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Axisymmetric Thermoelastic Interactions without Energy Dissipation in an Unbounded Body with Cylindrical Cavity

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Abstract. The linear theory of thermoelasticity without energy dissipation is employed to study thermoelastic interactions in a homogeneous and isotropic unbounded body containing a cylindrical cavity. The interactions are supposed to be due to a constant step in radial stress or temperature applied to the boundary of the cavity, which is maintained at a constant temperature or zero radial stress (as the case may be). By using the Laplace transform technique, it is found that the interactions consist of two coupled waves both of which propagate with a finite speed but with no attenuation. The discontinuities that occur at the wavefronts are computed. Numerical results applicable to a copper-like material are presented.

Mathematics Subject Classifications (1991): 73B30, 73D10, 73D15.

Key words: generalized thermoelasticity; energy dissipation; thermoelastic cylindrical waves.

1. Introduction

Thermoelasticity theories which admit a finite speed for thermal signals (second sound) have aroused much interest in the last three decades. In contrast to the conventional coupled thermoelasticity theory (CTE) based on a parabolic heat equation [1], which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories. Among these generalized theories, the extended thermoelasticity theory (ETE) proposed by Lord and Shulman [2] and the temperature-rate dependent thermoelasticity theory (TRDTE) developed by Green and Lindsay [3] have been subjected to a large number of investigations. In view of the experimental evidence available in favour of finiteness of heat propagation speed, generalized thermoelasticity theories are considered to be more realistic than the conventional thermoelasticity theory in dealing with practical problems involving very large heat fluxes at short intervals, like those occurring in laser units and energy channels. For a review of the relevant literature, see [4–6].

Recently, Green and Naghdi [7] proposed a new generalized thermoelasticity theory by including the 'thermal-displacement gradient' among the independent constitutive variables. An important feature of this theory, which is not present in other thermoelasticity theories, is that this theory does not accomodate dissipation of thermal energy. In the context of the linearized version of this theory, theorems on uniqueness of solutions have been established in [8, 9]; boundary-initiated one-dimensional waves in a half-space have been studied in [10, 11], plane harmonic waves in an unbounded body and Rayleigh waves in a half-space have been studied in [12, 13], and thermoelastic interactions in an unbounded body due to a line source have been studied in [14].

The purpose of the present paper is to study axisymmetric thermoelastic interactions in a homogeneous and isotropic unbounded thermoelastic solid containing a cylindrical cavity due to a uniform step in the radial stress or temperature applied to the boundary of the cavity by using the linear theory of thermoelasticity without energy dissipation (TEWOED) developed in [7]. We solve the governing equations by employing the Laplace transform technique. Since the second sound effects are short-lived [4], we perform the inverse Laplace transform operation for small time and derive expressions for displacement, temperature and radial and hoop stress fields. We find that the thermoelastic interactions consist of two coupled waves propagating with finite speeds, of which one is predominantly elastic and the other predominantly thermal, and that neither of these waves experiences attenuation. We further find that the displacement is continuous but the temperature and stresses are discontinuous at both the wavefronts. At the end of the paper we present some numerical results applicable to a copper-like material.

The counterparts of our problem in the contexts of CTE, ETE and TRDTE have been considered by Chattopadhyay *et al.* [15], Sharma [16] and Chandrasekharaiah and Keshavan [17] respectively. It must, however, be mentioned that in these works, transversely isotropic thermoelastic bodies have been considered and that in particular in [17] a unified system of equations that includes the governing equations of CTE, ETE and TRDTE as special cases have been employed. At the appropriate stages in our discussion here, we make a comparison of our results with those obtained in [15–17].

2. Basic Equations

According to TEWOED, the field equations for a homogeneous and isotropic thermoelastic body, in the absence of body forces and heat sources, are as follows [7]:

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\,\mathrm{div}\,\mathbf{u} - \gamma\nabla\theta = \rho\ddot{\mathbf{u}}, \tag{2.1}$$

$$c\ddot{\theta} + \gamma T_0\,\mathrm{div}\,\ddot{\mathbf{u}} = k^*\nabla^2\theta. \tag{2.2}$$

In these equations, $\mathbf{u}$ is the displacement vector, θ is the temperature-change above the uniform reference temperature T_0, ρ is the mass density, c is the specific heat, λ and μ are Lame's constants, $\gamma = (3\lambda + 2\mu)\beta^*$, β^* being the coefficient of volume expansion, and k^* is a material constant characteristic of the theory.

