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Effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection

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Summary. The effect of Coriolis force and different forms of basic temperature gradients on the onset of Marangoni ferroconvection in a horizontal layer of ferrofluid is investigated theoretically. The lower boundary is assumed to be rigid-isothermal, while the upper free boundary on which the surface tension acts is non-deformable and insulating to temperature perturbations. The Galerkin technique is used to obtain the critical stability parameters. It is shown that convection sets in as oscillatory motions provided that the Prandtl number is less than unity. A mechanism for suppressing or augmenting Marangoni ferroconvection by rotation, nonlinearity of magnetization and different forms of basic temperature gradients is discussed in detail. It is found that the inverted parabolic temperature profile indicates a reinforcement of stability, whereas the step function temperature profile indicates a diminution of stability. Comparisons of results between the present and the existing ones are made under the limiting conditions and good agreement is found.

1 Introduction

Convection in ferromagnetic fluids in the presence of a uniform magnetic field, called ferroconvection, is analogous to the Rayleigh-Benard convection in a horizontal layer of an ordinary viscous fluid heated from below and cooled from above. Ferroconvection in a horizontal layer of ferrofluid has been studied extensively because of its diverse applications, namely, energy conversion systems, liquid cooled loudspeakers, magnetic fluid seals and in many other engineering and technological applications [1]–[3]. Finlayson [4] was the first to study the linear stability of ferroconvection in a horizontal layer of ferrofluid heated from below in the presence of a uniform vertical magnetic field. Lalas and Carmi [5] have analyzed the same problem using the energy method. A similar analysis but with the fluid confined between ferromagnetic plates has been carried out by Gotoh and Yamada [6] using the linear stability analysis. Schwab et al. [7] have conducted experiments and their results are found to be in good agreement with [4]. Stiles and Kagan [8] have extended the problem to allow for the dependence of effective shear viscosity on temperature and colloid concentration. Recently, Kaloni and Lou [9] have studied convective instability in a horizontal layer of a magnetic fluid by considering the relaxation time and the rotational viscosity effects.

The effect of Coriolis force on ferroconvection has also been investigated because the ferrofluids are known to exhibit peculiar characteristics when set to rotation. Das Gupta and Gupta [10] have studied the convective instability in a rotating layer of ferrofluids between two free boundaries. Rudraiah and Sekhar [11] have analyzed the effect of uniform distribution of heat source on the onset of stationary ferroconvection. Venkatasubramanian and Kaloni [12] have discussed the effect of rotation on thermo-convective instability of a horizontal layer of ferrofluid confined between stress-free, rigid-paramagnetic and rigid-ferromagnetic boundaries for uniform temperature gradient. The corresponding problem for ferroconvection in a rotating porous medium is discussed by Sekar et al. [13], and Vaidyanathan et al. [14]. In the latter paper, the effect of magnetic field dependent viscosity is also taken into consideration. The weakly nonlinear instability of a rotating ferromagnetic fluid layer heated from below is discussed by Kaloni and Lou [15].

It is known that, apart from buoyancy, convective instability can also occur due to the local variation of surface tension when the fluid surface is free [16]. This type of convective instability is referred to as Marangoni convection. Much of the available literature on Marangoni convection has been concerned with viscous liquid layers and very little is known about Marangoni convection in ferrofluids, called Marangoni ferroconvection. In view of the fact that heat transfer is greatly enhanced due to convection, Marangoni ferroconvection offers new possibilities for applications in microgravity environments. Qin and Kaloni [17] have studied linear and nonlinear stability problems of combined buoyancy-surface tension effects in a ferrofluid layer heated from below. Recently, Shivakumara et al. [18] have discussed in detail the effect of different forms of basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces.

In view of the fact that rotation gives rise to interesting practical situations, the object of this paper is to study the combined effect of rotation and different forms of basic temperature gradients on the linear stability of Marangoni ferroconvection. In this study the lower rigid boundary is considered to be isothermal and the upper non–deformable free boundary is insulating to temperature perturbations. The resulting eigenvalue problem is solved numerically by employing the Galerkin technique. A comparative study is conducted to analyze the relative effects of different temperature profiles on the onset of convection and with the other works as well under the limiting conditions.

2 Formulation of the problem

We consider an infinite horizontal layer of an electrically non-conducting Boussinesq ferromagnetic fluid of depth d permeated by a uniform magnetic field acting normal to the boundaries. The layer is rotating uniformly about its vertical axis with angular velocity $\vec{\Omega} = \Omega \hat{k}$, which is bounded below by a rigid-isothermal surface and above by a non-deformable free-insulating surface. A temperature drop ΔT is acting across the boundaries and a Cartesian coordinate system (x, y, z) is used with the origin at the bottom of the surface and the z-axis vertically upwards. The surface tension σ is assumed to vary linearly with temperature as $\sigma = \sigma_0 - \sigma_T \Delta T$, where σ_0 is the unperturbed value and σ_T is the rate of change of surface tension with temperature.

