2015

Bi-Level Mathematical Modelling and Heuristics for Cellular Manufacturing Facility Layout Problem

Maral Zafar Allahyari
University of Windsor

Bi-Level Mathematical Modelling and Heuristics for Cellular Manufacturing Facility Layout Problem

by

Maral Zafar Allahyari

A Thesis
Submitted to the Faculty of Graduate Studies
through Industrial & Manufacturing Systems Engineering
in Partial Fulfillment of the Requirements for
the Degree of Master of Applied Science
at the University of Windsor

Windsor, Ontario, Canada

2014

DECLARATION OF PREVIOUS PUBLICATION

This thesis includes one original paper that has been previously submitted for publication in a peer reviewed conference, as follows:

Publication title and full citation	Publication Status
Allahyari Maral Zafar, Azab A., A Novel Bi-level Continuous Formulation for the Cellular Manufacturing System Facility layout Problem, 2014, 9th CIRP Conference on Intelligent Computation in Manufacturing Engineering.	published
Allahyari Maral Zafar, Azab A, Improved Bi-level Mathematical Programming and Heuristics for the Cellular Manufacturing Facility Layout Problem, 2015, 10th ASME Manufacturing Science and Engineering Conference	Submitted

I certify that I have the permission from the copyright owner(s) to include the above published material(s) in my thesis. I certify that the above material describes work completed during my registration as graduate student at the University of Windsor.

I certify that, to the best of my knowledge, my thesis does not infringe upon anyone's copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledged in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owner(s) to include such material(s) in my thesis and have included copies of such copyright clearances to my appendix.

I declare that this is a true copy of my thesis, including any final revisions, as approved by my thesis committee and the Graduate Studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.

CO-AUTHORSHIP DECLARATION

I hereby declare that this thesis incorporates material that is result of joint research, as follows:

"This thesis also incorporates the outcome of a joint research undertaken in collaboration with Dr. Ahmed Azab."

I am aware of the University of Windsor Senate Policy on Authorship and I certify that I have properly acknowledged the contribution of another researcher to my thesis, and have obtained written permission from each of the co-author to include the above material(s) in my thesis. I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.

ABSTRACT

In this thesis, a bi-level mixed-integer non-linear programming continuous model has been, developed for both intra-cell and inter-cell layout design sequentially. Facilities are assumed unequal sizes, and operation sequences and part demands are considered. The model includes overlap elimination, aisle, and block constraints. Since the model is nonlinear, the model has been linearized and solved exact. However, the facility layout problem is NP-hard; hence, novel heuristics and a meta-heuristic have been designed and implemented to solve the problem in a similar manner- both at intra- and inter-cellular levels. A real case study from the metal cutting inserts industry has been used where multiple families of inserts have been formed each with its distinguished master plan. C++ has been used for implementation of the algorithms. For mathematical programming, the model is being solved by the Xpress optimization tool using a branch-and-bound method to illustrate the performance of the model.

DEDICATION

به نام خداوند جان و خرد   کزین برتر اندیشه برنگذرد

IN the name of the Lord of both wisdom and mind, To nothing sublimer can thought be applied

All challenging work requires individual efforts as well as supports of others specially those ones who are in our heart.

I dedicate my work to

My Beloved Family

Whose encouragement, affection and prays facilitate my way to achieve my goal and honor.

Besides all, I appreciate efforts and hardworking of my respectable Teachers.

ACKNOWLEDGEMENTS

I want to thank my supervisor, Dr. Ahmed Azab for the wonderful support and guidance in completion of this thesis.

I would also acknowledge the guidance from committee members Dr. Baki and Dr. Tepe.

Moreover, I have to appreciate Saneeha’s help and support in write up of my thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDECIES

LIST OF ACRONYMS

MIP	Mixed Integer Programming
NLMIP	NonLinear Mixed Integer Programming
QAP	Quadratic Assignment Program
FLP	Facility Layout Problem
DFLP	Dynamic Facility Layout Problem
CMS	Cellular Manufacturing System
CM	Cellular Manufacturing
GF	Group Formation
CL	Cell Layout
GT	Group Technology
GS	Group Scheduling
MHC	Material Handling Cost
UA-FLP	Unequal sized Facility Layout Problem
SA	Simulate Annealing
GA	Genetic Algorithm
TS	Tabu Search

NOMENCLATURE

List of Nomenclature- Nonlinear Mixed Integer Model

$P = \{1,2,3,\dots,P\}$	Index set of part types
$M = \{1,2,3,\dots,M\}$	Index set of machine types
$C = \{1,2,3,\dots,C\}$	Index set of cell types
$O_p = \{1,2,3,\dots,O_p\}$	Index set of operations indices for part p
L	Horizontal dimension of shop floor
W	vertical dimension of shop floor
Y_{VALU}	Vertical dimension of upper side of aisle
Y_{VALL}	Vertical dimension of lower side of aisle
X_{HALLF}	Horizontal dimension of left side of aisle
X_{HALRT}	Horizontal dimension of right side of aisle
l_i	Length of machine *i*
w_i	Width of machine *i*
$\acute{l}_c$	Length of cell *c*
$\acute{w}_c$	Width of cell *c*
CA_j	Intracellular transfer unit cost for part *j*
CE_j	Intercellular transfer unit cost for part *j*
D_j	Demand quantity for part *j*

U_{joi} 1, if operation o of part *j* is done by machine *i*, otherwise 0

U'_{joc} 1, if operation o of part *j* is done by machine i which is located in cell c, otherwise 0

Q_{ic} 1, if machine i is assigned in cell c

x_i Horizontal distance between center of machine *i* and vertical reference line

y_i Vertical distance between center of machine *i* and horizontal reference line

x_c Horizontal distance between center of cell *c* and vertical reference line

y_c Vertical distance between center of cell *c* and horizontal reference line

Z_{iu} 1, if machine *u* is arranged in the same horizontal level as machine *i*, and 0 otherwise

$W_{c\acute{c}}$ 1, if cell c is arranged in the same horizontal level as cell $\acute{c}$ and 0 otherwise

Z_c 1, if cell c is arranged in out of horizontal aisle boundaries and 0 otherwise

W_c 1, if cell c is arranged in out of vertical aisle boundaries and 0 otherwise

List of Nomenclature- Linear Model

QX_{iu} 1, if horizontal dimension of facility i is greater than horizontal dimension of facility u

QY_{iu} 1, if vertical dimension of facility i is greater than vertical dimension of facility u

$QX_{c\acute{c}}$ 1, if horizontal dimension of cell c is greater than horizontal dimension of cell $\acute{c}$

$QY_{c\acute{c}}$ 1, if vertical dimension of facility c is greater than vertical dimension of facility $\acute{c}$

List of Nomenclature- Block Constraints

K	Number of blocks
$xblock_k$	The horizontal coordinate of block *k*
$yblock_k$	The vertical coordinate of block *k*
$lblock_k$	The length of block *k*
$wblock_k$	The width of block *k*
$\acute{Z}_{ck}$	1 if cell c is arranged in the same horizontal level as block k, and 0 otherwise

CHAPTER ONE: INTRODUCTION

1.1. Background

In this thesis, the facility layout problem (FLP) for particular class of manufacturing systems, where is cellular manufacturing system (CMS) has been tackled. In this section; the background and physics of the different elements pertaining to the problem at hand are explained. We start by explaining what is CMS; that is to be followed by definition of FLP and finally, some synopsis of the overall approach taken has been provided.

1.1.1. Cellular Manufacturing System

Cellular manufacturing system (CMS) layout has recently begun to receive heightened attention worldwide. Cellular Manufacturing (CM) is an application of GTCM is the combination of job shop and/or flow shop. In CM the site is divided physically into small groups which each are dedicated to parts which have similarities in process and operations requirement, machinery. The groups are called cells and the similar parts are named as part families. Generally speaking, each cell is designed to produce a part family. However, in the real world converting to pure CMS is impossible. Usually there are some parts that cannot be categorized in unique part family. Hence, the whole production process cannot be finished in one cell. Furthermore, there are some machine tools used as general utilization, these kinds of machine tools cannot be placed in specific cell. These kinds of parts and machine tools are placing in specific cells called reminder cell. There are some machine tools which cannot be assigned in specialized cell or reminder cell because of safety or economic issues such as those machine tools which produce too much heat that have to be placed in specific area of shop (Green & Sadowski, 1984).

The design of a CMS includes (1) cell formation (CF) – grouping parts which have similarities in design features or processing requirements into part families and associated machines into machine cells, (2) group layout – laying out machines within each cell (intra-cell layout) and cells with respect to each other (inter-cell layout), (3) group scheduling – scheduling parts and part families for production, and (4) resource allocation – assigning tools and human and materials resources.

An effective CMS implementation help any company improve machine utilization and quality; it also makes reduction in setup time, work-in-process inventory, material handling cost, part makespan, and expediting costs (Wemmerlov & Johnson, 1997; Ariafar *et al.,* 2011; Defersha and Chen, 2006).

1.1.2. Facility Layout Problem

Facility layout problem (FLP) is the arrangement of a given number of non-equal sized facilities within the given space. Good layout plan leads to improve machine utilization, part demand quality, efficient setup time, less work-in-process inventory and material handling cost. Generally speaking efficient layout design provides two main advantages, 1. Reduction of between %30 to %70 in total material handling cost (MHC), and 2. Designing layout is the long term plan. Hence, any changes in layout impose some expenditure such as shutting down production or service line, losing process time and so on. Thus, designing proper facility layout plan would prevent lots of costs (Yaman, 1993). As discussed in the literature the objective of FLP is minimizing total material handling cost (MHC) by considering these two constraints: 1. all facilities have to be placed within the site boundaries; and 2. facilities cannot overlap. There are three main parameters using in calculating MHC: f_{ij} the interaction or flows between facilities i and j; c_{ij} unit cost value for flows or interaction's movement between facilities i and j; the last one is r_{ij} closeness rating between facilities i and j (Meller and Gau, 1996). The comprehensive survey about FLP has been done by Drira *et. al.,* (2007).

1.1.3. Approach

Several algorithms have been developed for FLP problem. Traditional approach to FLP problem called discrete representation often addressed by quadratic assignment problem (QAP) with the objective of minimizing a given function cost. There are two main assumptions in QAP: firstly all facilities are equal size and shape; secondly the locations of facilities are known in a priori. However these kinds of assumptions are not applicable in real world case studies. This approach to FLP is not suited to represent the exact locations of facilities; and cannot formulate FLP especially when facilities are unequal size and shape or if there are different clearances between the facilities. The more proper approach to such kind of cases is continuous representation rather than discrete. There are two ways to solve this problem. Chronologically, the first one attempts at dividing each facility into smaller size unit blocks, where the total area of those blocks is approximately equal to the area of the facility. There are two drawbacks to this method: firstly the problem size is growing as the total number of blocks increase, and secondly the exact shapes of facilities are ignored. The second approach to continuous problem assumes the exact shape and dimensions of the facilities.

Table (1): FLP Discrete approach versus. FLP continuous approach

Approach	**Plant site**	**Distance**	**Facilities**	**Mathematical Formulation**
Discrete	Divided in rectangular blocks with same size and shape; *i.e.*, predetermined locations	Parameters **(Meller et al., 1999)**	Equal-sized	QAP
Continuous	No predetermined location, *i.e.*, no blocks	Variable	Unequal-sized	MIP

1.2. Motivation of thesis

FLP to CMS is focusing on the second step of design of CMS which by itself is two folds: inter-cell and intra-cell layout. The main objective of group layout is minimizing material handling cost (MHC) by arranging facilities in their corresponding cells and cells in floor. In this work, both demand and operation sequencing have been considered in optimizing the layout both at inter and intra cellular levels However, this was not the case with literature; there is a dearth of papers that happened to take a discrete approach which really did addressed those factors. Moreover, in this thesis we are adopting a continuous approach.

In this work, the detailed cellular layout problem has been addressed for both shop floor level and that at cellular level. In the literature, however for CMS significant of the work has solved only the block layout problem where layout problem at inter cellular was addressed.

The third motivation is taking more effective approach to FLP problem, *i.e.,* taking continuous approach. From proper engineering and practical outlook there is no predetermined location for facilities. By assuming specific locations for facilities, the chance to get more effective layout design is being decreased. Because lots of facilities' arrangements options are ignored. Moreover, by taking continuous approach the limitation of facilities' size is relaxed. Hence, in the developed model there is no restriction to the size of the facilities.

Finally, since FLP is a NP-hard problem, developing heuristics algorithms is the other motivation of this thesis. Designing heuristics algorithm for discrete FLP is easier than continuous approach. Because, the only operator needs is swap operator, *i.e.,* switching facilities' locations. Additionally, since locations are predetermined then overlap will happen among facilities and/ or facilities and site boundaries. However, when there is no location known in priori developing heuristic algorithm requires designing two operators such as move operator and swap operator- Move operator tries to decrease the distance between facilities. The chances of overlap in this kind of problems are high which requires designing variety of repair function to eliminate overlap.

1.3. Outline of the thesis

This thesis organized in the next few chapters. Problem statement and literature review is explained in chapter2. Chapter 3 includes mathematical modeling. The heuristics is presented in chapter 4. Chapter 5 includes the case study and computational results. Conclusion and future work are illustrated in chapter 6.

CHAPTER TWO: LITERATURE REVIEW

2.1. Problem definition

The facility layout problem (FLP) for cellular manufacturing system (CMS) is considered in this thesis. By taking systematic manufacturing outlook to FLP, the problem is to arrange facilities that are cells in the leader problem and machine tools in the follower problem in the continual planar site. The physics of the problem is as follows: machine tools, cells, and shop have rectangular shape with specific length and width.

To determine the flow rate operations sequence and different part's demand are considered. Each part compromises certain operations with specific sequence which is processed by dedicated machine tools. By predetermining group formation ahead of time; it has already known which machine belongs to which cell; and which operation of machine is processed in which cell; *i.e.*, operations of part j processed in cell k are known ahead of time. Speaking about the material flows between facilities the traffic within a cell is the material flow among the machines located in cell, and at shop floor (factory) level material flows between cells are actually the flows among the operations of parts on machines done in each cell. Therefore, the objective function in both levels is minimizing material handling cost (MHC) which is $flow \times distance$.

2.2. Relevant Literature Review

Literature review has two main folds, in the first part previous paper done in quadratics assignment problem (QAP) problem- discrete approach is reviewed, and then mixed integer programming (MIP) problem- continuous approach toward FLP problem is considered.

2.2.1. Discrete Approach

QAP is NP-complete problem which means that when the size of the problem is increasing that it cannot be solved by exact algorithm (Wilhelm and Ward, 1987). Hence, lots of efforts have been taken place to develop and apply heuristic and metaheuristic algorithm for this kind of problem. Wilhelm and Ward (1987) apply simulated annealing (SA) to solve QAP. Their results have been compared with the Computerized Relative Allocation of Facilities Technique (CRAFT), biased sampling and revised Hillier problem and showed better quality solutions.

Baykasoglu and Gindy (2001) apply the SA for dynamic layout problem, discrete approach. They claim their proposed algorithm finds better solution. They compared their proposed algorithm to the three works done such as Rosenblatt (1986), Conway and Vekataramanan (1994), and Balakrishnan and Cheng (2000). In the first comparison, their SA approach found optimum solution and revealed better solution than dynamic programming algorithm of Rosenblatt (1986). The second comparison has two experiments; first one done with no shifting cost and the SA algorithm found optimum solution and outperforms that Conway and Vekataramanan (1994) genetic algorithm. In this experiment relocation costs are included. The optimum solution was not found, however the results of SA showed a slight improvement than outputs of Conway and Vekataramanan (1994). Finally, in third comparison the data set obtained from Balakrishnan and Cheng (2000). They develop nonlinear genetic algorithm (NLGA). The comparison between SA-based approach and NLGA reveals the superiority of SA algorithm when the size of the problems is large. Since they have taken discrete approach to FLP, the only operator has been used in neighbourhood generation algorithm is the swap operator.

Tavakolli-Moghaddam *et al.,* (2005) develop a nonlinear mathematical modelling to solve the cell formation in dynamic environment in which demand varies in each time horizon. The strength point of their model is that it is a multi-objective model *i.e.*, considering more than one objective such as machine cost, operating cost, inter-cell material handling cost, and machine relocation cost. Three metaheuristic models such as genetic algorithm (GA), simulated annealing (SA), and tabu search (TS) have been used to solve this problem. The results show SA outperforms compare to the two metaheuristics.

Defersha and Chen (2006) provide a comprehensive manufacturing attributes used in CF design. They develop a MIP multi-objective mathematical programming for cell formation problem. The proposed model tries to minimize machine maintenance and overhead cost, machine procurement cost, inter-cell travel cost, machine operation and setup cost, tool consumption cost, and system configuration cost. The model incorporates several factors such as dynamic cell configurations, alternative routings, lot splitting, sequence of operations, multiple processing, tool consumption cost, set up cost, cell size limits, and machine adjacency constraints. They provide some numerical examples for small size of problem. No heuristic algorithm has been presented in their work.

Wu *et. al.,* (2006) propose a mathematical model to solve GF and GL (inter-cell and intra-cell) concurrently by minimizing total travel cost (inter and intra-cell) and the number of exceptional elements. They incorporate important factor such as part demand, machine capacity, operation sequence, transfer batch. Finally, a hierarchical genetic algorithm (HGA) is developed to solve the problem. In another study Wu *et al.,* (2007a) propose a HGA form manufacturing cells and determine the group layout of a CMS concurrently. The novelty of their presented algorithm is a new hierarchical chromosome structure, a selection scheme, and a group mutation operator.

Tavakoli-Moghaddam *et al.,* (2007) develop a nonlinear model for GF both inter-cell and intra-cell movement. The special feature of their work is that they are considering stochastic demand. They assume equal sized machine tools and cells; also unrestricted shop floor. It means that there is no restriction on the shape and dimensions of the shop floor. In order to prove their model, they use numerical example and no heuristic model has been developed.

Safaei *et. al.,* (2008) develop a mixed integer programming model which tries to minimize machine constant and variable costs, inter and intra material handling cost and reconfiguration costs. They present a hybrid model called mean field annealing and simulated annealing (MFA-SA) to solve the problem. MFA stands for mean field annealing which used to find the feasible initial solution for SA. Their work has some positive features such as considering all aspects of reconfiguration such as adding, removing and replacing machine tools. Moreover, maximum cell size and machine time capacity are the two main constraints considered in this model. These constraints make sense because it is not efficient to make one cell too crowed and the other one not as. Furthermore, machine capacity also is considered in this model. The other point is using operation sequence in calculating inter and intra material handling cost There are some drawbacks to the work as well. Firstly all machine tools assumed have equal size. Secondly, the other assumption is the equal distance between all cells and machine tools which is not happening in very realistic.

Airafar *et al.,* (2011) present a mathematical formulation to for facility layout plan in a hybrid cellular manufacturing system and develop a SA algorithm to solve the model. The interesting point of their model is that the demand varies during planning horizon. However, like as other QAP models they assume the equal size of facilities which is not applicable in real world cases. The other drawback to their model is that the shape and size of the shop floor is unrestricted, while it is not happen in any real case studies. In another study Airafar *et al.*, (2012) investigate the effect of demand variation on arrangement of facilities *i.e.* the demand has normal distribution. They develop a stochastic nonlinear integer programming by these assumptions that all facilities are equal sized, and there is no restriction on shape and dimension of shop floor. These two assumptions are the main limitations of the proposed model. No heuristic developed for solving the proposed model and the model solved by numerical examples.

Kia *et. al.,* (2012) present a mixed-integer non-linear programming model to integrate CF and GL simultaneously in dynamic. Another compromising aspect of this model is the utilization of multi-rows layout to locate machines in the cells configured with flexible shapes. The assumption used in this study is broad rang such as alternate process routings, operation sequence, processing time, production volume of parts, purchasing machine, duplicate machines, machine capacity, lot splitting, intra-cell layout, inter-cell layout, multi-rows layout of equal area facilities and flexible reconfiguration. Additionally, the objective of the integrated model is to minimize the total costs of intra and inter-cell material handling, machine relocation, purchasing new machines, machine overhead and machine processing. This study by looking at discrete approach to layout design is one of the comprehensive models. Finally they develop a SA algorithm to solve the model.

Recently, some efforts have been done to integrate all three aspects of CMS such as GF, GL and GS. Wu *et al.*, (2007b) propose a model to integrate the CF, GL and GS decisions concurrently. The objective function is minimizing the makespan. The model is solved by a hierarchical genetic algorithm. However their mathematical formulation is not clear enough. Firstly, they defined "machine position number index" and calculated the distance between two machines by subtracting the corresponding position numbers. The question here is that how they calculate the exact distance between machines and cells. The second critique to their work is that, they have not considered parts' demand or material movements among machines. They try to integrate the three main aspects of CMS just based on minimizing makespan. However, in reality there are several factors affecting CMS such as parts demand, inter-cell and intra-cell material movement that has to be considered. Third, their proposed model is static, so the dynamicity in the product mix and demand is not considered in their model. Finally, they have taken discrete approach to CMS design which means predetermined locations for machines, that by itself is a poor assumption.

2.2.2. Continuous Approach- MIP

The first MIP for FLP has been presented by Montreuil (1990). Herague and Kusiak (1991) develop the special case of Montreuli's model which the length, width, and orientation of facilities known in advance. They represent two models; one linear continuous and the second one linear mixed integer. They develop a heuristic method-penalty method to solve their models.

Alfa *et.al.*,(1992) develop a model to simultaneously solve group formation and intra-cell. The objective function is the summation of both inter-cell and intra-cell flow times distance-based. They develop SA/heuristic algorithm to solve their model.

SA has been used to find the initial solution, and then a heuristic approach based on penalty model developed to improve the solution. The main limitation of this model is that the cell locations are predetermined.

Bazargan-Lari and Kaebernick published few papers about design of cellular manufacturing (Kaebernick and Bazargan-Lari, 1996; Bazargan-Lari and Kaebernick, 1997, Bazargan-Lari, 1999, Bazargan- Lari *et al.,* 2000). Bazargan-Lari and Kaebernick (1997) present a continuous plane approach where different constraints such as cell boundaries, non-overlapping, closeness relationships, location restrictions/ preferences, orientation constraints, travelling distances have been considered. They develop a hybrid method which combined a nonlinear goal programming (NLGP) and simulated annealing for machine layout problem. They have combined all constraints as goals using goal programming (GP) formulas. Generally speaking GP divides those constraints into two main categories as absolute or hard and goal or soft constraints. Hard constraints are those that they have to be satisfied absolutely. It means that violation of any of them would yield to infeasibility. However, soft constraints can be compromise and be offset from desired set goals. They considered those constraints as three separate sets of objectives. The first priority level includes all set of absolute or hard objectives which have to be absolutely satisfied such as non-overlapped and cell boundaries constraints. The second and third priorities levels are preferences. The second priority is devoted to minimising area of the cells/ shop floor, satisfying closeness relationship, and orientation. Finally the third priority is to minimise the total travelling cost. Overall, the approach of Bazargan-Lari and Kaebernick is a combination of the NLGP and SA. They use the pattern search to solve their NLGP based on those three priorities. Since a pattern search is finding the local minimum, then they have been using SA to exit from the trap of local minimum. The core of their model is that they are generating alternative layout design by changing the order of priority levels 2 and 3 in each outer loop of SA algorithm. In other words, the starting point of new outer loop of SA is generated by the patter search algorithm. By changing the goal priority levels huge pool of efficient solutions are generating. To solve this issue they used what they called the filtering process to choose which sets of solutions have more different with the other ones. The logic behind this is giving decision makers the chance to consider how changing preferences' priorities would impact the solutions.

The other important piece of research was written by Imam and Mir (1993) and Mir and Imam (2001). Imam and Mir (1993) introduce a heuristic algorithm to place unequal sized rectangular facilities in continuous plane by introducing the new concept of "controlled coverage" by using "envelop blocks". In the initial solution facilities are randomly placed in plane in the envelop block the size of which is much larger than the actual size of facility and is calculated by multiplying magnification factor with the facilities' actual dimensions. Afterwards, during the heuristic iterations the sizes of envelop blocks are gradually decreased by decreasing the magnification factor until the dimensions of envelopes till became equal to the dimensions of their corresponding facilities. By this approach they were controlling the coverage of facilities together. The improvement iteration is based on the univariate search method. In this method only one of the $2n$ design variables which n is the number of facilities is changing at time. This change means moving facility horizontally or vertically along X-axis or Y-axis respectively. There are three draw backs to their method. Firstly, each iteration cycle is repeated $2n$ times, n times to move facilities horizontaly and then another n more times to move them vertically. The other drawback is that facilities are just allowed to move horizontally or vertically, there is no diagonal movement. Thirdly, there are no borders for the assumed continuous plane. However, in real world there is no plane without borders. The last drawback is related to magnification factor, they have not specified how large this factor has to be originally and by which fraction it has to be reduced in each iteration cycle.

Mir and Imam (2001) address to the second drawback mentioned above and try to improve their primary procedure. They develop a hybrid model by using SA for gaining the sub-optimal initial feasible solution and then they improved it by using steepest descent approach. As they also note the number of optimization iterations depends of the magnification factor by which the size of the envelope blocks reduces as magnification factor was being reduced. The algorithm stopped when magnification factor is equal to one. So it is obvious that the computational cost and time is quite dependent of magnification factor.

Tain *et. al.,* (2010) develop a mixed integer linear programming (MILP) to solve dynamic facility layout plan; *i.e.,* layout plan is not fixed for all period of time. They develop a GA to solve their model. Their work is quite unique. Once, the model is considering dynamicity to the FLP. Additionally, the rearrangement cost also is applied beside cost of material flow. They define rearrangement as changes in facility's coordinates or orientation. Finally, the budget constraint assumed for rearrangement cost. This approach has one drawback which is distracting the continuance aspect of their assumed FLP, because this method forces facilities to be placed within specific lines.

There are recent studies that have adopted a continuous approach (Arkat *et al.,* 2012 a, b). In the first study Arkat *et al.,* (2012 a)define two nonlinear mixed integer mathematical models. The first model developed to integrate cell formation problem with cell layout both inter-cell and intra-cell with the objective of minimizing total transportation cost of parts. The second model proposed to concurrently solve the formation of cells, cellular layout and cellular scheduling by minimising makespan. They develop a GA algorithm to solve the model. In the second study, Arkat *et al.,* (2012 b) present a multi-objectives mathematical modelling to solve CF, CL, and CS simultaneously. The two objectives are minimizing both total transportation cost and makespan cost. A multi-objective genetic algorithm (MOGA) is then developed to solve the problem. Using sequence of operation as well as considering non-overlap elimination constraints are the two strength points of their studies. However, there are two main drawbacks to their both models as are explaining below:

1) The authors have constrained the rectangular distance between centroids as following $|x_i - x_u| + |y_i - y_u| \geq 1$ to prevent equal area machines from overlapping each other. The authors have assumed that the machines are square and of length unity. However, this still does not rule out all possibilities of overlap, since simply if one has Δx and Δy of values greater than 0.5 and less than unity, one would still has

overlap between the two machines. Note that Δx and Δy are the difference in x and y coordinates between the centroids of the two machines named respectively. However, still this does not really rule out all possible scenarios where one would have overlap.

2) The constraints formulated do not really rule out the possibility of having non-rectangular cells as being claimed.

3) The constraints used to force machines to stay within shop floor boundaries are also not accurate. Since the x_i and y_i are centroid dimensions of each machine and we assume the length (h_i) and width (v_i) of machine i, hence end corner points for length of each machine would be $x_i - \frac{h_i}{2}$ and $x_i + \frac{h_i}{2}$ and the same for width $y_i - \frac{v_i}{2}$ and $y_i + \frac{v_i}{2}$. Therefore, if we assume W and L is vertical and horizontal distance of shop floor respectively, the boundary constraints would be $x_i + \frac{h_i}{2} \geq L,\ 0 \leq x_i - \frac{h_i}{2}$ for length and $y_i - \frac{v_i}{2} \geq 0$ and $y_i + \frac{v_i}{2} \leq W$ for width of shop floor.

4) Arkat *et al.*,(2012) have assumed that the machines have equal square area and cells are rectangle. However, in the real world these are poor assumptions.

2.3. Gap Analysis

Table (2) summarizes our findings and provides a comprehensive gap analysis. It is observed that FLP can be solved either by discrete approach or continuous approach. Discrete approach is the popular one because of its simplicity. The main assumptions considered in discrete approach are equal sized facilities, predetermined locations, and unrestricted shop. However, those are poor assumptions in the real world. Therefore, there is a need to develop a solution for FLP by assuming more realistic assumptions such as unequal sized facilities, restricted shop and no predetermined locations.

In real case studies all the area of the shop floor is not useable for arranging facilities in. For an example, there are aisles for material and human transportation where no other facilities can be located in, or there are fixed facilities and/or departments’ locations, input and output point locations and so on. These attributes in design of layout plan have not been considered in literature extensively. Aisle structures, fixed facilities’ positions and fixed department are considered in this work.

The problem has been considered in this thesis have manufacturing focus which is FLP toward cellular layout problem. Hence, considering manufacturing attributes such as operations’ sequence and part demand are so important. This was addressed by Kia et al., (2012); however the approach taken is discrete approach. Bazargan-Lari and Kaebernick (1997) have developed a comprehensive mathematical modeling and hybrid model for CMS. However they have not considered operation sequence in their studies. Mir and Imam (2001) also have developed a hybrid model for FLP; however firstly they do not take a manufacturing outlook into the problem. Hence, their approach is just placing facilities in continual plane site. Finally, Arkat *et al.*,(2012 a,b) has not applied part demands and moreover, all facilities assumed have unit square shape. Placing equal sized facilities are easier than unequal sized facilities.

Most of the literatures have taken discrete approach to FLP developed heuristics rather than the minor works done in continuous field. Developing heuristics for discrete problem is easier. Because locations are predetermined, the only operator needs is swap operator, *i.e.* switching facilities locations. Moreover, in discrete approach no overlap would happen between facilities. It can be concluded that how simple can be heuristics algorithm for discrete problem. However, in continuous problem since no location are known in priori the chances of overlap occurrence is high which requires designing variety of repair function to eliminate overlap.

