Microgravity Sci. Technol. (2015) 27:27–37
DOI 10.1007/s12217-014-9410-0

ORIGINAL ARTICLE

# Effect of Coriolis Force on Bénard–Marangoni Convection in a Rotating Ferrofluid Layer with MFD Viscosity

**C. E. Nanjundappa · I. S. Shivakumara · Jinho Lee**

Received: 28 May 2014 / Accepted: 16 December 2014 / Published online: 31 December 2014


**Abstract** The simultaneous effect of Coriolis force due to rotation and magnetic field dependent (MFD) viscosity on the onset of Bénard-Marangoni convection in a horizontal ferrofluid layer in the presence of a uniform vertical magnetic field is studied. The lower boundary is rigid while the upper free boundary is open to the atmosphere and at which the temperature-dependent surface tension effect is allowed for. The Galerkin technique is employed to extract the critical stability parameters numerically. The results show that the onset of Bénard-Marangoni ferroconvection is delayed with an increase in the MFD viscosity parameter $\Lambda$, Taylor number $Ta$, magnetic susceptibility $\chi$ and Biot number $Bi$ but opposite is the case with an increase in the value of magnetic number $M_1$ and nonlinearity of fluid magnetization $M_3$. Further, increase in $M_1$, $M_3$ and decrease in $\Lambda$, $Ta$, $\chi$ and $Bi$ is to decrease the size of the convection cells. Comparisons of results between the present and the existing ones are made under the limiting conditions and good agreement is found.

**Keywords** Bénard-Marangoni Ferroconvection · Coriolis force · MFD viscosity · Magnetic field · Biot number

C. E. Nanjundappa (✉)
Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore 560 056, India
e-mail: nanjundappace@gmail.com

I. S. Shivakumara
Department of Mathematics, Bangalore University, Bangalore 560 001, India
e-mail: shivakumarais@gmail.com

J. Lee
School of Mechanical Engineering, Yonsei University, Seoul 120-749, South Korea
e-mail: jinholee@yonsei.ac.kr

## Introduction

Ferrofluids (magnetic fluids) are commercially manufactured colloidal liquids usually formed by suspending mono–domain nano particles (their diameter is typically 10 nm) of magnetite in non–conducting liquids like heptanes, kerosene, water, etc. and they are also called magnetic nano fluids. The ferrofluid is a type of functional fluid whose flow and energy transport processes may be controlled by adjusting an external magnetic field, which makes it find a variety of applications in various fields such as electronic packing, mechanical engineering, aerospace, bioengineering, and thermal engineering. An authoritative introduction to this fascinating subject along with their applications is provided by Shliomis (1974), Rosensweig (1985) and Blums (2002).

The magnetization of ferrofluids depends on the magnetic field, temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is similar to buoyancy driven convection. A theoretical study on convective instability in a horizontal ferrofluid layer heated from below began with Finlayson (1970) and extensively continued over the years. The review articles by Ganguly et al. (2004), Odenbach (2004) and Nkurikiyimfura et al. (2013) have covered most of the developments in this field.

Convective instability in a ferrofluid layer can also be induced by surface tension forces provided it is a function of temperature and/or concentration. In view of the fact that heat transfer is greatly enhanced due to convection, ferroconvection offers new possibilities for applications in cooling of motors, loud speakers, transmission lines, and other equipments where magnetic field is already present.

If the ferrofluid layer has an upper surface open to atmosphere then the instability may be due to the combined effects of buoyancy as well as temperature–dependent surface tension forces, known as Bénard–Marangoni ferroconvection. A limited number of studies have addressed this type of instability problem in a horizontal ferrofluid layer (Qin and Kaloni 1994; Shivakumara et al. 2002; Hennenberg et al. 2005, 2006; Nanjundappa and Shivakumara 2008, Idris and Hashim 2010; Nanjundappa et al. 2010, 2011, 2013a, b) and also in a ferrofluid saturated porous layer (Shivakumara et al. 2010). Odenbach (1998) has investigated experimentally the stability of a free surface of a magnetic fluid subjected to a magnetic field parallel to the fluid surface under strongly reduced gravity.

Physically, all ferrofluids possesses a magnetic field-dependent (MFD) viscosity. The effect of an applied magnetic field is to align the magnetic moment of the particle with the magnetic field direction. As a result, larger gradients in the velocity field are expected surrounding a particle and thereby increases the viscous dissipation. In fact, the magnetic moment and magnetic field give rise to a magnetic torque hindering the free rotation of the particle and thereby increases the viscosity of the fluid. This is called rotational viscosity and was first predicted theoretically by Shliomis (1972) and subsequently it was verified with the results of McTague (1969) who measured the viscosity of diluted ferrofluids experimentally. Odenbach and Stork (1998) have contemplated that the concept of rotational viscosity is not valid for the description of the field-induced increase of viscosity in concentrated fluids at low shear rates. Shima et al. (2009) have reported that there will be an enhancement of viscosity in a stable magnetic nanofluid with magnetic field. The details can be found in the recent review article by Nkurikiyimfura et al. (2013). The above observed facts have tempted researchers to look in to the influence of MFD viscosity on the stability characteristics of ferroconvection. Vaidyanathan et al. (2002) considered linear variation of magnetic viscosity for small field variation, as a first approximation, and studied its impact on the onset of ferroconvection. Recently, the onset of Bénard-Marangoni ferroconvection has been analyzed with combined effects of basic cubic temperature profiles and magnetic field dependent (MFD) viscosity (Nanjundappa et al. 2014a, b), and the viscosity depending exponentially on temperature (Nanjundappa et al. 2014a, b).

The study of fluids in rotation is in itself an interesting topic for research. Ferrofluids are known to exhibit peculiar characteristics when they are set to rotation. Das Gupta and Gupta (1979) have studied convective instability in a rotating ferrofluid layer between two free boundaries. Venkatasubramanian and Kaloni (1994) have discussed the effect of rotation on thermo–convective instability of a horizontal layer of ferrofluid confined between stress–free, rigid–paramagnetic and rigid–ferromagnetic boundaries. Thermal convection in a rotating layer of a magnetic fluid is discussed by Auernhammer and Brand (2000). The weakly nonlinear instability of a rotating ferromagnetic fluid layer heated from below is studied by Kaloni and Lou (2004). Shivakumara and Nanjundappa (2006) have studied the effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection. Prakash and Gupta (2013) have qualitatively analyzed the complex growth rate of an arbitrary oscillatory motion of growing amplitude in ferroconvection rotating ferrofluid layer with MFD viscosity.

Although earlier works have been dealt with the influence of MFD viscosity on Bénard–Marangoni ferroconvection, but the effect of rotation has been overlooked despite its importance in understanding control of ferroconvection which is important in heat transfer applications involving ferrofluids. One of the reasons may be because of the numerical complications involved, since, as it shall be seen later, the resulting differential equation, with the inclusion of rotation, becomes tenth order, involving coupled boundary conditions and hence a numerical solution to the problem becomes increasingly difficult. The combined effect of rotation and MFD viscosity are important and the effect of these two factors cannot be independent on each other. The object of this paper is to explore the mutual effect of Coriolis, buoyancy, magnetic and surface tension forces on the linear stability of ferroconvection under the influence of MFD viscosity. The resulting eigenvalue problem is solved numerically by employing the Galerkin technique. A comparative study is conducted to investigate the implications of various forces on the onset of Bénard–Marangoni ferroconvection and also with other works under the limiting conditions to know the accuracy of the present numerical solution procedure.

## Mathematical Formulation

The physical configuration considered is as shown in Fig. 1. We consider a horizontal layer of an electrically non–conducting incompressible Boussinesq ferrofluid of depth $d$permeated by uniform applied magnetic field $H_0$acting in

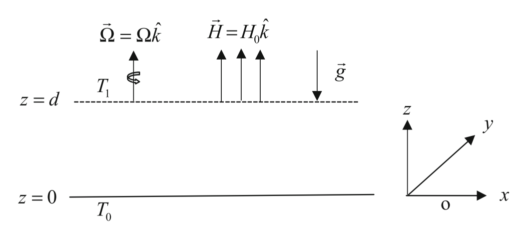


**Fig. 1** Physical configuration

the vertical direction. The layer is rotating uniformly about its vertical axis with angular velocity $\vec{\Omega} = \Omega \hat{k}$, which is bounded below by a rigid surface and above by a non–deformable free surface. A temperature drop $\Delta T$ is acting across the boundaries and a Cartesian co–ordinate system $(x, y, z)$ is used with the origin at the bottom of the surface and z–axis vertically upwards. The surface tension $\sigma$ is assumed to vary linearly with temperature as $\sigma = \sigma_0 - \sigma_T \Delta T$, where $\sigma_0$ is the unperturbed value and $-\sigma_T$ is the rate of change of surface tension with temperature. A linear variation in the viscosity with respect to magnetic field is sought after (Vaidyanathan et al. 2002) in the form $\eta = \eta_0(1+\vec{\delta}\cdot\vec{B})$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity considered to be isotropic and $\eta_0$ is the viscosity of fluid when the applied magnetic field is absent.