The relevant governing equations in the rotating frame of reference are [1], [4], [10]:

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right] = -\nabla p + \mu_0 \, (\vec{M} \cdot \nabla)\vec{H} + \mu \nabla^2 \vec{q} + 2\rho_0 \vec{q} \times \vec{\Omega} + \frac{\rho_0}{2} \nabla \left(\left| \vec{\Omega} \times \vec{r} \right|^2 \right), \tag{2}$$

$$\left[\rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right] \frac{DT}{Dt} + \mu_0 T \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H} \cdot \frac{D\vec{H}}{Dt} = k\nabla^2 T, \tag{3}$$

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = 0, \tag{4.1,2}$$

$$\vec{B} = \mu_0 \left(\vec{M} + \vec{H}\right), \tag{5}$$

$$\vec{M} = \frac{M}{H}(H,T)\vec{H}, \tag{6}$$

$$M = M_0 \ + \ \chi(H - H_0) \ - \ K(T - \bar{T}), \tag{7}$$

where $\vec{q} = (u, v, w)$ is the velocity, p is the pressure, T is the temperature, $\vec{H}$ is the magnetic field, $\vec{M}$ is the magnetization, $C_{V,H}$ is the specific heat at constant volume and magnetic field, $\vec{B}$ is the magnetic induction, μ is the coefficient of viscosity, μ_0 is the magnetic permeability of vacuum, ρ_0 is the reference density, k is the thermal conductivity, $\bar{T}$ is the average temperature, $\chi = (\partial M/\partial H)_{H_0,\bar{T}}$ is the magnetic susceptibility, $K = -(\partial M/\partial T)_{H_0,\bar{T}}$ is the pyromagnetic coefficient, $M_0 = M(H_0, \bar{T})$, and H_0 is the imposed uniform vertical magnetic field.

The basic state is given by

$$\begin{aligned} &\vec{q} = 0, \quad p = p_b(z), \quad -\frac{dT_b}{dz} = f(z), \\ &\vec{H}_b = \left[H_0 + \frac{K(T_b - \bar{T})}{1+\chi}\right]\hat{k}, \quad \vec{M}_b = \left[M_0 - \frac{K(T_b - \bar{T})}{1+\chi}\right]\hat{k}, \end{aligned} \tag{8}$$

where $\hat{k} = (0, 0, 1)$ is the unit vector in the z-direction, the subscript b denotes the basic state and $f(z)$ is the basic temperature gradient, such that $\int_0^d f(z)\,dz = -\frac{\Delta T}{d}$.

We shall analyze the stability of the basic state by introducing the following perturbations:

$$\vec{q} = \vec{q}', \quad p = p_b(z) + p', \quad T = T_b(z) + T', \quad \vec{H} = \vec{H}_b(z) + \vec{H}', \quad \vec{M} = \vec{M}_b(z) + \vec{M}', \tag{9}$$

where the primed quantities are perturbed ones and they are assumed to be small.

Substituting Eq. (9) into Eqs. (5) and (6) and using Eqs. (4) and (7), we get

$$H_i' + M_i' = \left(\frac{1 + M_0}{H_0}\right) H_i', \quad i = 1, 2, \tag{10}$$

$$H_3' + M_3' = (1 + \chi)H_3' - KT', \tag{11}$$

where we have assumed $K(T_b - \bar{T}) \ll (1 + \chi)H_0$.

Using Eq. (9) in Eq. (2), linearizing and then taking curl on the resulting equation (after neglecting primes), the z-component can be written as

$$\rho_0 \frac{\partial \xi}{\partial t} = \mu \nabla^2 \xi + 2\rho_0 \Omega \frac{\partial w}{\partial z}, \tag{12}$$

which is the vorticity transport equation and $\xi = \partial v/\partial x - \partial u/\partial y$ is the z-component of vorticity. Substituting Eq. (9) in Eq. (2), linearizing, taking curl twice and then using Eqs. (10) and (11) together with $\vec{H}' = \nabla \varphi'$, the z-component of the resulting equation can be written as (after neglecting the primes)

$$\left(\rho_0 \frac{\partial}{\partial t} - \mu \nabla^2\right) \nabla^2 w = -2\rho_0 \Omega \frac{\partial \xi}{\partial z} - \mu_0 K f(z) \frac{\partial}{\partial z}\left(\nabla_1^2 \varphi\right) + \frac{\mu_0 K^2 f(z)}{1+\chi} \nabla_1^2 T. \tag{13}$$

As before, using Eq. (9) in Eq. (3), and after linearizing, the equation obtained is (neglecting primes)

$$\rho_0 C_0 \frac{\partial T}{\partial t} - k\nabla^2 T - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right) = \left(\rho_0 C_0 - \frac{\mu_0 K^2 T_0}{(1+\chi)}\right) w f(z), \tag{14}$$

where $\rho_0 C_0 = \rho_0 C_{V,H} + \mu_0 K H_0$, $\nabla_1^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and $\nabla^2 = \nabla_1^2 + \partial^2/\partial z^2$.

