partial\eta(z)}{\partial z}\nabla^2\left(\frac{\partial w}{\partial z}\right) + \frac{1}{\varepsilon}\frac{\partial^2\eta(z)}{\partial z^2}\left(\nabla^2 w - 2\nabla_h^2 w\right) + \rho_0\alpha_t g\nabla_h^2 T \\ &-\mu_0 K\beta\frac{\partial}{\partial z}\left(\nabla_h^2\varphi\right) - \frac{\eta(z)}{k}\nabla^2 w - \frac{1}{k}\frac{\partial w}{\partial z}\frac{\partial\eta(z)}{\partial z} + \frac{\mu_0 K^2\beta}{1+\chi}\nabla_h^2 T \end{aligned} \tag{14}$$

where $\eta(z) = \eta_0 \exp\left[\gamma\beta\,(z - d/2) + \gamma\,(T_r - T_a)\right]$ and $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator. Equation (5), after using Eq. (11) and linearizing, takes the form (after dropping the primes)

$$(\rho_0 C)_1\frac{\partial T}{\partial t} - \mu_0 T_0 K\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial z}\right) = k_1\nabla^2 T + \left[(\rho_0 C)_2 - \frac{\mu_0 T_0 K^2}{1+\chi}\right] w\beta \tag{15}$$

where $(\rho_0 C)_1 = \varepsilon\rho_0 C_{V,H} + \varepsilon\mu_0 H_0 K + (1-\varepsilon)(\rho_0 C)_s$ and $(\rho_0 C)_2 = \varepsilon\rho_0 C_{V,H} + \varepsilon\mu_0 H_0 K$.

Equations (7.1, 2), after substituting Eq. (11) and using Eq. (13), may be written as (after dropping the primes)

$$\left(1 + \frac{M_0}{H_0}\right)\nabla_h^2\varphi + (1+\chi)\frac{\partial^2\varphi}{\partial z^2} - K\frac{\partial T}{\partial z} = 0. \tag{16}$$

As propounded by Vidal and Acrivos [36], it is not possible to prove the principle of exchange of stability analytically for the Marangoni convection problem due to the peculiar nature of one of the boundary conditions at the free surface. However, through numerical calculations, they have shown that the marginal state or neutral state for the Marangoni convection is indeed stationary. In discussing a similar problem for magnetic fluids, Weilepp and Brand [16] have demonstrated through their numerical calculations that there is no oscillatory instability and convection sets in as stationary convection. Stengel et al. [27] have opined that oscillatory convection is not a preferred mode of instability even in the case of viscosity varying with temperature. Based on these observations and also noting that there is no mechanism to set up oscillatory motion, the principle of exchange of stability is assumed to be valid for the problem considered. Accordingly, the normal mode expansion of the dependent variables is taken in the form

$$\{w, T, \varphi\} = \{W(z), \Theta(z), \Phi(z)\}\exp\left[i(\ell x + my)\right] \tag{17}$$

where $\ell$ and $m$ are wave numbers in the $x$ and $y$ directions, respectively.

On substituting Eq. (17) into Eqs. (14)–(16) and non-dimensionalizing the variables by setting

$$\left(x^*, y^*, z^*\right) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right),\ W^* = \frac{d}{\varepsilon\nu A}W,\ \Theta^* = \frac{\kappa}{\beta\nu d}\Theta,\ \Phi^* = \frac{(1+\chi)\,\kappa}{\mathrm{K}\beta\nu d^2}\Phi,\ \bar{f}(z) = \frac{\eta(z)}{\eta_0} \tag{18}$$

where $\nu = \eta_0/\rho_0$ is the kinematic viscosity, $\kappa = k_1/(\rho_0 C)_2$ is the effective thermal diffusivity and $A = (\rho_0 C)_1/(\rho_0 C)_2$, we obtain (after dropping the asterisks)

$$\begin{aligned} \bar{f}\left(D^2 - a^2\right)^2 W + 2D\bar{f}\left(D^2 - a^2\right)DW + D^2\bar{f}\left(D^2 + a^2\right)W - Da^{-1}\bar{f}\left(D^2 - a^2\right)W \\ -Da^{-1}D\bar{f}DW = R_t a^2\Theta - R_m a^2\left(D\Phi - \Theta\right), \end{aligned} \tag{19}$$

$$(D^2 - a^2)\,\Theta = -(1 - M_2 A)W, \tag{20}$$

$$(D^2 - a^2 M_3)\,\Phi - D\Theta = 0. \tag{21}$$

Here, $D = \mathrm{d}/\mathrm{d}z$ is the differential operator, $a = \sqrt{\ell^2 + m^2}$ is the overall horizontal wave number, $W$ is the amplitude of the vertical component of velocity, $\Theta$ is the amplitude of temperature, $\Phi$ is the amplitude of magnetic potential, $R_t = \alpha_t g \beta d^4/\nu\kappa A$ is the thermal Raleigh number, $R_m = R_t M_1 = \mu_0 K^2 \beta^2 d^4/(1 + \chi)\,\mu\kappa A$ is the magnetic Rayleigh number, $M_1 = \mu_0 K^2 \beta/(1+\chi)\,\alpha_t \rho_0 g$ is the magnetic number, $M_2 = \mu_0 T_a K^2/(1+\chi)\,\rho_0 C$ is the magnetic parameter, $M_3 = (1 + M_0/H_0)/(1+\chi)$ is the measure of nonlinearity of fluid magnetization parameter and $Da = k/\varepsilon d^2$ is the Darcy number. The typical value of $M_2$ for magnetic fluids with different carrier liquids turns out to be of the order of $10^{-6}$, and hence, its effect is neglected as compared to unity, and $\bar{f}(z)$ is given by

