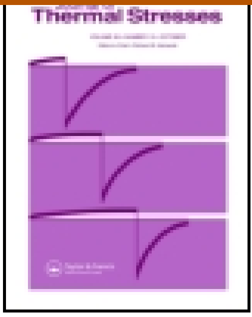

Journal of Thermal Stresses

ISSN: 0149-5739 (Print) 1521-074X (Online) Journal homepage: http://www.tandfonline.com/loi/uths20

MECHANOCALORIC EFFECT ON MAGNETOTHERMOELASTIC INTERACTIONS IN A CYLINDRICAL CONDUCTOR CARRYING AN ELECTRIC CURRENT

D. S. Chandrasekharaiah

MECHANOCALORIC EFFECT ON MAGNETOTHERMOELASTIC INTERACTIONS IN A CYLINDRICAL CONDUCTOR CARRYING AN ELECTRIC CURRENT

D. S. Chandrasekharaiah
Department of Mathematics
Bangalore University
Bangalore 560001, India

The mechanocaloric effect on magnetothermoelastostatic interactions in a cylindrical conductor carrying a uniform electric current is investigated. It is found that this effect reduces the Joule heating effect and induces non-linearity into the behavior of stresses. A condition under which the mechanocaloric effect nullifies the Joule effect is also obtained.

INTRODUCTION

In a recent paper Roetman [1] developed a theory for the basic equations of thermoelasticity, taking into account the mechanocaloric coupling effect. In this theory, unlike the coupled theory [2] and the generalized theory [3], a term involving the body force enters into the equation of heat conduction, and the displacement and thermal fields remain coupled together, even in static deformations. Our purpose in this paper is to utilize the heat equation obtained in [1] to study the interaction between the mechanical field and the thermal field produced in an electrically conducting thermoelastic cylinder permeated by a uniform axial current, assuming that the cylinder undergoes a plane static deformation. This problem was considered earlier by Yuan [4], using the classical form of the heat equation. Our analysis here shows that the inclusion of the mechanocaloric coupling reduces the Joule heating effect and introduces nonlinearity into the behavior of stresses. When the electrical conductivity, the magnetic permeability, the Poisson ratio, and the mechanocaloric coupling constant of the cylinder are connected by a particular relation [Eq. (17) below], the mechanocaloric effect nullifies the Joule effect, and the cylinder temperature does not change. When the mechanocaloric effect is neglected, our results reduce to those obtained in [4].

This work was carried out while the author was visiting the mathematics department of the Cranfield Institute of Technology, Cranfield, England, under the Younger Scientists Exchange Scheme between India and the United Kingdom.

Journal of Thermal Stresses, 1:193-198, 1978

0149-5739/78/020193-06$2.75

BASIC EQUATIONS

Consider a static plane deformation parallel to the xy plane of an electrically conducting thermoelastic solid, due to the presence of an electric current $\mathbf{J}$. The stresses associated with such a deformation are given by [4]

$$\tau_{11} = \frac{\partial^2 \psi}{\partial y^2} + \phi \qquad \tau_{22} = \frac{\partial^2 \psi}{\partial x^2} + \phi \qquad \tau_{12} = -\frac{\partial^2 \psi}{\partial x\, \partial y}$$
$$\tau_{33} = \nu(\tau_{11} + \tau_{22}) - E\alpha T \qquad \tau_{13} = \tau_{23} = 0 \tag{1}$$

where E = Young's modulus
ν = Poisson's ratio
α = coefficient of linear thermal expansion
$T = T(x, y)$ = temperature associated with the deformation
$\phi = \phi(x, y)$ and $\psi = \psi(x, y)$ are governed by the equations

$$\nabla^2 \phi = \mu_e J^2$$
$$\nabla^4 \psi = (\nu^* - 1)\mu_e J^2 - E^* \alpha^* \nabla^2 T \tag{2}$$

where μ_e is the magnetic permeability of the body, and

$$E^* = \frac{E}{1 - \nu^2} \qquad \nu^* = \frac{\nu}{1 - \nu} \qquad \alpha^* = (1 + \nu)\alpha \tag{3}$$

In the presence of the mechanocaloric effect, the equation governing T in the static case is given by [1]

$$k\nabla^2 T + Q + m\left[\operatorname{div} \mathbf{F} + \frac{2E}{3(1 + \nu)} \nabla^2 e\right] = 0 \tag{4}$$

where e = dilatation
$\mathbf{F}$ = external force per unit volume
Q = strength of internal heat sources
k = thermal conductivity
$m(> 0)$ = mechanocaloric coupling constant

In the absence of mechanocaloric coupling ($m = 0$), we readily see from Eq. (4) that the thermal field does not interact with $\mathbf{F}$ and e, and Eq. (4) reduces to the classical heat equation considered in [4].

