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Linear and weakly nonlinear triple diffusive convection in a couple stress fluid layer

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ABSTRACT

The effect of couple stresses on linear and weakly nonlinear stability of a triply diffusive fluid layer is investigated. Several departures not observed either in singly or doubly diffusive couple stress fluid layer have been identified while analyzing the linear stability of the problem. In contrast to the doubly diffusive couple stress fluid system, oscillatory convection is found to occur even if the diffusivity ratios are greater than unity. The presence of couple stress is to increase the threshold value of solute Rayleigh number beyond which oscillatory convection is preferred. Moreover, disconnected closed oscillatory neutral curves are identified for certain choices of physical parameters indicating the requirement of three critical values of Rayleigh number to specify the linear stability criteria instead of the usual single value. Besides, heart-shaped oscillatory neutral curves are also found to occur in some cases and the effect of couple stress parameter on some of these unusual behaviors is analyzed. A weakly nonlinear stability analysis is performed using modified perturbation technique and the stability of steady bifurcating non-trivial equilibrium solution is discussed. Heat and mass transfer are calculated in terms of Nusselt numbers and the influence of various physical parameters on the same is discussed in detail.

1. Introduction

Double diffusive convection is characterized by well mixed convecting layers and occurs if gradients of two competing stratifying agencies (heat and salt or any two solute concentrations) having different diffusivities are simultaneously present. One of the most interesting aspects of double diffusive instabilities is that even stabilizing overall density gradient can destabilize the system when the density gradients caused by individual components are opposed. It is observed that when the two individual diffusing components are opposed, salt fingers occur when the component with the smaller diffusivity is destabilizing, while oscillatory convection occurs when the faster diffusing component is destabilizing. The subject has attracted considerable interest over the last few decades and the significant developments took over have been largely due to its relevance and applications in many fields such as oceanography, astrophysics, geophysics and engineering. Copious literature is available on double diffusive convection and the topic has been reviewed extensively [1–7].

However, fluid dynamical systems cited above provide many examples of convective phenomena in which the density depends on three or more stratifying agencies having different molecular diffusivities. Thus one can expect multicomponent convection. As a first step towards understanding multicomponent convection, knowledge about how a triple diffusive system behaves differently from those of double diffusive systems is warranted. This is because, with the addition of a more slowly diffusing property to the bottom layer of a double diffusive system that would otherwise have produced a finger interface could cause a diffusive interface to form. Similarly, addition of the same property to the top layer of another system may change the resulting interface from a diffusive to a salt finger kind. The possibilities of existing of such interesting situations have prompted researchers to study convective instability in triple diffusive fluid systems both theoretically and experimentally.

Griffiths [8] was the first to investigate theoretically the linear stability of triple diffusive convection in a horizontal fluid layer, while Griffiths [9,10] and Turner [3] reported the related experimental works. Coriell et al. [11] and Noulty and Leaist [12] presented explicit situations in which triple diffusive convection has practical significance. Pearlstein et al. [13] performed a detailed study on the linear stability of a triply diffusive fluid layer. They have completely captured the physics of the onset of convection and showed that the triple diffusive system is capable of supporting several remarkable departures from what occurs in the singly and doubly diffusive fluid systems which were overlooked previously. Terrones and Pearlstein [14] generalized the linear stability

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Nomenclature

C_i (i = 1,2) solute concentration of the ith component
d depth of couple stress fluid layer
$\vec{g}$ acceleration due to gravity
$\hat{k}$ unit vector in the z-direction
ℓ, m wave numbers in the x and y directions
p pressure
$Pr = \nu/\kappa_t$ Prandtl number
$R_t = \beta_t g d^3 \Delta T/\nu\kappa_t$ thermal Rayleigh number
$R_{si} = \beta_{ci} g d^3 \Delta C_i/\nu\kappa_t$ (i = 1,2) solute Rayleigh number of the ith component
$\vec{q} = (u, v, w)$ velocity vector
t time
T temperature
(x,y,z) Cartesian co-ordinates

Greek symbols
α horizontal wave number
β_t thermal expansion coefficient
β_{ci} (i = 1,2) solute analog of β_t for the ith component
σ growth rate
κ_t thermal diffusivity
κ_{c1}, κ_{c2} solute analogs of κ_t
μ dynamic viscosity
μ_c couple stress viscosity
$\nu = \mu/\rho$ kinematic viscosity
$\psi(x,z,t)$ stream function
$\Lambda_c = \mu_c/\mu d^2$ couple stress parameter
ρ fluid density
ρ_0 reference density
$\tau_i = \kappa_{ci}/\kappa_t$ (i = 1,2) ratio of diffusivities of the ith component

analysis to an arbitrary number of components in a horizontal fluid layer. In the context of nonlinear stability analysis, Moroz [15] considered the linear stability problem originally discussed by Griffiths [8]. Lopez et al. [16] revealed the effect of rigid boundaries on convective instability in a triply diffusive fluid layer. The effects of cross-diffusion on the onset of convective instability in a horizontally unbounded triply cross-diffusive fluid layer have been investigated by Terrones [17]. Straughan and Walker [18] analyzed various aspects of penetrative convection in a triply diffusive fluid layer, while multicomponent convection – diffusion with internal heating or cooling in a fluid layer is considered by Straughan and Tracey [19].

All the aforementioned investigations on triple diffusive convection have been dealt with Newtonian fluids. In the study of many triply diffusive fluid dynamical problems mentioned above, the hypothesis of a Newtonian fluid will be too restrictive and cannot precisely describe the characteristics of the fluid flow involved therein. Therefore, probing the problems considering non-Newtonian effects are quite desirable and appropriate. Unlike Newtonian fluids, there are different kinds of non-Newtonian fluids and obviously they do not lend themselves to a unified treatment. In particular, polar fluids have received wider attention in recent years. These fluids deform and produce a spin field due to the microrotation of suspended particles. As far as these types of non-Newtonian fluids are concerned, there are two important theories proposed by Eringen [20] and Stokes [21] and these are, respectively, referred to as micropolar fluid theory and couple stress fluid theory. The micropolar fluids take care of local effects arising from microstructure and as well as the intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles setsup an antisymetric stress, known as couple stress, and thus forming couple stress fluid. The couple-stress fluid theory represents the simplest generalization of the classical viscous fluid theory that allows for polar effects and whose microstructure is mechanically significant in fluids. For such a special kind of non-Newtonian fluids, the constitutive equations are given by Stokes [21] which allows the sustenance of couple stresses in addition to usual stresses. This fluid theory shows all the important features and effects of couple stresses and results in equations that are similar to Navier–Stokes equations. Couple-stress fluids have applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates in a bath, exotic lubricants and colloidal fluids, electro-rheological fluids to mention a few.

Based on the formulation of Stokes [21], convective instability in a singly and doubly diffusive couple-stress fluid layer has been investigated in the recent past. Malashetty and Basavaraja [22] investigated the onset of Rayleigh–Benard convection in a layer of couple stress fluid under the influence of thermal/gravity modulation. The linear and non-linear double diffusive convection with Soret effect in couple stress liquids has been considered by Malashetty et al. [23], while Gaikwad et al. [24] reported the results on linear and non-linear double diffusive convection by considering both Soret and Dufour effects in couple stress liquids.

The effect of couple stresses on single and double diffusive fluid systems is considered in the past. Nonetheless, many fluid dynamical systems occurring in nature and engineering applications entail three or more diffusing components. Examples include molten polymers, salt solutions, slurries, geothermally heated lakes, magmas and their laboratory models, synthesis of chemical compounds and so on in which the fluid flow involved can be well characterized by couple stress fluid theory rather than Newtonian relationship. Moreover, there is all possibility of displaying variety of behavior by the fluid dynamical system, not observed in singly and doubly diffusive couple stress fluid systems, with three or more diffusing components. Under the circumstances, it is of interest to gain a general understanding of the manner in which the presence couple stresses affects the convective instability of a triply diffusive fluid layer. The main objective of the present study is therefore to investigate the effects of couple stresses on the linear and weakly nonlinear stability of a triply diffusive fluid layer and uncover the presence of couple stresses on some of the unusual behaviors of the system under certain conditions and also on the heat and mass transport.

2. Mathematical formulation

We consider an initially quiescent horizontal incompressible couple stress fluid layer of thickness d in which the density depends on three stratifying agencies namely, temperature T as well as solute concentrations C_1 and C_2 having different diffusivities. The density is assumed constant everywhere except in the body force and the off-diagonal contributions to the fluxes of the stratifying agencies are neglected. A Cartesian coordinate system (x,y,z) is used with the origin at the bottom of the fluid layer and the z-axis vertically upward. The gravity is acting vertically downwards with the constant acceleration, $\vec{g} = -g\hat{k}$ where $\hat{k}$ is the unit vector in the vertical direction. The lower boundary $z = 0$ of the fluid layer is maintained at higher temperature $T_0 + \Delta T$ and higher solute concentration $C_{i0} + \Delta C_i$ ($i = 1,2$), while the upper boundary $z = d$ is maintained at temperature T_0 and solute concentration C_{i0} ($i = 1,2$). Then the governing equations are as follows:

The continuity equation is:

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

where $\vec{q}$ is the velocity vector.

