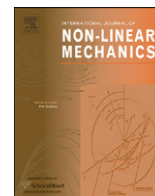




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Electrorheological Rayleigh–Taylor instability at the interface between a porous layer and thin shell with poorly conducting couple stress fluid

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ABSTRACT

This paper is concerned with the study of the Electrorheological Rayleigh–Taylor instability (ERTI) at the interface between a densely packed saturated poorly conducting couple stress porous layer accelerated by a lighter poorly conducting couple stress fluid in a thin shell in the presence of a transverse electric field and laser radiation. A simple theory based on fully developed flow approximations is used to derive the dispersion relation for the growth rate of ERTI. The cutoff and the maximum wave numbers and the corresponding maximum frequencies are obtained. It is shown that the effects of couple stress parameter and the electric field reduce the growth rate considerably compared to a non-conducting fluid in the absence of an electric field. These are favorable to control the surface instabilities in many practical applications discussed in this paper.

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1. Introduction

In this era of globalization, liberalization, modernization, increase in population and industries require more electric power, particularly an uninterrupted power supply. The depletion of fossil fuel and hydroelectric power due to atmospheric vagaries have strained adequate power supply for overall development of a country. Therefore, there is an urgent need of power supply from the unconventional sources of energy which are economically affordable and environmental friendly. Therefore, many unconventional methods like nuclear power, solar power, wind power and so on have been proposed to generate electric power. The United Nations, through International Atomic Energy Agency (IAEA), is proposing to generate an adequate electric power from Inertial Fusion Energy (IFE) by fusing the two hydrogen isotopes Deuterium–Tritium (DT), in Inertial Fusion Target (IFT), which is also one of the environmental friendly and unconventional methods of generating electric power. The nuclear picture of DT is such that there will be a strong repulsive force between them which prevents the natural fusion of DT. Therefore, there is a need of external agency like laser radiation to fuse them. The high

intensity laser radiation is impinged at the ablative surface of IFT to fuse DT. This radiation no doubt fuses DT but causes asymmetry in IFT which is one of the causes to reduce the efficiency of extraction of IFE (see [1,2]). To reduce this asymmetry of IFE target, there should be mechanisms to control surface instability. At present, several mechanisms (see [2–6]) have been used to reduce the growth rate of RTI at the interface between heavy fluid supported by a lighter fluid.

In addition to the study of IFE, we note that the interfacial instabilities continue to be the frontier area of research in understanding, control and exploitation of microfluidic devices. In the literature, the work on ordinary Newtonian poorly conducting fluid flows in the presence of an electric field, called Electrohydrodynamics (EHD) (see [7]), flowing through a micro-channel has been studied because of their applications in Science, Engineering and Technology, particularly in the effective design of artificial organs in biomedical engineering, solidification process in material science, heat transfer across barriers, friction between surfaces and their mitigation, failure of polymers and so on. Therefore, in biomedical engineering, for an effective design of artificial organ like cartilages in synovial joints, endothelium in Coronary Arteries Disease (CAD) involving poorly conducting fluids, it is necessary to control the growth rate of surface instabilities at the interface between cartilages and synovial fluid in synovial joints (SYJ), at the interface between endothelium and the body fluid in arteries in CAD, at the ablative surface of IFT in IFE. The available literature (see [8,2]) on surface instabilities is

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mainly concerned with the traditional Newtonian fluid approximations. During the degenerative changes either in synovial joints or in CAD, involving poorly conducting physiological fluids with suspension of nutrients, hyaluronic acid (HA), red blood cells (RBC), white blood cells (WBC), platelets and also in the process of fusing DT in IFE by laser radiation, the Newtonian fluid approximations cannot precisely describe the characteristics of the complex poorly conducting fluids described above. These poorly conducting fluids deform and produce a spin field due to their microrotation forming micropolar fluid developed by Eringen [9]. This micropolar fluid will be more effective in practical problems described above because it takes care of local effects arising from microstructure and as well as the intrinsic motions of microfluidics (see [10–13]). According to Eringen [9], the micropolar fluids may be regarded as non-Newtonian fluids like suspension of DT in IFE target. The spin field due to microrotation of these freely suspended particles like DT in IFE, HA in SYJ and RBC and so on in CAD sets up an antisymmetric stress known as couple stress and thus forming couple stress fluid (CSF). It is according to Eringen [9], a particular case of micropolar fluid, when microrotation balances with the natural vorticity of fluid. This poorly conducting couple stress fluid in the presence of an electric field can be regarded as Electrorheological Fluid (ERF).

We note that the DT in IFT, Synovial Fluid (SF) in SYJ and RBC, WBC and so on in CAD can be regarded as poorly conducting liquid crystals with conductivity, σ , increasing with the difference in temperature between the higher temperature at the ablative surface of IFT and lower temperature inside the IFE target. This difference in temperature causes difference in conductivity, $\nabla\sigma$. This $\nabla\sigma$ releases the charges from the nuclei forming distribution of charge density, ρ_e . These charges induce an electric field, \vec{E}_i called thermal electric field. If there is a need of higher strength of electric field, we can generate the applied electric field, \vec{E}_a , by embedding the electrodes of different potentials at the boundaries. The total electric field, $\vec{E} = \vec{E}_i + \vec{E}_a$ produces, according to Ohm's law, the current density, $\vec{J} = \sigma\vec{E}_i + \rho_e q_i$, which is the sum of conduction current and convective current and also produces an electric force, $\vec{f} = \rho_e \vec{E}$. This current, \vec{J} , acts as sensing and the force, \vec{f}_e , acts as actuation. These two properties make a poorly conducting couple stress fluid as smart material. This fluid in the presence of an electric field is called ERF introduced by Winslow [14] and Rajagopal and Wineman [15]. These ERFs have a number of possible technical applications in the various areas of microelectronics, industrial applications in addition to effective design of artificial organs in biomedical engineering and to increase the efficiency of the extraction of IFE as discussed above. Because of their importance considerable interest has been evinced, during the last decade, in the study of their experimental and theoretical point of view (see [15–17]).