Finally Eqs. (4.1, 2), after using Eqs. (10) and (11) together with $\vec{H}' = \nabla\varphi'$ become (neglecting primes)

$$\left(1 + \frac{M_0}{H_0}\right)\nabla_1^2 \varphi + (1+\chi)\frac{\partial^2 \varphi}{\partial z^2} - K\frac{\partial T}{\partial z} = 0. \tag{15}$$

We perform normal mode expansion of the dependent variables in the form

$$\{w, T, \varphi, \xi\} = \{W(z), \theta(z), \varphi(z), \xi(z)\}\exp[i(lx + my) + \omega t], \tag{16}$$

where l and m are wave numbers in the x- and y-direction, respectively, ω is the growth rate, $W(z), \theta(z)$, $\varphi(z)$ and $\xi(z)$ are the amplitudes of the z-component of the perturbation velocity, perturbation temperature, perturbation magnetization and the z-component of the perturbation vorticity, respectively.

Substituting Eq. (16) into Eqs. (12)–(15), we get

$$\left[\rho_0 \omega - \mu\left(D^2 - a^2\right)\right]\left(D^2 - a^2\right)W = a^2 \mu_0 K f(z) D\varphi - \frac{a^2 \mu_0 K^2 f(z)}{1+\chi}\theta - 2\rho_0 \Omega D\xi, \tag{17}$$

$$\omega\,\theta - \kappa(D^2 - a^2)\theta - \frac{\mu_0 K\, T_0}{\rho_0 C_0}\omega D\varphi = \left(1 - \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_0}\right) W f(z), \tag{18}$$

$$(1+\chi)D^2\varphi - \left(\frac{1+M_0}{H_0}\right)a^2\varphi - KD\theta = 0, \tag{19}$$

$$\rho_0\, \omega\, \xi = \mu(D^2 - a^2)\xi + 2\rho_0 \Omega\, DW, \tag{20}$$

where $D = d/dz$ is the differential operator and $a = \sqrt{l^2 + m^2}$ is the overall horizontal wavenumber.

Equations (17)–(20) are nondimensionalized by setting

$$\begin{aligned} &W^* = \frac{d}{\nu}W, \quad a^* = a\mathrm{d}, \quad \mathrm{D}^* = \mathrm{dD}, \quad \mathrm{t}^* = \frac{\nu}{d^2}t, \quad \xi^* = \frac{d^2}{\nu}\xi, \\ &\omega^* = \frac{d^2}{\nu}\omega, \quad \theta^* = \frac{\kappa}{\beta\nu d}\theta, \quad \varphi^* = \frac{(1+\chi)\kappa}{K\beta\nu d^2}\varphi, \quad f(z)^* = \frac{1}{\beta}f(z)\left(\beta = -\frac{\Delta \mathrm{T}}{d}\right). \end{aligned} \tag{21}$$

Thus Eqs. (17)–(20) become (after neglecting the asterisks)

$$\left(D^2 - a^2 - \omega\right)\left(D^2 - a^2\right)W = \mathrm{Ta}^{1/2} D\xi + Na^2 f(z)\theta - Na^2 f(z) D\varphi, \tag{22}$$

$$\left(D^2 - a^2 - \omega\mathrm{Pr}\right)\theta + \omega\mathrm{Pr}M_2 D\varphi = -(1 - M_2)\, W\, f(z), \tag{23}$$

$$\left(D^2 - a^2 M_3\right)\varphi - D\theta = 0, \tag{24}$$

$$\left(D^2 - a^2 - \omega\right)\xi = -\mathrm{Ta}^{1/2} DW. \tag{25}$$

Here $\mathrm{Ta}^{1/2} = 2\Omega d^2/\nu$ is the square root of the Taylor number, $N = \mu_0 K^2 \beta^2 d^4/(1+\chi)\kappa\mu$ is the magnetic Rayleigh number, $M_3 = (1 + M_0/H_0)/(1+\chi)$ is the measure of nonlinearity of magnetisation, $\mathrm{Pr} = \nu/\kappa$ is the Prandtl number, and $M_2 = \mu_0 T_0 K^2/\rho_0 C_0(1+\chi)$ is the nondimensional parameter and is neglected in the subsequent analysis since its value is negligible [4], and $f(z)$ is the nondimensional basic temperature gradient, such that $\int_0^1 f(z)\,dz = 1$.

