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Magneto-elastic transverse surface waves in an internal stratum

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ABSTRACT

Transverse surface waves in a stratum of uniform thickness, bounded on both sides by very deep layers of different materials, are investigated in the context of magneto-elasticity. Assuming that all the three materials are perfect conductors of electricity, it is found that the waves can exist for all orientations of the initial magnetic field and that the field increases, in general, the bounds for phase velocity. Other effects of the field on the physical properties of the material and on the phase velocity and frequency of waves are also considered in detail.

1. INTRODUCTION

THE study of wave motion in electrically conducting elastic solids in the presence of magnetic fields has aroused much interest in recent years, because of its possible applications in geophysics, plasma physics, aerospace engineering and astrophysics.[1-2] Among many problems investigated,[3] the propagation of surface waves has been considered by several authors. Kaliski and Rogula[4] have considered Rayleigh waves in a perfectly conducting half-space and have shown that in the presence of a transverse magnetic field, the wave velocity increases with the intensity of the field and approaches asymptotic values higher than the classical Rayleigh velocity. The same authors have considered[5] Rayleigh waves along and around a right circular cylinder and a cylindrical cavity in an infinite space under an axial magnetic field and have shown that the axial waves die out with depth more rapidly than waves in the non-magnetic case. Purushotham[6] has considered transverse waves in a perfectly conducting half-space and has shown that these waves can be propagated, unlike in classical elasticity theory, whenever the initial magnetic field is such that the electromagnetic radiation into the adjacent free space is not zero. Murthy[7] has considered Love waves and has modified the classical results for perfectly conducting materials in the cases when the initial magnetic field is aligned with or transverse to the direction of propagation. All these investigations have confirmed that the magnetic fields have very well pronounced effects on the propagation of surface waves in elastic solids.

The purpose of the present paper is to investigate another interesting problem. We consider here transverse surface waves in a stratum of uniform thickness bounded on both sides by half-spaces of different materials, in the presence of a uniform magnetic field. This problem is of geophysical interest and in the classical (non-magnetic) case, it has been investigated by Stoneley.[8] We assume that all the three bodies are perfectly conducting and differ only in their shear moduli and densities, and show that the wave motion is possible for all orientations of the initial magnetic field. The waves are, in general, dispersive as in classical elasticity and a particular wave which is non-dispersive can, however, exist when the initial magnetic field is oblique to the direction of propagation. The amplitude of this wave depends on the strength and orientation of the initial magnetic field and tends to zero as the magnetic field tends to zero or becomes aligned with or transverse to the direction of propagation. The magnetic field increases the bounds for phase velocity in general and the phase velocity, in all cases, remains greater than the shear wave velocity of the stratum, as in the non-magnetic case. Other interesting effects of the magnetic field observed in the course of our analysis are recorded at the end of the paper.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS

The linearized equations governing the induced magnetic field $\boldsymbol{h}$ and the displacement field $\boldsymbol{u}$ in an infinitely conducting elastic solid which is initially immersed in a uniform magnetic field H are given by[6]

$$\boldsymbol{h} = \text{curl } (\boldsymbol{u} \times \boldsymbol{H}) \tag{1}$$

$$\mu \nabla^2 \boldsymbol{u} + (\lambda + \mu) \text{ grad div } \boldsymbol{u} + K (\text{curl } \boldsymbol{h} \times \boldsymbol{H}) = \rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} \tag{2}$$

In these equations λ and μ denote the Lamé constants, ρ denotes the mass density and K denotes the magnetic permeability of the body.

The mechanical stresses τ_{ij} associated with $\boldsymbol{u}$ and the electromagnetic stresses τ^*_{ij} associated with $\boldsymbol{h}$ are given, in the cartesian tensor notations, by

$$\tau_{ij} = \lambda u_{k,k} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) \tag{3}$$

$$\tau^*_{ij} = K [H_i h_j + H_j h_i - H_k h_k \delta_{ij}] \tag{4}$$

We choose the coordinate axes such that the internal stratum occupies the space $0 \leqslant z \leqslant T$ and that the wave motion takes place in the positive

x-direction. We may then take the displacement vector associated with the transverse waves in the form

$$\boldsymbol{u} = (0, v, 0) \tag{5}$$

where v is a function of x, z and t only.