The literature on the study of RTI is mainly concerned, as stated above, with ordinary Newtonian fluid. Recently Rudraiah and Chandrashekhara [18] have studied the effect of hydrodynamic couple stress fluid, in the absence of electric field, on RTI and showed that this couple stress is very effective in reducing the asymmetry in IFE caused by fusing DT due to laser radiation and in reducing the side effects of haemolysis in biomedical engineering problems discussed above. So far, to our knowledge, much work has not been done on the study of ERTI using smart property of couple stress poorly conducting fluid, regarded as ERF, to reduce the growth rate of ERTI at the ablative surface of IFE target and also in the effective design of artificial organs in biomedical engineering in the presence of an electric field. The study of it is the main objective of this paper.

To achieve this objective, this paper is planned as follows. The required basic equations and the relevant boundary and surface conditions and the suitable approximations are discussed in Section 2. The electrical conductivity, the conduction temperature in the presence of radiation, electric potential, the electric field and the density of distribution of charges are also obtained in Section 2. Section 3 is devoted to find the velocity distribution. The dispersion relation derived analytically using linear stability analysis in Section 4. The important conclusions are drawn in the final Section 5.

2. Formulation of the Problem

We consider a target shell in the form of a thin film of an unperturbed thickness, h , filled with an incompressible poorly electrically conducting light couple stress fluid of uniform density, ρ_f , called region 1. It is bounded below by a rigid surface and above by an incompressible, poorly conducting couple stress heavy fluid saturated in a densely packed porous layer of large extent H of uniform density, ρ_p , called region 2. To generate an applied electric field the lower rigid boundary is embedded with the electrodes of higher potential at $y=0$ and at the interface the electrodes of lower potential are embedded at $y=h$. Here the suffixes f and p refer to the quantities in clear fluid and in porous layer, respectively. The assumption on densely packed fluid saturated porous layer with heavy couple stress poorly conducting fluid is almost static is needed for RTI. The assumption of the nominal surface as in the experimental work of Beavers and Joseph [19] and theoretical work of Rudraiah [20] is needed to maintain laminar flows as well as to use Saffman [21] slip condition. The fluid in the shell is set in motion by acceleration normal to the interface and small disturbances are amplified when the acceleration is directed from the lighter couple stress poorly conducting fluid in the shell to heavy poorly conducting couple stress fluid in the porous layer of large extent H compared to thin film of thickness h . This instability at the interface, by definition, is RTI. To investigate this RTI, we consider a rectangular coordinate system (x,y) as shown in Fig. 1 with x -axis parallel to the film and y -axis normal to it. Here $\eta(x,t)$ is the perturbed interface between the couple stress poorly conducting fluid in the shell and with the porous layer (see Fig. 1).

For the sake of clarity we first give general form of modified basic equations for a poorly conducting couple stress two dimensional incompressible fluid flow, modification in the sense of addition of electric force to be obtained from the general form of Maxwell equations.

The rheological properties of physiological fluids like synovial fluid in synovial joints, blood in arteries reveal that viscosity varies non-linearly with concentration of suspended particles exhibiting either shear thinning or shear thickening behavior. This is one of the non-Newtonian fluid flow properties, which have been studied in the literature (see [22,23]) but their work is silent about micromotions, microrotation and deformations. These are taken into account in this paper using couple stress fluid as a particular case of "micropolar fluid theory" developed by Eringen [9] as explained in Section 1 above. The basic equations for this fluid are:

Conservation of mass for an incompressible fluid:

$$\frac{\partial q_i}{\partial x_i} = 0 \quad (1)$$

Conservation of linear momentum:

$$\rho_f \frac{Dq_i}{Dt} = \frac{\partial \tau_{ij}}{\partial x_j} + \pi_i + \rho_f f_i + \rho_e E_i \quad (2)$$

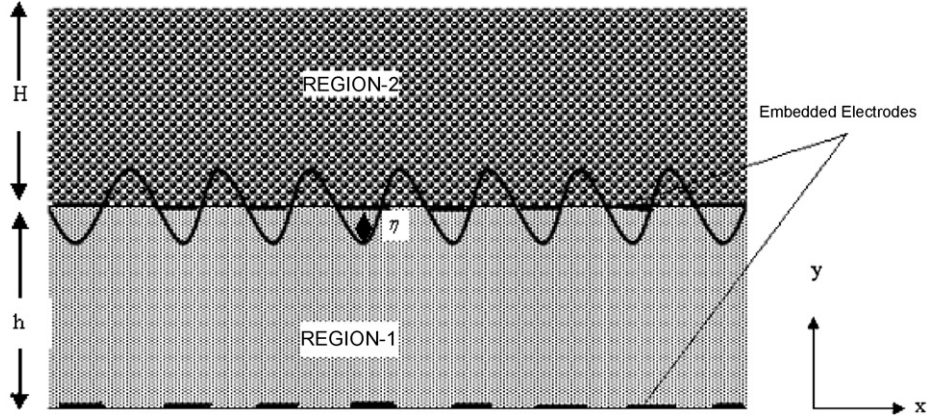


Fig. 1. Physical Configuration.