The momentum equation, following Stokes [21], is:

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right] = -\nabla p + \rho \vec{g} + \left(\mu - \mu_c \nabla^2 \right) \nabla^2 \vec{q}, \tag{2}$$

where p is the pressure, μ is the dynamic viscosity, μ_c is the couple stress viscosity, ρ is the fluid density and ρ_0 is the reference density. As propounded by Stokes [21], in case of polar fluids the action of one part of the body on its neighborhood cannot be represented by a force alone but rather by a force and couple. The last term on the right-hand side of Eq. (2) represents the effect of couple stresses in an incompressible fluid in the absence of body couples.

The energy equation is:

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = \kappa_t \nabla^2 T, \tag{3}$$

where κ_t is the thermal diffusivity.

The solute concentrations equations are:

$$\frac{\partial C_1}{\partial t} + (\vec{q} \cdot \nabla)C_1 = \kappa_{c1} \nabla^2 C_1, \tag{4}$$
$$\frac{\partial C_2}{\partial t} + (\vec{q} \cdot \nabla)C_2 = \kappa_{c2} \nabla^2 C_2, \tag{5}$$

where κ_{c1} and κ_{c2} are the solute analogs of κ_t.

The equation of state is:

$$\rho = \rho_0 [1 - \beta_t (T - T_0) + \beta_{c1}(C_1 - C_{10}) + \beta_{c2}(C_2 - C_{20})], \tag{6}$$

where β_t is the thermal volume expansion coefficient, β_{c1} and β_{c2} are the solute analogs of β_t. The basic state is quiescent and the solution for the steady-state satisfying the boundary conditions $T = T_0 + \Delta T$, $C_i = C_{i0} + \Delta C_i$ ($i = 1,2$) at $z = 0$ and $T = T_0$, $C_i = C_{i0}$ at $z = d$, is

$$\begin{aligned} &\vec{q}_b = 0, \quad T_b = T_0 + \Delta T(1 - z/d), \quad C_{ib} = C_{i0} + \Delta C_i (1 - z/d) \quad (i = 1,2), \\ &p_b(z) = p_0 - \rho_0 g \left[z - \beta_t \frac{\Delta T}{2d} z^2 + \beta_{c1} \frac{\Delta C_1}{2d} z^2 + \beta_{c2} \frac{\Delta C_2}{2d} z^2 \right], \end{aligned} \tag{7}$$

where the subscript b denotes the basic state. To study the stability of this quiescent basic state the variables are perturbed in the form

$$\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}', \quad T = T_b + T', \quad C_i = C_{ib} + C_i' \ (i = 1,2), \quad p = p_b + p', \\ &\rho = \rho_b + \rho', \end{aligned} \tag{8}$$

where $\vec{q}'$, T', C_i' $(i = 1,2)$, p' and ρ' are respectively the perturbed velocity, temperature, solute concentration pressure and fluid density. Substituting Eq. (8) into Eqs. (1)–(6) and rendering the equations to non-dimensional form by choosing d, κ_t/d, d^2/κ_t, $\mu\kappa_t/d^2$, ΔT and ΔC_i ($i = 1,2$) as the units of length, velocity, time, pressure, temperature and solute concentrations, we obtain (after dropping the primes for simplicity)

$$\nabla \cdot \vec{q} = 0, \tag{9}$$
$$\frac{1}{Pr}\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right] = -\nabla p + (R_t T - R_{s1} C_1 - R_{s2} C_2)\hat{k} + (1 - \Lambda_c \nabla^2)\nabla^2 \vec{q}, \tag{10}$$
$$\left[\frac{\partial}{\partial t} - \nabla^2\right] T + (\vec{q} \cdot \nabla) T = w, \tag{11}$$
$$\left[\frac{\partial}{\partial t} - \tau_1 \nabla^2\right] C_1 + (\vec{q} \cdot \nabla) C_1 = w, \tag{12}$$
$$\left[\frac{\partial}{\partial t} - \tau_2 \nabla^2\right] C_2 + (\vec{q} \cdot \nabla) C_2 = w, \tag{13}$$

where $R_t = \beta_t g d^3 \Delta T / \nu \kappa_t$ is the thermal Rayleigh number, $R_{s1} = \beta_{c1} g d^3 \Delta C_1 / \nu \kappa_t$ and $R_{s2} = \beta_{c2} g d^3 \Delta C_2 / \nu \kappa_t$ are the solute Rayleigh numbers, $\Lambda_c = \mu_c / \mu d^2$ is the couple stress parameter, $Pr = \nu / \kappa_t$ is the Prandtl number, $\tau_1 = \kappa_{c1}/\kappa_t$ and $\tau_2 = \kappa_{c2}/\kappa_t$ are the ratios of diffusivities. For the present configuration it is convenient to eliminate the pressure term from the equation of motion by taking the curl of Eq. (10) and to express the result in terms of the stream function $\psi(x,z,t)$ defined by

$$\vec{q} = (u, 0, w) = \left(\frac{\partial \psi}{\partial z}, 0, -\frac{\partial \psi}{\partial x} \right). \tag{14}$$

We then obtain the equations in the form

$$\left(\frac{1}{Pr} \frac{\partial}{\partial t} - \nabla^2 + \Lambda_c \nabla^4 \right) \nabla^2 \psi = -R_t \frac{\partial T}{\partial x} + R_{s1} \frac{\partial C_1}{\partial x} + R_{s2} \frac{\partial C_2}{\partial x} + \frac{1}{Pr} J(\psi, \nabla^2 \psi), \tag{15}$$
$$\left(\frac{\partial}{\partial t} - \nabla^2 \right) T = -\frac{\partial \psi}{\partial x} + J(\psi, T), \tag{16}$$
$$\left(\frac{\partial}{\partial t} - \tau_1 \nabla^2 \right) C_1 = -\frac{\partial \psi}{\partial x} + J(\psi, C_1), \tag{17}$$
$$\left(\frac{\partial}{\partial t} - \tau_2 \nabla^2 \right) C_2 = -\frac{\partial \psi}{\partial x} + J(\psi, C_2), \tag{18}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$ is the Laplacian operator and $J(\cdot\cdot,\cdot\cdot)$ stands for the Jacobian with respect to x and z.

The above equations are to be solved subject to appropriate boundary conditions. The boundaries are considered to be stress-free and perfect conductors of heat and solute concentrations. Accordingly, the boundary conditions are:

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^4 \psi}{\partial z^4} = T = C_1 = C_2 = 0 \quad \text{at} \quad z = 0, 1. \tag{19}$$

3. Linear stability analysis

The linear stability analysis proceeds by ignoring the Jacobian terms in Eqs. (15)–(18). In this section, the thresholds of both steady and oscillatory convection are predicted using linear theory. We assume the solution for ψ, T, C_1 and C_2 satisfying the respective boundary conditions in the form

$$\begin{aligned} &\psi = A_1 e^{\sigma t} \sin(\alpha x) \sin(\pi z), \\ &(T, C_1, C_2) = (A_2, A_3, A_4) e^{\sigma t} \cos(\alpha x) \sin(\pi z), \end{aligned} \tag{20}$$

where α is the horizontal wave number, σ is the growth rate and A_1 to A_4 are constants. Substituting Eq. (20) into Eqs. (15)–(18), eliminating the constants A_1 to A_4 and solving for R_t we obtain an expression in the form

$$\begin{aligned} R_t = &\frac{(\sigma + \delta^2)}{(\sigma + \tau_1 \delta^2)} R_{s1} + \frac{(\sigma + \delta^2)}{(\sigma + \tau_2 \delta^2)} R_{s2} \\ &+ \frac{\delta^2}{\alpha^2} (\sigma + \delta^2) \left\{ \frac{\sigma}{Pr} + \delta^2 + \Lambda_c \delta^4 \right\}, \end{aligned} \tag{21}$$

where $\delta^2 = \pi^2 + \alpha^2$. To investigate the stability of the system, now we set the real part of σ equal to zero and let $\sigma = i\omega$ in the above equation. After clearing the complex quantities from the denominator, Eq. (21) yields

$$R_t = \frac{(\omega^2+\tau_1\delta^4)}{(\omega^2+\tau_1^2\delta^4)}R_{s1} + \frac{(\omega^2+\tau_2\delta^4)}{(\omega^2+\tau_2^2\delta^4)}R_{s2} + \frac{\delta^2}{\alpha^2}\left\{\frac{-\omega^2}{Pr} + \delta^4(1+\Lambda_c\delta^2)\right\} + i\omega\delta^2 N, \quad (22)$$

where

$$N = \frac{(\tau_1-1)}{(\omega^2+\tau_1^2\delta^4)}R_{s1} + \frac{(\tau_2-1)}{(\omega^2+\tau_2^2\delta^4)}R_{s2} + \frac{\delta^2}{\alpha^2}\left\{\frac{1}{Pr} + (1+\Lambda_c\delta^2)\right\}. \quad (23)$$

Since R_t is a physical quantity, it implies either $\omega = 0$ ($N \neq 0$) or $N = 0$ ($\omega \neq 0$) in Eq. (22). The case $\omega = 0$ corresponds to stationary convection and the case $N = 0$ ($\omega \neq 0$) corresponds to oscillatory convection.