Equations (1) and (2) then yield

$$h_1 = h_3 = 0, \ h_2 = H_1 \frac{\partial v}{\partial x} + H_3 \frac{\partial v}{\partial z} \tag{6}$$

$$H_2 \frac{\partial h_2}{\partial x} = H_2 \frac{\partial h_2}{\partial z} = 0 \tag{7}$$

$$\mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) + K \left(H_1 \frac{\partial h_2}{\partial x} + H_3 \frac{\partial h_2}{\partial z}\right) = \rho \frac{\partial^2 v}{\partial t^2}. \tag{8}$$

Equations (7) are identically satisfied if we assume that the initial magnetic field lies in the zx-plane.

If we denote the angle at which the wave crosses the initial magnetic field by ϕ, we may write $H_1 = H \cos\phi$ and $H_3 = H \sin\phi$, where H denotes the magnitude of $\boldsymbol{H}$. Equations (6) and (8) then yield, on eliminating h_2, the equation

$$(1 + R\cos^2\phi) \frac{\partial^2 v}{\partial x^2} + (1 + R\sin^2\phi) \frac{\partial^2 v}{\partial z^2} + R \sin 2\phi \frac{\partial^2 v}{\partial x \partial z} = \frac{1}{S^2} \frac{\partial^2 v}{\partial t^2} \tag{9}$$

where we have put

$$R = \frac{KH^2}{\mu}, \quad S^2 = \frac{\mu}{\rho}. \tag{10}$$

Here S represents the shear wave velocity of the body and if we denote RS^2 by A^2, A gives the Alfvén speed.

Equations (3), (5) and (4), (6) now give

$$\left.\begin{aligned}
&\tau_{11} = \tau_{22} = \tau_{33} = \tau_{13} = 0, \quad \tau_{12} = \mu \frac{\partial v}{\partial x}, \\
&\tau_{23} = \tau = \mu \frac{\partial v}{\partial z} \quad \tau^*_{11} = \tau^*_{22} = \tau^*_{33} = \tau^*_{13} = 0, \\
&\tau^*_{12} = KH^2 \cos\phi \left(\cos\phi \frac{\partial v}{\partial x} + \sin\phi \frac{\partial v}{\partial z}\right) \\
&\tau^*_{23} = \tau^* = KH^2 \sin\phi \left(\sin\phi \frac{\partial v}{\partial z} + \cos\phi \frac{\partial v}{\partial x}\right)
\end{aligned}\right\} \tag{11}$$

Equations (6) and (11) determine the induced magnetic field and the mechanical and electromagnetic stresses, when the displacement field is known. To determine the displacement field we solve the equation (9) under appropriate conditions that are to be satisfied on the interfaces $z = 0, T$.

If we assume that there is welded contact between the bodies, the conditions to be satisfied at $z = 0, T$ are the continuity of (i) displacementsy (ii) total stresses, (iii) tangential components of the total electric field, and (iv) normal components of the magnetic induction. Since the total electric field in an infinitely conducting body is zero, the condition (iii) is identicall) satisfied.[9] Since $h_3 = 0$, condition (iv) implies the continuity of KH, which in turn implies the continuity of K, as H_3 is a constant. We may assume that this condition is identically satisfied, since the magnetic permeabilities of different solids vary little from each other, in most of the cases.[10] The remaining conditions (i) and (ii) may be expressed in the form

$$v = \text{continuous on } z = 0, \ T \tag{12}$$

$$\tau = \text{continuous on } z = 0, \ T. \tag{13}$$

Condition (13) reduces with the help of equation (11), to

$$\mu \frac{\partial v}{\partial z} + KH^2 \sin\phi \left(\sin\phi \frac{\partial v}{\partial z} + \cos\phi \frac{\partial v}{\partial x}\right) = \text{continuous on } z = 0, \ T.$$

By virtue of the continuity of v, this simplifies to

$$(\mu + KH^2 \sin^2\phi) \frac{\partial v}{\partial z} = \text{continuous on } z = 0, \ T \tag{14}$$

Our problem is now governed by the equation (9) and the boundary conditions (12) and (14).