$$\bar{f}(z) = \exp\left[B\,(z - 1/2) + (T_r - T_a)\,/\beta d\right] \tag{22}$$

where $B = \gamma\beta d$ is the dimensionless viscosity parameter. If $T_r = T_a$, then

$$\bar{f}(z) = \exp\left[B\,(z - 1/2)\right]. \tag{23}$$

The lower boundary is rigid-ferromagnetic while the upper free boundary at which the surface tension effects are accounted for is taken to be non-deformable and flat. In addition, both the boundaries are assumed to be perfectly insulated to temperature perturbations. The boundary conditions are then given by

$$W = DW = D\Theta = \Phi = 0 \qquad \text{at } z = 0, \tag{24}$$

$$W = \bar{f} D^2 W + Ma\, a^2 \Theta = D\Theta = D\Phi = 0 \quad \text{at } z = 1 \tag{25}$$

where $Ma = \sigma_T \Delta T d/\mu\kappa$ is the Marangoni number.

## 3 Method of solution

Equations (19)–(21) together with the boundary conditions constitute an eigenvalue problem with $R_t$ or $Ma$ as an eigenvalue. The eigenvalue problem is solved both numerically using the Galerkin method as well as analytically using a regular perturbation technique with the wave number as a perturbation parameter.

### 3.1 Solution by the Galerkin method

In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly, $W$, $\Theta$ and $\Phi$ are written as

$$W = \sum_{i=1}^{n} A_i\, W_i(z),\quad \Theta(z) = \sum_{i=1}^{n} C_i\, \Theta_i(z),\, \Phi(z) = \sum_{i=1}^{n} D_i\, \Phi_i(z) \tag{26}$$

where $A_i$, $C_i$ and $D_i$ are unknown constants to be determined. The base functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ are generally chosen such that they satisfy the boundary conditions but not the differential equations. We select the trial functions as

$$W_i = (z^4 - 5z^3/2 + 3z^2/2)T^*_{i-1},\, \Theta_i = z(1 - z/2)T^*_{i-1},\, \Phi_i = z^2(1 - 2z/3)T^*_{i-1} \tag{27}$$

where $T^*_i$'s are the modified Chebyshev polynomials. The above trial functions satisfy all the boundary conditions except the natural one, namely $\bar{f} D^2 W + Ma\, a^2 \Theta = 0$ at $z = 1$ but the residual from this condition is included as residual from the differential equation. Substituting Eq. (26) into Eqs. (19)–(21), multiplying momentum Eq. (19) by $W_j(z)$, energy Eq. (20) by $\Theta_j(z)$ and magnetization Eq. (21) by $\Phi_j(z)$; performing the integration by parts with respect to $z$ between $z = 0$ and $z = 1$ and using the boundary conditions, we obtain a system of linear homogeneous algebraic equations in $A_i$, $B_i$ and $C_i$. A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish, and this leads to a relation involving the parameters $R_t$, $Ma$, $R_m$, $Da^{-1}$, $M_1$, $M_3$, $B$ and $a$ in the form

$$F(R_t,\ Ma,\, R_m,\, Da^{-1},\, M_1,\, M_3,\, B,\, a) = 0. \tag{28}$$

The critical values of $R_{tc}$ or $Ma_c$ are found as functions of the wave number $a$ for various values of physical parameters. It is observed that the convergence is achieved with six terms in the series expansion of Eq. (26).

### 3.2 Solution by regular perturbation technique

It is known that for insulated boundary conditions the onset of convection corresponds to a vanishingly small wave number (i.e., unicellular convection). The numerical calculations carried out in the previous section also corroborate this fact. Therefore, an attempt is being made to exploit this fact to obtain an analytical formula for the onset of convection using a regular perturbation technique with wave number $a$ as a perturbation parameter. Accordingly, the variables W, $\Theta$ and $\Phi$ are expanded in powers of $a^2$ as

$$(W, \Theta, \Phi) = (W_0, \Theta_0, \Phi_0) + a^2(W_1, \Theta_1, \Phi_1) + \cdots . \tag{29}$$

Substituting Eq. (29) into Eqs. (19)–(21) and also in the boundary conditions, and collecting the terms of zeroth order, we obtain

$$D^4W_0 + 2BD^3W_0 + (B^2 - Da^{-1})D^2W_0 - BDa^{-1}DW_0 = 0, \tag{30.1}$$
$$D^2\Theta_0 + W_0 = 0, \tag{30.2}$$
$$D^2\Phi_0 + D\Theta_0 = 0 \tag{30.3}$$

with the boundary conditions

$$W_0 = DW_0 = 0 = D\Theta_0 = \Phi_0 \quad \text{at } z = 0, \tag{31.1}$$
$$W_0 = D^2W_0 = 0 = D\Theta_0 = D\Phi_0 \quad \text{at } \ = 1. \tag{31.2}$$

The solution to the zeroth order equations is found to be

$$W_0 = 0, \Theta_0 = 1 \quad \text{and} \quad \Phi_0 = 0. \tag{32}$$

The first-order equations are then

$$D^4W_1 + 2BD^3W_1 + (B^2 - Da^{-1})D^2W_1 - BDa^{-1}DW_1 = R_t(1 + M_1)e^{-B(z-1/2)} \tag{32.1}$$
$$D^2\Theta_1 + W_1 = 1, \tag{32.2}$$
$$D^2\Phi_1 - D\Theta_1 = 0 \tag{32.3}$$

with the boundary conditions

$$W_1 = DW_1 = \Phi_1 = D\Theta_1 = 0 \quad \text{at } z = 0, \tag{33.1}$$
$$W_1 = D^2W_1 + e^{-B(z-1/2)}Ma = D\Phi_1 = D\Theta_1 = 0 \quad \text{at } z = 1. \tag{33.2}$$