Table (2): Literature review summary

No.	References	Decision	Objective	Analytical model	Solution method	Comments/assumptions
1	Herague and Kusiak (1991)	FLP	$C_{ij} \times d_{ij} \times f_{ij}$	NLMIP	Heuristic method	Two models: linear continuous and linear mixed integer, unequal size facilities
2	Alfa *et al.*, (1992)	CF and Machine grouping	$C_{ij} \times d_{ij} \times f_{ij}$	0-1 nonlinear programming	Hybrid: SA combined with heuristic method	Concurrent, Fixed transfer distance, and machines' orientation
3	Imam and Mir (1993)	FLP	$f_{ij} \times d_{ij}$	FLOAT	Heuristic- univariate search technique	Concept of "Controlled Convergence"
4	Kaebernick and Bazargan-Lari (1996)	CF and CL	$f_{ij} \times d_{ij}$	GP	SA	Extension of the proposed model developed by Bazargan-Lari and Kaebernick (1997)

No.	References	Decision	Objective	Analytical model	Solution method	Comments/ assumptions
5	Bazargan-Lari and Kaebernick (1997)	Intra CL	$f_{ij} \times d_{ij}$ Rectangular distance	NLGP	Pattern Search and SA	Objectives and constraints: Feasibility of the solutions, minimizing the area of the cell, and minimizing the traveling cost. cell boundaries, closeness relationships, Location restrictions and preferences, orientation constraints No operation sequence has been considered
6	Bazargan-Lari (1999)	CL	$f_{ij} \times d_{ij}$	GP	SA	Demonstrate the capability of the model developed by Bazargan-Lari and Kaebernick (1997)
7	Bazargan-Lari et al., (2000)	CF and CL	$f_{ij} \times d_{ij}$	GP	Pattern Search and SA	Demonstrate the capability of the model developed by Bazargan-Lari and Kaebernick (1997)

No.	References	Decision	Objective	Analytical model	Solution method	Comments/assumptions
12	Wu et al., (2006)	CF and GL	$C_{ij} \times Differenct\ of\ Machines\ or\ Cells$ $Sequnce\ Position\ Number \times {D_j}/{B_j}$ Minimizing EE	Binary	GA	Concurrent, Minimizing total cost of movement, exceptional elements (EE), and backtracking movement. No numerical examples No distance concept applied
13	Wu et al., (2007b)	CF, GL, GS	Makespan of part j	MIP	GA	Concurrent No numerical examples, No distance concept applied
14	Wu et al., (2007a)	CF, GL, GS	$C_{ij} \times Differenct\ of\ Machines\ or\ Cells$ $Sequnce\ Position\ Number \times {D_j}/{B_j}$ Minimizing EE	Binary	GA	Concurrent, Minimizing EE No numerical examples No distance concept applied

No.	References	Decision	Objective	Analytical model	Solution method	Comments/assumptions
12	Wu et al., (2006)	CF and GL	$C_{ij} \times Differenct\ of\ Machines\ or\ Cells$ $Sequnce\ Position\ Number \times {}^{D_j}\!/_{B_j}$ Minimizing EE	Binary	GA	Concurrent, Minimizing total cost of movement, exceptional elements (EE), and backtracking movement. No numerical examples No distance concept applied
13	Wu et al., (2007b)	CF, GL, GS	Makespan of part j	MIP	GA	Concurrent No numerical examples, No distance concept applied
14	Wu et al., (2007a)	CF, GL, GS	C_{ij} $\times Differenct\ of\ Machines\ or\ Cells$ $Sequnce\ Position\ Number \times {}^{D_j}\!/_{B_j}$ Minimizing EE	Binary	GA	Concurrent, Minimizing EE No numerical examples No distance concept applied

No.	References	Decision	Objective	Analytical model	Solution method	Comments/assumptions
15	Tavakkoli-Moghaddam *et al.*, (2007)	GL	Number of trips between machines and cells × distance between location	Nonlinear 0-1 binary	Numerical Examples	Concurrent, Stochastic demand Discrete approach No Heuristic
16	Safaei et al., (2008)	GL	Machine constant cost and variable cost, Inter and intra material handling cost, Reconfiguration cost Number of trips × cost	NLMIP	Hybrid- MFA and SA	Maximum Cell size, Machine time capacity, Multi-purpose machine tools, Alternative process plan Equal sized machine tools Equal distance between cells and machine
17	Tian et al., (2010)	DFLP	$C_{ij} \times d_{ij} \times f_{ij}$	MILP	GA	Un-equal sized departments, Dynamic FLP, Budget constraint No operation sequence

No.	References	Decision	Objective	Analytical model	Solution method	Comments/assumptions
18	Airafar et al., (2011)	GL	$C_{ij} \times d_{ij} \times f_{ij}$	NLMIP	SA	Variable demand Equal sized facilities Not restricted shop floor Predetermined distance between locations No numerical examples
19	Ariafar et al., (2012)	GL	$C_{ij} \times d_{ij} \times f_{ij}$	NLMIP	Numerical Examples	Concurrent, Demand has normal distribution Equal sized facilities, Not restricted shop floor, Predetermined distance between locations
20	Gou et al., (2012)	CF and Inter-cell	$d_{kl} \times f_{ij}$	QAP	GA	Sequential Equal sized facilities, Not restricted shop floor, Predetermined distance between locations

No.	References	Decision	Objective	Analytical model	Solution method	Comments/assumptions
21	Arkat et al., (2012)	GF, GL, and CS	$C_{ij} \times d_{ij}$ Minimizing total completion time	MIP	GA	Concurrent, Operation sequence Equal sized machine tools No numerical examples Not considering parts' demand
22	Arkat et al., (2012)	GF, GL, and CS	$C_{ij} \times d_{ij}$ Minimizing total completion time	NLMIP	MOGA	Concurrent, Operation sequence Equal sized machine tools No numerical examples Not considering parts' demand
23	Kia et al., (2012)	GF and GL	$C_{ij} \times d_{ij} \times f_{ij}$	NLMIP	SA	Discrete Predetermined distance between locations Not considering facilities' size

CHAPTER THREE: MATHEMATICAL MODELING

At the core of the approach being taken, the group layout (GL) problem for CMS has been modeled and solved sequentially in steps. Group formation has been assumed to done a priori. A two-tier mixed integer non-linear programming model has been developed to solve the intra-cell and inter-cell layout sequentially at two different hierarchical levels, namely at the cellular and shop floor levels. The details are declared as follows.

3.1. Leader Problem- Intra-cell Layout

Since the Group Formation is done in advance, it is already known which machine is assigned to which cell. In this level the layout of group of facilities in their corresponding cell is being designed. Hence, the leader problem is the layout at the cell. The centroid of the facility is the reference for the coordinates of that facility. It has to be noted the origin for facilities' coordinates is their left bottom corner of their relative cell. Figure (1) represents the scheme of facilities regard to their corresponding cell.

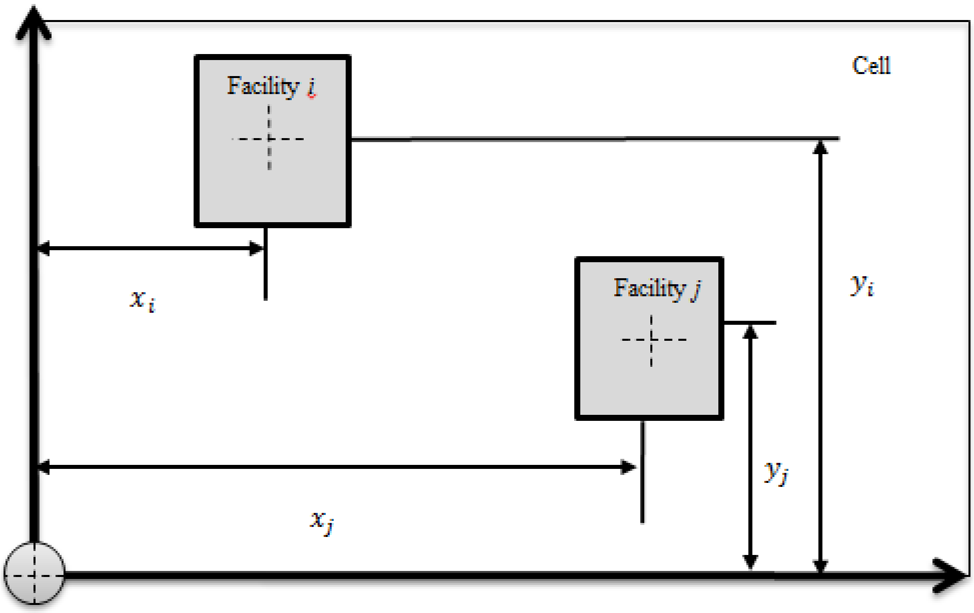


Figure (1): Scheme of facilities regard to their corresponding cell

It is important to note that initially when running the FLP for each cell (leader problem), an upper limits for the length and width of each cell are being defined by using constraints named as within-cell constraint. In other words, by assumption $l_c$and w_c as the length and width of cell c, the summation of the centroid horizontal dimension of each facility (x_i) and the half of length of that facility has to be less than equal to the length of the cell, and similarly the centroidal vertical dimensions of each facility (y_i) and the half of width of that facility has to be less than or equal to the width of the cell. Moreover, at leader level, the traffic at intra-level is the material flow among the machines (operations already assigned to machines) located in cell, the position of which the intercellular layout problem is yet to be determined.

3.2. Follower Problem- Inter-cell Layout

After the layout for all manufacturing cells have been finalized, the overall approach for the whole which is follower problem is being solved. Thus, follower is the layout for the whole shop (i.e. intercellular). The coordinates of cells are calculated based on the horizontal and vertical distance of the centroid of the cell to the origin of the whole shop which is left bottom corner of the shop. Similarly, the within constraints are applied in the follower problem as well. To illustrate, the cells have to be located within the boundaries of the whole shop. In other words, if shop has length *(L)* and width *(W)*, the summation of the centroid horizontal dimension of each cell ($\acute{x}_c$) and the half of length of that cell has to be less than equal to the length of the shop, and similarly the centroid vertical dimension of each cell ($\acute{y}_c$) and the half of width of that facility has to be less than or equal to the width of the shop. Moreover, the material flows in the follower level are inter-travel between cells. Since the Group Formation is done in advance, it is already known which operation of machine is processed in which cell; i.e., operations of part *j* processed in cell *k* are known ahead of time. Therefore, material flows between cells are actually the flows among the operations of parts on machines done in each cell. Figure (2) represents the scheme of cells regard to the shop.

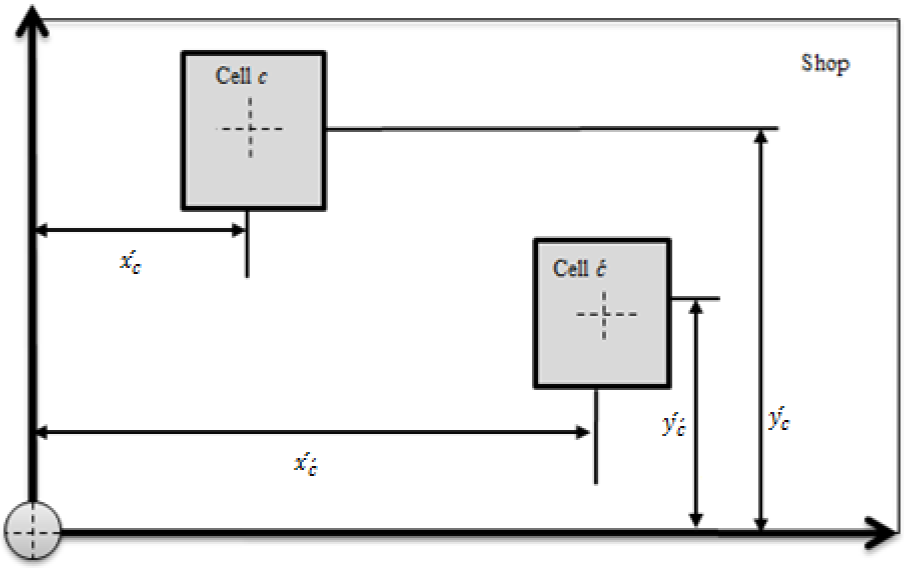


Figure (2): The scheme of cells regard to the shop

3.3 Problem Statement

The problem is to arrange facilities that are cells in the leader problem and machine tools in the follower problem in the continual planar site. The site has rectangle shape with specified length *(L)* and width *(W)*. Moreover, there is a horizontal aisle in the site by the same length as of site, however with two different vertical dimensions Y_{VALU} and Y_{VALL}. Aisle divides the site to two sections, upper and lower. No facilities could be arranged in this area. The objective is minimizing total travel-flow cost by considering shape, size and geometric characteristics constraints. Each facility has rectangle shape where its position determined by the coordinates of its center and its predetermined length and width. Hence, the facilities consider as rigid blocks. Facilities are not allowed to overlap each other and have to be assigned in their related boundaries area, which is the site's boundaries for follower problem that of cell for leader problem. The traditional Cartesian Coordinate System used shown in Figure (3), represents the scheme used in this paper. The following model has represented by (Allahyari & Azab, 2014).

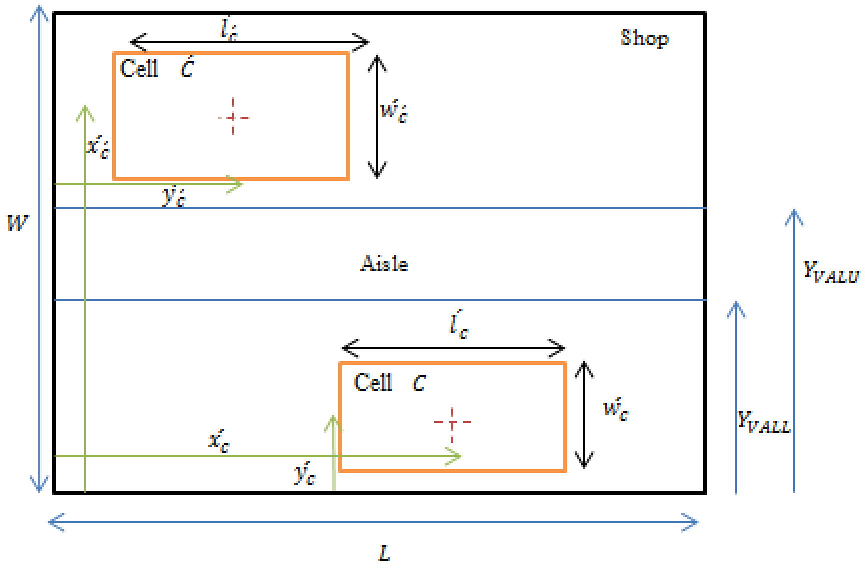


Figure (3): Scheme of shop

The problem is formulated under the following assumptions:

1. CF is known in advanced.
2. Machines are not in the same size.
3. Machines must be located within a given area.
4. Machines are not allowed overlap to each other.
5. Cell's dimensions and orientation are predetermined.
6. Each part type has a number of operations that must be processed based on its operation sequence readily available from the route sheet of parts. It should be noted that the process sequence of each parts are different.
7. The demand for each part type in known and is constant
8. Material handling devices moving the one part between machines.
9. Inter and intra-cell movements related to the part types have different costs is related to the distance traveled. We assume that the rectangular distance between each pair of machines' centroid.
10. In determining machine size and dimensions, the workspace required for operator usage and that needed to enforce between the different machines have been taken into account.

3.4. NonLinear Mixed Integer Programming Model (NLMIP)

The mathematical formulation represented as below:

Sets:

$P = \{1,2,3, \dots, P\}$	Index set of part types
$M = \{1,2,3, \dots, M\}$	Index set of machine types
$C = \{1,2,3, \dots, C\}$	Index set of cell types
$O_p = \{1,2,3, \dots, O_p\}$	Index set of operations indices for part p

Parameters:

L	Horizontal dimension of shop floor
W	vertical dimension of shop floor
Y_{VALU}	Vertical dimension of upper side of aisle
Y_{VALL}	Vertical dimension of lower side of aisle
X_{HALLF}	Horizontal dimension of left side of aisle
X_{HALRT}	Horizontal dimension of right side of aisle
l_i	Length of machine *i*
w_i	Width of machine *i*
l_c	Length of cell *c*
w_c	Width of cell *c*
CA_j	Intracellular transfer unit cost for part *j*
CE_j	Intercellular transfer unit cost for part *j*

D_j Demand quantity for part *j*

U_{joi} 1, if operation o of part *j* is done by machine *i*, otherwise 0

$\acute{U}_{joc}$ 1, if operation o of part *j* is done by machine i which is located in cell c, otherwise 0

Q_{ic} 1, if machine i is assigned in cell c

Decision variables:

x_i Horizontal distance between center of machine *i* and vertical reference line

y_i Vertical distance between center of machine *i* and horizontal reference line

$\acute{x}_c$ Horizontal distance between center of cell *c* and vertical reference line

$\acute{y}_c$ Vertical distance between center of cell *c* and horizontal reference line

Z_{iu} 1, if machine *u* is arranged in the same horizontal level as machine *i*, and 0 otherwise

$W_{c\acute{c}}$ 1, if cell c is arranged in the same horizontal level as cell $\acute{c}$ and 0 otherwise

Z_c 1, if cell c is arranged in out of aisle horizontal boundaries and 0 otherwise

W_c 1, if cell c is arranged in out of aisle vertical boundaries and 0 otherwise

The continuous bi-level programming problem is defined as: The **intra-cell** layout mathematical formulation to layout the different machines (machines here are the facilities) of every cell *c* at a time is as follows:

$$\text{Min } \sum_{j=1}^{P}\sum_{o=1}^{O_p-1}\sum_{\substack{i,u=1\\ i\neq u}}^{M} U_{joi}\, U_{jo+1u}(|x_i - x_u| + |y_i - y_u|)\; CA_j D_j \qquad (1)$$

s.t.

$$x_i + \frac{l_i}{2} \leq l_C \qquad i = 1, .., M \tag{2}$$

$$x_i - \frac{l_i}{2} \geq 0 \qquad i = 1, .., M \tag{3}$$

$$y_i + \frac{w_i}{2} \leq w_c \qquad i = 1, .., M \tag{4}$$

$$y_i - \frac{w_i}{2} \geq 0 \qquad i = 1, .., M \tag{5}$$

$$|x_i - x_u| \geq Z_{iu}(l_i + l_u)/2 \qquad i, u = 1, .., M \tag{6}$$

$$|y_i - y_u| \geq (1 - Z_{iu})(w_i + w_u)/2 \qquad i, u = 1, .., M \tag{7}$$

$$x_i, y_i \geq 0, Z_{iu} \text{ are binary} \qquad i, u = 1, .., M \tag{8}$$

Equation 1 declares the objective function of leader problem which is minimizes the total intra-cell transportation cost of parts. Equations 2 to 5 are within-site constraints that ensure each machine tool are assigned within the boundaries of its corresponding cell. Equations 6 and 7 force the overlap elimination for machine tools. Equation 8 represents the nature of the decision variables which are binary and non-negative.

Finally, the **inter-cell** layout problem tries to layout the different cells (cells here are the facilities) of the entire shop floor is as follows:

$$\text{Min} \sum_{j=1}^{P} \sum_{o=1}^{o_p - 1} \sum_{\substack{c,\acute{c}=1 \\ c \neq \acute{c}}}^{C} \acute{U}_{Joc} \, \acute{U}_{Jo+1\acute{c}} \, (|\acute{x}_c - \acute{x}_{\acute{c}}| + |\acute{y}_c - \acute{y}_{\acute{c}}|) \, CE_j D_j \tag{9}$$

s.t

$$\acute{x}_c + \frac{\acute{l}_c}{2} \leq L \qquad c = 1, .., C \tag{10}$$

$$\acute{x}_c - \frac{\acute{l}_c}{2} \geq 0 \qquad c = 1, .., C \tag{11}$$

$$\acute{y}_c + \frac{\acute{w}_c}{2} \le W \qquad c = 1, . . , C \tag{12}$$

$$\acute{y}_c - \frac{\acute{w}_c}{2} \ge 0 \qquad c = 1, . . , C \tag{13}$$

$$|\acute{x}_c - \acute{x}_{\acute{c}}| \ge W_{c\acute{c}}\left(\acute{l}_c + \acute{l}_{\acute{c}}\right)/2 \qquad c, \acute{c} = 1, . . , C \tag{14}$$

$$|\acute{y}_c - \acute{y}_{\acute{c}}| \ge (1 - W_{c\acute{c}})(\acute{w}_c + \acute{w}_{\acute{c}})/2 \qquad c, \acute{c} = 1, . . , C \tag{15}$$

Aisle Constraints:

Horizontal Aisle:

$$(\acute{y}_c + \acute{w}_c/2) - Y_{VALL} \le \text{ M } Z_c \tag{16}$$

$$Y_{VALU} - (\acute{y}_c - \acute{w}_c/2) \le \text{M } (1 - Z_c) \tag{17}$$

Vertical Aisle:

$$\left(\acute{x}_c - \acute{l}_c/2\right) - X_{HALRT} \le MW_c \tag{18}$$

$$X_{HALLF} - \left(\acute{x}_c + \acute{l}_c/2\right) \le M(1 - W_c) \tag{19}$$

$$\acute{x}_c, \acute{y}_c \ge 0, W_{c\acute{c}} , Z_c , W_c \text{ are binary} \qquad c = 1, . . , C \tag{20}$$

Equation 9 represents the objective function of follower program. The objective function minimizes the inter-cell transportation cost of parts. The within-site constraints are forced by the set of constraints 10 to 13; *i.e.* this constraints ensure cell are assigned within the boundaries of shop floor. Moreover, overlap elimination constraints are defined by constraints 14 and 15 which enforce the overlap elimination among cells. Equation 16 and 19 in the follower problem ensure that no cells would be assigned in the aisle boundaries. Finally, equation 20 specifies that the decision variables are binary and positive.

3.5. Linearization

Since both overlap eliminations constraints and objective functions have absolute terms, two terms are using for linealization.

3.5.1. Linearization of Objective Function

In order to linearize the absolute terms of leader problem's objective function, the linearized variables defined is such a term to satisfy equations (21) and (22).

$$|x_i - x_u| = x_{iu}^{+} - x_{iu}^{-} \tag{21}$$

$$|y_i - y_u| = y_{iu}^{+} - y_{iu}^{-} \tag{22}$$

The two above terms (21) and (22) are replaced by absolute terms in the objective function (equation 1). Moreover, those equations (21) and (22) are added to the constraints. Hence, the linearized objective function of leader problem would be:

$$\text{Min } \sum_{j=1}^{P} \sum_{o=1}^{O_p-1} \sum_{\substack{i,u=1 \\ i \neq u}}^{M} U_{joi}\ U_{jo+1u}\left((x_{iu}^{+} - x_{iu}^{-}\ ) + (y_{iu}^{+} - y_{iu}^{-})\right) CA_j D_j \tag{23}$$

Similarly; for linearizing follower problem's objective function (9) the new linearized variables defined which have to satisfied equations (24) and (25). These constraints are replaced in the nonlinear objective function:

$$|\acute{x}_c - \acute{x}_{\acute{c}}| = x_{c\acute{c}}^{\acute{+}} - x_{c\acute{c}}^{\acute{-}} \tag{24}$$

$$|\acute{y}_c - \acute{y}_{\acute{c}}| = y_{c\acute{c}}^{\acute{+}} - y_{c\acute{c}}^{\acute{-}} \tag{25}$$

Hence the linearized objective function of follower problem is:

$$\text{Min } \sum_{j=1}^{P} \sum_{o=1}^{O_p-1} \sum_{\substack{c,\acute{c}=1 \\ c \neq \acute{c}}}^{C} U_{Joc}^{\acute{}}\ U_{Jo+1\acute{c}}^{\acute{}}\ \left(\left(x_{c\acute{c}}^{\acute{+}} - x_{c\acute{c}}^{\acute{-}}\right) + \left(y_{c\acute{c}}^{\acute{+}} - y_{c\acute{c}}^{\acute{-}}\right)\right) CE_j D_j \tag{26}$$

3.5.2. Linearization of Constraints

The overlap elimination constraints of leader and follower problems (6)-(7) and (14)-(15) respectively have absolute terms which declare the nonlinearity nature of those constraints. In order to linearize, two variables are introduced:

Moreover, the following constraints substitute by constraints (6) and (7)

$$(x_i - x_u) + M \times QX_{iu} \geq Z_{iu}(l_i + l_u)/2 \qquad i, u = 1, .., M \qquad (27)$$

$$(x_i - x_u) - M \times (1 - QX_{iu}) \leq (-Z_{iu})(l_i + l_u)/2 \qquad i, u = 1, .., M \qquad (28)$$

$$(y_i - y_u) + M \times QY_{iu} \geq (1 - Z_{iu})(w_i + w_u)/2 \qquad i, u = 1, .., M \qquad (29)$$

$$(y_i - y_u) - M \times (1 - QY_{iu}) \leq -(1 - Z_{iu})(w_i + w_u)/2 \qquad i, u = 1, .., M \qquad (30)$$

Similarly; Furthermore, the four following constraints are replaced by constraints (14) and (15) in non-linear inter-cell problem:

$$(\acute{x}_c - \acute{x}_{\acute{c}}) + M \times QX_{c\acute{c}} \geq W_{c\acute{c}}\left(\acute{l}_c + \acute{l}_{\acute{c}}\right)/2 \qquad c, \acute{c} = 1, .., C \qquad (31)$$

$$(\acute{x}_c - \acute{x}_{\acute{c}}) - M \times (1 - QX_{c\acute{c}}) \leq (-W_{c\acute{c}})\left(\acute{l}_c + \acute{l}_{\acute{c}}\right)/2 \qquad c, \acute{c} = 1, .., C \qquad (32)$$

$$(\acute{y}_c - \acute{y}_{\acute{c}}) + M \times QY_{c\acute{c}} \geq (1 - W_{c\acute{c}})(\acute{w}_c + \acute{w}_{\acute{c}})/2 \qquad c, \acute{c} = 1, .., C \qquad (33)$$

$$(\acute{y}_c - \acute{y}_{\acute{c}}) - M \times (1 - QY_{c\acute{c}}) \leq -(1 - W_{c\acute{c}})(\acute{w}_c + \acute{w}_{\acute{c}})/2 \qquad c, \acute{c} = 1, .., C \qquad (34)$$

3.6. The Blocks Constraints

In some situations, some specific areas cannot be occupied by facilities such as inventory area. Moreover, in some certain conditions the locations of some facilities are fixed and cannot be changed based on the economic reasons or safety and so on. In these cases those areas or facilities are assumed as blocks with the exact length and width as well as coordinates. The figure (4) shows the scheme of block constraints.

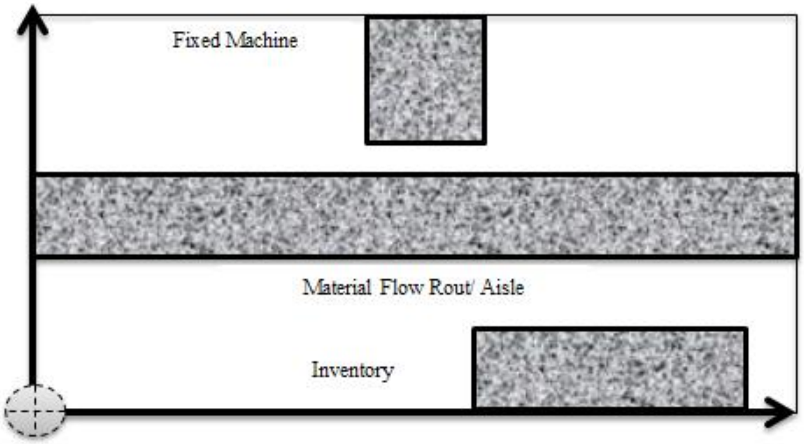


Figure (4): the scheme of block constraints

In order to consider those constraints, the below constraints are added NLMIP of follower problem:

$$|\acute{x}_c - xblock_k| \geq \acute{Z}_{ck}\left(\acute{l}_c + lblock_k\right)/2 \qquad c = 1,..,C, k = 1,..,K \tag{35}$$

$$|\acute{y}_c - yblock_k| \geq \left(1 - \acute{Z}_{ck}\right)(\acute{w}_c + wblock_k)/2 \quad c = 1,..,C, k = 1,..,K \tag{36}$$

Which:

K Number of blocks

$xblock_k$ The horizontal coordinate of block k

$yblock_k$ The vertical coordinate of block k

$lblock_k$ The length of block k

$wblock_k$ The width of block k

$\acute{Z}_{ck}$ 1 if cell c is arranged in the same horizontal level as block k, and 0 otherwise

Constraints (35) and (36) prevent overlap between the blocks and cells.

There are absolute terms in the constraints (35) and (36), in order to linearize these constraints, the following four constraints substitute with constraints (35) and (36) by defining two binary variables called XB_{ck} and YB_{ck}.

$$(\acute{x}_c - xblock_k) + MXB_{ck} \geq \acute{Z}_{cu}\left(\acute{l}_c + lblock_k\right)/2 \qquad c = 1,..,C, k = 1,..,K \tag{37}$$

$$(\acute{x}_c - xblock_k) - M \times (1 - XB_{ck}) \leq \left(-\acute{Z}_{cu}\right)(l_{\acute{c}} + lblock_k)/2 \quad c = 1,..,C, \qquad k = 1,..,K \tag{38}$$

$$(\acute{y}_c - yblock_k) + M \times YB_{ck} \geq \left(1 - \acute{Z}_{cu}\right)(\acute{w}_c + wblock_k)/2c = 1,..,C,\ k = 1,..,K \tag{39}$$

$$(\acute{y}_c - yblock_k) - M \times (1 - YB_{ck}) \leq -\left(1 - \acute{Z}_{cu}\right)(\acute{w}_c + wblock_k)/2\ c = 1,..,C, k = 1,..,K \tag{40}$$

3.7. Limitation of Study

From computational and optimization points of view, it is important to note that dividing and conquering the FLP for CMS does not produce the exact global optimal solution; i.e., the solution obtained would not really be the same exact global optimum solving the problem combined in one math model for the two different levels (that is assume the nonlinear model to be presented in this section is linearized). However, it is important to pinpoint that such models in the literature were complex enough that they were not really being attempted and solved for optimality using OR (Operations Research) exact methods and commercial OR software. Moreover, few of these models carried constraints that were formulated in a way that hindered the ability to solve them using these tools. To elaborate, one of the models had conditions on the decision variables associated with the overlap elimination constraints. Finally, some models even went further and overcomplicated the problem by introducing other elements such as the grouping and clustering that is needed ahead of time for cell formation, as well as the production scheduling of each cell. In our case, we find it far more efficient to solve the grouping problem beforehand clustering methods and else, and then to solve the layout problem at inter- and intra-cellular levels respectively. In next chapter, we approach the same problem using heuristics and metaheurisitcs, since the problem is NP-hard.

CHAPTER FOUR: HEURISTICS

4.1. Heuristic

In order to develop the feasible and efficient initial solution for the developed metaheuristic algorithm-simulated annealing- a novel heuristic algorithm has been developed. The major idea behind the developed heuristic algorithm is to minimize the possibility of overlaps between facilities by imposing distance between the centroid of the two consecutive facilities. To do this facilities are scattered in the site by taking radial movement. Figure (5) represents the scheme of radial movement. To illustrate, facilities are placed in the site along a radial at specific angle. As explained in order to make distance between the facilities a specific angle θ is defined and applied between the centroid of the two neighbor facilities. The angle θ is calculated by dividing 360^{o} over the total number of facilities. Hence, $\theta = \frac{360^{0}}{M}$. To start the heuristic algorithm, at first all facilities are placed on top of each other in the middle of the site which is divided into four equal size quadrants *i.e* Q_1, Q_2, Q_3, Q_4. The heuristic algorithm has compromised into two loops.

4.1.1. Outer Loop

In each iteration, one random facility called facility f_G is chosen as a target facility and placed within the site by taking radial movement. In other words, facility f_G is placed along the specific radius by taking the certain angle of the radial movement, called $\acute{\theta}$. The radius is the vector $\overrightarrow{r_f}$ with the origin of the centroid of the site and the end of the boundary of the corresponding quadrant in which the facility f_G is being placed. Furthermore, the angle $\acute{\theta}$ is calculated as:

$$\acute{\theta} = i \times \theta,\ i = 1,2,\dots,M \tag{1}$$

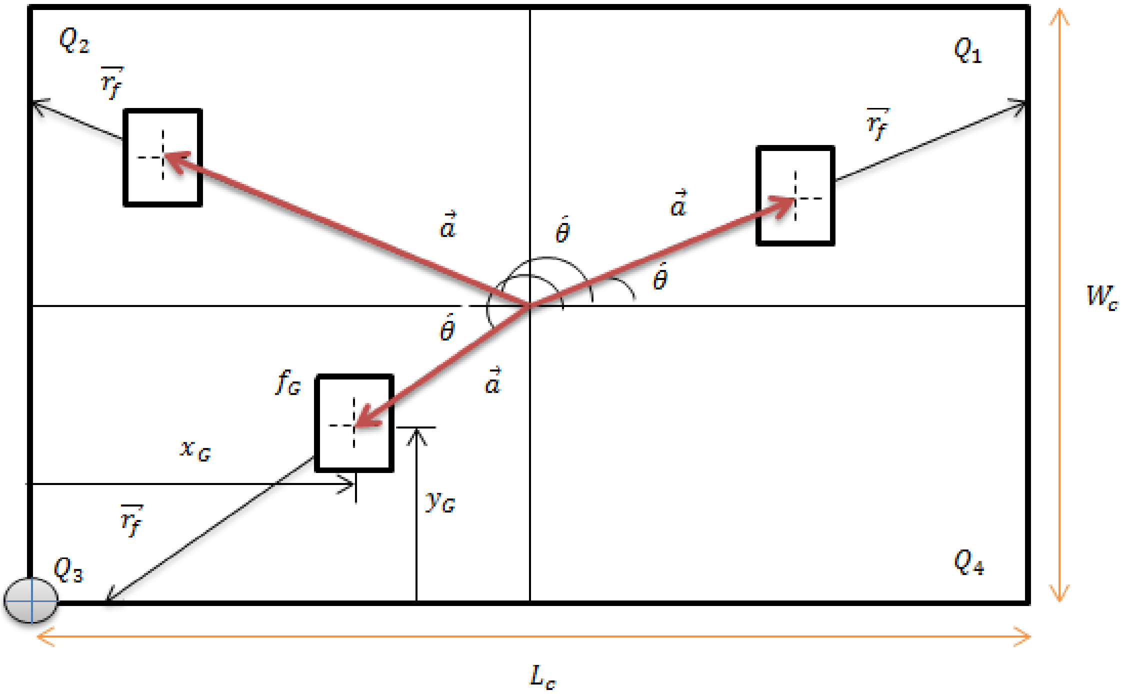


Figure (5): The scheme of radial movement

Facility, f_G, is placed by end of vector $\vec{a}$, which is a vector of random magnitude along vector's $\overrightarrow{r_f}$ direction. It has to be noted, the length of vector $\vec{a}$ is a random number between $\left[0, \left|\overrightarrow{r_f}\right| - r\right]$, r is the length of the diagonal of facility f_G. By this approach facility f_G is placed within the site. Table (3) and table (4) represent the calculation of length of vector $\overrightarrow{r_f}$ and the coordinates of f_G respectively.