The stress tensor $\mathbf{T}$ associated with $\mathbf{u}$ and θ is given by the following constitutive relation [7]:

$$\mathbf{T} = \lambda(\operatorname{div}\mathbf{u})\mathbf{I} + \mu(\nabla\mathbf{u} + \nabla\mathbf{u}^T) - \gamma\theta\mathbf{I}. \tag{2.3}$$

In all the above equations, the direct vector/tensor notation [18] is employed; also, an over-dot denotes the partial derivative with respect to the time variable t. Some of the symbols and the notation employed here are slightly different from those employed in [7].

In the present analysis we are concerned with an unbounded body having a cylindrical cavity. We choose the z-axis along the axis of the cavity and consider thermoelastic interactions which are symmetrical about the axis. Then the corresponding displacement vector has only the radial component $u = u(r, t)$, where r is the distance measured from the z-axis and the stress tensor has only two components σ_r and σ_φ which are normal stresses in the radial and transverse directions respectively. In this case, Equations (2.1) and (2.2) yield the following governing equations for u and θ:

$$(\lambda + 2\mu)\left[\frac{\partial^2 u}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{u}{r}\right)\right] - \gamma\frac{\partial\theta}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.4}$$

$$k^*\left[\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right] = c\frac{\partial^2\theta}{\partial t^2} + \gamma T_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right). \tag{2.5}$$

Also, the relation (2.3) yields the following expressions for σ_r and σ_φ:

$$\sigma_r = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda\frac{u}{r} - \gamma\theta, \tag{2.6}$$

$$\sigma_\varphi = \lambda\frac{\partial u}{\partial r} + (\lambda + 2\mu)\frac{u}{r} - \gamma\theta. \tag{2.7}$$

It is convenient to have the Equations (2.4) and (2.5) and expressions (2.6) and (2.7) rewritten in non-dimensional form. To this end, we consider the following transformations:

$$r' = \frac{1}{l}r, \quad t' = \frac{v}{l}t, \quad u' = \frac{1}{l}\frac{\lambda + 2\mu}{\gamma T_0}u,$$

$$\theta' = \frac{\theta}{T_0}, \quad \sigma'_r = \frac{1}{\gamma T_0}\sigma_r, \quad \sigma'_\varphi = \frac{1}{\gamma T_0}\sigma_\varphi. \tag{2.8}$$

Here, l is a standard length and v is a standard speed. Introducing (2.8) into (2.4)–(2.7) and suppressing primes, we obtain the following equations which are in non-dimensional form:

$$C_P^2\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]-C_P^2\frac{\partial\theta}{\partial r}=\frac{\partial^2 u}{\partial t^2}, \tag{2.9}$$

$$C_T^2\left[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right]=\frac{\partial^2\theta}{\partial t^2}+\varepsilon\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right), \tag{2.10}$$

$$\sigma_r=\frac{\partial u}{\partial r}+\eta\frac{u}{r}-\theta, \tag{2.11}$$

$$\sigma_\varphi=\eta\frac{\partial u}{\partial r}+\frac{u}{r}-\theta. \tag{2.12}$$

Here,

$$\begin{aligned} &C_P^2=\frac{\lambda+2\mu}{\rho v^2},\quad C_S^2=\frac{\mu}{\rho v^2},\quad C_T^2=\frac{k^*}{cv^2},\\ &\varepsilon=\frac{\gamma^2\theta_0}{c(\lambda+2\mu)},\quad \eta=\left(1-\frac{2C_S^2}{C_P^2}\right). \end{aligned} \tag{2.13}$$

We note that C_P and C_S respectively represent the non-dimensional speeds of purely elastic dilatational and shear waves, C_T is the non-dimensional speed of purely thermal waves and ε is the usual thermoelastic coupling parameter. Further, $0<\eta<1$.