The continuity equation for an incompressible Boussinesq fluid is

$$\nabla \cdot \vec{V} = 0. \tag{1}$$

where, $\vec{V}=(u, v, w)$ is the velocity vector.

The momentum equation for an incompressible ferrofluid with viscous force $2\nabla \cdot [\eta \underline{\underline{D}}]$ in the rotating frame of reference is (Chandrasekhar 1961; Auernhammer and Brand 2000)

$$\rho_0 \left[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} \right] = -\nabla \mathrm{P} + \rho_0 \left[ 1 - \alpha_t (T - \bar{T}) \right] \vec{g} + 2\nabla \cdot [\eta \underline{\underline{D}}] + \mu_0 (\vec{M} \cdot \nabla)\vec{H} + 2\rho_0 (\vec{V} \times \vec{\Omega}) \tag{2}$$

where, $T$ is the temperature, $\vec{H}$ is the magnetic field, $\vec{M}$ is the magnetization, $D = [\nabla \vec{V} + \nabla \vec{V}^T]/2$ is the rate of strain tensor, $P = p - \rho_0 \left| \vec{\Omega} \times \vec{r} \right|^2 /2$ is the modified pressure, $\rho_0 = \rho(\bar{T})$ is the density of fluid at the average temperature $\bar{T} = (T_0 + T_1)/2$, the coefficient $\mu_0 = 4\pi \times 10^{-7}$ Henry$m^{-1}$ is the magnetic constant and $\alpha_t$ is the thermal expansion coefficient. The fourth term on the right hand side of Eq. 2 describes the ponderomotive force which acts on a magnetized fluid in a non–uniform magnetic field (i.e., magnetized fluid tends to move in the direction of increasing magnetic field) while the last term is the Coriolis acceleration.

The energy equation for an incompressible ferrofluid which obeys Fourier's law is given by (Finlayson 1970)

$$\left[ \rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left( \frac{\partial \vec{M}}{\partial T} \right)_{V,H} \right] \frac{DT}{Dt} + \mu_0 T \left( \frac{\partial \vec{M}}{\partial T} \right)_{V,H} \cdot \frac{D\vec{H}}{Dt} = k_t \nabla^2 T \tag{3}$$

where, $k_t$ is the (constant) overall thermal conductivity, $C_{V,H}$ is the specific heat at constant volume and magnetic field, $H$ is the magnitude of $\vec{H} (H = |\vec{H}|)$, $M$ is the magnitude of $\vec{M} (M = |\vec{M}|)$ and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplacian operator.

Maxwell's equations, simplified for a non–conducting fluid with no displacement currents, become

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = 0 \quad \text{or} \quad \vec{H} = \nabla \varphi \tag{4}$$

where, $\vec{B}$ is the magnetic induction and $\varphi$ is the magnetic potential.

Further, $\vec{B}$, $\vec{M}$ and $\vec{H}$ are related by

$$\vec{B} = \mu_0 \left( \vec{M} + \vec{H} \right). \tag{5}$$

The magnetic equation of state is linearized about the magnetic field, $H_0$, and the average temperature, $\bar{T}$ to become

$$\vec{M} = [M_0 + \chi(H - H_0) - K(T - \overline{T})] \left( \frac{\vec{H}}{H} \right) \tag{6}$$

where, $K = -(\partial M/\partial T)_{H_0, \bar{T}}$ is the pyromagnetic co–efficient, $\chi = (\partial M/\partial H)_{H_0, \bar{T}}$ is the magnetic susceptibility and $M_0 = M(H_0, \bar{T})$ is the constant mean value of magnetization.

The basic steady state is quiescent and is given by

$$\left[ \vec{V}, P, \rho, T, \vec{H}, \vec{M} \right] = [0, P_b(z), \rho_b(z), T_b(z), \vec{H}_b(z), \vec{M}_b(z)] \tag{7}$$

where the subscript $b$ denotes the basic state. The unperturbed basic temperature, magnetic field and magnetization are found to be

$$T_b(z) = \bar{T} - \beta \left( z - \frac{d}{2} \right) \tag{8a}$$

$$\vec{H}_b(z) = \left[ H_0 + \frac{K(T_b - \bar{T})}{1 + \chi} \right] \hat{k} \tag{8b}$$

$$\vec{M}_b(z) = \left[ M_0 - \frac{K(T_b - \bar{T})}{1 + \chi} \right] \hat{k} \tag{8c}$$

where $\beta = (T_0 - T_1)/d$ is the temperature gradient.

To investigate the conditions under which the quiescent solution is stable against small disturbances, the basic state is perturbed in the form

$$\left[ \vec{V}, P, \eta, T, \vec{H}, \vec{M} \right] = \left[ \vec{V}', P_b(z) + P', \eta_b(z) + \eta', T_b(z) + T', H_b(z) + \vec{H}', M_b(z) + \vec{M}' \right] \tag{9}$$

where, $\vec{V}'$, $P'$, $\eta'$, $T'$, $\vec{H}'$ and $\vec{M}'$ are the perturbed variables and are assumed to be small.

Using Eq. 9 in Eqs. 5 and 6, and linearization gives

$$H_i' + M_i' = (1 + M_0/H_0)\, H_i', \quad i = 1, 2 \tag{10}$$

$$H_3' + M_3' = (1+\chi) \quad H_3' - K\ T', \tag{11}$$

where $K\,\beta\,d << (1+\chi)\,H_0$ is assumed.

Substituting Eq. 9 in Eq. 2, linearizing, using Eqs. 10 and 11 together with $\vec{H}' = \nabla\varphi'$, and taking curl twice the z–component of the resulting equation can be written as (after ignoring the primes)

$$\left[\rho_0 \frac{\partial}{\partial t} - \eta\,\nabla^2\right] \nabla^2 w = \rho_0 \alpha_t g\,\nabla_1^2 T - 2\,\rho_0\,\Omega\,\frac{\partial \xi}{\partial z} - \mu_0 K\beta \frac{\partial}{\partial z}(\nabla_1^2 \varphi) + \frac{\mu_0 K^2 \beta}{1+\chi}(\nabla_1^2 T). \tag{12}$$

where $\eta = \eta_0\,[1 + \delta\,\mu_0(M_0 + H_0)]$ and $\xi = \partial v/\partial x - \partial u/\partial y$is the z-component of vorticity arising due to the rotational effect and a separate equation has to be obtained for $\xi$.

Substituting Eq. 9 in Eq. 2, linearizing and taking curl on the resulting equation, the z-component gives an equation for $\xi$ in the form (after neglecting primes)

$$\rho_0\,\frac{\partial \xi}{\partial t} = \eta\,\nabla^2 \xi + 2\,\rho_0\,\Omega\,\frac{\partial w}{\partial z}. \tag{13}$$

Using Eqs. 7 and 8 in Eq. 3 as before, the equation obtained after linearizing and neglecting the primes is

$$\rho_0 C_0 \frac{\partial T}{\partial t} - \mu_0 K\ T_0 \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right) = \left(\rho_0 C_0 - \frac{\mu_0\,K^2\,T_0}{(1+\chi)}\right) w\,\beta + k_t \nabla^2 T \tag{14}$$

where, $\rho_0 C_0 = \rho_0 C_{V,H} + \mu_0 K\ H_0$.

