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SYSTEM RELIABILITY ESTIMATION IN MULTICOMPONENT EXPONENTIAL-LINDLEY STRESS-STRENGTH MODELS

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Abstract

A stress-strength model is formulated for a multi-component system consisting of k identical components. The k components of the system with random strengths $\left(X_1, X_2, \ldots, X_k\right)$ are subjected to one of the r random stresses $\left(Y_1, Y_2, \ldots, Y_r\right)$. The estimation of system reliability based on maximum likelihood estimates (MLEs) and Bayes estimators in k component system are obtained when the system is either parallel or series with the assumption that strengths and stresses follow Lindley distribution and Exponential distribution respectively. A simulation study is conducted to compare MLE and Bayes estimator through the mean squared errors of the estimators.

Key Words: Exponential Distribution, Lindley Distribution, Maximum Likelihood Estimators, Bayes Estimator, Stress-Strength Model, Multicomponent System, System Reliability.

1. Introduction

The reliability of a system is defined as the probability that it will perform satisfactorily at least for a specified period of time without a major break down. Epstein (1958) remarks that Exponential distribution plays an important role in life testing experiments. Lindley (1958) introduced Lindley distribution in the context of Bayesian statistics, as a counter example of fudicial statistics which is highly positively skewed for large values of its scale parameter. Recently, Pandit and Kantu (2013) addressed the problem of estimation of reliability in exponential stress-strength models. Gogoi and Borah (2012) have studied the estimation of reliability for multi-component stress-strength model with standby redundancy using Exponential, Gamma and Lindley distributions. A multi-component stress-strength model with standby system using mixture of two Exponential distributions is studied by Sandhya K., and Umamaheswari T. S. (2013). Sriwastav and Kakaty (1981) have assumed that all the components in the stress-strength model are identically distributed. But in general the distributions of stress and strength are different due to different physical conditions. However, to the best of our knowledge in case of multi-component stress- strength model no attempt has been made to use Lindley distribution for reliability assessment.

The present paper considers the estimation of reliability of Exponential-Lindley Stress-Strength model with multi-component system with more than two stresses. In this paper we consider the estimation of R_p= P[Max$\left(Y_1, Y_2, ..., Y_r\right)$ < Max$\left(X_1, X_2, ..., X_k\right)$] and R_s= P[Max$\left(Y_1, Y_2, ..., Y_r\right)$ <Min$\left(X_1, X_2, ..., X_k\right)$] when $X_1, X_2, ..., X_k$ are strengths subjected to one of the stresses $Y_1, Y_2, ..., Y_r$ assuming that $X_1, X_2, ..., X_k$ follow independent Lindley distribution and $Y_1, Y_2, ..., Y_r$ follow independent Exponential distribution.

In section 2, we derive the expression for system reliability of series and parallel systems for an Exponential-Lindley stress-strength model. The MLEs for the parameters and reliability functions with their asymptotic distributions are derived in section 3. In Section 4, the Bayes estimators are derived. Section 5 deals with evaluating the performance of the MLEs and Bayes estimators of reliability functions by estimating their mean squared errors (MSEs) through simulations. Some remarks and conclusions are given in section 6.

2. SYSTEM RELIABILITY

Consider a multi-component system with k identical components. Here, we assume that strengths of k components are subjected to one of the r stresses. Let $X_1, X_2, ..., X_k$ be strengths having Lindley distribution with parameter θ_1, subjected to one of the stresses $Y_1, Y_2, ..., Y_r$ that follow Exponential distribution with parameter θ_2.

The p.d.f. of X_i is given by

$$f_i(x) = \frac{\theta_1^2}{(1+\theta_1)}(1+x)e^{-\theta_1 x}, \qquad x>0\ ,\ \theta_1>0\ ,\ i=1,2,...,k$$

and

$$f_j(y) = \theta_2\, e^{-\theta_2 y}, \qquad y>0\ ,\ \theta_2>0\ ,\ j=1,2,...,r$$