The dilatation e is related to τ_{11}, τ_{22}, and T through the relation (see [4])

$$e = \frac{1 - \nu^*}{E^*} (\tau_{11} + \tau_{22}) + 2\alpha^* T$$

This relation yields, with the help of Eqs. (1) and (2),

$$\nabla^2 e = \frac{1 + \nu^*}{E^*} \left[(1 - \nu^*)\mu_e J^2 + E^*\alpha^*\nabla^2 T\right] \tag{5}$$

If there is no external force other than that due to magnetomechanical interaction, and if there is no heat distribution other than Joule heating, we may take

$$\mathbf{F} = \mu_e(\mathbf{J} \times \mathbf{H}) \qquad Q = \frac{1}{\sigma}J^2 \tag{6}$$

where σ = electrical conductivity of the body
$\mathbf{H}$ = magnetic field associated with $\mathbf{J}$

Using Maxwell's electromagnetic equations and Ohm's law, we may readily verify that

$$\text{div } \mathbf{F} = \mu_e \text{ div } (\mathbf{J} \times \mathbf{H}) = -\mu_e J^2 \tag{7}$$

With the aid of Eqs. (5)-(7), Eq. (4) may now be simplified to

$$\left(k + \frac{2}{3} mE^*\alpha^*\right) \nabla^2 T = \left[\frac{1}{3} m (1 + 2\nu^*) - \eta\right] \mu_e J^2 \tag{8}$$

where η is the magnetic viscosity of the body; that is, $\eta = (\mu_e \sigma)^{-1}$.

By eliminating $\nabla^2 T$ from the second of Eqs. (2) and Eq. (8), we get

$$\nabla^4 \psi = \frac{1}{1 + 2M} \left[\nu^* + \delta - (1 + 3M)\right] \mu_e J^2 \tag{9}$$

where

$$M = \frac{1}{3}\frac{mE^*\alpha^*}{k} \qquad \delta = \frac{E^*\alpha^*\eta}{k} \tag{10}$$

When $\mathbf{J}$ is known, the first of Eqs. (2) and Eqs. (8) and (9) may be solved under appropriate boundary conditions to determine ϕ, T, and ψ. The stresses then follow from Eqs. (1).

ANALYSIS

We now turn our attention to the main problem. Let R be the radius of the cylinder, and J_0 the magnitude of the axial uniform current that the cylinder carries. If we choose the z axis along the axis of the cylinder, we may write $\mathbf{J} = (0, 0, J_0)$ in cylindrical coordinates (r, θ, z) and assume that all field variables depend on r only, because of the axial symmetry.

The first of Eqs. (2) and Eq. (8) may now be integrated directly to give

$$\phi = \frac{1}{4}\mu_e J_0^2 r^2 + C_1 \tag{11}$$

$$T = \frac{1}{4E^*\alpha^*(1+2M)}\left[(1+2\nu^*)M - N\right] J_0^2 r^2 + C_2 \tag{12}$$

where C_1 and C_2 are constants of integration, and

$$N = \frac{E^*\alpha^*\eta}{k} = \frac{E^*\alpha^*}{k\mu_e\sigma} \tag{13}$$

When the mechanocaloric effect is not taken into account, the solution (12) reduces to

$$T = -\frac{1}{4}\frac{\eta J_0^2 r^2}{k} + C_2 \tag{14}$$

which agrees with the solution given in [4].

By comparing Eqs. (12) and (14), we may conclude that the mechanocaloric effect makes a significant contribution to the thermal distribution in the cylinder. In the special case of infinite conductivity (i.e., when $\sigma \to \infty$), Eqs. (12) and (14) reduce, respectively, to

$$\begin{aligned} T &= \frac{(1+2\nu^*)MJ_0^2r^2}{4E^*\alpha^*(1+2M)} + C_2 \\ T &= C_2 \end{aligned} \tag{15}$$

We readily see that a thermal distribution of the Fourier type, equivalent to that due to a source of strength

$$Q = -\frac{(1+2\nu^*)M}{(1+2M)E^*\alpha^*} < 0 \tag{16}$$

occurs when the mechanocaloric effect is taken into account, and no thermal distribution occurs otherwise. The mechanocaloric effect thus represents a "heat sink" within the cylinder.