Equation 4, after substituting Eq. 9 and using Eqs. 10 and 11, may be written as (after dropping the primes)

$$\left(1 + \frac{M_0}{H_0}\right) \nabla_1^2 \varphi + (1+\chi)\,\frac{\partial^2 \varphi}{\partial z^2} - K\,\frac{\partial T}{\partial z} = 0. \tag{15}$$

As is customary in convective instability analysis we assume the normal mode hypothesis or separation of variables. Each variable is expanded in the form

$$f(x,\ y,\ z,\ t) = f(z,\ t)\,e^{i\,(l\,x + m\,y)}. \tag{16}$$

where $\ell$ and $m$ are the wave numbers in the $x$ and $y$ directions, respectively.

Using Eq. 16, Eqs. 12 – 15 become

$$\left[\rho_0 \frac{\partial}{\partial t} - \eta\,(D^2 - a^2)\right] \left(D^2 - a^2\right) w = -a^2 \alpha_t\,g\,\theta + a^2 \mu_0 K\beta\,D\varphi - \frac{a^2 \mu_0 \mathrm{K}^2 \beta}{1+\chi}\theta - 2\,\rho_0 \Omega\,D\xi \tag{17}$$

$$\rho_0\,\frac{\partial \xi}{\partial t} = \eta\,(D^2 - a^2)\xi + 2\,\rho_0\,\Omega\,Dw \tag{18}$$

$$\frac{\partial \theta}{\partial t} - \kappa\left(D^2 - a^2\right)\theta - \frac{\mu_0\,K\,T_0}{\rho_0 C_0}\,\frac{\partial}{\partial t}\,(D\varphi) = \left(1 - \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_0}\right) w\,\beta \tag{19}$$

$$(1+\chi)^2 \varphi - (1 + M_0/H_0) a^2 \varphi - K\ D\theta = 0 \tag{20}$$

where, $D = d/dz$ is the differential operator and $a = \sqrt{\ell^2 + m^2}$ is the overall horizontal wave number. The form of above equations is simplified by introducing the following dimensionless variables:

$$(x*, y*, z*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \quad w* = w/(\nu/d), \quad t* = t/\left(d^2/\nu\right),\ \xi* = \xi/\left(\nu/d^2\right), \quad \theta* = \theta/(\beta\nu d/\kappa), \phi* = \phi/\left(K\beta\nu d^2/(1+\chi)\,\kappa\right), \tag{21}$$

where $\nu$ is the kinematic viscosity. Using Eq. 21, Eqs. 17 – 20 become (after neglecting the asterisks)

$$\left[(1+\Lambda)\left(\mathrm{D}^2 - a^2\right) - \frac{\partial}{\partial t}\right] \left(\mathrm{D}^2 - a^2\right) w = Ta^{1/2} D\xi + R_t a^2 \theta + R_m a^2 (\theta - \mathrm{D}\varphi) \tag{22}$$

$$\left[(1+\Lambda)(\mathrm{D}^2 - a^2) - \frac{\partial}{\partial t}\right] \xi = -Ta^{1/2}\,Dw \tag{23}$$

$$\left(\mathrm{D}^2 - a^2 - Pr\frac{\partial}{\partial t}\right)\theta + Pr\,M_2 \frac{\partial}{\partial t} D\varphi = -(1 - M_2)\,w \tag{24}$$

$$\left(\mathrm{D}^2 - a^2 M_3\right)\varphi - D\theta = 0. \tag{25}$$

Here, $R_t = \alpha_t\,g\,\beta\,d^4/\kappa\mu$ is the thermal Rayleigh number, $M_1 = \mu_0 K^2 \beta/(1+\chi)\alpha_t \rho_0 g$ is the magnetic number, $R_m (= R_t M_1) = \mu_0 K^2 \beta^2 d^4/(1+\chi)\kappa\mu$ is the magnetic Rayleigh number, $M_3 = (1 + M_0/H_0)/(1+\chi)$ is the measure of non–linearity of fluid magnetization parameter, $Ta = 4\,\Omega^2 d^4/\nu^2$ is the Taylor number, $\Lambda = \delta\,\mu_0(M_0 + H_0)$ is the non–dimensional magnetic field dependent viscosity parameter, $M_2 = \mu_0 T_0 K^2/\rho_0 C_0 (1+\chi)$ is the non–dimensional magnetic parameter and $\mathrm{Pr} = \nu/\kappa$ is the Prandtl number. The typical value of $M_2$ for ferrofluids with different carrier liquids turns out to be of the order of $10^{-6}$ and hence its effect is neglected as compared to unity. The effective viscosity of nanofluids as a function of applied magnetic field has been measured experimentally by Shima et al. (2009). It is observed that in magnetic field range of 82–283 G, the effective viscosity $\eta/\eta_0$ increases marginally from 1.6 to 2 at 0.078 vol % of magnetite nanoparticles.

In general, it has been observed that the effective viscosity varies from 1 to 1.65 when the vol% is varied from 0 to 0.08. All the above parameters affect the stability of the system in one way or the other, as the subsequent analysis only deals with the dimensionless variables. We set

$$\{w,\ \theta,\ \phi,\ \xi\}\ (z,\ t) = \{W(z),\ \Theta(z),\ \Phi(z),\ \xi(z)\}\ e^{\omega t} \tag{26}$$

where $\omega$ is the complex growth rate of disturbances.

Using Eq. 26, Eqs. 22 – 25 can be written as

$$\left[(1+\Lambda)\left(\mathrm{D}^2 - a^2\right) - \omega\right]\left(\mathrm{D}^2 - a^2\right)\mathrm{W} = Ta^{1/2}D\xi + \mathrm{R_t}a^2\Theta - \mathrm{R_m}a^2(\mathrm{D}\Phi - \Theta) \tag{27}$$

$$\left(\mathrm{D}^2 - a^2 - \Pr\omega\right)\Theta = -W \tag{28}$$

$$\left(\mathrm{D}^2 - a^2 M_3\right)\Phi - D\Theta = 0 \tag{29}$$

$$\left[(1+\Lambda)\left(D^2 - a^2\right) - \omega\right]\xi = -Ta^{1/2}DW. \tag{30}$$

The above equations are to be solved subject to appropriate boundary conditions.

At the lower rigid surface ($z = 0$)

$$W = DW = \Theta = \xi = 0 \tag{31a}$$

and at the upper free interface ($z = 1$)

$$W = (1+\Lambda)\,D^2W + a^2Ma\,\Theta = D\Theta + Bi\,\Theta = D\xi = 0 \tag{31b}$$

where, $Ma = \sigma_T\,\Delta T\,d/\mu\kappa$ is the Marangoni number. We remark that the Marangoni instability is a capillarity effect and it arises from the variation of the surface tension $\sigma$ at the upper surface with temperature. In addition, the Biot number, $Bi = h\,d/k_t$ arises from the Newton's heat transfer law due to cooling at the upper boundary. Note that $Bi$ for a perfectly heat conducting surface tends to infinity, and for an adiabatically insulated boundary tends to zero.

On the other hand, in the case of a finite magnetic permeability $\chi$ of the boundaries, the scalar magnetic potential must satisfy the following dimensionless boundary conditions (Stiles and Kagan 1990)

$$D\Phi - \Theta = \begin{cases} a\,\Phi/(1+\chi), & \text{at} \quad z = 0 \\ -a\,\Phi/(1+\chi), & \text{at} \quad z = 1. \end{cases} \tag{32}$$