The corresponding boundary conditions for the perturbed nondimensional variables take the form

$$W = DW = \varphi = \theta = \xi = 0 \quad \text{at} \quad z = 0, \tag{26}$$

$$W = D^2W + \mathrm{Ma}\,a^2\theta = D\varphi = D\theta = D\xi = 0 \quad \text{at } z = 1, \tag{27}$$

where $\mathrm{Ma} = \sigma_T \Delta T d/\mu\kappa$ is the Marangoni number.

To investigate the effect of nonuniform temperature gradients on the convective instability, the following types of basic temperature profiles are considered:

Model 1: Linear temperature profile:

$f(z) = 1.$

Model 2: Piecewise linear temperature profile heating from below:

$$f(z) = \begin{cases} \dfrac{1}{\varepsilon} & \text{for } 0 \le z < \varepsilon \\ 0 & \text{for } \varepsilon < z \le 1. \end{cases}$$

Model 3: Piecewise linear temperature profile cooling from above:

$$f(z) = \begin{cases} 0 & \text{for } 0 \le z < \varepsilon \\ \dfrac{1}{(1-\varepsilon)} & \text{for } \varepsilon < z \le 1. \end{cases}$$

Model 4: Step function temperature profile:

$f(z) = \delta(z - \varepsilon).$

Model 5: Inverted parabolic temperature profile:

$f(z) = 2(1 - z).$

Model 6: Parabolic temperature profile:

$f(z) = 2z.$

Regarding the applicability of the above temperature profiles in the situation considered, it may be noted that these can be achieved by sudden heating or cooling of boundaries and also by a uniform volumetric heat source (for details see [19]).

3 Method of solution

The governing equations (22)–(25) together with the boundary conditions (26) and (27) constitute an eigenvalue problem with Ma as an eigenvalue. To solve the resulting eigenvalue problem, the Galerkin technique is used. Accordingly, the variables are written in a series of basis functions as

$$W = \sum_{i=1}^{n} A_i\, W_i(z), \quad \theta(z) = \sum_{i=1}^{n} B_i\, \theta_i(z), \quad \varphi(z) = \sum_{i=1}^{n} C_i\, \varphi_i(z), \quad \xi = \sum_{i=1}^{n} E_i\, \xi_i(z), \tag{28}$$

where the trial functions $W_i(z)$, $\theta_i(z)$, $\varphi_i(z)$ and $\xi_i(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions, and A_i, B_i, C_i, and E_i are constants.

Substituting Eq. (28) into Eqs. (22)–(25), multiplying the resulting momentum equation by $W_j(z)$, the energy equation by $\theta_j(z)$, the magnetic potential equation by $\varphi_j(z)$ and the vorticity equation by $\xi_j(z)$, performing the integration by parts with respect to z between $z=0$ and $z=1$ and using the boundary conditions (26) and (27), we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}B_i + E_{ji}C_i + F_{ji}E_i = 0, \tag{29}$$

$$G_{ji}A_i + H_{ji}B_i = 0, \tag{30}$$

$$I_{ji}B_i + J_{ji}C_i = 0, \tag{31}$$

$$K_{ji}A_i + L_{ji}E_i = 0. \tag{32}$$

The coefficients $C_{ji} - L_{ji}$ involve the inner products of the basis functions and are given by

$$C_{ji} = \langle D^2W_jD^2W_i\rangle + (2a^2+\omega)\langle DW_j\,DW_i\rangle + a^2(a^2+\omega)\langle W_jW_i\rangle,$$

$$D_{ji} = -a^2N\langle f(z)\theta_jW_i\rangle + a^2\mathrm{Ma}\,DW_j(1)\theta_i(1),$$

$$E_{ji} = a^2N\langle f(z)W_jD\varphi_i\rangle,$$

$$F_{ji} = -Ta^{1/2}\langle W_jD\zeta_i\rangle,$$

$$G_{ji} = -\langle f(z)\theta_jW_i\rangle,$$

$$H_{ji} = \langle D\theta_jD\theta_i\rangle + (a^2+\omega\mathrm{Pr})\langle\theta_j\theta_i\rangle,$$

$$I_{ji} = \langle\varphi_jD\theta_i\rangle,$$

$$J_{ji} = \langle D\varphi_jD\varphi_i\rangle + a^2M_3\langle\varphi_j\varphi_i\rangle,$$

$$K_{ji} = -\mathrm{Ta}^{1/2}\langle\zeta_jDW_i\rangle,$$

$$L_{ji} = \langle D\zeta_jD\zeta_i\rangle + (a^2+\omega)\langle\zeta_j\zeta_i\rangle,$$

where the inner product is defined as $\langle\cdots\cdots\rangle = \int_0^1(\cdots)\,dz$.