Then the distribution function of $U = Max(X_1, X_2, ..., X_k)$ is given by

$$G_1(u) = P[U<u] = \left[1 - e^{-\theta_1 u}\left(1+\frac{\theta_1 u}{1+\theta_1}\right)\right]^k$$

and the distribution function of $V = Max(Y_1, Y_2, ..., Y_r)$ is given by

$$G_2(v) = P[V<v] = \left[1-e^{-\theta_2 v}\right]^r$$

Now in parallel system, the system reliability is

$$R_p = P[V<U]$$

$$= \int_0^\infty G_2(u)\,dG_1(u)$$

$$= k\theta_1 \int_0^\infty (1-e^{-\theta_2 u})^r \left(1+\frac{\theta_1 u}{1+\theta_1}\right)\left(1-e^{-\theta_1 u}\left(1+\frac{\theta_1 u}{1+\theta_1}\right)\right)^{k-1} e^{-\theta_1 u}\,du$$

$$-\frac{k\theta_1}{1+\theta_1}\int_0^\infty e^{-\theta_1 u}\left(1-e^{-\theta_2 u}\right)^r\left(1-e^{-\theta_1 u}\left(1+\frac{\theta_1 u}{1+\theta_1}\right)\right)^{k-1}du$$

$$=k\theta_1\sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{n=0}^{l+1}\left[\frac{\binom{k-1}{l}\binom{r}{m}\binom{l+1}{n}(-1)^{l+m}\left(\frac{\theta_1}{1+\theta_1}\right)^n\Gamma(n+1)}{\left((l+1)\theta_1+m\theta_2\right)^{n+1}}\right].$$

$$-\frac{k\theta_1}{1+\theta_1}\sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{p=0}^{l}\binom{k-1}{l}\binom{r}{m}\binom{l}{p}(-1)^{l+m}\left(\frac{\theta_1}{1+\theta_1}\right)^p\frac{\Gamma(p+1)}{\left((l+1)\theta_1+m\theta_2\right)^{p+1}}$$

On the similar lines, the distribution function of $W=Min(X_1,X_2,...,X_k)$ is given by

$$G_3(w)=1-e^{-\theta_1 wk}\left(1+\frac{\theta_1 w}{1+\theta_1}\right)^k$$

Then the system reliability for a series system is obtained as

$$R_{s}=P\left[V<W\right]$$

$$=r\theta_2\int_0^\infty e^{-(k\theta_1+\theta_2)v}\left(1-e^{-\theta_2 v}\right)^{r-1}\left(1+\frac{\theta_1 v}{1+\theta_1}\right)^k dv$$

$$=r\theta_2\sum_{l=0}^{k}\sum_{m=0}^{r-1}\binom{k}{l}\left[\frac{\binom{r-1}{m}(-1)^m\left(\frac{\theta_1}{1+\theta_1}\right)^l\Gamma(l+1)}{\left((m+1)\theta_2+k\theta_1\right)^{l+1}}\right]$$

As the reliability function of both series and parallel systems involve θ_1 and θ_2, first we consider the estimation of θ_1 and θ_2, using method of maximum likelihood and then the system reliability estimates are obtained in the next section.

3. MAXIMUM LIKELIHOOD ESTIMATORS FOR PARAMETERS θ_1, θ_2 AND RELIABILITY

Consider a k-component system, in which components are subjected to r stresses. Let $X_{i,1},X_{i,2},...,X_{i,k}\ (i=1,2,...,n)$ be a random sample of strengths of n systems, that are distributed as Lindley random variables with parameter θ_1 and $Y_{i,1},Y_{i,2},...,Y_{i,r}\ (i=1,2,...,n)$ be a random sample of stresses corresponding to n systems, that are Exponentially distributed with parameter θ_2.

The MLEs of R_p and R_s based on $\underline{\theta} = (\theta_1, \theta_2)$ are given by

$\hat{R}_p = R_p(\hat{\underline{\theta}})$, $\hat{R}_s = R_s(\hat{\underline{\theta}})$, where $\hat{\underline{\theta}} = (\hat{\theta}_1, \hat{\theta}_2)$.