The general solution of Eq. (32.1) is given by

$$W_1 = c_0 + c_1e^{-Bz} + c_2e^{\delta_1 z} + c_3e^{\delta_2 4z} + \frac{(R_t + R_m)}{BDa^{-1}}ze^{-B(z-1/2)} \tag{34}$$

where

$$\delta_1 = \frac{-B + \sqrt{B^2 + 4Da^{-1}})}{2},$$
$$\delta_2 = \frac{-B - \sqrt{B^2 + 4Da^{-1}})}{2},$$
$$c_0 = -c_1 - c_2 - c_3,$$
$$c_1 = \frac{\delta_1}{B}c_2 + \frac{\delta_2}{B}c_3 + \frac{(R_t + R_m)e^{B/2}}{B^2Da^{-1}},$$
$$c_2 = \frac{(R_t + R_m)\left[((B+1)e^{-B/2} - e^{B/2})\Delta_4 - (B - B^2)e^{-B/2}\Delta_2\right]}{BDa^{-1}(\Delta_2\Delta_3 - \Delta_1\Delta_4)} + \frac{Mae^{-B/2}\Delta_2}{(\Delta_2\Delta_3 - \Delta_1\Delta_4)},$$
$$c_3 = \frac{(R_t + R_m)\left[((B+1)e^{-B/2} - e^{B/2})\Delta_3 - (B - B^2)e^{-B/2}\Delta_1\right]}{BDa^{-1}(\Delta_2\Delta_3 - \Delta_1\Delta_4)} + \frac{Mae^{-B/2}\Delta_1}{(\Delta_2\Delta_3 - \Delta_1\Delta_4)},$$

with

$$\begin{aligned}
\Delta_1 &= B(1-e^{\delta_1}) + \delta_1(1-e^{-B}),\\
\Delta_2 &= B(1-e^{\delta_2}) + \delta_2(1-e^{-B}),\\
\Delta_3 &= \delta_1 B e^{-B} + \delta_1^2 e^{\delta_1},\\
\Delta_4 &= \delta_2 B e^{-B} + \delta_2^2 e^{\delta_1}.
\end{aligned}$$

From Eq. (32.2), it follows that

$$1 = \int_0^1 W_1 \mathrm{d}z. \tag{35}$$

Substituting for $W_1$ from Eq. (34) into Eq. (35) and carrying out the integration leads to an expression of the form

$$\frac{R_{tc}(1+M_1)}{F_{R_t}(B, Da^{-1})} + \frac{Ma_c}{F_{Ma}(B, Da^{-1})} = 1 \tag{36}$$

where

$$\begin{aligned}
F_{R_t}(B, Da^{-1}) &= B^3 Da^{-1}\left[\frac{\delta_2\lambda_1 F_1 + \delta_1\lambda_2 F_2}{\delta_1\delta_2(\Delta_2\Delta_3 - \Delta_1\Delta_4)} + (2-B)e^{B/2} - (2+B)e^{-B/2}\right]^{-1},\\
F_{Ma}(B, Da^{-1}) &= B^2 e^{-B/2}\left[\frac{\delta_2\lambda_1\Delta_2 + \delta_1\lambda_2\Delta_1}{\delta_1\delta_2(\Delta_2\Delta_3 - \Delta_1\Delta_4)}\right]^{-1}
\end{aligned}$$

with

$$\begin{aligned}
\lambda_1 &= \delta_1^2(1-B-e^{-B}) - B^2(1+\delta_1-e^{-B}),\\
\lambda_2 &= \delta_2^2(1-B-e^{-B}) - B^2(1+\delta_2-e^{-B}),\\
F_1 &= [(B+1)e^{-B/2} - e^{B/2}]\Delta_4 - [(B-B^2)e^{-B/2}]\Delta_2,\\
F_2 &= [(B+1)e^{-B/2} - e^{B/2}]\Delta_3 - (B-B^2)e^{-B/2}\Delta_1.
\end{aligned}$$

From Eq. (36), it is interesting to note that the parameter $M_3$ has no influence on the onset criterion. Since at the onset of convection $a_c = 0$ (very large wave length), one would expect that $M_3$ has no effect on the stability of the system. Besides, it can be seen that the parameters $M_1$ and $M_3$ have no influence on the onset of pure Marangoni ferroconvection ($R_t = 0$) in the absence of viscosity variation ($B = 0$). The numerical calculations carried out in the previous section also reflected the same behavior. It is interesting to check Eq. (36) under the limiting conditions. When $M_1 = 0$ and $Da^{-1} = 0$ (i.e., ordinary viscous fluid), Eq. (36) reduces to the result obtained by Char and Chen [37]. In the limit $B \to 0$ and when $M_1 = 0$, we recover the following result for an ordinary viscous fluid of constant viscosity (Garcia-Ybarra et al. [38]; Yang and Yang [39]):

$$\frac{R_{tc}(1+M_1)}{320} + \frac{Ma_c}{48} = 1. \tag{37}$$

When we set $Ma_c = 0$, Eq. (37) reduces to

$$R_{tc} = \frac{320}{1+M_1}. \tag{38}$$

The above result coincides with the one obtained by Nanjundappa and Shivakumara [40]. Equation (37) simply reduces to $Ma_c = 48$ when $R_{tc} = 0$ and $R_{tc} = 320$ when $Ma_c = 0 = M_1$, which are the known exact values for the viscous fluid layer.

