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Linear and nonlinear stability analysis of binary viscoelastic fluid convection

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ABSTRACT

The linear and weakly nonlinear stability analysis of the quiescent state in a viscoelastic fluid subject to vertical solute concentration and temperature gradients is investigated. The non-Newtonian behavior of the viscoelastic fluid is characterized using the Oldroyd model. Analytical expressions for the critical Rayleigh numbers and corresponding wave numbers for the onset of stationary or oscillatory convection subject to cross diffusion effects is determined. A stability diagram clearly demarcates non-overlapping regions of finger and diffusive instabilities. A Lorenz system is obtained in the case of the weakly nonlinear stability analysis. The effect of Dufour and Soret parameters on the heat and mass transports are determined and discussed. Due to consideration of dilute concentrations of the second diffusing component the route to chaos in binary viscoelastic fluid systems is similar to that of single-component (thermal) viscoelastic fluid systems.

1. Introduction

The study of non-Newtonian liquids has gained tremendous interest because of its usage as a working media in many engineering and industrial applications. Viscoelastic fluids which exhibit both solid and liquid properties have applications in such diverse fields as geothermal energy modeling, material processing, thermal insulation material, cooling of electronic devices, transport of chemical substances, crystal growth, injection molding and solar receivers. Other applications are found in the petroleum industry, chemical and nuclear industries, geophysics, bioengineering and so on. The rheological equation for viscoelastic liquid usually involve either one or two relaxation times. They possess both elasticity (associated with solids) and viscosity (associated with liquids) which leads to unique instability patterns such as overstability that is not predicted or observed in Newtonian fluids. Hence, Rayleigh–Bernard convection in a thin rectangular layer of viscoelastic fluid heated from below has been the focus of many studies over the past few decades, [1–8].

Vest and Arpaci [1] and Sokolov and Tanner [2] were among the first to study the linear stability of convection in a horizontal layer of an upper-convected Maxwell fluid, for which the stress exhibits an elastic response to strain characterized by a single viscous relaxation time. Li and Khayat [3,4] studied stationary and oscillatory instabilities in great detail for the Oldroyd-B viscoelastic model. Their study gave useful insight into pattern formation in viscoelastic fluid convection. Green [5] studied oscillatory convection in an elasticoviscous liquid. He found that a large restoring force sets up an oscillating convective motion in a thin layer of elasticoviscous fluid heated from below. The linear stability analysis of the Rayleigh–Benard convection problem in a Boussinesquian, viscoelastic fluid has investigated by Siddheshwar and Krishna [6]. They showed that thermodynamics and stability analysis dictates that the strain retardation time should be less than the stress relaxation

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time for convection to set in as oscillatory motions in high-porosity media. Recently, Siddheshwar et al. [7] studied nonlinear stability of thermal convection in a layer of viscoelastic liquid subject to gravity modulation. They used a novel transformation for the momentum equations as an alternative to the approach by Khayat that uses normal stresses explicitly in deriving the Lorenz system for the complex dynamics.

Sharma [8] studied the thermal instability of a layer of a uniformly rotating Oldroyd fluid and found that rotation has a destabilizing as well as a stabilizing effect in contrast to a Maxwell fluid (Bhatia and Steiner [9]).

Experimental studies by Kolodner [10] confirmed the existence of oscillatory convection in suspensions in annular geometry. The findings contradicted earlier beliefs that oscillatory convection can not be observed in viscoelastic liquids. Although he established a qualitative agreement in the oscillatory instability threshold with theoretical results, the critical oscillatory frequency was mismatched by several orders of magnitude. This shortcoming pointed to the fact that in theoretical studies binary fluid aspects are often neglected. Through a series of studies, Martinez-Mardones and co-workers [11–15] investigated Rayleigh–Benard convection in viscoelastic liquids taking binary aspects into consideration.

Double diffusive convection is a common feature in binary fluids with competition between heat and solute diffusivities. In such fluids density variations depend on both thermal and solutal gradients which diffuse at different rates. This leads to the formation of salt fingers or oscillations in the fluid layer (see, for example Stern [16,17]). Malashetty and Swamy [18] investigated the onset of double diffusive convection in a viscoelastic fluid layer. They found that there is a competition between the processes of thermal diffusion, solute diffusion and viscoelasticity that causes the convection to set in through an oscillatory mode rather than a stationary mode.

While heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are quite complex. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. Mass fluxes created by temperature gradients give rise to the thermal-diffusion or Soret effect. These effects are collectively known as the cross-diffusion effects. Both Dufour and Soret effects have been extensively studied in gases, while the Soret effect has been studied both theoretically and experimentally in liquids, see Mortimer and Eyring [19]. Studies have shown that in areas such as geosciences Dufour and Soret effects can be significant (see Knobloch [20] and the references therein). Awad et al. [21] performed a linear stability analysis of double-diffusive convection in a porosity porous medium saturated with a Maxwell fluid and subject to cross diffusion effects. Malashetty and Biradar [22] considered cross-diffusion effects on the onset of double-diffusive convection in a binary Maxwell fluid in a porous layer.

In this paper we use linear and weakly nonlinear stability analysis to investigate cross-diffusion effects in a binary viscoelastic fluid layer. The main objective of the paper is to study the Soret and Dufour effects on the onset of stationary and oscillatory convection in a viscoelastic fluid layer. We also study cross-diffusion effects on the heat and mass transports and also analyze the effect of the second diffusing component on chaos.

2. Mathematical formulation

We consider two-component convection in a viscoelastic liquid of Oldroyd-B type occupying a horizontal channel of infinite extent and depth d. A Cartesian coordinate system is taken with the lower plate in the xy-plane and z-axis vertically upwards. A temperature difference of ΔT and the concentration difference ΔC are maintained between lower and upper plates at $z = 0$ and $z = d$ respectively as shown in Fig. 1. The gravity $\vec{g} = -g\hat{k}$ is assumed act vertically downwards. The Boussinesq–Oberbeck approximation is assumed to be valid for the viscoelastic liquid considered (see Rajagopal et al. [23]). We also assume that there is coupling between the two diffusing components.

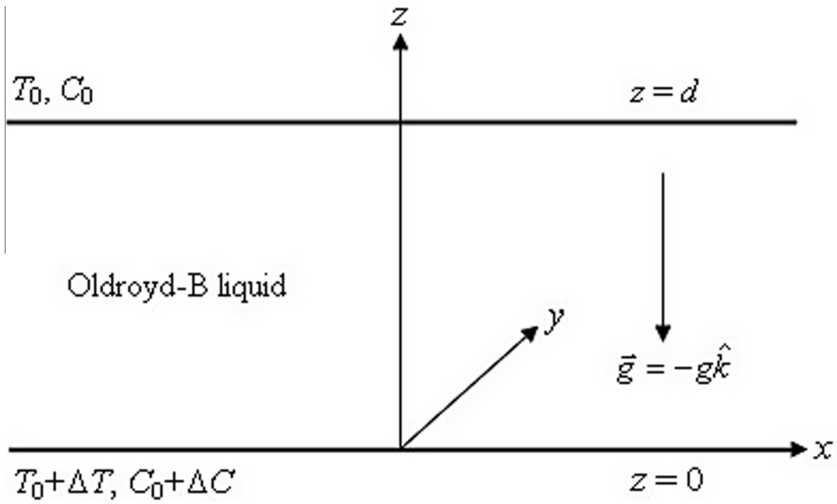


Fig. 1. Schematic diagram of the physical problem.

The governing equations for the Oldroyd-B liquid in the presence of cross diffusion effect are:

$$q_{i,i} = 0, \tag{1}$$

$$\rho_0 \frac{\partial q_i}{\partial t} = -p_{,i} - \rho g \delta_{i3} + \tau'_{ij,j}, \tag{2}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\tau'_{ij} = \left(1 + \lambda_2 \frac{\partial}{\partial t}\right)[\mu(q_{i,j} + q_{j,i})], \tag{3}$$

$$\frac{\partial T}{\partial t} + q_j T_{,j} = D_{11} T_{,jj} + D_{12} C_{,jj}, \tag{4}$$

$$\frac{\partial C}{\partial t} + q_j C_{,j} = D_{22} C_{,jj} + D_{21} T_{,jj}, \tag{5}$$

where q_i is the ith fluid velocity component, p is the pressure, ρ is the density, λ_1 is the stress relaxation coefficient, λ_2 is the strain retardation coefficient, μ is the fluid viscosity, T and C are respectively the temperature and solute concentrations, D_{11} and D_{22} are respectively the thermal and solutal diffusivities, D_{12} and D_{21} are parameters quantifying the contribution to the heat flux due to solutal gradient and to the mass flux due to temperature gradient respectively. The density ρ of the binary fluid depends on both the temperature T and the concentration C. For small density variations at a constant pressure, the density variation are modeled by the equation

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{6}$$

where α and α' are the coefficients of the thermal and solutal expansions respectively, T_0 and C_0 are taken as the reference state.

It is important to notice here the neglect of the convective derivatives in the momentum equation. This is a consequence of our assumption that thermally induced instabilities dominate hydrodynamic instabilities, i.e., the convective acceleration term is negligibly small in comparison with the heat advection term. This also means that we are considering small scale convective motions. Further, in view of the Boussinesq approximation, we have

$$\frac{1}{\rho_0}\frac{d\rho}{dt} \ll 1.$$

This has also been used in arriving at the continuity equation in the form (1). Added to this we note that the coefficient λ_1 of $\frac{1}{\rho_0}\frac{d\rho}{dt}$ is quite small (see Siddheshwar et al. [7] for further details). With this neglect it becomes apparent that convective and upper-convective terms cannot appear.

Eliminating τ'_{ij} between (2) and (3)

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left[\rho_0 \frac{\partial q_i}{\partial t} + p_i + \rho g \delta_{i3}\right] = \mu\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) q_{i,jj}. \tag{7}$$

The basic state of the fluid can be described by

$$q_{ib} = (0,\ 0,\ 0), \quad -\frac{d}{\Delta T}\frac{dT_b}{dz} = 1, \quad -\frac{d}{\Delta C}\frac{dC_b}{dz} = 1, \quad \rho = \rho_b(z) \quad \text{and} \quad p = p_b(z) \tag{8}$$

To determine the stability of the layer we disturb the basic state by an infinitesimal amplitude perturbation, and using (6) this yields:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{\partial q'_i}{\partial t} + \frac{1}{\rho_0} p'_{,i} - g\left(\alpha T' - \alpha' C'\right)\delta_{i3}\right] = \nu\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) q'_{i,jj}, \tag{9}$$

$$\frac{\partial T'}{\partial t} + q'_i T'_{,j} - w'\frac{\Delta T}{d} = D_{11} T'_{,jj} + D_{12} C'_{,jj}, \tag{10}$$

$$\frac{\partial C'}{\partial t} + q'_i T'_{,j} - w'\frac{\Delta C}{d} = D_{22} C'_{,jj} + D_{21} T'_{,jj}, \tag{11}$$

where $\nu = \frac{\mu}{\rho_0}$ is the kinematic viscosity.

