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Effect of temperature/gravity modulation on the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum

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Abstract

The effect of time-periodic temperature/gravity modulation at the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum is investigated by making a linear stability analysis. The Venezian approach is adopted in arriving at the critical Rayleigh and wave numbers for small amplitude temperature/gravity modulation. The temperature modulation is shown to give rise to sub-critical motion and gravity modulation leads to delayed convection. An asymptotic analysis is also presented for small and large frequencies. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Magneto-convection is concerned with the effect of a transverse magnetic field at the onset of convection in an electrically conducting fluid with suspended particles. In these fluids with suspended particles we have to consider the conservation of angular momentum in addition to the conservation of linear momentum.

Effective control of magneto-convection in fluids with internal angular momentum is important and this can be achieved by maintaining a non-uniform temperature gradient (see Refs. [1,2]). Such a temperature gradient may be generated by

(i) an appropriate type of heating or cooling at the boundaries,
(ii) injection/suction of fluid at the boundaries,
(iii) an appropriate distribution of the heat source, and
(iv) radiative heat transfer.

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These methods of controlling convection are mainly concerned with space-dependent temperature gradients. However, in practice, the non-uniform temperature gradient finds its origin in transient heating or cooling at the boundaries. Hence the basic temperature profile depends explicitly on position and time. This problem, called the thermal modulation problem, involves the solution of the energy equation under suitable time-dependent boundary conditions.

The effect of modulation on the stability of the flow of clean fluids between rotating cylinders was investigated experimentally [3]. In this experiment, the Newtonian fluid was confined to the narrow gap between two cylinders, with the outer cylinder held fixed while the inner cylinder was given an angular speed $\Omega + \Delta\Omega \cos \omega t$. Donnelly [3] found that the onset of instability was delayed by the modulation of the angular speed of the inner cylinder. He [3] interpreted his results as being due to a viscous wave penetrating the fluid and thereby altering the profile from an unstable one to a stable one. Since the problems of Taylor stability and Benard stability are very similar, Venezian [4] investigated the stability of a horizontal, viscous clean Newtonian fluid layer by considering thermally modulated boundaries. The paper by Rosenblat [5] is yet another effort in this direction for not so very small amplitudes. This is different from the approach of Venezian [4].

Another important class of natural convection problems is concerned with the difficulty in avoiding convection in the earth's gravitational field even when the basic temperature gradient is uniform and interfacial instabilities can be ignored. It is common knowledge that many extra-terrestrial experiments under microgravity conditions have been conducted to eliminate convection. Terrestrially simulated microgravity environment has also been considered by Ostrach [6], Knabe and Eilers [7], Alexander and Laudquist [8] and Alexander [9] in situations involving clean Newtonian fluids.

In the case of natural and simulated microgravity studies, time-dependent acceleration of sufficient amplitude due to manoeuvers and inherent mechanical vibrations lead to convection. This is called the g-jitter effect. Some of the important papers in this direction for clean Newtonian fluids are [10–13].

The unmodulated Rayleigh–Benard situation in fluids with internal angular momentum [14–19] has been investigated by many authors [1,2,20–29]. Inspite of immense possibility for practical applications, temperature/gravity modulation of convection in these fluids has not been considered. This is also true of the magneto-convection problem. In view of the mathematically challenging nature of this magneto-convection problem with modulation and also due to the numerous applications, we study the effect of temperature/gravity modulation at the onset of magneto-convection in a weak electrically conducting fluid with internal angular momentum. It appears that experimental work relating to the present paper has not been performed.

2. Mathematical formulation

Consider a layer of a Boussinesquian weak-electrically conducting fluid with internal angular momentum confined between two infinite horizontal walls distant 'L' apart (see Fig. 1). The uniform magnetic field is directed along the Z-axis. A cartesian coordinate system is taken with origin in the lower boundary and Z-axis vertically upwards. The basic governing equations are:

Continuity equation

$$\nabla \cdot \boldsymbol{q} = 0, \tag{1}$$

Conservation of linear momentum

$$\rho_{\mathrm{R}}\left[\frac{\partial \boldsymbol{q}}{\partial t} + (\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right] = -\nabla p - \rho g\hat{k} + (2\zeta + \eta)\nabla^2\boldsymbol{q} + \zeta\nabla\times\boldsymbol{\omega} + \boldsymbol{J}\times\boldsymbol{B}, \tag{2}$$

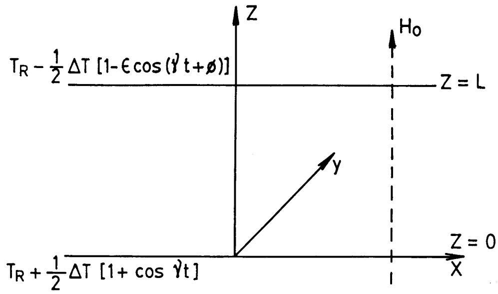

Fig. 1. Physical configuration.

Conservation of angular momentum

$$\rho_R I\left[\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{q}\cdot\nabla)\boldsymbol{\omega}\right] = (\lambda' + \eta')\nabla(\nabla\cdot\boldsymbol{\omega}) + \eta'\nabla^2\boldsymbol{\omega} + \zeta(\nabla\times\boldsymbol{q} - 2\boldsymbol{\omega}), \tag{3}$$

Conservation of energy

$$\frac{\partial T}{\partial t} + (\boldsymbol{q}\cdot\nabla)T = \frac{\beta}{\rho_R C_v}(\nabla\times\boldsymbol{\omega})\cdot\nabla T + \chi\nabla^2 T, \tag{4}$$

Equation of state

$$\rho = \rho_R[1 - \alpha(T - T_R)]\,, \tag{5}$$

Constitutive equations

$$\boldsymbol{J} = \sigma(\boldsymbol{q}\times\boldsymbol{B}), \tag{6}$$

$$\boldsymbol{B} = \mu\boldsymbol{H}, \tag{7}$$

where $\boldsymbol{q}$ is the velocity, $\boldsymbol{\omega}$ is the spin, T is the temperature, $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{J}$ is the current density, $\boldsymbol{B}$ is the magnetic induction vector, p is the pressure, ρ is the density, ρ_R is the density of the fluid at reference temperature $T = T_R$, g is the acceleration due to gravity, ζ is the coupling viscosity coefficient or vortex viscosity, η is the shear kinematic viscosity coefficients, I is the moment of inertia, λ' and η' are the bulk and shear spin velocity coefficients, β is the micropolar heat conduction coefficient, C_v is the specific heat, χ is the thermal conductivity, α is the coefficient of thermal expansion, σ is the electrical conductivity and μ is the magnetic permeability.

