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Convective instability of ferromagnetic fluids bounded by fluid-permeable, magnetic boundaries

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Abstract

Convective instability of a ferromagnetic fluid in a Rayleigh–Benard situation between fluid-permeable, magnetic boundaries and subject to an external constraint of a uniform, transverse magnetic field is studied. The fluid-permeable, magnetic boundaries require general boundary conditions on the velocity and the scalar magnetic potential. For these, the Garlerkin method predicts the critical eigenvalue to be between that of free–free and rigid–rigid boundaries. The paper also reaffirms the qualitative findings of earlier investigations which are, in fact, limiting cases of the present study.

1. Introduction

Convective fluid motion in ferromagnetic fluids has been the subject of intensive study because of the remarkable physical properties of the fluid and also due to practical applications [1]. Since the magnetisation of the fluid depends on the temperature as well as the magnetic field, convection may occur if, at least, a gradient of one of them is present. Convection in magnetic fluids due to infinitesimal perturbations has been studied by Finlayson [2] and Gotoh and Yamada [3] considering magnetic boundaries. Sekhar [4] has made an exhaustive study of the problem with non-uniform basic temperature profiles. Recently, the author [5] studied the problem with second sound waves. All these published research works deal with rigid or free boundaries. In this paper use has been made of a derived boundary condition which is intermediate between the free and rigid boundaries.

2. Mathematical formulation and solution

The onset of linear convective instability in a horizontal ferromagnetic liquid layer bounded by fluid-permeable, magnetic boundaries, as shown in Fig. 1, is considered. The basic state is quiescent and is perturbed by infinitesimal disturbances. The dimensionless perturbation equations for the Rayleigh–Benard situation in a Newtonian, Boussinesq and ferromagnetic fluid are [2]:

$$\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}(\nabla^2 W) = R(1+M_1)\nabla_1^2 + \nabla^4 W - RM_1\frac{\partial}{\partial Z}(\nabla_1^2\Phi), \tag{1}$$

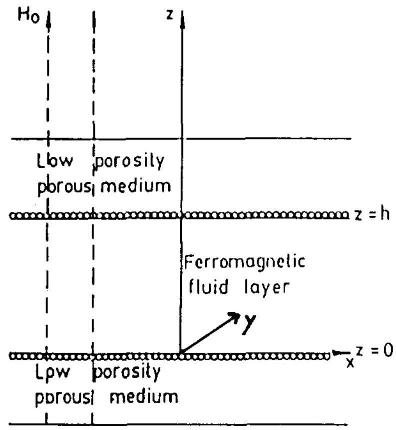

Fig. 1. Schematic diagram of the flow configuration.

$$\frac{\partial T}{\partial t} = W + \nabla^2 T, \tag{2}$$

$$\frac{\partial^2\Phi}{\partial Z^2} + M_3\nabla_1^2\Phi = \frac{\partial T}{\partial Z}, \tag{3}$$

where W is the vertical component of the velocity, T is the temperature, Φ is the scalar magnetic potential Pr and R are the dimensionless groups named after Prandtl and Rayleigh, ∇_1^2 and ∇^2 are the two- and three-dimensional Laplacian operators. M_1 and M_3 are magnetic parameters (see Ref. [2]). Eq. (2) does not contain the magnetic parameters for reasons given by Finlayson [2]. The analysis is made in terms of periodic waves as analysed by Chandrasekhar [6] and so W, T and Φ take the form

$$(\text{function of } Z)\exp(\sigma t + \mathrm{i}(lx+my)), \tag{4}$$

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where l and m are the x- and y-components of the horizontal wave number a, σ is the growth rate and $\mathrm{i} = \sqrt{-1}$. Substituting the form (4) into Eqs. (1)–(3), we obtain

$$\frac{\sigma}{\mathrm{Pr}}(\mathrm{D}^2 - a^2)W = -R(1 + M_1)a^2\Theta + (\mathrm{D}^2 - a^2)^2 W + RM_1 a^2 \mathrm{D}\Phi, \qquad (5)$$

$$\sigma\Theta = W + (\mathrm{D}^2 - a^2)\Theta, \qquad (6)$$

$$(\mathrm{D}^2 - a^2 M_3)\Phi = \mathrm{D}\Theta, \qquad (7)$$

where W, Θ, Φ are the respective amplitudes of the velocity, temperature and magnetic potential perturbations and $\mathrm{D} = \mathrm{d}/\mathrm{d}Z$.