Afterwards, overlap checking is considering. If any overlap happened between the target facility and site boundaries or between target facility and the previous placed facility the inner loop is performing.

Table (3): Length of vector $\overrightarrow{r_f}$, radial movement

$\boldsymbol{\acute{\theta}}$	$\lvert\overrightarrow{\boldsymbol{r_f}}\rvert$
$0 < \acute{\theta} \leq 45^o$	$\frac{L_C/2}{\text{Cos}\,\acute{\theta}}$
$45^o < \acute{\theta} < 90^o$	$\frac{W_C/2}{\text{Sin}\,\acute{\theta}}$
$90^o \leq \acute{\theta} \leq 135^o$	$\frac{W_C/2}{\text{Cos}(\acute{\theta} - 90)}$
$135^o < \acute{\theta} < 180^o$	$\frac{L_C/2}{\text{Sin}(\acute{\theta} - 90)}$
$180^o \leq \acute{\theta} \leq 235^o$	$\frac{L_C/2}{\text{Cos}(\acute{\theta} - 180)}$
$235^o < \acute{\theta} \leq 270^o$	$\frac{W_C/2}{\text{Sin}(\acute{\theta} - 180)}$
$270^o < \acute{\theta} \leq 315^o$	$\frac{W_C/2}{\text{Cos}(\acute{\theta} - 270)}$
$315^o < \acute{\theta} \leq 360^o$	$\frac{L_C/2}{\text{Sin}(\acute{\theta} - 270)}$

Table (4): Calculation of coordinates of target facility

Quadrant	Target Facility's Coordinates
Q_1	$x_G = {}^{L_C}/_2 + \lvert\vec{a}\rvert \times Cos(\acute{\theta})$ $y_G = {}^{W_C}/_2 + \lvert\vec{a}\rvert \times Sin(\acute{\theta})$
Q_2	$x_G = {}^{L_C}/_2 - \lvert\vec{a}\rvert \times Sin(\acute{\theta} - 90)$ $y_G = {}^{W_C}/_2 + \lvert\vec{a}\rvert \times Cos(\acute{\theta} - 90)$
Q_3	$x_G = {}^{L_C}/_2 - \lvert\vec{a}\rvert \times Cos(\acute{\theta} - 180)$ $y_G = {}^{W_C}/_2 - \lvert\vec{a}\rvert \times Sin(\acute{\theta} - 180)$
Q_4	$x_G = {}^{L_C}/_2 + \lvert\vec{a}\rvert \times Cos(\acute{\theta} - 270)$ $y_G = {}^{W_C}/_2 - \lvert\vec{a}\rvert \times Sin(\acute{\theta} - 270)$

4.1.2. Inner loop

In case of overlap, different repair functions are applied. The repair function is selected based on the type of overlap occurred. Repair function does two major performances, one is elimination of overlap between facilities and another is keeping facility within the boundaries of its corresponding quadrant. However, if the corresponding quadrant is too congested, the overlapped facilities can be placed partially in another quadrant. However, no facilities are allowed to violate site boundaries. In the first iteration of inner loop the overlap between facility f_G and the overlapped facility f_j is repaired. Afterwards, overlap checking performs for all facilities starting from the last placed facility to the first one to see if the repair(s) done in previous step has caused other overlaps or not. If no overlap happened the inner loop is end and algorithm goes back to the outer loop to place another facility, of course if any facility left. The scheme of overlap is shown in figure (6).

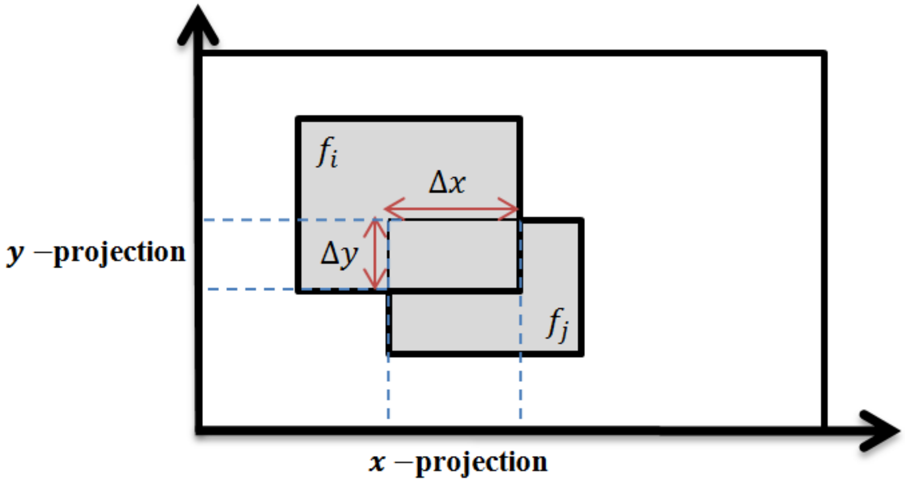


Figure (6): Scheme of overlap between two facilities

Hence, if

f_i The facility which its overlap with rest of facilities is under consideration

f_j The facility which has overlap with facility i

Δx x −projection of the overlap Δ

Δy y −projection of the overlap Δ

(x_i, y_i) The coordinates of facility f_i

(x_j, y_j) The coordinates of facility f_j

The Δx and Δy of the overlap Δ are defined as below:

$$\Delta x = \left(\frac{l_i + l_j}{2}\right) - |x_i - x_j| \quad (2)$$

$$\Delta y = \left(\frac{w_i + w_j}{2}\right) - |y_i - y_j| \quad (3)$$

The overlap called Δ is calculated as:

$$\Delta = \begin{cases} \Delta x \; if \; \Delta x \leq \Delta y \\ \Delta y \; if \; \Delta x \geq \Delta y \end{cases} \tag{4}$$

It means that if $\Delta x \leq \Delta y$, the overlap is repaired by removing overlap in x-projection. Similarly; if $\Delta x \geq \Delta y$, the overlap is eliminated in y-projection. Moreover, the overlap elimination function is taken by facility f_i, facility f_j or both.

Generally speaking the repair functions are designed based on different criteria such as:

1. The quadrant that facility f_i belongs to
2. The quadrant that facility f_j belongs to
3. The comparison between Δx and Δy

Firstly, If the distance_ left_between facility f_i and the boundaries of the site and the correspondent quadrant is less than the overlap Δ, then overlap elimination function is performed by moving facility f_i in upward or downward direction; otherwise, the facility f_j is taken into consideration. In other words, if the distance_left_between facility f_j and the boundaries of the site and the correspondent quadrant is less than the overlap Δ, then overlap elimination function is performed by moving facility f_j in upward or downward direction. However, if the distance left between the facility f_j and the boundaries of the site and its correspondent quadrant is not less than the overlap Δ, the overlap Δ would be shared between the both facilities f_i or f_j . Finally, if the overlap Δ is greater than the summation of distance_left_between of both facilities f_i and f_j, the vertical or horizontal movement is considered. The details regards to calculation of the distance_ left_between facility and the boundaries of site and quadrant shown in table (5).

Table (5): The distance_ left_ between facility f_i and the boundaries of site and quadrant

Quadrant	Upward Movement $(\boldsymbol{DLU_i})$	Downward Movement $(\boldsymbol{DLD_i})$
Q_1	$min\left\{\frac{W_C-\left(y_i+{w_i}/{2}\right)}{Sin\hat{\theta}},\frac{L_C-\left(x_i+{l_i}/{2}\right)}{Cos\hat{\theta}}\right\}$	$min\left\{\left\|\frac{\left(y_i-{w_i}/{2}\right)-{W_C}/{2}}{Sin\hat{\theta}}\right\|,\left\|\frac{\left(x_i-{l_i}/{2}\right)-{L_C}/{2}}{Cos\hat{\theta}}\right\|\right\}$
Q_2	$min\left\{\frac{W_C-\left(y_i+{w_i}/{2}\right)}{Cos(\hat{\theta}-90)},\frac{\left(x_i-{l_i}/{2}\right)}{Sin(\hat{\theta}-90)}\right\}$	$min\left\{\left\|\frac{\left(y_i-{w_i}/{2}\right)-{W_C}/{2}}{Cos(\hat{\theta}-90)}\right\|,\left\|\frac{{L_C}/{2}-\left(x_i+{l_i}/{2}\right)}{Sin(\hat{\theta}-90)}\right\|\right\}$
Q_3	$min\left\{\left\|\frac{{W_C}/{2}-\left(y_i+{w_i}/{2}\right)}{Sin(\hat{\theta}-180)}\right\|,\left\|\frac{{L_C}/{2}-\left(x_i+{l_i}/{2}\right)}{Cos(\hat{\theta}-180)}\right\|\right\}$	$min\left\{\left\|\frac{\left(x_i-{l_i}/{2}\right)}{Cos(\hat{\theta}-180)}\right\|,\left\|\frac{\left(y_i-{w_i}/{2}\right)}{Sin(\hat{\theta}-180)}\right\|\right\}$
Q_4	$min\left\{\left\|\frac{{W_C}/{2}-\left(y_i+{w_i}/{2}\right)}{Cos(\hat{\theta}-270)}\right\|,\left\|\frac{{L_C}/{2}-\left(x_i-{l_i}/{2}\right)}{Sin(\hat{\theta}-270)}\right\|\right\}$	$min\left\{\left\|\frac{L_C-\left(x_i+{l_i}/{2}\right)}{Sin(\hat{\theta}-270)}\right\|,\left\|\frac{\left(y_i-{w_i}/{2}\right)}{Cos(\hat{\theta}-270)}\right\|\right\}$

4.1.3. Steps of Heuristic

The steps of heuristic are as bellows:

Step 1: Place all facilities on top of each other in the centroid of the site. Set $k = 0$.

Step 2: Divide the site into four equal sized quadrants; and calculate the angle between facilities θ

Step 3: Outer loop

Step 3.1: Randomly choose one facility as target facility among those have not been placed yet and call it facility f_G. Set $k = k + 1$.

Step 3.2: Take radial movement.

Step 3.2.1: Calculate angle $\acute{\theta}$ of facility f_G and radial $\overrightarrow{r_f}$

Step 3.2.2: Find vector $\vec{a}$ along vector $\overrightarrow{r_f}$

Step 3.3: Place facility f_G at the end of vector $\vec{a}$ which centroid of facility f_G has distance equal to $|\vec{a}|$ to the centroid of the site. Find the new coordinate of facility f_G. The details are explained in table (4).

Step 3.4: Overlap checking; if there is any overlap between facility f_G and other facilities which have already been placed go to step 4 and set $u = k$; otherwise go to step 5.

Step 4: Inner loop

Step 4.1. $i = u - 1, j = i - 1$

Step 4.2. If $i \geq 2$ go to step 4.3; otherwise go to step 5.

Step 4.3. Specify the corresponding quadrants of the facilities f_i and f_j.

Step 4.4: Calculate the overlap Δ based on the comparison between Δx and Δy projections of the overlap between the two overlapped facilities f_i and f_j.

Step 4.5: Apply an appropriate repair function. The details brought in section 4.2.

Step 4.6. $j = j - 1$. If $j \geq 1$ then go to step 4.7; otherwise go to step 4.1.

Step 4.7: Overlap checking; if there is any overlap between facilities f_i and f_j go to step 4.3, otherwise go to step 4.6.

Step 5: If $k > M$ i.e. all facilities placed in the floor (cell) go to step 6; otherwise go to step 3. M is total number of facilities.

Step6: End

4.2. Repair Function

So far there are 126 different repair mechanisms designed for heuristic algorithm. It has to be noted based on comparison between horizontal and vertical projection of overlap between facility i, f_i and facility j, f_j the direction of repair movement is determined; *i.e.* upward movement or downward movement. The general steps of repair function mentioned in below:

4.2.1. General Steps of Repair Function

Step 1: Determine the quadrant in which facility $i,\ f_i$ has been placed

Step 2: Determine the quadrant in which facility $j,\ f_j$ has been placed

Step 3: Compare vertical coordinates of facility $i,\ f_i$ and facility , f_j, y_i and y_j respectively (If applicable)

Step 4: Compare horizontal coordinates of facility i, f_i and facility , f_j, x_i and x_j respectively (If applicable)

Step 5: Compare $x-$projection, Δx and $y-$projection, Δy of overlap, if $\Delta x \leq \Delta y$ then $\Delta = \Delta x$; otherwise $\Delta = \Delta y$.

Step 6: Determine appropriate overlap repair movement for facility i, f_i . If the movement direction is upward go to step 7, otherwise go to step 8.

Step 7: Calculate Distance-Left for facility i, called DLU_i . If $\Delta \leq DLU_i$ then move facility i, f_i upward by Δ value and go to step 17, otherwise go to step 9.

Step 8: Calculate Distance-Left for facility i, called DLD_i . If $\Delta \leq DLD_i$ then move facility i, f_i downward by Δ value and go to step 17, otherwise go to step 9.

Step 9: Determine appropriate overlap repair movement for facility j, f_j . If the movement direction is upward go to step 10, otherwise go to step 11.

Step 10: Calculate Distance-Left for facility j, called DLU_j . If $\Delta \leq DLU_j$ then move facility j, f_j upward by Δ value and go to step 17, otherwise go to step 12.

Step11: Calculate Distance-Left for facility j, called DLD_j . If $\Delta \leq DLD_j$ then move facility j, f_j downward by Δ value and go to step 17, otherwise go to step 12.

Step 12: If facility $i,\ \ f_i$ has to move upward, set $\acute{\Delta} = DLU_i$ and move it upward; otherwise, set $\acute{\Delta} = DLD_i$ and move it downward.

Step 13: Set $\acute{\Delta} = \Delta - \acute{\Delta}$. If facility $j,\ \ f_j$ has to move upward go to step 14, otherwise go to step 15.

Step14: If $\acute{\Delta} \leq DLU_j$ and move it upward by $\acute{\Delta}$ and go to step 17; otherwise go to step 16.

Step 15: If $\acute{\Delta} \leq DLD_j$ and move it upward by $\acute{\Delta}$ and go to step 17; otherwise go to step 16.

Step 16: Consider possibility of vertical or horizontal movement of facility $j,\ \ f_j$ and move it in appropriate direction. Go to step 17.

Step 17. Calculate new coordinates of both facility $i,\ \ f_i$ and facility $j,\ \ f_j$, Table (6) represents the calculation.

Step 18. End.

4.3. General Special Cases of Repair Function

Since the general idea of repair function is the same, the details for major special cases are represented below. Among those ones some of them are declared in details.

4.3.1. Facility $\boldsymbol{f_i}$ and facility $\boldsymbol{f_j}$ in quadrant $\boldsymbol{Q_1}$:

Firstly, the kind of overlap has to be determined. To do this, the vertical coordinate of f_i and f_j is compared together:

4.3.1.1. $\boldsymbol{y_i \geq y_j}$

4.3.1.1.1. $\boldsymbol{x_i \leq x_j}$

Based on the comparison between Δx and Δy, two special cases would exist as are explained below.

- **If** $\boldsymbol{\Delta x \geq \Delta y}$

Since $\Delta x \geq \Delta y$ the overlap Δ is set to y-projection of overlap, $\Delta = \Delta y$. As shown in figure (7), in order to eliminate overlap facility f_i has to move upward or facility f_j moves downward.

- Start with facility f_i ,

$$DLU_i = min\left\{\frac{W_c - \left(y_i + {}^{w_i}/_2\right)}{Sin\theta''}, \frac{L_c - \left(x_i + {}^{l_i}/_2\right)}{Cos\theta''}\right\} \tag{5}$$

If $\Delta \leq DLU_i$, facility f_i moves upward and new coordinate of facility f_i would be:

$$\acute{x_i} = x_i + \Delta \operatorname{Cos} \theta'' \tag{6}$$

$$\acute{y_i} = y_i + \Delta \tag{7}$$

- Otherwise, *i.e.* $\Delta > DLU_i$, moving facility f_j is considered:

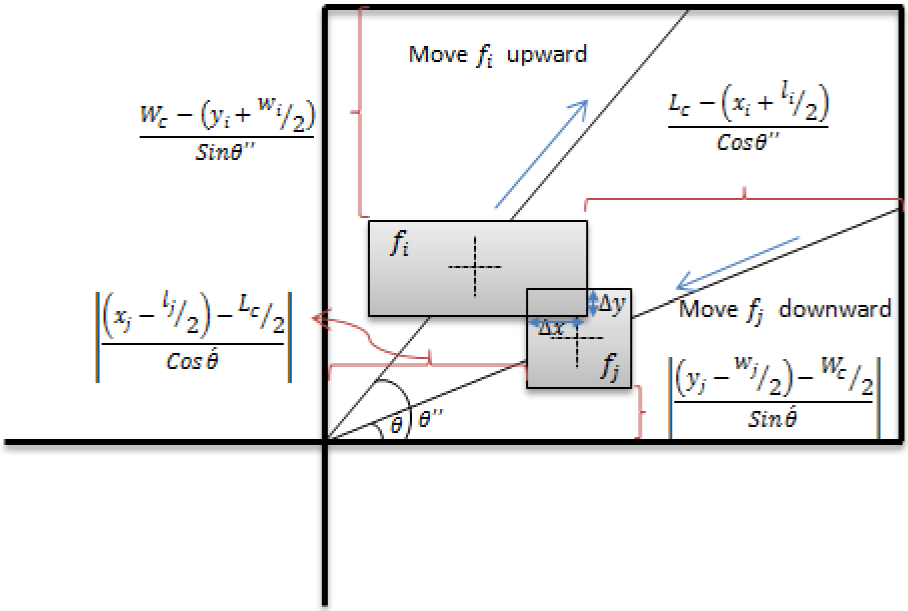
$$DLD_j = \ min\left\{\left|\frac{\left(y_j - {}^{w_j}/_2\right) - {}^{W_c}/_2}{Sin\acute{\theta}}\right|, \left|\frac{\left(x_j - {}^{l_j}/_2\right) - {}^{L_c}/_2}{Cos\acute{\theta}}\right|\right\} \tag{8}$$

If $\Delta \leq DLD_j$, facility f_j are moving downward and new coordinate of facility f_j would be:

$$\acute{x_j} = x_j - \Delta\,\mathrm{Cos}\,\acute{\theta} \tag{9}$$

$$\acute{y_j} = y_j - \Delta \tag{10}$$

- If $\Delta > DLD_j$, overlap is repaired by moving both facilities f_i and f_j, steps 12-16.



Move f_i upward
$\frac{W_c-(y_i+{}^{w_i}/_2)}{Sin\theta''}$
$\frac{L_c-(x_i+{}^{l_i}/_2)}{Cos\theta''}$
f_i
Δy
Δx
Move f_j downward
$\left|\frac{(x_j-{}^{l_j}/_2)-{}^{L_c}/_2}{Cos\acute{\theta}}\right|$
f_j
$\left|\frac{(y_j-{}^{w_j}/_2)-{}^{W_c}/_2}{Sin\acute{\theta}}\right|$
θ
θ''

Figure (7): Facility f_i and facility f_j in quadrant $Q_1, y_i \geq y_j, x_i \leq x_j, \Delta x \geq \Delta y$

- **If $\Delta x \leq \Delta y$**

Since $\Delta x \leq \Delta y$, the overlap Δ is set to y-projection of overlap, $\Delta = \Delta x$. As represented in figure (8), in order to eliminate overlap facility f_i has to move downward or facility f_j moves upward.

- Start with facility f_i,

$$DLD_i = min\left\{\left|\frac{(y_i-{}^{w_i}/_2)-{}^{W_C}/_2}{Sin\theta''}\right|, \left|\frac{(x_i-{}^{l_i}/_2)-{}^{L_C}/_2}{Cos\theta''}\right|\right\} \tag{11}$$

If $\Delta \leq DLD_i$, facility f_i are moving downward and new coordinate of facility f_i would be:

$$\acute{x}_\iota = x_i - \Delta \tag{12}$$

$$\acute{y}_\iota = y_i - \Delta cos\theta'' \tag{13}$$

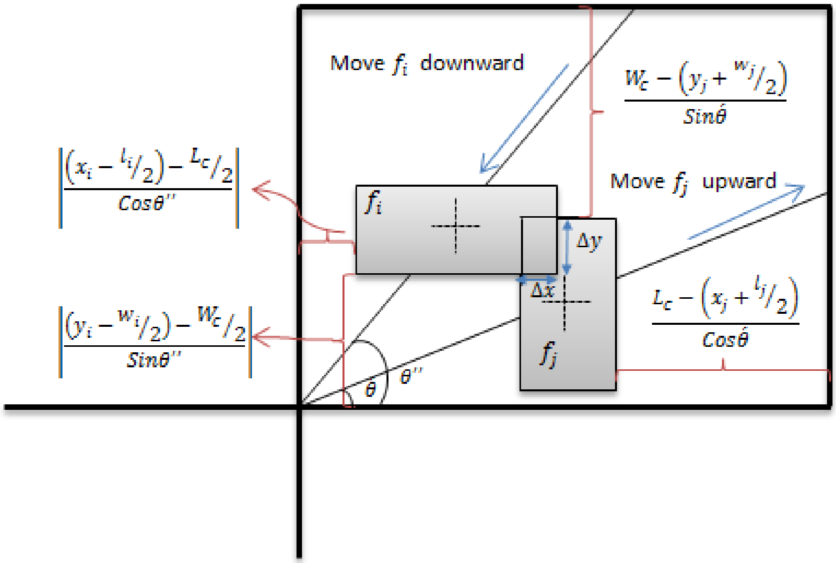

Move f_i downward
$\frac{W_c-\left(y_j+{}^{w_j}/_2\right)}{Sin\acute{\theta}}$
$\left|\frac{\left(x_i-{}^{l_i}/_2\right)-{}^{L_c}/_2}{Cos\theta''}\right|$
Move f_j upward
f_i
Δy
Δx
$\frac{L_c-\left(x_j+{}^{l_j}/_2\right)}{Cos\acute{\theta}}$
$\left|\frac{\left(y_i-{}^{w_i}/_2\right)-{}^{W_c}/_2}{Sin\theta''}\right|$
f_j
θ
θ''

Figure (8): Facility f_i and facility f_j in quadrant $Q_1, y_i \geq y_j, x_i \leq x_j, \Delta x \leq \Delta y$

- Otherwise, *i.e.* $\Delta > DLD_i$, moving facility f_j in upward direction is considered:

$$DLU_j = min\left\{\frac{W_C-\left(y_j+{}^{w_j}/_2\right)}{Sin\acute{\theta}}, \frac{L_C-\left(x_j+{}^{l_j}/_2\right)}{Cos\acute{\theta}}\right\} \tag{14}$$

If $\Delta \leq DLU_j$, facility f_j is moving upward and new coordinate of facility f_j would be:

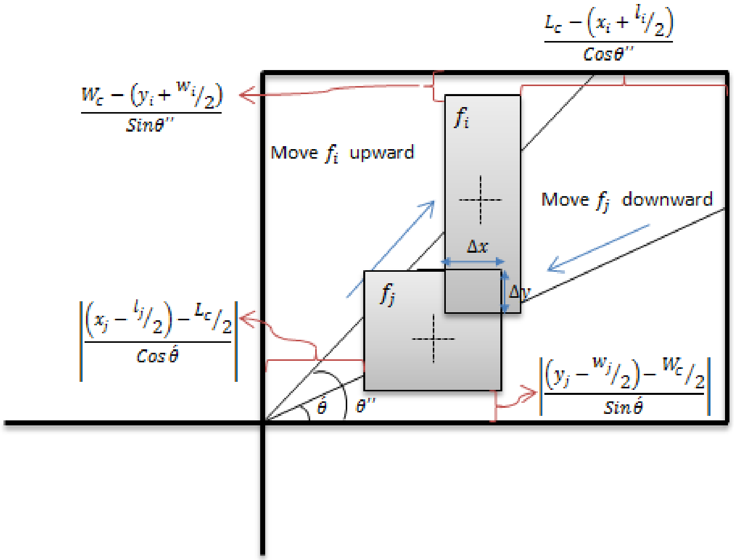
$$\acute{x}_\jmath = x_j + \Delta \tag{15}$$

$$\acute{y}_\jmath = y_j + \Delta\, Sin\acute{\theta} \tag{16}$$

- If $\Delta > DLU_j$, overlap is repaired by moving both facilities f_i and f_j, steps 12-16.

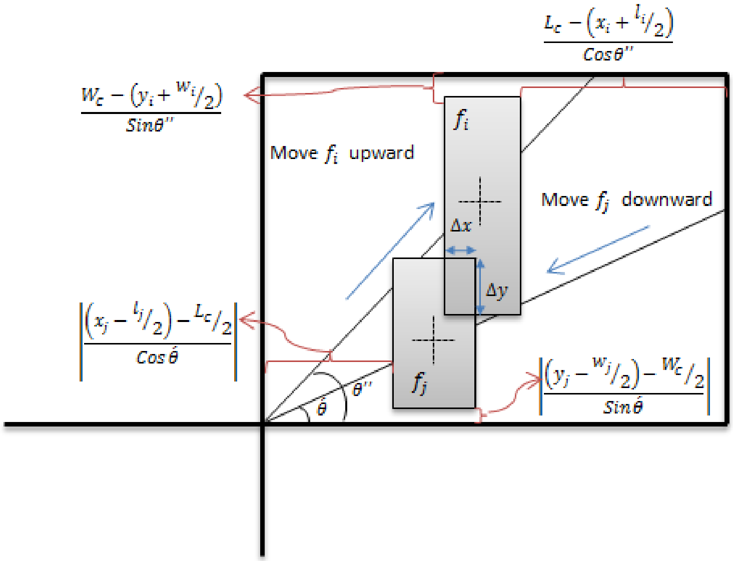
4.3.1.1.2. $x_i > x_j$

Either $\Delta x \geq \Delta y$ or $\Delta x < \Delta y$, there is one repair mechanism which is moving facility f_i upward or facility f_j downward. The figures (9) and (10) represent the scheme of this overlap case.



$\frac{L_c-(x_i+{}^{l_i}/_2)}{Cos\theta''}$
$\frac{W_c-(y_i+{}^{w_i}/_2)}{Sin\theta''}$
f_i
Move f_i upward
Move f_j downward
Δx
Δy
f_j
$\left|\frac{(x_j-{}^{l_j}/_2)-{}^{L_c}/_2}{Cos\acute{\theta}}\right|$
$\left|\frac{(y_j-{}^{w_j}/_2)-{}^{W_c}/_2}{Sin\acute{\theta}}\right|$
$\acute{\theta}$
θ''

Figure (9): Facility f_i and facility f_j in quadrant $Q_1, y_i \geq y_j, x_i \geq x_j, \Delta x \geq \Delta y$



$\frac{L_c-(x_i+{}^{l_i}/_2)}{Cos\theta''}$
$\frac{W_c-(y_i+{}^{w_i}/_2)}{Sin\theta''}$
f_i
Move f_i upward
Move f_j downward
Δx
Δy
$\left|\frac{(x_j-{}^{l_j}/_2)-{}^{L_c}/_2}{Cos\acute{\theta}}\right|$
θ''
$\acute{\theta}$
f_j
$\left|\frac{(y_j-{}^{w_j}/_2)-{}^{W_c}/_2}{Sin\acute{\theta}}\right|$

Figure (10): Facility f_i and facility f_j in quadrant $Q_1, y_i \geq y_j, x_i \geq x_j, \Delta x \leq \Delta y$

4.3.1.2. $y_i < y_j$

No case of overlap can be found in case of $x_i > x_j$. Hence, the only case has to be considered is when $x_i \leq x_j$. In this case either $\Delta x \leq \Delta y$ or $\Delta x > \Delta y$, one of the below three repair mechanisms:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figures (11) and (12) show this scheme.