For a weakly electrically conducting fluid the Lorentz force $\boldsymbol{J}\times\boldsymbol{B}$, on using the constitutive Eqs. (6) and (7), can be written as

$$\boldsymbol{J}\times\boldsymbol{B} = -\,\mu^2\sigma H_0^2\boldsymbol{q}, \tag{8}$$

where H_0 is the applied transverse magnetic field.

The no-spin condition is assumed for microrotation at the stress free isothermal boundary. Due to the Hartmann formulation, the magnetic field boundary conditions do not have any influence on the convection. The lower and upper walls are subjected to the temperatures

$$T(0, t) = T_R + \tfrac{1}{2}\Delta T\,[1 + \varepsilon\cos\gamma t] \tag{9}$$

and

$$T(L, t) = T_R - \tfrac{1}{2}\Delta T\,[1 - \varepsilon\cos(\gamma t + \varphi)], \tag{10}$$

respectively, where ε is the small amplitude, γ is the frequency and φ is the phase angle. Temperature modulation comes into picture in the present problem through these boundary conditions (9) and (10).

2.1. Basic state

The basic state of the fluid is quiescent and is described by

$$\boldsymbol{q}_{\mathrm{H}} = \boldsymbol{0}, \quad \boldsymbol{\omega}_{\mathrm{H}} = \boldsymbol{0}, \quad T = T_{\mathrm{H}}(z,t), \quad p = p_{\mathrm{H}}(z), \quad \rho = \rho_{\mathrm{H}}(z,t).$$

The temperature T_{H}, pressure p_{H}, and density ρ_{H} satisfy

$$\frac{\partial T_{\mathrm{H}}}{\partial t} = \chi \frac{\partial^2 T_{\mathrm{H}}}{\partial z^2}, \tag{11}$$

$$-\frac{\partial p_{\mathrm{H}}}{\partial z} = \rho_{\mathrm{H}} g \tag{12}$$

and

$$\rho_{\mathrm{H}} = \rho_0[1 - \alpha(T_{\mathrm{H}} - T_{\mathrm{R}})]. \tag{13}$$

The solution of Eq. (11) that satisfies the thermal boundary conditions (9) and (10) is

$$T_{\mathrm{H}} = T_{\mathrm{R}} + \frac{\Delta T}{2L}(L - 2Z) + \varepsilon\, \mathrm{Re}\{[a(\lambda)\mathrm{e}^{\lambda z/L} + a(-\lambda)\mathrm{e}^{-\lambda z/L}]\mathrm{e}^{-\mathrm{i}\gamma t}\}, \tag{14}$$

where

$$\lambda = (1 - \mathrm{i})\left(\frac{\gamma L^2}{2\chi}\right)^{1/2} \tag{15}$$

and

$$a(\lambda) = \frac{\Delta T}{2}\left[\frac{\mathrm{e}^{-\mathrm{i}\varphi} - \mathrm{e}^{-\lambda}}{\mathrm{e}^{\lambda} - \mathrm{e}^{-\lambda}}\right]$$

and Re stands for the real part.

2.2. Linear stability analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\begin{aligned} \boldsymbol{q} &= \boldsymbol{q}_{\mathrm{H}} + \boldsymbol{q}', \\ \boldsymbol{\omega} &= \boldsymbol{\omega}_{\mathrm{H}} + \boldsymbol{\omega}', \\ \rho &= \rho_{\mathrm{H}}(z) + \rho', \\ p &= p_{\mathrm{H}}(z) + p', \\ T &= T_{\mathrm{H}} + \theta. \end{aligned} \tag{16}$$

The prime indicates that the quantities are infinitesimal perturbations.

Substituting Eq. (16) into Eqs. (1)–(5), using Eq. (8) in Eq. (2), and using the basic state solution, we get linearized equations governing the infinitesimal perturbations in the form

$$\nabla\cdot\boldsymbol{q}' = 0, \tag{17}$$

$$\rho_{\mathrm{R}}\left[\frac{\partial \boldsymbol{q}}{\partial t}\right] = -\nabla p' - \rho' g\hat{k} + (2\zeta+\eta)\nabla^2\boldsymbol{q}' + \zeta\nabla\times\boldsymbol{\omega}' - \mu^2\sigma H_0^2\boldsymbol{q}', \tag{18}$$

$$\rho_{\mathrm{R}} I\left[\frac{\partial \boldsymbol{\omega}'}{\partial t}\right] = (\lambda'+\eta')\nabla(\nabla\cdot\boldsymbol{\omega}') + \eta'\nabla^2\boldsymbol{\omega}' + \zeta(\nabla\times\boldsymbol{q}' - 2\boldsymbol{\omega}'), \tag{19}$$

$$\frac{\partial\theta}{\partial t} = \left(\frac{\partial T_{\mathrm{H}}}{\partial z}\right)\left[\frac{\beta\Omega_z}{\rho_{\mathrm{R}} C_v} - W\right] + \chi\nabla^2\theta, \tag{20}$$

and

$$\rho' = -\alpha\rho_{\mathrm{R}}\theta. \tag{21}$$

The perturbation Eqs. (17)–(21) are non-dimensionalized using the following definitions:

$$(x^*, y^*, z^*) = \left(\frac{x}{L}, \frac{y}{L}, \frac{z}{L}\right),\quad t^* = \frac{\chi t}{L^2},\quad W^* = \frac{W'}{\chi/L},$$

$$\boldsymbol{\omega}^* = \frac{\boldsymbol{\omega}}{\chi/L^2},\quad \Omega^* = \frac{[\nabla\times\boldsymbol{\omega}']z}{\chi/L^3},\quad \theta^* = \frac{\theta}{\Delta T}. \tag{22}$$

