Numerical solution of the MHD Reynolds equation for squeeze film lubrication between two parallel surfaces

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ABSTRACT

Characteristic features of squeeze film lubrication between two rectangular plates, of which, the upper plate has a roughness structure, in the presence of a uniform transverse magnetic field are examined. The fluid in the film region is represented by a viscous, incompressible and electrically conducting couple-stress fluid. The thickness of the fluid film region is h and that of the roughness is h_s. The pressure distribution in the film region is governed by the modified Reynolds equation, which also incorporates the roughness structure and couple stress fluid. This Reynolds equation is solved using a novel multigrid method for all involved physical parameters. It is observed that the pressure distribution, load carrying capacity and squeeze film-time increase for smaller values of couple-stress parameter and for increasing roughness parameters and Hartmann number compared to the classical case.

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1. Introduction

The hydrodynamic lubrication theory for rough surfaces has been studied with considerable interest in recent years, because, all bearing surfaces are rough to some extent. All bearing surfaces develop roughness after having some run-in and wear. In some cases, contamination of lubricant is also one of the reasons to generate surface roughness through chemical degradation. Several approaches have been proposed in the literature to study the effect of surface roughness on bearing surfaces. Burton [1] modeled roughness by a Fourier series type approximation. Since, the surface roughness distribution is random in nature, a stochastic approach has to be adopted. Thus, Christensen [2] developed a stochastic theory for the study of rough surfaces in hydrodynamic lubrication. Since then many researchers have adopted and used extensively this approach to study roughness effect on bearing surfaces. For example, Prakash and Tiwari [3] used this theory to study the effect of surface roughness on the porous bearings. Gururajana and Prakash [4] have successfully used this theory to study the influence of roughness on narrow journal bearing in which bearing has a porous material and showed that roughness increases the pressure distribution in the fluid film region. Chiang et al. [5] have analysed the lubrication performance of rough finite journal bearings. Bujurke and Kudenatti [6] have numerically solved the modified Reynolds equation to explore the effects of surface roughness on articular cartilages in synovial joints by modeling them as the load sustaining bearings and showed that the pressure rise and load carrying capacity are more compared to the classical case.

The magnetohydrodynamic (MHD) flow of a fluid in a squeeze film lubrication is of interest, because, it prevents the unexpected variation of lubricant viscosity with temperature under sever operating conditions and also investigations into

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the effects of magnetic field in lubrication have been encouraging because of their importance in engineering applications, with obvious relevance to technology based world. The MHD lubrication in an externally pressurized thrust bearing has been investigated both theoretically and experimentally by Maki et al. [7]. Copious papers on MHD lubrication are available in the literature which include- MHD slider bearings ([8], [9]), MHD Journal bearings ([10], [11]), MHD squeeze film bearings ([12]). Hamza [13] has showed the effects of MHD on a fluid film squeezed between two rotating surfaces. Bujurke and Kudenatti [14] have theoretically explored the effect of roughness on the electrically conducting fluid in the rectangular plates, in which upper plate has a roughness structure. They modified the classical Reynolds equation to include the effects of roughness and magnetic field and solved it using a multigrid method. They showed that the effect of roughness and Hartmann number is to increase the pressure distribution and hence the load carrying capacity for increasing roughness and magnetic parameters. Hsu et al. [15] have studied MHD effects in circular disks with inertial effects and showed that the bearing characteristics are more pronounced for applied magnetic field.

Most of the theoretical investigations mentioned above, have been carried out by assuming the lubricant between two surfaces is Newtonian. Results of Newtonian fluid give a satisfactory understanding but this theory fails to give the effect of the non-Newtonian fluid. However, with development of modern industries, the importance of non-Newtonian fluid as a lubricant when squeeze film takes place, is important for the most of engineering applications. So, the effect of non-Newtonian fluid must be taken into account in the study of bearings. Polymer-thickened oils, greases, synovial fluid etc. are simple examples of non-Newtonian fluid. Stokes [16] introduced a microcontinuum theory for couple-stress fluids which is the simplest generalization of classical theory, and also accounts for polar effects such as presence of anti-symmetrical stresses, couple stresses and body couples etc. Naduvinamani et al. [17,18] have extensively studied the effect of couple-stress fluid on lubrication characteristics of various bearings such as journal and squeeze film bearings and showed that pressure distribution and load carrying capacity are more pronounced for couple-stress fluid compared to the Newtonian fluid. Lin and Hung [19] have investigated the combined effects of non-Newtonian couple-stress fluid and rotational inertia on the squeeze film behavior between rotating circular discs and showed that couple-stress fluid increases the load capacity and squeezing time of the bearing.

Thus, keeping the above discussion in mind, we investigate the effects of non-Newtonian couple-stress fluid and roughness on hydrodynamic squeeze film mechanisms in the presence of transverse magnetic field which have useful applications in understanding bearing characteristics.

Rest of the paper has been organized as follows. The required basic equations supplemented with appropriate boundary conditions are given and subsequently modified MHD Reynolds equation is derived in Section 2. To analyze the effect of roughness and couple-stress fluid, the Reynolds equation for longitudinal roughness is also derived in this section. In Section 3, the Reynolds equation is discretized with a finite difference method, and solved using multigrid method for fluid film pressure, load carrying capacity and squeeze film time. Predictions on bearing characteristics are given for varying couple-stress, roughness parameters, Hartmann number and aspect ratio in Section 4. Final section summarises the important findings and their usefulness in designing bearings.