Let a denote the (dimensionless) radius of the cavity. Then, if the body is initially held at rest in an undeformed state at the reference temperature and zero temperature-rate, the following initial conditions hold:

$$u=\frac{\partial u}{\partial t}=\theta=\frac{\partial\theta}{\partial t}=0\quad\text{at } t=0\quad\text{for } r\geqslant a. \tag{2.14}$$

If the thermoelastic interactions are caused by a uniform step in the radial stress applied to the boundary of the cavity which is held at the reference temperature, then the following boundary conditions hold:

$$\sigma_r=-\sigma_0H(t),\quad \theta=0\quad\text{for } r=a\quad\text{and } t>0.$$

Here, σ_0 is a positive constant and $H(t)$ is the Heaviside unit step function. On using (2.11), these conditions become

$$\frac{\partial u}{\partial r}+\eta\frac{u}{r}=-\sigma_0H(t)\quad\text{for } r=a\quad\text{and } t>0. \tag{2.15}$$

Alternatively, if the thermoelastic interactions are caused by a uniform step in temperature applied to the boundary of the cavity which is stress-free, then the following boundary conditions hold:

$$\theta = \theta_0 H(t), \quad \sigma_r = 0 \quad \text{for } r = a \quad \text{and } t > 0.$$

Here, θ_0 is a positive constant. On using (2.11), these conditions become

$$\frac{\partial u}{\partial r} + \eta \frac{u}{r} = \theta_0 H(t) \quad \text{for } r = a \quad \text{and } t > 0. \tag{2.16}$$

Equations (2.9) and (2.10) serve as a system of governing differential equations, (2.14) serve as a set of initial conditions and (2.15) or (2.16) (as the case may be) serves as a boundary condition for the determination of the non-dimensional displacement u and non-dimensional temperature θ. Once u and θ are determined, then σ_r and σ_φ can be computed with the aid of expressions (2.11) and (2.12).

We note that the boundary conditions (2.15) and (2.16) are strikingly similar. Accordingly, the two cases corresponding to (2.15) and (2.16) need not be studied separately; if we replace $-\sigma_0$ by θ_0 in the analysis corresponding to the condition (2.15), we obtain the analysis that corresponds to the condition (2.16). In what follows, we therefore confine ourselves to the analysis corresponding to the condition (2.15).

3. Transform Solution

Taking the Laplace transform of Equations (2.9) and (2.10) and expressions (2.11) and (2.12) under the homogeneous initial conditions (2.14), we obtain the following equations:

$$[C_P^2 D D_1 - s^2]\bar{u} = C_P^2 D\bar{\theta}, \tag{3.1}$$

$$[C_T^2 D_1 D - s^2]\bar{\theta} = \varepsilon s^2 D_1 \bar{u}, \tag{3.2}$$

$$\bar{\sigma}_r = D\bar{u} + \eta \frac{\bar{u}}{r} - \bar{\theta}, \tag{3.3}$$

$$\bar{\sigma}_\varphi = \eta D\bar{u} + \frac{\bar{u}}{r} - \bar{\theta}. \tag{3.4}$$

Here, an over bar denotes the Laplace transform of the corresponding function, s is the transform parameter, and

$$D = \frac{\mathrm{d}}{\mathrm{d}r}, \quad D_1 = \frac{\mathrm{d}}{\mathrm{d}r} + \frac{1}{r}.$$

The coupled Equations (3.1) and (3.2) can be decoupled (by eliminating $\bar{u}$ or $\bar{\theta}$) and put in the following form:

$$(DD_1 - m_1^2)(DD_1 - m_2^2)\bar{u} = 0, \tag{3.5}$$

$$(D_1 D - m_1^2)(D_1 D - m_2^2)\bar{\theta} = 0. \tag{3.6}$$

Here, m_1 and m_2 satisfy the biquadratic equation

$$C_P^2 C_T^2 m^4 - s^2\{C_T^2 + (1+\varepsilon)C_P^2\}m^2 + s^4 = 0. \tag{3.7}$$

Solving Equations (3.5) and (3.6) under the regularity conditions that $\bar{u}, \bar{\theta} \to 0$ as $r \to \infty$, we obtain the following expressions:

$$\bar{u} = A_1 K_1(m_1 r) + A_2 K_1(m_2 r), \tag{3.8}$$

$$\bar{\theta} = B_1 K_0(m_1 r) + B_2 K_0(m_2 r). \tag{3.9}$$

Here, K_1 and K_0 are modified Bessel functions of the second kind and of first and zeroth order respectively, and A_1, A_2, B_1, B_2 are arbitrary constants. Also, m_1 and m_2 are taken to have positive real parts.