## Method of Solution

Equations 27–30 together with the boundary conditions (31)–(32) constitute an eigenvalue problem with $R_t$ or $Ma$ as an eigenvalue. To solve the resulting eigenvalue problem, Galerkin method is used. Accordingly, the variables are written in a series of basis functions as

$$W = \sum_{i=1}^{n} A_i\,W_i(z),\quad \Theta(z) = \sum_{i=1}^{n} C_i\Theta_i(z),$$

$$\Phi(z) = \sum_{i=1}^{n} D_i\Phi_i(z),\quad \xi = \sum_{i=1}^{n} E_i\,\xi_i(z) \tag{33}$$

where, the trial functions $W_i(z)$, $\xi_i(z)$, $\theta_i(z)$ and $\Phi_i(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions and $A_i$, $C_i$, $D_i$ and $E_i$ are constants. Substituting Eq. 33 into Eqs. 27–30, multiplying Eq. 27 by $W_j(z)$, Eq. 28 by $\Theta_j(z)$, Eq. 29 by $\Phi_j(z)$ and Eq. 30 by $\xi_j(z)$, performing the integration by parts with respect to z between z = 0 and z = 1 and using the boundary conditions (31) and (32), we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}C_i + E_{ji}D_i + F_{ji}E_i = 0 \tag{34}$$

$$G_{ji}A_i + H_{ji}C_i = 0 \tag{35}$$

$$I_{ji}C_i + J_{ji}D_i = 0 \tag{36}$$

$$K_{ji}A_i + L_{ji}E_i = 0. \tag{37}$$

The coefficients $C_{ji} - L_{ji}$ involve the inner products of the basis functions and are given by

$$\begin{aligned}
C_{ji} &= (1+\Lambda)\left[\langle D^2W_jD^2W_i\rangle + 2a^2\langle DW_jDW_i\rangle + a^4\langle W_jW_i\rangle\right] \\
&\quad +\omega[\langle DW_jDW_i\rangle + a^2\langle W_jW_i\rangle] \\
D_{ji} &= -a^2(R_t+R_m)\,\langle\Theta_jW_i\rangle + a^2Ma\,DW_j(1)\,\Theta_i(1) \\
E_{ji} &= a^2R_m\,\langle W_jD\Phi_i\rangle \\
F_{ji} &= -Ta^{\frac{1}{2}}\langle W_jD\zeta_i\rangle \\
G_{ji} &= -\langle\Theta_jW_i\rangle \\
H_{ji} &= \langle D\Theta_jD\Theta_i\rangle + (a^2+\omega\,Pr)\langle\Theta_j\Theta_i\rangle + Bi\,\Theta_j(1)\Theta_i(1) \\
I_{ji} &= -\langle\Phi_jD\Theta_i\rangle \\
J_{ji} &= \frac{1}{4} - \frac{a}{2(1+\chi)} - a^2M_3\langle\Phi_j\Phi_i\rangle - \langle D\phi_j\ D\phi_i\rangle \\
K_{ji} &= -Ta^{\frac{1}{2}}\langle\zeta_jDW_i\rangle
\end{aligned}$$

where, the inner product is defined as $\langle\cdots\cdots\rangle = \int_0^1(\cdots)\,dz$. The above set of homogeneous algebraic equations can have a non–trivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} & F_{ji} \\ G_{ji} & H_{ji} & 0 & 0 \\ 0 & I_{ji} & J_{ji} & 0 \\ K_{ji} & 0 & 0 & L_{ji} \end{vmatrix} = 0. \tag{38}$$

The eigenvalues have to be extracted from the above characteristic equation. For this, we select the trial functions as

$$\mathrm{W_i} = \left(z^4 - 5z^3/2 + 3z^2/2\right) T*_{i-1}, \quad \Theta_i = \left(z - z^2/2\right) T*_{i-1},$$
$$\Phi_i = (z - 1/2)T*_{i-1}, \quad \xi_i = \left(z^2 - 2z^3/3\right) T*_{i-1} \tag{39}$$

where, $\mathrm{T}^*_{i-1}$'s ($i = 1,\ 2,\ 3$.........) are the modified Chebyshev polynomials such that they satisfy all the corresponding boundary conditions except the following conditions

$$(1+\Lambda)\, D^2W + a^2 Ma\, \Theta = D\Theta + Bi\, \Theta = 0 \text{ at } z = 1 \text{ and}$$
$$(1+\chi)\,(D\Phi - \Theta) \mp a\Phi = 0 \text{ at } z = 0,\ 1$$

but the residual from these equations are included as a residual from the differential equation. At this juncture, it would be instructive to look at the results for $i = j = 1$and for this order Eq. 38 gives the following characteristic equation

$$Ma = \frac{(\eta_1 + 8\omega\,\mathrm{Pr})}{1050a^2\langle\, W\,\Theta\,\rangle}\left[\frac{169\ Ta}{8\,(\eta_2 + 13\omega)} + \frac{(\eta_3 + \eta_4\omega)}{18}\right] + \frac{R_m\langle\, W D\Phi\rangle}{3\eta_5} - 4\,[R_t\langle W\,\Theta\rangle + R_m\langle\, W\,\Theta\rangle] \tag{40}$$

where,

$$\eta_1 = \left(8a^2 + 15Bi + 20\right)$$
$$\eta_2 = (1+\Lambda)\left(13a^2 + 42\right)$$
$$\eta_3 = (1+\Lambda)\left(19a^4 + 432a^2 + 4536\right)$$
$$\eta_4 = \left(19a^2 + 216\right) \quad \text{and}$$
$$\eta_5 = 3/4 + a^2M_3/12 + a/2(1+\chi).$$

To examine the stability of the system, we take $\omega = i\,\omega_i$ in Eq. 40 and clear the complex quantities, we obtain,

$$Ma = \frac{1}{1050a^2\langle\, W\,\Theta\rangle}\left[\frac{169Ta(\eta_1\eta_2 + 104\omega_i^2\,\mathrm{Pr})}{8\left(\eta_2^2 + 169\omega_i^2\right)} + \frac{(\eta_1\eta_3 - 8\omega_i^2\eta_4\,\mathrm{Pr})}{36}\right] + \frac{R_m\langle\, W\ D\Phi\rangle}{3\eta_5} - 4\,[R_t + R_m]\langle W\,\Theta\rangle + i\omega_i\,\Delta \tag{41}$$

where,

$$\Delta = \frac{1}{1050a^2\langle\, W\,\Theta\rangle}\left[\frac{169Ta(8\,\mathrm{Pr}\,\eta_2 - 13\eta_1)}{8(\eta_2^2 + 169\omega_i^2)} + \frac{(8\eta_3\,\mathrm{Pr} + \eta_1\eta_4)}{36}\right]. \tag{42}$$

Since $Ma$ *is* a physical quantity it must be real, so that it implies either $\omega_i = 0$or $\Delta = 0$ (i.e. $\omega_i \neq 0$) and accordingly the condition for steady and oscillatory onset is obtained.

The steady onset is governed by $\omega_i = 0$and it occurs at$Ma = Ma^s$, where

$$Ma^S = \frac{\eta_1}{1050a^2\langle\, W\,\Theta\rangle}\left[\frac{169Ta}{8\eta_2} + \frac{\eta_3}{36}\right] + \frac{R_m\langle\, W\ D\Phi\rangle}{3\eta_5} - 4[R_t + R_m]\langle W\,\Theta\rangle. \tag{43}$$

The oscillatory convection occurs at$Ma = Ma^0$, where

$$Ma^0 = \frac{\left(a_1a_4^2 + a_2a_4 + a_3\right)}{1050\ a_4a^2\langle\, W\,\Theta\rangle} + \frac{R_m\langle\, W\ D\Phi\rangle}{3\eta_5} \tag{44}$$
$$-4\,[R_t + R_m]\langle W\,\Theta\rangle. \tag{45}$$

Here,

$$a_1 = \left(\frac{1}{36}\,\eta_1\eta_2 - \frac{2}{117}\,\mathrm{Pr}\,\eta_2^2\right)$$
$$a_2 = \left(\frac{1}{36}\eta_1\eta_3 + \frac{2}{1521}\,\mathrm{Pr}\,\eta_4\eta_2^2 + 13Ta\,\mathrm{Pr}\right)$$
$$a_3 = -Ta\,\mathrm{Pr}\,\eta_4$$
$$a_4 = \left(\frac{\eta_1\eta_4 + 8\,\mathrm{Pr}\,\eta_3}{13\eta_1 - 8\,\mathrm{Pr}\,\eta_2}\right).$$

The corresponding frequency of oscillations is given by

$$\omega_i^2 = -\frac{\eta_2^2}{169} + \frac{117\ Ta}{2\eta_4}\left[\frac{1 - 2\beta_1\,\mathrm{Pr}}{1 + 2\beta_2\,\mathrm{Pr}}\right] \tag{46}$$

where, $\beta_1 = \frac{(1+\Lambda)4(42+13a^2)}{13(20+15Bi+4a^2)}$ and $\beta_2 = \frac{(1+\Lambda)[4536+432a^2+19a^4]}{(216+19a^2)\ (15Bi+20+8a^2)}$.