The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} & F_{ji} \\ G_{ji} & H_{ji} & 0 & 0 \\ 0 & I_{ji} & J_{ji} & 0 \\ K_{ji} & 0 & 0 & L_{ji} \end{vmatrix} = 0. \tag{33}$$

We select the trial functions as

$$W_i = z^{i+1} - z^{i+2}, \quad \theta_i = z^i - \frac{z^{i+1}}{2}, \quad \varphi_i = z^{i+1} - \frac{2}{3}z^{i+2} \quad \text{and} \quad \zeta_i = z^{i+1} - \frac{2}{3}z^{i+2}, \tag{34}$$

such that they satisfy all the corresponding boundary conditions except the one, namely $D^2W + \mathrm{Ma}\,a^2\theta = 0$ at $z=1$, but the residual from this equation is included as a residual from the differential equation.

At this juncture, it would be instructive to look at the results for $i=j=1$ and for this order Eq. (33) gives the following characteristic equation:

$$\mathrm{Ma} = -\frac{(\eta_1 + 2\,\omega \mathrm{Pr})}{1575\, a^2 \langle f(z) W\theta\rangle}\left[\frac{147\,\mathrm{Ta}}{(\eta_2 + 13\omega)} + 2(\eta_3 + \eta_4\omega)\right] - \frac{63 N\langle f(z) W D\varphi\rangle}{2\eta_5} - 2N\langle f(z) W\theta\rangle, \tag{35}$$

where $\eta_1 = 2\,a^2 + 5,\ \eta_2 = 42 + 13a^2,\ \eta_3 = a^4 + 28a^2 + 420,\ \eta_4 = 14 + a^2$ and $\eta_5 = 42 + 13\,M_3 a^2$.

To examine the stability of the system, we take $\omega = i\ \omega$ in Eq. (35) and clear the complex quantities from the denominator of Eq. (35), to get

$$\mathrm{Ma} = -\frac{1}{1575 a^2 \langle f(z) W\theta\rangle}\left[\frac{147\mathrm{Ta}(\eta_1\eta_2 + 26\omega^2\ \mathrm{Pr})}{(\eta_2^2 + 169\,\omega^2)} + 2(\eta_1\eta_3 -\ 2\,\omega^2\eta_4 \mathrm{Pr})\right]$$
$$- 2N\langle f(z) W\theta\rangle - \frac{63 N\langle f(z) W D\varphi\rangle}{2\eta_5} + i\omega\Delta, \tag{36}$$

where

$$\Delta = -\frac{1}{1575 a^2 \langle f(z)\, W\theta\rangle}\left[\frac{147\mathrm{Ta}(2\,\eta_2\,\mathrm{Pr} - 13\eta_1)}{(\eta_2^2 + 169\,\omega^2)} + 2(2\eta_3\,\mathrm{Pr} + +\,\eta_1\eta_4)\right]. \tag{37}$$

Since Ma is a physical quantity it must be real, so that it implies either $\omega = 0$ or $\Delta = 0(\omega \neq 0)$, and accordingly the condition for steady and oscillatory onset is obtained.

The steady onset is governed by $\omega = 0$ and it occurs at $\mathrm{Ma} = \mathrm{Ma}^s$, where

$$\mathrm{Ma}^s = -\frac{\eta_1}{1575\ a^2\ \langle f(z)\ W\ \theta\rangle}\left(\frac{147\ \mathrm{Ta}}{\eta_2} + 2\,\eta_3\right) - 2N\langle f(z) W\theta\rangle - \frac{63 N\ \langle f(z)\ W D\varphi\rangle}{2\eta_5}. \tag{38}$$

The oscillatory convection occurs at $\mathrm{Ma} = \mathrm{Ma}^0$, where

$$\mathrm{Ma}^0 = -\frac{2(a_1 a_4^2 + a_2 a_4 + a_3)}{1575\ a_4 a^2 \langle f(z) W\theta\rangle} - 2N\langle f(z) W\theta\rangle - \frac{63 N\langle f(z) W D\varphi\rangle}{2\eta_5}. \tag{39}$$

Here $a_1 = \eta_1\eta_2 - \frac{26}{169}\mathrm{Pr}\ \eta_2^2$, $a_2 = \eta_1\eta_3 + \frac{2}{169}\mathrm{Pr}\,\eta_4\eta_2^2 + \frac{147}{13}\mathrm{Ta}\,\mathrm{Pr}$, $a_3 = -\frac{147}{169}\mathrm{Ta}\,\mathrm{Pr}\,\eta_4$, and $a_4 = \frac{\eta_1\eta_4 + 2\,\mathrm{Pr}\,\eta_3}{13\eta_1 - 26\,\mathrm{Pr}\,\eta_2}$.