The problem defined through Eqs. (9)–(11) is non-dimensionalized using the following new variables

$$(x, y, z) = (x^*, y^*, z^*)d, \quad = \frac{d^2}{D_{11}} t^*, \quad P = \frac{d^2}{\mu D_{11}} p', \quad q'_i = \frac{D_{11}}{d} q^*_i, \quad T' = (\Delta T)\theta, \quad C' = (\Delta C)\phi. \tag{12}$$

By substituting (12) in Eqs. (9)–(11) and dropping asterisks for simplicity we obtain the following system:

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{1}{Pr}\frac{\partial q_i}{\partial t} + P_{,i} - Ra(\theta - N\phi)\delta_{i3}\right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right) q_{i,jj}, \tag{13}$$

$$\frac{\partial \theta}{\partial t} + q_j \theta_{,j} - w = \theta_{,jj} + Du\,\phi_{,jj}, \tag{14}$$

$$\frac{\partial \phi}{\partial t} + q_j \phi_{,j} - w = Le^{-1}\left(\phi_{,jj} + S\theta_{,jj}\right), \tag{15}$$

where *Pr* is the Prandtl number, *Ra* is the Rayleigh number, *N* is the buoyancy ratio, *Du* is the Dufour number, *S* is the Soret number and *Le* is the Lewis number. In addition, Λ_1 and Λ_2 are respectively the scaled stress-relaxation parameter (Deborah number) and the scaled strain-retardation parameter. These parameters are defined as

$$Pr = \frac{v}{D_{11}}, \quad Ra = \frac{\alpha g d^3 \Delta T}{v D_{11}}, \quad N = \frac{\alpha'}{\alpha}\frac{\Delta C}{\Delta T}, \quad Du = \frac{D_{12}}{D_{11}}\frac{\Delta C}{\Delta T},$$
$$S = \frac{D_{21}}{D_{22}}\frac{\Delta T}{\Delta C}, \quad Le = \frac{D_{11}}{D_{22}}, \quad \Lambda_1 = \lambda_1 \frac{D_{11}}{d^2}, \quad \Lambda_2 = \lambda_2 \frac{D_{11}}{d^2}.$$

Taking the curl twice on both sides of Eq. (13) yields

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 w - Ra\nabla_1^2(\theta - N\phi)\right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right)\nabla^4 w, \tag{16}$$

where ∇_1 and ∇ are the Laplacian operators in two- and three-dimensions respectively.

3. Linear stability analysis

To discuss the linear stability, we assume that the perturbed quantities can be expressed as follows

$$\begin{bmatrix} w(x,y,z,t) \\ \theta(x,y,z,t) \\ \phi(x,y,z,t) \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{bmatrix} \exp\{\sigma t + i\ell x + imy\} \tag{17}$$

where $W(z), \Theta(z)$ and $\Phi(z)$ are amplitudes, l and m are dimensionless wave numbers, with $k = l\hat{i} + m\hat{j}$. The quantity σ is a complex quantity given by $\sigma = \sigma_r + i\omega$ where σ_r, the growth rate and ω, the frequency of oscillations are real. Substituting Eq. (17) into (14)–(16), we get

$$\sigma\Theta - W = \left[\frac{d^2}{dz^2} - k^2\right](\Theta + Du\Phi), \tag{18}$$

$$\sigma\Phi - W = Le^{-1}\left[\frac{d^2}{dz^2} - k^2\right](\Phi + S\Theta), \tag{19}$$

$$\frac{\sigma}{Pr}\left[\frac{d^2}{dz^2} - k^2\right]W + k^2 Ra(\Theta - N\Phi) = \left(\frac{1 + \Lambda_2\sigma}{1 + \Lambda_1\sigma}\right)\left(\frac{d^2}{dz^2} - k^2\right)^2 W. \tag{20}$$

In general, a wide variety of boundary conditions may be applied to Eqs. (18)–(20), see Sekhar and Jayalatha [24] for a list of such conditions. Here, we make use of the usual stress-free, isothermal and isohaline boundary conditions:

$$W = \frac{d^2 W}{dz^2} = \Theta = \Phi = 0 \text{ at } z = 0, 1. \tag{21}$$

By assuming a periodic wave solution with sinusoidal variations in (W, Θ, Φ), we can set (see Chandrasekhar [25]);

$$(W, \Theta, \Phi) = (W_0, \Theta_0, \Phi_0,) \sin \pi z \tag{22}$$

where W_0, Θ_0 and Φ_0 are the amplitudes of the velocity, temperature and concentration perturbations. Clearly these satisfy the boundary conditions (21). Substituting (22) in Eqs. (18)–(20) we get

$$-W_0 + (\sigma + \delta^2)\Theta_0 + \delta^2 Du\Phi_0 = 0, \tag{23}$$

$$-W_0 + \delta^2 S Le^{-1}\Theta_0 + \left(\sigma + \delta^2 Le^{-1}\right)\Phi_0 = 0, \tag{24}$$

$$\left[\left(\frac{1 + \Lambda_2\sigma}{1 + \Lambda_1\sigma}\right)\delta^4 + \frac{\sigma}{Pr}\delta^2\right]W_0 - k^2 Ra\Theta_0 - k^2 RaN\Phi_0 = 0. \tag{25}$$

where $\delta^2 = \pi^2 + k^2$ is the total wave number. Eqs. (23)–(25) form a homogeneous system in W_0, θ_0 and ϕ_0:

$$\begin{bmatrix} \left(\frac{1+\Lambda_2\sigma}{1+\Lambda_1\sigma}\right)\delta^4 + \frac{\sigma}{Pr}\delta^2 & -k^2 Ra & k^2 RaN \\ -1 & \sigma + \delta^2 & \delta^2 Du \\ -1 & \delta^2 S Le^{-1} & \sigma + \delta^2 Le^{-1} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{26}$$

For a non-trivial solution to the above system, we require:

$$\begin{vmatrix} \left(\frac{1+\Lambda_2\sigma}{1+\Lambda_1\sigma}\right)\delta^4 + \frac{\sigma}{Pr}\delta^2 & -k^2 Ra & k^2 Rs \\ -1 & \sigma+\delta^2 & \delta^2 Du \\ -1 & \delta^2 S Le^{-1} & \sigma + \delta^2 Le^{-1} \end{vmatrix} = 0. \tag{27}$$

Here $Rs = RaN$ is a solute Rayleigh number. Solving the Eq. (27) for Ra we get

$$Ra = \frac{\left(\frac{\delta}{k}\right)^2 \left[\frac{\sigma}{Pr} + \left(\frac{1+\Lambda_2\sigma}{1+\Lambda_1\sigma}\right)\delta^2\right]\left[\varsigma\left(\sigma+\delta^2 Le^{-1}\right) - DuSLe^{-1}\delta^4\right] + Rs\left[\varsigma - \delta^2 S Le^{-1}\right]}{\left(\sigma+\delta^2 Le^{-1}\right) - \delta^2 Du}, \tag{28}$$

where $\varsigma = \sigma + \delta^2$.

3.1. Stationary instability (Finger regime)

We observe the onset of stationary convection (the exchange of stabilities) when $\sigma = 0$. In this case, from Eq. (28) we can get

$$Ra^{st} = Rs\left(\frac{Le - S}{1 - DuLe}\right) + \frac{\delta^6}{k^2}\left(\frac{1 - DuS}{1 - DuLe}\right), \tag{29}$$

where Ra is the Rayleigh number for exchange stability. The critical wave number k_c can be obtained by minimizing Ra with respect to k, that is, setting $\frac{\partial}{\partial k}(Ra) = 0$, we find k_c to be

$$k_c = \frac{\pi}{\sqrt{2}}. \tag{30}$$

The corresponding critical Rayleigh number is,

$$Ra_c^{st} = Rs\left(\frac{Le - S}{1 - DuLe}\right) + \frac{27\pi^4}{4}\left(\frac{1 - DuS}{1 - DuLe}\right). \tag{31}$$

It is to be noted here that in the absence of cross diffusion terms i.e., $Du = S = 0$ one obtains

$$Ra_c^{st} = RsLe + \frac{27\pi^4}{4}, \tag{32}$$

which coincides with the result reported by Malashetty and Swamy [18] in the case of double diffusive convection with out cross diffusion effects.

3.2. Oscillatory convection (Diffusive regime)

Setting $\sigma_r = 0$ we get $\sigma = i\omega$. Using this in Eq. (28) the Rayleigh number can be written as

$$Ra = \Delta_1 + i\omega\Delta_2, \tag{33}$$

where

$$\begin{aligned}\Delta_1 = \frac{\delta^4 Le^{-2} + \omega^2}{\delta^4\left(Le^{-1} - Du\right)^2 + \omega^2}\Bigg[\frac{\delta^2}{k^2}\Bigg\{\delta^4\left(\frac{1+\Lambda_1\Lambda_2\omega^2}{1+\Lambda_1^2\omega^2}\right)\cdot\left(1 + \frac{Du\left(\omega^2 - \delta^4 Le^{-1}\left(1 - DuS + Le^{-1}S\right)\right)}{\delta^4 Le^{-2}+\omega^2}\right) \\ - \omega^2\left(\frac{1}{Pr} + \frac{(\Lambda_2-\Lambda_1)\delta^2}{1+\Lambda_1^2\omega^2}\right)\cdot\left(1 - \frac{\delta^4 Du\left(1 + Le^{-1}(1-S)\right)}{\delta^4 Le^{-2}+\omega^2}\right)\Bigg\} + \frac{\delta^2\left(Le^{-1} - 1 + Le^{-1}S - Du\right)}{\delta^4 Le^{-2}+\omega^2}Rs\Bigg],\end{aligned} \tag{34}$$

$$\begin{aligned}\Delta_2 = \frac{\delta^4 Le^{-2} + \omega^2}{\delta^4\left(Le^{-1} - Du\right)^2 + \omega^2}\Bigg[\frac{\delta^4}{k^2}\Bigg\{\left(\frac{1+\Lambda_1\Lambda_2\omega^2}{1+\Lambda_1^2\omega^2}\right)\cdot\left(1 - \frac{\delta^4 Du\left(1 + Le^{-1}(1-S)\right)}{\delta^4 Le^{-2}+\omega^2}\right) + \left(\frac{1}{Pr} + \frac{(\Lambda_2-\Lambda_1)\delta^2}{1+\Lambda_1^2\omega^2}\right)\cdot\left(1 + \frac{Du\left(\omega^2 - \delta^4 Le^{-1}\left(1 - DuS + Le^{-1}S\right)\right)}{\delta^4 Le^{-2}+\omega^2}\right)\Bigg\} \\ + \frac{\delta^2\left(Le^{-1} - 1 + Le^{-1}S - Du\right)}{\delta^4 Le^{-2}+\omega^2}Rs\Bigg].\end{aligned} \tag{35}$$