Conservation of angular momentum:

Angular momentum is usually defined as the moment of the linear momentum. If r_i is the position vector of the particles and $p_i = \rho q_i$

$$(3)$$

is the linear momentum, then the angular momentum L_i is

$$L_i = \varepsilon_{ijk} r_j p_k \quad (4)$$

where

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{if } i, j, k \text{ take values in cyclic order} \\ -1 & \text{if } i, j, k \text{ take values in acyclic order} \\ 0 & \text{if two or all of } i, j, k \text{ take the same value} \end{cases}$$

is the Levi-Civita symbol.

The conservation of angular momentum, neglecting the body couple and contact couples, following Rudraiah [23], can be obtained by taking the cross product of r_i with Eq. (2) and using Eq. (3) in the form

$$\varepsilon_{ijk} r_i \frac{\partial p_k}{\partial t} + \varepsilon_{ijk} r_i \frac{\partial q_j}{\partial x_k} p_k = \varepsilon_{ijk} r_i \frac{\partial \tau_{jk}}{\partial x_k} + \varepsilon_{ijk} \rho_f r_i f_k + \rho_e r_i E_i \quad (5)$$

In Eqs. (1)–(5), $q_i (i=1, 2)$ is the velocity vector having the components (u, v) , τ_{jk} the stress tensor, $\rho_f f_k$ the body force, $\rho_e E_i$ the electric force, ρ_e the distribution of charge density and E_i is the electric field.

The conservation of charges for incompressible fluid:

$$\frac{\partial \rho_e}{\partial t} + q_i \frac{\partial \rho_e}{\partial x_i} + \frac{\partial J_i}{\partial x_i} = 0 \quad (6)$$

where J_i is the conduction current density given by

$$J_i = \sigma E_i \quad (7)$$

The Maxwell field equations are Gauss law is

$$\frac{\partial E_i}{\partial x_i} = \frac{\rho_e}{\varepsilon_0} \quad (8)$$

where ε_0 is the dielectric constant for free space.

In a poorly conducting fluid the induced magnetic field is negligible and there is no applied magnetic field, hence the Faradays law becomes

$$\frac{\partial E_i}{\partial x_j} - \frac{\partial E_j}{\partial x_i} = 0 \quad (9)$$

That is, the electric field is conservative, so that

$$E_i = -\frac{\partial \phi}{\partial x_i} \quad (10)$$

where ϕ is the electric potential.

In a poorly conducting fluid since $\sigma \ll 1$, any perturbation on it is negligible and increases with the conduction temperature, T_b , such that

$$\sigma = \sigma_0 [1 + \alpha_h (T_b - T_0)] \quad (11)$$

where T_0 is the atmospheric temperature, σ_0 that of σ at $T=T_0$, and α_h is the volumetric expansion coefficient of σ .

We note that the limitations encountered in the continuum theory are the lack of taking into account the microrotation of a fluid, for example hyaluronic acid (HA) molecules present in synovial fluid. In that case, the intrinsic motions of the microelements must be taken into account in deriving the required constitutive equations where the microelement motions and deformations play a significant role. In such situations, the Eringen's [9] "micropolar fluid theory" explained in Section 1 above is useful and he showed that the couple stress theory results as a special case of micropolar fluid theory if the microrotation vector is constrained equal to the fluid bulk vorticity throughout the flow field. Then, the constitute equations for couple stress fluid, following Stokes [24] as in Rudraiah [23], are:

$$\tau_{ij} = (-p + \mu' e_{kk}) \delta_{ij} + 2\mu e_{ij} \quad (12)$$

$$\tau'_{ij} = -2\lambda \Omega_{ij,kk} - \frac{\rho}{2} \varepsilon_{ijs} G_s \quad (13)$$

$$M_{ij} = 4\lambda \Omega_{j,i} + 4\lambda' \Omega_{ij} \quad (14)$$

where

$$e_{ij} = \frac{1}{2} \left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i} \right) \text{ is the strain tensor} \quad (15)$$

$$\Omega_i = \varepsilon_{ijk} q_{k,j} = \frac{1}{2} \left(\frac{\partial q_i}{\partial x_j} - \frac{\partial q_j}{\partial x_i} \right) \text{ is the vorticity tensor} \quad (16)$$

G_s is the angular velocity vector and M_{ij} is the body moment. Here the dimensions of μ' and μ are those of viscosity and the couple stress parameters λ and λ' are those of momentum. The ratio (λ/μ) has the dimensions of length squared.

For an incompressible fluid, in the absence of the body moments, the basic equations of motion for couple stress fluid,

using Eqs. (5), (12)–(14), following Rudraiah [23], are

$$\rho_f \left(\frac{\partial q_i}{\partial t} + q_i \frac{\partial q_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 q_i}{\partial x_i^2} - \lambda \frac{\partial^4 q_i}{\partial x_i^4} + \rho_e E_i \quad (17)$$

The conservation of energy, in the presence of laser radiation, in regions 1 and 2, are:

Region 1: For clear fluid

$$\frac{\partial T_f}{\partial t} + q_i \frac{\partial T_f}{\partial x_j} = K_f \frac{\partial^2 T_f}{\partial x_i^2} + I_0 \Omega e^{-\Omega y} \quad (18)$$