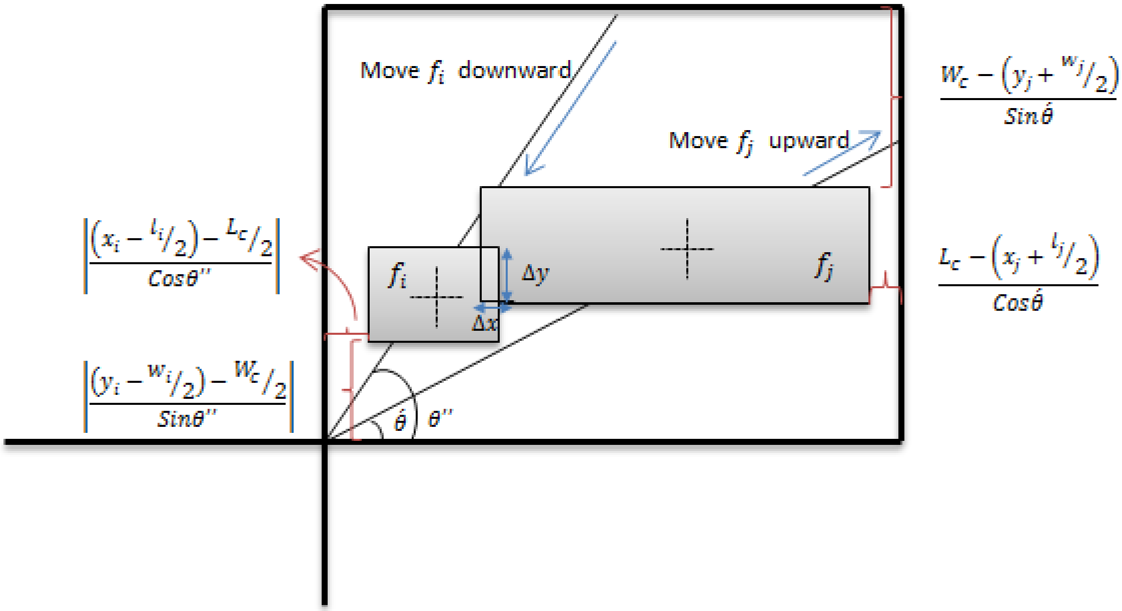

Move f_i downward
Move f_j upward
$\frac{W_c - (y_j + w_j/2)}{Sin\acute{\theta}}$
$\frac{L_c - (x_j + l_j/2)}{Cos\acute{\theta}}$
$\left|\frac{(x_i - l_i/2) - L_c/2}{Cos\theta''}\right|$
$\left|\frac{(y_i - w_i/2) - W_c/2}{Sin\theta''}\right|$
f_i
f_j
Δy
Δx
$\acute{\theta}$
θ''

Figure (11): Facility f_i and facility f_j in quadrant $Q_1, y_i \leq y_j, x_i \leq x_j, \Delta x \leq \Delta y$

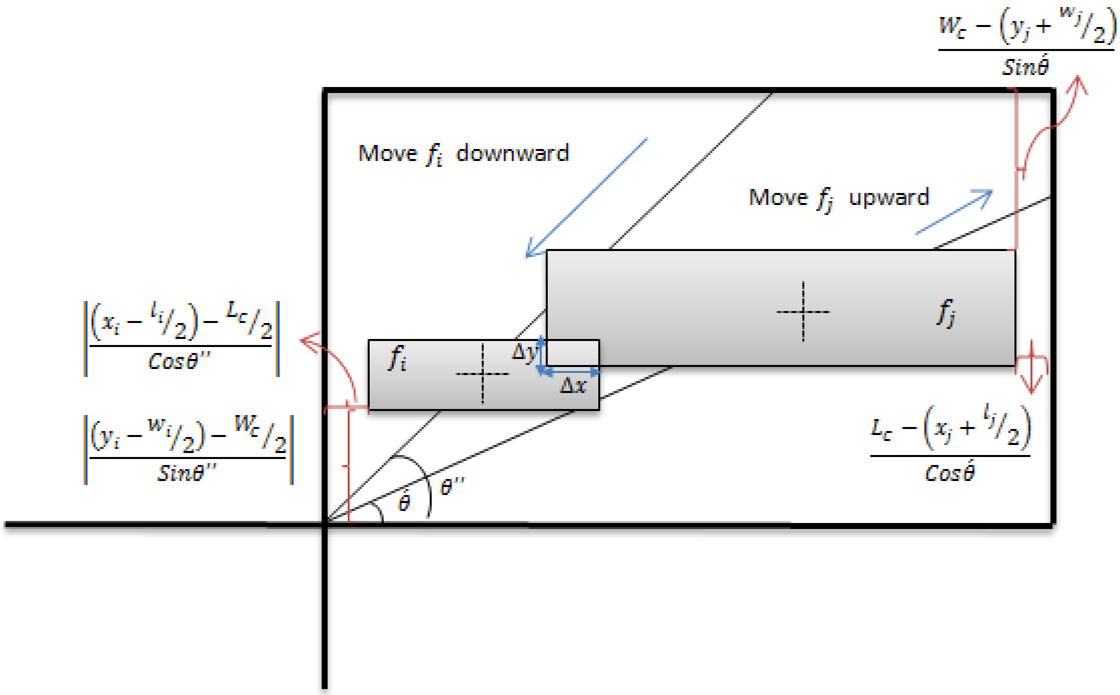

$\frac{W_c - (y_j + {}^{w_j}/_2)}{Sin\acute{\theta}}$
Move f_i downward
Move f_j upward
f_j
$|\frac{(x_i - {}^{l_i}/_2) - {}^{L_c}/_2}{Cos\theta''}|$
f_i
Δy
Δx
$|\frac{(y_i - {}^{w_i}/_2) - {}^{W_c}/_2}{Sin\theta''}|$
$\frac{L_c - (x_j + {}^{l_j}/_2)}{Cos\acute{\theta}}$
θ''
$\acute{\theta}$

Figure (12): Facility f_i and facility f_j in quadrant $Q_1, y_i \leq y_j, x_i \leq x_j, \Delta x \geq \Delta y$

4.3.2. Facility $\boldsymbol{f_i}$ in quadrant $\boldsymbol{Q_2}$:

When f_i is in Q_2, the overlapped facility is in either quadrant Q_2 or quadrant Q_1. In both cases at first horizontal coordinates of two facilities f_i and f_j are compared:

4.3.2.1. Facility $\boldsymbol{f_j}$ in quadrant $\boldsymbol{Q_1}$:

In this case definitely $x_i < x_j$; however comparison between the vertical coordinate of facility f_i and facility f_j make two different cases as explained below:

4.3.2.1.1. $\boldsymbol{y_i < y_j}$

Based on the comparison between *x*-projection and *y*-projection of the overlap two sub-cases are raised.

- **If $\boldsymbol{\Delta x \leq \Delta y}$**

Set $\Delta = \Delta x$. In order to eliminate overlap facility f_i has to move upward or facility f_j moves downward. Figure (13) represents the scheme of this case.

- Start with considering moving facility f_i in upward direction:

$$DLU_i = min\left\{\frac{W_C - \left(y_i + {}^{w_i}/_2\right)}{Cos(\theta'' - 90)}, \frac{\left(x_i - {}^{l_i}/_2\right)}{Sin(\theta'' - 90)}\right\} \qquad (17)$$

If $\Delta \leq DLU_i$, then move facility f_i upward. Hence, the new coordinates of facility f_i are:

$$\acute{x_\imath} = x_i - \Delta \qquad (18)$$

$$\acute{y_\imath} = y_i + \Delta\, cos(\theta'' - 90) \qquad (19)$$

- If $\Delta > DLU_i$, then calculate DLU_j for facility f_j

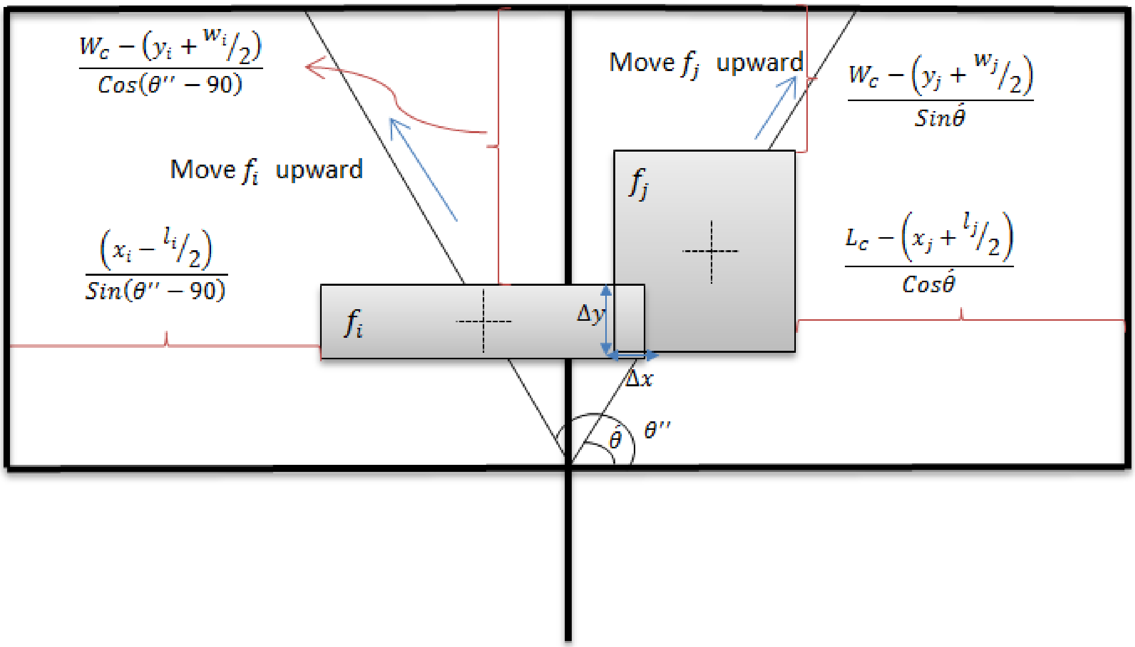
$$DLU_j = min\left\{\frac{W_C - \left(y_j + {}^{w_j}/_2\right)}{Sin\acute{\theta}}, \frac{L_C - \left(x_j + {}^{l_j}/_2\right)}{Cos\acute{\theta}}\right\} \qquad (20)$$

If $\Delta \le DLU_j$, then facility f_j moves upward. Hence, the new coordinates of facility f_j are:

$$\acute{x_j} = x_j + \Delta \tag{21}$$

$$\acute{y_\imath} = y_j + \Delta sin\acute{\theta} \tag{22}$$

- If $\Delta > DLU_j$, then movement both facilities f_i and f_j are considered, steps 12-16.



$\frac{W_c - (y_i + {}^{w_i}/_2)}{Cos(\theta'' - 90)}$
Move f_i upward
$\frac{(x_i - {}^{l_i}/_2)}{Sin(\theta'' - 90)}$
f_i
Δy
Δx
$\acute{\theta}$
θ''
Move f_j upward
$\frac{W_c - (y_j + {}^{w_j}/_2)}{Sin\acute{\theta}}$
f_j
$\frac{L_c - (x_j + {}^{l_j}/_2)}{Cos\acute{\theta}}$

Figure (13): Facility f_i in quadrant Q_2 and facility f_j in quadrant $Q_1, y_i \le y_j, , \Delta x \le \Delta y$

➢ **If $\Delta x > \Delta y$**

Set $\Delta = \Delta y$. In order to fix overlap, facility f_i has to move downward or facility f_j moves upward. Figure (14) represents the scheme of this case.

- Starting with facility f_i:

$$DLD_i = \left| \frac{(y_i - {}^{w_i}/_2) - {}^{W_C}/_2}{Cos(\theta'' - 90)} \right| \tag{23}$$

If $\Delta \le DLD_i$, facility f_i moves downward. Hence, the new coordinates of facility f_i are:

$$\acute{x}_\iota = x_i + \Delta sin(\theta'' - 90) \tag{24}$$

$$\acute{y}_\iota = y_i - \Delta \tag{25}$$

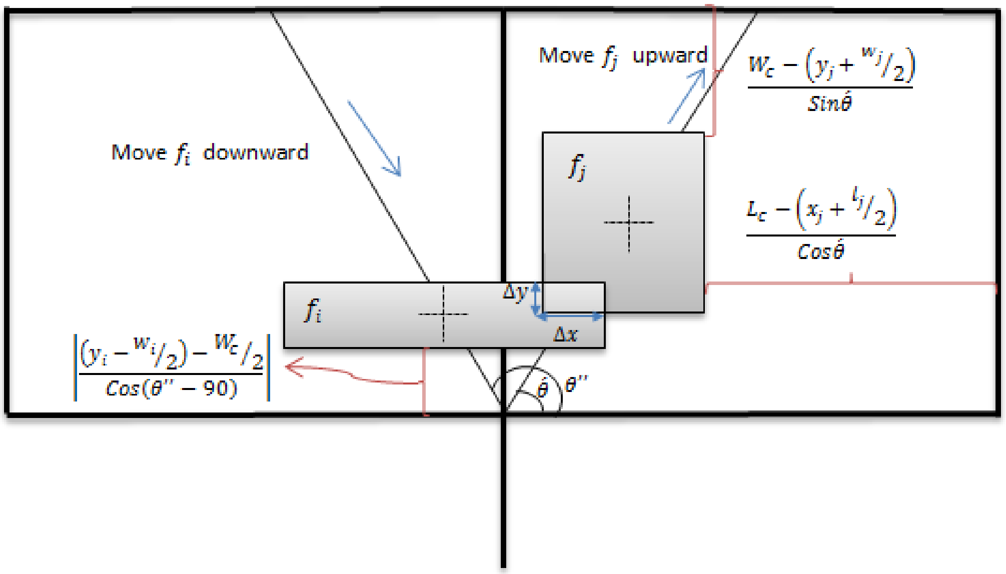

Move f_i downward
Move f_j upward
$\frac{W_c - (y_j + {}^{w_j}/_2)}{Sin\acute{\theta}}$
f_j
$\frac{L_c - (x_j + {}^{l_j}/_2)}{Cos\acute{\theta}}$
Δy
f_i
Δx
$\frac{\left|(y_i - {}^{w_i}/_2) - {}^{W_c}/_2\right|}{Cos(\theta'' - 90)}$
$\acute{\theta}$
θ''

Figure (14): Facility f_i in quadrant Q_2 and facility f_j is in quadrant $Q_1, y_i \le y_j, , \Delta x \ge \Delta y$

- If $\Delta > DLD_i$, then calculate DLU_j for facility f_j

$$DLU_j = min\left\{\frac{W_C - (y_j + {}^{w_j}/_2)}{Sin\acute{\theta}}, \frac{L_C - (x_j + {}^{l_j}/_2)}{Cos\acute{\theta}}\right\} \tag{26}$$

If $\Delta \le DLU_j$, facility f_j moves upward. Hence, the new coordinates of facility f_j are:

$$\acute{x}_j = x_j + \Delta\ Cos\acute{\theta} \tag{27}$$

$$\acute{y}_\iota = y_j + \Delta \tag{28}$$

- If $\Delta > DLU_j$, then movement both facilities f_i and f_j are considered, steps 12-16.

4.3.2.1.2. $\boldsymbol{y_i \geq y_j}$

Based on the comparison between *x*-projection and *y*-projection of the overlap two sub-cases are raised.

- **If $\boldsymbol{\Delta x \leq \Delta y}$**

Set $\Delta = \Delta x$. In order to remove overlap facility f_i has to move upward or facility f_j moves up. Figure (15) represents the scheme of this case.

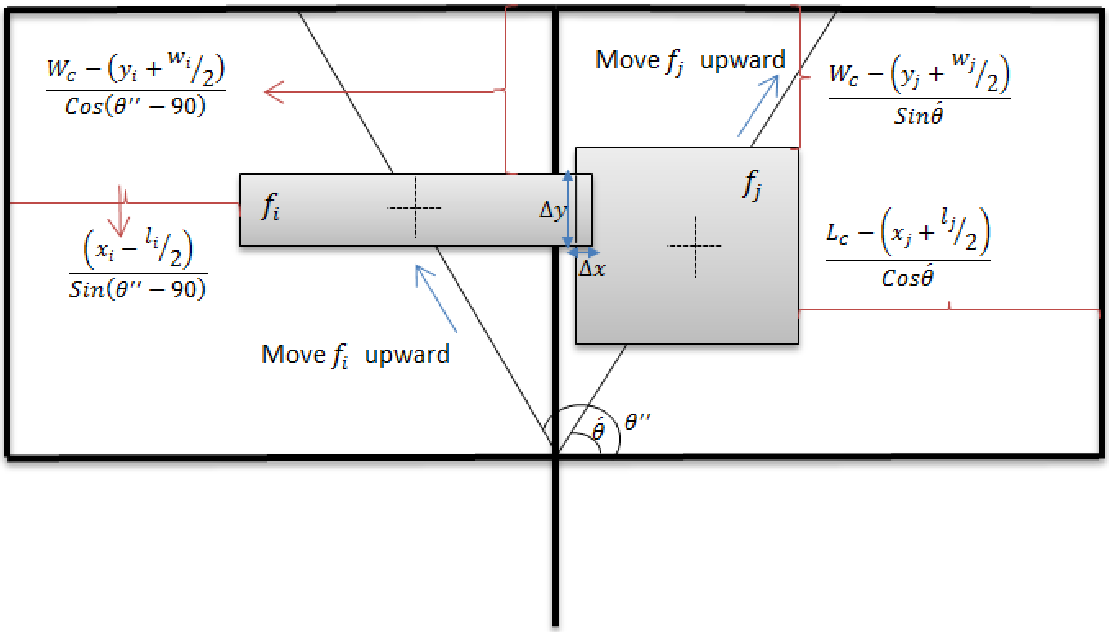
- Start with considering movement of facility f_i:

$$DLU_i = min\left\{\frac{W_C - \left(y_i + {}^{w_i}/_2\right)}{Cos(\theta'' - 90)}, \frac{\left(x_i - {}^{l_i}/_2\right)}{Sin(\theta'' - 90)}\right\} \tag{29}$$

If $\Delta \leq DLU_i$, then facility f_i moves upward. Hence, the new coordinates of facility f_i are:

$$\acute{x_i} = x_i - \Delta \tag{30}$$

$$\acute{y_i} = y_i + \Delta\, cos(\theta'' - 90) \tag{31}$$



$\frac{W_c - \left(y_i + {}^{w_i}/_2\right)}{Cos(\theta'' - 90)}$
Move f_j upward
$\frac{W_c - \left(y_j + {}^{w_j}/_2\right)}{Sin\acute{\theta}}$
f_j
f_i
Δy
Δx
$\frac{\left(x_i - {}^{l_i}/_2\right)}{Sin(\theta'' - 90)}$
$\frac{L_c - \left(x_j + {}^{l_j}/_2\right)}{Cos\acute{\theta}}$
Move f_i upward
$\acute{\theta}$
θ''

Figure (15): Facility f_i in quadrant Q_2 and facility f_j in quadrant $Q_1, y_i \geq y_j$, $\Delta x \leq \Delta y$

- If $\Delta > DLU_i$, then DLU_j for facility f_j is calculated:

$$DLU_j = min\left\{\frac{W_C-\left(y_j+{}^{w_j}/_2\right)}{Sin\acute{\theta}}, \frac{L_C-\left(x_j+{}^{l_j}/_2\right)}{Cos\acute{\theta}}\right\} \tag{32}$$

- If $\Delta \leq DLU_j$, then facility f_j moves upward. Hence, the new coordinates of facility f_j are:

$$\acute{x}_{J} = x_j + \Delta \tag{33}$$

$$\acute{y}_{\iota} = y_j + \Delta sin\acute{\theta} \tag{34}$$

- If $\Delta > DLU_j$, then movement both facilities f_i and f_j are considered, steps 12-16.

➢ **If $\Delta \boldsymbol{x} > \Delta \boldsymbol{y}$**

Set $\Delta = \Delta y$. In order to remove overlap facility f_i has to move upward or facility f_j moves downward. Figure (16) represents the scheme of this case.

- The details of moving facility f_i upward are brought in (29) to (31).
- If moving facility f_i in upward direction is impossible; *i.e.* $\Delta > DLU_i$, then the possibility of moving facility f_j downward is considering.

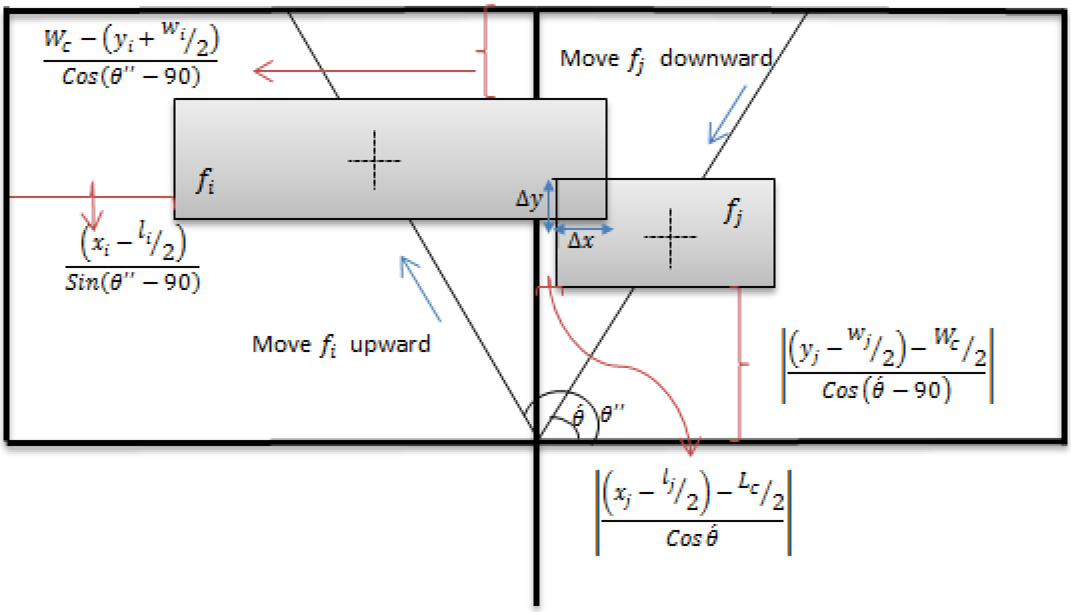
$$DLD_j = min\left\{\left|\frac{\left(y_j-{}^{w_j}/_2\right)-{}^{W_C}/_2}{Sin\acute{\theta}}\right|, \left|\frac{\left(x_j-{}^{l_j}/_2\right)-{}^{L_C}/_2}{Cos\acute{\theta}}\right|\right\} \tag{35}$$

If $\Delta \leq DLD_j$, then facility f_j is moved downward; and new coordinate would be:

$$\acute{x}_{J} = x_j - \Delta\, cos\acute{\theta} \tag{36}$$

$$\acute{y}_{\iota} = y_j - \Delta \tag{37}$$

- If $\Delta > DLD_j$, then movement both facilities f_i and f_j are considered, steps 12-16.



$\frac{W_c - (y_i + {}^{w_i}/_2)}{Cos(\theta'' - 90)}$
Move f_j downward
f_i
Δy
f_j
Δx
$\frac{(x_i - {}^{l_i}/_2)}{Sin(\theta'' - 90)}$
Move f_i upward
$\left|\frac{(y_j - {}^{w_j}/_2) - {}^{W_c}/_2}{Cos(\acute{\theta} - 90)}\right|$
$\acute{\theta}$
θ''
$\left|\frac{(x_j - {}^{l_j}/_2) - {}^{L_c}/_2}{Cos\,\acute{\theta}}\right|$

Figure (16): Facility f_i in quadrant Q_2 and facility f_j in quadrant Q_1, $y_i \geq y_j$, , $\Delta x > \Delta y$

4.3.2.2. Facility f_j in quadrant Q_2

The comparison between vertical coordinates of facility f_i and f_j makes two set of cases which are declared below:

4.3.2.2.1. $y_i \geq y_j$

If vertical coordinate of facility f_i is greater than vertical coordinated of facility f_j, definitely $x_i \leq x_j$. In both cases of $\Delta x \leq \Delta y$ and $\Delta x \geq \Delta y$ the overlap is fixed by applying one of these three mechanisms.

1. Move facility f_i upward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figures (17) and (18) represent these cases.

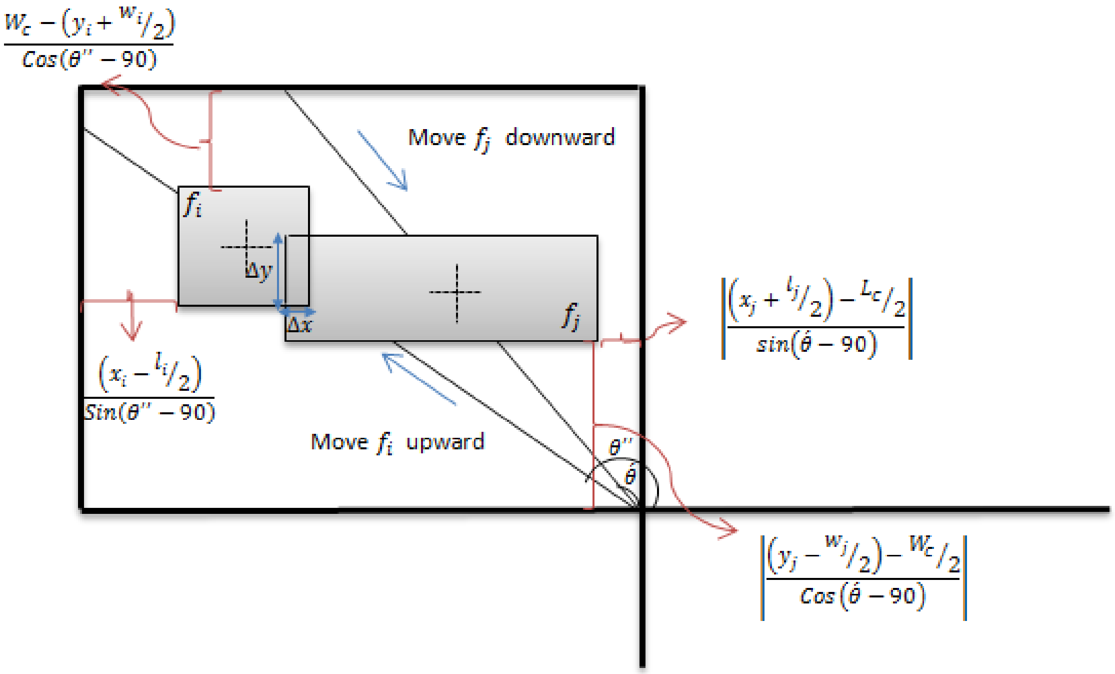

$\frac{W_c-(y_i+{}^{w_i}/_2)}{Cos(\theta''-90)}$
Move f_j downward
f_i
Δy
Δx
f_j
$\left|\frac{(x_j+{}^{l_j}/_2)-{}^{L_c}/_2}{sin(\acute{\theta}-90)}\right|$
$\frac{(x_i-{}^{l_i}/_2)}{Sin(\theta''-90)}$
Move f_i upward
θ''
$\acute{\theta}$
$\left|\frac{(y_j-{}^{w_j}/_2)-{}^{W_c}/_2}{Cos(\acute{\theta}-90)}\right|$

Figure (17): Facility f_i and facility f_j in quadrant $Q_2, y_i \geq y_j, x_i \leq x_j, \Delta x \leq \Delta y$

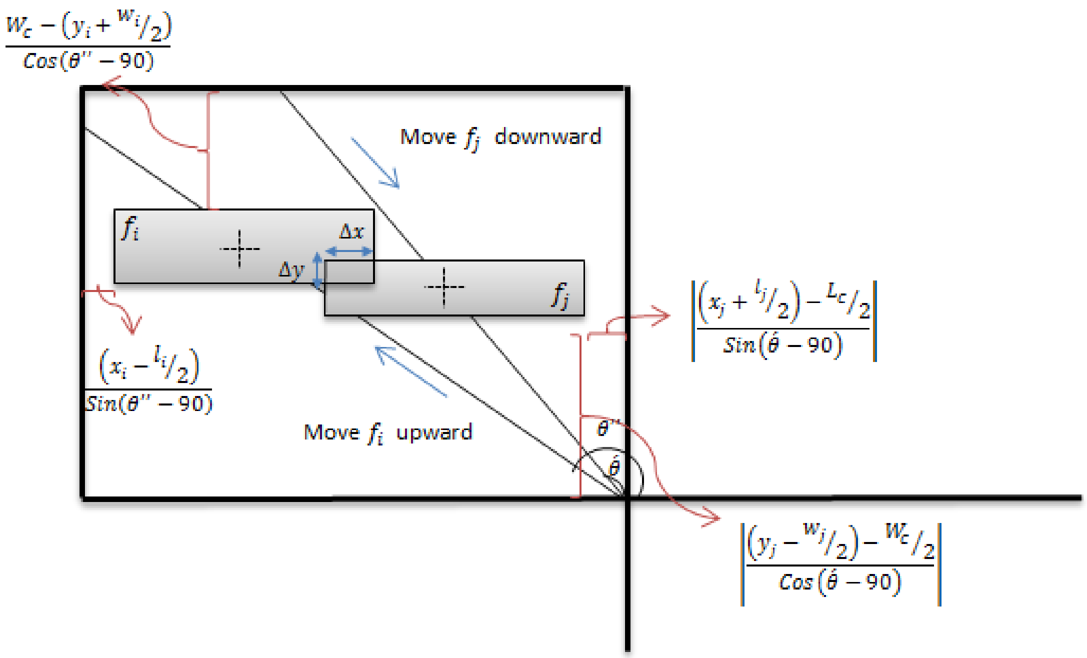

$\frac{W_c-(y_i+{}^{w_i}/_2)}{Cos(\theta''-90)}$
Move f_j downward
f_i
Δx
Δy
f_j
$\left|\frac{(x_j+{}^{l_j}/_2)-{}^{L_c}/_2}{Sin(\acute{\theta}-90)}\right|$
$\frac{(x_i-{}^{l_i}/_2)}{Sin(\theta''-90)}$
Move f_i upward
θ''
$\acute{\theta}$
$\left|\frac{(y_j-{}^{w_j}/_2)-{}^{W_c}/_2}{Cos(\acute{\theta}-90)}\right|$

Figure (18): Facility f_i and facility f_j in quadrant $Q_2, y_i \geq y_j, x_i \leq x_j, \Delta x > \Delta y$

4.3.2.2.2. $y_i < y_j$

Based on the comparison between vertical coordinates of facilities f_i and f_j two sets of sub-cases would exist such as:

❖ $\boldsymbol{x_i \leq x_j}$

Regard to the *x*-projection and *y*-projection of the overlap different repair functions would require.

➢ **If $\Delta x \geq \Delta y$**

Set $\Delta = \Delta y$ and the repair function is one of the functions below:

1. Move facility f_i downward by Δ
2. Move facility f_j upward Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figure (19) shows the scheme of this case.

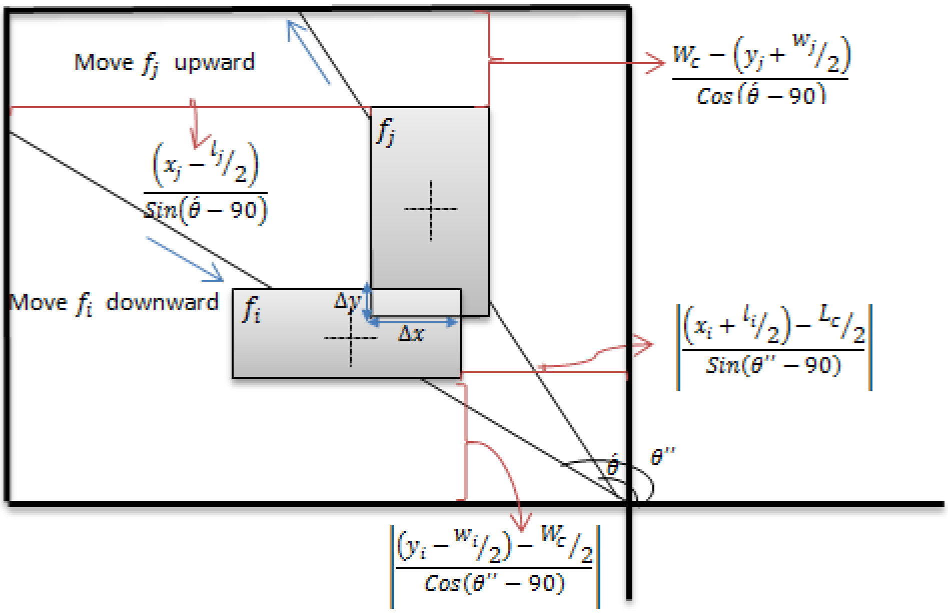

Move f_j upward
$\frac{W_c-(y_j+{}^{w_j}/_2)}{Cos(\acute{\theta}-90)}$
f_j
$\frac{(x_j-{}^{l_j}/_2)}{Sin(\acute{\theta}-90)}$
Move f_i downward
f_i
Δy
Δx
$\left|\frac{(x_i+{}^{l_i}/_2)-{}^{L_c}/_2}{Sin(\theta''-90)}\right|$
$\acute{\theta}$
θ''
$\left|\frac{(y_i-{}^{w_i}/_2)-{}^{W_c}/_2}{Cos(\theta''-90)}\right|$

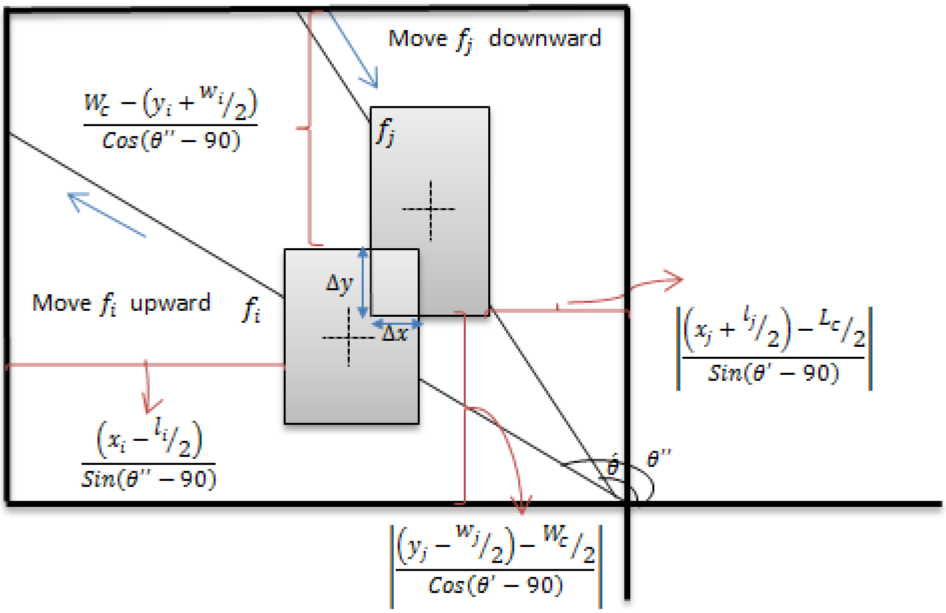
Figure (19): Facility f_i and facility f_j in quadrant $Q_2, y_i < y_j, x_i \leq x_j, \Delta x > \Delta y$

- **If $\Delta x < \Delta y$**

Set $\Delta = \Delta y$ and the repair function is one of the functions below:

1. Move facility f_i upward by Δ
2. Move facility f_j downward Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figure (20) shows the scheme of this case.