Operating curl twice on Eq. (18) and using Eq. (21), operating curl once on Eq. (19) and non-dimensionalizing the two resulting equations and Eq. (20), using Eq. (22), we get

$$\frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2 W) = R\nabla_1^2\theta + (1+N_1)\nabla^4 W + N_1\nabla^2\Omega_z - M^2\nabla^2 W, \tag{23}$$

$$\frac{N_2}{\Pr}\frac{\partial\Omega_z}{\partial t} = N_3\nabla^2\Omega_z - N_1\nabla^2 W - 2N_1\Omega_z, \tag{24}$$

$$\frac{\partial\theta}{\partial t} = \left(\frac{\partial T_0}{\partial z}\right)[N_5\Omega_z - W] + \nabla^2\theta, \tag{25}$$

where the asterisks have been dropped for simplicity and

$$R = \frac{\alpha g\Delta T\rho_{\mathrm{R}} L^3}{(\zeta+\eta)\chi}\ \text{(Rayleigh number)},\quad \Pr = \frac{(\zeta+\eta)}{\rho_{\mathrm{R}}\chi}\ \text{(Prandtl number)},$$

$$M^2 = \frac{\mu^2\sigma H_0^2 L^2}{(\zeta+\eta)}\ \text{(Hartmann number)},\quad N_1 = \frac{\zeta}{(\zeta+\eta)}\ \text{(Coupling parameter)},$$

$$N_2 = \frac{I}{L^2}\ \text{(Inertia parameter)},\quad N_3 = \frac{\eta'}{(\zeta+\eta)L^2}\ \text{(Couple stress parameter)},$$

$$N_5 = \frac{\beta}{\rho_0 C_v L^2}\ \text{(Micropolar heat conduction parameter)}.$$

In Eq. (26), $(\partial T_0/\partial z)$ is the non-dimensional form of $(\partial T_{\mathrm{H}}/\partial z)$, where

$$\frac{\partial T_0}{\partial z} = -1 + \varepsilon\,\mathrm{Re}\{[A(\lambda)\mathrm{e}^{\lambda z} + A(-\lambda)\mathrm{e}^{-\lambda z}]\mathrm{e}^{-\mathrm{i}\gamma t}\} \tag{26}$$

and

$$A(\lambda) = \frac{\lambda}{2}\left[\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right].$$

Eqs. (24)–(26) are solved subject to the conditions

$$W = \frac{\partial^2 W}{\partial z^2} = \Omega_z = \theta = 0 \quad \text{at } z = 0, 1. \tag{27}$$

Eliminating Ω_z and θ from Eqs. (23)–(25), we get an equation for $\boldsymbol{W}$ in the form

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2\left[\left(\frac{N_2}{\mathrm{Pr}}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1\right)\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t} - (1+N_1)\nabla^2 + M^2\right) + N_1^2\nabla^2\right]W$$
$$= R\nabla_1^2\left(\frac{\partial T_0}{\partial z}\right)\left[-N_5N_1\nabla^2 - \frac{N_2}{\mathrm{Pr}}\frac{\partial}{\partial t} + N_3\nabla^2 - 2N_1\right]W. \tag{28}$$

In dimensionless form, the velocity boundary conditions for solving Eq. (28) are obtainable from Eqs. (23)–(25) and Eq. (27) in the form

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = 0 \quad \text{at } z = 0, 1. \tag{29}$$

3. Stability analysis

We now seek the eigenfunction W and eigenvalues R of Eq. (28) for the basic temperature distribution (26) that departs from the linear profile $(\partial T_0/\partial z) = -1$ by quantities of order ε. Thus, the eigenvalues of the present problem differ from those of the ordinary Benard convection by quantities of order ε. We seek the solution of Eq. (28) in the form:

$$(R, W) = (R_0, W_0) + \varepsilon(R_1, W_1) + \varepsilon^2(R_2, W_2) + \text{- - - -} \tag{30}$$

The expansion (30) is substituted into Eq. (28) and the powers of ε are separated. Following Venezian [4], we can obtain R_0, R_1 and R_2 in the form:

$$R_0 = \frac{(1+N_1)N_3K_1^8 + N_1(2+N_1)K_1^6 + M^2[N_3K_1^2 + 2N_1]K_1^4}{a^2[(N_3 - N_5N_1)K_1^2 + 2N_1]}, \tag{31}$$

$$R_1 = 0 \tag{32}$$

and

$$R_2 = -\frac{R_0^2a^2}{2}\,\mathrm{Re}\sum\frac{|A_2|^2\,|B_n(\lambda)|^2}{|L_1(\gamma,n)|^2}\left[\frac{L_1(\gamma,n) + L_1^*(\gamma,n)}{2}\right], \tag{33}$$

where

$$A_2 = N_5N_1K_n^2 + i\gamma\frac{N_2}{\mathrm{Pr}} - N_3K_n^2 - 2N_1,$$

$$B_n(\lambda) = A(\lambda)g_{n1}(\lambda) + A(-\lambda)g_{n1}(-\lambda) = \frac{-2n\pi^2\lambda^2[e^{\lambda} - e^{-\lambda} + (-1)^n(e^{-\lambda-i\varphi} - e^{\lambda-i\varphi})]}{(e^{\lambda} - e^{-\lambda})[\lambda^2 + (n+1)^2\pi^2][\lambda^2 + (n-1)^2\pi^2]},$$

$$L_1(\gamma, n) = \left\{X_3(-K_n^4X_1 - \gamma^2K_n^2X_2 - R_0a^2X_3) - \frac{\gamma^2N_2}{\mathrm{Pr}}\left(K_n^2X_1 - K_n^4X_2 - \frac{R_0a^2N_2}{\mathrm{Pr}}\right)\right\}$$
$$+ \mathrm{i}\gamma\left\{X_3\left(K_n^2X_1 - K_n^4X_2 - \frac{R_oa^2N_2}{\mathrm{Pr}}\right) + \frac{N_2}{\mathrm{Pr}}(-K_n^4X_1 - \gamma^2K_n^2X_2 - R_0a^2X_3)\right\},$$