If we put

$$\zeta = az, \ \bar{\rho} = \rho/a, \ \bar{\mu} = (\mu + KH^2 \sin^2\phi)^{1/2} (\mu + KH^2 \cos^2\phi)^{1/2} \tag{15}$$

where

$$a = \left(\frac{\mu + KH^2 \cos^2\phi}{\mu + KH^2 \sin^2\phi}\right)^{1/2} \tag{16}$$

eqs (9), (12) and (14) assume the form

$$\bar{\mu}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial \zeta^2}\right) + KH^2 \sin 2\phi \frac{\partial^2 v}{\partial x \partial \zeta} = \bar{\rho} \frac{\partial^2 v}{\partial t^2} \tag{17}$$

$$\left.\begin{aligned} v &= \text{continuous} \\ \bar{\mu} \frac{\partial v}{\partial \zeta} &= \text{continuous} \end{aligned}\right\} \text{ on } \zeta = 0, \ aT \tag{18}$$

If $\phi = 0$ or $\pi/2$, eq. (17) reduces to

$$\bar{\mu}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial \zeta^2}\right) = \bar{\rho}\,\frac{\partial^2 v}{\partial t^2} \tag{19}$$

In the non-magnetic case, we have $\alpha = 1$ and eqs (19) and (18) reduce to

$$\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\,\frac{\partial^2 v}{\partial t^2} \tag{20}$$

$$\left.\begin{aligned} v &= \text{continuous} \\ \mu\,\frac{\partial v}{\partial z} &= \text{continuous} \end{aligned}\right\} \quad \text{on } z = 0,\ T \tag{21}$$

which are valid in the classical problem.[8]

Comparing eqs (18–19) with (20–21), we conclude that when the initial magnetic field is either aligned with the direction of propagation or perpendicular to the stratum, the effect of the field is equivalent to modifying the shear moduli and densities of all the three materials and the thickness of the stratum. These modifications are analysed in Section 4, in the context of the phase velocity equation.

3. Harmonic Waves: Bounds for Phase Velocity

We denote the bodies occupying the regions $z < 0$, $0 \leq z \leq T$ and $z > T$ by L_1, L_2 and L_3 respectively, and suffix all quantities associated with the body L_i with i, $i = 1, 2, 3$. For example, we use v_1 to denote v in L_1.

For waves of wavelength $2\pi/\gamma$ and phase velocity V, the amplitude of which decay with depth in the half-spaces L_1 and L_3 and is periodic in z in the stratum L_2, we seek a solution of eq. (9) in the form

$$v = \begin{cases} v_1 = A \exp\,[pz + i\gamma\,(x - vt)] & \text{in } L_1 \\ v_2 = (Be^{imz} + Ce^{inz}) \exp\,[i\gamma\,(x - Vt)] & \text{in } L_2 \\ v_3 = D \exp\,[-qz + i\gamma\,(x - Vt)] & \text{in } L_3 \end{cases} \tag{22}$$

where A, B, C and D are arbitrary constants, m and n are real ($\neq 0$) and p, q have positive real parts.