## 4 Results and Discussion

The influence of viscosity varying exponentially with temperature on the onset of coupled Bénard–Marangoni convection in a layer of ferrofluid saturated Brinkman porous medium is studied in the presence of a uniform vertical magnetic field. The lower rigid and upper free boundaries are assumed to be perfectly insulated to temperature perturbations. The critical eigenvalue $Ma_c$ or $R_{tc}$ and the corresponding wave number $a_c$ are computed numerically by the Galerkin method as well as analytically by employing a regular perturbation technique with wave number $a$ as a perturbation parameter for different values of $R_m$ or $M_1$, $Da^{-1}$and $B$. It is seen that the results obtained from these two methods are in good agreement. The values of the magnetic parameters chosen are based on the physical parameters for a commercially available magnetic fluid EMG 905 produced by Ferrofluidics [41]; density $\rho$[kg/m$^3$] $= 1.24 \times 10^3$, kinematic viscosity (27 °C)$\nu$ [m$^2$/s] $= 12 \times 10^{-6}$, thermal diffusivity $\kappa$ [m$^2$/s] $= 8 \times 10^{-8}$, heat capacity $c_p$ [J/kgK] $= 1.47 \times 10^3$, coefficient of thermal expansion $\alpha_t$ $[1/K] = 8.6\times10^{-4}$, susceptibility at low field $\chi = 1.9$, pyromagnetic coefficient at $H =$ 50 kA/m[A/Km] $= 110$ and mean particle diameter [nm] $= 10.2$. For such fluids, the magnetic parameters have the following order of magnitude: $M_1 \sim 10^{-4} - 10$ and $M_3 \geq 1$. The salient features of the physical parameters on the stability characteristics of the system are exhibited graphically in Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10. In general,

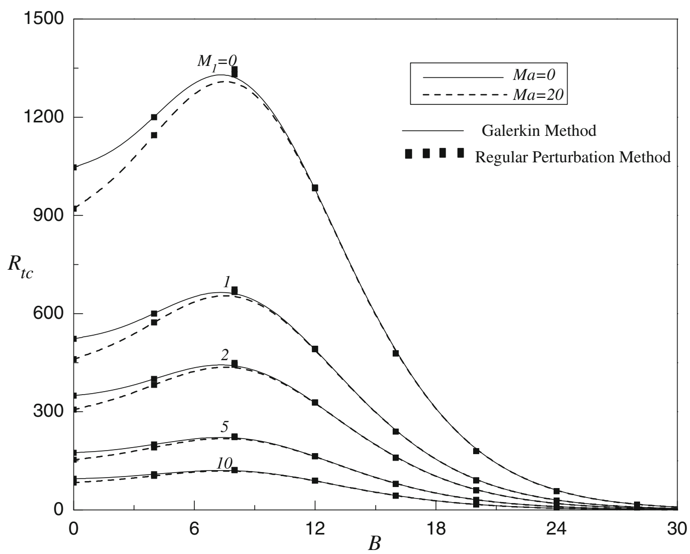



**Fig. 2** Critical Rayleigh number $R_{tc}$ versus temperature-dependent viscosity $B$ for different values of $M_1$ when $Da^{-1} = 50$

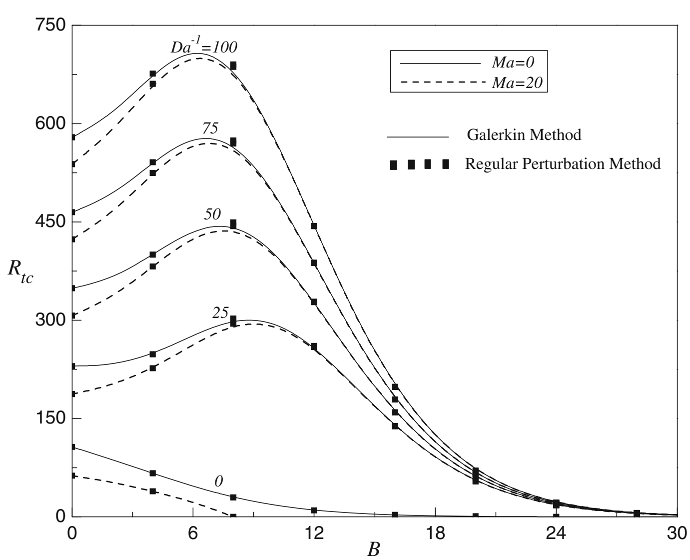



**Fig. 3** Critical Rayleigh number $R_{tc}$ versus temperature-dependent viscosity $B$ for different values of $Da^{-1}$ when $M_1 = 2$

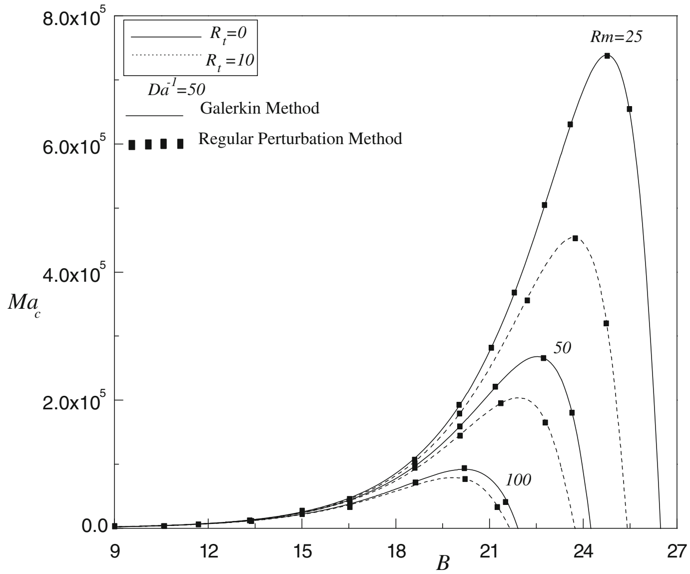



**Fig. 4** Critical Marangoni number $Ma_c$ versus temperature-dependent viscosity parameter $B$ for different values of $R_m$ when $Da^{-1} = 50$