$$X_1 = (N_3K_n^2 + 2N_1)((1+N_1)K_n^2 + M^2) - \frac{\gamma^2N_2}{\mathrm{Pr}^2} - N_1^2K_n^2,$$

$$X_2 = -\frac{1}{\mathrm{Pr}}[N_2((1+N_1)K_n^2 + M^2) + N_3K_n^2 + 2N_1],$$

$$X_3 = N_5N_1K_n^2 - N_3K_n^2 - 2N_1, \quad K_n^2 = n^2\pi^2 + a^2$$

and $L_1^*(\gamma, n)$ are the conjugates of $L_1(\gamma, n)$, respectively.

4. Minimum Rayleigh number for convection

The value of R obtained by this procedure is the eigenvalue corresponding to the eigenfunction $\boldsymbol{W}$ which, though oscillating, remains bounded in time. Since R is a function of the horizontal wave number $\boldsymbol{a}$ and the amplitude of perturbation ε, we have

$$R(a, \varepsilon) = R_0(a) + \varepsilon^2R_2(a) + \text{- - - -}. \tag{34}$$

It was shown by Alexander and Lundquist [8] that the critical value is determined to $O(\varepsilon^2)$ by evaluating R_0 and R_2 at $a = a_0$. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a = a_2$ minimizes R_2. To evaluate the critical value of R_2 (denoted by R_{2c}) one has to substitute $a = a_0$ in R_2, where a_0 is the value at which R_0 given by Eq. (31) is minimum.

We now evaluate R_{2c} for three cases:

(a) When the oscillating field is symmetric so that wall temperatures are modulated in phase with $\varphi = 0$. In this case,

$B_n(\lambda) = b_n$ or 0 accordingly as n is even or odd.

(b) When the wall temperature field is antisymmetric corresponding to out-of-phase modulation with $\varphi = \pi$. In this case,

$B_n(\lambda) = 0$ or b_n accordingly as n is even or odd.

(c) When only the temperature of the bottom wall is modulated, the upper plate being held at constant temperature with $\varphi = -\mathrm{i}\infty$. In this case,

$B_n(\lambda) = \dfrac{b_n}{2}$ for integer values of n.

The b_n's are given by

$$b_n = \frac{-4n\pi^2\lambda^2}{[\lambda^2 + (n+1)^2\pi^2][\lambda^2 + (n-1)^2\pi^2]}. \tag{35}$$

The variable λ defined in Eq. (15), in terms of the dimensionless frequency reduces to

$$\lambda = (1 - \mathrm{i})\left(\frac{\gamma}{2}\right)^{1/2}$$

and thus

$$|b_n|^2 = \frac{16n^2\pi^4\gamma^2}{[\gamma^2 + (n+1)^4\pi^4][\gamma^2 + (n-1)^4\pi^4]}. \tag{36}$$

Hence from Eq. (34) and using the above expression of $B_n(\lambda)$, we obtain the following expression for R_{2c}:

$$R_{2c} = -\frac{R_0^2 a_0^2}{2}\sum\frac{|A_2|^2|B_n(\lambda)|^2[L_1(\gamma,n) + L_1^*(\gamma,n)]}{2|L_1(\gamma,n)|^2}. \tag{37}$$

In Eq. (37) the summation extends over even values of n for case (a), odd values of n for case (b) and for all integer values of n for case (c). The infinite series (37) converges rapidly.

5. Limiting cases

The physical significance of R_{2c} as a function of γ can be well understood by examining the limiting cases for very small and very large values of γ.

***Case* (i)**: Very small frequency ($\gamma \ll 1$):

For $\gamma \ll 1$, the expression for R_{2c} (writing the summation for $n = 1$ and $n > 1$) reduces to

$$R_{2c} = R_{\mathrm{Pr}} - \frac{8R_{0c}^2 a^2 n^2}{(n^2-1)^4\pi^4}|A_2|^2 C_n^*\gamma^2, \tag{38}$$

where

$$R_{\mathrm{Pr}} = -\frac{R_{0c}^2 a_c^2}{2}C_1^*,$$

where

$$C_1^* = -(N_1N_5K_1^2 - N_3K_1^2 - 2N_1)K_1^2X_2 - \frac{(N_2/\mathrm{Pr})\,[K_1^2Y_1 - K_1^4X_2 - (R_0a^2N_2/\mathrm{Pr})]}{[K_1^2Y_1 - K_1^4X_2 - (R_0a^2N_2/\mathrm{Pr})]},$$

$$C_n^* = \frac{1}{(N_1N_5K_n^2 - N_3K_n^2 - 2N_1)\,[-K_n^4Y_n - R_0a^2(N_1N_5K_n^2 - N_3K_n^2 - 2N_1)]}\ (n > 1)$$

and

$$Y_n = (N_3K_n^2 + 2N_1)((1+N_1)K_n^2 + M^2) - N_1^2K_n^2.$$

In the case of symmetric modulation, the sum extends over only even values of n so that the expression for R_{2c} reduces to

$$R_{2c} = -\frac{8R_{0c}^2 a^2 n^2}{(n^2-1)^4\pi^4}|A_2|^2 C_n^*\gamma^2, \tag{39}$$

which is independent of Pr. The effect thus appears only for large values of γ.

In the case of antisymmetric and also for the case in which only lower wall temperature is modulated, R_{2c} is given by Eq. (38) with the convention that in the former case the sum extends over only odd values of n and in the latter case for all values of n.

***Case* (ii)**: Very large frequency ($\gamma \to \infty$):

As $\gamma \to \infty$, R_{2c} tends to zero, so that the effect of modulation disappears altogether.