The corresponding frequency of oscillations is given by

$$\omega^2 = -\frac{\eta_2^2}{169} + \frac{147\mathrm{Ta}}{26\eta_4}\left[\frac{1 - 2\beta_1\,\mathrm{Pr}}{1 + 2\beta_2\,\mathrm{Pr}}\right], \tag{40}$$

where

$$\beta_1 = \frac{42 + 13\,a^2}{65 + 26\,a^2} \quad \text{and} \quad \beta_2 = \frac{a^4 + 28a^2 + 420}{2a^4 + 33a^2 + 70}.$$

For the occurrence of oscillatory onset ω^2 should be positive and the necessary conditions for the same are

$$\mathrm{Pr} < \frac{(a^2 + 2.5)}{(a^2 + 3.23)} \quad \text{and} \quad \mathrm{Ta} > \frac{26}{24843}\eta_2^2\eta_4\left[\frac{1 + 2\beta_2\,\mathrm{Pr}}{1 - 2\beta_1\,\mathrm{Pr}}\right]. \tag{41}$$

It is thus evident that for the oscillatory onset to exist the Prandtl number Pr should be less than unity as observed in the classical viscous liquids. But for most of the ferrofluids, whether they are water based or any other organic liquid based, the Prandtl number is greater than unity and hence the overstability is not a preferred mode of instability [12], [15].

In what follows we restrict ourselves to the case of steady onset and put $\omega = 0$ in Eq. (33). Now this determinantal equation leads to the characteristic equation giving the Marangoni number Ma as a function of the wavenumber a, basic temperature gradient $f(z)$ and the parameters N, M_3 and Ta. The inner products involved in the determinant were evaluated analytically in order to avoid errors in the numerical integration. The critical Marangoni numbers Ma_{ci} $(i = 1 \text{ to } 6)$ are obtained by minimizing respectively Ma_i $(i = 1 \text{ to } 6)$ with respect to the wavenumber a and thermal depth ε (in the case of Models 2–4). Computations reveal that the convergence in finding Ma_{ci} $(i = 1 \text{ to } 6)$ crucially depends on the value of Ta, and for higher value of Ta more terms in the expansion given by Eq. (28) were found to be required. The results presented here are for $i = j = 8$, the order at which the convergence is achieved, in general.

4 Results and discussion

The linear stability theory is used to investigate the combined effects of Coriolis force and different forms of basic temperature gradients on Marangoni ferroconvection in a rotating ferrofluid layer. The lower boundary is taken to be rigid-isothermal and the upper non-deformable free boundary is insulating to temperature perturbations. The Galerkin technique is used to find the eigenvalues as this technique is found to be more convenient to tackle different basic temperature profiles.

To know the validity of our solution procedure, first the critical values (Ma_c, a_c, ε_c) obtained from the present study under the limiting conditions are compared with those of Vidal and Acrivos [20] and Lebon and Cloot [19] in Tables 1 and 2, respectively. The results tabulated in Table 1 for different values of Ta are for $f(z) = 1$ and $N = 0$ (i.e., Marangoni convection for non-ferrofluids), while the results tabulated in Table 2 are for $N = 0$, $Ta = 0$ and for different basic temperature profiles.

From these tables, it is evident that there is an excellent agreement between the present and the previously published results. Further, it may be noted that an increase in the value of Ta is

Table 1. Comparison of Ma_c and a_c for different values of Ta when $N = 0$

Ta	Vidal and Acrivos [20]		Present study	
	Ma_c	a_c	Ma_c	a_c
0	80	2.0	79.61	1.99
10^2	92	2.2	91.31	2.17
10^3	164	3.0	163.11	2.97
10^4	457	5.0	456.23	4.99
10^5	1400	8.6	1398.36	8.86

Table 2. Comparison of Ma_c, a_c and ε_c for different basic temperature profiles when $N = 0$ and $Ta = 0$

Nature of temperature profile	Lebon and Cloot [19]			Present study		
	Ma_c	a_c	ε_c	Ma_c	a_c	ε_c
Linear temperature profile	79.61	1.99	–	79.61	1.993	–
Piecewise linear profile heating from below	78.1	2.03	0.96	78.1	1.980	0.959
Piecewise linear profile cooling from above	42.62	2.25	0.675	42.63	2.259	0.678
Step function profile	34.3	2.305	0.815	34.27	2.226	0.813

Table 3. Values of Ma_{ci} ($i = 1$–6) for different values of Ta and N when $M_3 = 1$

