



Theoretical data of External Bremsstrahlung radiation cross-section of bone

H.C. Manjunatha*, B. Rudraswamy

Department of Physics, Bangalore University, Bangalore 560 056, Karnataka, India

Received 17 February 2006; received in revised form 17 May 2006; accepted 1 October 2006

Abstract

Theoretical data of external Bremsstrahlung (EB) radiation cross section of bone is estimated using tabulated results of EB cross section given for various elements at various photon and electron energies. This data may be useful in the analysis of Bremsstrahlung imaging which is the technique applied in medical therapy.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: External Bremsstrahlung; Radiation; Cross section; Bone

1. Introduction

Mayuki et al. (1997) reported that $^{89}\text{SrCl}_2$ is being widely used as a palliative treatment for patients with bone pain caused by bone metastases. For this purpose, it is essential to know the spectral distribution of beta particles of ^{89}Sr when placed within the bone. As betas produce EB when it interacts with the bone material, it is easy to treat patients by studying EB spectrum externally, which is popularly known as Bremsstrahlung imaging which lead to EB application in medical therapy. NaI scintillation camera is used to study this technique. The theoretical data of EB estimated for bones may be useful for the analysis of this Bremsstrahlung imaging. The photo-electron Bremsstrahlung was investigated by Chettle (1989) in the context of quantitative analysis of lead in bone. Shivaramu (1990) measured EB energy yield for various compounds and he estimated the modified (Z_{mod}) and mean (Z_{mean}) atomic numbers for various compounds using the theoretical formulae given by Markowicz and VanGriken (1984). Accordingly, Z_{mod} for a compound (or mixture/alloy) for

EB process is given by

$$Z_{\text{mod}} = \frac{\sum_i^l \frac{W_i Z_i^2}{A_i}}{\sum_i^l \frac{W_i Z_i}{A_i}}, \quad (1)$$

where l indicates the number of elements in the compound (or mixture/ alloys) and W_i , A_i , and Z_i are the weight fraction, atomic weight, and atomic number of the i th element present in the compound, respectively. He estimated EB yields of elements and compounds using the measured spectrum. The experimental (Z_{eff}) for various compounds were obtained using Lagrange interpolation formula from the measured yield data of elements and compounds and reported that effective (Z_{eff}) is closer to Z_{mod} than Z_{mean} . Chapman (1967) evaluated the theoretical γ -ray attenuation coefficient of germanium for energy range 0.05–15 MeV based on the attenuation coefficients of five elements tabulated by Grodstein (1957) whose atomic numbers are adjacent to the atomic number of germanium, using the following Lagrange interpolation method as described in detail by Sokolnikoff and Redheffer (1958):

$$X_{Z'} = \sum Z_Z \left\{ \prod_{Z' \neq Z} (Z' - Z) \right\} / \left\{ \prod_{z \neq Z} (z - Z) \right\}, \quad (2)$$

*Corresponding author.

E-mail address: manjunathhc@rediffmail.com (H.C. Manjunatha).

where z is the atomic number of the element of known attenuation coefficient X_z adjacent to the atomic number Z' of the element whose attenuation coefficient $x_{Z'}$ is desired and Z are atomic numbers of other elements of known attenuation coefficients adjacent to Z' .

Table 1
Compact and cortical bone composition

Element	Fractional weight of the element present in the compound	
	Compact bone	Cortical bone
H	0.063984	0.047234
C	0.278	0.14433
N	0.0270	0.04199
O	0.410016	0.446096
Mg	0.0020	0.00270
P	0.070	0.10497
S	0.002	0.00315
Ca	0.147	0.20993
Zn	—	0.00011

2. Method

Lagrange interpolation method is a general one which can be used not only for γ -ray attenuation coefficient but also for EB cross section. In the present study, we have extended the above method to evaluate EB cross section in compounds such as bone compact and bone cortical whose compositions are given in Table 1. To estimate EB cross section in a compound, (2) can be suitably written as

$$\sigma_{Z'} = \sum \sigma_z \left[\frac{\prod_{Z' \neq Z} (Z' - Z)}{\prod_{z \neq Z} (z - Z)} \right], \tag{3}$$

where z is the atomic number of the element of known EB cross section σ_z adjacent to the atomic number Z' of the compound whose EB cross section $\sigma_{Z'}$ is desired and Z are atomic numbers of other elements of known EB cross sections adjacent to Z' and Z_{mod} is taken as Z' .