6. Subcritical instability

The critical value of Rayleigh number R_c is determined to order of ε^2, by evaluating R_{0c} and R_{2c} and is of the form

$$R_c = R_{0c} + \varepsilon^2 R_{2c}, \tag{40}$$

where R_{0c} and R_{2c} can be obtained from Eqs. (31) and (33), respectively.

If R_{2c} is positive, supercritical instability exists and R_c has a minimum at $\varepsilon = 0$. When R_{2c} is negative, subcritical instabilities are possible. In this case, we have from Eq. (40)

$$\varepsilon^2 < \frac{R_{0c}}{R_{2c}}. \tag{41}$$

Now we can calculate the maximum range of ε, by assigning values to the physical parameters involved in the above condition. Thus, the range of the amplitude of modulation, which causes subcritical instabilities in different physical situations, can be explained.

We have thus far investigated the effect of thermal modulation on the onset of convection. Reiterating what was mentioned in the introduction to the paper, we now address ourselves to the important problem of gravity modulation in the next section using the analysis of the previous sections.

7. Gravity modulation

Under the influence of a periodically varying vertical gravity field

$$g = g_0(1 + \delta \cos \gamma_1 t), \tag{42}$$

where g_0 is the mean gravity, δ is the small amplitude of gravity modulation, γ_1 is the frequency and t is the time. The time fluctuating gravity is referred to as g-jitter.

The governing equations for the Boussinesquian, weak electrically conducting fluid with internal angular momentum are essentially the same Eqs. (1)–(8) but with 'g' given by Eq. (42).

The basic state of the fluid is quiescent and is described by

$$\boldsymbol{q}_H = \boldsymbol{0}, \quad \boldsymbol{\omega}_H = \boldsymbol{0}, \quad \rho = \rho_H(z), \quad T = T_H(z), \quad p = P_H(z). \tag{43}$$

This clearly differs from the one in the thermally modulated case. Pressure p_H and the density ρ_H satisfy Eqs. (12) and (13) whereas T_H satisfies

$$\frac{d^2 T_H}{dz^2} = 0. \tag{44}$$

Following the analysis in the previous section, the linearized perturbation equations, on using Eq. (42) and non-dimensionalization, yield

$$\frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2 W) = R(1 + \delta \cos \gamma_1 t)\nabla_1^2 \theta + (1 + N_1)\nabla^4 W + N_1 \nabla^2 \Omega_z - M^2 \nabla^2 W, \tag{45}$$

$$\frac{N_2}{\Pr}\frac{\partial \Omega_z}{\partial t} = N_3 \nabla^2 \Omega_z - N_1 \nabla^2 W - 2N_1 \Omega_z, \tag{46}$$

$$\frac{\partial \theta}{\partial t} = W - N_5 \Omega_z + \nabla^2 \theta, \tag{47}$$

where the parameters N_1, N_2, N_3, N_5, R, Pr and M^2 are as in the earlier sections. Eq. (47) is essentially Eq. (25).

Following the analysis of Venezian [8] for the velocity boundary conditions (29) we obtain R_0 and R_2 in the form,

$$R_0 = \frac{(1+N_1)N_3K_1^8 + N_1(2+N_1)K_1^6 + M^2[N_3K_1^2 + 2N_1]K_1^4}{a^2[(N_3 - N_5N_1)K_1^2 + 2N_1]} \tag{48}$$

and

$$R_2 = \frac{R_0^2 a^2}{2}\,\mathrm{Re}\sum \frac{|A_2|^2}{|L_1(\gamma_1, n)|^2}\left[\frac{L_1(\gamma_1, n) + L_1^*(\gamma_1, n)}{2}\right], \tag{49}$$

where

$$L_1(\gamma, n) = \left\{ -X_3(-K_n^4X_1 - \gamma^2K_n^2X_2 - R_0a^2X_3) + \frac{\gamma^2N_2}{\mathrm{Pr}}\left(K_n^2X_1 - K_n^4X_2 - \frac{R_0a^2N_2}{\mathrm{Pr}}\right)\right\}$$
$$+ \mathrm{i}\gamma\left\{ -X_3\left(K_n^2X_1 - K_n^4X_2 - \frac{R_0a^2N_2}{\mathrm{Pr}}\right) - \frac{N_2}{\mathrm{Pr}}(-K_n^4X_1 - \gamma^2K_n^2X_2 - R_0a^2X_3)\right\}.$$

In Eq. (49), $L_1^*(\gamma, n)$ are the conjugates of $L_1(\gamma, n)$, respectively.

8. Discussion and conclusion

In the paper we make an analytical study of the effects of temperature/gravity modulation and transverse magnetic field at the onset of convection in weak electrically conducting fluids with suspended particles. We first discuss the results in respect of temperature modulation.

In the case of thermal modulation the amplitude is small compared with the imposed steady temperature difference. The validity of the results obtained here depends on the value of the modulating frequency γ. When $\gamma \ll 1$, the period of modulation is large and hence the disturbance grows to such an extent as to make finite amplitude effects important. When $\gamma \to \infty$, $R_{2c} \to 0$, thus the effect of modulation becomes small. In view of this, we choose only moderate values of γ in our present study. Before we embark on the discussion of results depicted by the graphs, we must note that the presence of suspended particles in the otherwise clean fluid is to increase its viscosity. This follows from the well-known Einstein relation for suspended particles

$$\mu = \mu_0(1 + 2.5\alpha\varphi),$$

where μ and μ_0 are the viscosities of suspension (i.e. clean fluid + suspended particles) and clean fluid, respectively, α is the shape factor and φ is the volume fraction of the suspended particles. In view of this we consider values of Prandtl number of unclean fluids higher than those of clean fluids. We now discuss the results arrived at in the paper.