The MLEs of θ_1 and θ_2 are obtained as

$$\hat{\theta}_1 = \frac{\left(1-\bar{x}\right)+\sqrt{\left(\bar{x}^2+6\bar{x}+1\right)}}{2\bar{x}}, \qquad \text{where} \quad \bar{x} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{k} x_{i,j}}{nk},$$

$$\hat{\theta}_2 = \frac{nr}{\sum_{i=1}^{n}\sum_{j=1}^{r} y_{i,j}}$$

The asymptotic variances of the MLEs of $\hat{\theta}_1$ and $\hat{\theta}_2$ are given by

$$V(\hat{\theta}_1) = \frac{\theta_1^2 (1+\theta_1)^2}{nk\left(\theta_1^2 + 4\theta_1 + 2\right)}, \qquad V(\hat{\theta}_2) = \frac{\theta_2^2}{nr}.$$

The MLEs $\hat{\underline{\theta}}$ are consistent asymptotically normal with mean $\underline{\theta}$ and variance-covariance matrix $diag\left(\frac{\theta_1^2\left(1+\theta_1\right)^2}{n\,k\left(\theta_1^2+4\theta_1+2\right)}, \frac{\theta_2^2}{n\,r}\right)$. Since $\hat{R}_p$ and $\hat{R}_s$ are functions of $\hat{\underline{\theta}}$, asymptotic distribution of $\hat{R}_p$ and $\hat{R}_s$ are given as below.

The distribution of $\hat{R}_p$ is AN $\left(R_p, B_p' \Lambda B_p\right)$ and that of $\hat{R}_s$ is AN $\left(R_s, B_s' \Lambda B_s\right)$ where

$$B_p' = \left(\frac{\partial R_p}{\partial \theta_1}, \frac{\partial R_p}{\partial \theta_2}\right), \; B_s' = \left(\frac{\partial R_s}{\partial \theta_1}, \frac{\partial R_s}{\partial \theta_2}\right) \text{ and } \Lambda = \frac{1}{n} diag\left(\frac{\theta_1^2 (1+\theta_1)^2}{k\left(\theta_1^2+4\theta_1+2\right)}, \frac{\theta_2^2}{r}\right).$$

Here

$$\frac{\partial R_p}{\partial \theta_1} = -k\sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{n=0}^{l+1} K_1\,\Gamma(n+1)L_1 - k\sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{p=0}^{l}\Gamma(p+1)L_2$$

$$\frac{\partial R_p}{\partial \theta_2} = -k\theta_1\sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{n=0}^{l+1} K_1\, m\Gamma(n+2)L_3 + k\sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{p=0}^{l} K_2\, m\Gamma(p+2)L_4$$

$$\frac{\partial R_s}{\partial \theta_1} = -r\theta_2\sum_{l=0}^{k}\sum_{m=0}^{r-1}\binom{k}{l}\binom{r-1}{m}(-1)^m\,\Gamma(l+1)L_5$$

$$\frac{\partial R_s}{\partial \theta_2} = r\sum_{l=0}^{k}\sum_{m=0}^{r-1}\binom{k}{l}\binom{r-1}{m}(-1)^m\,\Gamma(l+1)L_6$$

where $K_1 = \binom{k-1}{l}\binom{r}{m}\binom{l+1}{n}(-1)^{l+m}$; $K_2 = \binom{k-1}{l}\binom{r}{m}\binom{l}{p}(-1)^{l+m}$

$$L_1 = \left(\frac{\theta_1^n (1+\theta_1)^{n-1} ((l+1)\theta_1 + m\theta_2)^n \left((l+1)n\theta_1^2 - m\theta_2\theta_1 - m(n+1)\theta_2\right)}{((l+1)\theta_1 + m\theta_2)^{2(l+1)} (1+\theta_1)^{2n}} \right)$$

$$L_2 = \left(\frac{(p+1)\theta_1^p (1+\theta_1)^p ((l+1)\theta_1 + m\theta_2)^p \left((l+1)\theta_1^2 - m\theta_2\right)}{((l+1)\theta_1 + m\theta_2)^{2(p+1)} (1+\theta_1)^{2(p+1)}} \right)$$

$$L_3 = \left(\frac{\theta_1}{1+\theta_1}\right)^n \left(\frac{1}{((l+1)\theta_1 + m\theta_2)^{n+2}}\right); \qquad L_4 = \left(\frac{\theta_1}{1+\theta_1}\right)^{p+1} \left(\frac{1}{((l+1)\theta_1 + m\theta_2)^{p+2}}\right)$$

$$L_5 = \left(\frac{\theta_1^{l-1} (1+\theta_1)^{l-1} (k\theta_1 + (m+1)\theta_2)^l \left((l+1)k\,\theta_1^2 + k\theta_1 - l(m+1)\theta_2\right)}{(k\theta_1 + (m+1)\theta_2)^{2(l+1)} (1+\theta_1)^{2l}} \right);$$

$$L_6 = \left(\frac{\theta_1}{1+\theta_1}\right)^l \left(\frac{1}{(k\theta_1 + (m+1)\theta_2)^{l+1}}\right)$$