By solving Equation (3.7), we find that

$$m_\alpha = \frac{s}{V_\alpha}, \tag{3.10}$$

where

$$V_\alpha = \frac{1}{\sqrt{2}}[\{C_T^2 + (1+\varepsilon)C_P^2\} + (-1)^{\alpha+1}\Delta]^{1/2} \tag{3.11}$$

with

$$\Delta = [\{C_T^2 - (1+\varepsilon)C_P^2\}^2 + 4\varepsilon C_P^2 C_T^2]^{1/2} \tag{3.12}$$

$$= V_1^2 - V_2^2. \tag{3.13}$$

Here and in the expressions that follow, the index α takes values 1, 2.

Since $\bar{u}$ and $\bar{\theta}$ are coupled together, A_α and B_α cannot be independent. By substituting for $\bar{u}$ and $\bar{\theta}$ from (3.8) and (3.9) in Equation (3.2) and equating the corresponding coefficients with the aid of the identities

$$\frac{\mathrm{d}}{\mathrm{d}z}\{K_0(z)\} = -K_1(z), \quad \frac{\mathrm{d}}{\mathrm{d}z}\{K_1(z)\} = -\frac{1}{z}K_1(z) - K_0(z),$$

we find that

$$B_\alpha = \frac{\varepsilon s^2 m_\alpha}{s^2 - C_T^2 m_\alpha^2} A_\alpha. \tag{3.14}$$

Taking the Laplace transform of the boundary condition (2.15) and substituting for $\bar{u}$ from (3.8) in the resulting expression, we obtain the following expression for A_α:

$$A_\alpha = (-1)^\alpha \frac{(s^2 - C_T^2 m_\alpha^2)}{s\Gamma} \sigma_0 m_{3-\alpha} K_0(m_{3-\alpha} a). \tag{3.15}$$

Here,

$$\begin{aligned}\Gamma \; = \; & [m_1 K_0(m_1 a)(s^2 - C_T^2 m_2^2)] \left[\frac{(1-\eta)}{a} K_1(m_2 a) + m_2 K_0(m_2 a)\right] \\ & -[m_2 K_0(m_2 a)(s^2 - C_T^2 m_1^2)] \\ & \times \left[\frac{(1-\eta)}{a} K_1(m_1 a) + m_1 K_0(m_1 a)\right].\end{aligned} \tag{3.16}$$

Substituting for A_α from (3.15) and for B_α from (3.14) in (3.8) and (3.9), we obtain explicit expressions for $\bar{u}$ and $\bar{\theta}$. Taking the inverse Laplace transform of these resulting expressions, we obtain u and θ. However, determining u and θ for arbitrary t is a tedious task. Since the second-sound effects are short lived [4], it may be sufficient to obtain and analyse the solutions for small t. This is done by taking s to be large.

We note that when s is large, m_α, then given by (3.10), are also large. Hence in our computations we set

$$K_0(m_\alpha r) \approx K_1(m_\alpha r) = \left(\frac{\pi}{2m_\alpha r}\right)^{1/2} \mathrm{e}^{-m_\alpha r}. \tag{3.17}$$

4. Derivation of the Solution

With the aid of (3.10) and (3.17), expressions (3.15) and (3.16) simplify to the following form:

$$A_\alpha = \left(\frac{2m_\alpha a}{\pi}\right)^{1/2} \left[\frac{C_\alpha}{s^2} + \frac{E_\alpha}{s^3}\right] \mathrm{e}^{(s/V_\alpha)a}. \tag{4.1}$$