Region 2: For porous layer

$$\frac{\partial T_p}{\partial t} + q_i \frac{\partial T_p}{\partial x_j} = K_p \frac{\partial^2 T_p}{\partial x_i^2} + I_0 \Omega e^{-\Omega y} \quad (19)$$

where T_f and T_p are the temperatures in clear fluid and in fluid saturated porous layer, respectively, Ω is the absorption frequency, K_f and K_p are the thermal diffusivities of fluid in thin shell and in porous layer, respectively. In region 2, the poorly conducting couple stress fluid is very heavy and densely packed porous layer so that we assume it as almost static and has uniform density and temperature distribution. From the energy equation, by assuming the fluid in a porous layer is static and is in thermal equilibrium, we get

$$\frac{dT_b^2}{dy^2} = -\frac{I_0 \Omega e^{-\Omega y}}{K} \quad (20)$$

The boundaries are assumed to be conducting and isothermal so that we have the boundary conditions:

$$T_b = T_0 \quad \text{at } y = 0 \quad (21)$$

$$T_b = T_1 \quad \text{at } y = 1 \quad (22)$$

Solving Eq. (20) using conditions (21) and (22) and using Eq. (11) with the assumption that the absorption frequency ($\Omega \cong 10^6/\text{m}$) of laser radiation has to decay in a short distance of order $1/\Omega$, we get,

$$\sigma = \sigma_0 [1 + \alpha_h \Delta T y] \cong \sigma_0 e^{2y} \quad (23)$$

where

$$\alpha = \alpha_h \Delta T \quad \text{and} \quad \Delta T = T_1 - T_0 \quad (24)$$

Further, we assume the frequency of charge distribution is smaller than the corresponding relaxation frequency of the electric field so that the convective current $\rho_e q_i$ and the time derivative of ρ_e are negligible compared to $\nabla \cdot (\sigma E)$, in Eq. (6). Then we get

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \alpha \frac{\partial \phi}{\partial y} = 0 \quad (25)$$

Satisfying the boundary conditions,

$$\phi = \frac{Vx}{h} \quad \text{at } y = 0 \quad (26)$$

$$\phi = \frac{V(x-x_0)}{h} \quad \text{at } y = h \quad (27)$$

These conditions arise due to the embedded electrodes of different potentials at $y=0$ and h as shown in Fig. 1 and permits a linear variation of ϕ with respect to x .

Solving Eq. (25), using conditions (26) and (27), after making them dimensionless using the scales V for potential and h for length, we get

$$\phi = x - \frac{x_0(1-e^{-2y})}{(1-e^{-2})} \quad (28)$$

with this, using Eqs. (7)–(10), we get,

$$E_x = -\frac{\partial \phi}{\partial x} = -1 \quad (29)$$

$$E_y = -\frac{\partial \phi}{\partial y} = \frac{x_0 \alpha e^{-2y}}{1-e^{-2}} \quad (30)$$

and

$$\rho_e = -\varepsilon_0 \frac{\partial^2 \phi}{\partial y^2} = -\frac{x_0 \varepsilon_0 \alpha^2 e^{-2y}}{1-e^{-2}} \quad (31)$$

From these equations we obtain,

$$\rho_e E_x = \frac{x_0 \varepsilon_0 \alpha^2 e^{-2y}}{1-e^{-2}} \quad (32)$$

and

$$\rho_e E_y = -\frac{\varepsilon_0 x_0^2 \alpha^3 e^{-2y}}{(1-e^{-2})^2} \quad (33)$$

3. The solution of the problem

In this section we obtain the velocity and temperature distributions by using the suitable boundary and surface conditions:

3.1. Velocity Distribution

To study the problem of RTI posed in this paper, we consider the steady and unidirectional flow of lighter couple stress poorly conducting fluid in region 1 accelerating the heavy dense couple stress poorly conducting fluid saturated in densely packed porous layer in region 2 in the presence of laser radiation with $\eta(x,t)$ as the elevation of the interface at $y=h$. To obtain the required equations, following Rudraiah [23], we use the following combined lubrication and Stokes approximations:

1. The thickness h of the film is much smaller than the thickness H of densely packed porous layer above the film, that is

$$h \ll H \quad (34)$$

2. The non-dimensional Strouhal number, S , which is the measure of the local acceleration to the inertial acceleration, is $S = L/UT \ll 1$

Here $U = (v/L)$ the characteristic velocity, $v = (\mu/\rho)$ the kinematic viscosity, $L = \sqrt{\gamma/\delta}$ the characteristic length, γ the surface tension, $\delta = g(\rho_p - \rho_f)$ and $T = (\mu\gamma/h^3\delta^2)$ is the characteristic time scale. These assumptions enabled us to neglect the local acceleration term $\partial q/\partial t$ in Eq. (17).

3. We consider high viscous couple stress fluid so that the inertial acceleration term in Eq. (17) can be neglected compared with the viscous term.
4. The interface elevation η is assumed to be small compared with film thickness h , that is

$$\frac{\eta}{h} \ll 1$$

These assumptions enabled us to use the creeping flow approximations which allow us to neglect certain terms in the perturbation equations to obtain linear equations for the interface elevation. Under these assumptions, for two dimensional flow, the basic Eqs. (17) and (1) respectively, using Eqs. (32) and (33) and making them dimensionless using the scales, h for length, δh for

pressure, $\delta h^2/\mu_f$ for velocity, $\mu_f\gamma/h^3\delta^2$ for time, V/h for electric field, $\epsilon_0 V/h$ for density of the charges, $\theta_{f,p}T_o$ for temperatures, V for electric potential, take the form

$$0 = -\beta^2 \frac{\partial p}{\partial x} + \beta^2 \frac{\partial^2 u}{\partial y^2} - \frac{\partial^4 u}{\partial y^4} + \frac{w_e \beta^2 \alpha_0^2 x_0 e^{-\alpha y}}{(1-e^{-\alpha})} \tag{36}$$

$$0 = -\frac{\partial p}{\partial y} - \frac{w_e x_0^2 \alpha^3 e^{-2\alpha y}}{(1-e^{-\alpha})^2} \tag{37}$$

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{38}$$

where $\beta^2 = (\mu_f h^2/\lambda)$ is the couple stress parameter, $w_e = (\epsilon_0 V^2 \alpha^2 x_0 / \delta h^2)$ is the electric number which represents physically the measures the electric energy to kinetic energy of the system.