4. BAYES ESTIMATION OF RELIABILITY FUNCTION

In this section Bayes estimator of R_p and R_s are derived by considering the prior distribution of the parameters θ_1 and θ_2 as,

$$g(\theta_1) = \frac{b^a}{\Gamma(a)} e^{-b\theta_1}\, \theta_1^{a-1} \qquad \theta_1 > 0,\ a \ge 0, b > 0$$

$$g(\theta_2) = \frac{1}{\Gamma(\alpha)} e^{-\theta_2}\, \theta_2^{\alpha-1} \qquad \theta_2 > 0,\ \alpha \ge 0\ .$$

The Bayes estimator of Reliability function $\hat{R}_{pB}$ is obtained as the posterior expectation of $\hat{R}_p$ is given by

$$\hat{R}_{pB} = \int_0^\infty \int_0^\infty R_p\, f\left(\theta_1, \theta_2 \middle| x_{i,j}, y_{i,j}\right) d\theta_1 d\theta_2$$

where

$$f\left(\theta_1, \theta_2 \middle| x_{i,j}, y_{i,j}\right) = \frac{\theta_1^{2n_1k+a-1}\, \theta_2^{n_2r+\alpha-1}}{\Gamma(n_2 r + \alpha)(1+\theta_1)^{n_1k}\, I} A_1\, A_2$$

$$A_1 = \left(1 + \sum_{i=1}^{n_2} \sum_{j=1}^{r} y_{i,j}\right)^{n_2 r + \alpha};$$

$$A_2 = e^{-\left(b+\sum_{i=1}^{n_1}\sum_{j=1}^{k} x_{i,j}\right)\theta_1 - \left(1+\sum_{i=1}^{n_2}\sum_{j=1}^{r} y_{i,j}\right)\theta_2}$$

$$I = \int_0^\infty e^{-\left(b+\sum_{i=1}^{k}\sum_{j=1}^{n_1} x_{i,j}\right)\theta_1} \frac{\theta_1^{2n_1k+a-1}}{(1+\theta_1)^{n_1k}} d\theta_1$$

Therefore,

$$\hat{R}_{pB} = \frac{A_1\, k}{I^*\Gamma(n_2 r+\alpha)} \sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{n=0}^{l+1} \binom{l+1}{n}\binom{k-1}{l}\binom{r}{m}(-1)^{l+m}\,\Gamma(n+1) I_1 - \frac{A_1\, k}{I^*\Gamma(n_2 r+\alpha)} \sum_{l=0}^{k-1}\sum_{m=0}^{r}\sum_{p=0}^{l} \binom{k-1}{l}\binom{r}{m}\binom{l}{p}(-1)^{l+m}\,\Gamma(p+1) I_2$$

where

$$I_1 = \int_0^\infty\int_0^\infty \frac{A_2\, \theta_1^{2n_1k+a+n}\, \theta_2^{n_2r+\alpha-1}}{((l+1)\theta_1+m\theta_2)^{n+1}(1+\theta_1)^{n_1k+n}} d\theta_1\, d\theta_2 \quad \text{and}$$

$$I_2 = \int_0^\infty\int_0^\infty \frac{A_2\, \theta_1^{2n_1k+a+p}\, \theta_2^{n_2r+\alpha-1}}{((l+1)\theta_1+m\theta_2)^{p+1}(1+\theta_1)^{n_1k+p+1}} d\theta_1\, d\theta_2$$