2. Formulation of the problem

The model consists of flow of viscous isothermal and incompressible electrically conducting couple-stress fluid between two rectangular plates in which the upper plate has a roughness structure. The physical configuration of the problem is shown in Fig. 1. The upper rough plate approaches the lower smooth plate with a constant velocity $\frac{dH}{dt}$. A uniform transverse magnetic field M_0 is applied in the z-direction. The upper and lower plates are separated by thickness H, then, the total film thickness is made up of two parts as

$$H = h(t) + h_s(x, y, \xi), \tag{1}$$

where $h(t)$ is the height of the nominal smooth part of the film region, and h_s is part due to the surface asperities measured from the nominal level, which is a randomly varying quantity of zero mean, and ξ is the index parameter determining a definite roughness structure. In addition to the usual assumptions of lubrication theory, we assume fluid inertia is negligible, and except the Lorentz force, the body forces are also neglected. Under these assumptions, the governing equations in Cartesian co-ordinate system are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2a}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma M_0^2 u, \tag{2b}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \sigma M_0^2 v, \tag{2c}$$

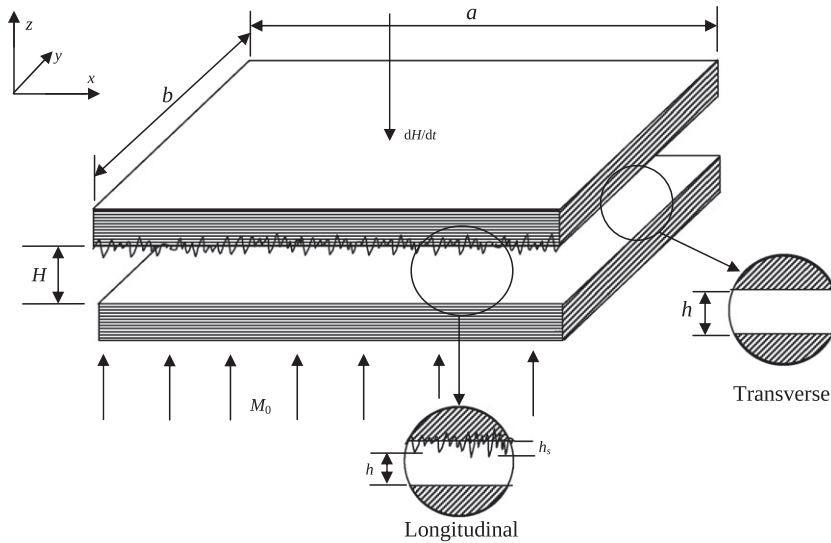$$\frac{\partial p}{\partial z} = 0, \tag{2d}$$

Fig. 1. The physical configuration of squeeze film lubrication between two rectangular plates in the presence of a transverse magnetic field M_0. The film thickness H contains height of nominal smooth part and roughness. The upper plate approaches the lower one with constant velocity $\frac{dH}{dt}$.

where u, v and w are the velocity components in x, y and z directions respectively, p is the pressure, σ is electrical conductivity of the fluid, M_0 is the impressed magnetic field and μ is viscosity of the fluid. Introduction of η in Eqs. (2b) and (2c) above, is due to polar additives in the non-polar lubricant. The quantity $\frac{\eta}{\mu}$ has the dimension of length-squared which characterizes the material length of the fluid. The case, $\eta \to 0$, the above equations reduce to the classical Newtonian fluid.

The relevant boundary conditions for the velocity components are

$$u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0 \quad \text{at } z = 0, \tag{3a}$$

$$u = 0, \quad v = 0, \quad w = \frac{dH}{dt}, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0 \quad \text{at } z = H, \tag{3b}$$

where the last two conditions in (3b) indicate the vanishing of couple-stress fluid at the bearing surfaces, and $\frac{dH}{dt}$ is the velocity of an upper plate approaching the lower plate. Making use of the solutions of (2b) and (2c) for u and v using appropriate boundary conditions in (3b) in the continuity equation and integrating it from 0 to H using the conditions $w = 0$ at $z = 0$ and $w = \frac{dH}{dt}$ at $z = H$, we get the modified Reynolds equation for the unknown pressure distribution in the fluid film region as

$$\frac{\partial}{\partial x}\left(F(H, K_3, K_4)\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(F(H, K_3, K_4)\frac{\partial p}{\partial y}\right) = \frac{\mu K_2(K_3^2 - K_4^2)}{K_1}\frac{dH}{dt}, \tag{4}$$

where

$$F(H, K_3, K_4) = H(K_3^2 - K_4^2) - \frac{2K_3^2}{K_4}\tanh(K_4 H/2) + \frac{2K_4^2}{K_3}\tanh(K_3 H/2),$$

$$K_3 = \sqrt{\frac{K_1}{2} + \frac{1}{2}\sqrt{K_1^2 - 4K_2}}, \quad K_4 = \sqrt{\frac{K_1}{2} - \frac{1}{2}\sqrt{K_1^2 - 4K_2}}, \tag{5}$$

$$K_1 = \tau_c^2, \quad K_2 = \frac{M^2}{h_0^2}\tau_c^2, \quad M = \sqrt{\frac{\sigma}{\mu}}M_0 h_0. \tag{6}$$