For the onset of oscillatory convection we require $\Delta_2 = 0$ $(\omega \neq 0)$ from which we obtain the quadratic equation for ω^2 in the form

$$a_2\left(\omega^2\right)^2 + a_1\omega^2 + a_0 = 0, \tag{36}$$

where

$$a_0 = \delta^6\Big[Le^{-2}Pr - DuPr\Big\{1 + Le^{-1}(1-S)\Big\} + \{1 + (\Lambda_2 - \Lambda_1)\delta^2 Pr\}\Big\{Le^{-2} - DuLe^{-1}\Big(1 - DuS + Le^{-1}S\Big)\Big\}\Big] + PrRsk^2\Big(Le^{-1} - 1 + Le^{-1}S - Du\Big)$$

$$a_1 = \delta^2 Pr\Big[1 + \Lambda_1\Lambda_2\delta^4\Big\{Le^{-2} - Du\Big(1 + Le^{-1}(1-S)\Big)\Big\}\Big] + \delta^2(1+Du)\{1 + (\Lambda_2 - \Lambda_1)\delta^2 Pr\} + \delta^6\Lambda_1^2\Big\{Le^{-2} - DuLe^{-1}\Big(1 - DuS + Le^{-1}S\Big)\Big\} + PrRsk^2\Lambda_1^2\Big(Le^{-1} - 1 + Le^{-1}S - Du\Big)$$

$$a_2 = \Lambda_1\delta^2[\Lambda_1(1+Du) + \Lambda_2 Pr]$$

and the oscillatory Rayleigh number is given by

$$Ra^{os} = \frac{\delta^4 Le^{-2} + \omega^2}{\delta^4\Big(Le^{-1} - Du\Big)^2 + \omega^2}\left[\frac{\delta^2}{k^2}\left\{\delta^4\left(\frac{1 + \Lambda_1\Lambda_2\omega^2}{1 + \Lambda_1^2\omega^2}\right)\cdot\left(1 + \frac{Du\Big(\omega^2 - \delta^4 Le^{-1}\Big(1 - DuS + Le^{-1}S\Big)\Big)}{\delta^4 Le^{-2} + \omega^2}\right) - \omega^2\left(\frac{1}{Pr} + \frac{(\Lambda_2 - \Lambda_1)\delta^2}{1 + \Lambda_1^2\omega^2}\right)\cdot\left(1 - \frac{\delta^4 Du\Big(1 + Le^{-1}(1-S)\Big)}{\delta^4 Le^{-2} + \omega^2}\right)\right\} + \frac{\delta^2\Big(Le^{-1} - 1 + Le^{-1}S - Du\Big)}{\delta^4 Le^{-2} + \omega^2}Rs\right] \quad (37)$$

It is to be noted here that in the absence of cross diffusion terms i.e., $Du = S = 0$, Eqs. (31) to (37) coincide with that reported by Malashetty and Swamy [18].

3.3. Conditions for finger and diffusive instabilities

The conditions for the onset of finger instability in the case of double-diffusive convection are given by

$$Ra < 0; \quad Rs < 0; \quad -Rs > -\left(\frac{Le^{-1} - Du}{1 - SLe^{-1}}\right)Ra + \frac{\delta^6 Le^{-1}}{k^2}\left(\frac{1 - SDu}{1 - SLe^{-1}}\right). \quad (38)$$

Now, for $Ra < 0$, that is, when the density gradient is statically stable with respect to faster diffusing component, $\alpha T_{,z}$ is positive. The third inequality in (38) can be further written as,

$$\Big(Le^{-1} - Du\Big) - \frac{\alpha' S_{,z}}{\alpha T_{,z}}\Big(1 - SLe^{-1}\Big) > \frac{\delta^6 Le^{-1}}{Rak^2}(1 - SDu), \quad (39)$$

where

$$\frac{\alpha' S_{,z}}{\alpha T_{,z}} = \frac{Rs}{Ra}.$$

The hydrostatic stability is assumed by $-Ra + Rs > 0$, i.e., $-\alpha T_{,z} + \alpha' S_{,z} < 0$. For $Ra \gg \delta^6/k^2$, the inequality (39) takes the form

$$\Big(Le^{-1} - Du\Big) - \frac{\alpha' S_{,z}}{\alpha T_{,z}}\Big(1 - SLe^{-1}\Big) > 0. \quad (40)$$

Thus the condition for the formation of fingers in the presence of cross diffusion terms are

$$\alpha T_{,z} > 0; \quad \alpha' S_{,z} > 0; \quad \Big(Le^{-1} - Du\Big) > \left|\frac{\alpha' S_{,z}}{\alpha T_{,z}}\right|\Big(1 - SLe^{-1}\Big). \quad (41)$$

The condition for the onset of diffusive instability are given by

$$Ra > 0; \quad Rs > 0; \quad Ra < \left[\frac{1 + \Lambda_1\delta^2\Big(1 - SLe^{-1}\Big)}{1 + \Lambda_1\delta^2\Big(Le^{-1} - Du\Big)}\right]Rs + \frac{\delta^2}{k^2}\left[\frac{\chi}{1 + \Lambda_1\delta^2\Big(Le^{-1} - Du\Big)}\right], \quad (42)$$

where $\chi = Le^{-1}Pr^{-1}(1 - SDu)(1 + Pr\delta^2\Lambda_2) + (1 + Le^{-1})$.

Now, for $Ra > 0$, that is, when the density gradient is statically stable with respect to the faster diffusing component, $\alpha T_{,z}$ is negative. For $Ra \gg \delta^2/\beta^2$, the last inequality in (42) takes the form

$$\Big[1 + \Lambda_1\delta^2\Big(Le^{-1} - Du\Big)\Big] - \frac{\alpha' S_{,z}}{\alpha T_{,z}}\Big[1 + \Lambda_1\delta^2\Big(1 - SLe^{-1}\Big)\Big] < 0. \quad (43)$$

Thus the conditions for the onset of diffusive instability in the presence of cross diffusion terms are given by

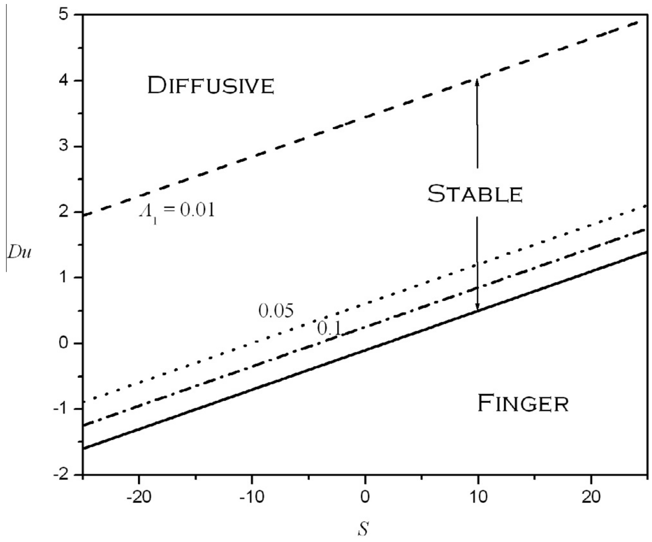


Fig. 2. Stability boundaries as a function of *Du* and *S* for different values of Λ_1 with $\frac{\alpha' S_z}{\alpha T_z} = 0.3$.

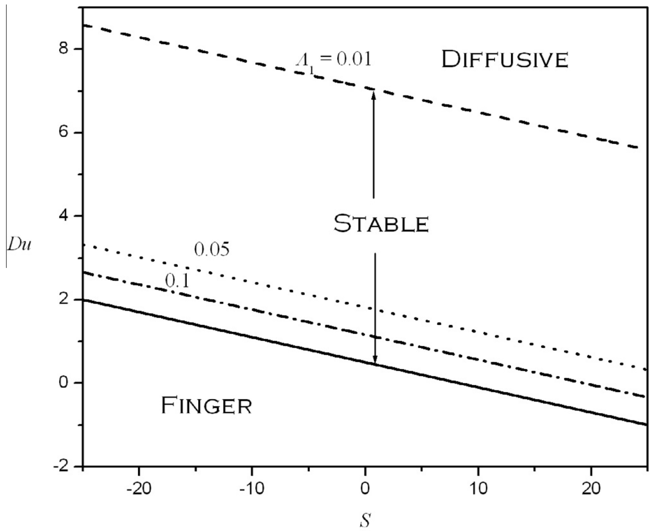


Fig. 3. Stability boundaries as a function of *Du* and *S* for different Λ_1 with $\frac{\alpha' S_z}{\alpha T_z} = -0.3$.

$$\alpha T_{,z} < 0; \quad \alpha' S_{,z} < 0; \quad \left|\frac{\alpha' S_{,z}}{\alpha T_{,z}}\right| \left[1 + \Lambda_1 \delta^2 \left(1 - SLe^{-1}\right)\right] > \left[1 + \Lambda_1 \delta^2 \left(Le^{-1} - Du\right)\right]. \tag{44}$$

It should be noted through Eqs. (41) and (44) that the conditions for finger and diffusive instabilities are independent of the stress relaxation parameter Λ_2. The stability boundaries as indicated by Eqs. (41) and (44) are shown in Figs. 2 and 3 for fixed values of Rs/Ra, subject to the static stability constraint $-Ra + Rs > 0$. The boundaries in case of diffusive instability are shown for different values of Λ_1. The two stability boundaries are parallel lines through the points

$$(Le, Le^{-1}) \quad \text{and} \quad \left(\frac{1 + \Lambda_1 \delta^2}{\Lambda_1 \delta^2}, \frac{1 + \Lambda_1 \delta^2 Le^{-1}}{\Lambda_1 \delta^2}\right),$$

which have slope equal to Rs/Ra.

The stability boundaries show that the finger and diffusive instabilities may not occur simultaneously even though both types of instability may occur in concentration gradients that are normally conducive to the other type of instability. The two types of instabilities may occur even when both components have stabilizing effects. It should also be noted that increasing values of Λ_1 results in expanding the diffusive instability regime or reducing the stable region.