These equations have to be solved using the following non-dimensional form of the boundary and surface conditions on velocity:

The no-slip boundary conditions at the rigid surface are

$$u = 0 \text{ and } v = 0 \text{ at } y = 0 \tag{39}$$

The Saffman [21] slip condition at the interface between porous layer and clear fluid layer is

$$\frac{\partial u}{\partial y} = -\alpha_1 \sigma_p u \text{ at } y = 1 \tag{40}$$

where $\sigma_p = h/\sqrt{k}$ is the porous parameter and α_1 is the slip coefficient.

The couple stress boundary conditions are

$$\frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = 0 \text{ and } y = 1 \tag{41}$$

The dynamic surface condition, at the interface, is

$$p = -\eta(\delta(1) - w_e) - \frac{1}{B} \frac{\partial^2 \eta}{\partial x^2} \text{ at } y = 1 \tag{42}$$

where $B = \delta h^2/\gamma$ is the Bond number and $\delta(1) = \theta_f(1) - \theta_p(1)$.

The kinematic surface condition, at the interface, is

$$v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \text{ at } y = 1 \tag{43}$$

For the linear case the above condition (43) reduces to the form

$$v = \frac{\partial \eta}{\partial t} \text{ at } y = 1 \tag{44}$$

Solution of Eq. (36), satisfying conditions (39)–(41), is

$$u = C \cosh \beta y + D \sinh \beta y + \frac{y^2}{2} \frac{\partial p}{\partial x} - \frac{A y}{\beta^2} + \frac{w_e \beta^2 x_0 e^{-\alpha y}}{(\alpha^2 - \beta^2)(1 - e^{-\alpha})} + \frac{1}{\beta^2} \left(\frac{\partial p}{\partial x} - B_0 \right) \tag{45}$$

where

$$A = \frac{\beta^2}{(1 + \alpha_1 \sigma_p)} \left\{ a_1 \frac{\partial p}{\partial x} - \frac{w_e x_0 \alpha^2 a_2}{(1 - e^{-\alpha})(\alpha^2 - \beta^2)} + w_e x_0 \alpha_1 \sigma_p \right\},$$

$$a_1 = \frac{(2 + \alpha_1 \sigma_p)}{2} - \frac{(\cosh \beta - 1)}{\beta \sinh \beta}, \quad a_2 = \frac{\beta(e^{-\alpha} \cosh \beta - 1)}{\sinh \beta} + \frac{\beta^2 e^{-\alpha}}{\alpha^2},$$

$$B_0 = \frac{-w_e x_0 \beta^2}{1 - e^{-\alpha}}, \quad C = -\frac{1}{\beta^2} \left[\frac{\partial p}{\partial x} + \frac{w_e \beta^2 x_0 \alpha^2}{(\alpha^2 - \beta^2)(1 - e^{-\alpha})} \right] \text{ and}$$

$$D = -\frac{1}{\beta^2} \left[\frac{(1 - \cosh \beta)}{\sinh \beta} \frac{\partial p}{\partial x} + \frac{w_e \beta^2 x_0 \alpha^2 (e^{-\alpha} - \cosh \beta)}{(\alpha^2 - \beta^2)(1 - e^{-\alpha}) \sinh \beta} \right]$$

3.2. Temperature Distribution

To find the effect of laser radiation on heat transfer in a thin film past a porous layer, we assume, as in Section 3.1, fully developed flow with basic state as $\vec{q} = (u(y), 0)$, $T_{f,p} = T_{f,p}(y)$, and making Eqs. (18) and (19) dimensionless using the scales mentioned in Section 3.1, we get,

$$0 = \frac{\partial^2 \theta_f}{\partial y^2} + s_f e^{-\Omega y} \tag{46}$$

$$0 = \frac{\partial^2 \theta_p}{\partial y^2} - s_p e^{-\Omega y} \tag{47}$$

where $s_f = (I_o \Omega h^3 / T_o k_f)$ and $s_p = (I_o \Omega h^3 / T_o k_p)$.

To find the solutions of Eqs. (46) and (47), we consider the following two cases for a poorly conducting fluid in porous layer.

Case 1: This fluid to be homogeneous.

Case 2: This fluid to be Boussinesq fluid.

For a poorly conducting couple stress homogeneous incompressible fluid in thin-film and in porous layer, we assume that the temperature, T_p , in the porous layer is uniform. Then Case 1: The boundary conditions on temperature for this case are:

Set 1:

$$\theta_f = 1 \text{ at } y = 0 \tag{48}$$

$$\theta_f = \theta_1 \text{ at } y = 1 \tag{49}$$

Set 2:

$$\theta_f = 1 \text{ at } y = 0 \tag{49}$$

$$\frac{\partial \theta_f}{\partial y} = -B_i(\theta_b - 1) \text{ at } y = 1 \tag{50}$$

where $B_i = (h_c h / k_f)$, is the Biot number and $\theta_f = \theta_b$ at $y = 1$.