The dimensional parameter $\tau_c\left(= \sqrt{\frac{\mu}{\eta}}\right)$ is the couple-stress parameter and is responsible to give effect of couple-stress fluid. It is assumed that the effects of couple stresses are more pronounced for smaller values of the couple-stress parameter, and when τ_c is large, its effects are almost negligible. The parameter M is the Hartmann (or Magnetic) number, and gives the effect of magnetic field on squeeze film lubrication in which h_o is the initial film thickness. For including the effect of roughness, we take expectation of the Reynolds equation (4) as

$$\frac{\partial}{\partial x}\left(E\left(F(H, K_3, K_4)\frac{\partial p}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(E\left(F(H, K_3, K_4)\frac{\partial p}{\partial y}\right)\right) = \frac{\mu K_2(K_3^2 - K_4^2)}{K_1}\frac{dE(H)}{dt}, \tag{7}$$

where the expectancy operator $E(\cdot)$ is defined by

$$E(\cdot) = \int_{-\infty}^{\infty} (\cdot) f(h_s) dh_s \tag{8}$$

and $f(h_s)$ is the probability density function of the stochastic variable h_s. In many engineering application, bearing surfaces show a roughness height distribution which is Gaussian in nature. Therefore, polynomial form which approximates the Gaussian is chosen in the analysis. Such a probability density function is given by Christensen [2]

$$f(h_s) = \begin{cases} \frac{35}{32c^7}(c^2 - h_s^2)^3, & \text{if } -c < h_s < c; \\ 0, & \text{elsewhere,} \end{cases} \tag{9}$$

where c is the total range of random film thickness variable and function terminates at $c = \pm 3\sigma$ and σ is being the standard deviation.

In the context of rough surfaces, there are two types of roughness patterns which are of special interest. The modified Reynolds equation for both longitudinal and transverse roughness structure is derived with help of Christensen [2] stochastic theory. The one-dimensional longitudinal structure where the roughness has the form of long narrows ridges and valleys running in the x-direction and the one-dimensional transverse structure where roughness striations run in the y-direction in the form of long narrows and valleys.

2.1. Longitudinal roughness

In this case, the film thickness takes the form

$$H = h(t) + h_s(x, \xi),$$

then, Eq. (7) becomes

$$\frac{\partial}{\partial x}\left(E(F(H, K_3, K_4))\frac{\partial E(p)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{E(1/F(H, K_3, K_4))}\frac{\partial E(p)}{\partial y}\right) = \frac{\mu K_2(K_3^2 - K_4^2)}{K_1}\frac{dE(H)}{dt}. \tag{10}$$

2.2. Transverse roughness

In this case, the film thickness takes the form

$$H = h(t) + h_s(y, \xi),$$

then, Eq. (7) becomes

$$\frac{\partial}{\partial x}\left(\frac{1}{E(1/F(H, K_3, K_4))}\frac{\partial E(p)}{\partial x}\right) + \frac{\partial}{\partial y}\left(E(F(H, K_3, K_4))\frac{\partial E(p)}{\partial y}\right) = \frac{\mu K_2(K_3^2 - K_4^2)}{K_1}\frac{dE(H)}{dt}. \tag{11}$$

However, our present study is confined to one-dimensional longitudinal roughness since the one roughness structure can be obtained from other by just rotation of coordinate axes. Therefore, the modified Reynolds equation (10) for one-dimensional longitudinal roughness structure is considered for further analysis. In order to solve modified Reynolds equation for the pressure, the following boundary conditions are used

$$E(p) = 0, \quad \text{at } x = 0, a \quad \text{and} \quad y = 0, b, \tag{12}$$

where a and b are dimensions of plates in x and y directions respectively.

From Eq. (8), we have

$$E(H) = h. \tag{13}$$

Introduce non-dimensional parameters and variables as follows

$$\bar{x} = \frac{x}{a}, \quad \bar{y} = \frac{y}{b}, \quad \alpha = \frac{a}{b}, \quad \bar{H} = \frac{H}{h_0}, \quad C = \frac{c}{h_0}, \quad \tau = \tau_c h_0, \quad p = -\frac{E(p)h_0^3}{\mu a^2 \frac{dh}{dt}}, \tag{14}$$

where C is the non-dimensional roughness parameter, τ represents non-dimensional couple-stress parameter, p the non-dimensional fluid film pressure, α is the aspect ratio, then Eqs. (10) and (12) after dropping the overhead bars, become

$$\frac{\partial}{\partial x}\left(E(F(H, K_3, K_4))\frac{\partial p}{\partial x}\right) + \frac{1}{\alpha^2}\frac{\partial}{\partial y}\left(\frac{1}{E(1/F(H, K_3, K_4))}\frac{\partial p}{\partial y}\right) = -M^2(K_3^2 - K_4^2), \tag{15}$$

where

$$E(F(H, K_3, K_4)) = \frac{35}{32C^7} \int_{-C}^{C} F(H, K_3, K_4)(C^2 - h_s^2)^3 dh_s, \tag{16}$$

$$E(1/F(H, K_3, K_4)) = \frac{35}{32C^7} \int_{-C}^{C} \frac{(C^2 - h_s^2)^3}{F(H, K_3, K_4)} dh_s, \tag{17}$$

$$K_3 = \frac{\tau}{\sqrt{2}} \left(1 + \sqrt{1 - \frac{4M^2}{\tau^2}} \right)^{1/2},$$

$$K_4 = \frac{\tau}{\sqrt{2}} \left(1 - \sqrt{1 - \frac{4M^2}{\tau^2}} \right)^{1/2}$$

and boundary conditions for the pressure field are given by

$$p = 0, \quad \text{at } x = 0, 1 \quad \text{and} \quad y = 0, 1. \tag{18}$$

Solution of the above modified Reynolds equation is too complicated to be solved analytically, because it involves two integral expressions (16) and (17) which are not solvable in closed form. In view of this, we resort to the multigrid method of solving the Reynolds equation (15).