For most of the ferrofluids, whether it is water based or any other organic liquid based, Prandtl number is very much greater than unity. For higher values of $Pr$ from Eq. 46 it is observed that $\omega_i^2 < 0$. Hence, overstability is not a preferred mode of instability. In what follows we restrict ourselves to the case of steady onset and put $\omega_i = 0$in Eq. 38. A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish and this leads to a relation involving the parameters $R_t$, $Ma$, $R_m$, $M_1$, $M_3$, $\Lambda$, $Bi$, $Ta$, $\chi$ and $a$ in the form

$$f(R_t,\ Ma,\ R_m,\ M_1,\ M_3,\ \ \Lambda,\ Bi,\ Ta,\ \chi,\ a) = 0. \tag{47}$$

The critical values of $R_{tc}$or $Ma_c$are found as a function of wave number $a$ for various values of physical parameters. The results presented here are for $i = j = 8$ the order at which the convergence is achieved, in general.

## Results and Discussion

A linear stability analysis has been performed to investigate the influence of rotation and MFD viscosity on coupled Bénard–Marangoni ferroconvection in a rotating ferrofluid layer. The lower boundary is rigid, while the upper free boundary is open to the atmosphere at which the temperature-dependent surface tension effect is considered. The resulting eigenvalue problem is solved using the Galerkin method with either thermal Rayleigh number ($R_t$) or Marangoni number ($Ma$) as the eigenvalue for stationary convection. Computations reveal that the convergence in obtaining critical values of $Ma$ and $R_t$ with respect to the wave number crucially depends on the value of Taylor number$Ta$. For higher values of $Ta$, more number of terms in the Galerkin expansion was found to be required. The results presented here are for $i = j = 8$, the order at which the convergence is achieved, in general. The critical Marangoni number $Ma_c$is determined as a function of wave number $a$ by taking all other parameters as given. The results thus obtained for different values of physical parameters are presented in Tables 1, 2 and 3 and graphically in Figs. 2, 3, 4, 5, 6, 7, 8, 9 and 10.

In order to validate the numerical solution procedure used, first the critical values ($Ma_c$, $a_c$) obtained from the present study under the limiting conditions are compared with the previously published results of Vidal and Acrivos (1966) in Table 1. The results tabulated in Table 1 for different values of $Ta$ are for $Bi = R_t = R_m = \Lambda = \chi = 0$ which correspond to Marangoni convection for classical viscous fluids. In order to compare the results of the present analysis with those of Qin and Kaloni (1994), a new magnetic parameter $S\,(= \mu_0 K^2 \kappa\, \nu/(1+\chi)\, \rho_0 g^2 \alpha_t^2 d^4)$ was introduced in the analysis to compute the results. The critical values obtained for different values of $Ma$ with values of magnetic parameter $S\,(= 10^{-4})$ and Biot number $Bi$ (= 0, 10) are exhibited in Table 2. From the values presented in Tables 1 and 2, it is evident that there is an excellent agreement between the results of the present study and the previously published ones. This verifies the applicability and accuracy of the method used in solving the convective instability problem considered.

The tight coupling between buoyancy, surface tension, magnetic and Coriolis forces is exhibited quantitatively by tabulating the values of triplets ($R_{tc}$, $Ma_c$, $R_{mc}$) for different values of $Ta$ with $\Lambda = 0.2$, $Bi = 2$ and $\chi = 0$ in Table 3. From the table, it can be seen that an increase in the nonlinearity of fluid magnetization $M_3$ is to decrease $R_{mc}$ but only marginally and thus it has a destabilizing effect on the stability of the system. This may be due to the fact that the application of magnetic field makes the ferrofluid to acquire larger magnetization which in turn interacts with the imposed magnetic field and releases more energy to drive the flow faster. Hence, the system becomes unstable with a smaller temperature gradient as the value of $M_3$ increases. From the Table 3, we note that an increase in $M_3$ is to increase $a_c$ and hence its effect is to decrease the dimension of convection cells. The results for $R_t = 0$ correspond to those for Marangoni ferroconvection with magnetic Rayleigh number as an eigenvalue. Besides, as $M_3$ increases, $R_{mc}$ decreases and the results reduce to that of classical Bénard–Marangoni problem for ordinary viscous fluids as $M_3 \to \infty$. That is, $R_{mc} = R_{tc}$ as $M_3 \to \infty$.

The neutral stability curves ($Ma$ against$a$) for two values of$M_1$, $M_3$, $Bi$, $\Lambda$ and $\chi$ are shown in Figs. 2–6 for different values of $Ta$. The neutral curves exhibit single but different minimum with respect to the wave number $a$ and their shape is identical in the form to that of Bénard–Marangoni problem in a rotating ferrofluid layer. For increasing $M_1$ (see Fig. 2), $M_3$ (see Fig. 3), $\chi$ (see Fig. 4), decreasing $Bi$ (see Fig. 5) and $\Lambda$ (see Fig. 6), the neutral curves are slanted towards the higher wave number region. From the figures, it is also seen that increasing $Ta$ is to shift the neutral curves towards the higher wave number region. Moreover, the effect of increasing$M_1$, $M_3$ and $\chi$ as well as decreasing$Bi$, $\Lambda$and $Ta$ is to decrease the region of stability.

The salient characteristics of the physical parameters are exhibited graphically in Figs. 7–10 for various values of Taylor number $Ta$. Figures 7–10 show the locus of

**Table 1** Comparison of $Ma_c$and $a_c$ for different values of $Ta$ when $R_m = 0$, $\Lambda = 0$ and $R_t = 0$

| $Ta$ | Vidal and Acrivos (1966) | | Present study | |
|---|---|---|---|---|
| | $Ma_c$ | $a_c$ | $Ma_c$ | $a_c$ |
| 0 | 80 | 2.0 | 79.61 | 1.99 |
| $10^2$ | 92 | 2.2 | 91.31 | 2.17 |
| $10^3$ | 164 | 3.0 | 163.11 | 2.97 |
| $10^4$ | 457 | 5.0 | 456.21 | 4.99 |
| $10^5$ | 1400 | 8.6 | 1400.45 | 8.82 |

**Table 2** Comparison of critical values of $R_{tc}$ and $R_{mc}$ for different values of $Ma$ and $Bi$ when $\Lambda = 0$, $M_3 = 1$, $Ta = 0$ and $S = 10^{-4}$

| $Bi$ | $Ma$ | Present analysis | | Qin and Kaloni (1994) | |
|---|---|---|---|---|---|
| | | $R_{tc}$ | $R_{mc}$ | $R_{tc}$ | $R_{mc}$ |
| 0 | 0 | 637.875 | 40.688 | 652.87 | 42.624 |
| | 10 | 566.418 | 32.083 | 572.11 | 32.731 |
| | 30 | 416.358 | 17.335 | 414.72 | 17.199 |
| | 50 | 256.414 | 6.575 | 254.06 | 6.455 |
| | 70 | 85.9213 | 0.738 | 85.67 | 0.734 |
| | 79.61 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0 | 934.009 | 87.237 | 892.06 | 79.577 |
| | 100 | 748.641 | 56.046 | 721.01 | 51.981 |
| | 200 | 540.996 | 29.268 | 526.21 | 27.690 |
| | 300 | 306.831 | 9.414 | 301.89 | 9.114 |
| | 350 | 177.771 | 3.160 | 176.10 | 3.101 |
| | 413.44 | 0.000 | 0.000 | 0.000 | 0.000 |

**Table 3** Critical instability parameters $R_{tc}$ and $R_{mc}$ for different values of $Ma$ and $Ta$ when $\Lambda$ =0.2, $Bi = 2$ and $\chi = 0$