We have estimated Z_{mod} as 9.1970 and 11.6977 using (1) and EB cross section using (3) for compact bone and

Table 2
EB cross sections of compact bone

T	k/T							
	0.1	0.2	0.4	0.6	0.8	0.9	1.0	
0.001	999000000	510600000	259300000	172100000	127500000	112800000	101100000	
0.005	64500000	31800000	14750000	9080000	6340000	5463000	4789000	
0.01	18640000	8860000	3895000	2739000	1784000	1498000	1160000	
0.02	5198000	2389000	999000	561800	362700	302700	260600	
0.05	965400	422480	163000	85320	50816	39881	32490	
0.1	27800	11700	42330	21030	11631	8510	6350	
0.5	22410	8660	2793	1210	540	337	160	
1.0	10049	3816	1218	517	218	129	47	
2.0	4642	1952	640	280	121	70	19	
5.0	2179	890	309	145	70	41	7	
10.0	1139	483	179	89	47	29	3	

Table 3
EB cross sections of cortical bone

T	k/T							
	0.1	0.2	0.4	0.6	0.8	0.9	1.0	
0.001	1410000000	720000000	371000000	250000000	189000000	168000000	151000000	
0.005	97000000	48700000	23000000	14300000	10100000	8800000	7700000	
0.01	28900000	13900000	6300000	3780000	2610000	2240000	1960000	
0.02	8220000	3800000	1630000	940000	620000	520000	461000	
0.05	1540000	680000	279000	143000	88000	71000	59000	
0.1	444000	188000	69000	35500	20200	14900	9900	
0.5	35800	13900	4570	2000	910	580	310	
1.0	15800	6118	1970	840	360	220	94	
2.0	7950	3100	1020	450	190	110	37	
5.0	3390	1390	480	230	110	670	13	
10.0	1760	750	280	140	70	47	6	

cortical bone, respectively. σ_z for compact bone is estimated as follows:

Compact bone is composed of the following elements: N, O, F, Ne, Na, Mg with $Z = 7, 8, 9, 10, 11,$ and $12,$ respectively. Seltzer and Berger (1986) defined the total scaled EB energy weighted cross-section as $(\beta^2/Z^2)k(d\sigma/dk)$ ($d\sigma/dk$), where $\beta = k/T, k$ and T are the photon and electron energies in MeV, respectively and $(d\sigma/dk)$ is the EB cross section per unit energy. In the present study, $(\beta^2/Z^2)k(d\sigma/dk)$ is considered as σ_z whose tabulated results for the above elements are given by him. Substituting the known Z and the tabulated results of σ_z in (2), we obtain:

$$\begin{aligned} \sigma_z = & \sigma_7 \left[\frac{(9.197 - 8)(9.197 - 9)(9.197 - 10)(9.197 - 11)(9.197 - 12)}{(7 - 8)(7 - 9)(7 - 10)(7 - 11)(7 - 12)} \right] \\ & + \sigma_8 \left[\frac{(9.197 - 7)(9.197 - 9)(9.197 - 10)(9.197 - 11)(9.197 - 12)}{(8 - 7)(8 - 9)(8 - 10)(8 - 11)(8 - 12)} \right] \\ & + \sigma_9 \left[\frac{(9.197 - 7)(9.197 - 8)(9.197 - 10)(9.197 - 11)(9.197 - 12)}{(9 - 7)(9 - 8)(9 - 10)(9 - 11)(9 - 12)} \right] \\ & + \sigma_{10} \left[\frac{(9.197 - 7)(9.197 - 8)(9.197 - 9)(9.197 - 11)(9.197 - 12)}{(10 - 7)(10 - 8)(10 - 9)(10 - 11)(10 - 12)} \right] \\ & + \sigma_{11} \left[\frac{(9.197 - 7)(9.197 - 8)(9.197 - 9)(9.197 - 10)(9.197 - 12)}{(11 - 7)(11 - 8)(11 - 9)(11 - 10)(11 - 12)} \right] \\ & + \sigma_{12} \left[\frac{(9.197 - 7)(9.197 - 8)(9.197 - 9)(9.197 - 11)(9.197 - 12)}{(12 - 7)(12 - 8)(12 - 9)(12 - 10)(12 - 11)} \right], \end{aligned}$$

σ_z (milli barn/MeV) estimated for various k and T is tabulated in Table 2. Similarly σ_z for cortical bone is tabulated in Table 3. Variations of the above theoretical EB cross-section data with photon and electron energies are shown in Figs. 1–4.