4. Weakly nonlinear stability analysis

In this section we study the nonlinear stability analysis using a minimal truncated representation of a Fourier series that consists of two terms. As the linear stability analysis fails to provide insight about the convection amplitudes and the rate of heat and mass transfer we revert to the nonlinear stability analysis. We restrict ourselves to the case of two-dimensional rolls, so that all the physical quantities are independent of y. We introduce the stream function ψ such that $u = -\partial\psi/\partial z,\ w = \partial\psi/\partial x$ into Eq. (13), eliminate pressure term to obtain

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2\psi\right)-Ra\frac{\partial}{\partial x}(\theta-N\phi)\right]=\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\psi, \tag{45}$$

$$\frac{\partial\theta}{\partial t}+\frac{\partial(\psi,\theta)}{\partial(x,z)}-\frac{\partial\psi}{\partial x}=\nabla^2\theta+Du\nabla^2\phi, \tag{46}$$

$$\frac{\partial\phi}{\partial t}+\frac{\partial(\psi,\phi)}{\partial(x,z)}-\frac{\partial\psi}{\partial x}=Le^{-1}\left(\nabla^2\phi+S\nabla^2\theta\right), \tag{47}$$

Following Siddheshwar et al. [7] we rearrange Eq. (45) as two first-order equations in time as follows:

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2\psi\right)=Ra\frac{\partial}{\partial x}(\theta-N\phi)+\Lambda\nabla^4\psi+M, \tag{48}$$

with M satisfying the equation given below

$$\frac{\partial M}{\partial t}=-M+(1-\Lambda)\nabla^4\psi. \tag{49}$$

Here $\Lambda=\frac{\Lambda_2}{\Lambda_1}$ is the ratio of scaled stress-retardation parameter to that of scaled-relaxation parameter. A minimal double Fourier series which describes the finite amplitude convection is given by

$$\psi(x,z,t)=A_1(t)\sin(kx)\sin(\pi z), \tag{50}$$

$$\theta(x,z,t)=A_2(t)\cos(kx)\sin(\pi z)+A_3(t)\sin(2\pi z), \tag{51}$$

$$\phi(x,z,t)=A_4(t)\cos(kx)\sin(\pi z)+A_5(t)\sin(2\pi z), \tag{52}$$

$$M(x,z,t)=A_6(t)\sin(kx)\sin(\pi z), \tag{53}$$

where the amplitudes $A_i,\ i=1,2,\ldots,6$ are time dependent and are to be determined from the dynamics of the system. Substituting Eqs. (50)–(53) into the coupled non-linear partial differential Eqs. (46)–(49) and equating the coefficients of like terms we obtain the following Lorenz system:

$$\frac{dX_1}{d\tau}=Pr[X_2-NX_4-\Lambda X_1-(1-\Lambda)X_6], \tag{54}$$

$$\frac{dX_2}{d\tau}=R'X_1-X_2-DuX_4-X_1X_3, \tag{55}$$

$$\frac{dX_3}{d\tau}=\frac{X_1X_2}{2}-bX_3-bDuX_5, \tag{56}$$

$$\frac{dX_4}{d\tau}=R'X_1-Le^{-1}X_4-Le^{-1}SX_2-X_1X_5, \tag{57}$$

$$\frac{dX_5}{d\tau}=\frac{X_1X_4}{2}-bLe^{-1}X_5-bLe^{-1}SX_3, \tag{58}$$

$$\frac{dX_6}{d\tau}=\frac{1}{\Gamma}(X_1-X_6), \tag{59}$$

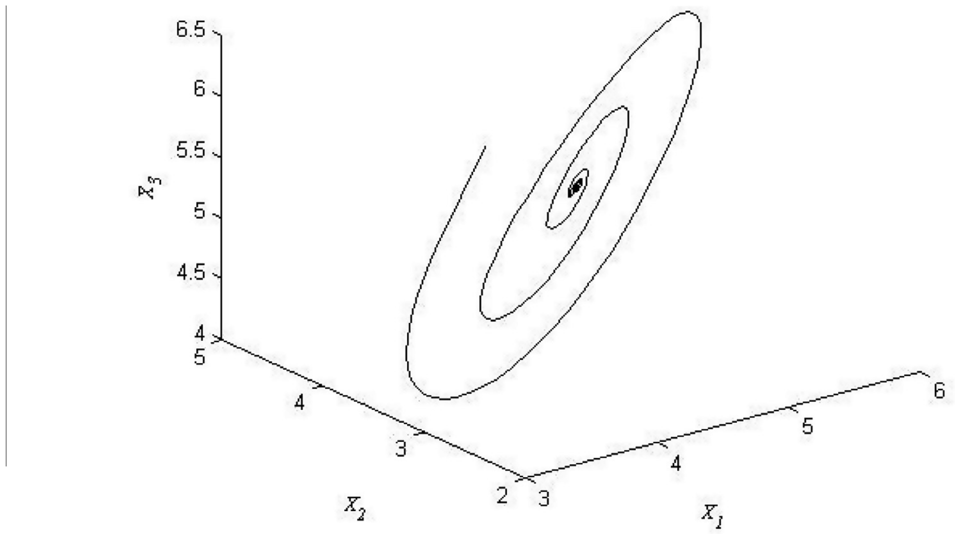
where

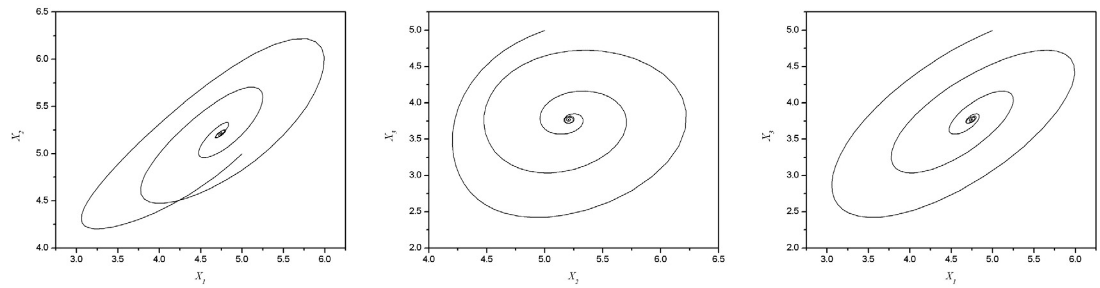
$$X_1=\frac{\pi k}{\delta^2}A_1,\quad (X_2,X_3,X_4,X_5)=\pi R'(A_2,-A_3,A_4,-A_5),\quad X_6=\frac{\pi k}{(1-\Lambda)\delta^6}A_6,$$

$$\tau=\delta^2 t,\quad R'=\frac{k^2}{\delta^2}Ra,\quad b=\frac{4\pi^2}{\delta^2},\quad \Gamma=\Lambda_1\delta^2\quad\text{and}\quad \delta^2=k^2+\pi^2.$$

The solutions of Eqs. (54)–(59) are uniformly bounded in time and possess many properties of the full problem. Also, the system (54)–(59) is dissipative with the volume in the phase-space contracting at a uniform rate given by

$$\frac{\partial}{\partial X_1}\left(\frac{dX_1}{d\tau}\right)+\frac{\partial}{\partial X_2}\left(\frac{dX_2}{d\tau}\right)+\frac{\partial}{\partial X_3}\left(\frac{dX_3}{d\tau}\right)+\frac{\partial}{\partial X_4}\left(\frac{dX_4}{d\tau}\right)+\frac{\partial}{\partial X_5}\left(\frac{dX_5}{d\tau}\right)+\frac{\partial}{\partial X_6}\left(\frac{dX_6}{d\tau}\right)$$
$$=-\left[\Lambda Pr+\frac{1}{\Gamma}+\left(1+\frac{1}{Le}\right)(1+b)\right]. \tag{60}$$





(a) **(b)** **(c)**

Fig. 4. Phase portraits at $R' = 5,\ Le = 2,\ Pr = 10,\ Rs = 100,\ \Lambda = 0.6,\ \Lambda_1 = 0.5,\ Du = 0.2$ and $S = 0.3$.

Consequently, the trajectories are attracted to a set of measure zero in the phase space. In particular, they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor. We have a well-developed theory for a Lorenz system of third order (see references [26–30]). However, for Lorenz systems of higher dimensions or Lorenz like systems, one has to resort to computational analysis. Extensive computation reveals that the trajectories are never attracted to a strange attractor though they are attracted to limit cycles, see Fig. 4.

From Eq. (60) we infer that if a set of initial points in the phase space occupies a region $V(0)$ at time $\tau = 0$, then after some time τ, the end points of the corresponding trajectories will fill a volume

$$V(\tau) = V(0)\exp\left\{-\left(\Lambda Pr + \frac{1}{\Gamma} + \left(1 + \frac{1}{Le}\right)(1+b)\right)\tau\right\}. \tag{61}$$

The Lorenz system arising due to thermal convection in viscoelastic liquids in the absence of cross diffusion effects has been studied extensively by Khayat [30–32]. In the present study we observe similar patterns for the onset of chaos, see Fig. 5. The cross diffusion terms have nothing to contribute to the non-linear dynamics of the thermal convection. This is due to the fact that they contribute only linear terms to the Lorenz system which may perhaps change the location of critical points. For this reason we restrict our analysis of the Lorenz system to heat and mass transfer considerations. However, we make a qualitative discussion of the influence of various factors on chaos in the binary viscoelastic fluid system.

5. Heat and mass transports

The rate of heat and mass transport per unit area, respectively, denoted by H and J are given by

$$H = -D_{11}\left\langle \frac{\partial T_{total}}{\partial z}\right\rangle_{z=0} - D_{12}\left\langle \frac{\partial C_{total}}{\partial z}\right\rangle_{z=0}, \tag{62}$$

$$J = -D_{22}\left\langle \frac{\partial C_{total}}{\partial z}\right\rangle_{z=0} - D_{21}\left\langle \frac{\partial T_{total}}{\partial z}\right\rangle_{z=0}, \tag{63}$$

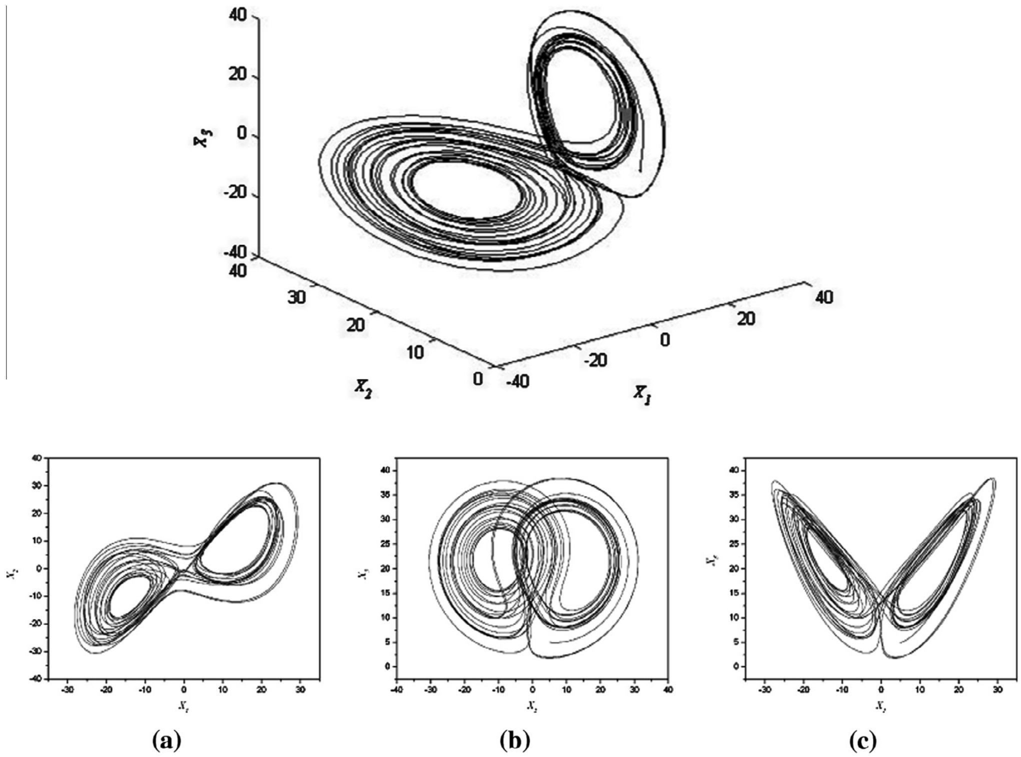


Fig. 5. Phase portraits at $R' = 23,\ Le = 2,\ Pr = 10,\ Rs = 100, \Lambda = 0.6,\ \Lambda_1 = 0.5,\ Du = 0.2$ and $S = 0.3$.