| $Ta$ | $Ma$ | $R_m = 0$ | | $R_t = 0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $M_3 = 1$ | | $M_3 = 15$ | | $M_3 = 25$ | | $M_3 \to \infty$ | |
| | | $R_{tc}$ | $a_c$ | $R_{mc}$ | $a_c$ | $R_{mc}$ | $a_c$ | $R_{mc}$ | $a_c$ | $R_{mc}$ | $a_c$ |
| 0 | 0 | 997.524 | 2.392 | 1256.036 | 2.453 | 1052.822 | 2.465 | 1033.453 | 2.444 | 997.524 | 2.392 |
| | 50 | 747.670 | 2.358 | 934.177 | 2.398 | 789.697 | 2.413 | 774.991 | 2.397 | 747.670 | 2.358 |
| | 100 | 478.679 | 2.348 | 592.955 | 2.371 | 505.603 | 2.382 | 496.178 | 2.373 | 478.679 | 2.348 |
| | 150 | 189.353 | 2.363 | 232.373 | 2.371 | 199.833 | 2.376 | 196.168 | 2.372 | 189.353 | 2.363 |
| | 180.815 | 0.0 | 2.386 | 0.0 | 2.386 | 0.0 | 2.386. | 0.0 | 2.386 | 0.0 | 2.386 |
| $10^2$ | 0 | 1098.155 | 2.544 | 1376.386 | 2.612 | 1153.609 | 2.611 | 1133.925 | 2.591 | 1098.155 | 2.544 |
| | 50 | 851.795 | 2.509 | 1059.861 | 2.556 | 895.292 | 2.561 | 879.862 | 2.546 | 851.795 | 2.509 |
| | 100 | 585.018 | 2.496 | 721.991 | 2.524 | 614.901 | 2.531 | 604.290 | 2.520 | 585.018 | 2.496 |
| | 150 | 296.462 | 2.463 | 362.621 | 2.519 | 311.418 | 2.523 | 306.094 | 2.518 | 296.462 | 2.463 |
| | 197.515 | 0.0 | 2.540 | 0.0 | 2.540 | 0.0 | 2.540 | 0.0 | 2.540 | 0.0 | 2.540 |
| $10^3$ | 0 | 1763.335 | 3.272 | 2153.833 | 3.373 | 1822.971 | 3.320 | 1800.921 | 3.304 | 1763.345 | 3.272 |
| | 50 | 1538.723 | 3.239 | 1869.601 | 3.320 | 1590.632 | 3.282 | 1571.433 | 3.268 | 1538.732 | 3.239 |
| | 100 | 1290.302 | 3.218 | 1558.476 | 3.280 | 1333.555 | 3.254 | 1317.549 | 3.242 | 1290.310 | 3.218 |
| | 150 | 1015.721 | 3.212 | 1218.796 | 3.255 | 1049.386 | 3.239 | 1036.916 | 3.230 | 1015.727 | 3.212 |
| | 300.275 | 0.0 | 3.307 | 0.0 | 3.307 | 0.0 | 3.307 | 0.0 | 3.307 | 0.0 | 3.307 |
| $5 \times 10^3$ | 0 | 3547.423 | 4.383 | 4616.724 | 3.566 | 3620.804 | 4.417 | 3592.942 | 4.405 | 3547.435 | 4.383 |
| | 50 | 3363.088 | 4.363 | 4355.888 | 3.566 | 3431.565 | 4.395 | 3405.549 | 4.384 | 3363.099 | 4.363 |
| | 100 | 3158.108 | 4.346 | 4075.692 | 3.566 | 3221.322 | 4.376 | 3197.289 | 4.365 | 3158.118 | 4.346 |
| | 150 | 2929.717 | 4.333 | 3774.269 | 3.566 | 2987.289 | 4.360 | 2965.387 | 4.351 | 2929.727 | 4.333 |
| | 529.256 | 0.0 | 4.559 | 0.0 | 4.559 | 0.0 | 4.559 | 0.0 | 4.559 | 0.0 | 4.559 |

the critical Marangoni number $Ma_c$ and the critical thermal Rayleigh number $R_{tc}$ for different $Bi$, $\Lambda$, $M_1$ and $\chi$ respectively. Besides, from these figures, it is obvious that the curves are slightly convex and there is a strong coupling between $R_{tc}$ and $Ma_c$, and an increase in the thermal Rayleigh number has a destabilizing effect on the system. Thus, when the buoyancy force is predominant, the surface tension force becomes negligible and vice–versa. From the figures, it is also seen that the extent to which the surface tension effect is diminished due to buoyancy force depends on the strength of rotation. The critical thermal Rayleigh $R_{tc}$ and Marangoni numbers $Ma_c$ increase with an increase in the Taylor number and this indicates the presence of Coriolis force due to rotation is to suppress the Bénard–Marangoni ferroconvection. For Taylor number $Ta \leq 10^3$, the effect of Coriolis force is not so significant, while for $Ta > 10^3$

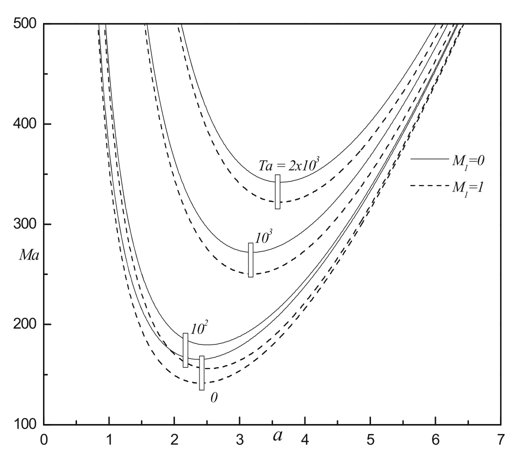


**Fig. 2** Plot of $Ma$ versus $a$ for different values of $Ta$ with two values of $M_1$ for $M_3 = 1$, $Bi = 2$, $\Lambda = 0.2$, $\chi = 0$ and $R_t = 100$

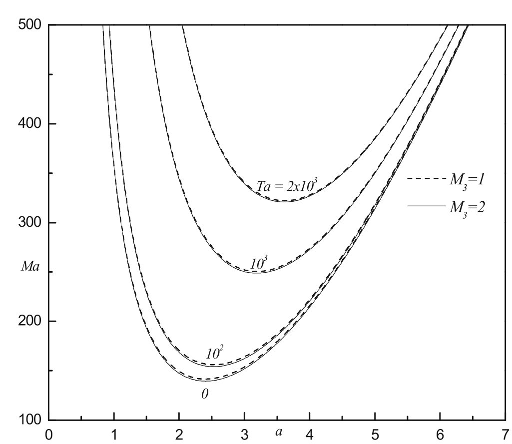


**Fig. 3** Plot of $Ma$ versus $a$ for different values of $Ta$ with two values of $M_3$ for $M_1 = 2$, $Bi = 2$, $\Lambda = 0.2$, $\chi = 0$ and $R_t = 100$

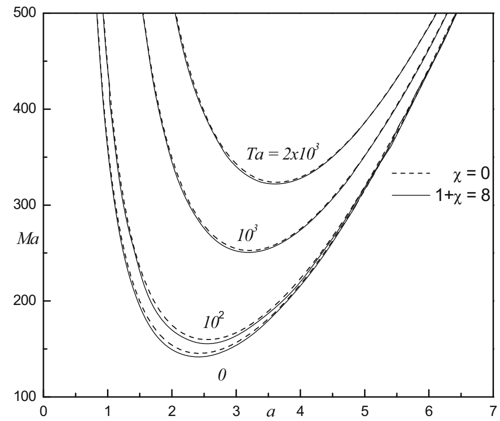


**Fig. 4** Plot of $Ma$ versus $a$ for different values of $Ta$ with two values of $\chi$ for $M_3 = 1$, $M_1 = 2$, $Bi = 2$, $\Lambda = 0.2$and $R_t = 100$

a rapid increase in the critical Rayleigh/ Marangoni number could be seen. As $Ta \to \infty$ the Bénard–Marangoni ferroconvection ceases to exist and the corresponding critical $R_{tc}$ and $Ma_c$ become infinite.

The plots in Fig. 7 represent the locus of $Ma_c$ and $R_{tc}$ for different values of $Ta$ with two values of Biot number $Bi$ (= 1 and 2) when $\Lambda = 0.2$, $M_3 = 1$, $M_1 = 2$ and $\chi = 0$. On the vertical axis, $R_{tc}$ =0 and the results correspond to those for pure Marangoni ferroconvection. From the figure it is evident that an increase in the value of heat transfer coefficient $Bi$ (i.e., Biot number) is to increase $R_{tc}$ as well as $Ma_c$ and thus its effect is to delay the onset of Bénard–Marangoni ferroconvection. This may be attributed to the fact that with increasing $Bi$, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable.

