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Brief note

A NEW ANALYTICAL PROCEDURE FOR SOLVING THE NON-LINEAR DIFFERENTIAL EQUATION ARISING IN THE STRETCHING SHEET PROBLEM

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The paper discusses a new analytical procedure for solving the non-linear boundary layer equation arising in a linear stretching sheet problem involving a Newtonian/non-Newtonian liquid. On using a technique akin to perturbation the problem gives rise to a system of non-linear governing differential equations that are solved exactly. An analytical expression is obtained for the stream function and velocity as a function of the stretching parameters. The Clairaut equation is obtained on consideration of consistency and its solution is shown to be that of the stretching sheet boundary layer equation. The present study throws light on the analytical solution of a class of boundary layer equations arising in the stretching sheet problem.

Key words: differential equations, Clairaut equation, Newtonian liquid, stretching sheet, suction/injection.

1. Introduction

Stretching sheet problems typically arise in polymer extrusion processes that involve cooling of continuous strips extruded from a die by drawing them through a stagnant liquid (see Fig.1). The stretching imparts a uni-directional orientation to the extrudate, thereby improving its mechanical properties (see Fisher, 1976). The stagnant liquid is basically meant to cool the stretching sheet. The extruded sheet (film) can be a needed product in a plastic or glass industry or in industries dealing with artificial fibre. In some cases, this needed product is a perforated one. Ever since the pioneering works of Sakiadis (1961) and Crane (1970), several researchers have investigated this problem considering different aspects (see Andersson 2002; 1992; 1995; Liao, 2003; Liao and Pop, 2004; Magyari *et al.*, 2003; Magyari and Keller, 2001; 2002; Rajagopal, 1984; Siddheshwar and Mahabaleswar, 2005; Siddheshwar *et al.,* 2005 and references therein). From the practical point of view, therefore, the stretching assumed as being non-linearly proportional to the axial distance seems more natural than the linear one (Kumaran and

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Ramanaiah, 1996 and Vajravelu, 2001 and references therein). In the present paper, as a first step in the general modelling exercise, we make use of a simple super-linear stretching model, viz., quadratic stretching. The importance of this paper lies in obtaining a closed form solution to the linear stretching sheet problem using the quadratic stretching problem.

2. Mathematical formulation and solution of the Crane (1970) problem with suction (injection) using a new analytical procedure

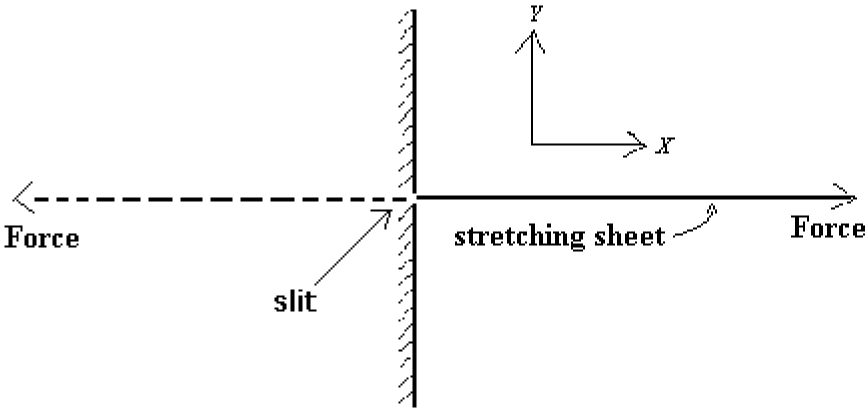

Fig.1. Schematic of the stretching sheet problem.

The steady and incompressible isothermal flow of a Newtonian liquid caused by a stretching sheet is considered and shown schematically in Fig.1. In these problems by applying two equal and opposite forces along the sheet (x-axis), it is intended to stretch the sheet with a speed proportional to the distance from the fixed origin $x = 0$. The resulting motion of the otherwise quiescent liquid is thus caused solely by the stretching sheet. To solve the problem a Cartesian coordinates system (x, y) is introduced with the origin at the slot, the x-axis along the direction of stretching and y-axis normal to it. Under these assumptions, the boundary-layer flow along the stretching sheet is governed by the equations (see Crane, 1970)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}\,. \tag{2.2}$$