Move f_j downward
$\frac{W_c - (y_i + {}^{w_i}/_2)}{Cos(\theta'' - 90)}$
f_j
Move f_i upward
f_i
Δy
Δx
$\left|\frac{(x_j + {}^{l_j}/_2) - {}^{L_c}/_2}{Sin(\theta' - 90)}\right|$
$\frac{(x_i - {}^{l_i}/_2)}{Sin(\theta'' - 90)}$
$\acute{\theta}$
θ''
$\left|\frac{(y_j - {}^{w_j}/_2) - {}^{W_c}/_2}{Cos(\theta' - 90)}\right|$

Figure (20): Facility f_i and facility f_j in quadrant $Q_2, y_i < y_j, x_i \leq x_j, \Delta x \leq \Delta y$

- $\boldsymbol{x_i > x_j}$

Regardless to the *x*-projection and *y*-projection of the overlap different repair functions would require.

1. Move facility f_i downward by Δ
2. Move facility f_j upward Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figures (21) and (22) show the schemes of these cases.

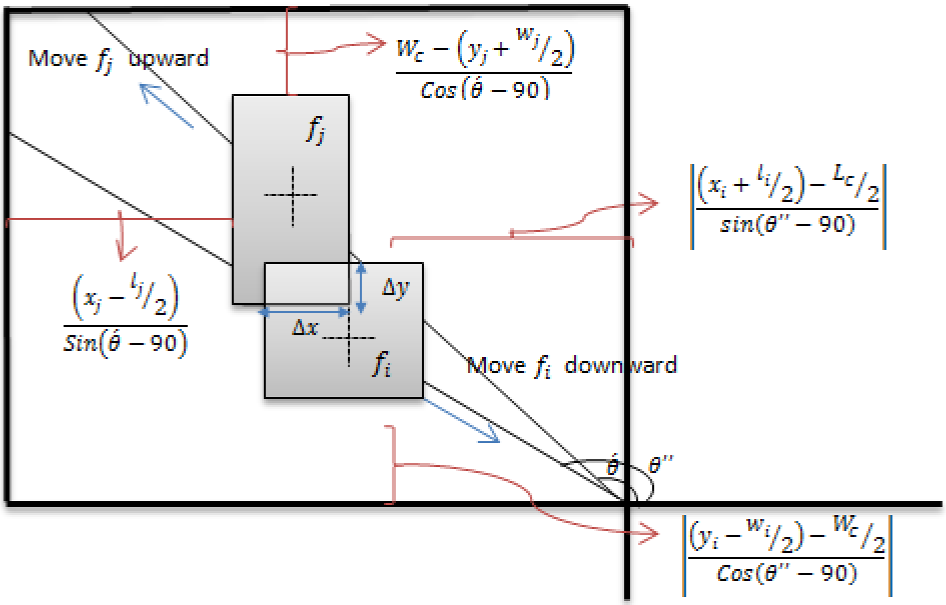

Move f_j upward
$\frac{W_c - (y_j + {}^{w_j}/_2)}{Cos(\acute{\theta} - 90)}$
f_j
$\left|\frac{(x_i + {}^{l_i}/_2) - {}^{L_c}/_2}{sin(\theta'' - 90)}\right|$
$\frac{(x_j - {}^{l_j}/_2)}{Sin(\acute{\theta} - 90)}$
Δy
Δx
f_i
Move f_i downward
$\acute{\theta}$
θ''
$\left|\frac{(y_i - {}^{w_i}/_2) - {}^{W_c}/_2}{Cos(\theta'' - 90)}\right|$

Figure (21): Facility f_i and facility f_j in quadrant $Q_2, y_i < y_j, x_i \geq x_j, \Delta x > \Delta y$

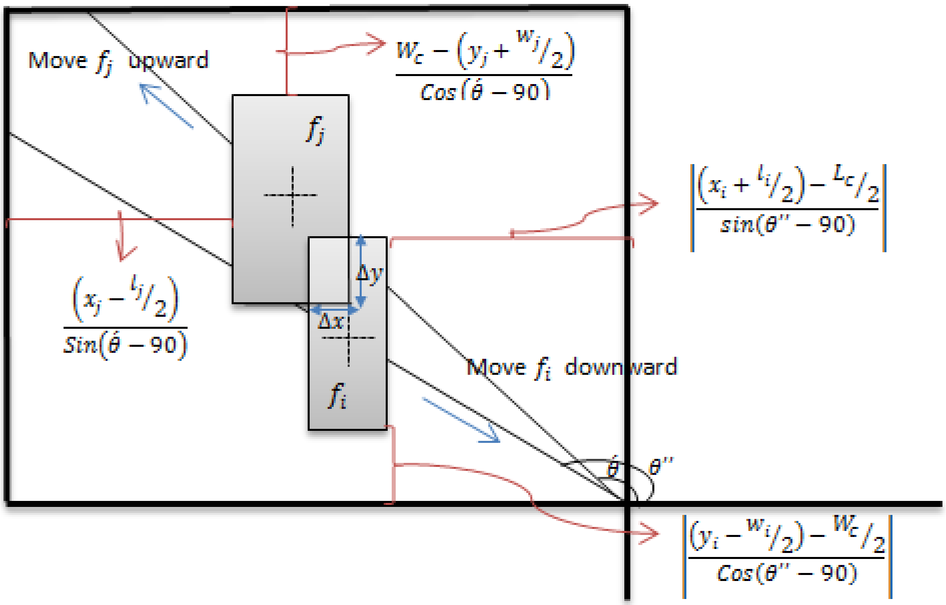

Move f_j upward
$\frac{W_c - (y_j + {}^{w_j}/_2)}{Cos(\acute{\theta} - 90)}$
f_j
$\left|\frac{(x_i + {}^{l_i}/_2) - {}^{L_c}/_2}{sin(\theta'' - 90)}\right|$
$\frac{(x_j - {}^{l_j}/_2)}{Sin(\acute{\theta} - 90)}$
Δy
Δx
f_i
Move f_i downward
$\acute{\theta}$
θ''
$\left|\frac{(y_i - {}^{w_i}/_2) - {}^{W_c}/_2}{Cos(\theta'' - 90)}\right|$

Figure (22): Facility f_i and facility f_j in quadrant $Q_2, y_i < y_j, x_i \geq x_j, \Delta x \leq \Delta y$

4.3.3. Facility f_i in quadrant Q_3

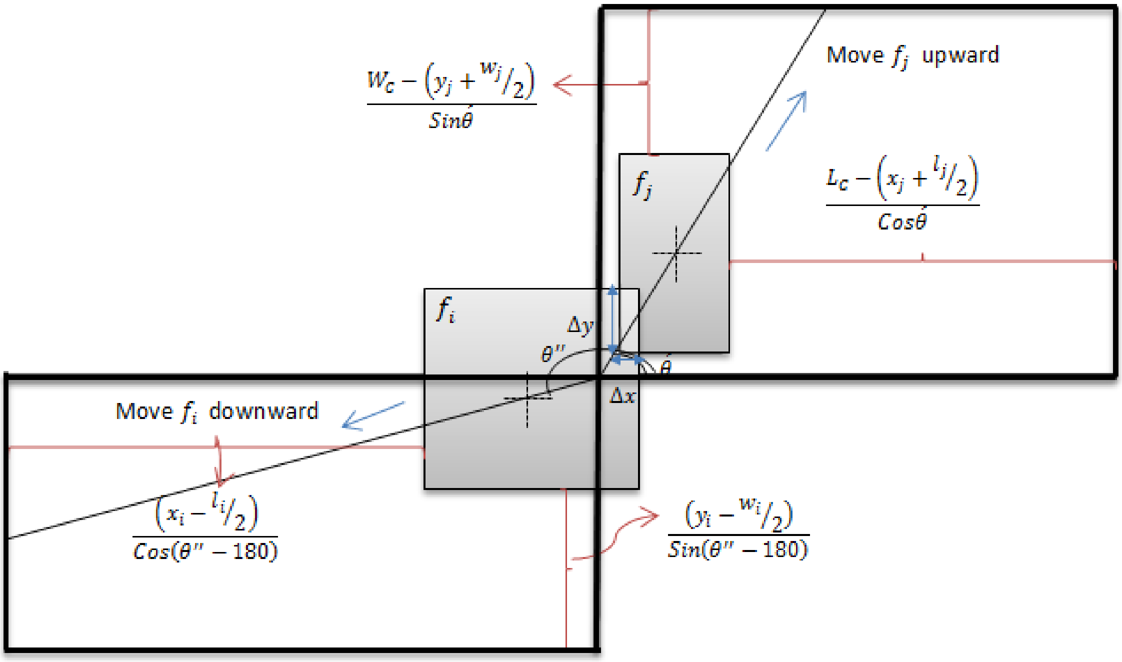
When facility f_i is in quadrant Q_3, the overlapped facility is in one of quadrant Q_3, Q_2 or Q_1.

4.3.3.1. Facility f_j in quadrant Q_1

In this case obviously $y_i < y_j$ and $x_i < x_j$. Hence, in both cases of $\Delta x \geq \Delta y$ and $\Delta x < \Delta y$ overlap is fixed by using one of these below functions:

1. Move facility f_i downward by Δ
2. Move facility f_j upward Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figures (23) and (24) show the schemes of these cases.



$\frac{W_C-(y_j+{}^{w_j}/_2)}{Sin\theta}$
Move f_j upward
$\frac{L_C-(x_j+{}^{l_j}/_2)}{Cos\theta}$
f_j
f_i
Δy
θ''
θ
Δx
Move f_i downward
$\frac{(x_i-{}^{l_i}/_2)}{Cos(\theta''-180)}$
$\frac{(y_i-{}^{w_i}/_2)}{Sin(\theta''-180)}$

Figure (23): Facility f_i in quadrant Q_3 and facility f_j in quadrant Q_1,, $\Delta x \leq \Delta y$

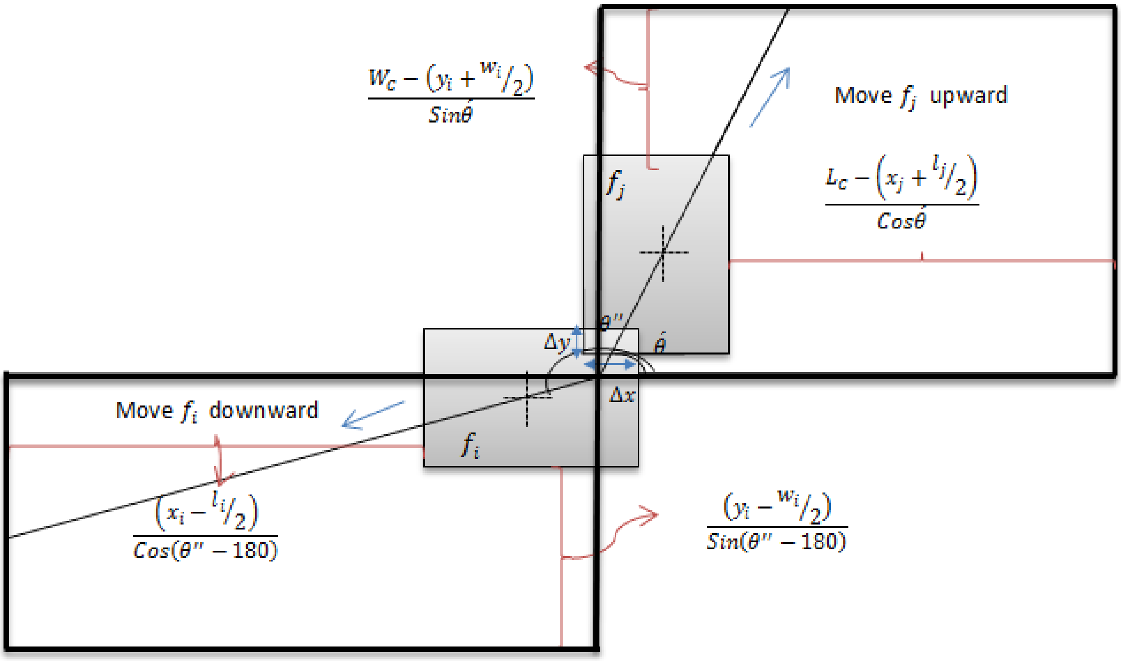

$\frac{W_c - (y_i + {}^{w_i}/_2)}{Sin\acute{\theta}}$
Move f_j upward
$\frac{L_c - (x_j + {}^{l_j}/_2)}{Cos\acute{\theta}}$
f_j
θ''
Δy
$\acute{\theta}$
Δx
Move f_i downward
f_i
$\frac{(x_i - {}^{l_i}/_2)}{Cos(\theta'' - 180)}$
$\frac{(y_i - {}^{w_i}/_2)}{Sin(\theta'' - 180)}$

Figure (24): Facility f_i is in quadrant Q_3 and facility f_j is in quadrant Q_1, $\Delta x > \Delta y$

4.3.3.2. Facility f_j in quadrant Q_2

Since f_i is in the third quadrant and f_j is in the second quadrant, the vertical coordinate of facility f_i is less than the vertical coordinate of facility f_j ; *i.e.* $y_i < y_j$. Based on the comparison between horizontal coordinates of facility f_i and f_j , two sets of repair functions would be defined as follows:

❖ $\boldsymbol{x_i \leq x_j}$

➢ **If $\Delta x < \Delta y$**

Set $\Delta = \Delta x$. The overlap is eliminated by using one of the below functions:

1. Move facility f_i downward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

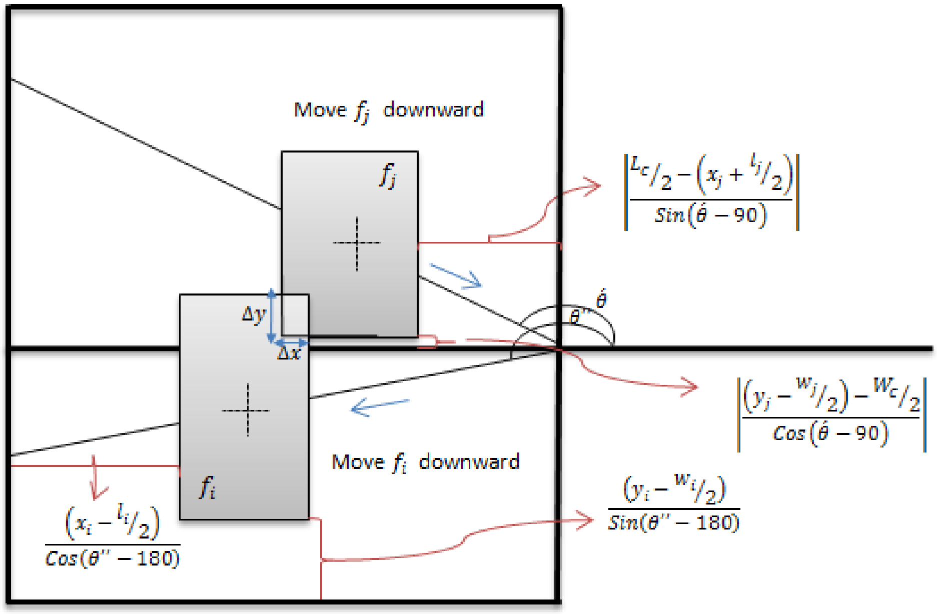

Move f_j downward
f_j
$\left|\frac{L_c/2-(x_j+l_j/2)}{Sin(\acute{\theta}-90)}\right|$
$\acute{\theta}$
θ''
Δy
Δx
$\left|\frac{(y_j-w_j/2)-W_c/2}{Cos(\acute{\theta}-90)}\right|$
Move f_i downward
f_i
$\frac{(y_i-w_i/2)}{Sin(\theta''-180)}$
$\frac{(x_i-l_i/2)}{Cos(\theta''-180)}$

Figure (25): Facility f_i in quadrant Q_3 and facility f_j in quadrant Q_2 , $x_i \leq x_j$, $\Delta x \leq \Delta y$

- **If $\Delta x \geq \Delta y$**

Set $\Delta = \Delta y$. The overlap would be eliminated by using one of the functions below:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figure (26) represents the scheme of this case.

- $\boldsymbol{x_i > x_j}$

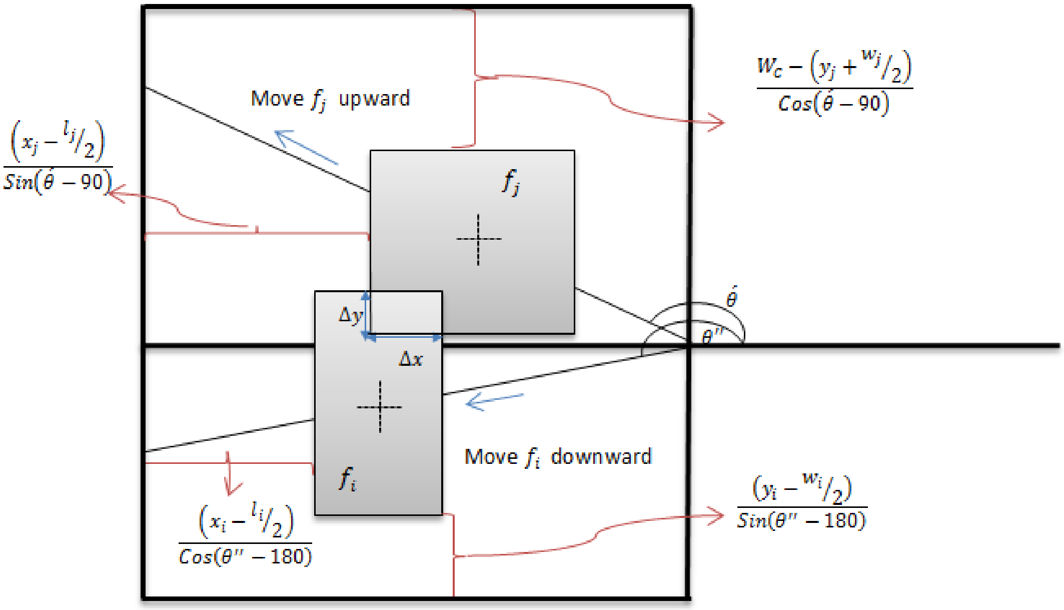
- **If $\Delta x \geq \Delta y$**

Set $\Delta = \Delta y$. The overlap is eliminated by using one of the functions below:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ

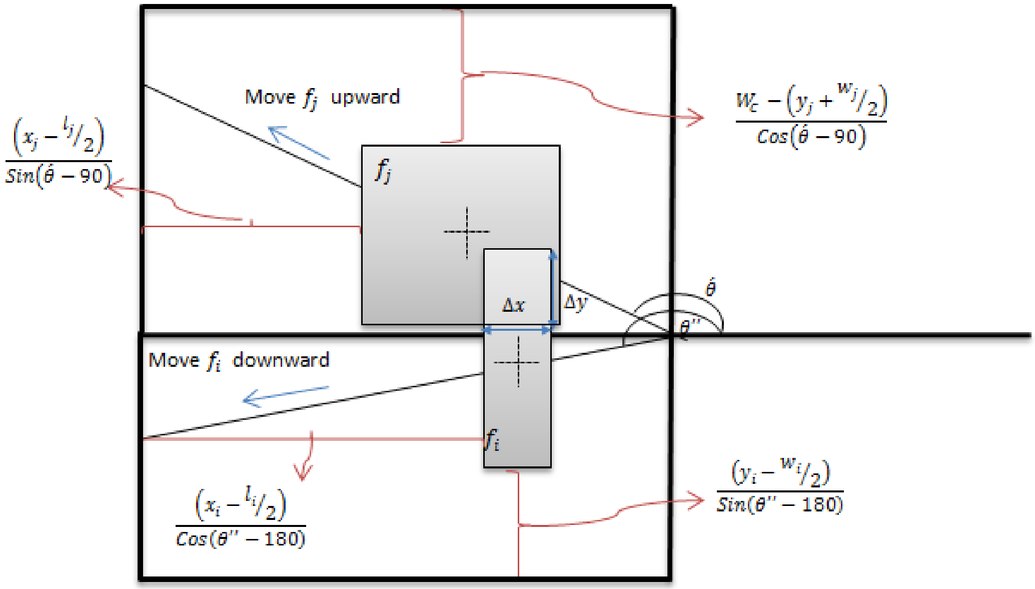
3. Move both facilities f_i and f_j, steps 12-16.

Figure (27) represents the scheme of this case.



$\frac{\left(x_j - {}^{l_j}/_2\right)}{Sin(\acute{\theta} - 90)}$
Move f_j upward
$\frac{W_c - \left(y_j + {}^{w_j}/_2\right)}{Cos(\acute{\theta} - 90)}$
f_j
Δy
Δx
$\acute{\theta}$
θ''
Move f_i downward
f_i
$\frac{\left(x_i - {}^{l_i}/_2\right)}{Cos(\theta'' - 180)}$
$\frac{\left(y_i - {}^{w_i}/_2\right)}{Sin(\theta'' - 180)}$

Figure (26): Facility f_i in quadrant Q_3 and facility f_j in quadrant Q_2 , $x_i \leq x_j, \Delta x \geq \Delta y$



$\frac{\left(x_j - {}^{l_j}/_2\right)}{Sin(\acute{\theta} - 90)}$
Move f_j upward
$\frac{W_c - \left(y_j + {}^{w_j}/_2\right)}{Cos(\acute{\theta} - 90)}$
f_j
Δx
Δy
$\acute{\theta}$
θ''
Move f_i downward
f_i
$\frac{\left(x_i - {}^{l_i}/_2\right)}{Cos(\theta'' - 180)}$
$\frac{\left(y_i - {}^{w_i}/_2\right)}{Sin(\theta'' - 180)}$

Figure (27): Facility f_i in quadrant Q_3 and facility f_j is in quadrant Q_2 , $x_i \geq x_j, \Delta x \geq \Delta y$

- **If $\Delta x < \Delta y$**

Set $\Delta = \Delta x$. The overlap is eliminated by using one of the functions below:

1. Move facility f_i upward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figure (28) represents the scheme of this case.

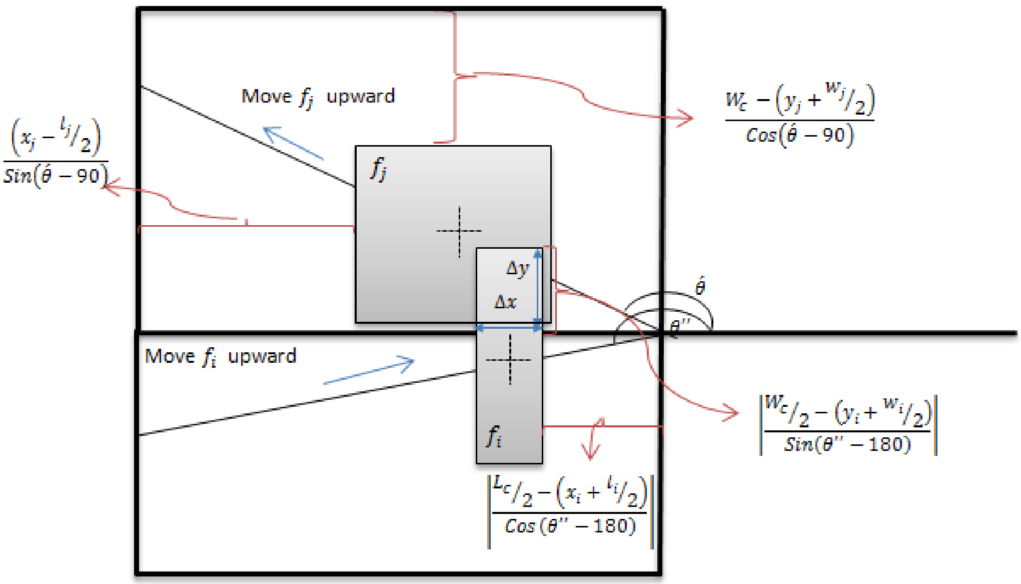

Move f_j upward
$\frac{W_c - (y_j + {}^{w_j}/_2)}{Cos(\acute{\theta} - 90)}$
$\frac{(x_j - {}^{l_j}/_2)}{Sin(\acute{\theta} - 90)}$
f_j
Δy
Δx
$\acute{\theta}$
θ''
Move f_i upward
$\left|\frac{{}^{W_c}/_2 - (y_i + {}^{w_i}/_2)}{Sin(\theta'' - 180)}\right|$
f_i
$\left|\frac{{}^{L_c}/_2 - (x_i + {}^{l_i}/_2)}{Cos\,(\theta'' - 180)}\right|$

Figure (28): Facility f_i in quadrant Q_3 and facility f_j in quadrant Q_2 , $x_i \geq x_j$, $\Delta x \leq \Delta y$

4.3.3.3. Facility f_j is in quadrant Q_3

4.3.3.3.1. $y_i \geq y_j$

No case can be found in which $x_i < x_j$. In case of $x_i \geq x_j$ for both $\Delta x \geq \Delta y$ and $\Delta x < \Delta y$, the overlap is fixed by applying one of these repair functions:

1. Move facility f_i upward by Δ
2. Move facility f_j downward by Δ

3. Move both facilities f_i and f_j, steps 12-16.

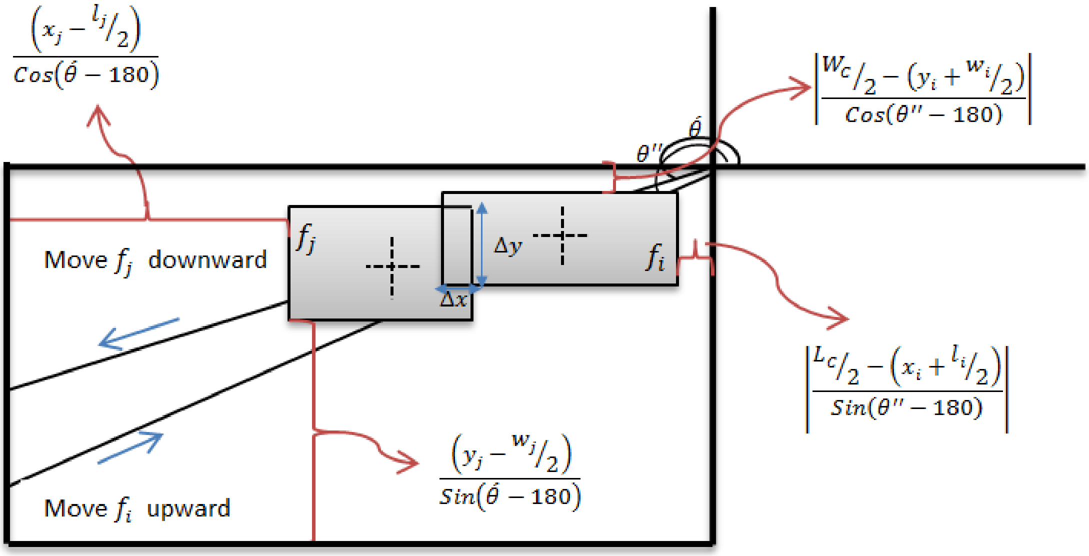

$\frac{(x_j - l_j/2)}{Cos(\acute{\theta} - 180)}$
$\left|\frac{W_c/2 - (y_i + w_i/2)}{Cos(\theta'' - 180)}\right|$
$\acute{\theta}$
θ''
f_j
Δy
f_i
Move f_j downward
Δx
$\left|\frac{L_c/2 - (x_i + l_i/2)}{Sin(\theta'' - 180)}\right|$
$\frac{(y_j - w_j/2)}{Sin(\acute{\theta} - 180)}$
Move f_i upward

Figure (29): Facility f_i and facility f_j in quadrant Q_3 , $y_i \geq y_j$, $x_i \geq x_j$, $\Delta x \leq \Delta y$

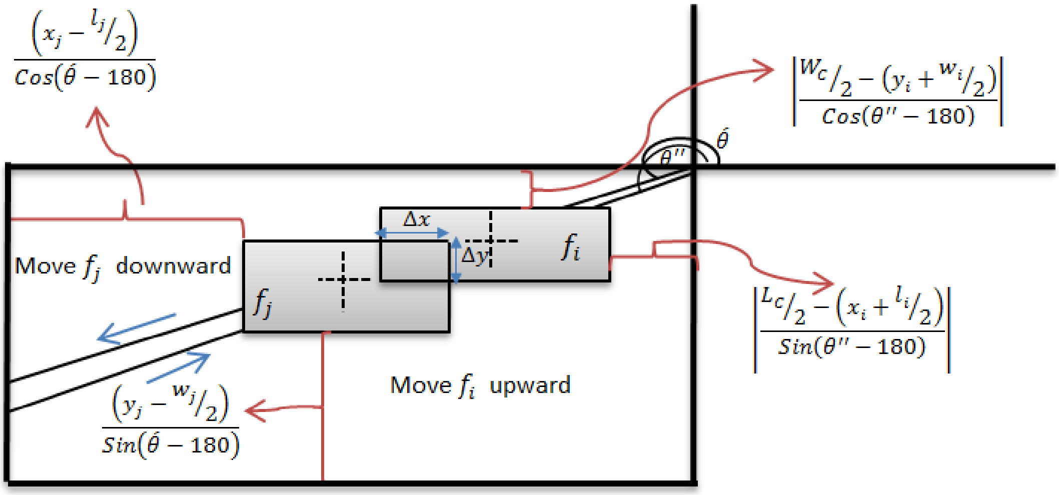

$\frac{(x_j - l_j/2)}{Cos(\acute{\theta} - 180)}$
$\left|\frac{W_c/2 - (y_i + w_i/2)}{Cos(\theta'' - 180)}\right|$
θ''
$\acute{\theta}$
Δx
Move f_j downward
Δy
f_i
f_j
$\left|\frac{L_c/2 - (x_i + l_i/2)}{Sin(\theta'' - 180)}\right|$
Move f_i upward
$\frac{(y_j - w_j/2)}{Sin(\acute{\theta} - 180)}$

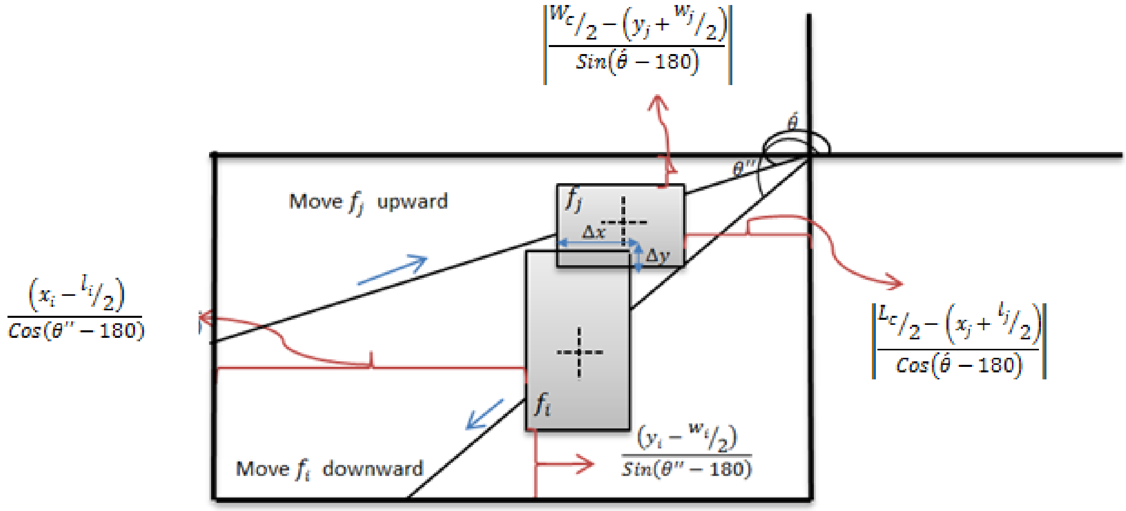
Figure (30): Facility f_i and Facility f_j in quadrant Q_3 , $y_i \geq y_j$, $x_i \geq x_j$, $\Delta x \geq \Delta y$

4.3.3.3.2. $y_i < y_j$

❖ $x_i < x_j$

In both cases of $\Delta x \geq \Delta y$ and $\Delta x < \Delta y$, the overlap is repaired by applying one of the below functions:

1. Move f_i downward by Δ
2. Move f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.



$\left|\frac{W_c/2-(y_j+w_j/2)}{Sin(\acute{\theta}-180)}\right|$
$\acute{\theta}$
θ''
Move f_j upward
f_j
Δx
Δy
$\frac{(x_i-l_i/2)}{Cos(\theta''-180)}$
$\left|\frac{L_c/2-(x_j+l_j/2)}{Cos(\acute{\theta}-180)}\right|$
f_i
$\frac{(y_i-w_i/2)}{Sin(\theta''-180)}$
Move f_i downward

Figure (31): Facility f_i and facility f_j in quadrant Q_3 , $y_i \leq y_j$, $x_i \leq x_j$, $\Delta x \geq \Delta y$

❖ $x_i \geq x_j$

Based on the comparison between x- projection and y- projection of overlap, there are two sets of repair functions.