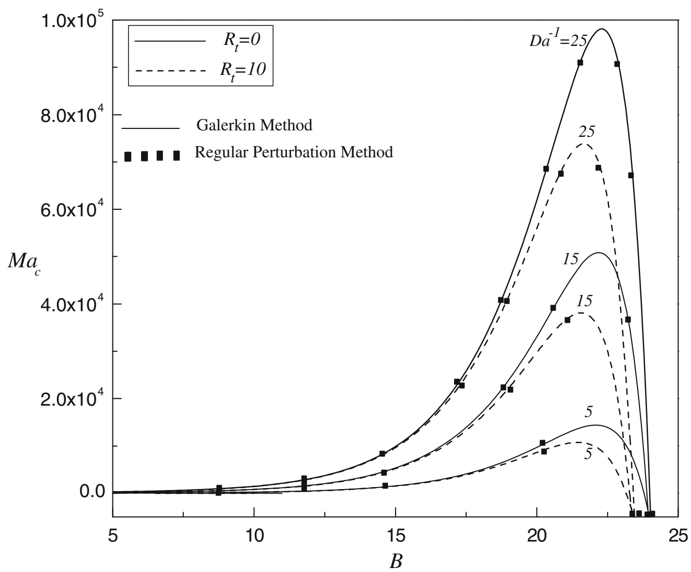



**Fig. 5** Critical Marangoni number $Ma_c$ versus temperature-dependent viscosity parameter $B$ for different values of $Da^{-1}$ when $R_m = 50$

the nonlinearity of the fluid magnetization parameter $M_3$ is found to have no influence on the criterion for the onset of Bénard–Marangoni ferroconvection.

The variation in the critical Rayleigh number $R_{tc}$ as a function of viscosity parameter $B$ for different values of magnetic parameter $M_1$ when $Da^{-1} = 50$ is illustrated in Fig. 2. The results are presented for two values of $Ma = 0$ (i.e., in the absence of surface tension force) and 20 (i.e., in the presence of surface tension force) in the figure. The figure clearly represents the strong influence of $B$ and $M_1$ on the onset of coupled Bénard–Marangoni ferroconvection. The viscosity parameter $B$ shows a dual effect on the stability characteristics of the system. That is, $R_{tc}$ increases initially with $B$, attains a maximum value depending on the strength of surface tension force and then starts decreasing with further increase in the value of $B$. At the maximum value of $R_{tc}$, a sublayer starts to form, and the thickness of this sublayer and $R_{tc}$ gets reduced with increasing $B$. Increasing $M_1$ is to diminish the initial increasing trend of $R_{tc}$ with $B$ and also to hasten the onset

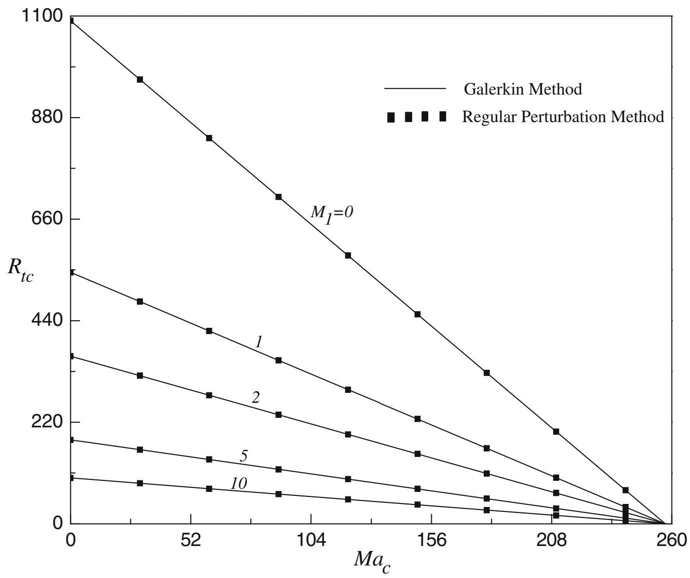



**Fig. 6** Plot of locus of critical Rayleigh number $R_{tc}$ versus critical Marangoni number $Ma_c$ for different values of $M_1$ for $Da^{-1} = 50$ and $B = 2$