The boundary conditions to obtain the particular solution of the above equations are

$$u = \alpha x \qquad \text{at} \qquad y = 0, \tag{2.3a}$$

$$v = v_c \qquad \text{at} \qquad y = 0, \tag{2.3b}$$

$$u = 0 \qquad \text{as} \qquad y \to \infty \tag{2.3c}$$

where u and v are the velocity components in the x and y directions, respectively, α and v_c are constants. Equation (2.3b) signifies suction (or injection) at the perforated sheet. The condition in Eq.(2.3c) implies that

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the liquid has no motion in the *x*-direction as $y \to \infty$, that in turn means that the stretching of the sheet does not induce dynamics at distances far away from the sheet. A new analytical procedure to obtain the analytical solution of the problem will now be presented. This idea was put forth by the first author in lecture at a national conference (Siddheshwar, 2004). To this end, we introduce a quadratic term in the stretching condition Eq.(2.3a) and this give rise to the quadratic stretching sheet problem considered by Kumaran and Ramanaiah (1996). It has so happened that there is a mathematical commonality between our work and that of Kumaran and Ramanaiah (1996) up to the point of formulation, though the motivation for the two are entirely different. The focus in our work is on a new analytical procedure for solving the linear stretching sheet problem whereas their work is a typical example of a non-linearly stretching sheet.

The introduction of the quadratic term in Eq.(2.3a) gives us the conditions

$$u = \alpha\, x + \beta x^2 \qquad \text{at} \qquad y = 0, \tag{2.4a}$$

$$v = v_c + \delta x \qquad \text{at} \qquad y = 0, \tag{2.4b}$$

$$u = 0 \qquad \text{as} \qquad y \to \infty \tag{2.4c}$$

where β and δ are constants, and their very small magnitude facilitates our remaining within the purview of the boundary layer approximation. A linear term has been introduced in the boundary condition on *v* due to the fact that when the stretching in the *x*-direction is non-linear then that gives rise to a ‘suction-like’ lift effect at each point on the sheet. As we see later this has also to do with the requirements of the continuity equation. If one chooses to forget about the physics of the problem for a moment and considers the mathematical point of view then the terms involving β and δ in Eq.(2.4) can be taken as mild perturbations on the Crane (1970) problem with suction.

The equations and boundary conditions are made dimensionless using the following definition

$$(X,Y) = \sqrt{\frac{\alpha}{\nu}}(x,y), \qquad (U,V,V_c) = \frac{(u,v,v_c)}{\sqrt{\alpha\nu}}, \qquad \beta^* = \frac{\beta}{\alpha}\sqrt{\frac{\nu}{\alpha}}, \qquad \delta^* = \frac{\delta}{2\alpha}. \tag{2.5}$$

Equations (2.1) and (2.2) in the non-dimensional form read as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2.6}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2}. \tag{2.7}$$

Introducing the stream function $\psi(X,Y)$, we get

$$U = \frac{\partial \psi}{\partial Y}, \qquad V = -\frac{\partial \psi}{\partial X}. \tag{2.8}$$

Using Eqs (2.8) in Eq.(2.7), we get the non-linear partial differential equation

$$\frac{\partial^3 \psi}{\partial Y^3} + \frac{\partial \psi}{\partial X}\frac{\partial^2 \psi}{\partial Y^2} - \frac{\partial \psi}{\partial Y}\frac{\partial^2 \psi}{\partial X \partial Y} = 0. \tag{2.9}$$

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The boundary conditions to be satisfied by ψ can be obtained from Eqs (2.4), (2.5) and (2.8) as

$$\frac{\partial\psi}{\partial Y} = X + \beta^* X^2 \qquad \text{at} \qquad Y = 0, \tag{2.10a}$$

$$\frac{\partial\psi}{\partial X} = V_c + 2\delta^* X \qquad \text{at} \qquad Y = 0, \tag{2.10b}$$

$$\frac{\partial\psi}{\partial Y} = 0 \qquad \text{as} \qquad Y \to \infty. \tag{2.10c}$$