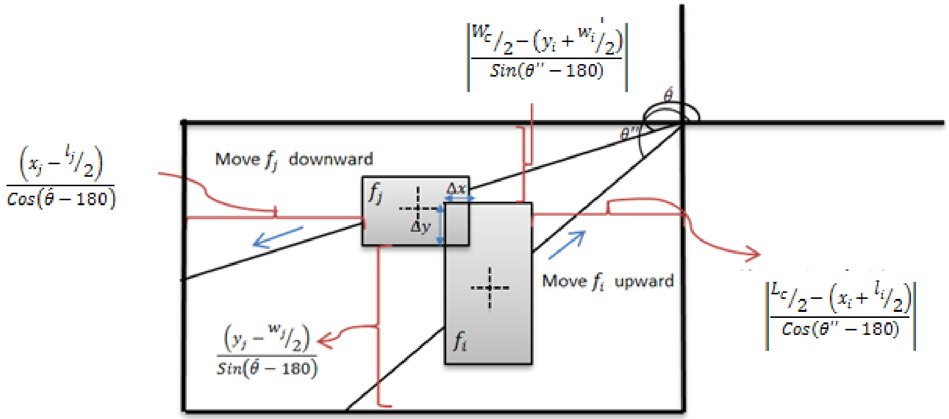
➢ **If $\Delta x \leq \Delta y$**

Set Δ= Δx . The overlap is fixed by applying one of the below functions:

1. Move facility f_i upward by Δ

2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

Figure (32) shows the scheme of this case.



$\left|\frac{W_c/2-(y_i+w_i/2)}{Sin(\theta''-180)}\right|$
$\acute{\theta}$
θ''
$\frac{(x_j-l_j/2)}{Cos(\acute{\theta}-180)}$
Move f_j downward
f_j
Δx
Δy
Move f_i upward
$\left|\frac{L_c/2-(x_i+l_i/2)}{Cos(\theta''-180)}\right|$
$\frac{(y_j-w_j/2)}{Sin(\acute{\theta}-180)}$
f_i

Figure (32): Facility f_i and facility f_j in quadrant Q_3 , $y_i \le y_j$, $x_i \ge x_j$, $\Delta x \le \Delta y$

- **If $\Delta x > \Delta y$**

Set $\Delta = \Delta y$. The overlap is fixed by applying one of the below functions:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

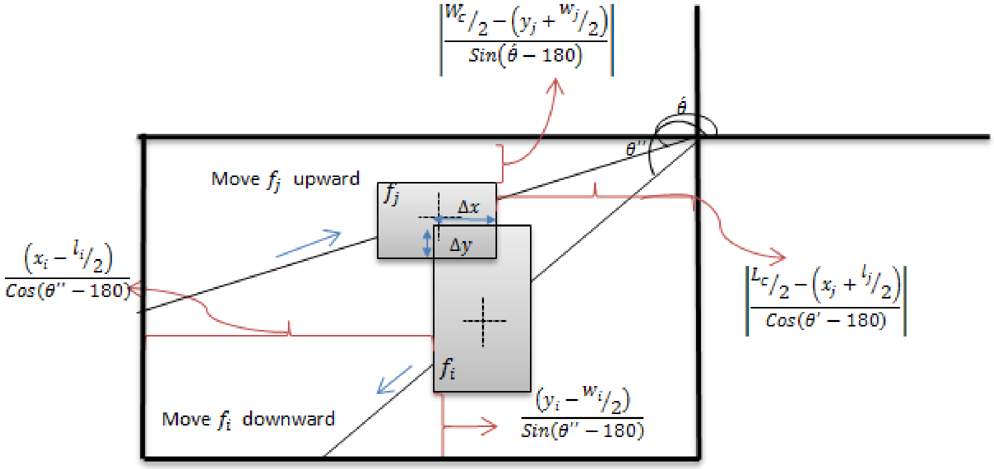

$\left|\frac{W_c/2-(y_j+w_j/2)}{Sin(\acute{\theta}-180)}\right|$
$\acute{\theta}$
θ''
Move f_j upward
f_j
Δx
Δy
$\frac{(x_i-l_i/2)}{Cos(\theta''-180)}$
$\left|\frac{L_c/2-(x_j+l_j/2)}{Cos(\theta'-180)}\right|$
f_i
$\frac{(y_i-w_i/2)}{Sin(\theta''-180)}$
Move f_i downward

Figure (33): Facility f_i and facility f_j in quadrant Q_3 , $y_i \le y_j$, $x_i \ge x_j$, $\Delta x > \Delta y$

4.3.4. Facility $\boldsymbol{f_i}$ in quadrant $\boldsymbol{Q_4}$

Based on in which of the quadrants Q_1, Q_2or Q_3 facility f_j has been located, different repair function is defined.

4.3.4.1. Facility $\boldsymbol{f_j}$ in quadrant $\boldsymbol{Q_1}$

In both cases of $\Delta x \geq \Delta y$ and $\Delta x < \Delta y$, repair function is one of the below functions:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

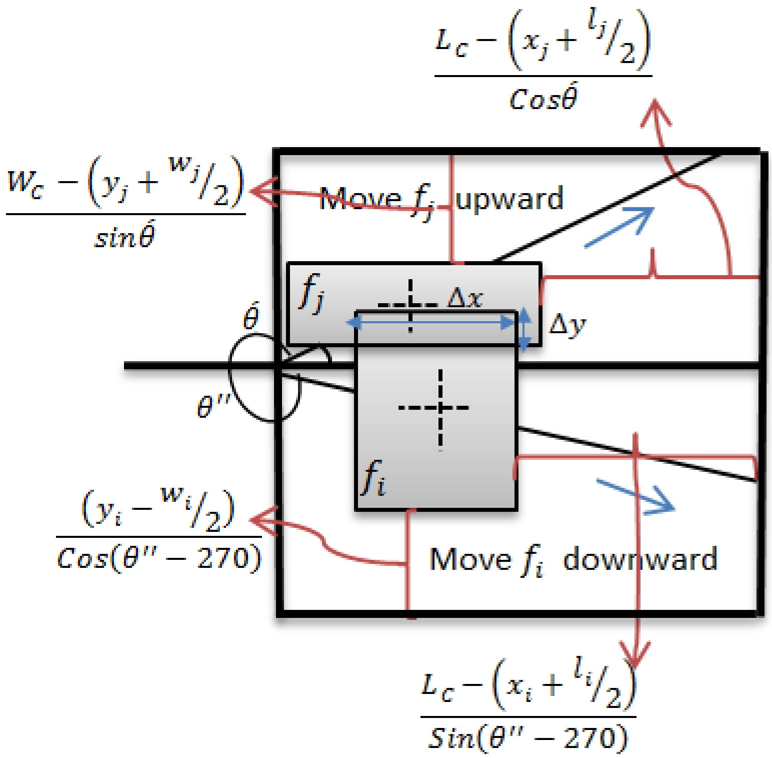
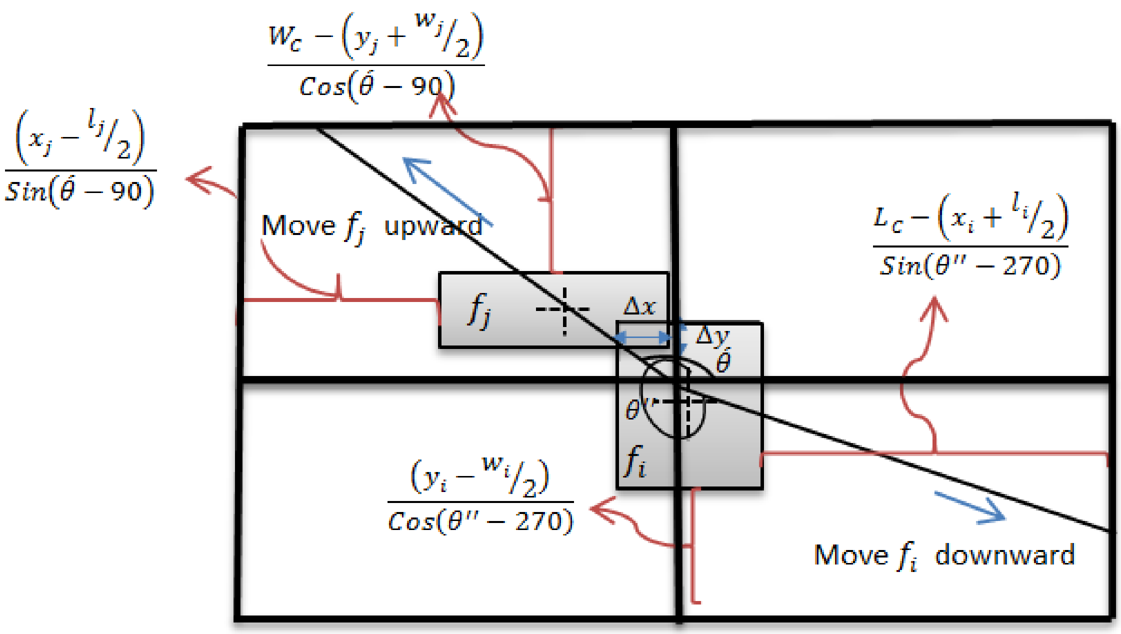

$\frac{L_C - \left(x_j + {}^{l_j}/_2\right)}{Cos\acute{\theta}}$
$\frac{W_C - \left(y_j + {}^{w_j}/_2\right)}{sin\acute{\theta}}$
Move f_j upward
f_j
Δx
Δy
$\acute{\theta}$
θ''
f_i
$\frac{\left(y_i - {}^{w_i}/_2\right)}{Cos(\theta'' - 270)}$
Move f_i downward
$\frac{L_C - \left(x_i + {}^{l_i}/_2\right)}{Sin(\theta'' - 270)}$

Figure (34): Facility f_i in quadrant Q_4 and facility f_j in quadrant Q_1 ,$x_i \geq x_j, \Delta x \geq \Delta y$

4.3.4.2. Facility f_j in quadrant Q_2

In both cases of $\Delta x \geq \Delta y$ and $\Delta x < \Delta y$, repair function is one of the below functions:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.



$\frac{W_C - (y_j + {}^{w_j}/_2)}{Cos(\acute{\theta} - 90)}$
$\frac{(x_j - {}^{l_j}/_2)}{Sin(\acute{\theta} - 90)}$
Move f_j upward
$\frac{L_C - (x_i + {}^{l_i}/_2)}{Sin(\theta'' - 270)}$
f_j
Δx
Δy
$\acute{\theta}$
θ''
f_i
$\frac{(y_i - {}^{w_i}/_2)}{Cos(\theta'' - 270)}$
Move f_i downward

Figure (35): Facility f_i in quadrant Q_4 and facility f_j in quadrant Q_2 , $x_i \geq x_j, \Delta x \geq \Delta y$

4.3.4.3. Facility f_j in quadrant Q_3

Obviously, in this case horizontal coordinate of facility f_j, x_j is smaller than horizontal coordinate of facility f_i, x_i. Based on the comparison between y_i and y_j, there are two sets of repair functions:

4.3.4.3.1. $y_j \geq y_i$

Regard to *x*-projection and *y*-projection of overlap different sub-case would be defined. Figure (34) shows this case.

➢ **If $\Delta x \leq \Delta y$**

Set $\Delta = \Delta x$. To eliminate overlap one of the below functions has to be applied:

1. Move facility f_i downward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

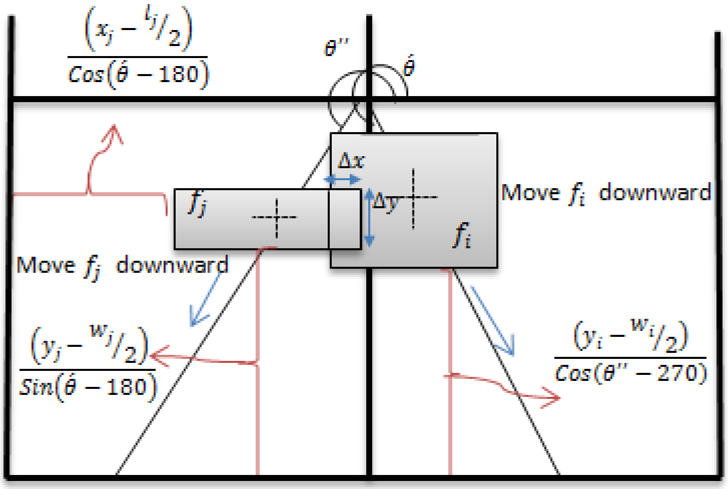

$\frac{\left(x_j - {}^{l_j}/_2\right)}{Cos(\acute{\theta} - 180)}$
θ''
$\acute{\theta}$
Δx
f_j
Δy
Move f_i downward
f_i
Move f_j downward
$\frac{\left(y_j - {}^{w_j}/_2\right)}{Sin(\acute{\theta} - 180)}$
$\frac{\left(y_i - {}^{w_i}/_2\right)}{Cos(\theta'' - 270)}$

Figure (36): Facility f_i is in quadrant Q_4 and facility f_j is in quadrant Q_3 , $y_i \geq y_j$, $\Delta x \leq \Delta y$

➢ **If $\Delta x > \Delta y$**

Set $\Delta = \Delta y$. To eliminate overlap one of the below functions has to be applied:

1. Move facility f_i upward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

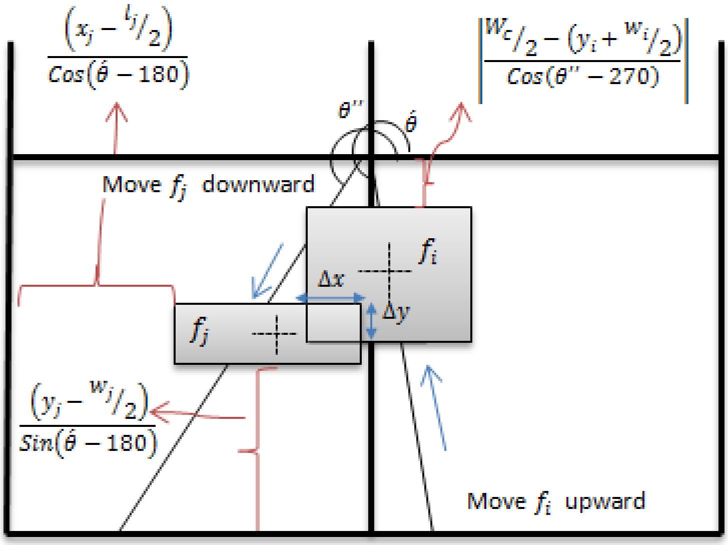

$\frac{\left(x_j - {}^{l_j}/_2\right)}{Cos(\acute{\theta} - 180)}$
$\left|\frac{{}^{W_c}/_2 - \left(y_i + {}^{w_i}/_2\right)}{Cos(\theta'' - 270)}\right|$
θ''
$\acute{\theta}$
Move f_j downward
f_i
Δx
Δy
f_j
$\frac{\left(y_j - {}^{w_j}/_2\right)}{Sin(\acute{\theta} - 180)}$
Move f_i upward

Figure (37): Facility f_i in quadrant Q_4 and facility f_j in quadrant Q_3 , $y_i \geq y_j, \Delta x \geq \Delta y$

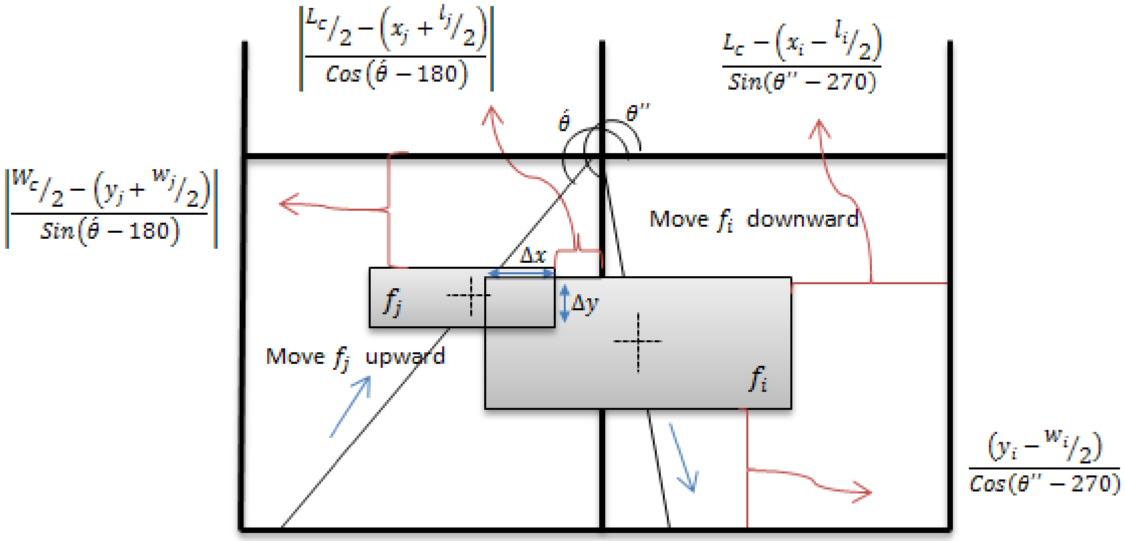
4.3.4.3.2. $y_i < y_j$

Based on the x-projection and y-projection of overlap different sub-cases would be defined. Figure (37) shows this case.

- **If $\Delta x \geq \Delta y$**

Set $\Delta = \Delta y$. To eliminate overlap one of the below functions has to be applied:

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.



$\left|\frac{L_c/2-(x_j+l_j/2)}{Cos(\acute{\theta}-180)}\right|$
$\frac{L_c-(x_i-l_i/2)}{Sin(\theta''-270)}$
$\acute{\theta}$
θ''
$\left|\frac{W_c/2-(y_j+w_j/2)}{Sin(\acute{\theta}-180)}\right|$
Move f_i downward
Δx
Δy
f_j
f_i
Move f_j upward
$\frac{(y_i-w_i/2)}{Cos(\theta''-270)}$

Figure (38): Facility f_i in quadrant Q_4 and facility f_j in quadrant Q_3 , $y_i \leq y_j, \Delta x \geq \Delta y$

- **If $\Delta x < \Delta y$**

Set $\Delta = \Delta x$. To eliminate overlap one of the below functions has to be applied:

1. Move facility f_i downward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

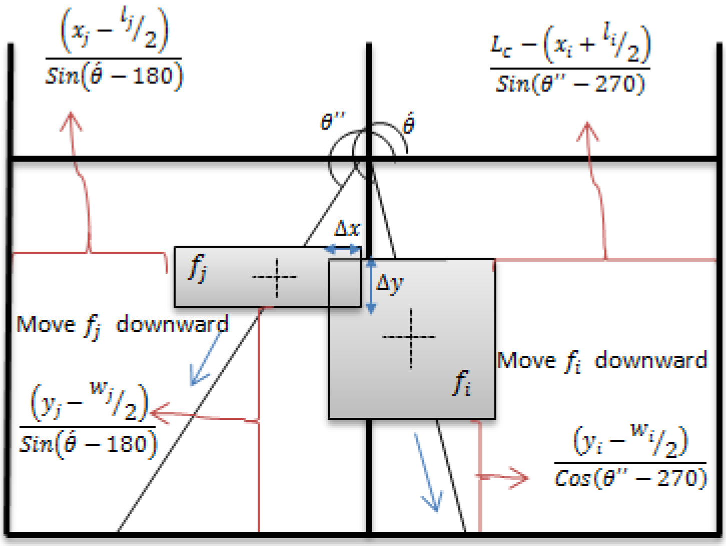

$\frac{(x_j - l_j/2)}{Sin(\acute{\theta} - 180)}$
$\frac{L_c - (x_i + l_i/2)}{Sin(\theta'' - 270)}$
θ''
$\acute{\theta}$
Δx
f_j
Δy
Move f_j downward
f_i
Move f_i downward
$\frac{(y_j - w_j/2)}{Sin(\acute{\theta} - 180)}$
$\frac{(y_i - w_i/2)}{Cos(\theta'' - 270)}$

Figure (39): Facility f_i in quadrant Q_4 and facility f_j in quadrant Q_3 , $y_i \le y_j, \Delta x \le \Delta y$

4.3.4.4. Facility f_j in quadrant Q_4

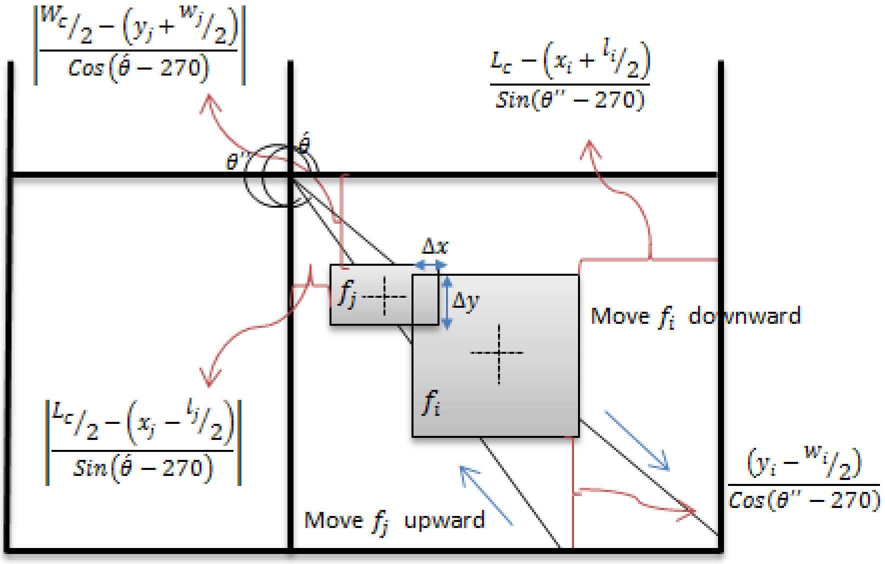
Regards to the comparison of horizontal dimensions of facility f_i and f_j, main set of repair functions are defined.

4.3.4.4.1. $y_i \le y_j$

When $y_i \le y_j$, the horizontal; dimension of facility f_i , x_i cannot be smaller than horizontal dimension of f_j, x_j. Thus, the only case remains is when $x_i \ge x_j$.

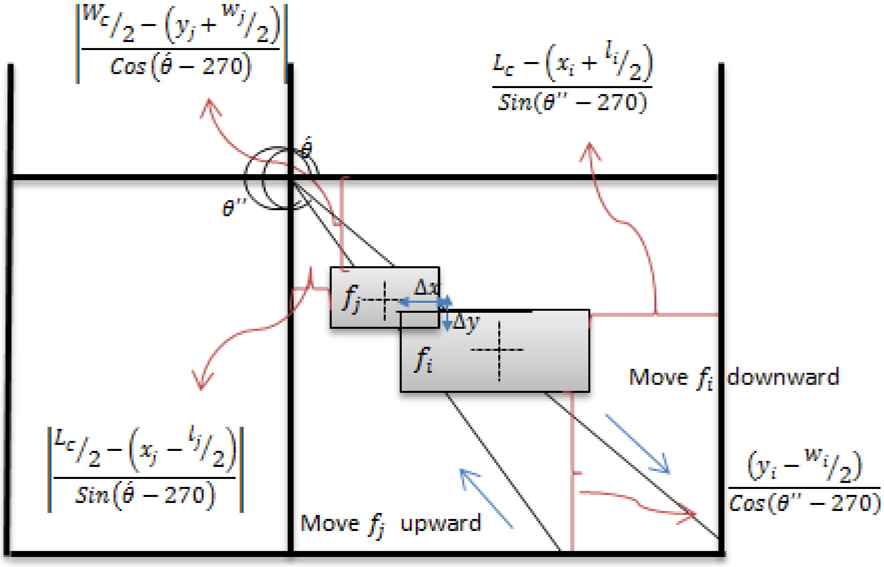
Either $\Delta x \le \Delta y$ or $\Delta x > \Delta y$, three repair functions are designed and overlap would be fixed by applying one of them .

1. Move facility f_i downward by Δ
2. Move facility f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.



$\left|\frac{W_c/2-(y_j+w_j/2)}{Cos(\acute{\theta}-270)}\right|$
$\frac{L_c-(x_i+l_i/2)}{Sin(\theta''-270)}$
θ''
$\acute{\theta}$
Δx
f_j
Δy
Move f_i downward
f_i
$\left|\frac{L_c/2-(x_j-l_j/2)}{Sin(\acute{\theta}-270)}\right|$
$\frac{(y_i-w_i/2)}{Cos(\theta''-270)}$
Move f_j upward

Figure (40): Facility f_i and facility f_j in quadrant Q_4, $y_i \leq y_j, \Delta x \leq \Delta y$



$\left|\frac{W_c/2-(y_j+w_j/2)}{Cos(\acute{\theta}-270)}\right|$
$\frac{L_c-(x_i+l_i/2)}{Sin(\theta''-270)}$
$\acute{\theta}$
θ''
f_j
Δx
Δy
f_i
Move f_i downward
$\left|\frac{L_c/2-(x_j-l_j/2)}{Sin(\acute{\theta}-270)}\right|$
$\frac{(y_i-w_i/2)}{Cos(\theta''-270)}$
Move f_j upward

Figure (41): Facility f_i and facility f_j in quadrant Q_4, $y_i \leq y_j, \Delta x \geq \Delta y$

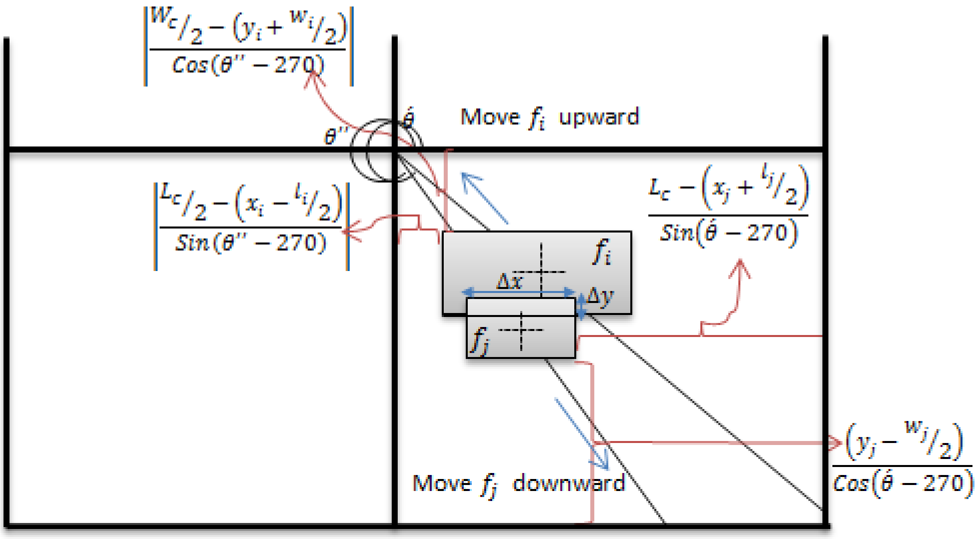
4.3.4.4.2. $y_i > y_j$

- ❖ $x_i > x_j$

➢ **If $\Delta x > \Delta y$**

Three repair functions are designed and overlap would be fixed by applying one of them.

1. Move facility f_i upward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.



$\left|\frac{W_c/2-(y_i+w_i/2)}{Cos(\theta''-270)}\right|$
$\acute{\theta}$
Move f_i upward
θ''
$\left|\frac{L_c/2-(x_i-l_i/2)}{Sin(\theta''-270)}\right|$
$\frac{L_c-(x_j+l_j/2)}{Sin(\acute{\theta}-270)}$
f_i
Δx
Δy
f_j
$\frac{(y_j-w_j/2)}{Cos(\acute{\theta}-270)}$
Move f_j downward

Figure (42): Facility f_i and facility f_j are in quadrant Q_4, $y_i \geq y_j, x_i \geq x_j, \Delta x \geq \Delta y$

➢ **If $\Delta x \leq \Delta y$**

Three repair functions are designed and overlap would be fixed by applying one of them.

1. Move f_i downward by Δ
2. Move f_j upward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

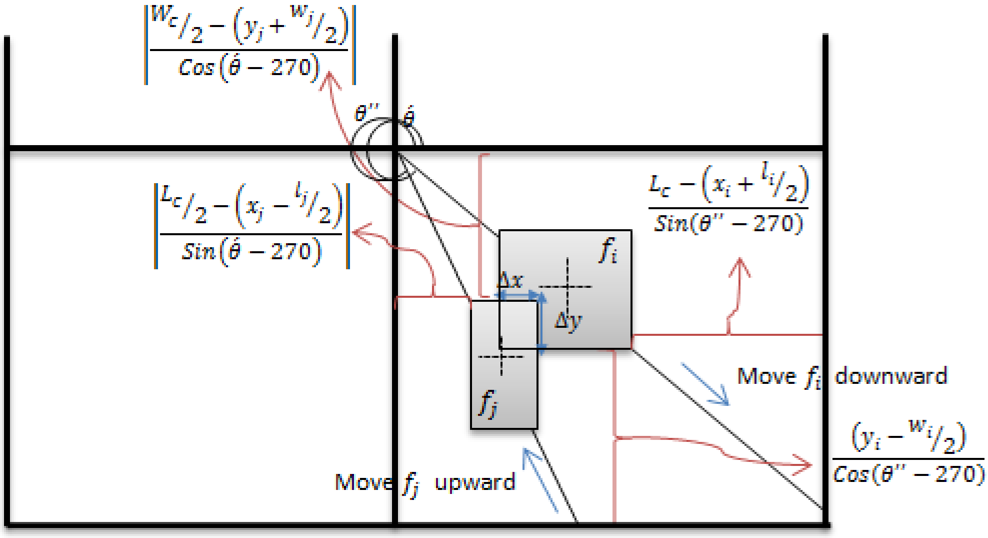

$\left|\frac{W_c/2-(y_j+w_j/2)}{Cos(\acute{\theta}-270)}\right|$
θ''
$\acute{\theta}$
$\left|\frac{L_c/2-(x_j-l_j/2)}{Sin(\acute{\theta}-270)}\right|$
$\frac{L_c-(x_i+l_i/2)}{Sin(\theta''-270)}$
f_i
Δx
Δy
f_j
Move f_i downward
$\frac{(y_i-w_i/2)}{Cos(\theta''-270)}$
Move f_j upward

Figure (43): Facility f_i and facility f_j in quadrant Q_4, $y_i \geq y_j, x_i \geq x_j, \Delta x \leq \Delta y$

- $\boldsymbol{x_i \leq x_j}$

In both case of $\Delta x > \Delta y$ and $\Delta x \leq \Delta y$, three repair functions are designed and overlap would be fixed by applying one of them .