where the angular bracket corresponds to a horizontal average and z is the dimensionless space variable. The total temperature and concentrations T_{total} and C_{total} are given by

$$T_{total} = T_0 - (\Delta T)z + (\Delta T)\theta(t, x, z), \tag{64}$$

$$C_{total} = C_0 - (\Delta C)z + (\Delta C)\phi(t, x, z). \tag{65}$$

Substituting Eqs. (51) and (52) in Eqs. (64) and (65), respectively, and using the resultant equations in Eqs. (62) and (63), we get

$$H = \Delta T[D_{11}(1 - 2\pi A_3) + D_{12}(1 - 2\pi A_5)], \tag{66}$$

$$J = \Delta C[D_{22}(1 - 2\pi A_5) + D_{21}(1 - 2\pi A_3)]. \tag{67}$$

The Nusselt and Sherwood numbers are respectively define by

$$Nu = \frac{H}{D_{11}\Delta T} = (1 - 2\pi A_3) + Du(1 - 2\pi A_5), \tag{68}$$

$$Sh = \frac{J}{D_{22}\Delta C} = (1 - 2\pi A_5) + S(1 - 2\pi A_3). \tag{69}$$

Using on the scaled variable $(X_3, X_5) = -\pi R'(A_3, A_5)$ we obtain

$$Nu = \left(1 + \frac{2}{R'}X_3\right) + Du\left(1 + \frac{2}{R'}X_5\right), \tag{70}$$

$$Sh = \left(1 + \frac{2}{R'}X_5\right) + S\left(1 + \frac{2}{R'}X_3\right). \tag{71}$$

6. Results and discussion

The onset of two-component convection in a binary viscoelastic fluid layer that is heated and salted from below has been investigated using the linear theory. A minimal representation of Fourier series has been used for a weakly non-linear stability analysis that results in a sixth-order generalized Lorenz model. The viscoelastic fluid has been assumed to subscribe to the Oldroyd-B description. Analytical expressions for the critical Rayleigh number and the corresponding wavenumbers for

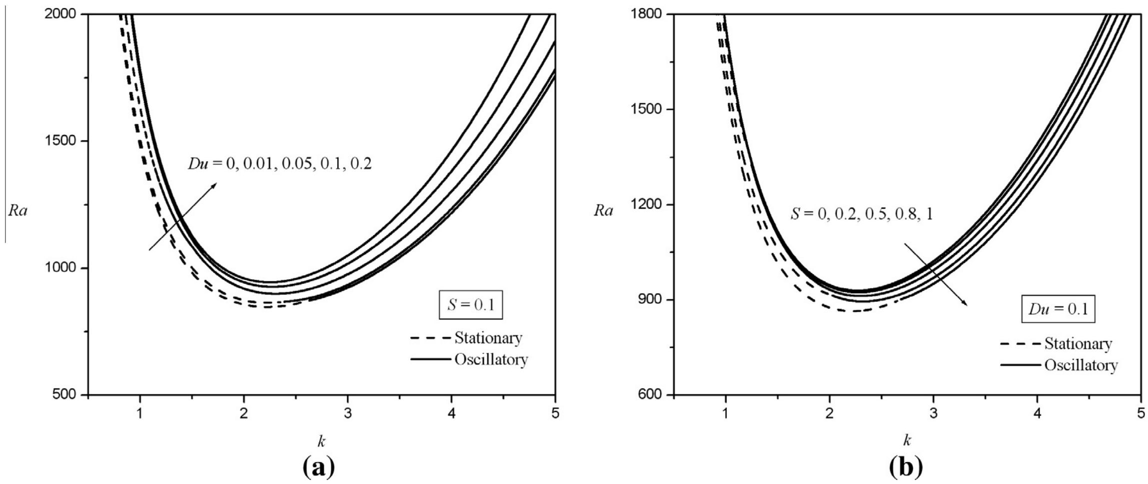


Fig. 6. Neutral stability curves for different values of (a) Du and (b) S with $\Lambda_1 = 0.3,\ \Lambda_2 = 0.25,\ Pr = 10,\ Rs = 100$ and $Le = 2$.

Table 1
Values of critical wavenumber k_c and critical Rayleigh number Ra_c in case of overstable mode for $Pr = 10,\ Rs = 100$ and $Le = 5$.

S	Du	Newtonian $\Lambda_1 = 0.1$ $\Lambda_2 = 0.1$		Maxwell $\Lambda_1 = 0.1$ $\Lambda_2 = 0$		Oldroyd $\Lambda_1 = 0.5$ $\Lambda_2 = 0.3$		Rivlin–Erickson $\Lambda_1 = 0.001$ $\Lambda_2 = 0.05$	
		k_c	Ra_c	k_c	Ra_c	k_c	Ra_c	k_c	Ra_c
−0.5	0	2.225	896.615	5.479	206.273	2.275	598.655	2.168	978.089
	0.2	2.225	880.745	5.392	170.202	2.258	585.260	2.180	904.099
	0.5	2.225	857.975	5.310	135.109	2.258	563.222	2.202	813.189
	0.8	2.225	836.362	5.256	112.172	2.258	542.197	2.214	740.111
	1	2.225	822.555	5.220	100.826	2.258	528.938	2.225	698.869
0	0	2.225	897.524	5.511	216.535	2.269	599.264	2.168	982.505
	0.2	2.225	881.497	5.408	179.670	2.258	587.502	2.191	902.757
	0.5	2.225	858.501	5.327	143.151	2.258	565.784	2.214	804.666
	0.8	2.225	836.675	5.284	118.987	2.258	544.672	2.236	725.730
	1	2.225	822.730	5.265	106.957	2.258	531.295	2.247	681.152
0.5	0	2.225	898.433	5.543	226.798	2.281	599.706	2.168	986.921
	0.2	2.225	882.249	5.420	189.332	2.270	589.926	2.191	901.404
	0.5	2.225	859.028	5.334	151.487	2.258	568.627	2.225	796.099
	0.8	2.225	836.988	5.299	126.139	2.258	547.463	2.247	711.236
	1	2.225	822.906	5.289	113.444	2.258	533.985	2.270	663.252

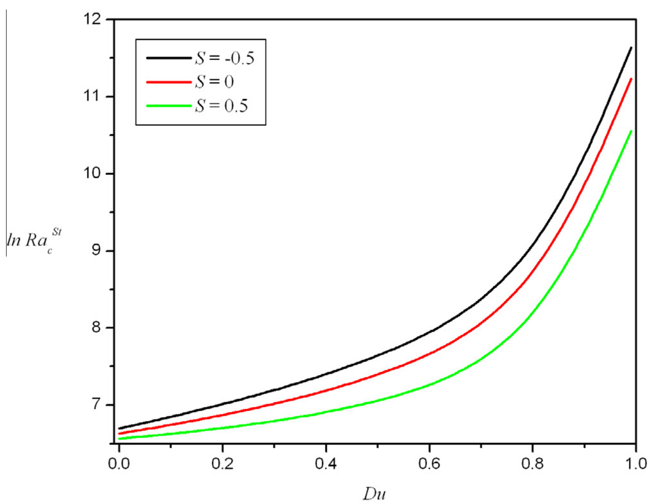


Fig. 7. $\ln Ra_c^{St}$ as a function of Du for different values of S with $Rs = 100$ and $Le = 1$.

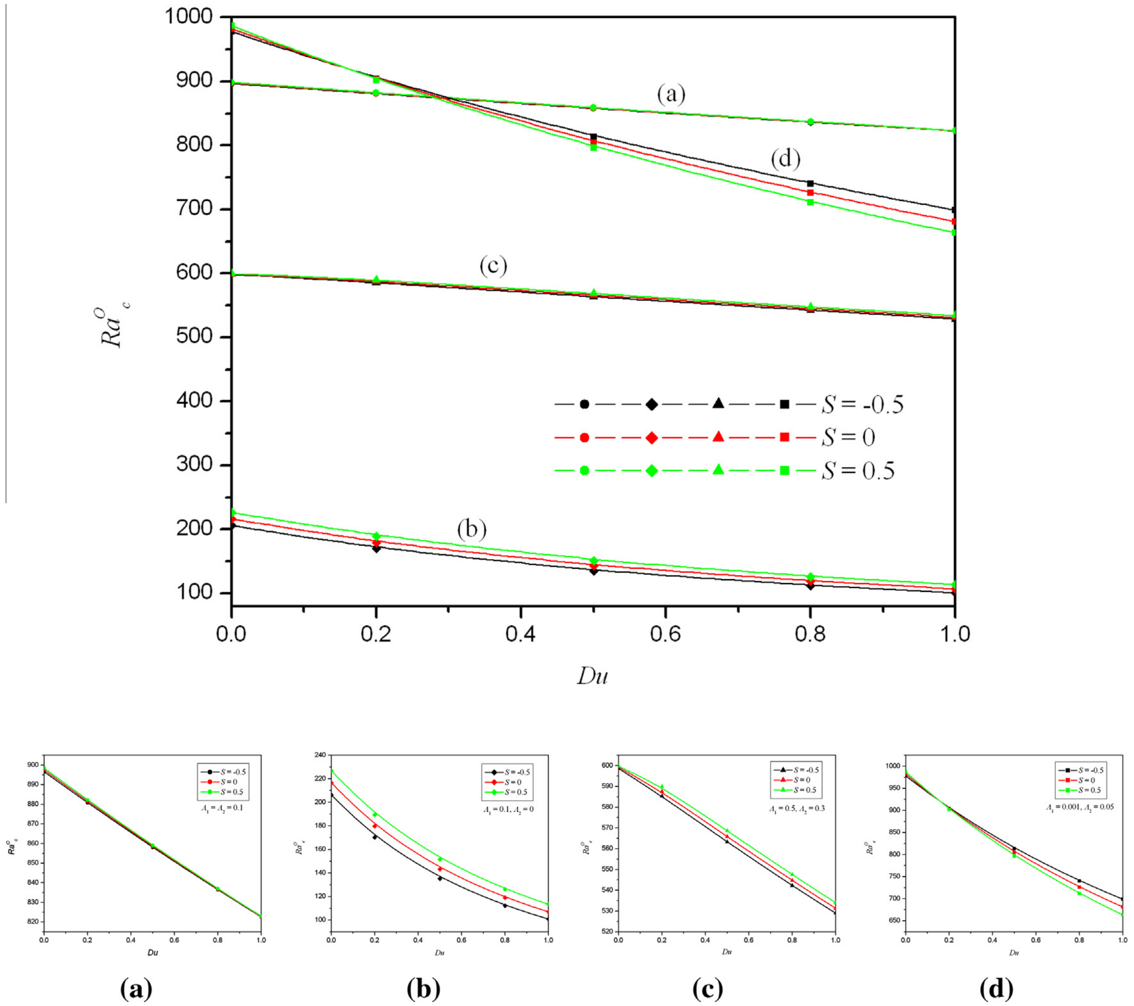


Fig. 8. Ra_c^O as a function of *Du* for different values of *S* in (a) Newtonian, (b) Maxwell (c) Oldroyd and (d) Rivlin–Erickson fluid cases with $Pr = 10,\ Rs = 100$ and $Le = 5$.

the onset of stationary or oscillatory convection subject to cross diffusion effects were determined using linear stability theory. Heat and mass transports were quantified with the help of weakly non-linear theory. A Lorenz system is obtained in the case of the weakly nonlinear stability analysis.