3. Multigrid solution

The modified Reynolds equation (15) is of elliptic type in nature, which is too complicated to be solved analytically, hence, we solve it numerically using finite difference based multigrid method. Derivative terms in Eq. (15) have been approximated using a standard second order finite difference scheme. Expressions (16) and (17) have been numerically integrated using Simpson's 1/3rd rule. The number of grids in each directions is taken to be 257×257. Thus, there are 257×257 number of unknowns and hence equations in the problem. As it is difficult to solve these many number of equations, we resort to the convergent accelerator multigrid method for the solution of the discretized Reynolds equation (15). The method provides us with a simple way to compute the pressure distribution. In the multigrid method, few Gauss–Seidel iterations are applied for smoothing the errors; half weighting restriction operator is used for transferring the calculated residual to the coarser grid level. Repeat this procedure till we reach the coarsest level with just single grid, and solve it exactly. Next, bilinear interpolation operator is used to prolongate the solution obtained at the coarsest level to next finer grid level. Repeat this till original the finest level is reached. The convergent solution for the pressure is obtained when the pressures at two consecutive finest levels are almost same up to 10^{-6}. Full numerical results using 257×257 grid level have been compared with those obtained with 513×513 grid level, the pressure distribution between the two are graphically indistinguishable, thus, former grid level is adopted for further computation.

4. Results and discussion

Application of multigrid method for solution of the Reynolds equation to investigate the effects of various physical parameters such as surface roughness, couple-stress fluid and Hartmann number etc. between two rectangular plates in the transverse magnetic field has revealed important features of squeeze film lubrication. The characteristics of squeeze film bearings are obtained as functions of couple-stress fluid τ, roughness parameter C, aspect ratio α and Hartmann number M. Multigrid solution to the Reynolds equation (15) exists when the quantities K_3 and K_4 are real and condition for K_3 and K_4 to become real is $\tau > 2M$. In this particular study, we focus on the parameter range for $C = 0.1 - 0.7$, $\tau = 5 - 20$, $M = 1 - 8$ and $\alpha = 0.1 - 10$ as these values are chosen to be in the range of parameters that have been used extensively in previous studies. In order to compare our results with those of previous models, the present analysis corresponds to the Newtonian fluid ($\tau \to \infty$ studied by Bujurke and Kudenatti [14]) and to the classical case ($C \to 0$, $\tau \to \infty$) studied by Lin [20].

4.1. Pressure distribution

In order to compare the effect of the couple-stress fluid with that of Newtonian case, the variations of pressure distribution p with rectangular co-ordinates x and y are shown in Figs. 2a, 2b, 2c, 2d for different values of couple-stress parameter τ keeping other parameters fixed. It is observed from these Figures that, for small values of couple-stress parameter, the built up pressure in fluid film region is higher than that of larger one. The effect of couple-stress fluid is to increase the built-up pressure. As τ increases its value, the pressure rise starts to decrease in which the fluid loses its non-Newtonian characteristics. At sufficiently large value of τ (i.e. $\tau = 1000$, in the study) the fluid becomes Newtonian and hence there is no variation of pressure rise.

Also, as in Fig. 2b, keeping τ constant ($\tau = 10$) and varying the Hartmann number from $M = 2$ to $M = 4$, Fig. 2e plots the pressure distribution, and it is found that the magnetic field enhances the pressure rise in the fluid film region. This is
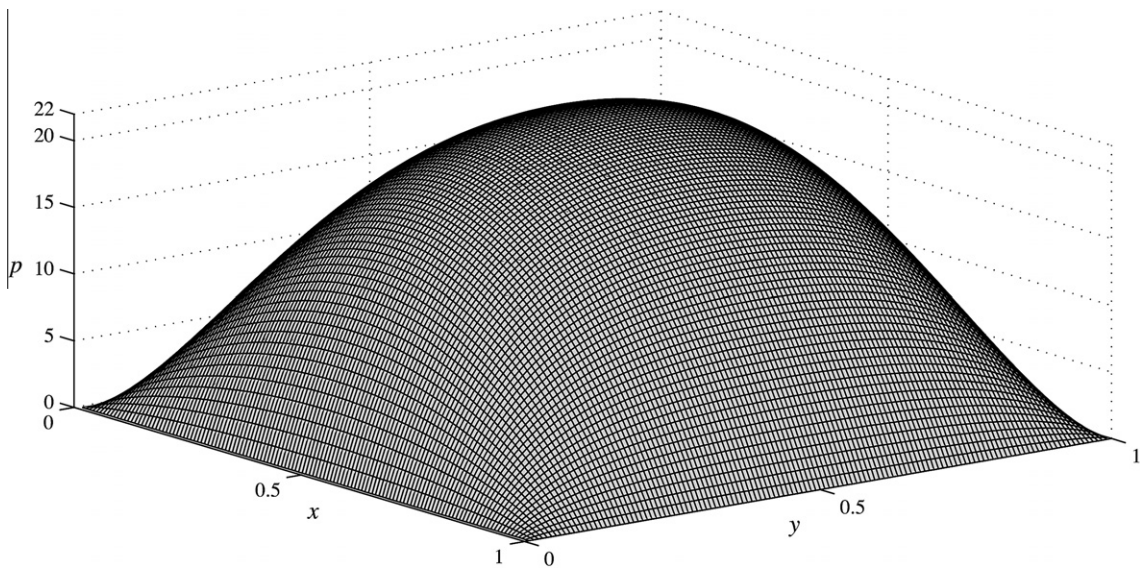
Fig. 2a. Variation of distribution of pressure p for $\tau = 5$, $C = 0.4$, $M = 2.0$ and $\alpha = 1.0$.
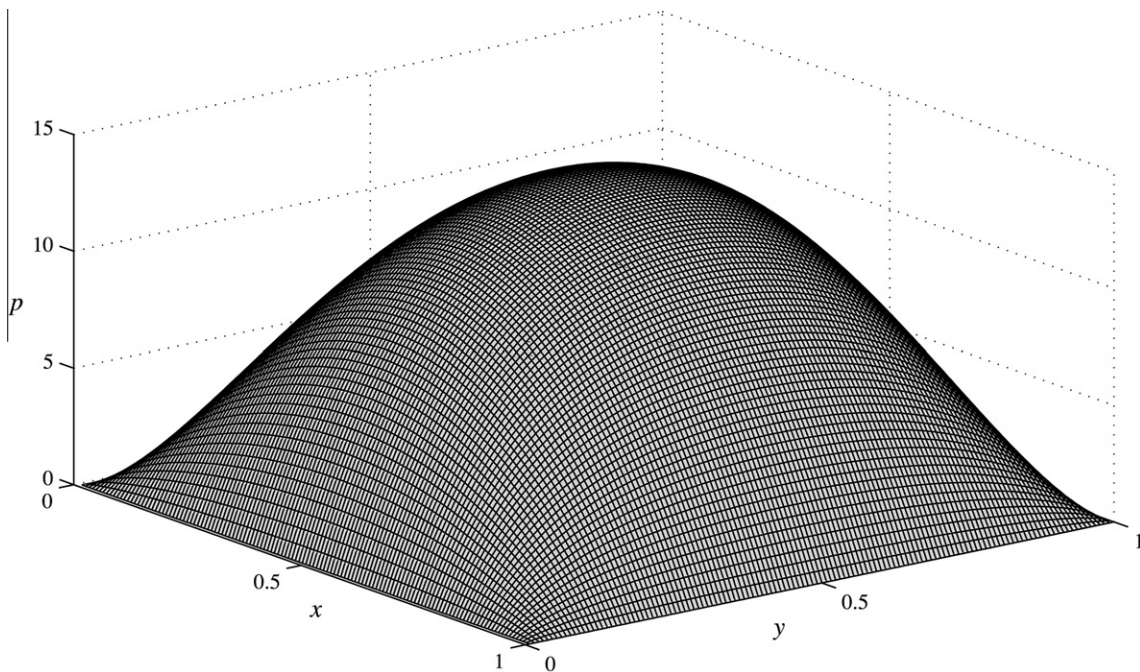