The solution to Eq.(2.9), subject to Eq.(2.10), may be taken as

$$\psi = X f(Y) - \beta^* X^2 g(Y). \tag{2.11}$$

Using Eq.(2.11) in Eq.(2.8), we get

$$U = X f'(Y) - \beta^* X^2 g'(Y),$$

$$V = -f(Y) + 2\beta^* X g(Y). \tag{2.12}$$

Clearly the continuity Eq.(2.6) is satisfied by U and V of Eq.(2.12) only when the second term in the expression for V is present. This justifies the term δx in Eq.(2.4b). In Eq.(2.11) if we take $\beta^* = 0$, then the equation signifies the Crane (1970) form of solution. Now a word on the presence of $g(Y)$ is needed. It is imperative that such a term arises in view of the boundary condition (2.10a). The idea behind the introduction of such a term is the following: If we formulate a consistency condition by which a non-linear ODE and linear ODE admit a common solution, then that consistency condition will itself end up being - after a suitable change of variables- a single ODE of lower order than the *2* ODEs we start with and that lower order ODE can in fact be solved to obtain the desired common solution to the original equation.

Substituting Eq.(2.11) into Eq.(2.9) and equating the coefficients of X, X^2 and X^3, the following system of ordinary differential equations is obtained

$$\frac{d^3 f}{dY^3} + f\frac{d^2 f}{dY^2} - \left(\frac{df}{dY}\right)^2 = 0, \tag{2.13}$$

$$\frac{d^3 g}{dY^3} + f\frac{d^2 g}{dY^2} + 2g\frac{d^2 f}{dY^2} - 3\frac{df}{dY}\frac{dg}{dY} = 0, \tag{2.14}$$

and

$$g\frac{d^2 g}{dY^2} - \left(\frac{dg}{dY}\right)^2 = 0. \tag{2.15}$$

On using Eq.(2.11) in Eqs (2.10a, b, c), the boundary conditions on f and g can be obtained in the following form

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$$f(0) = -V_c\,, \qquad \frac{df}{dY}(0) = 1, \qquad \frac{df}{dY}(\infty) \to 0, \tag{2.16a}$$

$$g(0) = s_1, \qquad \frac{dg}{dY}(0) = -1, \qquad \frac{dg}{dY}(\infty) \to 0 \tag{2.16b}$$

where $s_1 = \dfrac{\delta^*}{\beta^*}$. Equations (2.13) and (2.16a) form the governing non-linear boundary value problem (BVP) of the linear stretching sheet problem of Crane (1970) but with suction or injection. The solution of this BVP is the main focus of the paper.

Observing Eqs (2.13)-(2.15), we find that we have an equation each for f and g, and one equation involving both f and g. On solving Eqs (2.15) and (2.16b) for g and substituting the same in Eq.(2.14) we get a second order linear differential equation in f. Thus we will have two equations for f before us:
(i) a third-order non-linear differential equation,
(ii) a second-order linear differential equation and this warrants the need to address questions of consistency.

In what follows we adopt this approach. Solving Eq.(2.15) for g subject to the first and third boundary conditions in Eq.(2.16b), we get

$$g(Y) = s_1\, e^{-Y/s_1}\,, \tag{2.17}$$

and this automatically satisfies the remaining condition, viz., $\dfrac{dg}{dY}(0) = -1$. Substitution of Eq.(2.17) in Eq.(2.14) results in the second-order linear differential equation

$$2 s_1^2\, \frac{d^2 f}{dY^2} + 3 s_1 \frac{df}{dY} + f = \frac{1}{s_1}\,. \tag{2.18}$$

The solution of Eq.(2.18), subject to Eq.(2.16a), is

$$f(Y) = -\left(\frac{2s_1^2 - s_1 V_c - 1}{s_1}\right) e^{-Y/s_1} + 2\left(\frac{s_1^2 - s_1 V_c - 1}{s_1}\right) e^{-Y/2s_1} + \frac{1}{s_1}\,. \tag{2.19}$$

For consistency the solution of Eq.(2.18) has to be a solution of Eq.(2.13). Equation (2.19) can be a solution of the non-linear differential Eq.(2.13) only if

$$s_1^2 - s_1 V_c - 1 = 0\,. \tag{2.20}$$

In order to have the physically realistic exponentially decaying function $\dfrac{df}{dY}$ as $Y \to \infty$, it is imperative that $s_1 > 0$. An introduction of the positive root $s_1 = \dfrac{V_c + \sqrt{V_c^2 + 4}}{2}$ in Eq.(2.19) yields the solution of the Crane (1970) problem as