Here,

$$C_\alpha = \sigma_0 \left(1 - \frac{C_T^2}{V_\alpha^2}\right) \frac{V_\alpha^3 V_{3-\alpha}^2}{C_T^2 (V_\alpha^2 - V_{3-\alpha}^2)},$$

$$E_\alpha = \sigma_0 \frac{1-\eta}{a} \left(1 - \frac{C_T^2}{V_\alpha^2}\right) \frac{(V_\alpha V_{3-\alpha} + C_T^2)}{(C_T^2)^2 (V_{3-\alpha} - V_\alpha)} \frac{V_\alpha^4 V_{3-\alpha}^3}{(V_\alpha + V_{3-\alpha})^2}. \tag{4.2}$$

Substituting for A_α from (4.1) and for B_α from (3.14) in expressions (3.8) and (3.9), we obtain $\bar{u}$ and $\bar{\theta}$. Substituting the resulting expressions into (3.3) and (3.4), we get $\bar{\sigma}_r$ and $\bar{\sigma}_\varphi$. Taking the inverse Laplace transforms of the expressions for $\bar{u}, \bar{\theta}, \bar{\sigma}_r$ and $\bar{\sigma}_\varphi$ so obtained, we obtain (after some lengthy calculations) the following solutions for u, θ, σ_r and σ_φ, valid for small values of t:

$$\begin{aligned} u(r,t) \; &= \; \left[\left(t - \frac{R}{V_1}\right) C_1 + \frac{1}{2}\left(t - \frac{R}{V_1}\right)^2 E_1\right] H_1 + \left[\left(t - \frac{R}{V_2}\right) C_2 \right. \\ & \left. + \frac{1}{2}\left(t - \frac{R}{V_2}\right)^2 E_2\right] H_2, \end{aligned} \tag{4.3}$$

$$\theta(r,t) = \left[F_1 + \left(t - \frac{R}{V_1}\right) G_1\right] H_1 + \left[F_2 + \left(t - \frac{R}{V_2}\right) G_2\right] H_2, \tag{4.4}$$

$$\begin{aligned} \sigma_r(r,t) \; &= \; -\left[\mathrm{I}_1 + \left(t - \frac{R}{V_1}\right) L_1 + \left(t - \frac{R}{V_1}\right)^2 M_1\right] H_1 \\ & -\left[\mathrm{I}_2 + \left(t - \frac{R}{V_2}\right) L_2 + \left(t - \frac{R}{V_2}\right)^2 M_2\right] H_2 \end{aligned} \tag{4.5}$$

$$\begin{aligned} \sigma_\varphi(r,t) \; &= \; -\left[N_1 + \left(t - \frac{R}{V_1}\right) P_1 + \left(t - \frac{R}{V_1}\right)^2 Q_1\right] H_1 \\ & -\left[N_2 + \left(t - \frac{R}{V_2}\right) P_2 + \left(t - \frac{R}{V_2}\right)^2 Q_2\right] H_2. \end{aligned} \tag{4.6}$$

Here, $R = r - a$, and

$$\begin{aligned} & F_\alpha = \frac{\sigma_0 \varepsilon V_\alpha^2 V_{3-\alpha}^2}{C_T^2 (V_\alpha^2 - V_{3-\alpha}^2)}, \\ & G_\alpha = \sigma_0 \varepsilon \frac{1-\eta}{a} \frac{(V_\alpha V_{3-\alpha} + C_T^2)}{(C_T^2)^2 (V_{3-\alpha} - V_\alpha)} \frac{V_\alpha^3 V_{3-\alpha}^3}{(V_\alpha + V_{3-\alpha})^2}, \\ & \mathrm{I}_\alpha = \frac{C_\alpha}{V_\alpha} + F_\alpha, \qquad L_\alpha = \frac{1-\eta}{r} C_\alpha + \frac{E_\alpha}{V_\alpha} + G_\alpha, \\ & M_\alpha = \frac{1-\eta}{2r} E_\alpha, \qquad N_\alpha = \eta \frac{C_\alpha}{V_\alpha} + F_\alpha, \\ & P_\alpha = \eta \frac{E_\alpha}{V_\alpha} - \frac{1-\eta}{r} C_\alpha + G_\alpha, \qquad Q_\alpha = -\frac{(1-\eta)}{2r} E_\alpha, \\ & H_\alpha = \left[\frac{a}{r}\right]^{1/2} H\left(t - \frac{R}{V_\alpha}\right). \end{aligned} \tag{4.7}$$