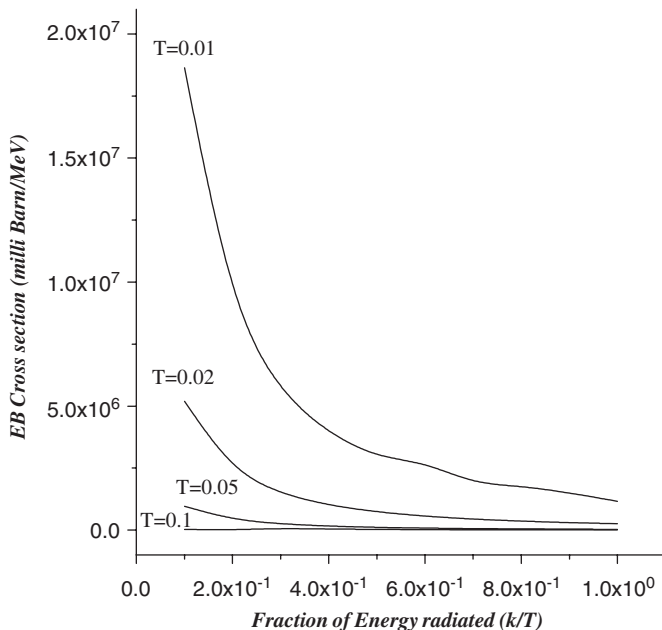


Fig. 1. EB cross-sectional dependence on photon energy at low incident electron energy for compact bone.

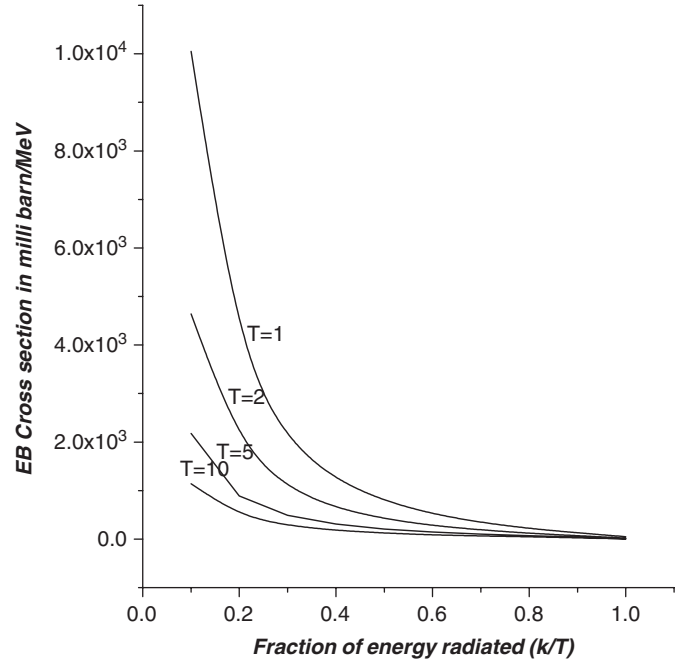


Fig. 2. EB cross-sectional dependence on photon energy at high incident electron energy for compact bone.