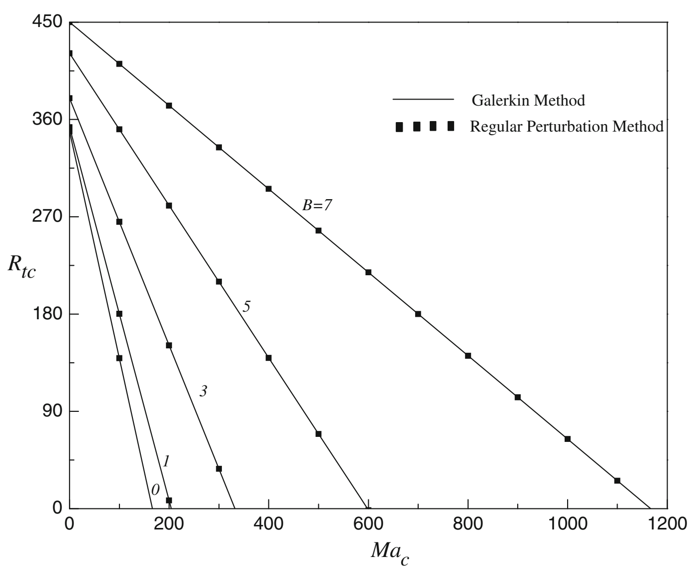



**Fig. 7** Plot of locus of critical Rayleigh number $R_{tc}$ versus critical Marangoni number $Ma_c$ for different values of $B$ for $Da^{-1} = 50$ and $M_1 = 2$

of ferroconvection due to an increase in the destabilizing magnetic force. Moreover, $R_{tc}$ decreases quite rapidly at first, and then, slowly and finally, the curves of different $M_1$ merge as the value of $B$ is becoming extremely large because the thickness of the sublayer reduces infinitesimally. It is more so with an increase in the value of $M_1$, and this is due to additive reinforcement of destabilizing magnetic force. The results for $M_1 = 0$ correspond to an ordinary viscous fluid, and it is observed that higher heating is required to have instability in this case. Furthermore, the presence of surface tension force facilitates the onset of ferroconvection but only up to certain values of $B$, exceeding which the effect of surface tension is found to be of no consequence on the onset criterion. The combined effect of surface tension and magnetic forces is to reinforce together and to advance the onset of ferroconvection compared to their effect in isolation. Irrespective of the values of $M_1$, the critical Rayleigh number $R_{tc}$ attains its maximum value $(R_{tc})_{\max}$ with respect to $B$ at a fixed value of $B = 7.4316$ when $Ma = 0$ and at $B = 7.6083$ when $Ma = 20$.

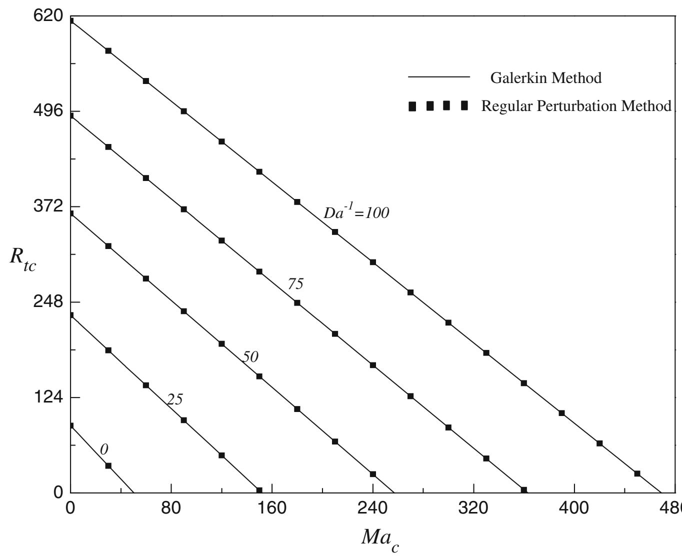
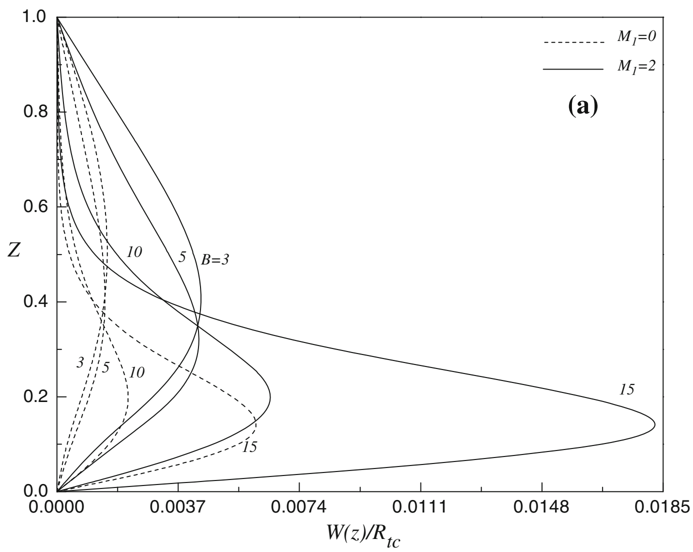



**Fig. 8** Plot of locus of critical Rayleigh number $R_{tc}$ versus critical Marangoni number $Ma_c$ for different values of $Da^{-1}$ for $B = 2$ and $M_1 = 2$

Figure 3 shows the variation in $R_{tc}$ as a function of $B$ for various values of $Da^{-1}$ with $M_1 = 2$ for two values of $Ma = 0$ and 20. For the case of $Da^{-1} = 0$ (i.e., in the absence of porous medium), the critical Rayleigh number $R_{tc}$ decreases monotonically with $B$, and the variation in the critical Rayleigh number between $Ma = 0$ and 20 is more noticeable for all values of $B$. For nonzero values of $Da^{-1}$, however, the curves of $R_{tc}$ coalesce in the presence as well as in the absence of surface tension force with increasing $B$ although initially the effect of increasing $Da^{-1}$ is found to delay the onset of ferroconvection. The values of $B$ at which $R_{tc}$ attains its maximum value $(R_{tc})_{\max}$ for different values of $Da^{-1}$ when $Ma = 0$ and 20 for $M_1 = 2$ are tabulated in Table 1. From the table, it is seen that the values of $B$ decrease with increasing $Da^{-1}$ but increase with increasing $Ma$.