From the solutions (4.3)–(4.6) we observe that each of u, θ, σ_r, and σ_φ is made up of two parts and that each part corresponds to a wave propagating with a finite speed, the speed of the wave corresponding to the first part being V_1 and that corresponding to the second part being V_2. Using expressions (3.11)–(3.13), we find that

(i) $V_1 > (C_P, C_T) > V_2$,
(ii) If $C_P > C_T$, then $V_1 > C_P > C_T > V_2$ and $V_1 \to C_P, V_2 \to C_T$ as $\varepsilon \to 0$,
(iii) If $C_T > C_P$, then $V_1 > C_T > C_P > V_2$ and $V_1 \to C_T, V_2 \to C_P$ as $\varepsilon \to 0$.

Accordingly, the interactions being considered consist of two coupled waves, one following the other; the faster wave has its speed equal to V_1 and the slower wave has its speed equal to V_2. If $C_P > C_T$, the faster wave is a predominantly elastic wave (or the e-wave) and the slower is a predominantly thermal wave (or the θ-wave). On the other hand, if $C_T > C_P$, the faster wave is the θ-wave and the slower is the e-wave.

From the solutions (4.3)–(4.6), we observe that neither the e-wave nor the θ-wave experiences decay with the distance (attenuation). That this is *not* the case in CTE, ETE and TRDTE is an interesting fact to record; in these theories, the waves do experience attenuation [15–17]. The absence of attenuation is, evidently, a characteristic feature of TEWOED. From (4.3)–(4.6), we also note that all of u, θ, σ_r and σ_φ are identically zero for $r > tV_1$; this means that, at a given instant of time $t^* > 0$, the points of the region $r > a$ that are beyond the faster wavefront $r = V_1 t^*$ do not experience any disturbance. This observation verifies that, like ETE and TRDTE, TEWOED is also a generalized thermoelasticity theory.

By direct inspection of the solutions (4.3)–(4.6), we find that u is continuous, whereas θ, σ_r and σ_φ are discontinuous, at both the wavefronts. The discontinuities in θ, σ_r and σ_φ are given as follows:

$$
\begin{aligned}
[\theta]_\alpha &= \left[\frac{a}{r}\right]^{1/2} F_\alpha, \\
[\sigma_r]_\alpha &= -\left[\frac{a}{r}\right]^{1/2} \mathrm{I}_\alpha, \\
[\sigma_\varphi]_\alpha &= -\left[\frac{a}{r}\right]^{1/2} N_\alpha.
\end{aligned}
\tag{4.8}
$$

Here, $[\,]_\alpha$ denotes the discontinuity of the function across the wavefront $t = R/V_\alpha$.

In view of expressions (4.7), we note that the discontinuities given by (4.8) are all constants. This is not so in ETE and TRDTE; in these theories the discontinuities decay exponentially with distance from the boundary [16, 17]. It is the absence of attenuation in TEWOED that brings out this difference between TEWOED and ETE and TRDTE.

The first expression in (4.8) exhibits another interesting phenomenon. This expression and the expression for F_α contained in (4.7) show that the temperature

Table I. Discontinuities in temperature and stresses at the wave fronts

θ-wavefront	e-wavefront
$[\theta]_1 = 0.000734$	$[\theta]_2 = -0.000842$
$[\sigma_r]_1 = -0.000773$	$[\sigma_r]_2 = -0.952595$
$[\sigma_\varphi]_1 = -0.000755$	$[\sigma_\varphi]_2 = -0.497424$