Putting v_1 for v in (9) from (22) and noting that p cannot be purely imaginary, we get the following condition on V:

$$\left.\begin{aligned} &V^2 < \epsilon_1 S_1^2 \quad \text{strictly}, \\ &\epsilon_1 = (1 + R_1)/(1 + R_1 \sin^2\phi),\ 1 \leqslant \epsilon_1 \leqslant 1 + R_1 \end{aligned}\right\} \tag{23}$$

Also, the admissible value for p is given by

$$p = \gamma\,(1 + R_1 \sin^2\phi)^{-1}\left[-\tfrac{1}{2}iR_1 \sin 2\phi + (1 + R_1 \sin^2\phi)^{1/2} \times \left(\epsilon_1 - \frac{V^2}{S_1^2}\right)^{1/2}\right] \tag{24}$$

Similarly, we get the following further conditions on V:

$$\left.\begin{aligned} &V^2 \geqslant \epsilon_2 S_2^2 \\ &\epsilon_2 = (1 + R_2)/(1 + R_2 \sin^2\phi),\ 1 \leqslant \epsilon_2 \leqslant 1 + R_2 \end{aligned}\right\} \tag{25}$$

$$\left.\begin{aligned} &V^2 < \epsilon_3 S_3^2 \text{ strictly,} \\ &\epsilon_3 = (1 + R_3)/(1 + R_3 \sin^2\phi),\ 1 \leqslant \epsilon_3 \leqslant 1 + R_3 \end{aligned}\right\} \tag{26}$$

The admissible values for q, m and n are then given by

$$q = \gamma\,(1 + R_3 \sin^2\phi)^{-1}\left[\tfrac{1}{2}\, iR_3 \sin 2\phi + (1 + R_3 \sin^2\phi)^{1/2} \times \left(\epsilon_3 - \frac{V^2}{S_3^2}\right)^{1/2}\right] \tag{27}$$

$$\left.\begin{aligned} &m = -a + b,\ n = -a - b \\ &a = \tfrac{1}{2}\gamma\,(1 + R_2 \sin^2\phi)^{-1} R_2 \sin 2\phi \\ &b = \gamma\,(1 + R_2 \sin^2\phi)^{-1/2}\left(\frac{V^2}{S_2^2} - \epsilon_2\right)^{1/2} \end{aligned}\right\} \tag{28}$$

It is important to note that in view of eq. (28), the equality ($V^2 = \epsilon_2 S_2^2$) in eq. $(25)^1$ can hold good only when the initial magnetic field is not absent and cuts the direction of propagation obliquely.

With the expressions for p, q, m and n given by eqs (24), (27) and (28), eq. (22) gives a solution of required type, of the eq. (9), provided conditions (23), (25) and (26) are satisfied. The phase velocity of waves of required type, if they exist, therefore satisfies the following inequalities.

$$\epsilon_2 S_2^2 \leqslant V^2 < \epsilon_1 S_1^2,\quad \epsilon_2 S_2^2 \leqslant V^2 < \epsilon_3 S_3^2.$$

Combining these together, we get

$$\epsilon_2 S_2^2 \leqslant V^2 < \min\,(\epsilon_1 S_1^2,\ \epsilon_3 S_3^2). \tag{29}$$

In the absence of the initial magnetic field, we have $\epsilon_i = 1$, and the inequality (29) reduces to

$$S_2^2 < V^2 < \min\,(S_1^2,\ S_3^2) \tag{30}$$

This agrees with the one proposed by Stoneley[8] in the classical case.

Suppose the initial magnetic field is aligned with the direction of propagation, *i.e.*, $\phi = 0$. Then we have $\epsilon_i = 1 + R_i$. Inequality (29) then reduces to

$$(1 + R_2) S_2^2 < V^2 < \min [(1 + R_1) S_1^2, (1 + R_3) S_3^2]. \tag{31}$$

Comparing (31) with (30), we see that the initial magnetic field increases the lower bound for V from S_2 to $S_2 (1 + R_2)^{1/2}$ and the upper bound from S_1 to $S_1 (1 + R_1)^{1/2}$ or S_3 to $S_3 (1 + R_3)^{1/2}$, depending on whether $S_1 (1 + R_1)^{1/2}$ is less than or greater than $S_3 (1 + R_3)^{1/2}$. It is clear that the increase in the lower bound of V^2 is equal to the square of the Alfvén speed of the stratum and that in the upper bound is equal to the square of the Alfvén speed of one of the half-spaces adjacent to the stratum.