Fig. 2b. Variation of distribution of pressure p for $\tau = 10$, $C = 0.4$, $M = 2.0$ and $\alpha = 1.0$.

because of the application of uniform magnetic field normal to the flow reduces the velocity of the lubricant. Thus, the larger amount of fluid is retained in the film region, and this results in an increase in the pressure rise. Thus, an increase of application of magnetic field leads to reduce the velocity of the fluid consequently pressure rise increases.

4.2. Non-dimensional load carrying capacity

Once the fluid film pressure is obtained, hydrodynamic bearing characteristics such as load carrying capacity can be evaluated. The load carrying capacity W of the bearing surface per unit area in a non-dimensional form is
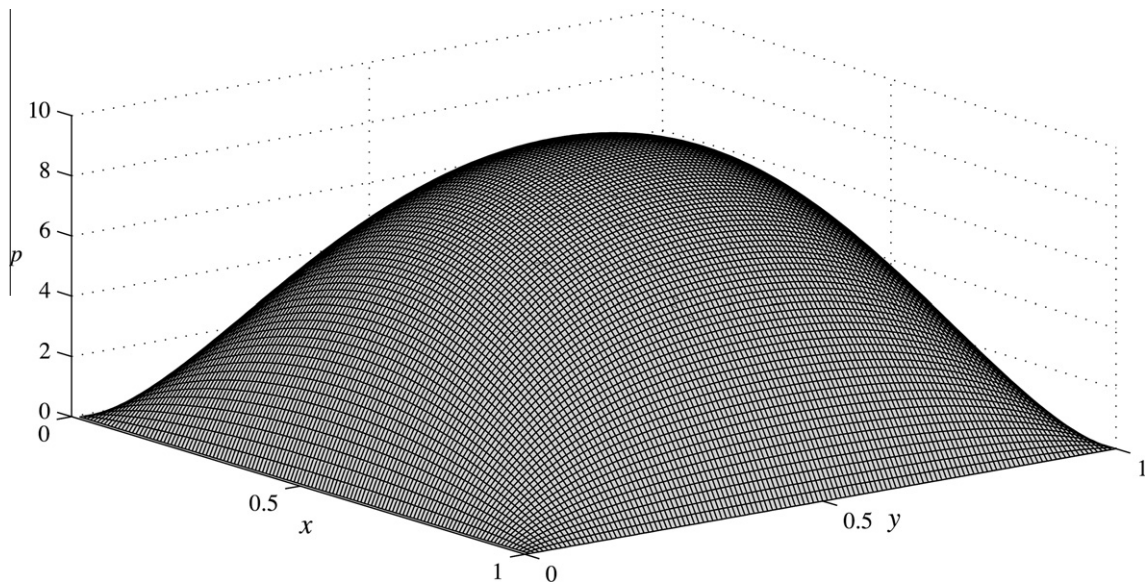
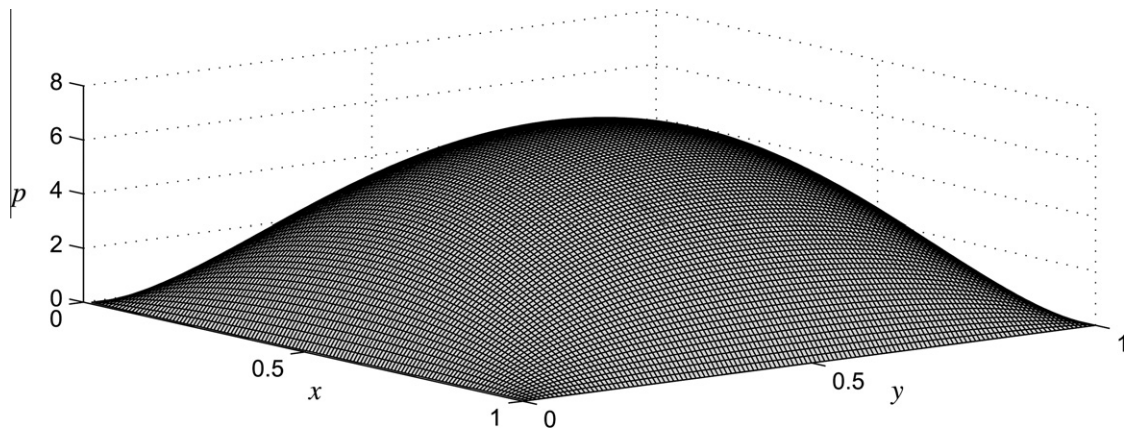**Fig. 2c.** Variation of distribution of pressure p for $\tau = 15$, $C = 0.4$, $M = 2.0$ and $\alpha = 1.0$.