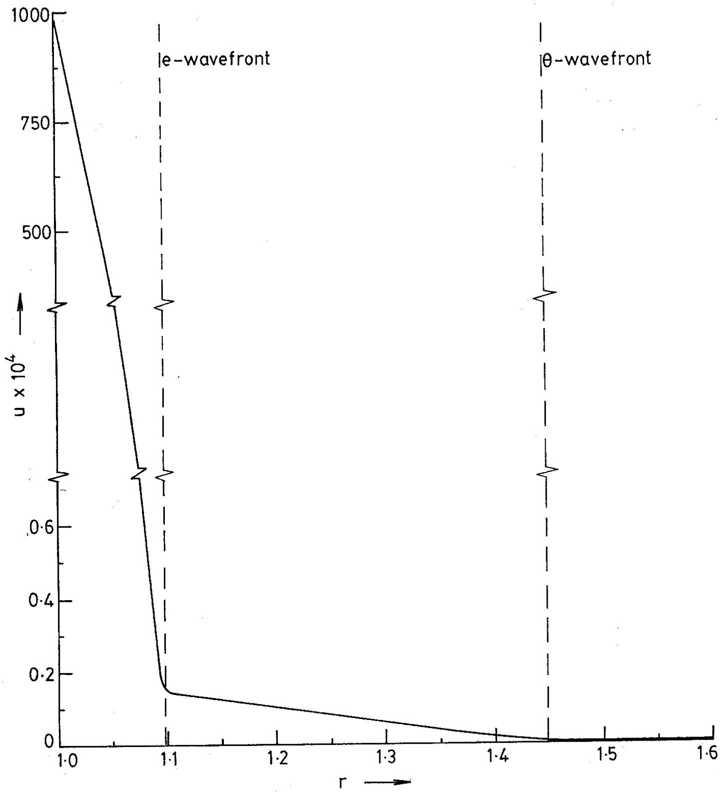

Figure 1. Variation of u with r at $t = 0.1$.

is discontinuous at both the wavefronts in spite of the fact that the boundary load is purely mechanical in nature. This means that a discontinuous mechanical load applied to the boundary does generate discontinuities in temperature. This phenomenon is present in ETE as well [16] but is absent in TRDTE [17]; according to TRDTE, the temperature is continuous when the applied load is purely mechanical in nature [17]. A similar observation has been made in the half-space problem in the context of TEWOED [10].

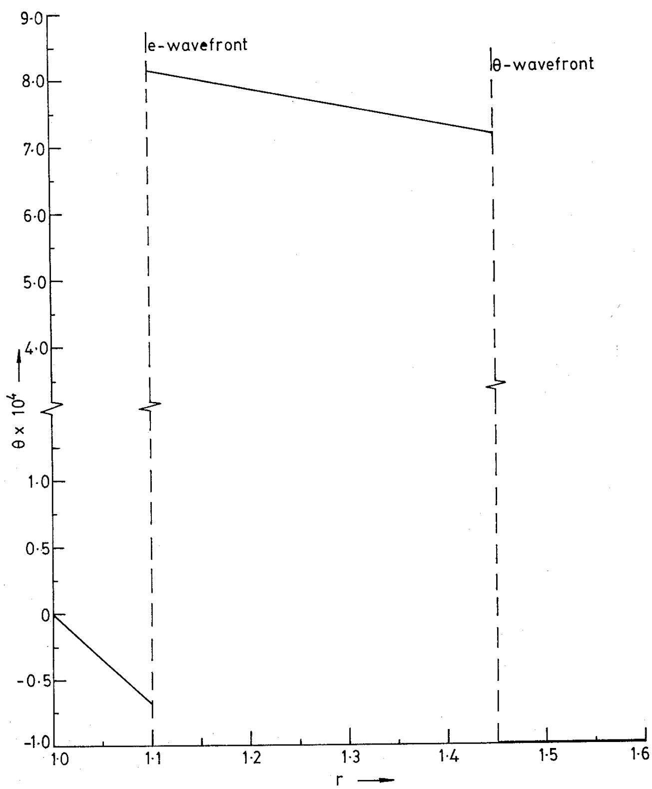

Figure 2. Variation of θ with r at $t = 0.1$.