Fig. 2 is the plot of R_{2c} versus γ for different values of Prandtl number Pr and fixed value of Hartmann number M^2, in respect of modulation in-phase. We observe that as Pr increases, R_{2c} becomes more and more negative. We can infer from this that the effect of increasing the concentration of the suspended particles is to destabilize the system. This means that fluids with suspended particles are more vulnerable than clean fluids to destabilization by modulation. It is appropriate to note here that Pr does not affect the R_0-part of R (see

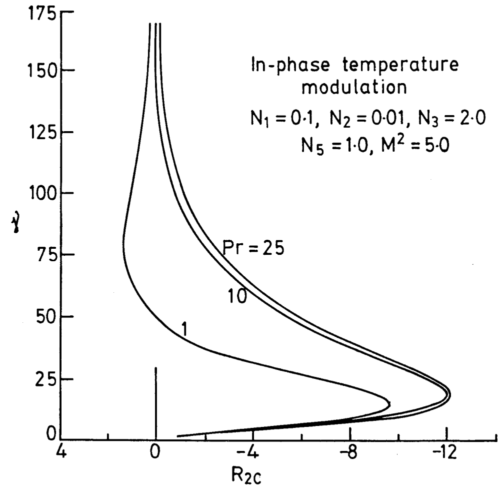

Fig. 2. Plot of R_{2c} versus frequency of modulation γ for different values of Prandtl number Pr.

Table 1
Variation of R_{0c}, R_{2c} for $M^2 = 5.0$ (In-phase modulation)

N_1	N_2	N_3	N_5	R_{0c}	$\mathrm{Pr} = 1, \gamma = 15$ R_{2c}	$\mathrm{Pr} = 5, \gamma = 20$ R_{2c}
0.1	0.01	2.0	1.0	991.21	− 9.7195	− 11.7070
0.5				1585.10	− 15.3590	− 17.6107
1.0				2855.07	− 25.9921	− 28.9214
1.5				5712.08	− 43.3225	− 47.4542
0.1	0.01	2.0	1.0	991.21	− 9.7195	− 11.7070
	0.1			991.21	− 9.5039	− 11.6549
	0.5			991.21	− 8.4460	− 11.4052
	1.0			991.21	− 6.9337	− 11.0729
0.1	0.01	2.0	1.0	991.21	− 9.7195	− 11.7070
		4.0		966.15	− 9.4878	− 11.4187
		6.0		958.04	− 9.4124	− 11.3244
		8.0		954.03	− 9.3751	− 11.2778
		10.0		951.64	− 9.3529	− 11.2499
0.1	0.01	2.0	0.5	965.96	− 9.4733	− 11.4121
			1.0	991.21	− 9.7195	− 11.7070
			1.5	1071.81	− 9.9789	− 12.0210
			2.0	1045.88	− 10.2524	− 12.3515

Eqs. (30)–(33)). It affects only R_2. R_0 is the Rayleigh number of the unmodulated system. It is also observed that for low concentration of the suspended particles supercritical motion is possible and for high concentration only subcritical motion is possible. Thus, in the case of fluids with suspended particles subcritical motions are more probable than supercritical motion. Table 1 also bears out this fact.

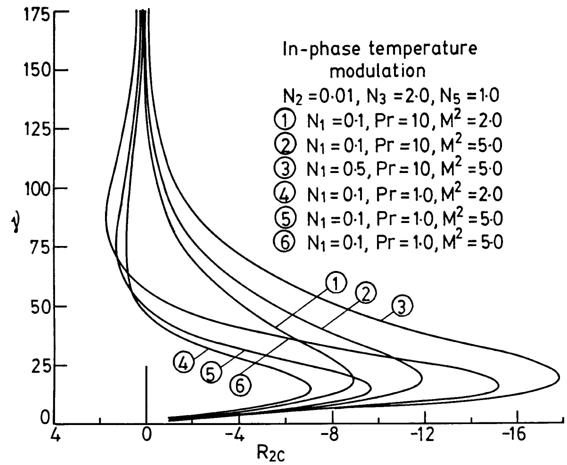

Fig. 3. Plot of R_{2c} versus γ for different values of coupling parameter N_1, Prandtl number Pr and Hartmann number M^2.

Fig. 3 is the plot of R_{2c} versus γ for different values of coupling parameter N_1, Pr and M^2, in the case of in-phase modulation. Increase in N_1 implies increase in the concentration of suspended particles. In the case of fluids with suspended particles part of the energy is consumed by these particles in forming the gyrational velocity. Hence in the figure we observe that as N_1 increases R_{2c} becomes more and more negative. This essentially reiterates the conclusion in the context of Fig. 2. It is interesting to note that for a given value of N_1, R_{2c} decreases for small values of γ and increases for moderate values of γ. Thus small values of γ destabilize and moderate values of γ stabilize the system. This is due to the fact that when the frequency of modulation is low, the effect of modulation on the temperature field is felt throughout the fluid layer. If the plates are modulated in-phase, the temperature profile consists of the steady straight line section plus a parabolic profile which oscillates in time. As the amplitude of the modulation increases, the parabolic part of the profile becomes more and more significant. It is known that a parabolic profile is subject to finite amplitude instabilities so that convection occurs at lower Rayleigh numbers than those predicted by the linear theory. Fig. 3 also gives the effect of Hartmann number on modulation. Hartmann number is the ratio of Lorentz force to viscous force and is thus a measure of the relative importance of the two forces (see Ref. [30]). We observe from the figure that increase in M^2 is to make R_{2c} more and more negative. In making conclusions from the figure, we should also recollect that N_1 and M^2, unlike Pr, influence R_{0c}. We find that R_{0c} increases with increases in N_1 and M^2. This important result is documented in Table 1. From the table it is clear that the increase in R_{0c} with N_1 is more marked than that of $|R_{2c}|$ with N_1.

Fig. 4 is the plot of R_{2c} versus γ for different values of inertia parameter N_2, Pr and M^2, in the case of in-phase modulation. Increase in N_2 is representative of the increase in inertia of the fluid due to the suspended particles. Thus, as is to be expected, we find that as N_2 increases R_{2c} becomes less and less negative thereby stabilizing the system. Since N_2 essentially arises with the acceleration term, it does not have any influence on R_{0c}. It influences only R_{2c}.