The variation in critical Marangoni number $Ma_c$ as a function of viscosity parameter $B$ is represented for $R_t = 0$ (i.e., in the absence of gravitational force) and 10 (i.e., in the presence of gravitational force) in Fig. 4 when $Da^{-1} = 50$ and for different values of magnetic Rayleigh number $R_m$, while in Fig. 5 for different values of $Da^{-1}$ when $R_m = 50$. As can be seen from Fig. 4, the curves of $Ma_c$ for different values of $R_m$ as well as for two values of $R_t$ join together in the beginning but separate apart with increasing $B$, and in that case, the effect of increasing $R_m$ and $R_t$ is found to advance the onset of ferroconvection. As observed in Figs. 2 and 3, $Ma_c$ also attains its peak value at some intermediate value of $B$ depending on the values of $R_m$, $Da^{-1}$ and $R_t$. From Fig. 5, it is seen that increasing $Da^{-1}$ is to increase the peak value of $Ma_c$ and to delay the onset of Marangoni ferroconvection. Moreover, the curves of $Ma_c$ for different values of $Da^{-1}$ coalesce at higher values of $B$ and take distinct values for $R_t = 0$ and 10. The value of $B$ at which $Ma_c$ attains its maximum value $(Ma_c)_{\max}$ is tabulated in Table 2 for different values of $R_m$ with $Da^{-1} = 50$ for two values of $R_t = 0$ and 10. From the table, it is seen that the values of $B$ decrease with increasing $R_m$ and $R_t$.

To know the simultaneous presence of buoyancy, surface tension and magnetic forces on the stability of the system, the locus of $R_{tc}$ and $Ma_c$ is exhibited in Fig. 6 for different values of $M_1$ with $B = 2$ and $Da^{-1} = 50$, in Fig. 7 for different values of $B$ with $M_1 = 2$ and $Da^{-1} = 50$, while in Fig. 8 for different values of $Da^{-1}$ with $M_1 = 2$ and $B = 2$. From the figures, it is observed that there is a strong coupling between the critical Rayleigh and the Marangoni numbers and the curves are slightly convex. That is, when the buoyancy force is predominant, the surface tension force becomes negligible and vice-versa. From Fig. 6, it is seen that an increase in the value of $M_1$ (i.e., magnetic force) is to decrease the value of $R_{tc}$ as well as $Ma_c$, and thus, its effect is to hasten the onset of ferroconvection. The curves of different $M_1$ converge to the same value $Ma_c = 258$ when $R_{tc} = 0$ indicating $M_1$ has no effect on the onset of ferroconvection when it is only due to surface tension forces. Figure 7 shows that for the range of values of $B$ considered, an increase in the value of $B$ is to increase both $Ma_c$ and $R_{tc}$. From Fig. 8, it is seen that an increase in the value of $Da^{-1}$ is to increase the value of $R_{tc}$ as well as $Ma_c$, and



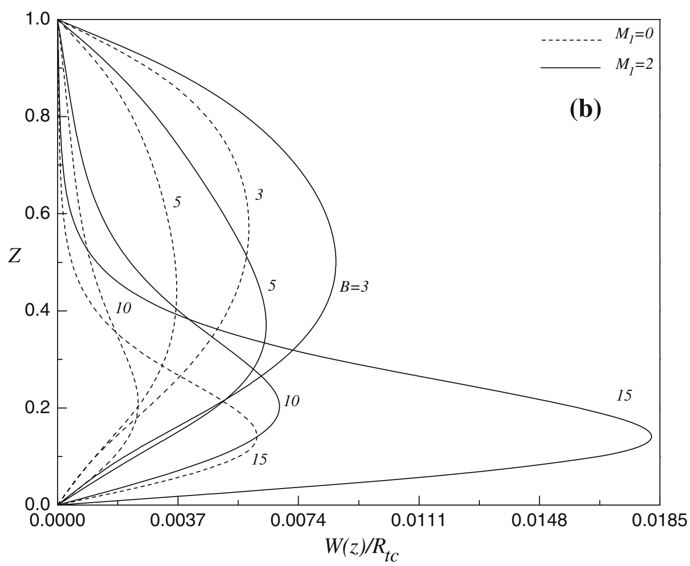




**Fig. 9** Perturbed velocity eigenfunction for different values of $B$ for **a** $Ma = 0$ and **b** $Ma = R_{tc}$ when $Da^{-1} = 50$

thus, its effect is to delay the onset of Bénard–Marangoni ferroconvection. Also, we note that $Ma_c < R_{tc}$ always.

Figure 9a, b show the perturbed vertical velocity eigenfunction for $Ma = 0$ and $Ma = R_{tc}$, respectively, for different values of $B$. The results presented in these figures are for two values of $M_1 = 0$ and 2 when $Da^{-1} = 50$. As can be seen, the shape of the eigenfunction is parabolic in nature, and increasing $M_1$ is to increase the vigor of the ferrofluid flow; hence, its effect is to hasten the onset of ferroconvection. With increasing $B$, a slightly skewed shape of the eigenfunction downward is formed compared to the normal shapes observed at lower values of $B$. For larger values of $B$, the perturbed vertical velocity vanishes at the upper part of the porous layer, and an apparent sublayer, in which the onset of ferroconvection takes place, is observed. Further inspection of the figures reveals that the presence of surface tension force is to enhance the flow at a fixed lower value of$B$. At higher values of $B$, however, the velocity eigenfunction remains almost unchanged indicating that the surface tension has no influence on the onset criterion. The results depicted in Fig. 10a, b are respectively for $Ma = 0$ and $Ma = R_{tc}$ when $Da^{-1} = 25$ and 100 with $M_1 = 2$. It is observed that increasing $Da^{-1}$ is to inhibit the fluid flow, and hence, its effect is to delay the onset of Bénard–Marangoni ferroconvection. The other observations are similar those observed in Fig. 9a, b.