5. Numerical Results

With the view of illustrating the theoretical results obtained in the preceding section, we now present some numerical results. For this purpose, we choose a copper-like material for which

$$C_P^2 = 1, \quad C_T^2 = \frac{1}{0.05}, \quad C_S^2 = 0.2387, \quad \varepsilon = 0.0168.$$

For this material, we have $C_T > C_P$. As such, the faster wave happens to be the θ-wave and the slower wave the e-wave. By using expressions (3.11) and (3.12), we find that the (dimensionless) speeds of these waves are $V_1 = 4.474113$ and $V_2 = 0.999558$ respectively. By taking $a = 1$ and $\sigma_0 = 1$, we analyse the behaviour of u, θ, σ_r and σ_φ at (dimensionless) time $t = 0.1$. We find that at this instant of time, the faster wavefront (θ-wavefront) is positioned at $r = r_1 = 1 + tV_1 = 1.447411$ and the slower wavefront (e-wavefront) at $r = r_2 = 1 + tV_2 = 1.099956$. We have computed the discontinuities in θ, σ_r and σ_φ at the wavefronts by using expressions (4.7) and (4.8) and the results are summarized in Table I. We have also computed

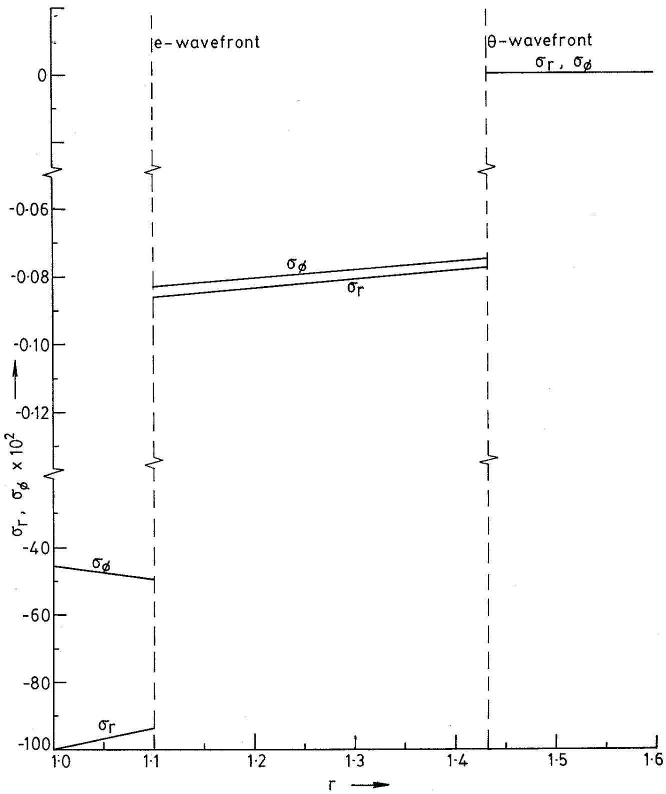

Figure 3. Variations of σ_r and σ_φ with r at $t = 0.1$.

the values of u, θ, σ_r and σ_φ at $t = 0.1$ for $r \geqslant 1$, by using the solutions (4.3)–(4.6). These values are displayed in Figures 1–3. From these Figures we find that the displacement, temperature and stresses are all identically zero beyond the θ-wavefront. This indicates that the effects of disturbances are confined to the domain $1 \leqslant r \leqslant tV_1$, as predicted by the theoretical results obtained earlier.

Figure 1 shows that u is continuous at all positions including those of the wavefronts, as predicted by the theoretical results obtained in the preceding section. We further note that u decreases steadily in the domain $r \geqslant 1$, the maximum value of u, equal to 0.095201 (approx), occurring on the boundary of the cavity.

Figure 2 shows that θ is negative between the boundary and the location of the slower wavefront (e-wavefront) and is positive between the location of the slower and faster wavefronts. In each of these intervals (viz, in $1 < r < 1.099956$ and $1.099956 < r < 1.447411$), θ decreases steadily. The maximum value of θ, equal to 0.000811 (approx), occurs at the point that lies just beyond the slower front.

Figure 3 shows that both σ_r and σ_φ are compressive throughout the domain of influence (viz, $1 \leqslant r \leqslant 1.447411$). Between the boundary of the cavity and the location of the slower wavefront, the *magnitude* of σ_r decreases while that of σ_φ

increases. Between the locations of the slower and faster wavefronts the *magnitudes* of both σ_r and σ_φ decrease. Whereas $|\sigma_r|$ is maximum, equal to 1, on the boundary of the cavity, $|\sigma_\varphi|$ is maximum, equal to 0.498227 (approx), at the point that lies just behind the slower wavefront. We further find that $|\sigma_\varphi| \approx 0.452209$ on the boundary of the cavity.

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