The characterization of chaotic binary convection in a two-relaxation-time viscoelastic liquid is quite difficult and prohibitive due to the fact that we need to tackle the six-dimensional nonlinear dynamical system. This is further complicated by the appearance of four new parameters – Lewis number, solutal Rayleigh number, Dufour and Soret parameters. It is now well known that for thermal convection in an Oldroyd B fluid, the route to and from chaos is similar to that in a Newtonian fluid but with viscoelastic chaotic behavior characterized by a higher fractal dimension than the Newtonian chaotic behavior. In view of the fact that a second diffusing component is of dilute concentration in a viscoelastic fluid, it becomes apparent that parameters arising due to the second diffusing component marginally alter the quantitative picture and with the qualitative aspect intact. Thus, in effect, it means that the results discussed by Abu-Ramadan et al. [33] hold good in the present paper as well. Added to all the above observations made so far it is important to note that the scaled amplitudes X_3 and X_5 are connected with the Nusselt and Sherwood numbers. In the light of the comments made above, we restrict ourselves to the analysis of heat and mass transports and draw appropriate inferences from the same. But first we make some general conclusions from the local linear stability analysis.

The primary objective of this study was to determine the effects of the Soret and Dufour parameters on the stability of viscoelastic fluid layer heated and salted from below. Consequently, we have not shown the effects of other parameters such as the Lewis number, Prandtl number and solutal Rayleigh number whose significance has been widely studied in the literature on double diffusive convection (see Malashetty and Swamy [18]). With this in mind the parameters' values were chosen to be in keeping with previous works and focused our study on cross diffusion effects.

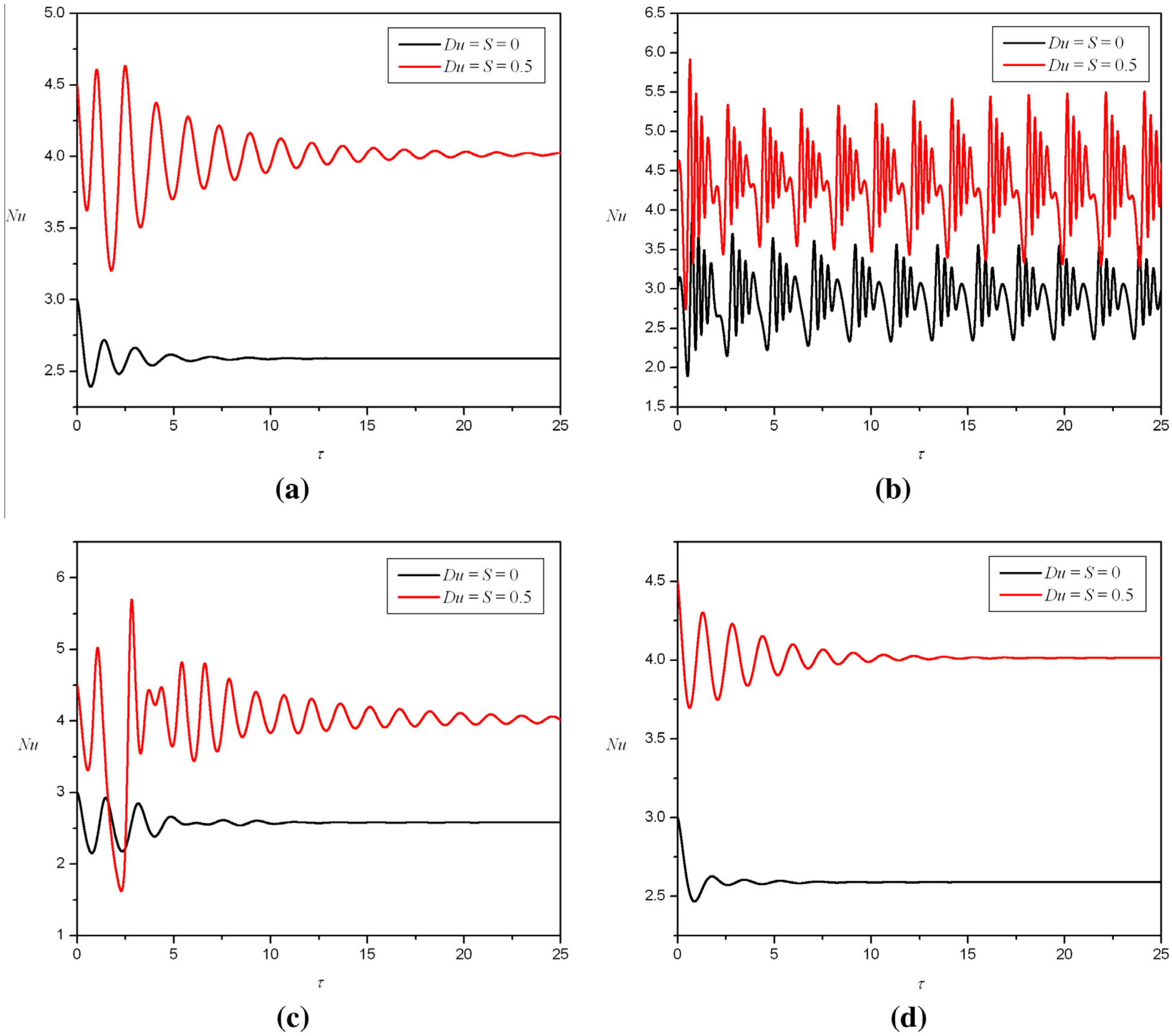


Fig. 9. Variation of Nu with τ for $R' = 5$ in (a) Newtonian, (b) Maxwell (c) Oldroyd and (d) Rivlin–Erickson fluid cases.

We begin with the discussion of cross diffusion effects on the stationary and oscillatory modes of convection. Figs. 6(a) and (b) show, respectively, the effect of Dufour and Soret parameters on the neutral stability curves in the $Ra - k$ plane. From Fig. 6(a) it is clear that Du has a stabilizing effect on the onset of convection as increasing values of Du results in increasing the critical Rayleigh number. Also, for small values of Du the convection initially sets in the stationary mode till it reaches a critical value Du_c beyond which it switches to oscillatory mode. Similar observations can be made with regards to the effect of Soret parameter on the neutral stability curves Fig. 6(b). In contrast to effects of Du the Soret number has a destabilizing effect on the onset of convection. Here, also we observe the bifurcation between stationary and oscillatory modes of convection. Initially the convection sets in the oscillatory mode till the value of S reaches a critical value S_c beyond which it switches to stationary mode of convection. The critical values of wavenumber k_c and overstable Rayleigh number Ra_c are tabulated in Table 1 for different values of Du and S. For Newtonian case one can observe that the critical wavenumber remains the same while for other three kinds of fluids it is sensitive to the parameter values considered.

It should be noted from Eq. (31) that the stationary critical Rayleigh number is independent of the viscoelastic parameters and therefore our results on stationary convection has to be the same as that of a Newtonian binary fluid layer (see Rudraiah and Siddheshwar [34] for a discussion of these results). The stationary critical Rayleigh number and critical wave number are independent of viscoelastic parameters because of the absence of base flow in the present case. This is in contrast to viscoelastic Taylor–Couette flow, where the base flow depends on viscoelastic parameters, thus leading to critical conditions that are influenced by elastic effects. It should also be noted from Eq. (31) that the stationary mode of convection is prevalent only when $Du < Le^{-1}$. Fig. 7 illustrates the effect of Soret and Dufour parameters on critical stationary Rayleigh number. It is evident from this figure that the $Ra_c^{St} - Du$ curve is steep for all values of the Soret parameter S. The effect of increasing S is to reduce the magnitude of Ra_c^{St} and this is a classical result (see Rudraiah and Siddheshwar [34]).

Unlike Ra_c^{St}, the oscillatory Rayleigh number, Ra_c^O, depends on the viscoelastic parameters Λ_1 and Λ_2. Fig. 8 shows Ra_c^O as a function of Du for different values of S, for Newtonian, Maxwell, Oldroyd-B and Rivlin–Erickson fluids. In the case of a Newtonian fluid ($\Lambda_1 = \Lambda_2$) the Dufour parameter has a destabilizing effect on Ra_c^O while the effect of Soret parameter is almost negligible as observed from Fig. 8(a). The onset of convection in Maxwell fluid ($\Lambda_2 = 0$) is more sensitive to the choice of Du and S than the other three fluids. Fig. 8(b) clearly shows that the Maxwell fluid succumbs to instability faster than the four fluids considered. The general observation on the cross diffusion effects on Ra_c^O made in the case of Maxwell fluid holds good for an Oldroyd fluid ($\Lambda_1 \neq 0,\ \Lambda_2 \neq 0$) and Rivlin–Erickson fluid ($\Lambda_1,\ \Lambda_2 \ll 1$ and $\Lambda \gg 1$) as can be seen from Figs. 8(c) and 8(d). Fig. 8(d) shows that for $Du < Du_c, S$ increases Ra_c^O whereas for $Du > Du_c$ it reduces Ra_c^O. This effect of S is not seen in the case of Newtonian, Maxwell and Oldroyd fluids. The critical value of Du for the mixed behavior of S in case of Rivlin–Erickson fluid was found to be $Du_c = 0.184$. As a summary, one can infer the following from Fig. 8:

(i) In the absence of cross diffusion effect ($Du = S = 0$):

$$(Ra_c^O)_{Maxwell} < (Ra_c^O)_{Oldroyd} < (Ra_c^O)_{Newtonian} < (Ra_c^O)_{Rivlin-Erickson}.$$