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$$f(Y)=\frac{1}{s_1}-s_1^2 e^{-Y/s_1} \ . \tag{2.21}$$

It will now be shown that the solution 'assumed' by Gupta and Gupta (1977) for the linear stretching sheet problem is, in fact, Eq.(2.21). Equation (2.20) on rearrangement gives us

$$\frac{1}{s_1}=s_1-V_c \, . \tag{2.22}$$

A substitution of Eq.(2.22) in the first term on the RHS of Eq.(2.21), and simplification, yields the Gupta and Gupta (1977) solution

$$f\left(Y\right)=\frac{1-e^{-sY}}{s}-V_c \tag{2.23}$$

where $s=\frac{1}{s_1}$. Using this new procedure the well-known solution of Eq.(2.23) 'assumed' by Gupta and Gupta (1977) stands 'derived' by rigor.

It is our endeavour now to extract some more information from the above. To this end, Eq.(2.21) is differentiated and Eq.(2.17) is used in the resulting equation to get

$$g(Y)=\frac{1}{s}\frac{df}{dY}. \tag{2.24}$$

With this relation consistency is addressed afresh by going back to the point where consistency is yet to be considered. A substitution of Eq.(2.24) in Eqs (2.14) and (2.15), yields the following two equations

$$\frac{d}{dY}\left[\frac{d^3 f}{dY^3}+f\frac{d^2 f}{dY^2}-\left(\frac{df}{dY}\right)^2\right]=0 \, , \tag{2.25}$$

$$\left(\frac{d^2 f}{dY^2}\right)^2-\frac{df}{dY}\frac{d^3 f}{dY^3}=0 \, . \tag{2.26}$$

There are three equations for f now, viz., Eqs (2.13), (2.25) and (2.26). Questions on consistency need to be addressed at this point. Obviously, Eq.(2.25) can be obtained from Eq.(2.13). We are thus left with two Eq.(2.13) and (2.26) for f.

The work of Kumaran and Ramanaiah (1996) does take us up to this point but then it adopts a different route due to its focused attention on solving a quadratic stretching sheet problem. Our path breaks with theirs here to take a different route in arriving at an analytical solution by consistency argument and exploiting the autonomous nature of the system of equations for f.

In a straight-forward manner one can rearrange Eq.(2.26) as follows

$$\left(\frac{f''}{f'}\right)'=0. \tag{2.27}$$

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Now integrating Eq.(2.27) twice, a first-order equation is obtained as

$$f' = -A_1 f + B_1, \tag{2.28}$$

whose solution satisfying Eq.(2.16a) is Eq.(2.23).

Alternately equating the expression of $\dfrac{d^3 f}{dY^3}$ from the two Eqs (2.13) and (2.26), the following autonomous differential equation (ADE) is arrived at after rearrangement

$$\left(\frac{d^2 f}{dY^2}\right)^2 + f\frac{df}{dY}\frac{d^2 f}{dY^2} - \left(\frac{df}{dY}\right)^3 = 0. \tag{2.29}$$

Equation (2.29) is now transformed to a lower order equation by taking

$$\frac{df}{dY} = F, \tag{2.30}$$

and treating f as an independent variable. Differentiating Eq.(2.30) with respect to Y yields the following expression for $\dfrac{d^2 f}{dY^2}$

$$\frac{d^2 f}{dY^2} = \frac{dF}{df}\frac{df}{dY} = F\frac{dF}{df}. \tag{2.31}$$

A substitution of Eqs (2.30) and (2.31) into Eq.(2.29) and rearrangement results in the Clairaut differential equation

$$F = \left(\frac{dF}{df}\right)f + \left(\frac{dF}{df}\right)^2, \tag{2.32}$$

whose solution is

$$F = \frac{df}{dY} = A\,f + A^2 \tag{2.33}$$

where A is a constant. In conjunction with the first two conditions in Eq.(2.16a), the above equation yields the consistency condition Eq.(2.20) and so we take $A = -\dfrac{1}{s_1}$ in keeping with the third condition of Eq.(2.18).