Fig. 2d. Variation of distribution of pressure p for $\tau \to \infty$, $C = 0.4$, $M = 2.0$ and $\alpha = 1.0$.

$$W = \int_0^1 \int_0^1 p(x,y)dxdy. \tag{19}$$

Results of load carrying capacity as a function of various parameters are shown in Figs. 3–5. Fig. 3 shows the load carrying capacity W as a function of aspect ratio α for different couple-stress parameters τ keeping other parameters constant as shown in its figure caption. In figure, the dashed curve represents the case of Newtonian fluid. Couple-stress fluid increases the load carrying capacity compared to corresponding Newtonian case. As explained in the previous section, effect of couple-stress fluid is to increase the pressure rise in the fluid region, and therefore load carrying capacity also increases. As $\tau \to \infty$, load carrying capacity also decreases. However, as aspect ratio α increases from 0.1 to 10, the load carrying capacity also increases, and this trend is observed for all values of τ. Variations of load carrying capacity W as a function of Hartmann number M for different roughness parameters C are shown in Fig. 4. It is observed that the effect of roughness is to increase the load carrying capacity compared to smooth case ($C \to 0$). At particular value of M, the roughness increases load carrying capacity and as M increases, again it increases. The plausible reason for this to happen is that, the presence of surface asperities on bearing surface reduce the velocity of the fluid, and also reduce sidewise leakage of the fluid and as explained earlier, magnetic field also reduces the flow. As a result, the load carrying capacity of the bearing surfaces enhances for increasing values of both C and M.
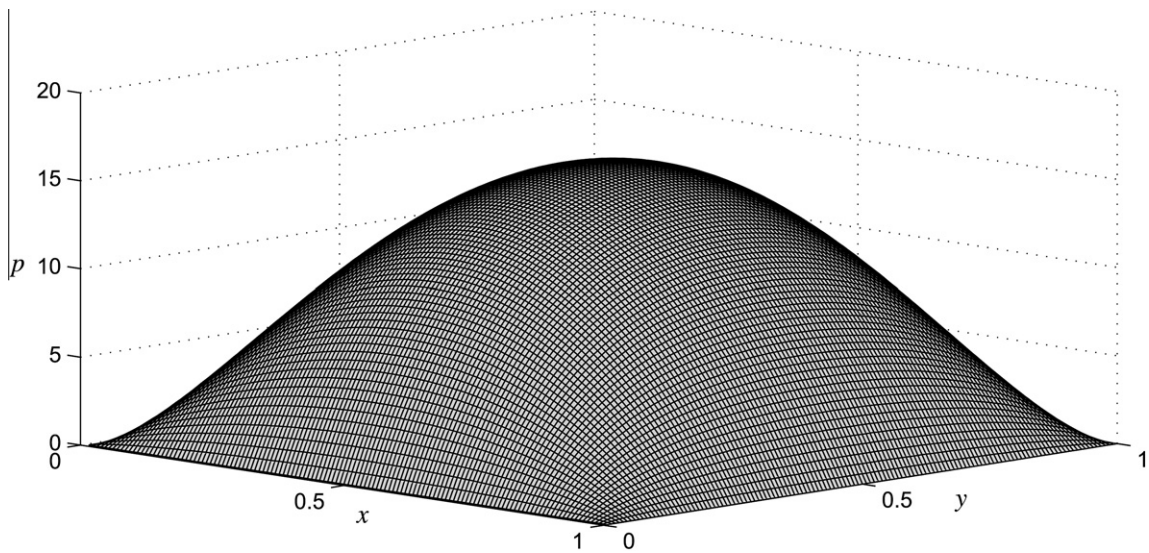
Fig. 2e. Variation of distribution of pressure p for $\tau = 10$, $C = 0.4$, $M = 4.0$ and $\alpha = 1.0$.
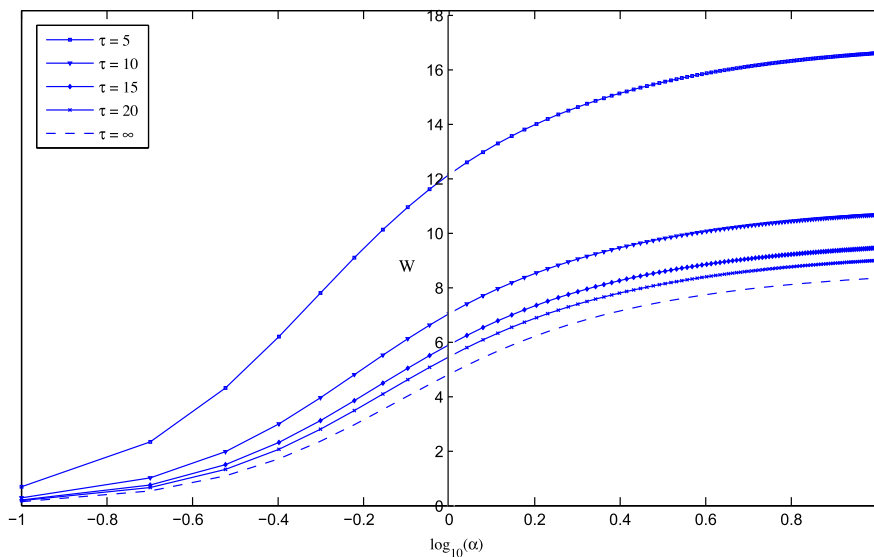