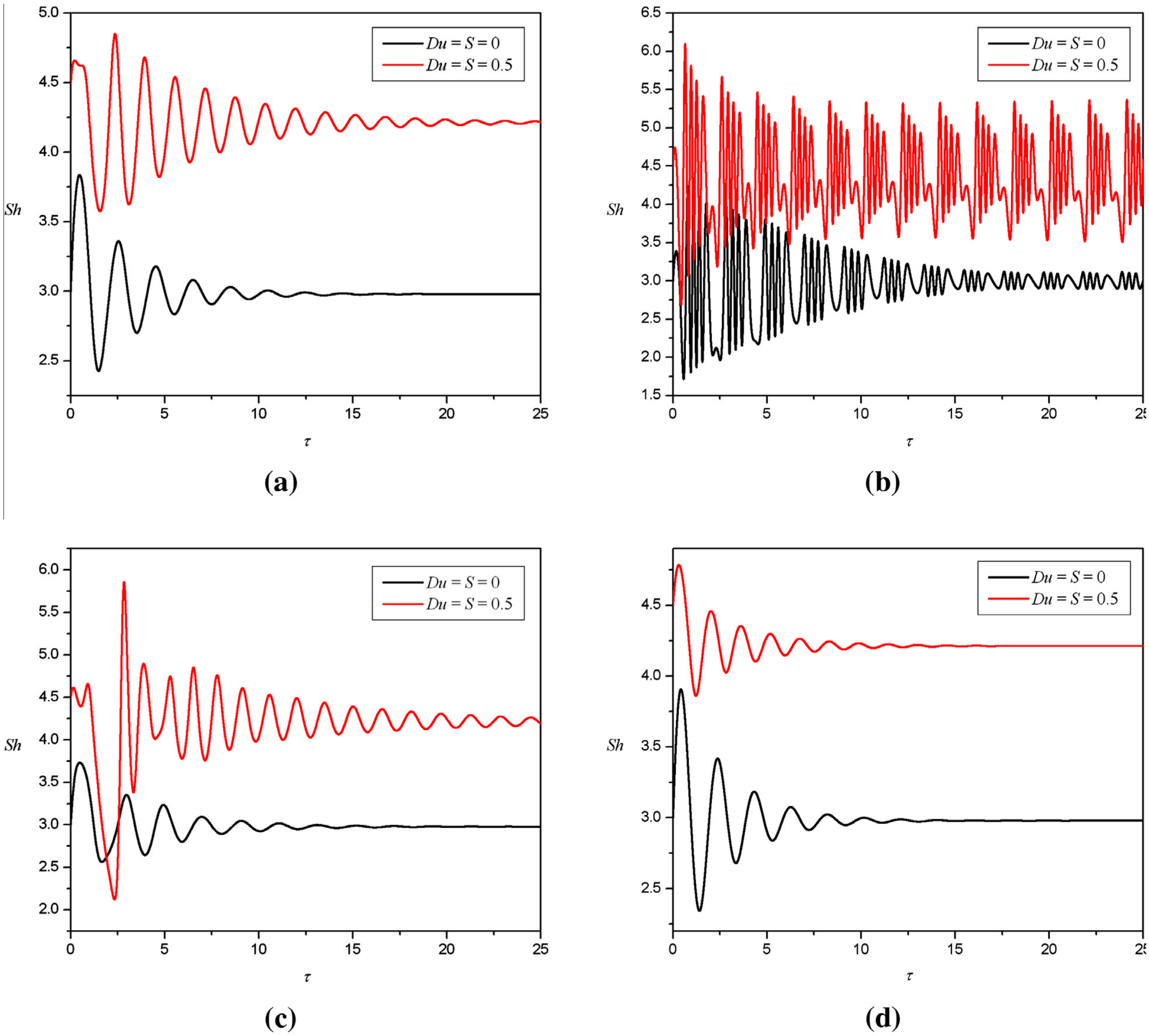


Fig. 10. Variation Sh with τ for $R' = 5$ in (a) Newtonian, (b) Maxwell (c) Oldroyd and (d) Rivlin–Erickson fluid cases.

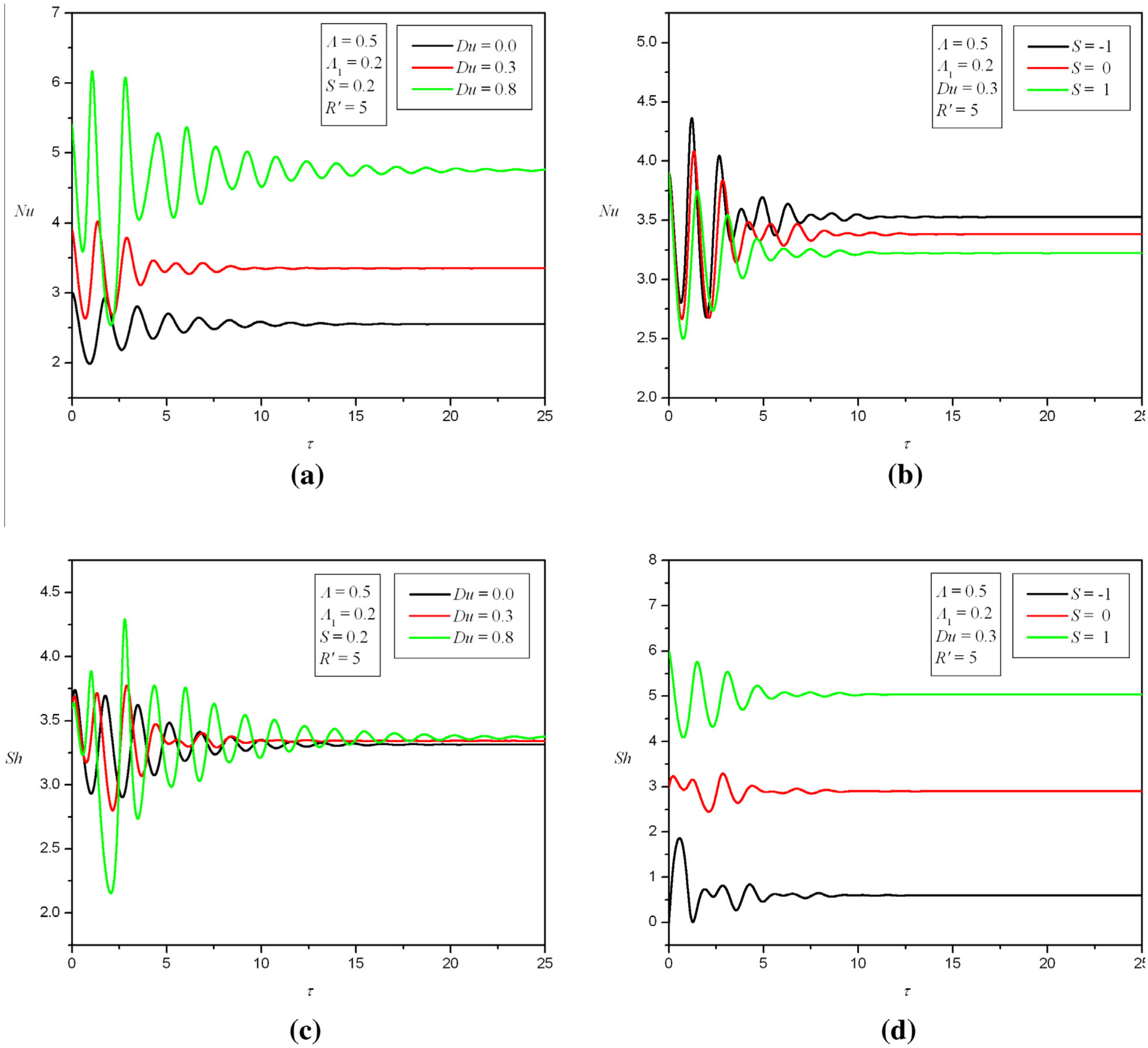


Fig. 11. Variation of Nu and Sh with τ in case of Oldroyd-B fluid for $R' = 5$.

(ii) In the presence of cross diffusion effects, we have

$$(Ra_c^O)_{Maxwell} < (Ra_c^O)_{Oldroyd} < (Ra_c^O)_{Newtonian} < (Ra_c^O)_{Rivlin-Erickson}, \quad \text{for } Du < 0.25,$$

$$(Ra_c^O)_{Maxwell} < (Ra_c^O)_{Oldroyd} < (Ra_c^O)_{Rivlin-Erickson} < (Ra_c^O)_{Newtonian}, \quad \text{for } Du < 0.25.$$

At this point we also note that direct, sub-critical and super-critical Hopf and Co-dimension two bifurcations can be studied in systems as the one considered in this study but our focus is mainly on the influence of the second-diffusing component and cross-diffusion on heat and mass transports. We now initiate discussion of the results from our nonlinear study. From Fig. 9 that are plots of Nu versus τ for four different fluids and for $R' = 5$, the following general conclusion can be made using Nu_m (a mean defined in any time interval):

(i) In the absence of cross diffusion effect ($Du = S = 0$):

$$(Nu_m)_{Maxwell} > (Nu_m)_{Oldroyd} > (Nu_m)_{Newtonian} > (Nu_m)_{Rivlin-Erickson}.$$

(ii) In the presence of cross diffusion effects, we have

$$(Nu_m)_{Maxwell} > (Nu_m)_{Oldroyd} > (Nu_m)_{Newtonian} > (Nu_m)_{Rivlin-Erickson}, \quad \text{for } Du < 0.25,$$

$$(Nu_m)_{Maxwell} > (Nu_m)_{Oldroyd} > (Nu_m)_{Rivlin-Erickson} > (Nu_m)_{Newtonian}, \quad \text{for } Du < 0.25.$$

A similar conclusion as above can be made on the mean Sherwood number from Fig. 10. It is thus clear that the presence of the cross diffusion enhances both *Nu* and *Sh* in all the four types of fluids chosen. In the case of a Maxwell fluid, unlike the other three fluids, *Nu* and *Sh* do not level-off to the steady state values as τ elapses. This is indicative of the fact that early chaos is precipitated in a single-relaxation-time fluid of the Maxwell type as compared to that in Newtonian, Oldroyd and Rivlin–Erickson fluids. These results concur with those reported by Siddheshwar et al. [7].

The individual effects of *Du* and *S* on the heat and mass transfer in the case of an Oldroyd-B type viscoelastic fluid are shown in Figs. 11 and 12 for $R' = 5$ and $R' = 28$ corresponding to the Khayat–Lorenz dynamics. It is clear from Figs. 11(a) and (c) that the Dufour parameter helps in enhancing both heat and mass transfer. The Soret parameter reduces heat transfer and increases mass transfer as can be seen via Figs. 11 (b) and (d) respectively. The same trends are observed in other three types of fluids considered and the plots are not shown for reasons of space. Extensive computation reveals that chaos sets in at $R' = 28$ and beyond, and hence these are depicted by Fig. 12.