1. Move facility f_i upward by Δ
2. Move facility f_j downward by Δ
3. Move both facilities f_i and f_j, steps 12-16.

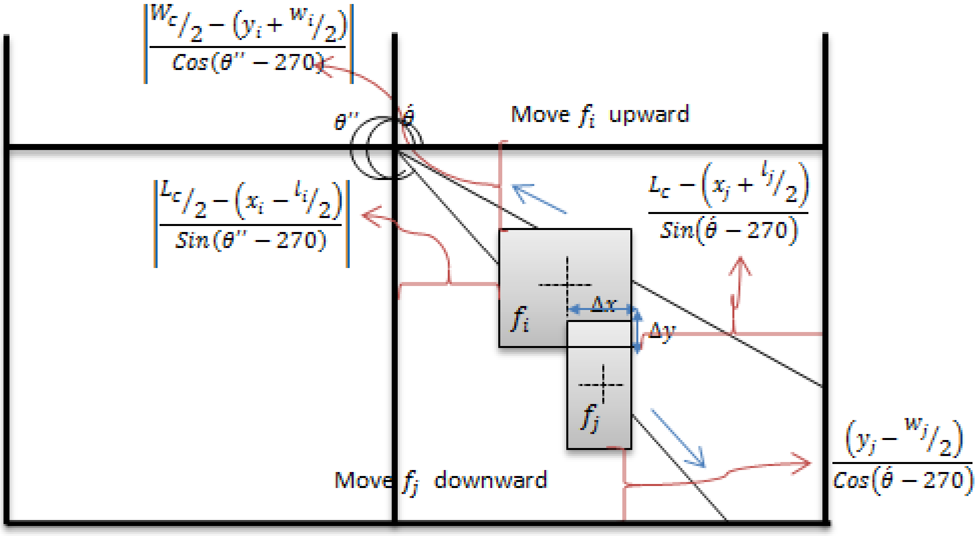

$\left|\frac{W_c/2-(y_i+w_i/2)}{Cos(\theta''-270)}\right|$
θ''
$\acute{\theta}$
Move f_i upward
$\left|\frac{L_c/2-(x_i-l_i/2)}{Sin(\theta''-270)}\right|$
$\frac{L_c-(x_j+l_j/2)}{Sin(\acute{\theta}-270)}$
f_i
Δx
Δy
f_j
$\frac{(y_j-w_j/2)}{Cos(\acute{\theta}-270)}$
Move f_j downward

Figure (44): Facility f_i and facility f_j in quadrant Q_4, $y_i \geq y_j, x_i \leq x_j, \Delta x \geq \Delta y$

Table (6): New coordinates of f_i after applying repair function

Target Facility's Coordinates				
Movement	**Upward Movement**		**Downward Movement**	
Quarter	$\Delta x \geq \Delta y$	$\Delta x < \Delta y$	$\Delta x \geq \Delta y$	$\Delta x < \Delta y$
Q1	$\acute{x}_\imath = x_i + \Delta \times Cos(\acute{\theta})$ $\acute{y}_\imath = y_i + \Delta$	$\acute{x}_\imath = x_i + \Delta$ $\acute{y}_\imath = y_i + \Delta \times Sin(\acute{\theta})$	$\acute{x}_\imath = x_i - \Delta \times Cos(\acute{\theta})$ $\acute{y}_\imath = y_i - \Delta$	$\acute{x}_\imath = x_i - \Delta$ $\acute{y}_\imath = y_i - \Delta \times Sin(\acute{\theta})$
Q2	$\acute{x}_\imath = x_i - \Delta \times Sin(\acute{\theta} - 90)$ $\acute{y}_\imath = y_i + \Delta$	$\acute{x}_\imath = x_i - \Delta$ $\acute{y}_\imath = y_i + \Delta \times Cos(\theta - 90)$	$\acute{x}_\imath = x_i + \Delta \times Sin(\acute{\theta} - 90)$ $\acute{y}_\imath = y_i - \Delta$	$\acute{x}_\imath = x_i + \Delta$ $\acute{y}_\imath = y_i - \Delta \times Cos(\acute{\theta} - 90)$
Q3	$\acute{x}_\imath = x_i + \Delta \times Cos(\acute{\theta} - 180)$ $\acute{y}_\imath = y_i + \Delta$	$\acute{x}_\imath = x_i + \Delta$ $\acute{y}_\imath = y_i + \Delta \times Sin(\acute{\theta} - 180)$	$\acute{x}_\imath = x_i - \Delta \times Cos(\acute{\theta} - 180)$ $\acute{y}_\imath = y_i - \Delta$	$\acute{x}_\imath = x_i - \Delta$ $\acute{y}_\imath = y_i - \Delta \times Sin(\acute{\theta} - 180)$
Q4	$\acute{x}_\imath = x_i - \Delta \times Sin(\acute{\theta} - 270)$ $\acute{y}_\imath = y_i + \Delta$	$\acute{x}_\imath = x_i - \Delta$ $\acute{y}_\imath = y_i + \Delta \times Cos(\acute{\theta} - 270)$	$\acute{x}_\imath = x_i + \Delta \times Sin(\acute{\theta} - 270)$ $\acute{y}_\imath = y_i - \Delta$	$\acute{x}_\imath = x_i + \Delta$ $\acute{y}_\imath = y_i - \Delta \times Cos(\acute{\theta} - 270)$

4.4. Improved Heuristic

Since the efficiency of metaheuristic algorithm depends on the quality of initial solution so designing the good initial solution is very important. If there is no sufficient distance between facilities, the two main operators such as move and swap operators of metaheuristic algorithm would not work properly. In order to overcome this issue the specific distance between any two facilities is forcing. The length of distance between the new facility and the pervious facility is equal to the width of the pervious facility. To do this, two new vectors along the vector $\overrightarrow{r_c}$ of the new facility are being constructed with the floor/ cell taken as origin. The first vector $\overrightarrow{r_1}$ is based on the lower boundary of previous facility and the second one $\overrightarrow{r_2}$ is based on the upper boundary of previous facility.

$$\overrightarrow{r_1 r_2} = \overrightarrow{r_2} - \overrightarrow{r_2} \qquad (38)$$

In this way we are forcing the distance equal to the length of vector $\overrightarrow{r_1 r_2}$ which is $|\overrightarrow{r_1 r_2}|$.

Table (7): Length of vectors $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$

Quadrant	$\boldsymbol{\theta}$ of new facility	$\lvert\overrightarrow{\boldsymbol{r_1}}\rvert$	$\lvert\overrightarrow{\boldsymbol{r_2}}\rvert$
Q_1	$0 < \theta < 90^o$	$\frac{(y_i - {w_i}/{2}) - ({YCell}/{2})}{Sin\theta}$	$\frac{(y_i + {w_i}/{2}) - ({YCell}/{2})}{Sin\theta}$
Q_2	$\theta = 90^o$	$(y_i - {w_i}/{2})$	$(y_i + {w_i}/{2})$
Q_2	$90^o < \theta < 180^o$	$\frac{(y_i - {w_i}/{2}) - ({YCell}/{2})}{\lvert Cos(\theta - 90)\rvert}$	$\frac{(y_i + {w_i}/{2}) - ({YCell}/{2})}{\lvert Cos(\theta - 90)\rvert}$
Q_2	$\theta = 180^o$	$x_i + {l_i}/{2}$	$x_i - {l_i}/{2}$
Q_3	$\theta - \frac{360^o}{M} = 180^o$	$\frac{(x_i + {l_i}/{2})}{\lvert Cos(\theta - 180)\rvert}$	$\frac{(x_i - {l_i}/{2})}{\lvert Cos(\theta - 180)\rvert}$
Q_3	$180^o < \theta < 270^o$	$\frac{(y_i + {w_i}/{2})}{\lvert Sin(\theta - 180)\rvert}$	$\frac{(y_i - {w_i}/{2})}{\lvert Sin(\theta - 180)\rvert}$
Q_3	$\theta = 270^o$	$(y_i + {w_i}/{2})$	$(y_i - {w_i}/{2})$
Q_4	$270^o < \theta < 360^o$	$\frac{(y_i + {w_i}/{2})}{Cos(\theta - 270)}$	$\frac{(y_i - {w_i}/{2})}{Cos(\theta - 270)}$
Q_4	$\theta = 360^o$	$x_i - {l_i}/{2}$	$x_i - {l_i}/{2}$

So the logic behind this approach is that the new facility cannot be placed in vector $\overrightarrow{r_1 r_2}$ and it can located in either $\overrightarrow{r_2}$ or $\overrightarrow{r_2 r_c}$ which is chosen randomly. It means that coordinates of facility can be either in $[o, |r_1|]$ or $[|r_2|, |r_c - r_2|]$. It should be noted which point of one of the two intervals would be the coordinate of the facility is selected randomly. However there is one point here, since the reference of facility is its centroid, let's say $\theta = 235$ it means that facility has to be placed in third quadrant Q_3 and the point 0 is randomly chose as the placement of the facility so some part of facility would be step over to the other quadrants. In order to prevent this problem the diagonal of facility is calculated as

$$r_f = \sqrt{{}^{li}/_{2}{}^{2} + {}^{wi}/_{2}{}^{2}} \tag{39}$$

And then those intervals have to be modified to $\left[|r_f|, |r_1 - r_f|\right]$ or $\left[|r_2 + r_f|, |r_c - r_2 - r_f|\right]$. Generally the approach toward the selecting coordinate is completing random based. It means that at first it is going to be checked which interval is qualified to occupying with facility. To do this the length of each interval has to be greater than equal to the two times of corresponding facility's diagonal, *i.e.*

$$|r_f| - |r_1 - r_f| \geq 2 \times r_f \tag{40}$$

$$|r_2 + r_f| - |r_c - r_2 - r_f| \geq 2 \times r_f \tag{41}$$

If both intervals are qualified then one of them is selected randomly. Otherwise if one of them is only qualified that one is chosen. The worst case is happening when none of them are qualified, in this case one of them is choosing randomly or one with less difference is chosen randomly.

More detail about the algorithm is part of future work. However, it has to be mentioned this improved heuristic algorithm has been developed and the implementation and verification are in the process.

4.5. Metaheuristic Algorithm

4.5.1. Simulated Annealing

Simulated Annealing is a stochastic neighborhood search technique which was initially developed by Metropolis (1953) and applied to combinatorial problems by Kirkpatrick et al. (1983) firstly.

To begin with, the basic of SA is based on statistical mechanics and comes from the similarity between the annealing of solids process and the solving method of combinatorial problem. If each feasible solution to the combinatorial optimization problem as a configuration of atoms and the objective function value of corresponding feasible solution as the energy of the system, then the optimal solution of combinatorial optimization problem is as like as the lowest energy state of the physical system (Golden and Skiscim, 1986). The core of heuristic algorithms for solving combinatorial problem is based on continual improvement, moving from one solution to another one in order to decrease the objective function in one iteration to next one. The same procedure is taking in quenching the system from high to low temperature in order to reach the required quality.

4.5.2 Why not using greedy algorithms?

The main difference between simulated annealing SA and local search algorithms which called "greedy algorithms", is that the greedy algorithms start with initial solution and try to improve solution repeatedly until no improvement is possible. In greedy algorithm the solution traps in local minimum or maximum solution. In other words, greedy algorithms searches for solution in downhill direction and it accepts new solutions if the new objective function value has improvement in comparison to the current one. In this case there is no chance to escape from that local optima region and exploring new region. However, SA takes another approach. SA is not just searching in downhill region. On the contrary SA is occasionally accepting worst solution by this hope that it backs out existing downhill direction and finding better solution in further steps. This action of SA is called hill climbing.

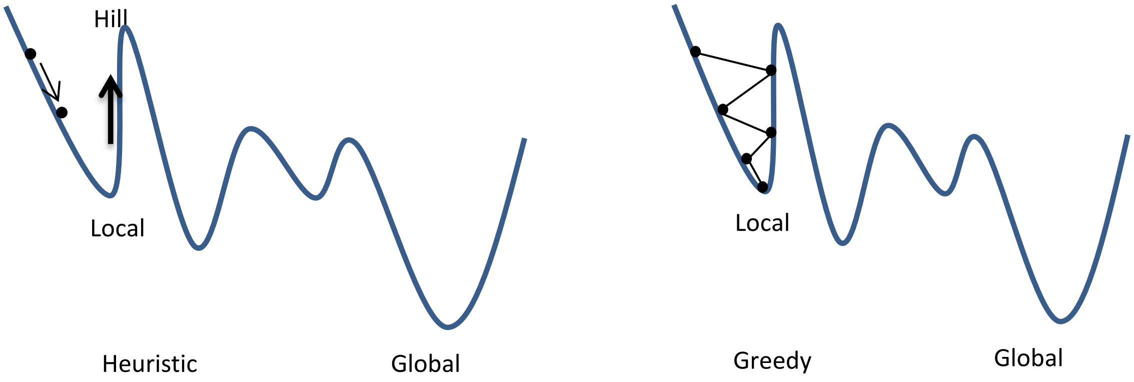


Figure (45): Heuristic v.s. Greedy algorithm

4.5.3. SA Procedures

Generally speaking all heuristic algorithms such as SA are taking exploration procedure which means moving from one solution to another one. However there are a couple of points have to be answered before applying simulated annealing such as:

1. How new solution is generated?
2. How many solutions have to be tested?
3. When the algorithm has to stop?

The following is trying to answer all of above.

4.5.4. The elements of SA algorithm

The core of SA algorithm is Metropolise algorithm which allows uphill moves sometimes. Metropolise algorithm has four main elements (Press et al., 2007, Kirkpatrick et al.,1983)

1. Initial solution and description of system configuration

   It is the starting point of SA algorithm. There are two main approaches to generating initial solution. One is generating initial solution randomly; by taking this approach feasibility of initial solution has to be considered. The second approach is getting feasible initial solution by adapting greedy algorithms or another heuristic algorithm. It has to be noted that initial solution should not be too good because escaping from its local optimum is hard.

2. Configuration changes:

   By moving from one configuration to another one new neighborhood solution is generated. These changes happen by defining some operators which responsible to make changes in current solution.

3. Objective function that represent the quantitative measurement of goodness of a system.

   After finding any neighbor the difference between objective value of new solution (E_{n+1}) and of the current solution (E_n) is calculated. If ($\Delta E < 0$) it means the objective value of neighborhood solution is showing improvement in comparison to the objective value of the current solution found so far ($\Delta E < 0$). Hence, the current one will be accepted as the new best solution. On the other hand, if ($\Delta E \geq 0$) the new solution is accepted with a certain probability. By this approach SA tries to exit from local optima region in which it trap. The probability is based on the so-called Boltzmann probability distribution,

$$Prob(\Delta E) \sim exp(-\Delta E / k_b T) \qquad (42)$$

   T is the parameter and k_b is the Boltzmann's constant which is not required when Metropolis algorithm is applying to combinatorial problems (Wilhelm and Ward, 1987). The acceptance probability of new solution depends on two factors, one is how large is this difference. The bigger difference, the less chance of accepting this new solution. The second criterion is a control parameter (temperature). It should be noted if the initial temperature is not large enough or it decreases dramatically the chances that the algorithm traps at local optima is high.

4. Annealing Schedule/ Cooling Schedule

   The annealing schedule determines four rules:

4.1. Initial temperature: Since the annealing of solids is the basic of SA approach, initial temperature is the melting point of SA algorithm and it should be defined in such a way that the solutions generated by high acceptance probability approximately close to one. Kirkpatrick (1983) noted that the initial temperature has to be large enough that %80 of generated solutions are accepted. Kia et al., (2012) and Baykasoglu and Gindy

(2001) defined initial solution high enough in such a way that %95 of generated candidates can be accepted by using following equation:

$$T_0 = \frac{Objv_j - Objv_i}{ln(0.95)} \qquad (43)$$

$Objv_j$ and $Objv_i$ are the objective values of two random solution i and j respectively. It should be noted initial solution T_0 is generated once at the beginning of SA algorithm.

4.2.Temperature length

4.3. Termination: There are different approaches to stopping criteria such as

- A Specific number of iteration
- Exact final temperature
- No improvement for a number of iteration

Based on the literature review done, there are different approached for choosing SA parameters:

Table (8): SA parameters

Author	Initial temperature (T_0)	Cooling rate (α)	Temperature reduction	Loop length	
				inner	outer
Bazargan-Lari and Kaebernick (1997)	10	0.9	$t_i = 10(0.9)^{i-1}$	$\acute{N} \times n$	K
Baykasoglu and Gindy (2001)	$T_{in} = \frac{f_{min} - f_{max}}{lnP_c = \ln(0.95)}$	$\propto = \left(\frac{lnP_c}{lnP_f}\right)^{1/(eL_{max}{}^{-1})}$	$T_{el+1} = \alpha T_{e1}$	I_L>LMC	el_{max}calculated
Heragu and Alfa (1992)	999	0.90	$T = rT$	Epoch concept $\acute{N} \times n$	K
Wilhelm and Ward (1985)	10	0.9	$t_i = 10(0.9)^{i-1}$	Epoch concept $\acute{N} \times n$	K

Epoch: Predetermined specific number of successful pairwise interchanges at each temperature

$\acute{N}$: Predetermined integer

n: Total number of facilities

K: Predetermined integer- the total number of temperature steps

4.5.5. The General Pseudo Code of SA

```
Set initial solution $X_0$; $X^* = X_0$, $X_C = X_0$

Compute Objective function value (Energy) $X_0$: $E(X_0)$; $E^* = E(X_0)$, $EC = E(X_0)$

 Set initial temperature $T_0$; $T = T_0$

Repeat

    For $i = 1\ to\ L$ do

        Generate new neighborhood solution, $X_i$

        Compute energy change $\Delta E = EC - E(X_i)$

        If $(\Delta E < 0)$ then

                Accept the new solution and set $X^* = X_i, X_C = X_i$ and
                $EC = E(X_i)$, $E^* = E(X_i)$

        Else

                 Generate random variable $rn = random(0,1)$

        If $rn < e^{-(\Delta E/T)}$ then

            Accept the new solution and set $X_C = X_i$ and $EC = E(X_i)$

        Else

            Reject the solution

        End-if

    End-for
    Set new temperature $T_{i+1} =\propto T_i$
Until the stopping criteria
  Return  $X^*$ and $E^*$
```

Figure (46) shows the flowchart of a general simulated annealing algorithm.

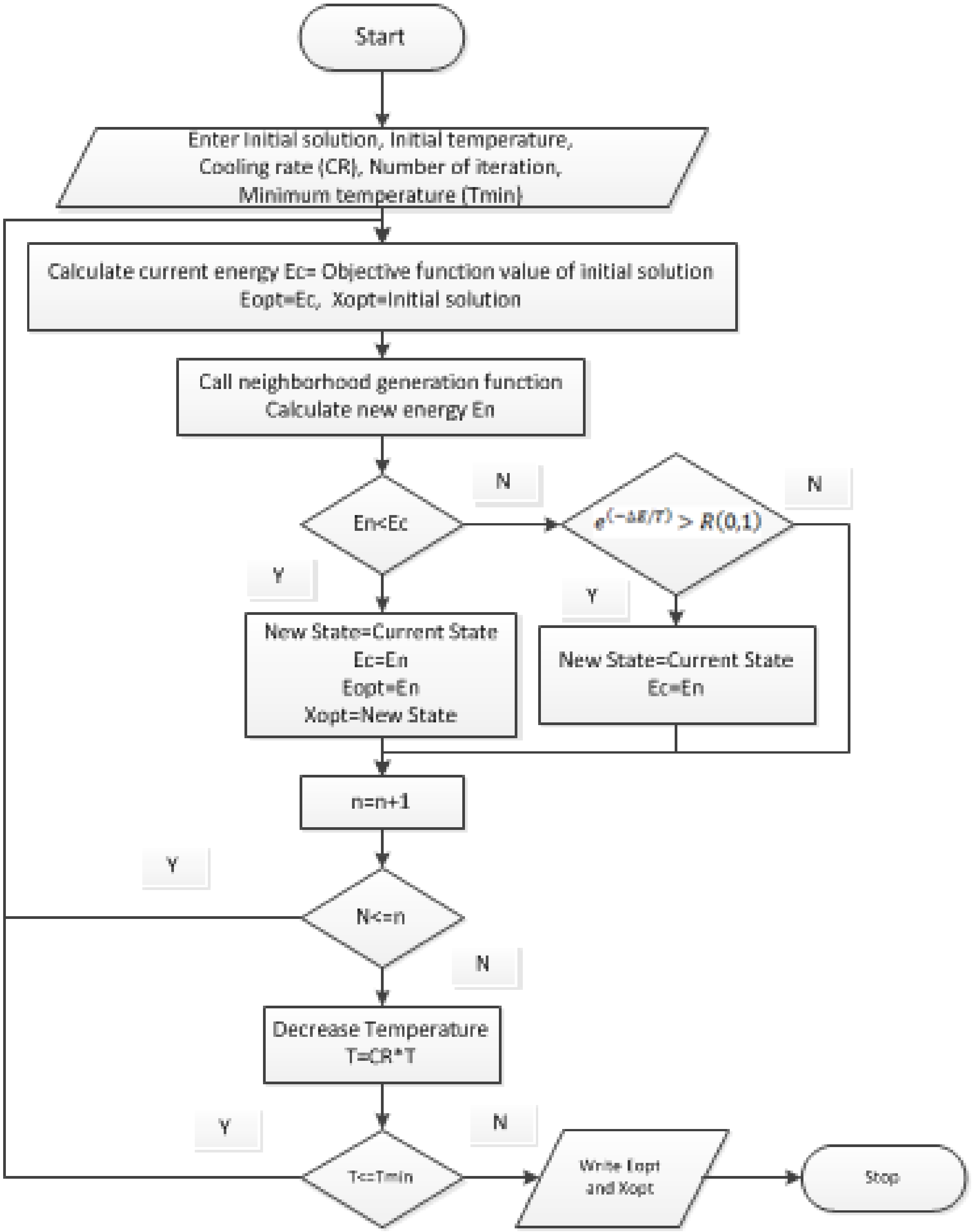

Start
Enter Initial solution, Initial temperature,
Cooling rate (CR), Number of iteration,
Minimum temperature (Tmin)
Calculate current energy Ec= Objective function value of initial solution
Eopt=Ec, Xopt=Initial solution
Call neighborhood generation function
Calculate new energy En
N
En<Ec
$e^{(-\Delta E/T)} > R(0,1)$
N
Y
Y
New State=Current State
Ec=En
Eopt=En
Xopt=New State
New State=Current State
Ec=En
n=n+1
Y
N<=n
N
Decrease Temperature
T=CR*T
Y
N
T<=Tmin
Write Eopt
and Xopt
Stop

Figure (46): Flowchart of Simulated Annealing

4.5.6. Developed Simulated Annealing for FLP

4.5.6.1. Neighbourhood solution scheme

In order to generate new neighbourhood solution two main operators namely move operator and swap operator have been developed. Move operator tries to make facilities close to each other; and also swap operator switches the location of the two facilities. The details about these two operators explained below.

4.5.6.1.1. Move Operator

The developed move operator tries to reduce distances between the facilities. The logic behind this algorithm is decreasing the distance between one facility-called In-Context facility which is chosen randomly and the closest facility towards that. By moving the In-Context facility toward its closest facility the possibility of overlap between In-Context facility and the rest of facilities is decreased. Main point here is that how much the maximum_movable_ distance is. Maximum_movable_ distance is the maximum length which if In-Context facility moved toward its closest facility no overlap will happen between them. The steps of move operator algorithm are explained below:

1. Randomly choose one facility, called In-Context facility f_G.
2. The Euclidean distance between the centroid of In-Context facility f_G and the rest of facilities are calculated.

$$Dis_{Gi} = \sqrt{(X_G - X_i)^2 - (Y_G - Y_i)^2} \qquad \forall\ i = 1,2, \dots, M \ and \ i \neq G \qquad (44)$$

3. Facilities are sorted based on the distances found in step 3 in the descending order. The first one among the above set would be the closest facility f_C to the In-Context facility f_G.
4. Divide the In-context facility f_G into four equal-sized quadrants by the origin of its centroid.
5. Find in which quadrant of In-Context facility f_G the closest facility f_C is located.

6. At this point the maximum_movable_ distance $\left|\overrightarrow{C\acute{C}}\right|$ is calculated. For finding this distance two points C and $\acute{C}$ have to be found. C is the conjunction of vector $\overrightarrow{r'}$ and the closest boundary of In-Context facility f_G to the closest facility f_C; and $\acute{C}$ is the conjunction of vector $\overrightarrow{r''}$ and the closest boundary of closest facility to In-Context facility. To do this, these concepts are defined:

   $\overrightarrow{O'O''}$: Vector between centroids of In-Context facility f_G and closest facility f_C .

   $\left|\overrightarrow{C\acute{C}}\right|$: Maximum_movable_distance

   θ_1: The angle between vector $\overrightarrow{O'O''}$ and horizontal line

   θ_2: The angle between vector $\overrightarrow{O'O''}$ and vertical line

   $\overrightarrow{r'}$: Vector from centroid of In-Context facility O' to the closest boundary of In-Context facility f_G toward the closet facility f_C.

   $\overrightarrow{r''}$: Vector from centroid of the closest facility O'' to the closet boundary of the closest facility f_C toward the In-Context facility f_G .

   $$\theta_1 = tan^{-1}\frac{|Opposite\ side|}{|Adjacent\ side|} = tan^{-1}\frac{|Y_G - Y_C|}{|X_G - X_C|} \quad (45)$$

   $$\theta_2 = tan^{-1}\frac{|Opposite\ side|}{|Adjacent\ side|} = tan^{-1}\frac{|X_G - X_C|}{|Y_G - Y_C|} \quad (46)$$

   Also: $\theta_2 = 90 - \theta_1$

   Where X_G and Y_G are vertical and horizontal coordinates of centroid of In-Context facility f_G respectively. Similarly; X_C and Y_C are vertical and horizontal coordinates of centroid of In-Context facility f_C respectively.

   It has to be noted, the length of both vectors $\overrightarrow{r'}$ and $\overrightarrow{r''}$ depends on their corresponding angles θ_1 and θ_2. Figures (47) and (48) illustrate this topic.

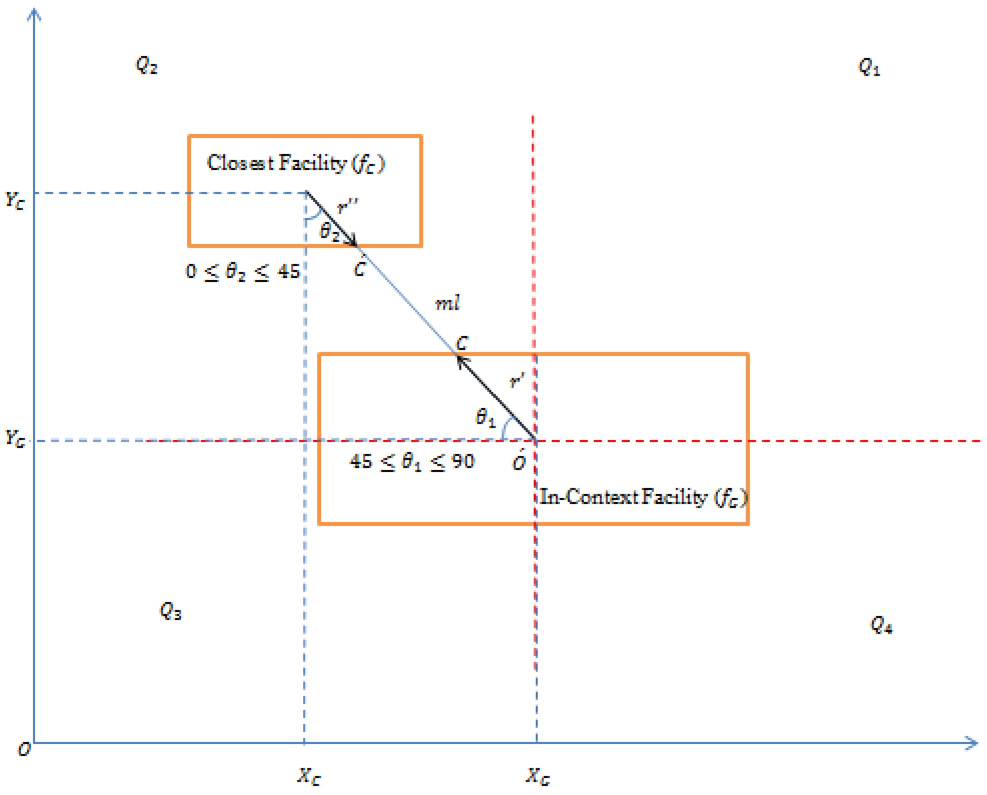


Figure (47): Angle calculation in move operator(I)

$$|\vec{r'}| = \begin{cases} \dfrac{Adjacent\ side}{Cos\theta_1} = \dfrac{L_G/_2}{Cos\theta_1} & if \quad 0 \le \theta_1 \le 45^0 \\ \dfrac{Opposite\ side}{Sin\theta_1} = \dfrac{W_G/_2}{Sin\theta_1} & if \quad 45^0 \le \theta_1 \le 90^0 \end{cases} \tag{47}$$

$$|\vec{r''}| = \begin{cases} \dfrac{Adjacent\ side}{Cos\theta_2} = \dfrac{W_C/_2}{Cos\theta_2} & if \quad 0 \le \theta_2 \le 45^0 \\ \dfrac{Opposite\ side}{Sin\theta_2} = \dfrac{L_C/_2}{Sin\theta_2} & if \quad 45^0 \le \theta_2 \le 90^0 \end{cases} \tag{48}$$

Where L_G and W_G are length and width of In-Context facility f_G respectively. Similarly; L_C and W_C are length and width of In-Context facility f_C respectively.

Based on in which quadrant closing facility is located, C and $\acute{C}$ coordinates are calculating by equations shown in table (9).

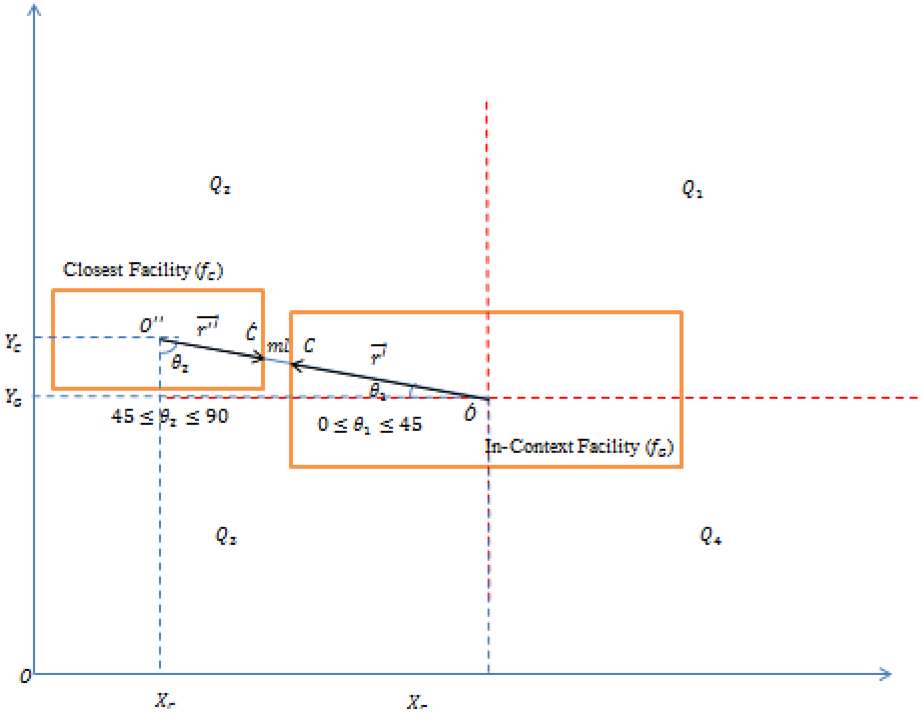


Figure (48): Concept of Angle in move operator (II)

Table (9): C and $\acute{C}$ coordinates

	Coordinates	
Quadrant	C	$\acute{C}$
1	$(X_G + r'Cos\theta_1, Y_G + r'Sin\theta_1)$	$(X_i - r''Cos\theta_2, Y_i - r''Sin\theta_2)$
2	$(X_G - r'Cos\theta_1, Y_G + r'Sin\theta_1)$	$(X_i + r''Cos\theta_2, Y_i - r''Sin\theta_2)$
3	$(X_G - r'Cos\theta_1, Y_G - r'Sin\theta_1)$	$(X_i + r''Cos\theta_2, Y_i + r''Sin\theta_2)$
4	$(X_G + r'Cos\theta_1, Y_G - r'Sin\theta_1)$	$(X_i - r''Cos\theta_2, Y_i + r''Sin\theta_2)$

Hence, the length of vector $\left|\overrightarrow{C\acute{C}}\right|$ is:

$$\left|\overrightarrow{C\acute{C}}\right| = \sqrt{(X_C - X_{C'})^2 - (Y_C - Y_{C'})^2} \qquad (49)$$

7. At this point the length of the movement, called ml is the random number in interval $\left(0, \left|\overrightarrow{C\acute{C}}\right|\right]$. Furthermore, the direction of movement is along the vector $\overrightarrow{C\acute{C}}$.
8. If the closest facility is adjacent to the facility f_G, find the other closest facility and go to step 5, otherwise go to step 9.
9. Finally, new coordinates of In-Context facility f_G is calculated and shown in table (10).