Fig. 3. Variation of non-dimensional load carrying capacity W with the aspect ratio α for different values of couple-stress parameter τ keeping other parameters constant (i.e. $C = 0.4$, $M = 2.0$).

Fig. 5 depicts, the variations of load carrying capacity W over the aspect ratio α for different values of roughness and for two sets of couple-stress parameters. It is interesting to note that, there exist a critical value $\alpha_c(\tau)$ (say) of the aspect ratio α at which the effect of roughness vanishes. At the critical value α_c ($\alpha_c = 1.752$ for $\tau = 5$ and $\alpha_c = 2.231$ for $\tau = 10$), for $\alpha > \alpha_c$, the effect of roughness is to increase the load carrying capacity and for $\alpha < \alpha_c$, the trend reverses. This trend is observed for both values of τ. And also the load carrying capacity decreases for increasing couple-stress parameter, and increases for increasing roughness parameter.

4.3. Non-dimensional squeeze time-height relation

Finally, another most important characteristics of the squeeze film bearings is the squeeze film time, i.e., the squeezing time taken by the upper plate to reach a film thickness h, that can be determined in non-dimensional form as
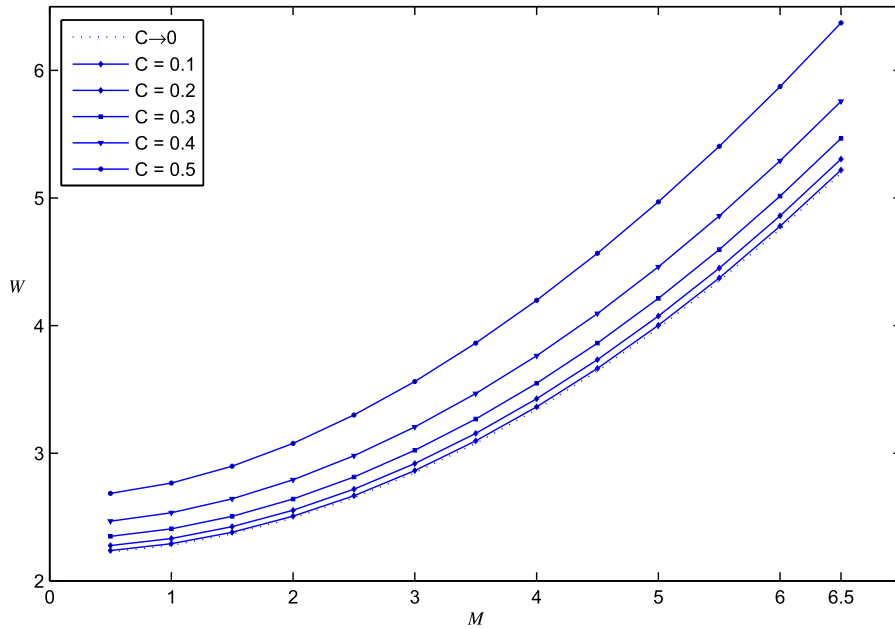
Fig. 4. Variation of non-dimensional load carrying capacity W with the Hartmann number M for different values of roughness parameter C with $\tau = 10$, $\alpha = 1.0$.
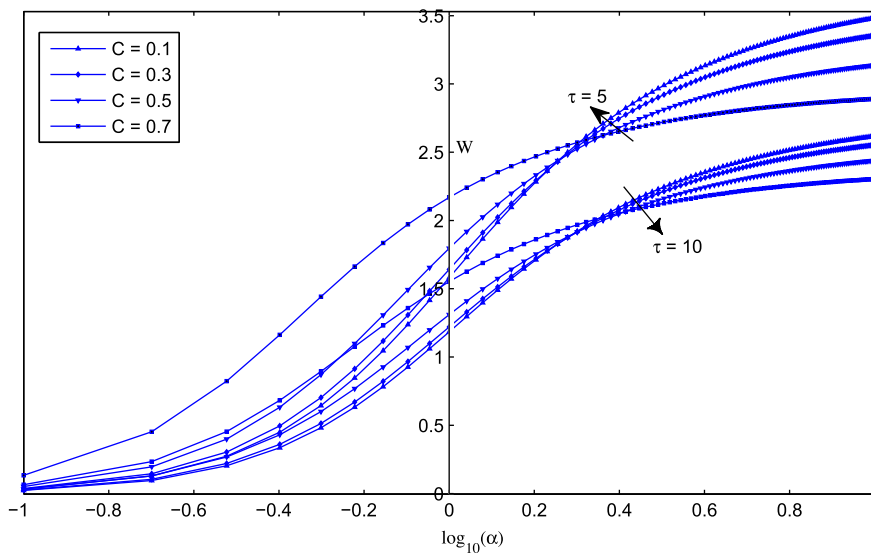