Solving Eq. (46) using set 1 boundary conditions (48) and (49), we get

$$\theta_f = \frac{s_f}{\Omega^2} (1 - e^{-\Omega y}) + \frac{s_f (e^{-\Omega} - 1)}{\Omega^2} y + (\theta_1 - 1) y + 1 \tag{51}$$

In this case

$$\delta(1) = \theta_b - \theta_p \tag{52}$$

where θ_p is the constant temperature of the fluid saturated porous layer.

Case 2: The boundary conditions on temperature, for Boussinesq fluid in the thin film as well as in the porous layer, are

$$\theta_f = 1 \text{ at } y = 0 \tag{53}$$

$$\frac{\partial \theta_f}{\partial y} = -B_i(\theta_f - 1) \text{ at } y = 1 \tag{54}$$

$$\frac{\partial \theta_p}{\partial y} = -B_i(\theta_p - 1) \text{ at } y = 1 \tag{55}$$

$$\frac{\partial \theta_p}{\partial y} \rightarrow 0 \text{ as } y \rightarrow \infty \tag{56}$$

The solutions of Eqs. (46) and (47) using the this boundary conditions (53)–(56) are

$$\theta_f = 1 - \frac{s_f (e^{-\Omega y} - 1)}{\Omega^2} + \frac{B_i s_f (e^{-\Omega} - 1)}{\Omega^2 (B_i + 1)} y - \frac{s_f e^{-\Omega}}{\Omega (B_i + 1)} y \tag{57}$$

and

$$\theta_p = 1 + \frac{S_p}{\Omega^2} e^{-\Omega y} + \frac{S_p e^{-\Omega}}{\Omega^2 B_i} (B_i - \Omega) \tag{58}$$

Now using Eqs. (57) and (58), we obtain an expression for $\delta(1)$ in the following form:

$$\delta(1) = \theta_f(1) - \theta_p(1) \tag{59}$$

That is

$$\delta(1) = \frac{S_f}{\Omega^2} \left[\frac{1 - (1 + \Omega)e^{-\Omega}}{B_i + 1} \right] - \frac{S_p e^{-\Omega} (2B_i - \Omega)}{\Omega^2 B_i} \tag{60}$$

4. Dispersion Relation

Integrating Eq. (38) over the limits 0 to 1 and using conditions (39), we get

$$v(1) = - \int_0^1 \frac{\partial u}{\partial x} dy \tag{61}$$

This, using Eq. (45) and integrating, we get

$$v(1) = \left\{ \frac{1}{3} - \left[\frac{4 + \beta^2}{4\beta^2} - \frac{2(\cosh\beta - 1)}{\beta^3 \sinh\beta} + \frac{2(\cosh\beta - 1) - \beta \sinh\beta}{4\beta \sinh\beta (1 + \alpha_1 \sigma_p)} \right] \right\} \frac{\partial^2 p}{\partial x^2} \tag{62}$$

From Eq. (44), using the normal mode solution of the form $\eta = \eta_0 e^{ilx + nt}$ and using Eqs. (42) and (62), we get the dispersion relation in the form

$$n = \left\{ \frac{1}{3} - \left[\frac{4 + \beta^2}{4\beta^2} - \frac{2(\cosh\beta - 1)}{\beta^3 \sinh\beta} + \frac{2(\cosh\beta - 1) - \beta \sinh\beta}{4\beta \sinh\beta (1 + \alpha_1 \sigma_p)} \right] \right\} l^2 \left(\delta(1) - w_e - \frac{l^2}{B} \right) \tag{63}$$

where n is the growth rate in the presence of couple stress, electric field and laser radiation. In the absence of electric field, couple stress, porous layer and laser radiation that is as $w_e \rightarrow 0$, $\lambda \rightarrow 0$ ($\beta \rightarrow \infty$), $\sigma_p \rightarrow 0$ and $\delta(1) \rightarrow 1$, the dispersion relation (63) reduces to

$$n_b = \frac{l^2}{3} \left(1 - \frac{l^2}{B} \right) \tag{64}$$

which coincides with the expression given by Babchin et al. [25]. Then Eq. (63) can be written as

$$n = n_b - \left[\frac{4 + \beta^2}{4\beta^2} - \frac{2(\cosh\beta - 1)}{\beta^3 \sinh\beta} + \frac{2(\cosh\beta - 1) - \beta \sinh\beta}{4\beta \sinh\beta (1 + \alpha_1 \sigma_p)} \right] l^2 \left(\delta(1) - w_e - \frac{l^2}{B} \right) \tag{65}$$

5. Results and Conclusions

The linear ERTI of an incompressible viscous poorly conducting couple stress fluid in a thin film in the presence of an electric field and laser radiation, bounded below by a rigid surface and above by densely packed porous layer consisting of heavy couple stress poorly conducting fluid of high density is studied using linear stability analysis combined with the normal mode solution. The dispersion relation given by Eq. (65) following [8,2] can be expressed in the form:

$$n = n_b - \beta^* l v_a \tag{66}$$

where

$$v_a = \left[\frac{4 + \beta^2}{4} - \frac{2(\cosh\beta - 1)}{\beta \sinh\beta} + \frac{2\beta(\cosh\beta - 1) - \beta \sinh\beta}{4 \sinh\beta (1 + \alpha_1 \sigma_p)} \right] l \left(\delta(1) - w_e - \frac{l^2}{B} \right) \tag{67}$$

is the transverse velocity at the interface, $\beta^* = (1/\beta^2)$, n_b is given by Eq. (64).