The horizontal layer of ferromagnetic fluid is confined between two plane, fluid-permeable, isothermal, magnetic surfaces and hence the boundary conditions are (see Appendix for derivation of Eqs. (8) and (9)):

$$W = \mathrm{D}^2 W - \mathrm{Da_s} \mathrm{D}W = 0 \quad \text{at } Z = 0, \qquad (8)$$

$$W = \mathrm{D}^2 W + \mathrm{Da_s} \mathrm{D}W = 0 \quad \text{at } Z = 1, \qquad (9)$$

$$\Theta = 0 \quad \text{at } Z = 0 \text{ and } Z = 1, \qquad (10)$$

$$(1 + \chi)\mathrm{D}\Phi - a\Phi = 0 \quad \text{at } Z = 0, \qquad (11)$$

$$(1 + \chi)\mathrm{D}\Phi + a\Phi = 0 \quad \text{at } Z = 1, \qquad (12)$$

where $\mathrm{Da_s} = \alpha h / \sqrt{K}$ is the slip-D'Arcy number (α is the slip coefficient [7], K is the permeability of the bounding porous media) and χ is the magnetic susceptibility. Eqs. (8) and (9) are derived from the Beavers and Joseph slip condition [7], the continuity equation and the normal mode solution (4). The Maxwell stresses do not appear in these equations because of the assumed boundary conditions on the magnetic field. The eigenvalue R for stationary convection is obtained by applying the Galerkin method. Oscillatory convection is ruled out because the principle of exchange of stability is valid. The trial functions used for W, Θ and Φ are:

$$W = Z^4 - 2Z^3 + \frac{\mathrm{Da_s}}{2 + \mathrm{Da_s}} Z^2 + \frac{2}{2 + \mathrm{Da_s}} Z,$$

$$\Theta = Z(1 - Z), \quad \Phi = M_L Z^2 - M_L Z + 1,$$

where $M_L = a/(1 + \chi)$. The results are discussed below.

3. Results and discussion

A linear stability analysis is performed of the convective instability of a ferromagnetic fluid in a Rayleigh–Benard situation between fluid-permeable, magnetic boundaries and subject to an external constraint of a uniform, transverse magnetic field. Since the boundary conditions are complicated, the Galerkin method is used to obtain the critical eigenvalue which is stationary to small changes in the trial functions [2]. The analysis predicts the critical eigenvalue to be between that of free–free and rigid–rigid boundaries. The reasoning is that the fluid-permeable boundaries allow for slipping of the magnetic fluid and give scope for greater mobility of the fluid near the boundaries unlike the rigid case. However, the mobility is less than that near free boundaries. Thus, we have the following result for the critical eigenvalue R_C:

$$R_{\text{Free}} \leq R_{\text{Permeable}} \leq R_{\text{Rigid}}.$$

The equality sign is understandable because R_{Free} and R_{Rigid} can be obtained from $R_{\text{Permeable}}$ in the limits $\mathrm{Da_s} \to 0$ and $\mathrm{Da_s} \to \infty$ respectively. Fig. 2 is a plot of the critical Rayleigh number R_C and magnetic number N_C $(= R_C M_1)$ versus the slip-D'Arcy number $\mathrm{Da_s}$. When M_1 is very large, N_C is the governing parameter in place of R_C. The figure comprehensively shows the bridge between the results of free–free and rigid–rigid boundaries constructed on the assumption of fluid-permeable boundaries. The figure also reaffirms the predicted destabilising nature of M_1 and M_3 and the stabilising nature of χ [2].

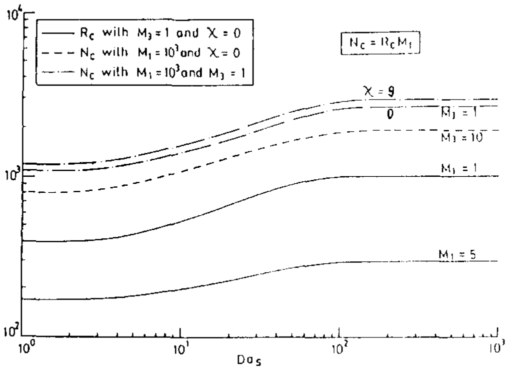

Fig. 2. Plot of critical Rayleigh number R_c and magnetic number N_c vs. slip-D'Arcy number $\mathrm{Da_s}$.