3.1. Stationary convection ($\omega = 0, N \neq 0$)

The stationary convection occurs at the thermal Rayleigh number $R_t = R_t^s$, where

$$R_t^s = \frac{R_{s1}}{\tau_1} + \frac{R_{s2}}{\tau_2} + \frac{(1+\Lambda_c\delta^2)\delta^6}{\alpha^2}. \quad (24)$$

For a doubly diffusive system ($R_{s2} = 0$, say), Eq. (24) reduces to

$$R_t^s = \frac{R_{s1}}{\tau_1} + \frac{(1+\Lambda_c\delta^2)\delta^6}{\alpha^2}, \quad (25)$$

which coincides with the result obtained by Malashetty et al. [23]. In the absence of couple stresses ($\Lambda_c = 0$), Eq. (24) reduces to

$$R_t^s = \frac{R_{s1}}{\tau_1} + \frac{R_{s2}}{\tau_2} + \frac{\delta^6}{\alpha^2} \quad (26)$$

and coincides with Griffiths [8] and Pearlstein et al. [13]. We note that R_t^s given by Eq. (24) attains its critical value at $\alpha^2 = \alpha_c^2$, where α_c^2 satisfies the quadratic equation

$$3\Lambda_c(\alpha_c^2)^2 + 2(\Lambda_c\pi^2+1)\alpha_c^2 - \pi^2(\Lambda_c\pi^2+1) = 0. \quad (27)$$

It is seen that the critical wave number is independent of additional diffusing components as observed in the case of classical Newtonian fluids but depends on the couple stress parameter.

We note that

$$\alpha_c^2 \sim \pi^2/2 \quad \text{as } \Lambda_c \to 0, \qquad \alpha_c^2 \sim \pi^2/3 \quad \text{as } \Lambda_c \to \infty. \quad (28)$$

Thus increase in the value of couple-stress parameter is to decrease the critical wave number. In other words, the presence of couple stresses is to increase the size of convection cells.

3.2. Oscillatory convection ($\omega \neq 0, N = 0$)

The expression $N = 0$ yields a dispersion relation of the form

$$\Delta_1(\omega^2)^2 + \Delta_2(\omega^2) + \Delta_3 = 0, \quad (29)$$

where

$$\Delta_1 = \delta^2(Pr+1+Pr\Lambda_c\delta^2), \quad (30)$$

$$\Delta_2 = \delta^6(Pr+1+Pr\Lambda_c\delta^2)(\tau_1^2+\tau_2^2) + PrR_{s1}\alpha^2(\tau_1-1) + PrR_{s2}\alpha^2(\tau_2-1), \quad (31)$$

$$\Delta_3 = \delta^{10}(Pr+1+Pr\Lambda_c\delta^2)\tau_1^2\tau_2^2 + PrR_{s1}\alpha^2\delta^4\tau_2^2(\tau_1-1) + PrR_{s2}\alpha^2\delta^4\tau_1^2(\tau_2-1). \quad (32)$$

Eq. (29) shows that, for a suitable combination of parameters R_{s1}, R_{s2}, Λ_c, τ_1, τ_2 and Pr, it is possible to have as many as two different real positive values of ω^2 at the same α. In that case, for each one of these frequency values ($\omega^2 > 0$), there is a corresponding real value of the thermal Rayleigh number on the oscillatory neutral curve. From the Descartes' rule of signs, in order for Eq. (29) to have two positive roots, it is necessary that, $\Delta_2 < 0$ and $\Delta_3 > 0$. Then from Eqs. (31) and (32) it follows, respectively, that

$$\delta^6(Pr+1+Pr\Lambda_c\delta^2)(\tau_1^2+\tau_2^2) + PrR_{s1}\alpha^2(\tau_1-1) < PrR_{s2}\alpha^2(1-\tau_2) \quad (33)$$

and

$$PrR_{s2}\alpha^2(1-\tau_2) < \delta^6(Pr+1+Pr\Lambda_c\delta^2)\tau_2^2 + PrR_{s1}\alpha^2\frac{\tau_2^2}{\tau_1^2}(\tau_1-1). \quad (34)$$

From Eqs. (33) and (34), it follows that

$$0 < \delta^6(Pr+1+Pr\Lambda_c\delta^2)\tau_1^2 < \frac{PrR_{s1}\alpha^2}{\tau_1^2}(\tau_1-1)(\tau_2^2-\tau_1^2), \quad (35)$$

which is equivalent to satisfying one of the conditions

$$\tau_2 > \tau_1 > 1 \quad \text{or} \quad \tau_2 < \tau_1 < 1. \quad (36)$$

It is interesting to note that oscillatory convection is possible even if the diffusivity ratios are greater than unity; a result which is not true in the case of double diffusive convection in a couple stress fluid layer (i.e., when $R_{s1} = 0$ or $R_{s2} = 0$). If $R_{s2} = 0$ (say) then it is observed that oscillatory convection occurs provided

$$\omega^2 = \frac{(1-\tau_1)PrR_{s1}\alpha^2}{\delta^2\{Pr(1+\Lambda_c\delta^2)+1\}} - \tau_1^2\delta^6 > 0. \quad (37)$$

Thus the necessary conditions for the occurrence of oscillatory convection are

$$\tau_1 < 1 \quad \text{and} \quad R_{s1} > \frac{\delta^6\tau_1^2\{pr(1+\Lambda_c\delta^4)+1\}}{Pr\alpha^2(1-\tau_1)}. \quad (38)$$

From Eq. (38) it is evident that in a doubly diffusive couple stress fluid layer oscillatory convection occurs only when the ratio of diffusivities is less than unity and also the solute Rayleigh number exceeds a threshold value. We also note that the presence of couple stresses is to increase the threshold value of R_{s1} for the occurrence of oscillatory convection.

We can obtain important information about the neutral stability curves in the (R_t, α)-plane by locating the bifurcation points at which the steady and oscillatory neutral curves meet. These will occur on the steady neutral curve at wave number α_b for which $\omega = 0$ is a root of Eq. (29). Thus$\Delta_3(\alpha_b) = 0$, or equivalently

$$Pr\Lambda_c(\alpha_b^2)^4 + (Pr+1+4Pr\Lambda_c\pi^2)(\alpha_b^2)^3 + 3\pi^2(Pr+1+2Pr\Lambda_c\pi^2)(\alpha_b^2)^2 + \left[3\pi^4\left(Pr+1+\frac{4}{3}Pr\Lambda_c\pi^2\right) + Pr\frac{R_{s1}}{\tau_1^2}(\tau_1-1) + Pr\frac{R_{s2}}{\tau_2^2}(\tau_2-1)\right](\alpha_b^2) + \pi^6(Pr+1+Pr\Lambda_c\pi^2) = 0. \quad (39)$$

From the above equation, it can be seen that if the coefficient of (α_b^2) is negative then there are two sign changes and hence by Descartes' rule of signs it follows that there will be either two or no bifurcation points.

When $N = 0$, from Eq. (22), we note that oscillatory convection occurs at $R_t = R_t^o$, where

$$R_t^o = \frac{(\omega^2+\tau_1\delta^4)}{(\omega^2+\tau_1^2\delta^4)}R_{s1} + \frac{(\omega^2+\tau_2\delta^4)}{(\omega^2+\tau_2^2\delta^4)}R_{s2} + \frac{\delta^2}{\alpha^2}\left\{\frac{-\omega^2}{Pr} + \delta^4(1+\Lambda_c\delta^2)\right\} \quad (40)$$

and ω^2 is given by Eq. (29). For any chosen parametric values, the critical value of R_t^o with respect to the wave number, denoted by R_{tc}^o, is determined as follows. Eq. (29) is solved first to determine the positive values of ω^2. If there are none, then no oscillatory convection is possible. If there is only one positive value of ω^2 then the critical value of R_t^o with respect to the wave number is computed numerically from Eq. (40). If there are two positive values of ω^2, then the least of R_t^o amongst these two ω^2 is retained to find the critical value of R_t^o with respect to the wave number.