Suppose the initial magnetic field is perpendicular to the stratum, *i.e.*, $\phi = \pi/2$. Then we have $\epsilon_i = 1$. Inequality (29) then reduces to

$$S_2^2 < V^2 < \min (S_1^2, S_3^2)$$

from which it is clear that the initial magnetic field has no effect on the bounds of V.

Suppose the initial magnetic field is oblique to the direction of propagation. Then, since we have $1 < \epsilon_i < 1 + R_i$, the initial magnetic field increases the bounds in this case also. It is interesting to note that, in this case, unlike in other cases, V can reach its lower bound, *viz.*, $\epsilon_2^{\frac{1}{2}} S_2$.

From the above discussion it follows that the initial magnetic field increases the bounds for phase velocity, in general. The increase depends not only on the physical properties of the bodies and the strength of the initial magnetic field, but also on the orientation of the field. The increase is maximum when the field is aligned with the direction of propagation and it is zero when the field is perpendicular to the stratum. The phase velocity can reach its lower bound only when the magnetic field is not absent and it cuts the direction of propagation obliquely. In all cases, including the non-magnetic case, the phase velocity remains greater than the shear wave velocity of the stratum.

4. Equation for Phase Velocity

We now derive the equation determining V, by applying the boundary conditions (12) and (14) to the solution (22) of eq. (9).

The conditions (12) and (14) together with eqs. (22) give

$$(pM_1 - imM_2) B + (pM_1 - inM_2) C = 0 \tag{32}$$

$$(qM_3 + imM_2)\, Be^{im\tau} + (qM_3 + inM_2)Ce^{in\tau} = 0 \tag{33}$$

where we have put

$$\begin{aligned} M_i &= \mu_i (1 + R_i \sin^2 \phi) \\ &= \rho_i S_i^2 + K_i H^2 \sin^2 \phi,\ i = 1,\ 2,\ 3 \end{aligned} \tag{34}$$

on using eqs (10).

Since the arbitrary constants B, C cannot vanish together, the determinant of the coefficients of these in eqs (32)-(33) should vanish. This gives the following equation:

$$\frac{(pM_1 - imM_2)\ (qM_3 + inM_2)}{(pM_1 - inM_2)\ (qM_3 + imM_2)} = e^{2ib\tau}. \tag{35}$$

This equation determines the phase velocity V when the expressions for p, q, m, n and M_i are substituted from eqs (24), (27), (28) and (34). Since the simplification of eq. (35) does not yield any interesting result as such, we restrict ourselves to show that this equation has a root for V which satisfies the inequality (29), for all orientations of the initial magnetic field.

In the absence of the initial magnetic field, (35) reduces, after simplification, to

$$\tanh\ (r_2 T) = \frac{-\rho_2 r_2 S_2^2\ (\rho_1 r_1 S_1^2 + \rho_3 r_3 S_3^2)}{\rho_2^2 r_2^2 S_2^4 + \rho_1 \rho_3 r_1 r_3 S_1^2 S_3^2} \tag{36}$$

where

$$r_i = r\left(1 - \frac{V^2}{S_i^2}\right)^{1/2}$$

This is identical with the equation obtained by Stoneley[8] in the classical case, apart from a change in notations. He has shown that eq. (36) gives a value of V which satisfies the inequality

$$S_2^2 < V^2 < \min\ (S_1^2,\ S_3^2)$$

and has concluded that the waves of the required type can exist.