Similarly, the Bayes estimator of Reliability function $\hat{R}_{sB}$ is obtained as the posterior expectation of $\hat{R}_s$ is given by

$$\hat{R}_{sB} = \frac{A_1\, r}{I^*\Gamma(n_2 r+\alpha)} \sum_{l=0}^{k}\sum_{m=0}^{r-1} \binom{k}{l}\binom{r-1}{m}(-1)^m\,\Gamma(l+1) I_3,$$

where $I_3 = \int_0^\infty\int_0^\infty \frac{A_2\, \theta_1^{2n_1k+a+l-1}\, \theta_2^{n_2r+\alpha}}{(k\theta_1+(m+1)\theta_2)^{l+1}(1+\theta_1)^{n_1k+l}} d\theta_1\, d\theta_2$

5. SIMULATION STUDY

A simulation study is conducted to evaluate MSE's of reliabilities of series and parallel systems with different strengths and stresses. A simulation study of 100,000 samples of different sizes are generated for different values of k, the number of strengths, r, the number of stresses and the parameters (θ_1, θ_2, α, b, a) as specified in tables.

Based on the simulation study for the parameters considered, the values of MLE and Bayes estimators for R_p and R_s with their MSEs are presented in Table 5.1, 5.2, 5.3 and 5.4 for different combinations of (θ_1, θ_2, k, r, α, b, a). The actual values of R_p and R_s are also given which can be compared with estimates.

n_1	n_2	$\hat{R}_p$	$\hat{R}_s$	$\hat{R}_{pB}$	$\hat{R}_{sB}$	$MSE\left(\hat{R}_p\right)$	$MSE\left(\hat{R}_s\right)$	$MSE\left(\hat{R}_{pB}\right)$	$MSE\left(\hat{R}_{sB}\right)$
10	10	0.5117	0.2043	0.5107	0.2105	0.0124	0.005	0.0112	0.0049
	20	0.508	0.1999	0.5108	0.2068	0.0088	0.0034	0.0082	0.0035
	25	0.5069	0.1989	0.5106	0.2059	0.008	0.0031	0.0075	0.0032
	26	0.505	0.1967	0.5088	0.2046	0.008	0.003	0.0075	0.0031

Table 5.1: MLE's, Bayes estimators and MSE for estimates of R_p and R_s

$(\theta_1 = 0.5 \quad \theta_2 = 0.3 \quad k = 2 \quad r = 2 \quad \alpha = 0.4 \;\; b = 0.6, a = 0.2$, $R_p = 0.5084,\; R_s = 0.1951)$

n_1	n_2	$\hat{R}_p$	$\hat{R}_s$	$\hat{R}_{pB}$	$\hat{R}_{sB}$	$MSE\left(\hat{R}_p\right)$	$MSE\left(\hat{R}_s\right)$	$MSE\left(\hat{R}_{pB}\right)$	$MSE\left(\hat{R}_{sB}\right)$
10	10	.5123	.2047	.5052	.2069	.0124	.005	.0112	.0047
	20	.5082	.1999	.5111	.2069	.0087	.0034	.0081	.0034
	25	.5064	.1985	.51	.2055	.008	.0031	.0075	.0031

Table 5.2: MLE's, Bayes estimators and MSE for estimates of R_p and R_s

$(\theta_1 = 0.5,\; \theta_2 = 0.3,\; k = 2,\; r = 2,\; \alpha = 0.4, b = 0.3, a = 0.6 \;\; R_p = 0.5084,\; R_s = 0.1951)$

n_1	n_2	$\hat{R}_p$	$\hat{R}_s$	$\hat{R}_{pB}$	$\hat{R}_{sB}$	$MSE\left(\hat{R}_p\right)$	$MSE\left(\hat{R}_s\right)$	$MSE\left(\hat{R}_{pB}\right)$	$MSE\left(\hat{R}_{sB}\right)$
10	10	0.6876	0.3932	0.6761	0.3916	0.0109	0.0092	0.0097	0.0082
	20	0.6827	0.3851	0.6786	0.3876	0.0071	0.0057	0.0065	0.0054
	30	0.6799	0.3816	0.6784	0.3853	0.0058	0.0046	0.0055	0.0044
	40	0.6809	0.3819	0.6806	0.3862	0.0051 7	0.0041	0.0049	0.004