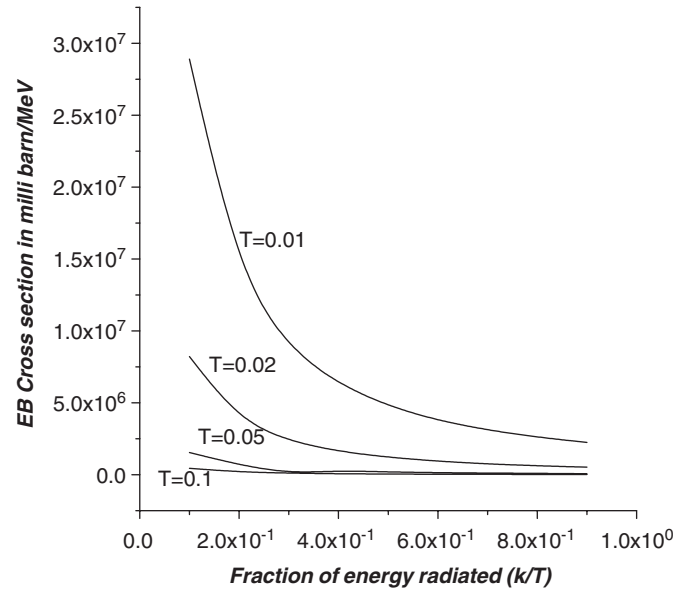


Fig. 3. EB cross-sectional dependence on photon energy at low incident electron energy for cortical bone.

3. Results and discussions

The theoretical EB cross-sectional data estimated is found to be higher for cortical bone than compact bone at all emitted photon and incident electron energies, which is in accordance with Z^2 dependence of EB cross section. This theoretical data estimated for cortical bone and compact

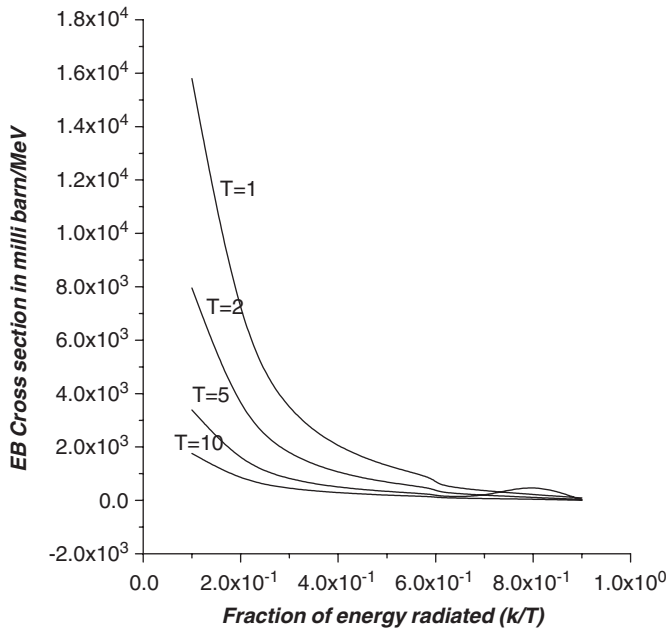


Fig. 4. EB cross-sectional dependence on photon energy at high incident electron energy for cortical bone.

bone may be useful to obtain theoretical EB spectrum which can be further used in the analysis of Bremsstrahlung imaging which is the technique followed by the previous

worker in the treatment of bone metastases. The present theoretical technique may be extended to evaluate theoretical cross section of any other compounds, which may be used to estimate the theoretical EB energy yield, which may be further compared with the measured yield.

Reference

- Chapman, G.T., 1967. Gamma ray attenuation coefficients for germanium. *Nucl. Instrum. Methods* 52, 101.
- Chettle, D.R., 1989. Photoelectron Bremsstrahlung-analytical possibilities. *Phys. Med. Biol.* 35, 259–264.
- Grodstein, G.W., 1957. X-ray attenuation coefficients from 10 keV to 100 MeV. NBS, 583.
- Markowicz, A.A., VanGriken, R.E., 1984. Composition dependence of Bremsstrahlung background in electron-probe X-ray microanalysis. *Annu. Chem.* 56, 2049.
- Shivaramu, 1990. Modified Kramer's law for Bremsstrahlung produced by complete beta particle absorption in thick targets and compounds. *J. Appl. Phys.* 68, 1225–1228.
- Sokolnikoff, I.S., Redheffer, R.M., 1958. *Math. Phys. Mod. Eng.*, 699.
- Seltzer, S.D., Berger, M.J., 1986. Bremsstrahlung energy spectra from electrons with kinetic energy 1 keV to 10 GeV incident on screened nuclei and orbital electrons of neutral atoms with $Z = 1-100$. *Atom. Data. Nucl. Data Tables* 35, 345–418.
- Mayuki, U., Hiroto, N., Motoji, M., Hiroshi, S., Yataka, M., Nobuyoshi, F., Kenji, K., 1997. Strontium-89 therapy and imaging with Bremsstrahlung in bone metastases. *Clin. Nucl. Med.* 22, 605–609.