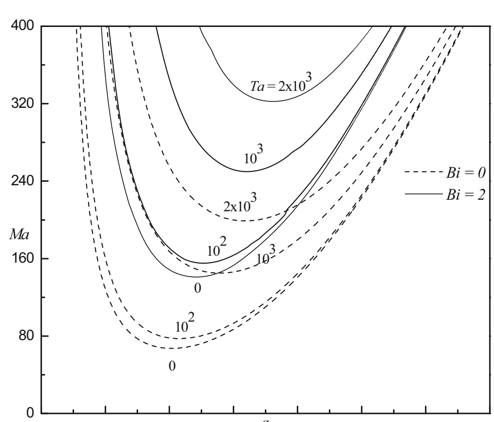


**Fig. 5** Plot of $Ma$ versus $a$ for different values of $Ta$ with two values of $Bi$ for $M_1 = 2$, $M_3 = 1$, $\Lambda = 0.2$, $\chi = 0$ and $R_t = 100$

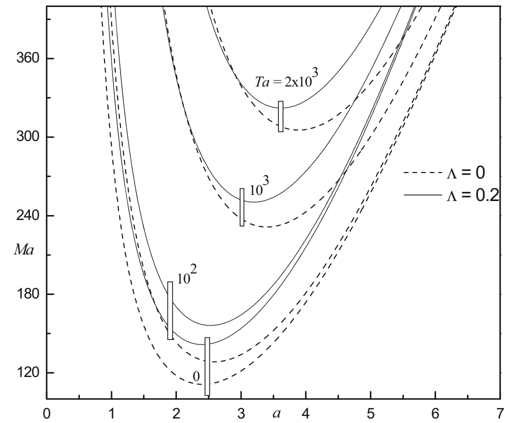


**Fig. 6** Plot of $Ma$ versus $a$ for different values of $Ta$ with two values of $\Lambda$ for $M_1 = 2$, $M_3 = 1$, $Bi = 2$, $\chi = 0$ and $R_t = 100$

Figure 8 shows the locus of $Ma_c$ and $R_{tc}$ for different values of $Ta$ with two values of MFD viscosity parameter $\Lambda$ (= 0 and 0.5) when $M_3 = 1$, $M_1 = 2$, $Bi = 2$ and $\chi = 0$. From the figure, the amount to which the surface tension effect is diminished due to $R_{tc}$ however, depends on the MFD viscosity parameter $\Lambda$. From the figure, it is seen that $R_{tc}$ and $Ma_c$ increase with an increase in the MFD viscosity parameter $\Lambda$. Thus the existence of MFD viscosity gives rise to a resistive–type force. This force has the tendency to slow down the motion of the fluid in the boundary layer, thus inducing the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of Bénard–Marangoni ferroconvection. Thus, MFD viscosity promotes stabilization.

The locus of $R_{tc}$ and $Ma_c$ is shown in Fig. 9 for different values of $Ta$ with two values of $M_1$ (= 0 and 1) when $M_3 = 1$, $Bi = 2$, $\Lambda = 0.2$ and $\chi = 0$. The size of $M_1$ is related to the importance of magnetic forces

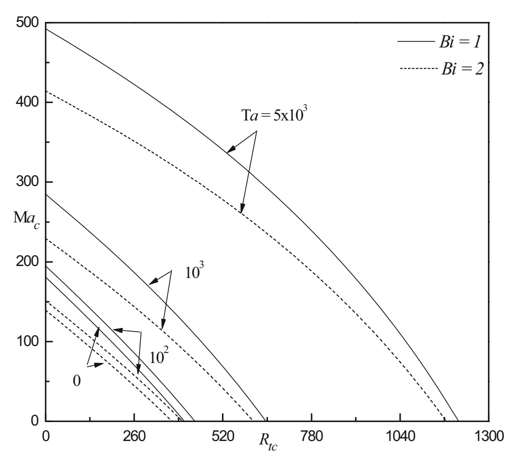


**Fig. 7** Locus of $Ma_c$ versus $R_{tc}$for different values of $Ta$ and $Bi$ when $\Lambda = 0.2$, $M_3 = 1$, $M_1 = 2$ and $\chi = 0$

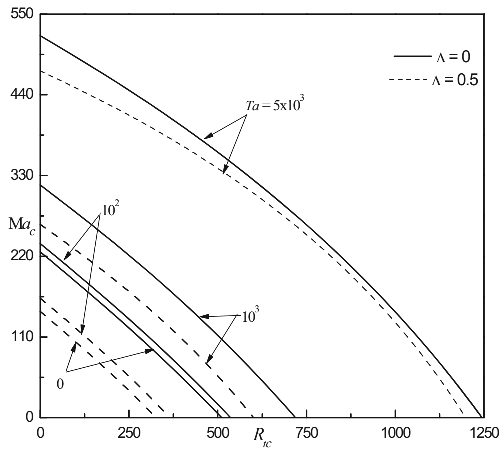


**Fig. 8** Locus of $Ma_c$ versus $R_{tc}$ for different values of $Ta$ and $\Lambda$ when $Bi = 2$, $M_3 = 1$, $M_1 = 2$ and $\chi = 0$

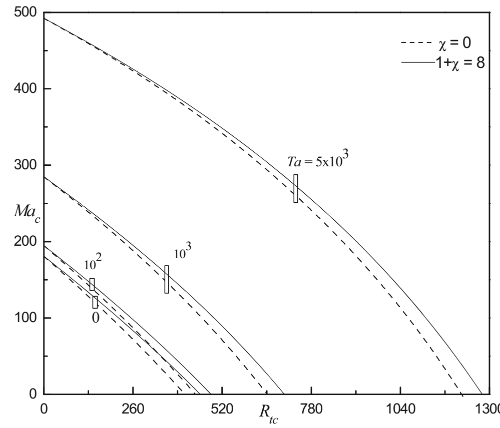


**Fig. 10** Locus of $Ma_c$ versus $R_{tc}$ for different values of $Ta$ and $\chi$ when $M_3 = 1$, $M_1 = 2$, $Bi = 2$ and $\Lambda = 0.2$

as compared to buoyancy forces. The case $M_1 = 0$ corresponds to convective instability in an ordinary viscous fluid layer. From the figure, it is seen that an increase in $M_1$ leads to decrease the values of $R_{tc}$ and $Ma_c$ suggesting that the ferrofluids carry heat more efficiently than the ordinary viscous fluids. This is due to an increase in the destabilizing magnetic force with increasing $M_1$, which favors the fluid to flow more easily. That is to say that the buoyancy and magnetic forces are complementary to each other.

Figure 10 represents the locus of $R_{tc}$ and $Ma_c$ for different values of $Ta$ with two values of $\chi$ (= 0 and 7) when $M_3 = 1$, $M_1 = 2$, $Bi = 2$ and $\Lambda = 0.2$. It is seen that $R_{tc}$ and $Ma_c$ increases with an increase in magnetic susceptibility $\chi$ and hence its effect is to delay the onset of Bénard–Marangoni ferroconvection. Moreover, the system is found to be more stable if the boundaries are paramagnetic with $\chi = 7$ as compared to the case of $\chi = 0$. This

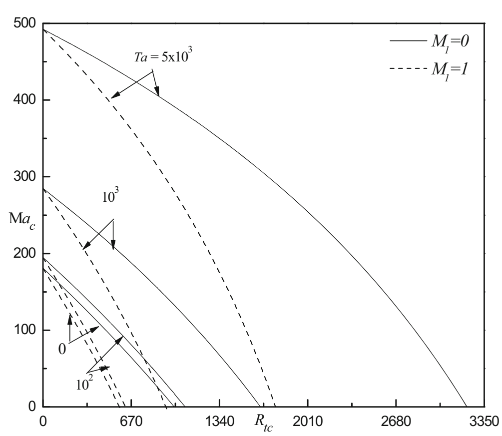


**Fig. 9** Locus of $Ma_c$ versus $R_{tc}$ for different values of $Ta$ and $M_1$ when $M_3 = 1$, $Bi = 2$, $\Lambda = 0.2$ and $\chi = 0$

result is in accordance with the one obtained by Gotoh and Yamada (1982). However, the influence of magnetic susceptibility on the stability of the system goes on diminishing as the value of $\chi$ increases.