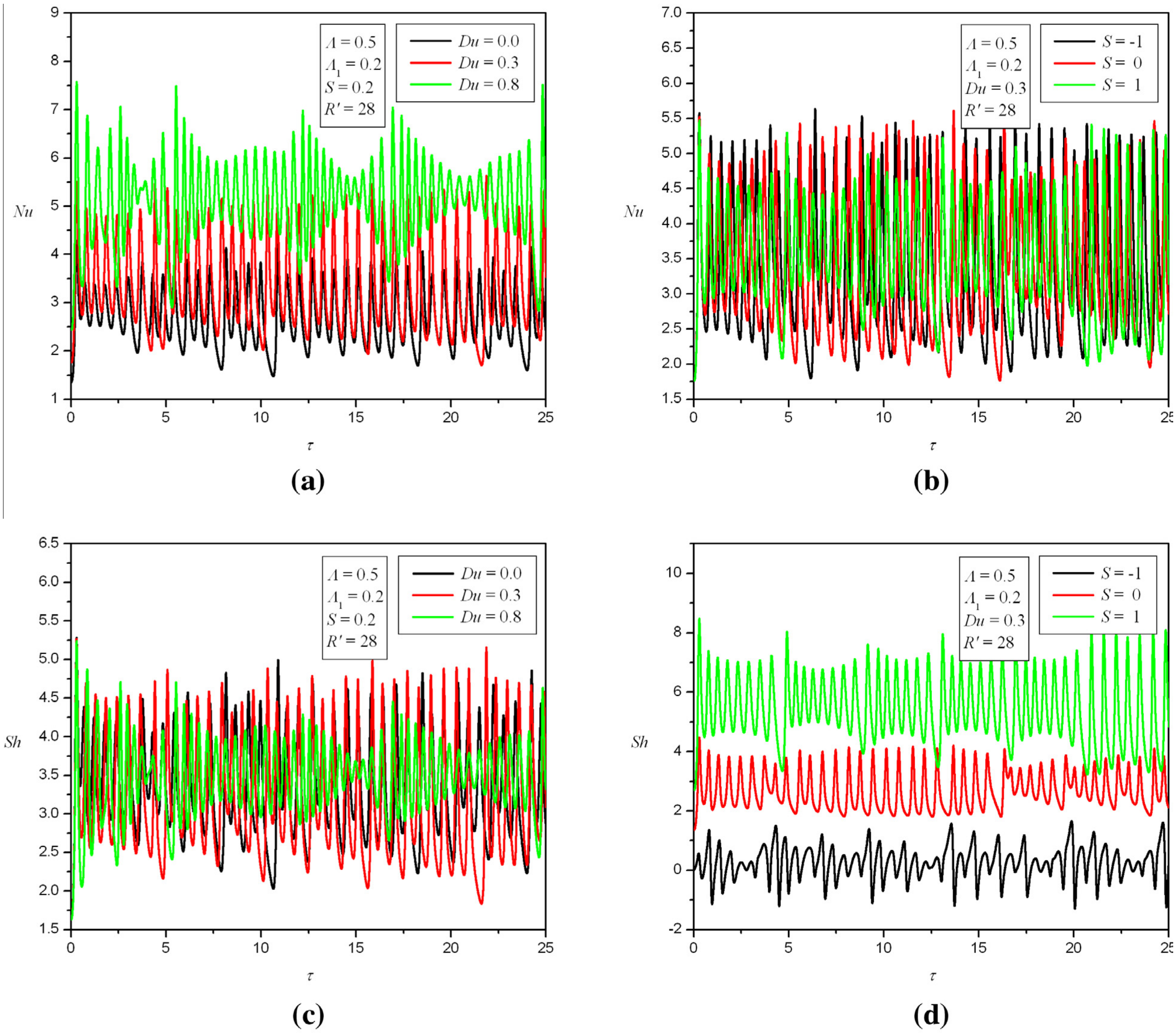


Fig. 12. Variation of *Nu* and *Sh* with τ in case of Oldroyd-B fluid for $R' = 28$.

7. Conclusion

Dufour and Soret parameters have opposite influence on the onset of stationary binary convection in viscoelastic fluids with Ra_c^{St} increasing with increase in *Du*. In the case of overstability for all four fluids except Rivlin–Erickson, however, the Dufour and Soret effects work in tandem in influencing Ra_c^O. The effect of Soret parameter is to increase Ra_c^O in the three fluids but shows a mixed influence in the case of Rivlin–Erickson fluid. The stability boundaries show that finger and diffusive instabilities may not occur simultaneously even though both types of instability may occur in concentration gradients that are normally conducive to the other type of instability. The two types of instabilities may occur even when both components have stabilizing effects. The effect of increasing the Soret number is to reduce *Nu* and increase *Sh* while increase in *Du* results in an increase in both *Nu* and *Sh*. The results for Maxwell, Rivlin–Erickson and Newtonian fluids are obtained as limiting cases of the present general study involving an Oldroyd fluid. The route to chaos in the binary viscoelastic fluid system is similar to that of the single-component viscoelastic fluid system due to the consideration in the study of dilute concentration of the second component.

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References

[1] C.M. Vest, V.S. Arpaci, Overstability of a viscoelastic fluid layer heated from below, J. Fluid Mech. 36 (1969) 613–623.
[2] M. Sokolov, R.I. Tanner, Convective stability of a general viscoelastic fluid heated from below, Phys. Fluids 15 (1972) 534–539.
[3] Z. Li, R.E. Khayat, Three-dimensional thermal convection of viscoelastic fluids, Phys. Rev. E 71 (2005) 0663051-11.
[4] Z. Li, R.E. Khayat, Finite-amplitude Rayleigh–Benard convection and pattern selection for viscoelastic fluids, J. Fluid Mech. 529 (2005) 221–251.
[5] T. Green, Oscillating convection in an elasticoviscous liquid, Phys. Fluids 11 (1968) 1410–1413.
[6] P.G. Siddheshwar, C.V. Srikrishna, Rayleigh–Benard convection in a visoelasic fluid-filled high-porosity medium with nonuniform basic temeparture gradient, JMMS 25 (9) (2001) 609–619.
[7] P.G. Siddheshwar, G.N. Sekhar, G. Jayalatha, Effect of time-periodic vertical oscillations of the Rayleigh–Benard system on nonlinear convection in viscoelastic liquids, J. Non-Newtonian Fluid Mech. 165 (2010) 1412–1418.
[8] R.C. Sharma, Effect of rotation on thermal instability of a viscoelastic fluid, Acta Physiol. Hung. 40 (1976) 11–17.
[9] P.K. Bhatia, J.M. Steiner, Convective instability in a rotating viscoelastic fluid layer, ZAMM 52 (1972) 321–327.
[10] P. Kolodner, Oscillatory convection in viscoelastic DNA suspensions, J. Non-Newtonian Fluid Mech. 75 (1998) 167–192.
[11] J. Martinez-Mardones, R. Tiemann, D. Walgraef, Rayleigh–Benard convection in binary viscoelastic fluid, Physica A 283 (2000) 233–236.
[12] J. Martinez-Mardones, R. Tiemann, D. Walgraef, Thermal convection thresholds in viscoelastic solutions, J. Non-Newtonian Fluid Mech. 93 (2000) 1–15.
[13] J. Martinez-Mardones, R. Tiemann, D. Walgraef, Amplitude equation for stationary convection in a binary viscoelastic fluid, Physica A 327 (2003) 29–33.
[14] D. Laroze, J. Martinez-Mardones, J. Bragard, Thermal convection in a rotating binary viscoelastic liquid mixture, Eur. Phys. J. Spec. Top. 146 (2007) 291–300.
[15] D. Laroze, J. Martinez-Mardones, J. Bragardc, C. Peirez-Garcia, Realistic rotating convection in a DNA suspension, Physica A 385 (2007) 433–438.
[16] M.E. Stern, The 'salt fountain' and thermohaline convection, Tellus 12 (1960) 172–175.
[17] M.E. Stern, Collective instability of salt fingers, J. Fluid Mech. 35 (1969) 209–218.
[18] M.S. Malashetty, M. Swamy, The onset of double diffusive convection in a viscoelastic fluid layer, J. Non-Newtonian Fluid Mech. 165 (2010) 1129–1138.
[19] R.G. Mortimer, H. Eyring, Elementary transition state theory of the Soret and Dufour effects, Proc. Nat. Acad. Sci. USA 77 (1980) 1728–1731.
[20] E. Knobloch, Convection in binary fluids, Phys. Fluids 23 (9) (1980) 1918–1920.
[21] F.G. Awad, P. Sibanda, S.S. Motsa, On the linear stability analysis of a Maxwell fluid with double-diffusive convection, Appl. Math. Model. 34 (2010) 3509–3517.
[22] M.S. Malashetty, B.S. Biradar, The onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross-diffusion effects, Phys. Fluids 23 (2011) 064102.
[23] K.R. Rajagopal, M. Ruzika, A.R. Srinivasa, On the Oberbeck–Boussinesq approximation, Math. Models Methods Appl. Sci. 6 (8) (1996) 1157–1167.
[24] G.N. Sekhar, G. Jayalatha, Elastic effects on Rayleigh–Bénard convection in liquids with temperature-dependent viscosity, Int. J. Thermal Sci. 49 (2010) 67–75.
[25] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Clarendon, Oxford, 1961.
[26] E.N. Lorenz, Deterministic non-periodic flow, J. Atmos. Sci. 20 (1963) 130–141.
[27] R.C. Hilborn, Chaos and Nonlinear Dynamics, Oxford University Press, New York, 1961.
[28] P.G. Drazin, W.H. Reid, Hydrodynamic Stability, Cambridge University Press, Cambridge, 1981.
[29] C. Sparrow, The Lorenz Equations: Bifurcations, chaos and strange attractors, Lecture Notes in Applied Mathematics, vol. 41, Springer-Verlag, New York, 1981.
[30] R.E. Khayat, Chaos and overstability in the thermal convection of viscoelastic fluids, J. Non-Newtonian Fluid Mech. 53 (1994) 227–255.
[31] R.E. Khayat, Fluid elasticity and transition to chaos in thermal convection, Phys. Rev. E 51 (1995) 380–399.
[32] R.E. Khayat, Non-linear overstability in the thermal convection of viscoelastic fluids, J. Non-Newtonian Fluid Mech. 58 (1995) 331–356.
[33] E. Abu-Ramadan, J.M. Hay, R.E. Khayat, Characterization of chaotic thermal convection of viscoelastic fluids, J. Non-Newtonian Fluid Mech. 115 (2003) 79–113.
[34] N. Rudraiah, P.G. Siddheshwar, A weak nonlinear stability analysis of double diffusive convection with crossp-diffusion in a fluid-saturated porous medium, Heat Mass Transf. 33 (1998) 287–293.