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Appendix A. Derivation of a general boundary condition on W

The Beavers–Joseph (BJ) condition is extended here to arrive at an appropriate boundary condition at the fluid-permeable surface considered in the present problem. It is assumed that there is a trickling flow of a suitable carrier fluid through the densely packed porous beds and the horizontal magnetic fluid layer between the beds is almost quiescent with limited dynamics seen only in a thin, horizontal magnetic fluid layer adjacent to the porous–fluid interface. This weak basic flow can be neglected for all practical purposes while studying the stability of Rayleigh–Benard convection in the magnetic fluid layer.

Using the terminology of BJ, let Q_1, Q_2 be the horizontal D'Arcy velocity components in porous media and u_p, v_p the horizontal slip velocity components at the interface. Let w_p be the vertical component of the porous-media velocity resulting from gravity. Let u_b, v_b, w_b be the components of velocity of the basic flow in the channel limited to a thin, horizontal magnetic fluid layer adjacent to the interface. w_b can be taken to be independent of X and Y in this thin layer.

The dimensionless form of the BJ slip condition at the interface can now be written as

$$\frac{\partial u_b}{\partial Z} = \mathrm{Da_s}(u_p - Q_1), \quad \frac{\partial v_b}{\partial Z} = \mathrm{Da_s}(v_p - Q_2) \quad \text{at } Z = 0, \tag{13a}$$

$$\frac{\partial u_b}{\partial Z} = -\mathrm{Da_s}(u_p - Q_1), \quad \frac{\partial v_b}{\partial Z} = -\mathrm{Da_s}(v_p - Q_2) \quad \text{at } Z = 1, \tag{13b}$$

and

$$u_p = u_b, \quad v_p = v_b, \quad w_p = w_b \quad \text{at } Z = 0,\ 1. \tag{13c}$$

$\mathrm{Da_s}$ is the slip-D'Arcy number (see main text).

We now consider the boundary at $Z = 0$ to obtain the general boundary condition. Similarly, one can obtain the required condition at $Z = 1$. The analysis in the paper is limited to infinitesimal perturbations of the almost quiescent magnetic fluid layer. Denoting the perturbations by primed quantities, one can write

$$\frac{\partial u'}{\partial Z} = \mathrm{Da_s} u'_p, \quad \frac{\partial v'}{\partial Z} = \mathrm{Da_s} v'_p, \quad u' = u'_p, \quad v' = v'_p, \quad w' = 0, \quad \text{at } Z = 0, \tag{14}$$

where it has been assumed that the D'Arcy velocity components Q_1 and Q_2, and w_p remain unperturbed. The thermal condition and a properly chosen carrier fluid ensure such a situation, it is assumed. In Eq. (14), the subscript 'b' has been dropped for the velocity perturbations of the magnetic fluid layer.

Using the periodic wave solution (4) in (14), we get the conditions on the amplitudes of the velocity perturbation as

$$\frac{\mathrm{d}U}{\mathrm{d}Z} = \mathrm{Da_s} U, \quad \frac{\mathrm{d}V}{\mathrm{d}z} = \mathrm{Da_s} V, \quad W = 0, \quad \text{at } Z = 0. \tag{15}$$

Now, consider the continuity equation

$$\frac{\partial u'}{\partial X} + \frac{\partial v'}{\partial Y} + \frac{\partial w'}{\partial Z} = 0. \tag{16}$$

Using the periodic wave solution (4) in (16), we get on rearrangement

$$lU + mV = \mathrm{i}\frac{\mathrm{d}W}{\mathrm{d}Z}. \tag{17}$$

Eq. (17) applies at the boundaries also. Combining (15) and (17), we arrive at the general boundary condition on W used in the paper. The major assumption in the entire derivation is that the mutual mixing of the pure carrier fluid in the porous media with the magnetic fluid in the channel does not alter the dynamics on either side of the interface. However, if the carrier fluids in the two regions are immiscible or slowly mixing, then the general boundary conditions are better realised.

References

[1] R.E. Rosensweig, Ferrohydrodynamics (Cambridge University Press, Cambridge, 1985).
[2] B.A. Finlayson, J. Fluid Mech. 40 (1970) 753.
[3] K. Gotoh and M. Yamada, J. Phy. Soc. Jpn. 51 (1982) 3042.
[4] G.N. Sekhar, Convection in Magnetic Fluids, PhD Thesis, Bangalore University, India (1990).
[5] P.G. Siddheshwar, Jpn. Soc. Magn. Fluids 25 (1993).
[6] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability (Clarendon Press, London, 1961).
[7] G.S. Beavers and D.D. Joseph, J. Fluid Mech. 30 (1967) 197.