4. Weakly nonlinear stability analysis

The linear theory presented in the previous section does not give any information about the stability of bifurcating finite amplitude solution. In this section, we discuss this aspect for the steady case using modified perturbation theory. For the steady case, Eqs. (15)–(18) take the form

$$(\Lambda_c\nabla^4 - \nabla^2)\nabla^2\psi + R_t\frac{\partial T}{\partial x} - R_{s1}\frac{\partial C_1}{\partial x} - R_{s2}\frac{\partial C_2}{\partial x} = \frac{1}{Pr}J(\psi, \nabla^2\psi), \quad (41)$$

$$\frac{\partial \psi}{\partial x} - \nabla^2 T = J(\psi, T), \quad (42)$$

$$\frac{\partial \psi}{\partial x} - \tau_1\nabla^2 C_1 = J(\psi, C_1), \quad (43)$$

$$\frac{\partial \psi}{\partial x} - \tau_2\nabla^2 C_2 = J(\psi, C_2). \quad (44)$$

The dependent variables (i.e., ψ, T, C_1 and C_2) and the thermal Rayleigh numbers R_t are expanded in terms of a small parameter ε identified with the amplitude, such that

$$\psi = \varepsilon\psi_1 + \varepsilon^2\psi_2 + \cdots, \qquad T = \varepsilon T_1 + \varepsilon^2 T_2 + \cdots,$$
$$C_1 = \varepsilon C_{11} + \varepsilon^2 C_{12} + \cdots C_2 = \varepsilon C_{21} + \varepsilon^2 C_{22} + \cdots,$$
$$R_t = R_t^s + \varepsilon R_{t1}^s + \varepsilon^2 R_{t2}^s + \cdots \quad (45)$$

while the other parameters $R_{s1}, R_{s2}, \Lambda_c, \tau_1$ and τ_2 are taken as given. At each stage in the expansion, a column vector may be defined as

$$\vec{\zeta}_n = [\psi_n, T_n, C_{1n}, C_{2n}]^t, \quad n = 1, 2, 3\ldots \quad (46)$$

Substituting Eqs. (45) and (46) into Eqs. (41)–(44), we note that at leading order in ε the equations are linear homogeneous and can be written in the form

$$L\vec{\zeta}_1 = 0, \quad (47)$$

where L is a self-adjoint differential operator given by

$$L = \begin{pmatrix} (\Lambda_c\nabla^4 - \nabla^2)\nabla^2 & R_t^s\frac{\partial}{\partial x} & -R_{s1}\frac{\partial}{\partial x} & -R_{s2}\frac{\partial}{\partial x} \\ -R_t^s\frac{\partial}{\partial x} & R_t^s\nabla^2 & 0 & 0 \\ R_{s1}\frac{\partial}{\partial x} & 0 & R_{s1}\tau_1\nabla^2 & 0 \\ R_{s2}\frac{\partial}{\partial x} & 0 & 0 & R_{s2}\tau_2\nabla^2 \end{pmatrix}. \quad (48)$$

Eq. (47) represents the eigenvalue problem which is discussed in Section 3.1. The eigenvalue is

$$R_t^s = \frac{R_{s1}}{\tau_1} + \frac{R_{s2}}{\tau_2} + \frac{(1 + \Lambda_c\delta^2)\delta^6}{\alpha^2} \quad (49)$$

and the eigenfunction is

$$\vec{\zeta}_1 = 2\sqrt{2}\left(\frac{\delta}{\alpha}\right)\begin{pmatrix} \sin\alpha x \sin\pi z \\ \frac{\alpha}{\delta^2}\cos\alpha x \sin\pi z \\ \frac{\alpha}{\delta^2}\cos\alpha x \sin\pi z \\ \frac{\alpha}{\delta^2}\cos\alpha x \sin\pi z \end{pmatrix}, \quad (50)$$

where the normalization is chosen for subsequent convenience. Since the operator L is self-adjoint, the identity

$$\left\langle \vec{\zeta}_1^t, L\vec{\zeta}_n \right\rangle = \left\langle \vec{\zeta}_n^t, L\vec{\zeta}_1 \right\rangle = 0 \quad (51)$$

holds for all n, where $\langle\cdots\rangle = \int_0^1 \int_0^{\pi/\alpha}(\cdots)dx\,dz$.

To the second order in ε, the equations are inhomogeneous and are given by

$$L\vec{\zeta}_2 = \begin{pmatrix} R_{t1}^s\frac{\partial T_1}{\partial x} - \frac{J(\psi_1, \nabla^2\psi_1)}{Pr} \\ R_t^s J(\psi_1, T_1) \\ R_{s1}J(\psi_1, C_{11}) \\ R_{s2}J(\psi_1, C_{21}) \end{pmatrix}. \quad (52)$$

Using Eq. (50), it is found that $J(\psi_1, \nabla^2\psi_1) = 0$. The solvability condition on this in-homogeneous equation is obtained by applying Eqs. (51) and (52) and then it is found that $R_{t1}^s = 0$ as expected from the symmetry of the problem. If we impose the orthogonality condition

$$\left\langle \vec{\zeta}_1^t, L\vec{\zeta}_n \right\rangle = 0 \ (n \neq q1), \quad (53)$$

then the solution of Eq. (52) is

$$\vec{\zeta}_2 = \begin{pmatrix} 0 \\ -\frac{1}{\pi}\sin 2\pi z \\ -\frac{1}{\tau_1^3\pi}\sin 2\pi z \\ -\frac{1}{\tau_2^3\pi}\sin 2\pi z \end{pmatrix}. \quad (54)$$

At third order, now we have

$$L\vec{\zeta}_3 = \begin{pmatrix} R_{t2}^s\frac{\partial T_1}{\partial x} \\ R_t^s J(\psi_1, T_2) \\ R_{s1}J(\psi_1, C_{12}) \\ R_{s2}J(\psi_1, C_{22}) \end{pmatrix}. \quad (55)$$

From Eqs. (51) and (55), the solvability condition yields

$$R_{t2}^s = \frac{(\tau_1^2 - 1)}{\tau_1^3}R_{s1} + \frac{(\tau_2^2 - 1)}{\tau_2^3}R_{s2} + \frac{(1 + \Lambda_c\delta^2)\delta^6}{\alpha^2}. \quad (56)$$

This is the first non-trivial finite amplitude Rayleigh number and note that R_{t2}^s may be either positive or negative. The finite amplitude solution is said to be stable (i.e. supercritical) if $R_{t2}^s > 0$ and unstable (i.e. sub-critical) if $R_{t2}^s < 0$ when $\omega^2 < 0$. In the absence of additional diffusing components (i.e.,$R_{s1} = 0 = R_{s2}$), we find that $R_{t2}^s = R_t^s$, and hence sub-critical instability is not possible. For double diffusive Newtonian fluid case (i.e., $\Lambda_c = 0 = R_{s2}$), Eq. (56) coincides with that of Nagata and Thomas [25].

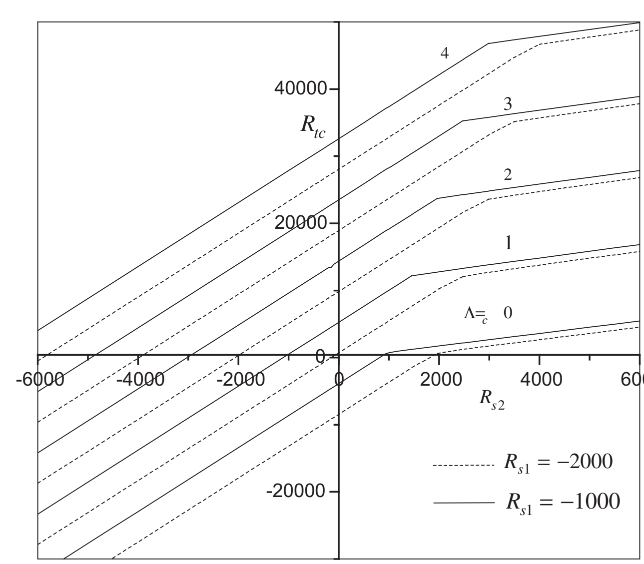

Fig. 1. Variation of R_{tc} as a function of R_{s2} for different values of Λ_c for $Pr = 10.2$, $\tau_1 = 0.22$ and $\tau_2 = 0.21$.