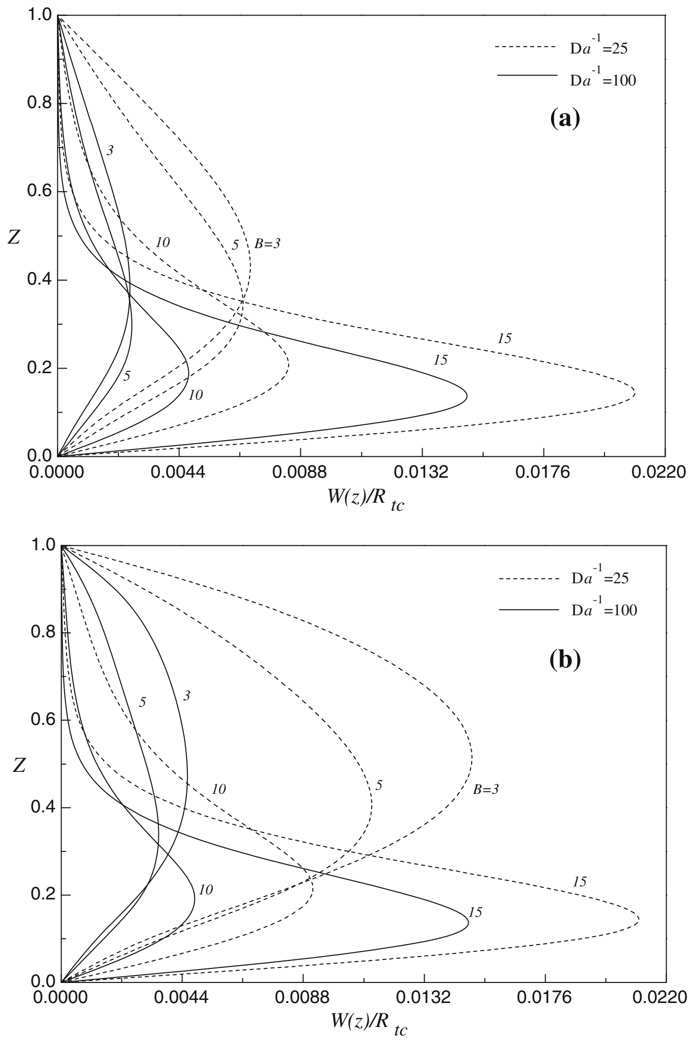



**Fig. 10** Perturbed velocity eigenfunction for different values of $B$ for **a** $Ma = 0$ and **b** $Ma = R_{tc}$ when $M_1 = 2$

**Table 1** Values of $(R_{tc})_{\max}$ occurring at $B$ for different values of $Da^{-1}$ when $Ma = 0$ and 20 with $M_1 = 2$

| $Ma$ | $Da^{-1}$ | $(R_{tc})_{\max}$ | $B$ |
|---|---|---|---|
| 0 | 25 | 306.070 | 8.893 |
| | 50 | 451.108 | 7.431 |
| | 75 | 587.629 | 6.736 |
| | 100 | 719.693 | 6.325 |
| 20 | 25 | 300.306 | 9.103 |
| | 50 | 444.680 | 7.608 |
| | 75 | 580.836 | 6.885 |
| | 100 | 712.670 | 6.453 |

**Table 2** Values of $(Ma_c)_{\max}$ occurring at $B$ for different values of $R_m$ when $R_{tc} = 0$ and 10 with $Da^{-1} = 50$

| $R_{tc}$ | $R_m$ | $(Ma_c)_{\max}$ | $B$ |
|---|---|---|---|
| 0 | 50 | 283,134.00 | 22.732 |
| | 100 | 97,144.96 | 20.367 |
| | 200 | 30,688.20 | 17.816 |
| 10 | 50 | 214,991.00 | 22.124 |
| | 100 | 83,383.00 | 20.000 |
| | 200 | 28,172.10 | 17.630 |

## 5 Conclusions

The onset of coupled Bénard–Marangoni ferroconvection in a ferrofluid saturated Brinkman porous layer with viscosity varying exponentially with temperature in the presence of a uniform vertical magnetic field has been studied. The viscosity parameter $B$ exhibits a dual effect on the stability characteristics of the system. It shows a stabilizing effect on the system initially, but displays a reverse trend after exceeding a certain value of $B$ depending on the choices of parametric values. That is, the critical Rayleigh or Marangoni number attains its maximum value at some intermediate values of $B$. The nonlinearity of fluid magnetization parameter $M_3$ has no effect on the onset of coupled Bénard–Marangoni ferroconvection in a porous medium. The effect of an increase in the value of magnetic Rayleigh number $R_m$ and the Darcy number $Da$ is to hasten the onset of coupled Bénard–Marangoni ferroconvection. The buoyancy, surface tension and magnetic forces reinforce each other in hastening the onset of ferroconvection. The buoyancy and magnetic forces show no influence on the onset of ferroconvection up to moderate values of $B$, while the surface tension force exhibits no influence on the onset criterion at higher values of $B$. The critical eigenvalues obtained by a regular perturbation technique and computed numerically using the Galerkin method complement with each other, indicating the analytical solutions obtained are exact.

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