Setting $n=0$ in the dispersion relation (63), we get the critical wave number, l_{ct} , as

$$l_{ct} = \sqrt{(\delta(1) - w_e) B} \tag{68}$$

The maximum wave number, l_m , obtained from Eq. (63), by setting $(\partial n / \partial l) = 0$ is

$$l_m = \frac{l_{ct}}{\sqrt{2}} \tag{69}$$

Relations (68) and (69) are true even in the absence of couple stress, electric field and laser radiation effects and for convenience we call them as classical results denoted by suffix b . Substituting Eq. (69) into Eq. (63), we get the maximum growth rate in the form

$$n_m = \frac{B(\delta(1) - w_e)^2}{4} \left\{ \frac{1}{3} - \left[\frac{4 + \beta^2}{4\beta^2} - \frac{2(\cosh\beta - 1)}{\beta^3 \sinh\beta} + \frac{2(\cosh\beta - 1) - \beta \sinh\beta}{4\beta \sinh\beta (1 + \alpha_1 \sigma_p)} \right] \right\} \tag{70}$$

Similarly, using Eqs. (64) and (68), we get the classical maximum growth rate as

$$n_{m_b} = \frac{B}{12} \tag{71}$$

The ratio of the growth rates, $G_m = n_m / n_{m_b}$, is obtained from Eqs. (70) and (71) as

$$G_m = (\delta(1) - w_e)^2 \left\{ 1 - 3 \left[\frac{4 + \beta^2}{4\beta^2} - \frac{2(\cosh\beta - 1)}{\beta^3 \sinh\beta} + \frac{2(\cosh\beta - 1) - \beta \sinh\beta}{4\beta \sinh\beta (1 + \alpha_1 \sigma_p)} \right] \right\} \tag{72}$$

where $\delta(1)$ is given by Eq. (60). We note that in the limit $\lambda \rightarrow 0$ ($\beta^2 \rightarrow \infty$), $w_e \rightarrow 0$ and $\delta(1) \rightarrow 1$, this ratio of growth rates given by Eq. (72) reduces to

$$G_m = \frac{4 + \alpha_1 \sigma_p}{4(1 + \alpha_1 \sigma_p)} \tag{73}$$

Table 1

Reduction in growth rate for different values β Couple stress parameter (For varying electric number w_e and slip parameter α_1).

β	w_e	$\delta(1)$	α_1	σ_p	G_m	Reduction in growth rate (%)
0.1	1	0.5	0.001	10	0.00392986	99.607
0.75					0.0116312	98.8369
1.5					0.0399001	96.01
2.25					0.0730293	92.6971
0.1	2	0.5	0.0001	10	0.00230254	99.7697
0.75					0.10476	89.524
1.5					0.361254	63.8746
2.25					0.661414	33.8586

Table 2

Reduction in growth rate for different values β couple stress parameter (for varying $\delta(1)$ and porous parameter σ_p).

β	w_e	$\delta(1)$	α_1	σ_p	G_m	Reduction in growth rate (%)
0.1	1	0.5	0.001	5	0.00115059	99.8849
0.75					0.0116228	98.8377
1.5					0.0400291	95.9971
2.25					0.073283	92.6717
0.1	1	0.75	0.001	10	0.000982464	99.9018
0.75					0.00290781	99.7092
1.5					0.00997502	99.0025
2.25					0.0182573	98.1743

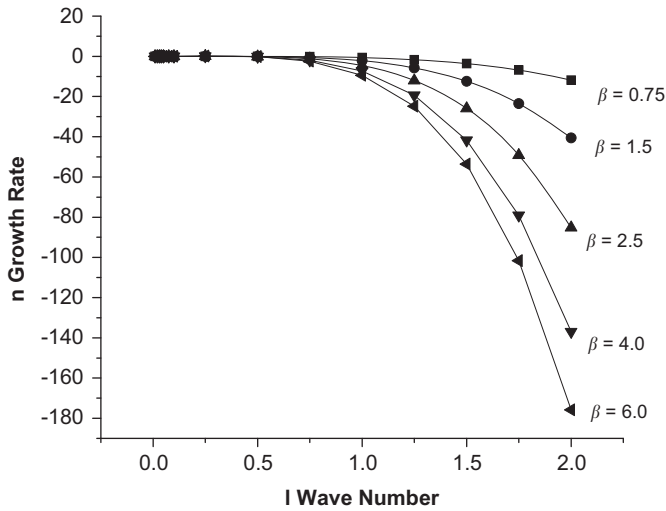


Fig. 2. Graph of n versus l for different values of β .

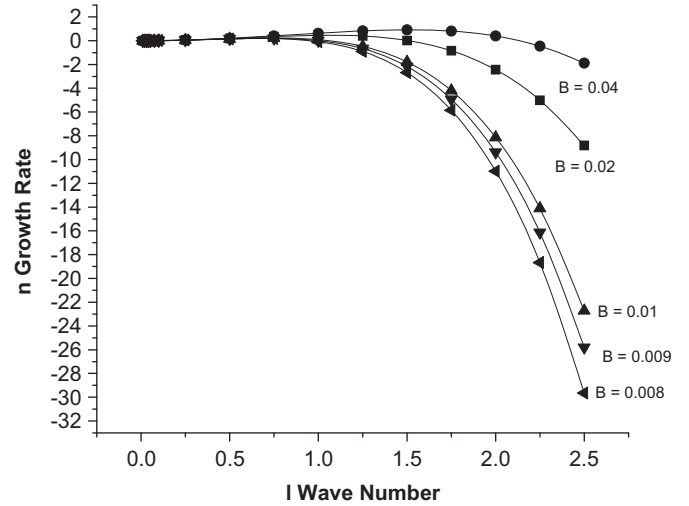


Fig. 5. Graph of n versus l for different values of B .