Fig. 5. Variation of non-dimensional load carrying capacity W with the aspect ratio α for different values of roughness parameter C for two sets of couple-stress parameters with $M = 2.0$.

$$T = \int_h^1 \int_0^1 \int_0^1 p(x,y) dx dy dh. \tag{20}$$

Results for squeeze film time are obtained as a function of couple-stress parameter τ and magnetic parameter M. Figs. 6 and 7 depict the variations of squeezing time T with the film thickness h for various values of τ and M respectively keeping other parameters constant. It is observed from Fig. 6 that the effect of couple-stress fluid is to enhance the squeezing time compared to the case $\tau = 25$ which represents the almost Newtonian fluid case. As discussed in the previous section, we directly claim that couple-stress fluid offers the delayed squeezing time of the upper plate which reduces the coefficient of friction. Similar results can also observed from Fig. 7. The applied magnetic field helps to reduce the velocity of the fluid in the film region which in turn results into higher squeezing time. As parameter M increases, the squeezing time also
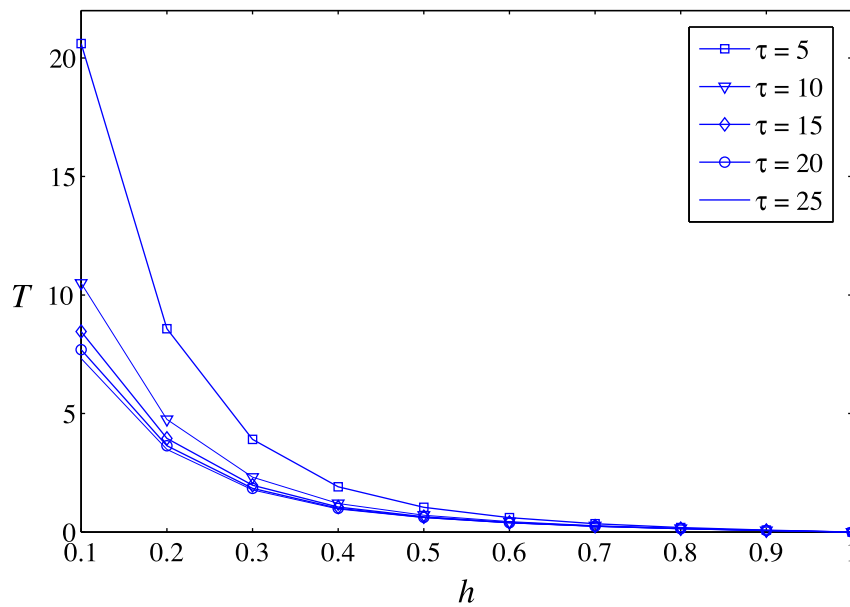
Fig. 6. Variation of non-dimensional squeeze film time T with film thickness h for different values of couple-stress parameter τ with $C = 0.4$, $M = 2.0$, and $\alpha = 1$.
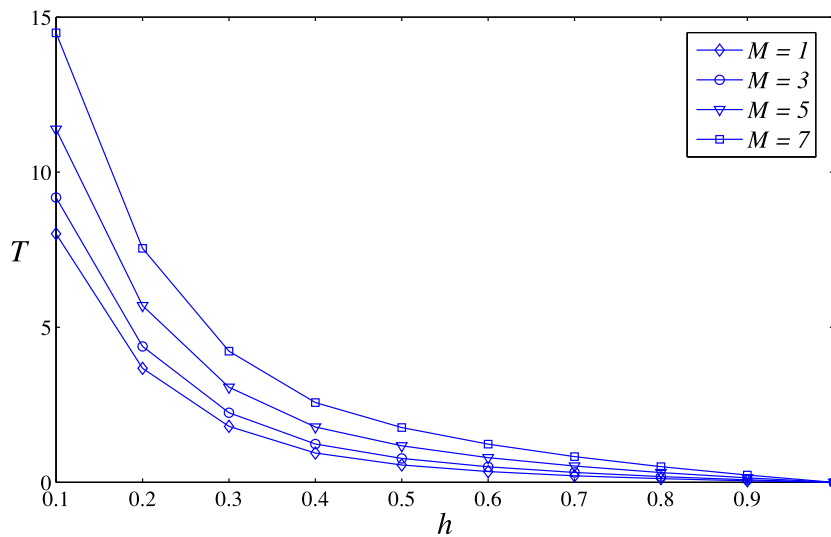

Fig. 7. Variation of non-dimensional squeeze film time T with film thickness h for different values of magnetic parameter M with $C = 0.4$, $\tau = 15$, and $\alpha = 1$.

increases. Furthermore, the smaller film thickness enhances the squeeze time for all values of τ and M. Applied magnetic field together with couple-stress fluid promote the squeezing time of the upper plate, and help to reduce the coefficient of friction and rate of wear of the plates.

5. Conclusions

On the basis of Stokes [16] micro-continuum theory for couple-stress fluid and Christensen [2] stochastic model for roughness, the effect of surface roughness on MHD squeeze film between two rectangular plates lubricated with electrically conducting couple-stress fluid in the presence of transverse magnetic field is explored. Finite difference based multigrid method is found to be accurate for the solution of modified form of Reynolds equation. Fifth place decimal convergent solution for all bearing characteristics is obtained. Investigations of effects of couple-stress fluid and roughness in magnetic fields show that these improve the lubricating characteristics of bearing surfaces i.e. the pressure distribution, load capacity and

squeeze film time have almost doubled compared to the corresponding classical case. It is expected that these findings help the design engineers to choose the appropriate roughness parameters for given magnetic field and lubricant to enhance the normal functioning of the bearing life.

Acknowledgments

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