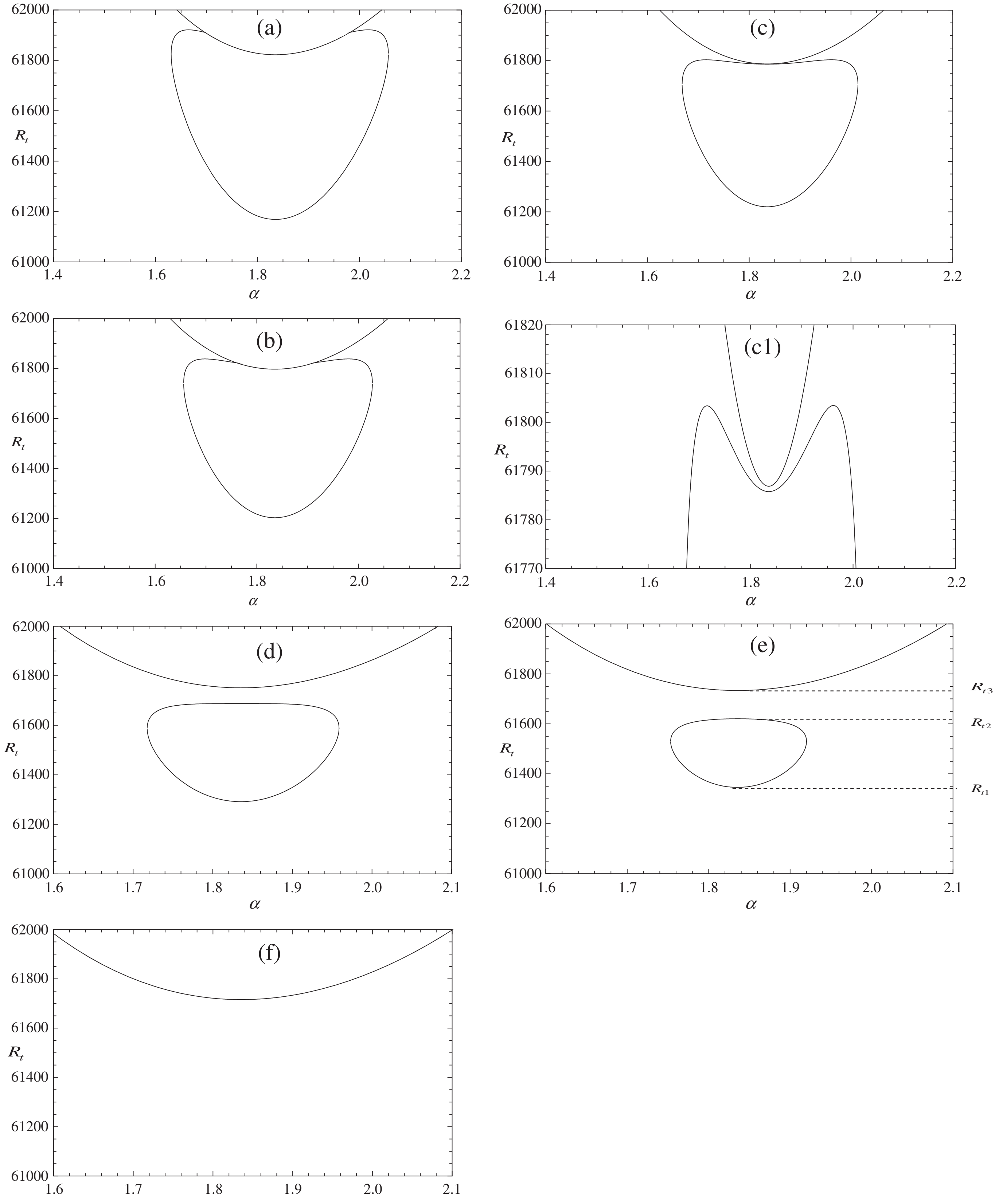

Fig. 2. Neutral curves for various R_{s2} with R_{s1} = 43000, Pr = 625, τ_1 = 0.8125, τ_2 = 0.28125 and Λ_c = 1: (a) R_{s2} = −255 (b) R_{s2} = −262 (c) R_{s2} = −265 (c1) R_{s2} = −265 (expanded) (d) R_{s2} = −275 (e) R_{s2} = −280 (f) R_{s2} = −285.

5. Heat and mass transport

The vigor of convection can be measured in terms of either heat/mass flux. However, it is convenient to introduce a normalized heat/ mass flux, given by the Nusselt numbers

$$Nu_t = -\left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0}, \tag{57}$$

where $T_{total} = 1 - z + T = 1 - z + \varepsilon T_1 + \varepsilon^2 T_2 + \cdots$ and the angular brackets denote the horizontal average. Substituting for T_1 and $T_2$from Eqs. (50) and (54), respectively then Eq. (57) gives

$$Nu_t = 1 + \frac{2(R_t - R_t^s)}{R_{t2}^s}. \tag{58}$$

Similarly, the solute Nusselt numbers are defined and are given by

$$Nu_{s1} = 1 + \frac{2(R_t - R_t^s)}{R_{t2}^s \tau_1^3}, \tag{59}$$

$$Nu_{s2} = 1 + \frac{2(R_t - R_t^s)}{R_{t2}^s \tau_2^3}. \tag{60}$$

In the absence of convection (i.e., $R_t = R_t^s$), the heat/mass transfer is only by conduction and in that case $Nu_t = 1 = Nu_{s1} = Nu_{s2}$.

6. Results and discussion

The effects of couple stresses on linear and weakly nonlinear stability of a triply diffusive fluid layer are investigated. Several departures not observed either in singly or doubly diffusive couple stress fluid layer have been identified in analyzing the linear stability problem. A study on the weakly nonlinear stability of the system indicates that subcritical bifurcation is possible for a certain choice of physical parameters and the heat and mass transport are calculated in terms of Nusselt numbers.

To know the effect of couple stress on the linear stability of the system, the critical Rayleigh numbers R_t^c (least among R_{tc}^s and R_{tc}^o) calculated for different values of Λ_c are shown in Fig. 1 as a function of R_{s2} for two values of $R_{s1} = -1000$ and -2000 by setting the property ratios at $Pr = 10.2$, $\tau_1 = 0.22$ and $\tau_2 = 0.21$. The negative value of R_{s1} or R_{s2} corresponds to the respective component is destabilizing. From the figure it is seen that R_t^c is a piecewise linear function of R_{s2}. The portion of each stability boundary lying to the right of the discontinuity in slope corresponds to oscillatory convection, while to the left the bifurcation is of direct type (steady convection). The slope of R_t^c against R_{s2} plot is more pronounced with increasing Λ_c and $|R_{s1}|$. The lowest locus for two values of R_{s1} considered corresponds to the Newtonian fluid case (i.e., $\Lambda_c = 0$). Thus the presence of couple stress is to stabilize the fluid against both steady and oscillatory convection. Besides, increasing the negative value of R_{s1} from -1000 to -2000 is to hasten the onset of convection irrespective of R_{s2} is positive or negative. Further inspection of the figures reveals that for negative values of R_{s2} only steady convection is preferred; while for positive values of R_{s2} both steady and oscillatory types of convection are possible. The value of R_{s2} at which the preferred mode of instability changes is increased with increasing Λ_c and $|R_{s1}|$. Thus the presence of an additional stabilizing/destabilizing diffusing component and couple stress exhibits a profound influence on the stability characteristics of the system.

It is important to study systematically the topology of neutral curves to unveil some of the striking features of a triply diffusive couple stress fluid dynamical system. Necessary information about the same can be obtained by locating the bifurcation points. A bifurcation point on the stationary neutral curve is one at which the oscillatory neutral curve intersects with the stationary neutral curve and the frequency on the oscillatory neutral curve approaches zero as the intersection is approached. For chosen parametric values, the bifurcation points can be located by solving Eq. (39). There may exist zero, one or two bifurcation points. Accordingly, three types of neutral curves are identified in the (R_t, α) plane depending on the choices of physical parameters namely (i) only stationary neutral curve (ii) both stationary and oscillatory neutral curves connecting at one or two bifurcation points and (iii) a stationary neutral curve with disconnected closed oscillatory neutral curve having no bifurcation points.

Fig. 2(a)–(f) exhibit the successive neutral stability curves for various negative values of R_{s2} ranging from -255 to -285 for the transport property ratios $Pr = 625$, $\tau_1 = 0.8125$, $\tau_2 = 0.28125$ which are appropriate for an aqueous NaCl–KCl – Sucrose system [8]. The results presented here are for $R_{s1} = 43000$ and $\Lambda_c = 1$. Fig. 2(a)–(c) display the neutral curves for $R_{s2} = -255$, -262 and -265, respectively. It is seen that the oscillatory neutral curve is connected to the stationary neutral curve at two bifurcation points initially which move closer together as $|R_{s2}|$ is increased and in fact detaches from the stationary neutral curve. This fact is made clear in Fig. 2(c1) which is on the expanded scale of Fig. 2(c). This figure shows that the oscillatory neutral curve gets disconnected from the stationary neutral curve. Besides, the oscillatory neutral curve

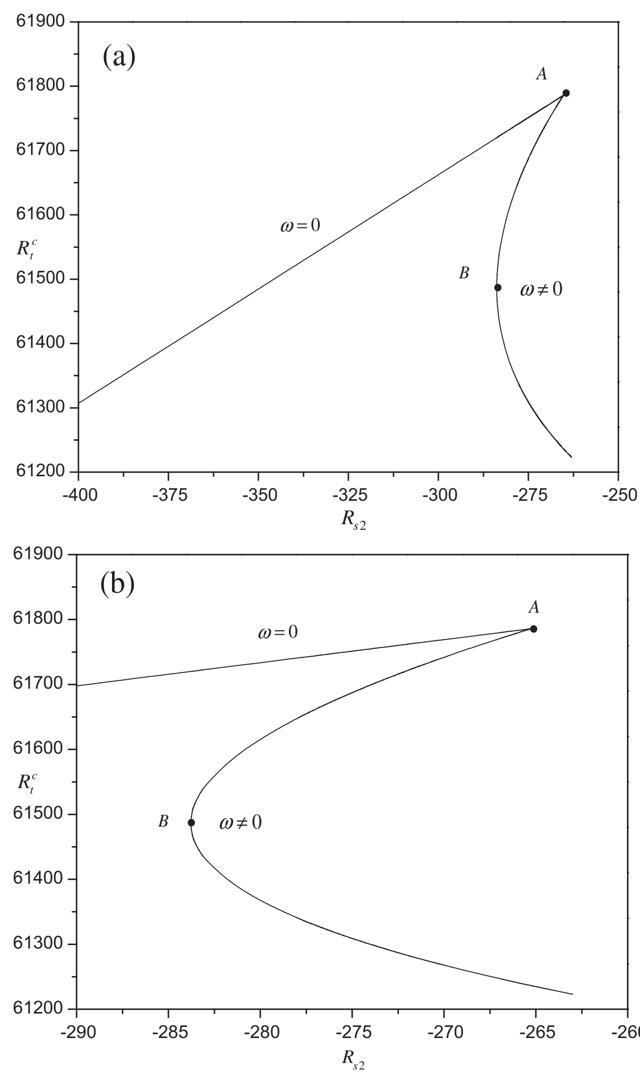

Fig. 3. Stability boundary for $R_{s1} = 43000$, $Pr = 625$, $\tau_1 = 0.8125$, $\tau_2 = 0.28125$, $\Lambda_c = 1$. (b) Expanded view of the multivalued region.