Ta	N	Ma_{c1}	a_c	Ma_{c2}	a_c	Ma_{c3}	a_c	Ma_{c4}	a_c	Ma_{c5}	a_c	Ma_{c6}	a_c
0	0	79.61	1.993	78.16	1.978	42.63	2.259	34.27	2.226	130.91	1.844	56.91	2.075
	100	68.57	1.969	66.34	1.949	30.93	2.117	17.96	1.969	119.05	1.845	45.06	2.028
	183.625	59.22	1.951	56.18	1.929	20.33	2.034	**0.0**	1.905	108.84	1.847	35.02	1.939
	334.900	42.06	1.924	37.13	1.901	**0.0**	1.934			90.35	1.854	16.56	1.900
	468.234	26.67	1.906	19.58	1.890					73.82	1.865	**0.0**	1.939
	610.840	9.95	1.892	**0.0**	1.898					55.88	1.882		
	694.631	**0.0**	1.888							45.19	1.895		
	1038.800									**0.0**	1.987		
10	0	80.85	2.012	79.40	1.998	43.09	2.284	34.66	2.251	133.45	.862	57.71	2.095
	100	69.86	1.989	67.78	1.970	31.49	2.141	18.56	1.994	121.67	1.863	45.91	2.049
	186.840	60.18	1.970	57.12	1.948	20.56	2.055	0.0	1.927	110.97	1.865	35.52	2.014
	340.530	42.80	1.944	37.82	1.920	**0.0**	1.955			92.16	1.873	16.84	1.960
	476.595	27.13	1.926	19.94	1.911					75.25	1.885	**0.0**	1.921
	621.740	10.14	1.914	**0.0**	1.921					56.94	1.903		
	707.263	**0.0**	1.910							45.99	1.917		
	1055.630									**0.0**	2.015		
10^2	0	91.31	2.166	89.74	2.152	46.81	2.485	37.79	2.452	155.29	2.001	64.33	2.254
	100	80.69	2.145	78.37	2.127	35.99	2.339	23.24	2.199	143.24	2.003	52.95	2.213
	213.930	68.29	2.124	65.01	2.103	22.40	2.225	0.0	2.098	129.33	2.008	39.76	2.172
	387.710	49.06	2.101	43.67	2.078	**0.0**	2.120			107.75	2.021	19.24	2.121
	547.229	31.01	2.087	22.93	2.076					87.52	2.039	**0.0**	2.086
	713.110	11.83	2.083	**0.0**	2.105					66.00	2.676		
	813.660	**0.0**	2.085							52.69	2.090		
	1193.430									**0.0**	2.234		
10^3	0	163.11	2.971	160.43	2.952	66.36	3.605	54.48	3.541	329.35	2.718	107.81	3.086
	100	154.23	2.956	151.13	2.936	59.04	3.461	44.59	3.365	316.94	2.721	98.48	3.059
	405.813	125.98	2.924	121.10	2.896	32.34	3.135	**0.0**	3.080	277.49	2.741	68.93	2.996
	716.400	95.47	2.912	44.03	2.944	**0.0**	2.995			234.75	2.780	37.30	2.965
	1062.670	59.07	2.932	25.59	3.093					183.20	2.853	**0.0**	2.973
	1334.430	28.41	2.974	**0.0**	3.132					138.95	2.941		
	1570.140	**0.00**	3.033							96.88	3.047		
	2025.880									**0.0**	3.352		

to increase Ma_c and a_c, and thus having a stabilizing effect on the system (see Table 1). Also, in the notation of the present study, it is noted that $\mathrm{Ma}_{c4} < \mathrm{Ma}_{c3} < \mathrm{Ma}_{c2} < \mathrm{Ma}_{c4}$ indicating that the non-uniform basic temperature gradients promote instability (see Table 2).

The results obtained for the complete problem for different basic temperature profiles (i.e., Models 1–6) are tabulated in Tables 3 and 4 for $M_3 = 1$ and 2, respectively, for different values of Ta and N. A glance at the tabulated values reveals that the magnetic Rayleigh number has a destabilizing effect on the system. In fact there is a strong coupling between the magnetic Rayleigh and the Marangoni numbers. That is, when the buoyancy is predominant the surface tension effect becomes negligible and the extent to which the surface tension effect is diminished due to N depends on the form of basic temperature gradient $f(z)$, the nonlinearity of magnetization as well as on the strength of rotation. It can be seen that the surface tension effect diminishes for Ta $= 0$, 10 and 10^2 at a lesser value of N for $M_3 = 1$, while for Ta $\geq 10^3$ a similar effect is noticed for $M_3 = 2$. Nonetheless, the critical Marangoni number increases with an increase in the Taylor number and this indicates the presence of Coriolis force due to rotation is to reduce the intensity of Marangoni ferroconvection. A comparison of critical Marangoni numbers among different forms of basic temperature profiles shows that

Table 4. Values of Ma_{ci} ($i = 1–6$) for different values of Ta and N when $M_3 = 2$