Fig. 5 is the plot of R_{2c} versus γ for different values of couple stress parameter N_3, Pr and M^2, in the case of in-phase modulation. The role played by the shear stress in the conservation of linear momentum is played by couple stress in the conservation of angular momentum equation. Increase in N_3 signifies decrease in gyrational velocities. Hence, as N_3 increases, we observe that R_{2c} becomes less and less negative. Table 1 gives the effect of N_3 on R_{0c}. Clearly R_{0c} changes more markedly than R_{2c} with N_3.

Fig. 6 is the plot of R_{2c} versus γ for different values of micropolar heat conduction parameter N_5, Pr and M^2, in case of in-phase modulation. An increase in N_5 implies that the heat induced into the system also

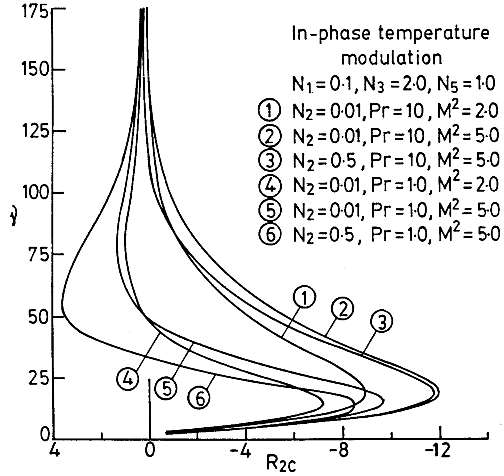

Fig. 4. Plot of R_{2c} versus γ for different values of inertia parameter N_2, Pr and M^2.

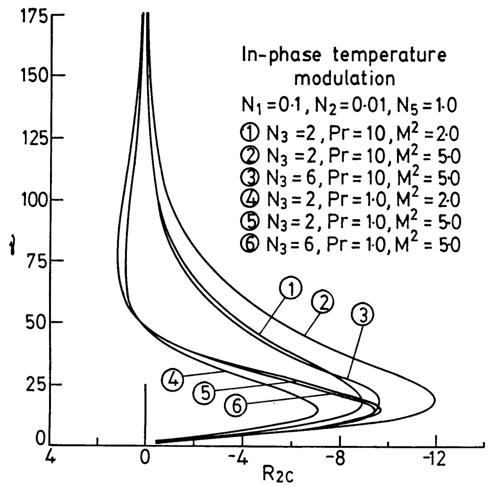

Fig. 5. Plot of R_{2c} versus γ for different values of couple stress parameter N_3, Pr and M^2.

increases resulting in reduced heat transfer from bottom to top. As a result, we find from the figure that as N_5 increases R_{2c} becomes more and more negative. Further, Table 1 shows a more marked influence of N_5 on R_{0c} than on R_{2c}.

Fig. 7 is a plot of R_{2c} versus γ for different values of Pr and fixed value of other parameters in respect of out-of-phase modulation. We find that even though R_{2c} decreases with increase in Pr it does not become negative. Thus subcritical motion is ruled out in the case of out-of-phase modulation. Why this is so is explained in one of the succeeding paragraphs.

We now discuss the results pertaining to out-of-phase modulation. Comparing Figs. 3 and 8, Figs. 5 and 10, and Figs. 6 and 11, respectively we can conclude that R_{2c} is positive for out-of-phase whereas for in-phase it is negative. Thus N_1, N_3 and N_5 have opposing influences in in-phase and out-of-phase modulations.

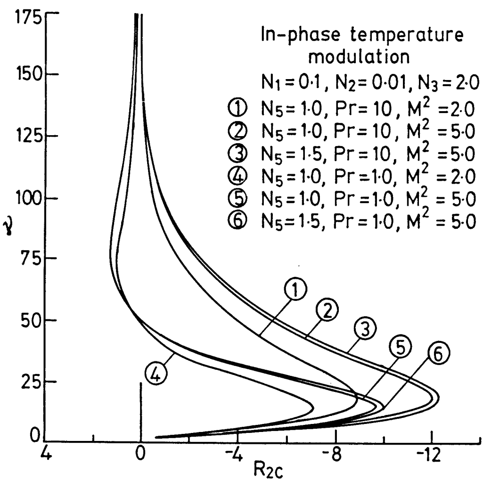

Fig. 6. Plot of R_{2c} versus γ for different values of micropolar heat conduction parameter N_5, Pr and M^2.

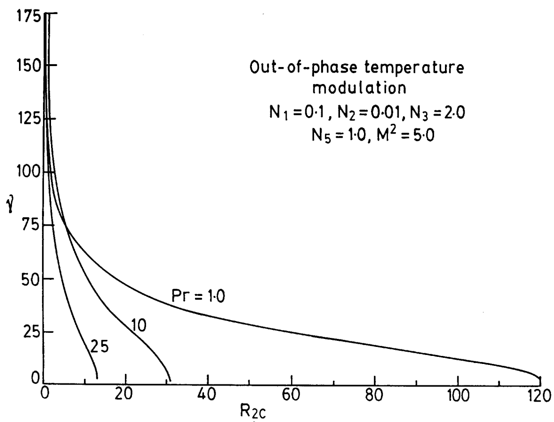

Fig. 7. Plot of R_{2c} versus γ for different values of Pr.

However, N_2 has identical influence on R_{2c} in both in-phase and out-of-phase modulations and these can be seen in Figs. 4 and 9. The above results are due to the fact that in the case of out-of-phase modulation the temperature field has essentially a linear gradient varying in time, so that the instantaneous Rayleigh number is supercritical for half a cycle and subcritical during the other half cycle (see Ref. [4]).

The above results on the effect of various parameters on R_{2c} for out-of-phase modulation do not qualitatively change for the case of temperature modulation of just the lower boundary. This is illustrated with the help of Fig. 12. The physical explanation is the same as in out-of-phase modulation (see Ref. [4]).

We find the following limiting cases in respect of temperature modulation:

1. $\lim_{N_1 \to 0}$ [Results of the present study] → [Results of Ref. [31]]

2. $\lim_{\substack{N_1 \to 0 \\ M^2 \to 0}}$ [Results of the present study] → [Results of Ref. [4].