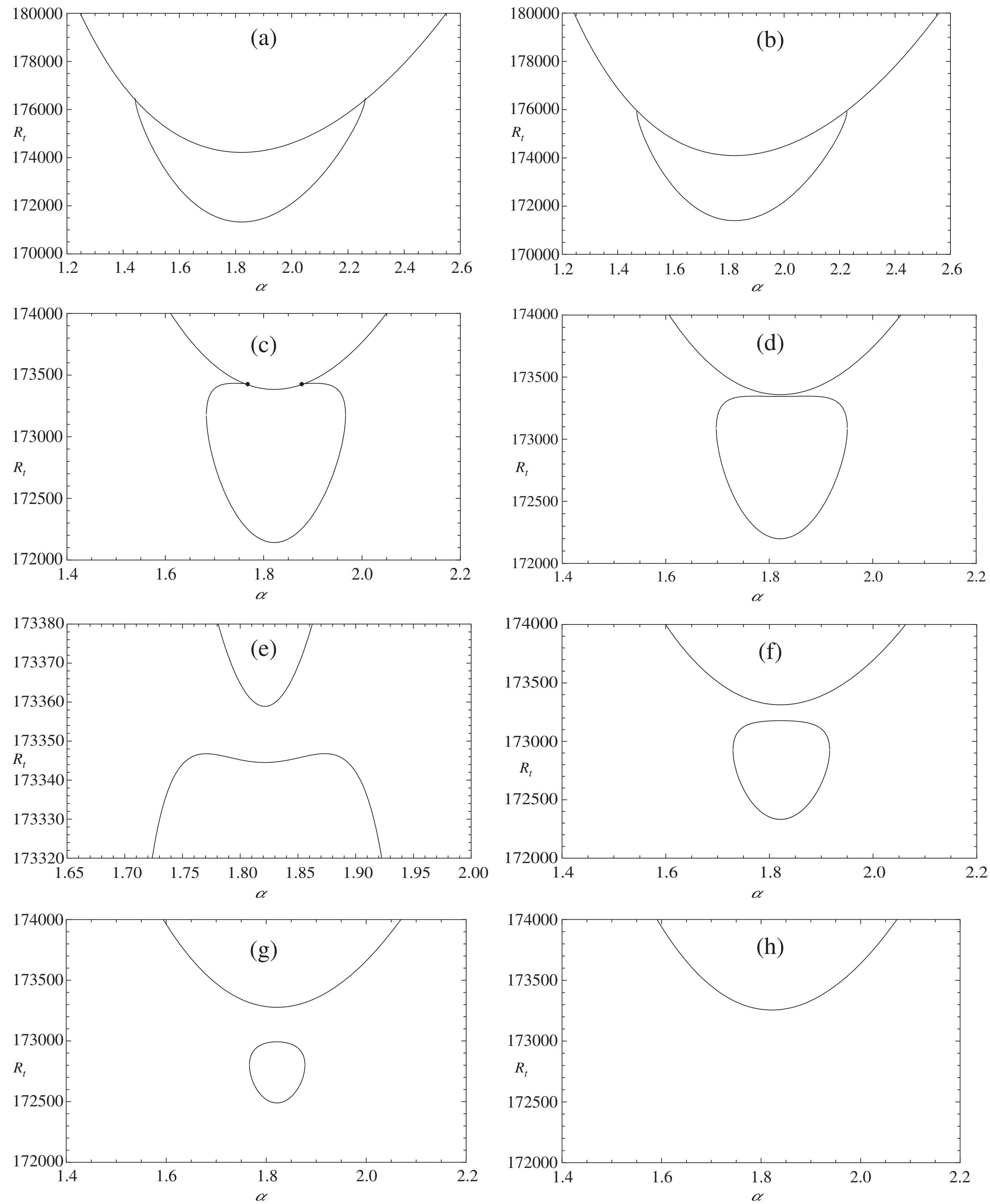

Fig. 4. Neutral curves for various R_{s2} with R_{s1} = 120000, Pr = 625,τ_1 = 0.8125, τ_2 = 0.28125, Λ_c = 3: (a) R_{s2} = −425 (b) R_{s2} = −460 (c) R_{s2} = −660 (d) R_{s2} = −667 (e) R_{s2} = −667 (expanded) (f) R_{s2} = −680 (g) R_{s2} = −690 (h) R_{s2} = −696.

loses its single-valued character in certain range of wave number (two onset frequencies exist for a fixed value of wave number) with twin maxima moving above the minimum of the stationary neutral curve. Beyond the twin maxima there is only one value of R_t and only one onset frequency. Till this stage the linear stability of the system can be conveniently determined by a single value of R_t. With further increase in the value of $|R_{s2}|$, the closed heart-shaped disconnected oscillatory neutral curve with twin maxima moves further below the minimum of the stationary neutral curve as shown in Fig. 2(d) and (e) for R_{s2} = −275 and −280, respectively. Thus there are two distinct disturbances, with incommensurable values of the frequency and wave number that simultaneously become unstable at the same Rayleigh number. The significance of this type of disconnected oscillatory neutral curve is the requirement of three critical values of R_t to specify the linear stability criteria instead of the usual single value. From Fig. 2(e) it is seen that there is a range of thermal Rayleigh numbers $R_{t2} < R_t < R_{t3}$ for which all solutions, oscillatory or steady, of the linear disturbance equations are stable at any wave number. Thus, the linear stability criteria involve three values of R_t and may be stated as follows. For $R_t < R_{t1}$, and $R_{t2} < R_t < R_{t3}$, the layer is linearly stable. For $R_{t1} < R_t < R_{t2}$, and $R_t > R_{t3}$, the layer is unstable. Fig. 2(f) shows that the closed oscillatory neutral curve collapses to a point and subsequently disappears, leaving only the stationary neutral curve when $R_{s2} = -285$.

The stability boundaries for the same transport ratios considered in Fig. 2 ($Pr = 625, \tau_1 = 0.8125, \tau_2 = 0.28125$ and $\Lambda_c = 1$) are shown in Fig. 3(a) and (b). The value of R_{s2} is varied and R_{s1} is fixed at 43000. As can be seen from Fig. 3(a), the stability boundary can be viewed separately in three regions depending on the values of R_{s2}. To the right of the cusp (A) the onset is of oscillatory type for $R_{s2} > -265$ and moreover it occurs at a lower single value of R_t^c than does stationary instability. To the left of the point of infinite slope (B) the onset is of steady type and again there is a single value of R_t^c. In Fig. 3(b), the region between the points A and B is shown enlarged. It is observed that the single-valued stationary and oscillatory portions of the stability boundary do not merely intersect, but instead give rise to a multivalued (R_t^c, R_{s2})-curve. For $-283.7832 < R_{s2} < -265$, it is seen that three critical values of R_t^c are needed to specify the linear stability criteria.

Fig. 4(a)–(h) display the evolution of neutral curves for the property ratios $Pr = 625$, $\tau_1 = 0.8125$, $\tau_2 = 0.28125$ considered earlier but for an increased value of $\Lambda_c = 3$. For this case, the pattern of disconnected neutral curves was observed when R_{s1} takes the value 120000 and R_{s2} is varied ranging from -425 to -696. The oscillatory neutral curve is connected to the stationary neutral curve at