In view of the above analysis, we may conclude that if the cylinder is a good conductor of electricity, the presence of mechanocaloric coupling decreases the Joule heating. In fact, we may verify from Eqs. (10) and (13) that if σ, ν, μ_e, and m are connected by the relation

$$\sigma = \frac{3(1-\nu)}{\mu_e m(1+\nu)} \tag{17}$$

Eq. (12) yields $T = C_2$. Therefore, no thermal distribution occurs in the cylinder. This leads us to conclude that when relation (17) holds, the mechanocaloric effect nullifies the Joule effect.

We now take up the solution of Eq. (9). Because of the axial symmetry, this equation reduces to

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr}\right)\right]\right\} = \frac{[\nu^* + \delta - (1 + 3M)]\,\mu_e J_0^2}{1 + 2M}$$

By integrating this and noting that ψ is to be finite for $r = 0$, we get the following solution for ψ:

$$\psi = a_1 + a_2 r^2 + \frac{[\nu^* + \delta - (1 + 3M)]\,\mu_e J_0^2 r^4}{64(1 + 2M)} \tag{18}$$

If the boundary of the cylinder is stress free, we have $\tau_{rr} = 0$ on $r = R$. Equations (11) and (18), together with this boundary condition, yield the following expressions for stresses:

$$\tau_{rr} = \frac{1}{r}\frac{\partial\psi}{\partial r} + \phi = -\frac{\nu^* + 3 + \delta + 5M}{16(1 + 2M)}\left(1 - \frac{r^2}{R^2}\right)\mu_e J_0^2 R^2$$

$$\tau_{\theta\theta} = \frac{\partial^2\psi}{\partial r^2} + \phi = -\frac{\nu^* + 3 + \delta + 5M}{16(1 + 2M)}\left[1 - \frac{1 + 3(\nu^* + \delta) - M}{\nu^* + \delta + 3 + 5M}\,\frac{r^2}{R^2}\right]\mu_e J_0^2 R^2 \tag{19}$$

$$\tau_{r\theta} = 0$$

By examining these expressions we see that, unlike the classical situation [4], the stresses developed by the Lorentz force *cannot* be separated from those caused by the thermal distribution, because of the presence of $1 + 2M$ in the common denominators. Hence we may infer that the mechanocaloric coupling introduces nonlinearity into the behavior of stresses in the cylinder. However, this disappears when the mechanocaloric coupling effect tends to vanish, and expressions (19) then reduce to those obtained in [4].

We further verify from Eq. (19) that τ_{rr} is compressive everywhere and that $\tau_{\theta\theta}$ may be compressive, tensile, or zero as in the classical case. The expression for $\tau_{\theta\theta}$ on the boundary $r = R$ is given by

$$\tau_{\theta\theta}\big|_{r=R} = -\frac{1}{8}\left(1 - \frac{\delta + \nu^* - M}{1 + 2M}\right)\mu_e J_0^2 R^2 \tag{20}$$

In the special case when σ, μ_e, ν, and m are connected by relation (17), the expressions for τ_{rr} and $\tau_{\theta\theta}$ given by (19) reduce to

$$\tau_{rr} = -\frac{\nu^* + 3}{16}\left(1 - \frac{r^2}{R^2}\right)\mu_e J_0^2 R^2 \tag{21}$$

$$\tau_{\theta\theta} = -\frac{\nu^* + 3}{16}\left(1 - \frac{1 + 3\nu^*}{\nu^* + 3}\,\frac{r^2}{R^2}\right)\mu_e J_0^2 R^2 \tag{22}$$

It is readily seen that these stresses are due to the action of the Lorentz force only and are independent of the thermal field. This conforms with our earlier observation that when relation (17) holds, the mechanocaloric coupling effect and the Joule effect cancel each other and the cylinder experiences no change in temperature.

REFERENCES

1. E. L. Roetman, The Mechano-Caloric Effect in Thermoelastic Problems, *J. Eng. Math.*, vol. 10, pp. 355–362, 1976.
2. P. Chadwick, Thermoelasticity, the Dynamic Theory, in I. N. Sneddon & R. Hill (eds.), *Progress in Solid Mechanics*, vol. I, North Holland, Amsterdam, pp. 263–328, 1960.
3. H. W. Lord and Y. Shulman, A Generalized Dynamic Theory of Thermoelasticity, *J. Mech. Phys. Solids*, vol. 15, pp. 299–309, 1967.
4. K. Yuan, Magnetothermoelastic Stresses in an Infinitely Long Cylindrical Conductor Carrying a Uniformly Distributed Axial Current, *Appl. Sci. Res.*, vol. 26, pp. 307–314, 1972.

Communicated by R. S. Dhaliwal on February 3, 1978

Request reprints from D. S. Chandrasekharaiah.