We now suppose that the initial magnetic field is aligned with the direction of propagation ($\phi = 0$). Then eq. (35) reduces to

$$\tanh\ (r_2' T') = \frac{-\rho_2' r_2'\ (S_2')^2\ [\rho_1' r_1'\ (S_1')^2 + \rho_3' r_3'\ (S_3')^2]}{(\rho_2' r_2')^2\ (S_2')^4 + \rho_1'\ \rho_3'\ r_1' r_3'\ (S_1' S_3')^2} \tag{37}$$

where we have put

$$r_i' = \gamma\left(1 - \frac{V^2}{S_i^2\,(1 + R_i)}\right)^{1/2},\ \rho_i' = \rho_i(1 + R_i)^{-1/2},$$

$$S_i' = S_i\,(1 + R_i)^{\frac{1}{2}},\ T' = T\,(1 + R_2)^{1/2},$$

Comparing equations (37) with (36), we see that the two equations are identical, provided we replace ρ_i by ρ_i', S_i by S_i' and T by T' in eq. (36). We note that replacing ρ_i by ρ_i' and S_i by S_i' is equivalent to replacing μ_i by $\mu_i(1+R_i)^{1/2}$. It follows therefore that the effect of the initial magnetic field, in this case, is equivalent to decreasing the densities and increasing the shear moduli of all the three bodies, and also increasing the thickness of the stratum. This is essentially what we observed in section 2, through eqs (18)–(21). Since eq. (36) has a root for V in the range $S_2^2 < V^2 < \min(S_1^2, S_3^2)$, it follows from the above comparison, that eq. (37) has a root for V in the range

$$S_2^2(1+R_2) < V^2 < \min\,[S_1^2(1+R_1), S_3^2(1+R_3)].$$

This may, however, be verified directly from eq. (37) also.

Next, we suppose that the initial magnetic field is perpendicular to the stratum ($\phi = \pi/2$). Equation (35) then reduces to

$$\tanh(r_2 T'') = \frac{-\rho_2'' r_2 S_2^2(\rho_1'' r_1 S_1^2 + \rho_3'' r_3 S_3^2)}{(\rho_2'' r_2)^2 S_2^4 + \rho_1'' \rho_3'' r_1 r_3 S_1^2 S_3^2} \tag{38}$$

where

$$\rho_i'' = \rho_i(1+R_i)^{1/2}, \quad T'' = T(1+R_2)^{-1/2}.$$

Comparing eq. (38) with (36) we see that the two equations are identical, provided we replace ρ_i by ρ_i'' and T by T'' in eq. (36) without changing S_i. It follows therefore that the effect of the initial magnetic field, in this case, is equivalent to increasing the densities as well as the shear moduli of all the three bodies, and decreasing the thickness of the stratum. This is essentially what we observed in section 2. This comparison between eqs (36) and (38) lead us to conclude that eq. (38) has a root for V in the range $S_2 < V < \min(S_1, S_3)$. This may also be verified directly.

We now suppose that the initial magnetic field is oblique to the direction of propagation. In this case the possible range for V is [*vide* eq. (29)].

$$\epsilon_2 S_2^2 \leq V^2 < \min\;(\epsilon_1 S_1^2,\; \epsilon_3 S_3^2).$$

We readily verify that $V = \epsilon_2^{1/2} S_2$ satisfies this inequality and also eq. (35). It follows therefore that eq. (35) has an admissible root for V when the magnetic field is oblique to the direction of propagation also.

Thus eq. (35) has an admissible root for V for all orientations of the initial magnetic field and we may therefore conclude that waves of the required type are possible for all directions of the initial magnetic field.

We observe from eq. (35) that V depends on γ in general because of the presence of γ in the expression for b, and V can be independent of γ only when $b = 0$. We further verify that b can be zero iff $V = \epsilon_2^{\frac{1}{2}} S_2$ and, as we have already seen, this is a possible phase velocity only when the initial magnetic field is not absent and acts obliquely to the direction of propagation. It follows therefore that the waves are in general dispersive and a non-dispersive wave can arise only when the initial magnetic field is not zero and acts obliquely to the direction of propagation.

5. ANALYSIS OF AMPLITUDE FUNCTIONS

The amplitude of waves in the regions $-\infty < z < 0$, $0 \leq z \leq T$ and $T < z < \infty$ are respectively given by A exppz, B exp$imz + C$ expinz and D exp $(-qz)$. We now analyse the effects of the initial magnetic field on these functions by considering again different orientations of the field.