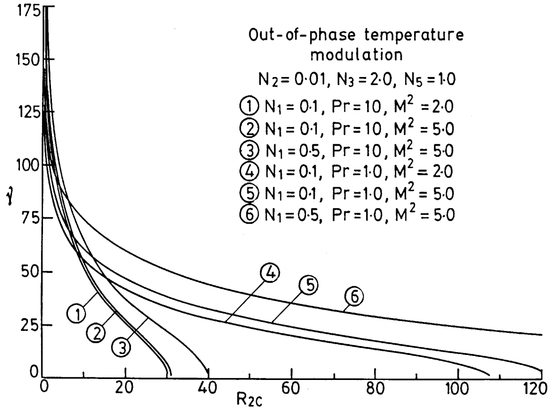

Fig. 8. Plot of R_{2c} versus γ for different values of N_1, Pr and M^2.

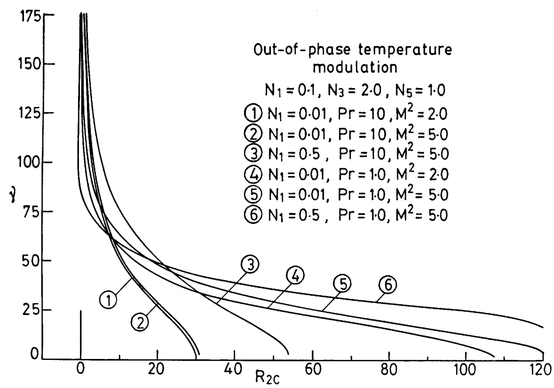

Fig. 9. Plot of R_{2c} versus γ for different values of N_2, Pr and M^2.

In the first limiting case M^2 is to be identified with $1/Pl$ (porous parameter) of Ref. [31]. This clearly shows the analogy between magnetoconvection and convection in porous media involving clean fluids.

We have so far discussed results on temperature modulation. We now discuss results of gravity modulation. Unlike temperature modulation which is at the boundaries, gravity modulation affects the entire bulk of the fluid between the bounding plates. Fig. 13 shows that the effect of various parameters on R_{2c} is qualitatively similar to that of out-of-phase temperature modulation. One interesting result of gravity modulation in contrast to temperature modulation is that at large Prandtl numbers R_{2c} can become negative. This is due to the opposing influences of viscosity and buoyancy force on convection.

The following limiting case applies in respect of gravity modulation:

$$\lim_{N_1 \to 0} \text{[Results of the present study]} \to \text{[Results of Ref. [32]]}.$$

In this limiting case M^2 is to be identified with σ^2 (porous parameter) of Ref. [32].

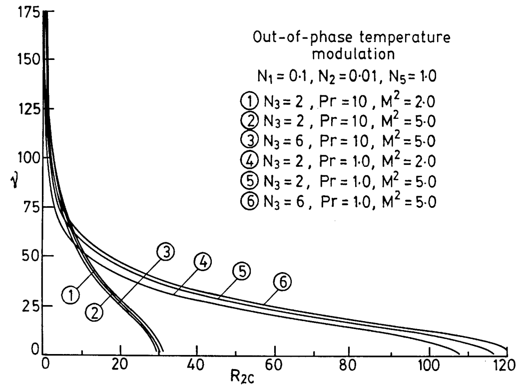

Fig. 10. Plot of R_{2c} versus γ for different values of N_3, Pr and M^2.

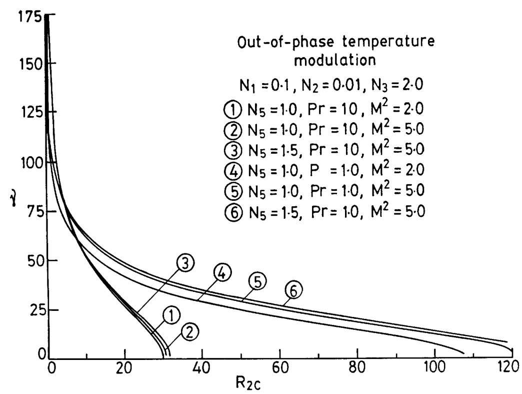

Fig. 11. Plot of R_{2c} versus γ for different values of N_5, Pr and M^2.

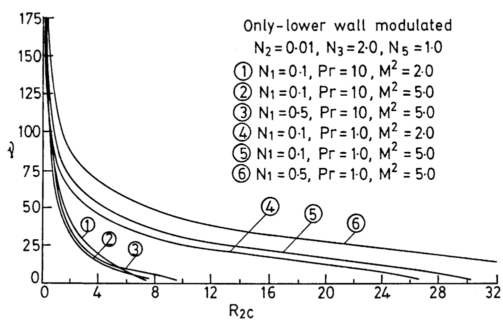

Fig. 12. Plot of R_{2c} versus γ for different values of N_1, Pr and M^2.

The results of the study throw light on an external means of controlling magnetoconvection, either advancing or delaying convection by temperature modulation, in a weak electrically conducting fluid with internal angular momentum. The results of gravity modulation, in general, indicate that g-jitter or gravity modulation leads to delay of convection only. It is also observed that for large frequencies, the effects of

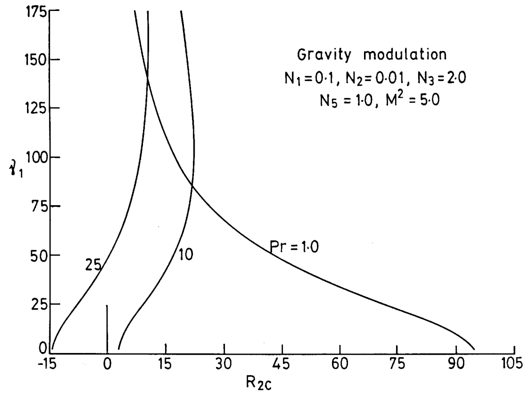

Fig. 13. Plot of R_{2c} versus γ for different values of Pr.

temperature and gravity modulation disappear. From the study we may also conclude that suspended particles scale down the effect of temperature/gravity modulation.

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