Table 5.3: MLE's, Bayes estimators and MSE for estimates of R_p and R_s

$(\theta_1 = 0.5 \quad \theta_2 = 0.3 \quad k = 2 \quad r = 1 \quad \alpha = 0.4 \;\; b = 0.6, a = 0.2$, $R_p = 0.683,\; R_s = 0.3795)$

n_1	n_2	$\hat{R}_p$	$\hat{R}_s$	$\hat{R}_{pB}$	$\hat{R}_{sB}$	$MSE\left(\hat{R}_p\right)$	$MSE\left(\hat{R}_s\right)$	$MSE\left(\hat{R}_{pB}\right)$	$MSE\left(\hat{R}_{sB}\right)$
10	10	0.6877	0.3934	0.6716	0.3874	0.0109	0.0092	0.0099	0.0080
	20	0.6821	0.3847	0.6733	0.3828	0.0072	0.0057	0.0068	0.0053
	30	0.6796	0.3813	0.6733	0.3806	0.0058	0.0045	0.0056	0.0043
	40	0.6807	0.3817	0.6756	0.3815	0.0051	0.004	0.0049	0.0039

Table 5.4: MLE's, Bayes estimators and MSE for estimates of R_p and R_s
($\theta_1 = 0.5$ $\theta_2 = 0.3$ $k = 2$ $r = 1$ $\alpha = 0.4$ $b = 0.3$ $a = 0.6$ $R_p = 0.6830$ $R_s = 0.3795$)

n_1	n_2	$\hat{R}_p$	$\hat{R}_s$	$\hat{R}_{pB}$	$\hat{R}_{sB}$	$MSE\left(\hat{R}_p\right)$	$MSE\left(\hat{R}_s\right)$	$MSE\left(\hat{R}_{pB}\right)$	$MSE\left(\hat{R}_{sB}\right)$
10	10	0.9191	0.6878	0.9033	0.6715	0.0024	0.0084	0.003	0.0077
	20	0.9199	0.6839	0.9113	0.6766	0.0015	0.0055	0.0018	0.0052
	30	0.9195	0.6814	0.9134	0.6772	0.0013	0.0046	0.0014	0.0044
	40	0.9201	0.6816	0.9153	0.6790	0.0011	0.0041	0.0012	0.0039

Table 5.5: MLE's, Bayes estimators and MSE for estimates of R_p and R_s
($\theta_1 = 0.3$ $\theta_2 = 0.5$ $k = 2$ $r = 1$ $\alpha = 0.4$ $b = 0.6$ $a = 0.2$ $R_p = 0.9253$ $R_s = 0.6853$)

n_1	n_2	$\hat{R}_p$	$\hat{R}_s$	$\hat{R}_{pB}$	$\hat{R}_{sB}$	$MSE\left(\hat{R}_p\right)$	$MSE\left(\hat{R}_s\right)$	$MSE\left(\hat{R}_{pB}\right)$	$MSE\left(\hat{R}_{sB}\right)$
10	10	0.9191	0.6877	0.9012	0.6676	0.0024	0.0083	0.0032	0.0079
	20	0.9195	0.6833	0.9089	0.6721	0.0015	0.0054	0.0019	0.0053
	30	0.9208	0.6833	0.9125	0.6752	0.0012	0.0045	0.0015	0.0044
	40	0.9200	0.6823	0.9120	0.6742	0.0013	0.0045	0.0015	0.0045

Table 5.6: MLE's, Bayes estimators and MSE for estimates of R_p and R_s
($\theta_1 = 0.3$ $\theta_2 = 0.5$ $k = 2$ $r = 1$ $\alpha = 0.4$ $b = 0.3$ $a = 0.6$ $R_p = 0.9253$ $R_s = 0.6853$)

6. SOME REMARKS AND CONCLUSIONS

1. In this paper, we have considered Exponential-Lindley as stress-strength model for estimating the parameters and reliability functions.
2. The parameters are estimated using the method of maximum likelihood (ML) and then the ML estimators for reliability function for parallel and series system is obtained, when the system has k independent strength and r independent stresses.
3. The Bayes estimators are also derived for the reliability of series and parallel systems with respect to conjugate priors and their MSE's are evaluated using simulation. It can be observed that MSEs of Bayes estimators are slightly lower than MSEs of MLE's. Hence, MLE's are comparable with Bayes estimators.

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