Suppose the field is aligned with the direction of propagation. Then from eqs (24) and (27) we see that the values of p and q are greater than their corresponding values in the non-magnetic case. Accordingly, the amplitudes of waves in the half-spaces L_1 and L_3 decay with depth more rapidly in this case than in the non-magnetic case. Further, the value of b in this case is less than its value $S_2 = ir_2$ in the non-magnetic case. Since in the non-magnetic case the amplitude in the stratum has its period equal to $2\pi/S_2$, we may conclude that the initial magnetic field increases the period of amplitude in the stratum, in this case.

Suppose the initial magnetic field is perpendicular to the stratum. Then we see that [eqs (24), (27)–(28)] the values of p, q, and b are less than their corresponding values in the non-magnetic case. Accordingly, the amplitude in the half-spaces L_1 and L_3 decays with depth slowly, compared to the non-magnetic case and the period of amplitude in the stratum remains greater than that in the non-magnetic case.

Suppose the initial magnetic field is oblique to the direction of propagation. Then, unlike in other cases including the non-magnetic case, p and q are complex numbers; the imaginary part of these depend on the strength and orientation of the initial magnetic field, *vide* equations (24), (27). It therefore follows that the initial magnetic field introduces a periodic part into the amplitude in the half-spaces L_1 and L_3 in this case, unlike in all other cases where it is purely exponential. Further, since the strength and orientation of the initial magnetic field also influences the real parts of p and q as well as m and n, the decay of amplitude in the bodies L_1 and L_3 and the period of amplitude in the stratum also get affected by the field.

We have already seen that non-dispersive waves with phase velocity $V = \epsilon_2^{\frac{1}{2}} S_2$ is possible when the initial magnetic field is oblique to the direction of propagation. The amplitude of this wave in the stratum is given by

$$(B + C) \exp\ [(\tfrac{1}{2} \sin 2\phi)/(1 + R_2 \sin^2\phi)] irz.$$

Clearly, for a given wavelength, the amplitude depends on the strength and orientation of the initial magnetic field and becomes zero whenever the initial magnetic field is either zero or whenever it is aligned with the direction of propagation or perpendicular to the stratum. A wave with this amplitude in the stratum, therefore, cannot be encountered in pure elastic and other magneto-elastic cases.

6. CONCLUSIONS

Under the assumptions made, the waves of required type can exist for all orientations of the initial magnetic field; their phase velocity remains greater than the shear wave velocity of the stratum in all cases.

The initial magnetic field increases, in general, the bounds for phase velocity. This increase depends on the orientation of the field and it is maximum when the field is aligned with the direction of propagation. There is no increase in the bounds when the field is perpendicular to the stratum.

When the initial magnetic field is aligned with the direction of propagation, the effect of the field is equivalent to increasing the shear moduli and decreasing the densities of all the three bodies and also increasing the thickness of the stratum. The waves in the half-spaces adjacent to the stratum decay more rapidly with depth when compared to the non-magnetic case, and the period of amplitude in the stratum remains greater than that in the classical case.

When the initial magnetic field is perpendicular to the stratum, the effect of the field is equivalent to increasing the densities as well as the shear moduli of all the three bodies and decreasing the thickness of the stratum. The waves in the half-spaces adjacent to the stratum decay slowly when compared to the non-magnetic case, and the period of amplitude in the stratum remains greater than that in the non-magnetic case.

When the initial magnetic field is oblique to the stratum, the field introduces a periodic part into the amplitude in the half-spaces adjacent to the stratum and affects its decay. The field also affects the period of amplitude in the stratum.

A wave motion that cannot be encountered in the classical theory can be observed when the initial magnetic field is oblique to the direction of propagation. The amplitude of this wave depends entirely on the wavelength and strength and orientation of the initial magnetic field and, unlike waves in other cases, this wave is non-dispersive in nature.

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