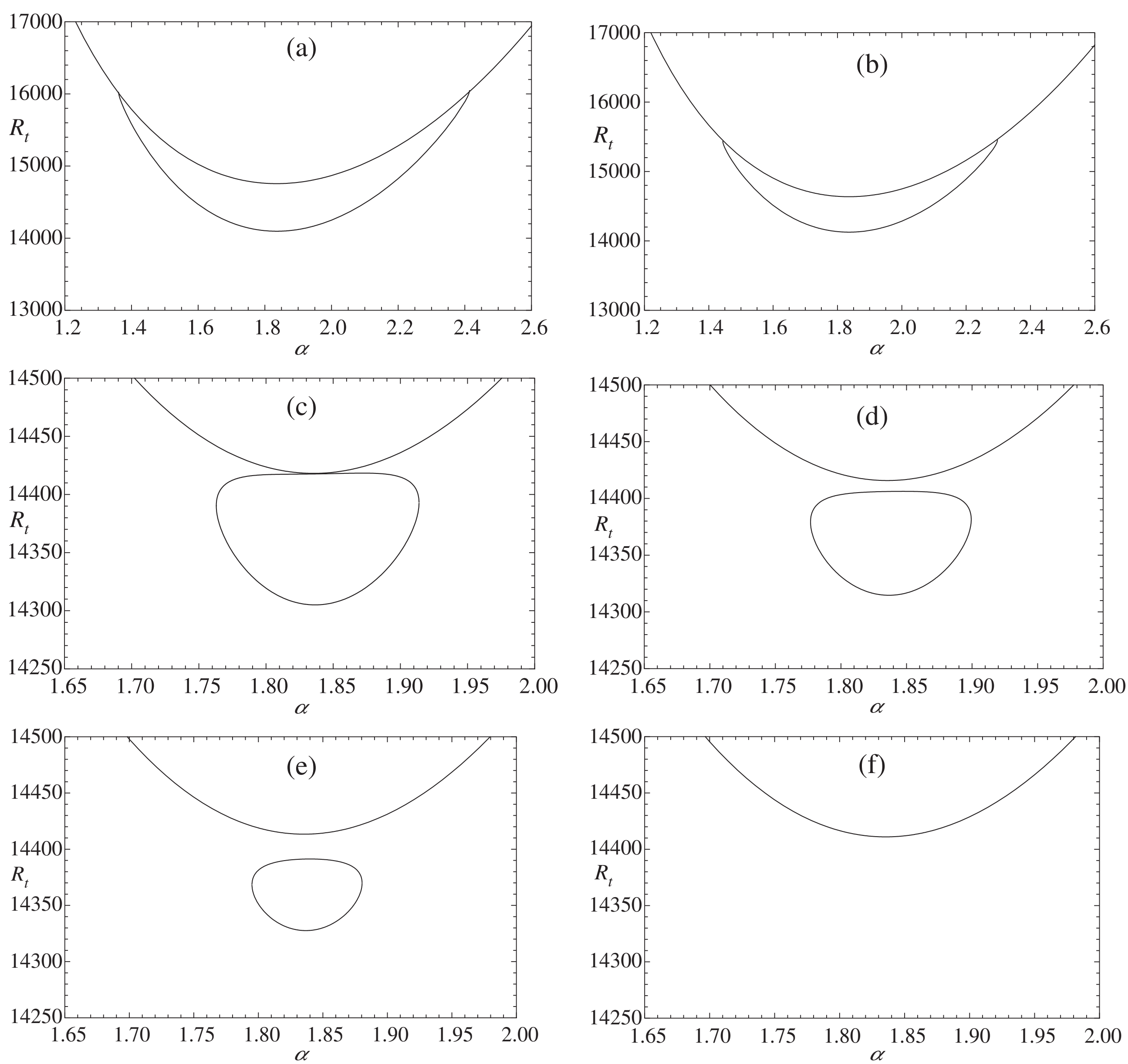

Fig. 5. Neutral curves for various R_{s2} with $R_{s1} = 7600$, $Pr = 10.2$, $\tau_1 = 0.22$, $\tau_2 = 0.21$ and $\Lambda_c = 1$: (a) $R_{s2} = -6215$ (b) $R_{s2} = -6240$ (c) $R_{s2} = -6286$ (d) $R_{s2} = -6286.5$ (e) $R_{s2} = -6287$ (f) $R_{s2} = -6287.5$.

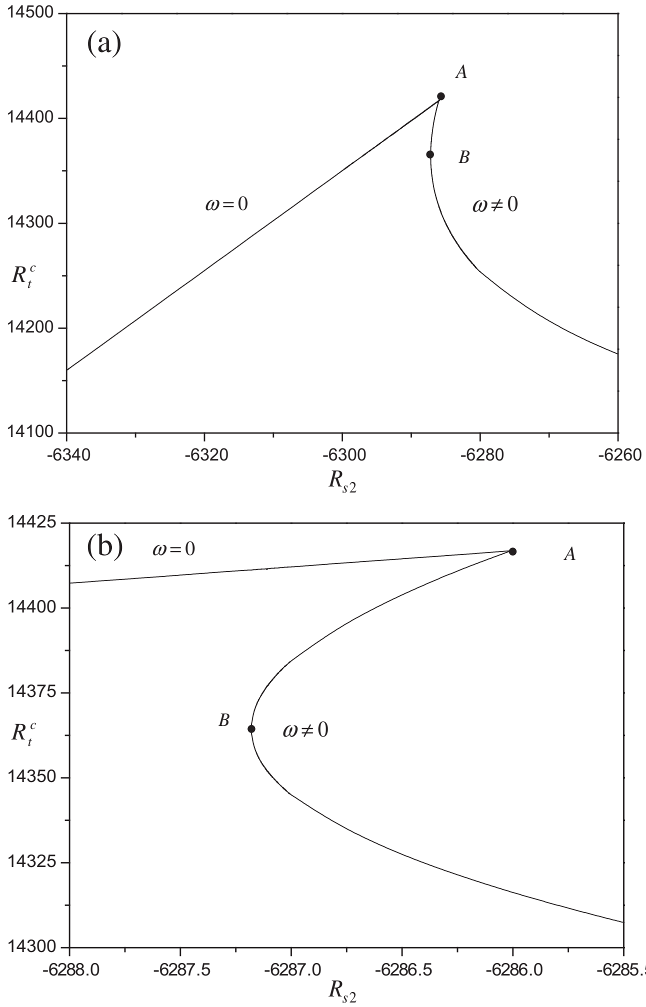

Fig. 6. (a) Stability boundary for $R_{s1} = 7600$, $Pr = 10.2$, $\tau_1 = 0.22$, $\tau_2 = 0.21$, $\Lambda_c = 1$. (b) Expanded view of the multivalued region.

two bifurcation points (Fig. 4(a)–(c)) and gets disconnected with increasing $|R_{s2}|$ (Fig. 4(c)–(g)). In contrast to $\Lambda_c = 1$ case, the oscillatory neutral curve loses its heart shape as the value of $|R_{s2}|$ is increased and becomes a closed convex curve in the process. The closed convex oscillatory neutral curve goes on diminishing and eventually disappears leaving behind only the stationary neutral curve at $R_{s2} = -696$ (Fig. 4(h)).

The evolution of neutral curves for another set of transport property ratios $Pr = 10.2$, $\tau_1 = 0.22$, $\tau_2 = 0.21$ and $\Lambda_c = 1$ is illustrated in Fig. 5(a)–(f). Here R_{s1} is fixed at 7600 and R_{s2} is varied ranging from -6215 to -6287.5. The scenario is same as the one observed in Fig. 4(a)–(h). Initially, the oscillatory and stationary neutral curves are connected at two bifurcation points (Fig. 5(a) and (b)) and with increasing $|R_{s2}|$ they get disconnected (Fig. 5(c)–(e)). It is also seen that the oscillatory neutral curve loses its heart shape and becomes a closed convex curve. The closed convex oscillatory neutral curve goes on diminishing and eventually disappears leaving behind only the stationary neutral curve at $R_{s2} = -6287.5$ (Fig. 5(f)). The corresponding stability boundary is presented in Fig. 6(a) and (b). In this case, the onset is of oscillatory type for $R_{s2} > -6285.9$ and steady for $R_{s2} < -6287.1806$. However, three critical values of R_t^c are needed to specify the linear stability criteria for $-6287.1806 < R_{s2} < -6285.9$.

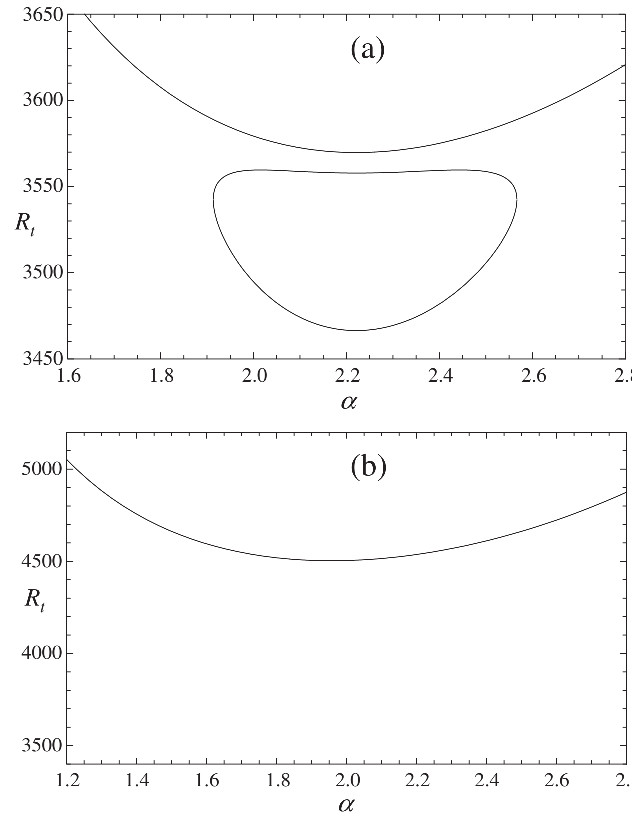

Fig. 7. Neutral curves (a) $\Lambda_c = 0$ (Newtonian fluid case) (b) $\Lambda_c = 0.1$ for $Pr = 464$, $\tau_1 = 0.5907$, $\tau_2 = 0.5670$, $R_{s1} = 13730.72$ and $R_{s2} = -11528.58$.