## Conclusions

The simultaneous effect of rotation and magnetic field dependent (MFD) viscosity on the onset of Bénard–Marangoni convection in a horizontal layer of ferrofluid in the presence of a uniform vertical magnetic field is investigated. From the foregoing study, the following conclusions may be drawn:

1. The neutral stability curves for various values of physical parameters exhibit that the onset of Bénard–Marangoni problem in a rotating ferrofluid layer retains its unimodal shape with one distinct minimum which defines the critical Marangoni number and the corresponding wave number. Moreover, the effect of increasing $M_1$, $M_3$, $\chi$ as well as decreasing $\Lambda$, $Ta$ and $Bi$ is to decrease the region of stability.
2. The critical thermal Rayleigh number $R_{tc}$ and the Marangoni number $Ma_c$ increase with an increase in the Taylor number $Ta$ and this indicates the presence of Coriolis force due to rotation is to reduce the intensity of Bénard–Marangoni convection in a rotating ferrofluid layer.
3. The effect of increasing the value of Biot number $Bi$ and MFD viscosity parameter $\Lambda$ is to delay, while increasing the value of magnetic parameter $M_1$ is to advance the onset of Bénard–Marangoni ferroconvection.
5. The buoyancy force (Bénard) and surface tension force (Marnagoni) complement with each other and it is

always found that $Ma_c < R_{tc}$; a result in accordance with ordinary viscous fluids.

6. As $M_3$ increases, $R_{mc}$ decreases and the results reduce to that of classical Bénard–Marangoni problem for ordinary viscous fluid as $M_3 \rightarrow \infty$. That is, $R_{mc} = R_{tc}$ as $M_3 \rightarrow \infty$

**Acknowledgments** One of the authors (CEN) gratefully acknowledges the financial support received in the form of a VTU research grant scheme from Visvesvaraya Technological University, Belgaum, Karnataka State, India. The authors wish to thank the reviewers for their helpful suggestions.

## References

Auernhammer, G.K., Brand, H.R.: Thermal convection in a rotating layer of a magnetic fluid. The European Phys. J. B **16**, 157–168 (2000)

Blums, E.: Heat and mass transfer phenomena. In: Odenbach, S. (ed.) Ferrofluids: Magnetically Controllable Fluids and Their Applications. Springer-Verlag, Berlin, Heidelberg, New York (2002)

Chandrasekhar, S.: Hydrodynamics and Hydromagnetic Stability. Oxford University, Claredon Press, London (1961)

Das Gupta, M., Gupta, A.S.: Convective instability of a layer of a ferromagnetic fluid rotating about a vertical axis. Int. J. Eng. Sci. **17**, 271–277 (1979)

Finlayson, B.A.: Convective instability of ferromagnetic fluids. J. Fluid Mech. **40**(4), 753–767 (1970)

Ganguly, R., Sen, S., Puri, I.K.: Heat transfer augmentation using a magnetic fluid under the influence of a line dipole. J. Magn. Magn. Mater. **271**, 63–73 (2004)

Gotoh, K., Yamada, M.: Thermal convection in a horizontal layer of magnetic fluids. J. Phys. Soc. Jpn. **51**, 3042–3048 (1982)

Hennenberg, M., Weyssow, B., Slavtchev, S., Alexandrov, V., Desaive, T.: Rayleigh-Marangoni-Bénard instability of a ferrofluid layer in a vertical magnetic field. J. Magn. Magn. Mater. **289**, 268–271 (2005)

Hennenberg, M., Weyssow, B., Slavtchev, S., Scheld, B.: Coupling between stationary Marangoni and Cowley-Rosensweig instabilities in a deformable ferrofluid layer. Fluid Dyn. Mater. Proc. **1**, 101–107 (2006)

Idris, R., Hashim, I.: Effects of controller and cubic temperature profile on onset of Benard–Marangoni convection in ferrofluid. Int. Comm. Heat Mass Trans. **37**, 624–628 (2010)

Kaloni, P.N., Lou, J.X.: Weakly nonlinear instability of a ferromagnetic fluid rotating about a vertical axis. J. Magn. Magn. Mater. **284**, 54–68 (2004)

McTague, J.P.: Magnetoviscosity of magnetic colloids. J. Chem. Phys. **51**, 133–136 (1969)

Nanjundappa, C.E., Shivakumara, I.S.: Effect of velocity and temperature boundary conditions on convective instability in a ferrofluid layer. ASME J. Heat Transf. **130**, 1045021–1045025 (2008)

Nanjundappa, C.E., Shivakumara, I.S., Arunkumar, R.: Bénard–Marangoni ferroconvection with magnetic field dependent viscosity. J. Magn. Magn. Mater. **322**, 2256–2263 (2010)

Nanjundappa, C.E., Shivakumara, I.S., Arunkumar, R.: Onset of Bénard-Marangoni ferroconvection with internal heat generation. Microgravity Sci. Technol. **23**, 29–39 (2011)

Nanjundappa, C.E., Shivakumara, I.S., Arunkumar, R.: Onset of Marangoni–Bénard ferroconvection with temperature dependent viscosity. Microgravity Sci. Technol. **25**, 103–112 (2013a)

Nanjundappa, C.E., Shivakumara, I.S., Srikumar, K.: On the penetrative Benard–Marangoni convection in a ferromagnetic fluid layer. Aero. Sci. Tech. **27**, 57–66 (2013b)

Nanjundappa, C.E., Shivakumara, I.S., Savitha, B.: Onset of Bénard–Marangoni ferroconvection with a convective surface boundary condition: The effects of cubic temperature profile and MFD viscosity. Int. Comm. Heat Mass Trans. **51**, 39–44 (2014a)

Nanjundappa, C.E., Prakash, H.N., Shivakumara, I.S., Lee, J.: Effect of temperature dependent viscosity on the onset of Bénard-Marangoni ferroconvection. Int. Comm. Heat Mass Trans. **51**, 25–30 (2014b)

Nkurikiyimfura, I., Wang, Y., Pan, Z.: Heat transfer enhancement by magnetic nanofluids-A review. Ren. Sustain. Energy Rev. **21**, 548–561 (2013)

Odenbach, S.: On the stability of a free surface of a magnetic fluid under microgravity. Adv. Space Res. **22**, 1169–1173 (1998)

Odenbach, S.: Recent progress in magnetic fluid research. J. Phys. Condens. Matter **16**, 1135–1150 (2004)

Odenbach, S., Stork, H.: Shear dependence of þeld-induced contributions to the viscosity of magnetic ßuids at low shear rates. J. Magn. Magn. Mater **183**, 188–194 (1998)

Prakash, J., Gupta, S.: On arresting the complex growth rates in ferromagnetic convection with magnetic field dependent viscosity in a rotating ferrofluid layer. J. Magn. Magn. Mater. **345**, 201–207 (2013)

Qin, Y., Kaloni, P.N.: Nonlinear stability problem of a ferromagnetic fluid with surface tension effect. Eur. J. Mech. B/Fluids **13**, 305–321 (1994)

Rosensweig, R.E.: Ferrohydrodynamics. Cambridge University Press, Cambridge (1985)

Shima, P.D., Philip, J., Raj, B.: Magnetically controllable nanofluid with tunable thermal conductivity and viscosity. Appl. Phys. Letts. **95**, 1331121-3 (2009)

Shivakumara, I.S., Nanjundappa, C.E.: Effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection. Acta Mech. **182**, 113–124 (2006)

Shivakumara, I.S., Rudraiah, N., Nanjundappa, C.E.: Effect of non-uniform basic temperature gradient on Rayleigh–Bénard–Marangoni convection in ferrofluids. J. Magn. Magn. Mater **248**, 379–395 (2002)

Shivakumara, I.S., Lee, J., Nanjundappa, C.E., Ravisha, M.: Brinkman–Benard–Marangoni convection in a magnetized ferrofluid saturated porous layer. Int. J. Heat Mass Trans. **53**, 5835–5846 (2010)

Shliomis, M.I.: Magnetic fluids. Soviet Phys. Uspekhi (Engl Trans.) **17**, 153–169 (1974)

Shliomis, M.I.: Effect of viscosity of magnetic suspensions. Soviet Physics JETP **34**, 1291–1294 (1972)

Stiles, P.J., Kagan, M.J.: Thermoconvective instability of a ferrofluid in a strong magnetic field. J. Colloid Interface Sci. **134**, 435–449 (1990)

Vaidyanathan, G., Sekar, R., Ramanathan, A.: Effect of magnetic field dependent viscosity on ferroconvection in rotating medium Indian. J. Pure Applied Phys. **40**, 159–165 (2002)

Venkatasubramanian, S., Kaloni, P.N.: Effects of rotation on the thermoconvective instability of a horizontal layer of ferrofluids. Int. J. Eng. Sci. **32**, 237–256 (1994)

Vidal, A., Acrivos, A.: The influence of Coriolis force on surface tension driven convection. J. Fluid Mech. **26**, 807–818 (1966)