Ta	N	Ma_{c1}	a_c	Ma_{c2}	a_c	Ma_{c3}	a_c	Ma_{c4}	a_c	Ma_{c5}	a_c	Ma_{c6}	a_c
0	0	79.61	1.993	78.16	1.978	42.63	2.259	34.27	2.226	130.91	1.844	56.91	2.075
	100	68.98	1.979	66.77	1.959	31.21	2.137	18.60	2.069	119.32	1.851	45.44	2.044
	194.744	58.76	1.969	55.64	1.947	19.47	2.065	**0.0**	2.071	108.19	1.860	34.41	2.019
	342.630	42.53	1.959	37.52	1.938	**0.0**	2.003			90.54	1.879	16.89	1.989
	482.545	26.85	1.957	19.42	1.952					73.49	1.904	**0.0**	1.971
	624.260	10.63	1.964	**0.0**	2.996					55.83	1.938		
	715.481	**0.0**	1.973							44.21	1.966		
	1045.52									**0.0**	2.126		
10	0	80.85	2.012	79.39	1.998	43.09	2.284	34.67	2.251	133.45	1.862	57.71	2.095
	100	70.26	1.999	68.05	1.979	31.77	2.162	19.13	2.093	121.84	1.896	46.28	2.064
	197.797	59.74	1.989	56.60	1.967	19.72	2.087	**0.0**	2.096	110.4	1.879	34.93	2.039
	348.120	43.28	1.979	38.22	1.959	**0.0**	2.025			92.35	1.898	17.18	2.011
	490.752	27.31	1.978	19.77	1.974					74.91	1.925	**0.0**	1.993
	634.641	10.85	1.986	**0.0**	2.022					56.89	1.959		
	727.742	**0.0**	1.973							44.97	1.989		
	1060.770									**0.0**	2.155		
10^2	0	91.31	2.166	89.74	2.152	46.81	2.485	37.79	2.452	155.29	2.001	64.33	2.254
	100	80.98	2.154	78.74	2.137	36.19	2.357	23.33	2.284	143.54	2.009	53.25	2.228
	223.195	68.02	2.145	64.69	2.124	21.75	2.261	**0.0**	2.295	128.83	2.023	39.35	2.202
	394.100	49.59	2.140	44.12	2.121	**0.0**	2.200			107.93	2.049	19.62	2.179
	559.742	31.22	2.147	22.68	2.148					87.05	2.084	**0.0**	2.171
	720.941	12.82	2.165	**0.0**	2.221					66.05	2.131		
	830.388	**0.0**	2.185							51.33	2.172		
	1185.020									**0.0**	2.379		
10^3	0	163.11	2.971	160.43	2.953	66.36	3.605	54.48	3.541	329.35	2.718	107.81	3.086
	100	154.34	2.963	151.27	2.943	59.08	3.469	44.63	3.348	316.99	2.726	98.55	3.068
	392.315	127.60	2.950	122.89	2.923	33.68	3.189	**0.0**	3.358	279.22	2.758	70.41	3.034
	711.215	96.29	2.959	88.32	2.935	**0.0**	3.102			234.71	2.814	37.76	3.033
	1054.550	59.65	3.003	43.28	3.035					181.78	2.909	**0.0**	3.076
	1287.194	32.69	3.054	**0.0**	3.274					141.94	3.001		
	1546.810	**0.0**	3.136							92.15	3.138		
	1939.690									**0.0**	3.423		

$\mathrm{Ma}_{c4} < \mathrm{Ma}_{c3} < \mathrm{Ma}_{c6} < \mathrm{Ma}_{c2} < \mathrm{Ma}_{c1} < \mathrm{Ma}_{c5}$

for both ferro and non-ferrofluids. That is, the system is most unstable (i. e., augments convection) in the case of step function type of basic temperature gradient because the jump in temperature occurs nearer the less restrictive free surface, whereas the inverted parabolic type of basic temperature gradient makes the system more stable. Thus, it is possible to control Marangoni ferroconvection effectively by the choice of different forms of basic temperature gradients including the effect of Coriolis force due to rotation. From the tables it is also noted that the critical wavenumber decreases with the increase in the value of N but the decreasing trend with N is not so significant, but an increase in the value of Ta significantly increases the critical wavenumber. Hence the effect of an increase in Ta and decrease in N is to reduce the size of the convection cells.

The variation of critical Marangoni numbers Ma_{c2}, Ma_{c3} and Ma_{c4} as a function of thermal depth ε showed that all of them decrease at first to a minimum value, and then increase steadily with increasing ε. The critical thermal depth ε_c obtained numerically for piecewise linear and step function temperature profiles are listed in Table 5 for different values of Ta and N. It can be seen that an increase in Ta is to increase ε_c, while an increase in N is to decrease the same.

Table 5. Values of ε_c for different values of Ta and N when $M_3 = 1$

Nature of temperature Profiles	Ta	$N = 0$ ε_c	$N = 100$ ε_c	$N = 150$ ε_c
Piecewise linear profile for heating from below	1	0.959	0.948	0.941
	10	0.960	0.949	0.943
	10^2	0.964	0.955	0.951
	10^3	0.980	0.977	0.976
Piecewise linear profile for cooling from above	1	0.678	0.617	0.592
	10	0.680	0.620	0.595
	10^2	0.701	0.656	0.622
	10^3	0.787	0.760	0.745
Step function profile	1	0.812	0.749	0.724
	10	0.814	0.751	0.726
	10^2	0.827	0.770	0.745
	10^3	0.877	0.853	0.837

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