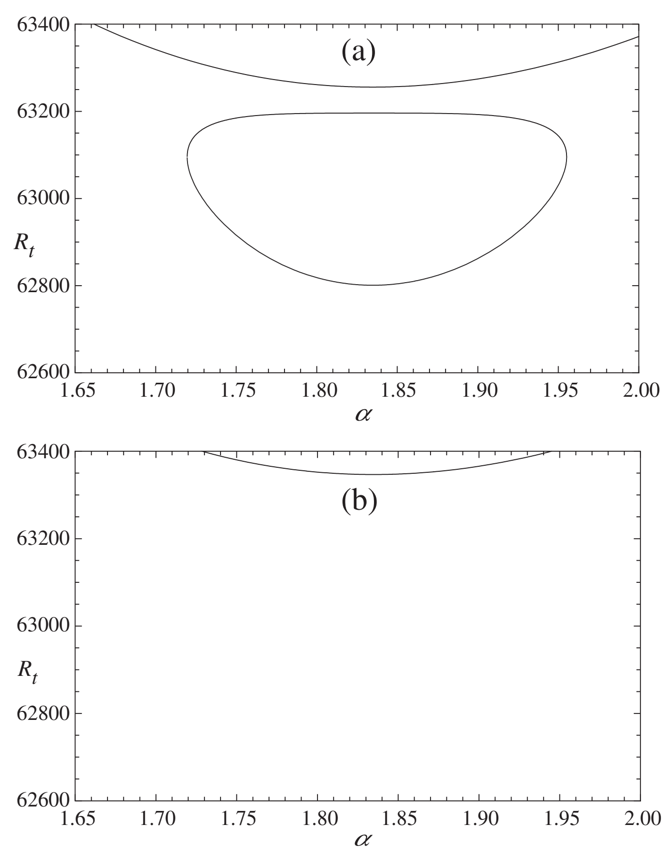

Fig. 8. Neutral curves for $R_{s1} = 44000$, $R_{s2} = -275$, $Pr = 625$, $\tau_1 = 0.8125$, $\tau_2 = 0.28125$ with (a) $\Lambda_c = 1.03$ (b) $\Lambda_c = 1.04$.

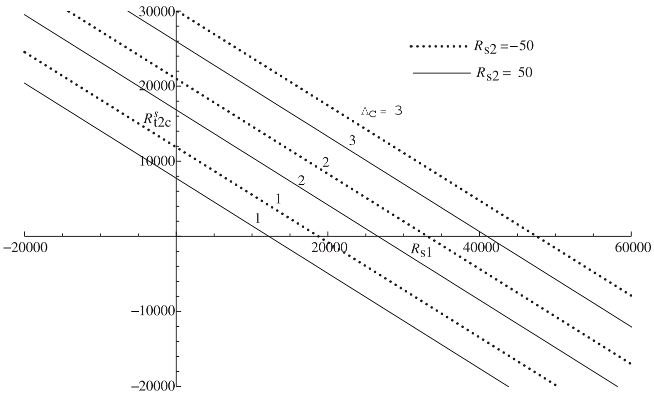

Fig. 9. Variation of R^s_{t2c} as a function of R_{s1} for $\tau_1 = 0.8125$, $\tau_2 = 0.28125$.

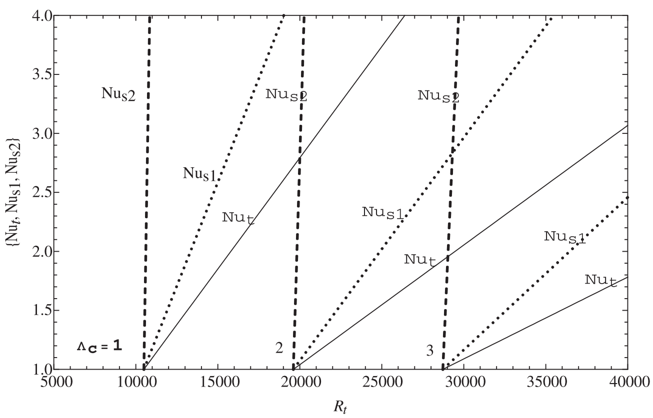

Fig. 10. Variation of Nusselt numbers for $R_{s1} = 2000$, $R_{s2} = -500$, $\tau_1 = 0.8125$ and $\tau_2 = 0.28125$ for different values of Λ_c.

The sensitivity of the system to the presence of couple stress is demonstrated in Figs. 7 and 8 for two sets of parameters. Fig. 7(a) shows disconnected heart-shaped oscillatory neutral curve in the (R_t, α)-plane for a Newtonian fluid ($\Lambda_c = 0$) for the parameters $Pr = 464$, $\tau_1 = 0.5709$, $\tau_2 = 0.5670$, $R_{s1} = 13730.72$ and $R_{s2} = -11528.58$. Whereas for the same parametric values in the presence of couple stress with $\Lambda_c = 0.1$, it is seen that the onset of convection switches over to stationary as shown in Fig. 7(b). Fig. 8(a) and (b) illustrate the effect of small variation in Λ_c on the stability characteristic of the system for $Pr = 10.2$, $\tau_1 = 0.22$, $\tau_2 = 0.21$, $R_{s1} = 44000$ and $R_{s2} = -275$. It is seen that the oscillatory neutral curve is disconnected from the stationary one when $\Lambda_c = 1.03$ (Fig. 8(a)) but it disappears when $\Lambda_c = 1.04$ (Fig. 8(b)) indicating the instability sets in via stationary convection.

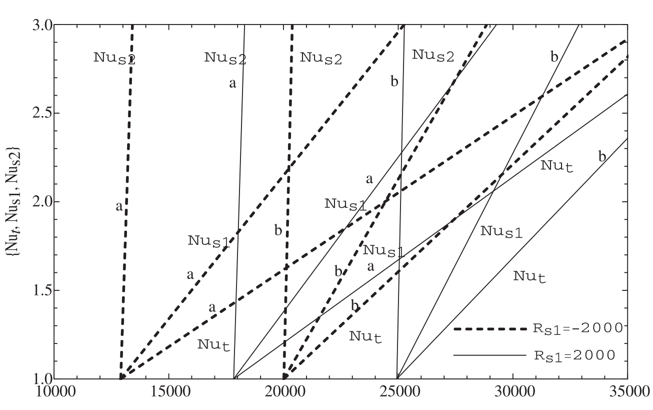

Fig. 11. Variation of Nusselt numbers for $\tau_1 = 0.8125$, $\tau_2 = 0.28125$ and $\Lambda_c = 2$ for two values of $R_{s2} = -1000$ (denoted by a) and 1000 (denoted by b).

The critical value of steady bifurcating non-trivial equilibrium solution R^s_{t2} obtained with respect to α_c for a fixed value of Λ_c is denoted by R^s_{t2c}. In Fig. 9, R^s_{t2c} is plotted as a function of R_{s1} for $R_{s2} = -50$ and 50, $\Lambda_c = 1$, 2 and 3 when $\tau_1 = 0.8125$ and $\tau_2 = 0.28125$. From the figure it is observed that the bifurcation of non-trivial equilibrium steady solution becomes subcritical at higher values of R_{s1} with increasing Λ_c. Besides, the range of R_{s1} beyond which R^s_{t2c} is negative increases when R_{s2} is destabilizing ($R_{s2} < 0$), while opposite is the trend if R_{s2} is stabilizing. Moreover, the non-trivial equilibrium steady solution bifurcates always super critically when $R_{s1} \leqslant 0$.

The variation of Nusselt numbers Nu_t, Nu_{s1} and Nu_{s2} as a function of Rayleigh number R_t is shown in Figs. 10 and 11 for the property ratios $\tau_1 = 0.8125$ and $\tau_2 = 0.28125$. From the figures it is observed that the Nusselt numbers increase with increasing thermal Rayleigh number. Fig. 10 shows the results for different values of Λ_c when $R_{s1} = 2000$ and $R_{s2} = -500$. The results for $\Lambda_c = 0$ corresponds to the Newtonian fluid case. It is clear that the Nusselt numbers decrease with increasing Λ_c. Thus, the effect of increasing couple stress parameter is to suppress convection and hence to decrease the rate of heat and mass transfer. Fig. 11 illustrates the results for $R_{s1} = 2000$ and -2000 respectively for two values of $R_{s2} = -1000$ and 1000 and for a fixed value of $\Lambda_c = 2$. From the figures it is obvious that heat and mass transfer are increased when the diffusing components are destabilizing. Also, it is observed that $Nu_t < Nu_{s1} < Nu_{s2}$.

7. Conclusions

The effect of couple stresses on linear and weakly nonlinear triple diffusive convection is investigated. It is observed that the presence of couple stresses show a significant influence on the preferred mode of bifurcation. Even small variations in the couple stress parameter allow the onset to be via stationary convection rather than oscillatory convection and vice versa. This reiterates the importance of considering couple stresses in the study of convective instability problems. The results of the foregoing study may be summarized as follows:

(i) Oscillatory convection is possible even if the diffusivity ratios are greater than unity; a result of contrast compared to doubly diffusive fluid systems. The presence of couple stress is to increase the threshold value of solute Rayleigh number for the existence of oscillatory convection.
(ii) Requirement of three critical Rayleigh numbers to specify the linear stability criteria instead of the usual single value. That is, existence of two non-zero frequencies at the same wave number.
(iii) Existence of a heart-shaped oscillatory neutral curve with twin maxima, in some cases. That is, occurrences of two distinct disturbances with incommensurable values of the frequency and wave number which simultaneously become unstable at the same Rayleigh number. This corresponds to the multivalued nature of the stability boundaries.
(iv) Existence of finite range of Rayleigh number in which the system is linearly stable, in addition to usual infinite range.
(v) The non-trivial finite amplitude steady solution bifurcates either super critically or subcritically depending on the choices of physical parameters.

(vi) Heat and mass transfer decrease with increasing couple stress parameter and increase when the diffusing components are destabilizing.

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