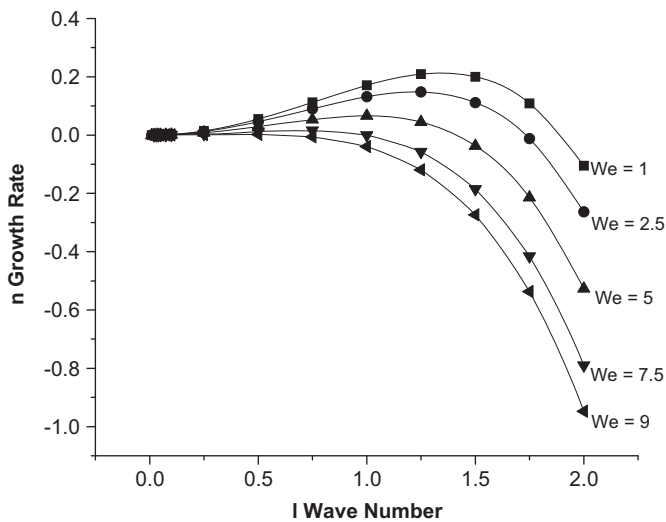


Fig. 3. Graph of n versus l for different values of We .

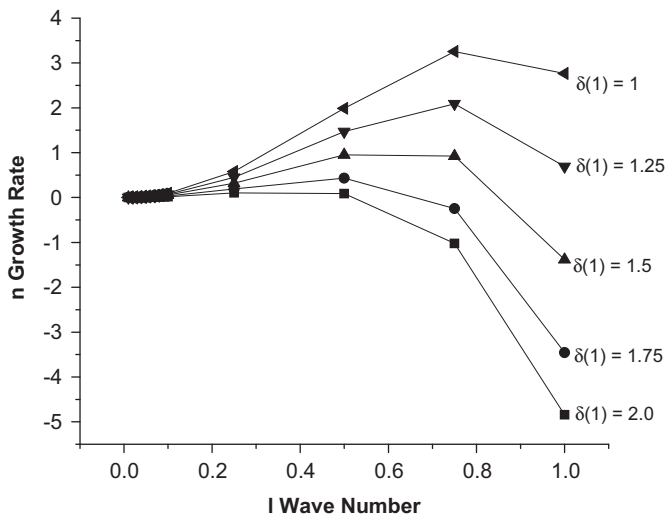


Fig. 4. Graph of n versus l for different values of $\delta(1)$.

this coincides with the one given by Rudraiah [2] in the absence of couple stress and electric field effects, called NR-formula.

This, G_m given by Eq. (72), is computed for different values of the couple stress parameter β , electric number w_e , porous parameter σ_p and absorption frequency Ω (expressed in difference in temperatures $\delta(1)$ at the interface) at the interface of the laser radiation and the results are tabulated in Tables 1 and 2.

From these tables, we note that the reduction of growth rate is 99% in the presence of laser radiation; couple stress and electric field, compared to 45% reduction predicted by Takabe et al. [8] and 79% by Rudraiah [2] in the absence of these quantities.

Also the growth rate n given by Eq. (63) is always real. This n is computed for different values of β , w_e , B and $\delta(1)$ and the values are depicted in Figs. 2–5, these Figs. 2–5 represent the dispersion relation n versus cutoff wave number l .

Fig. 2 represents the graph of the growth rate n versus wave number l for different values of β and with fixed values of $w_e=1$, $\delta(1)=10$, $B=0.02$ and $\sigma=0.001$. This figure shows decrease in β (increase in λ couple stress parameter) the negative growth rate n decreases. Particularly for $\beta=0.75$ the growth rate n has almost approaches zero and hence stabilizes the interface elevation.

Fig. 3 represents the graph of the growth rate n versus wave number l for different values of w_e and with fixed values of $\beta=1$, $\delta(1)=10$, $B=0.02$ and $\sigma=0.001$. This graph shows that for an increase in w_e the growth rate n decreases. Particularly for $w_e=9$ the growth rate n is very small and hence stabilizes the interface elevation.

Fig. 4 represents the graph of the growth rate n versus wave number l for different values of $\delta(1)$ and with fixed values of $\beta=1$, $w_e=1$, $B=0.02$ and $\sigma=0.001$. This figure shows that for an increase in $\delta(1)$ the growth rate n decreases. Particularly for $\delta(1)=1$ the growth rate n is very small and hence stabilizes the interface elevation.

Fig. 5 represents the graph of the growth rate n versus wave number l for different values of Bond number B with fixed values of $\beta=1$, $w_e=1$, $\delta(1)=10$ and $\sigma=0.001$. This figure shows that decrease in B (increase in surface tension) increases the negative growth rate. Particularly for $B=0.04$ the growth rate n is very small and hence stabilizes the interface elevation.

Finally, from these graphs, we predict that the reduction of maximum growth rate in presence of an electric field, laser radiation and couple stress. Hence, we conclude that the electric field, laser radiation and couple stress are more effective in reducing the asymmetry of IFE caused by fusing DT by laser

radiation compared to classical growth rate in the absence of electric field and couple stress parameter. This conclusion on the reduction of growth rate of the ERTI mode is more effective in the design of a suitable IFT for increasing the efficiency of extraction of IFE by reducing the asymmetry caused by laser radiation. These conclusions are also useful in the effective design of artificial organs in biomedical engineering to prevent the side effects like hemolysis.

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