The solution of the linear differential Eq.(2.33) for $f(Y)$ is

$$f(Y) = B\,e^{-Y/s_1} - \frac{1}{s_1} \tag{2.34}$$

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where B is a constant. The particular solution can be obtained by using any two boundary conditions in Eq.(2.16a) and this automatically satisfies the remaining boundary condition. The particular solution is found to be the same as Eq.(2.23). By doing what we did we have proved that the derived solution of the third-order, non-linear, ordinary differential equation can, in fact, be obtained from a first-order linear differential equation of the Clairaut type. At this stage let us summarize what we did and explain why we did it, and then answer some new questions.

- We formulated a quadratic stretching sheet problem that results in a system of differential equations with approximate boundary conditions.
- The linear stretching sheet problem statement is a part of the above system of equations. The analytical solution of the nonlinear BVP of the linearly stretching sheet problem is obtained after duly going into questions of consistency. The 'derived' solution is shown to be the classical Gupta and Gupta (1977) solution.
- Analysing the solution of the system of 3 equations for f and g, we draw a connection between f and g and use this to obtain equations for f. One resulting equation happens to be the first derivative of Eq.(2.13).
- Dealing with two equations for f and addressing consistency equations we arrive at the Clairaut Eq.(2.32) and at the result that the solution of this equation is, in fact, the solution of the BVP problem of the Crane (1970) problem with suction. It is important to note here that we obtained the Clairaut equation by going into questions of consistency.

An inspection of Eq.(2.22) shows that the solution $f(Y)$ satisfies the differential equation

$$\frac{df}{dY} + sf = 1 - sV_c\,, \qquad f(0) = -V_c\,. \tag{2.35}$$

It can be easily verified that this equation is satisfied by $f(Y)$ in the stretching sheet problems involving Walters' liquid B cooling liquid for both electrically non-conducting and conducting liquids. This leads to the inference that $f(Y) = \frac{1-e^{-sY}}{s} - V_c$ is the solution of the entire class of boundary layer equations arising in a stretching sheet problem, s being different for each. Table 1 documents the value of s for different stretching sheet problems.

Table 1. Value of s for different continuum.

Cooling liquid	Value of s
Newtonian liquid (with suction or injection) Gupta and Gupta (1977)	$s = \frac{-V_c + \sqrt{V_c^2 + 4}}{2}$
Weak electrically conducting Newtonian liquid (with suction or injection) Dandapat *et al.* (2004)	$s = \frac{-V_c + \sqrt{V_c^2 + 4(1+Q)}}{2}$
Walters' liquid B (with suction or injection) Siddappa and Abel (1986)	Minimum of positive roots of $(V_c s - 1)(1 + k_1 s^2) + s^2 = 0$
Weak electrically conducting Walters' liquid B (with suction or injection) Ariel (1994)	Minimum of positive roots of $(V_c s - 1)(1 + k_1 s^2) + (s^2 - Q) = 0$

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3. Results and discussion

The paper discusses a new method of solving the linear stretching sheet problem by considering a perturbed system over the linear problem. In the course of the method we have obtained the analytical solution of the governing non-linear differential equation of the linear stretching sheet problem.

The required solution turns out to be the solution of the Clairaut equation and also alternately that of a linear differential equation of first order (see Eq.(2.33)). The problem demonstrates a general formulation and solution of a class of stretching sheet problems involving various cooling liquids. It further stresses the fact that there is no merit in solving many different, related stretching sheet problems in isolation.

4. Conclusion

Considering a quadratic stretching sheet problem (a perturbed system) it is possible to obtain the analytical solution of the linear stretching sheet problem.

Nomenclature

f – similarity function
u – velocity component along the sheet [$m\ s\text{-}1$]
v – velocity component normal to the sheet [$m\ s\text{-}1$]
v_w – suction/injection
x – coordinate along the sheet [m]
y – coordinate normal to the sheet [m]
α,β – constant rate of stretching [s^{-1}]
η – similarity variable
μ – dynamic viscosity [$kg\ m^{-1}s^{-1}$]
υ – kinematic fluid viscosity [$m^2\ s^{-1}$]
ρ – density [$kg\ m^{-3}$]
ψ – stream function [$m^2\ s^{-1}$]

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