Table (10): New coordinate of f_G after move

	New coordinates of target facility	
Direction	X_G	Y_G
Quadrant 1	$X_G + ml * Cos\theta_1$	$Y_G + ml * Sin\theta_1$
Quadrant 2	$X_G - ml * Cos\theta_1$	$Y_G + ml * Sin\theta_1$
Quadrant 3	$X_G - ml * Cos\theta_1$	$Y_G - ml * Sin\theta_1$
Quadrant 4	$X_G + ml * Cos\theta_1$	$Y_G - ml * Sin\theta_1$

4.5.6.1.2 Swap Operator

The second operator of the developed SA is swap operator which is switching positions of two the facilities. The point here is how swap two facilities together that with the minimum possibility of overlap. To do that, the new concepts called free zone is defined. To apply this concept, a random facility called f_G is chosen and the available free space around this facility called FZ_G is determined by applying the maximum_movable_distance concept introduced in move operator. It has to be noted the centroid of free zone FZ_G is the same as centroid of the facility f_G. If there is any facility whose area is greater than the area of the facility f_G and less than the area of free zone FZ_G then that facility is qualified for swapping. By swapping this facility with facility f_G the possibility of occurrence of overlap is minimized. Moreover, if there is more than one facility which are qualified to swap with the facility f_G, one facility is chosen randomly. The figure (48) shows the scheme of free zone concept. The algorithm below explained swap operator's steps in details:

1. One facility is chosen randomly, called facility f_G
2. The closest facility to the f_G is determined-details mentioned in move operator.
3. Maximum_movable_distance is calculated.
4. Free zone FZ_G of facility f_G is determined.
5. Areas of facility f_G and FZ_G are calculated.
6. Among the rest of facilities those ones whose areas are greater than the area of facility f_G and less than the area of free zone FZ_G are found.
7. Randomly one facility among those facilities is found in step 6 is chosen, call it f_i.
8. Swap facility f_G to the facility f_i.
9. Calculated the new coordinates of both f_G and f_i.
10. End

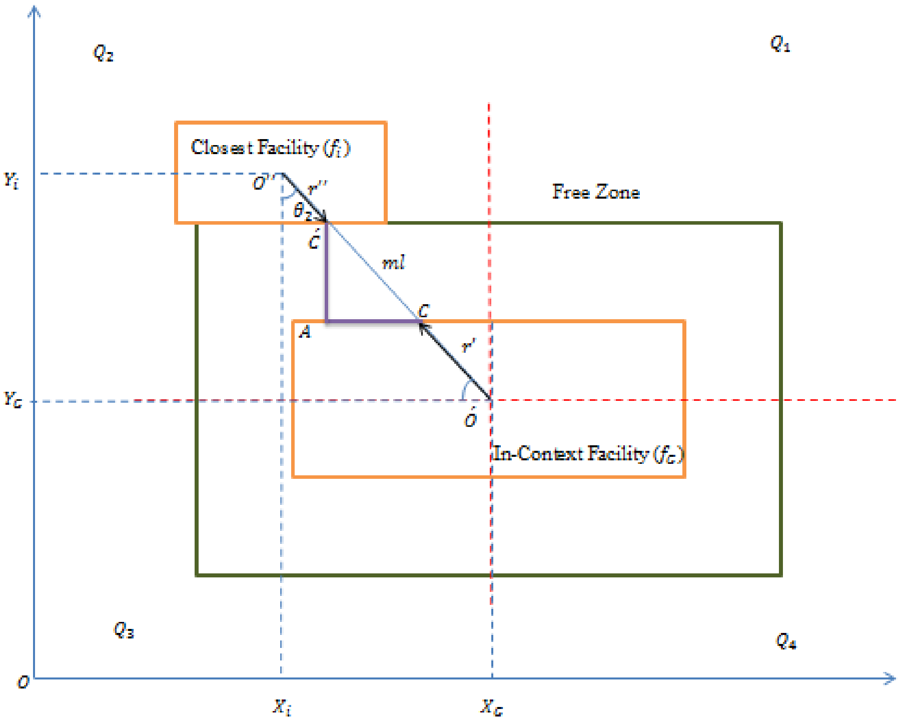


Figure (49): Free zone concept

Assume:

L_G: Length of the f_G

W_G: Width of the f_G

ml: Maximum movable distance

LFZ: Length of the *FZ*

WFZ: Width of the *FZ*

AFZ: Area of *FZ*

$$AC = min\left(\left(X_G - {}^{L_G}/_{2}\right), ml \times Cos\theta_1\right) \qquad (50)$$

$$\acute{A}C = min\left(\left(Y_G - {}^{W_G}\!/_{2}\right), ml \times Sin\theta_1\right) \qquad (51)$$

$$LFZ = L_G + 2AC \qquad (52)$$

$$WFZ = W_G + 2A\acute{C} \qquad (53)$$

$$AFZ = LFZ \times WFZ \qquad (54)$$

4.5.6.2. Aisle Constraints

In case of aisle, the operators move and swap, vary. The details are brought below.

4.5.6.2.1. Move Operator:

The move operator has the same procedure as the move operator developed in case of no aisle. Hence, in case of aisle one facility is chosen randomly f_G and moves to its closest facility f_C. Afterwards, the possibility of overlap between aisle and new position of facility f_G called $\acute{f}_G$ is considering. If any overlap happened, it has to be fixed. To do that, two repair functions have been developed.

4.5.6.2.2. Before-Aisle Repair Function:

The idea behind this function is if there is any overlap between $\acute{f}_G$ and aisle happens, the facility $\acute{f}_G$ moves back exactly before the aisle. To illustrate, $\acute{f}_G$ backs to the back of boundary of aisle which it passed over. The figures (50) and (51) represent this overlap conditions in both cases of vertical; and horizontal aisle.

The steps of the move operator with aisle constraints are explained as follows:

Step 1. Move facility f_G toward its closest facility. Calculate new coordinates of facility f_G and call it facility $\acute{f}_G$.

Step 2. Check overlaps possibility between $\acute{f}_G$ and aisle

Step 3. If there is any overlap, take appropriate repair function

Step 4. Find the coordinates of $\acute{f}_G$- details shown in table (11)-(12)

Step 5. End

Repair Function- Horizontal Aisle

- Facility f_G is lower side of the aisle is:

$$Rep = \left(\left(\acute{y}_G + {}^{W_G}\!/_2\right) - \left(Y_A - {}^{w_A}\!/_2\right)\right)\Big/_{Sin\theta} \tag{55}$$

- Facility f_G is upper side of the aisle:

$$Rep = \left(\left(Y_A + {}^{W_A}\!/_2\right) - \left(\acute{y}_G - {}^{w_G}\!/_2\right)\right)\Big/_{Sin\theta} \tag{56}$$

Repair Function- Vertical Aisle

- Facility f_G is in the left side of the aisle:

$$Rep = \left(\left(\acute{x}_G + {}^{l_G}\!/_2\right) - \left(X_A - {}^{L_A}\!/_2\right)\right)\Big/_{Cos\theta} \tag{57}$$

- Facility f_G is in the right side of the aisle:

$$Rep = \left(\left(X_A + {}^{L_A}\!/_2\right) - \left(\acute{x}_G - {}^{l_G}\!/_2\right)\right)\Big/_{Cos\theta} \tag{58}$$

Table (11): Revised coordinate based on Before-Aisle repair function - horizontal aisle

Horizontal Aisle	$\boldsymbol{x_{f_G} < x_{\acute{f}_G}}$	$\boldsymbol{x_{f_G} \geq x_{\acute{f}_G}}$
$y_{f_G} < Y_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$
$y_{f_G} > Y_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$

Table (12): Revised coordinate based on Before-Aisle repair function -vertical aisle

Vertical Aisle	$\boldsymbol{y_{f_G} < y_{\acute{f}_G}}$	$\boldsymbol{y_{f_G} \geq y_{\acute{f}_G}}$
$x_{f_G} < X_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$
$x_{f_G} > X_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$

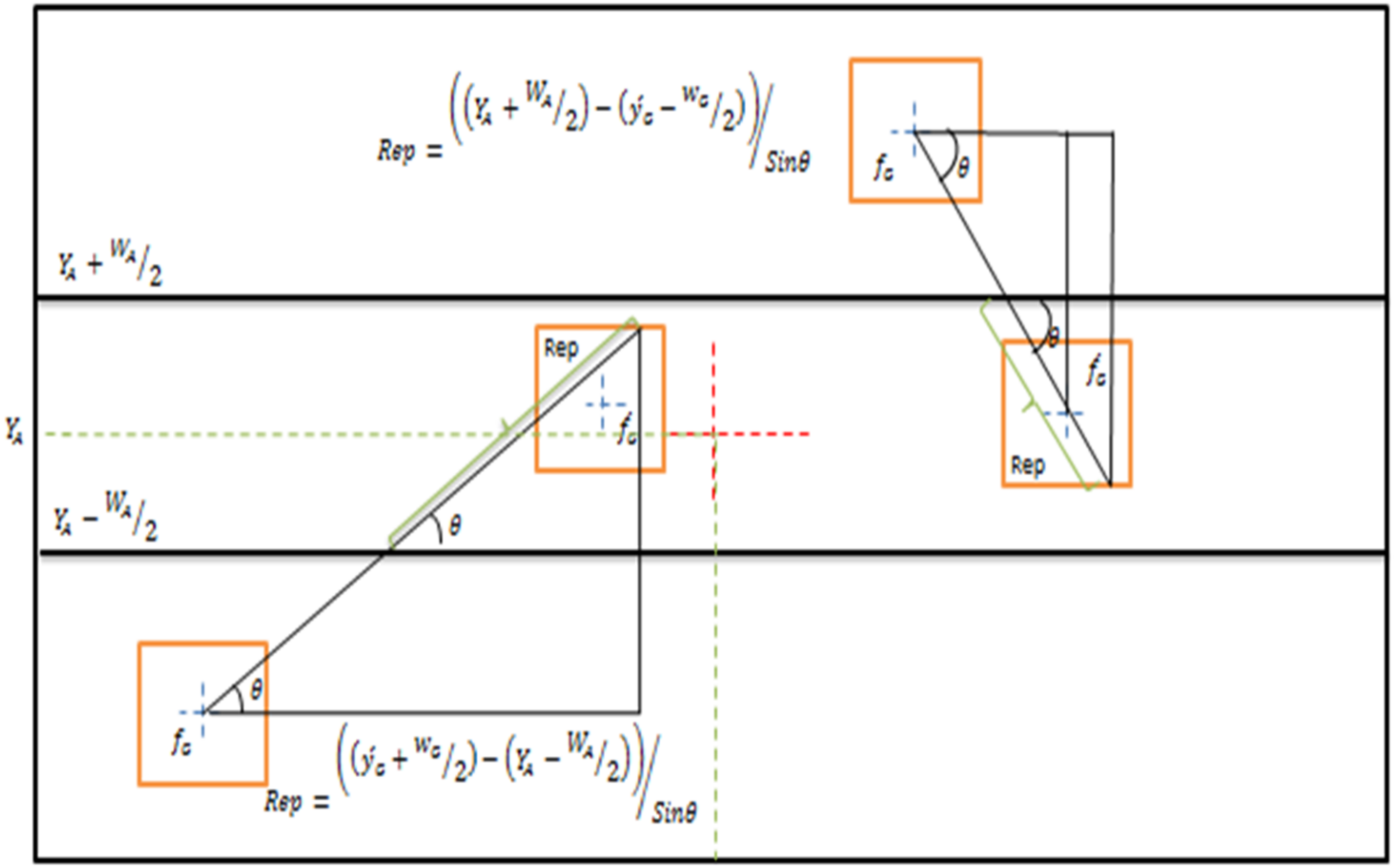
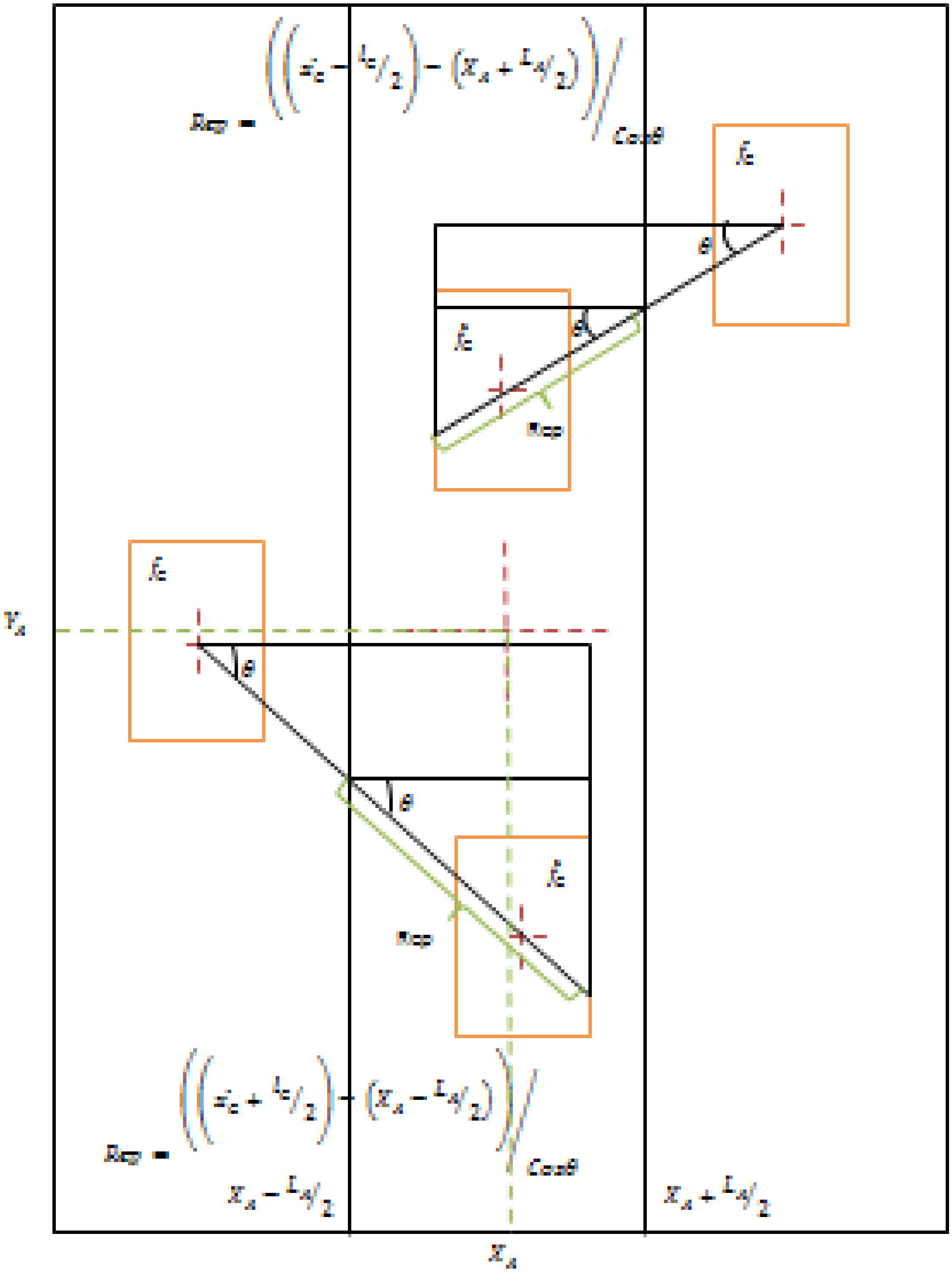

$Rep = \left(\left(Y_A + {}^{W_A}/_2\right) - \left(\acute{y}_G - {}^{w_G}/_2\right)\right) / {}_{Sin\theta}$
f_G
θ
$Y_A + {}^{W_A}/_2$
Rep
θ
f_G
Y_A
f_G
Rep
$Y_A - {}^{W_A}/_2$
θ
θ
f_G
$Rep = \left(\left(\acute{y}_G + {}^{w_G}/_2\right) - \left(Y_A - {}^{W_A}/_2\right)\right) / {}_{Sin\theta}$

Figure (50): Before-Aisle-Move operator for horizontal aisle



$R_{cp} = \left(\left(x'_c - {L_c}/{2}\right) - \left(X_A + {L_A}/{2}\right)\right) / Cos\theta$
f_c
θ
θ
f_c
R_{cp}
f_c
Y_A
θ
θ
f_c
R_{cp}
$R_{cp} = \left(\left(x'_c + {L_c}/{2}\right) - \left(X_A - {L_A}/{2}\right)\right) / Cos\theta$
$X_A - {L_A}/{2}$
$X_A + {L_A}/{2}$
X_A

Figure (51): Before-Aisle-Move operator for vertical aisle

4.5.7. Developed SA Algorithm

In this thesis the parameters taken by Bazargan-Lari and Kaebernick (1997) have been used in the developed SA algorithm:

I. Initial temperature: 10

II. Cooling rate: 0.9

III. Temperature reduction: $t_i = 10(0.9)^{i-1}$

IV. Outer loop: 25

V. Inner loop: $100 \times M,\ M$ is the total number of facilities

4.5.8. Improved Repair Function

It has to be noted this section is explaining about the new repair function which has been developed, coded; however the implementation and verification are still in process.

4.5.8.1. Improved Move Operator, After-Aisle

The previous move operator replaces the facility right after the side of the aisle which it has passed over. However, by taking that operator most of the facilities are kept in the side of the site where they originally have been located. In order to overcome this drawback, the improved move operator developed. The new operator called After-Aisle-Operator since it moves overlapped facility right after the aisle. The scheme of (52) and (53) represents this concept for both vertical and horizontal aisles.

The steps of the move operator with aisle constraints are explained as follows:

Step 1. Move facility f_G toward its closest facility.

Step 2. Check overlap possibility between $\acute{f}_G$ and aisle.

Step 3. If there is any overlap, take appropriate repair function

Step 4. Find the coordinates of $\acute{f}_G$- details shown in table (13) and (14)

Step 5. End

Repair Function- Horizontal Aisle

- Facility f_G is lower side of the aisle is:

$$Rep = \left(\left(Y_A + {}^{w_A}/_{2}\right) - \left(\acute{y}_G - {}^{W_G}/_{2}\right)\right) \Big/ _{Sin\theta} \qquad (59)$$

- Facility f_G is upper side of the aisle:

$$Rep = \left(\left(\acute{y}_G + {}^{w_G}/_{2}\right) - \left(Y_A - {}^{W_A}/_{2}\right)\right) \Big/ _{Cos\theta} \qquad (60)$$

Repair Function- Vertical Aisle

- Facility f_G is in the right side of the aisle:

$$Rep = \left(\left(\acute{x}_G + {}^{l_G}/_{2}\right) - \left(X_A - {}^{L_A}/_{2}\right)\right) \Big/ _{Cos\theta} \qquad (61)$$

- Facility f_G is in the left side of the aisle:

$$Rep = \left(\left(X_A + {}^{L_A}\!/_{2}\right) - \left(\acute{x}_G - {}^{l_G}\!/_{2}\right)\right)\Big/_{Cos\theta} \tag{62}$$

Table (13): Revised coordinate based on After-Aisle repair function -horizontal aisle

Horizontal Aisle	$\boldsymbol{x_{f_G} < x_{\acute{f}_G}}$	$\boldsymbol{x_{f_G} \geq x_{\acute{f}_G}}$
$y_{f_G} < Y_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$
$y_{f_G} > Y_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$

Table (14): Revised coordinate based on After-Aisle repair function -vertical aisle

Vertical Aisle	$\boldsymbol{y_{f_G} < y_{\acute{f}_G}}$	$\boldsymbol{y_{f_G} \geq y_{\acute{f}_G}}$
$x_{f_G} < X_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$
$x_{f_G} > X_L$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} + Rep \times sin\theta$	$x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times cos\theta$ $x_{\acute{f}_G} = x_{\acute{f}_G} - Rep \times sin\theta$

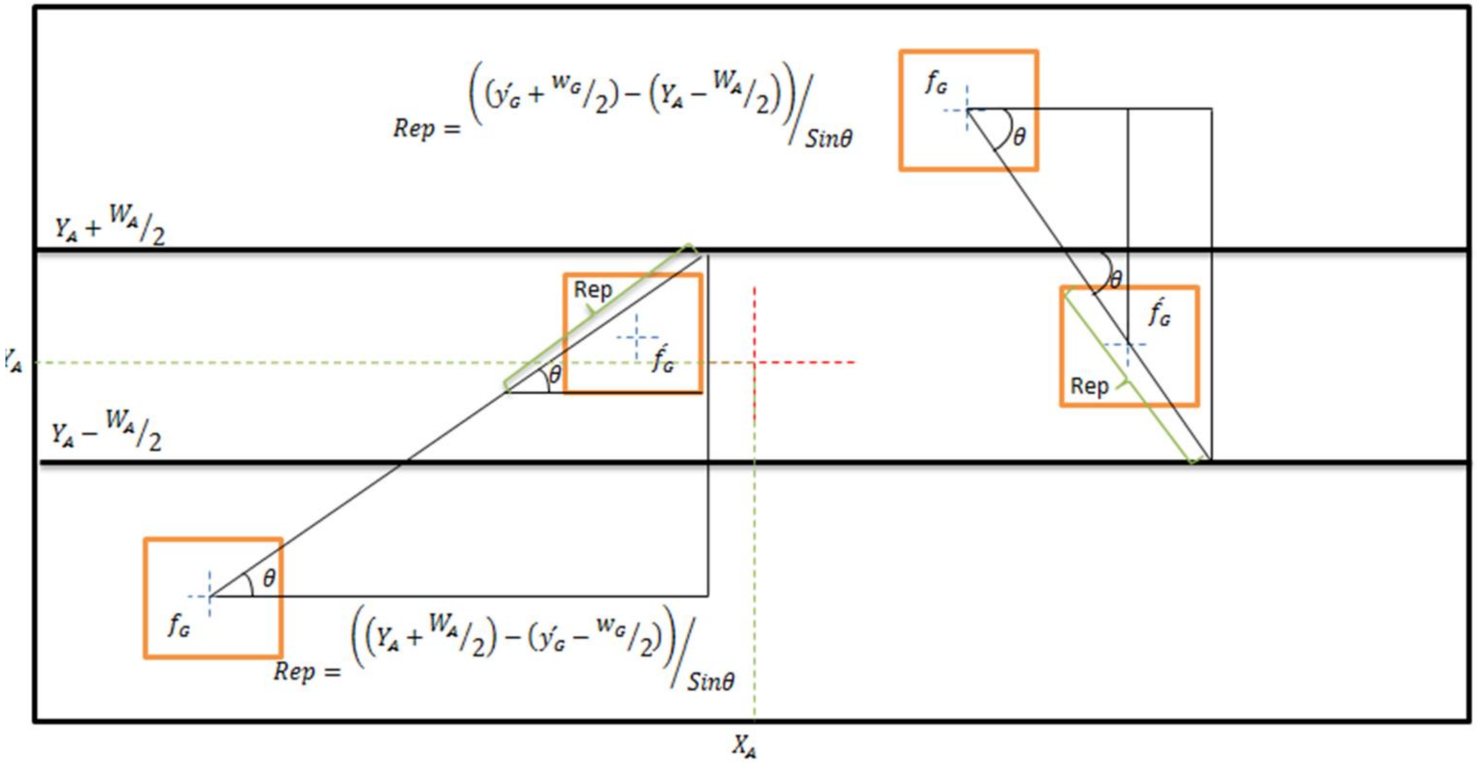

$Rep = \left(\left(\acute{y}_G + {}^{w_G}/_2 \right) - \left(Y_A - {}^{W_A}/_2 \right) \right) \Big/ _{Sin\theta}$
f_G
θ
$Y_A + {}^{W_A}/_2$
Rep
θ
$\acute{f}_G$
θ
$\acute{f}_G$
Rep
Y_A
$Y_A - {}^{W_A}/_2$
θ
f_G
$Rep = \left(\left(Y_A + {}^{W_A}/_2 \right) - \left(\acute{y}_G - {}^{w_G}/_2 \right) \right) \Big/ _{Sin\theta}$
X_A

Figure (52): After-Aisle-Operator for horizontal aisle

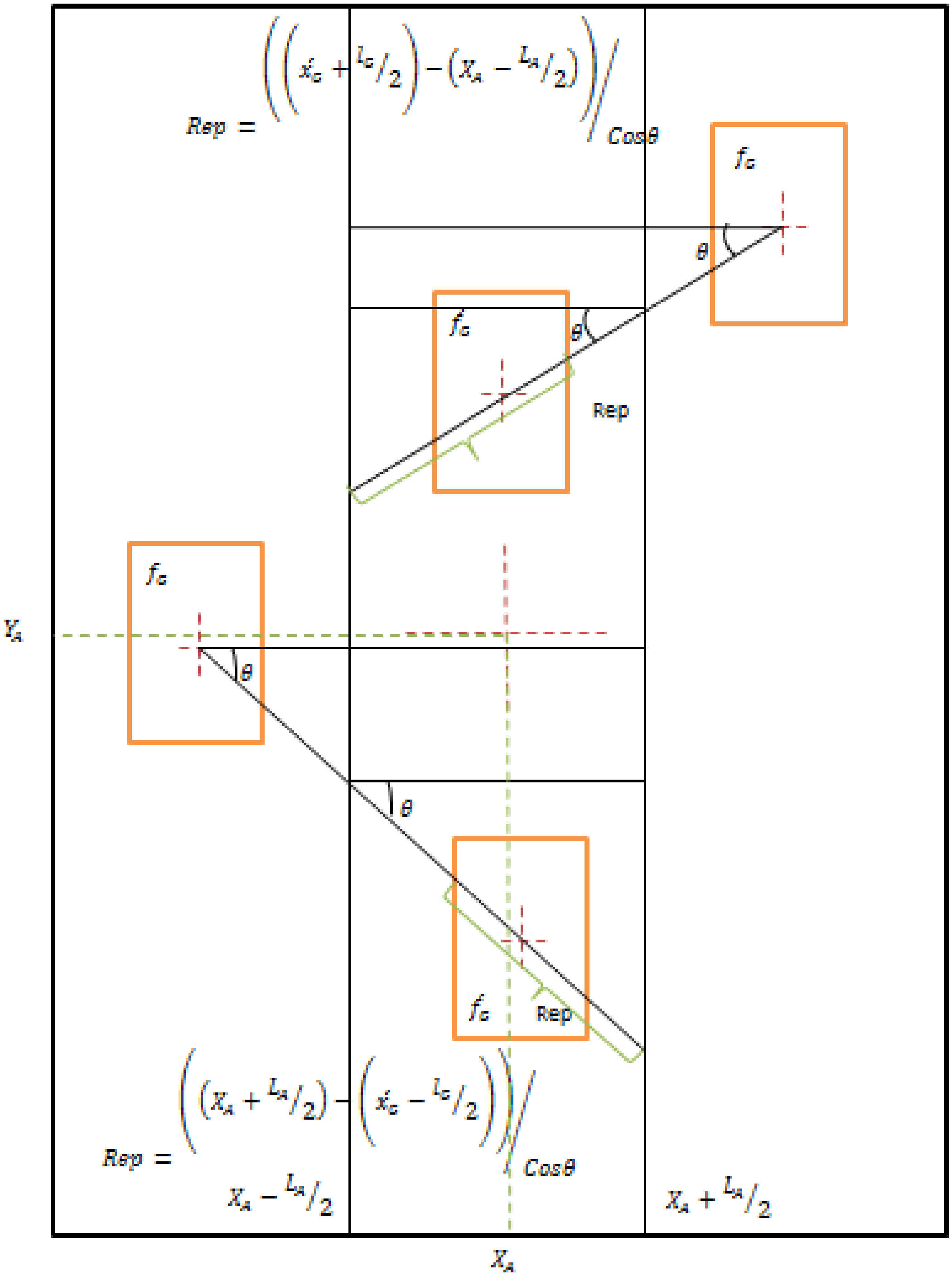

$Rep = \left(\left(\acute{x}_G + {}^{L_G}/_2\right) - \left(X_A - {}^{L_A}/_2\right)\right) / Cos\theta$
f_G
θ
f_G
θ
Rep
f_G
Y_A
θ
θ
f_G
Rep
$Rep = \left(\left(X_A + {}^{L_A}/_2\right) - \left(\acute{x}_G - {}^{L_G}/_2\right)\right) / Cos\theta$
$X_A - {}^{L_A}/_2$
$X_A + {}^{L_A}/_2$
X_A

Figure (53): After-Aisle-Operator for vertical aisle

It has to be noted because of aisle in the site, the closest facility to the target facility is usually chosen among those facilities where are in the same side of aisle as target facility. Hence, in order to have efficient algorithm a improved swap operator defined here. Moreover, the ratio of swap operator to mover operator in case of aisle has to be greater than the case when there is no aisle in the site.

4.5.8.2. Improved Swap Operator- Swap-Aisle

The improved swap operator is based on Free Zone concept; however, the aisle boundaries are participating in making free zone for target facility. The steps of improved-swap operator are as follows:

Step 1: Randomly choose a facility, called f_G

Step 2: Find the closet facility to the f_G, called f_C

Step 3: Calculate the area of Free Zone, called *FZ*

Step 4: Randomly choose one facility whose area is greater than equal to the area of f_G and less than equal to the area of *FZ*

Step 5: Switch the location of the two facilities

Step 6: Find the new coordinated of the two facilities

Step 7: End

CHAPTER FIVE: CASE STUDY

5.1 Company Description

The case study is company X which is Carbide Tool Inc manufactures and distributes metalcutting tools. The company is dedicated to develop specialized Carbide, PCD (Polycrystalline diamond) and CBN (*Cubic Boron Nitride*) inserts, as well as multitask tooling for the aerospace, automotive and mold-die industries. Since the metal cutting tools are small and the operations time done on them are short enough, the volume of production each day is large enough. One of the factors that facilitates production and leads it into the proper way is the layout of facilities. The current layout is job shop and is not efficient and optimal enough. After several meeting with plant manager and group, they concluded cellular manufacturing system (CMS) is the best option for them. The group formation was discussed with plant manager and performed. Moreover, the machine tools were assigned to their respective cells, which followed a product layout.

5.1.1 Products and Machine tools

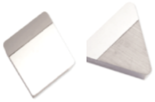
Five different kinds of family of cutting insert tools are produced namely, Dog bone, S shape, Triangular, Diamond, and Top Notch.

Dog Bone

Triangular

Top Notch

S Shape



Diamond

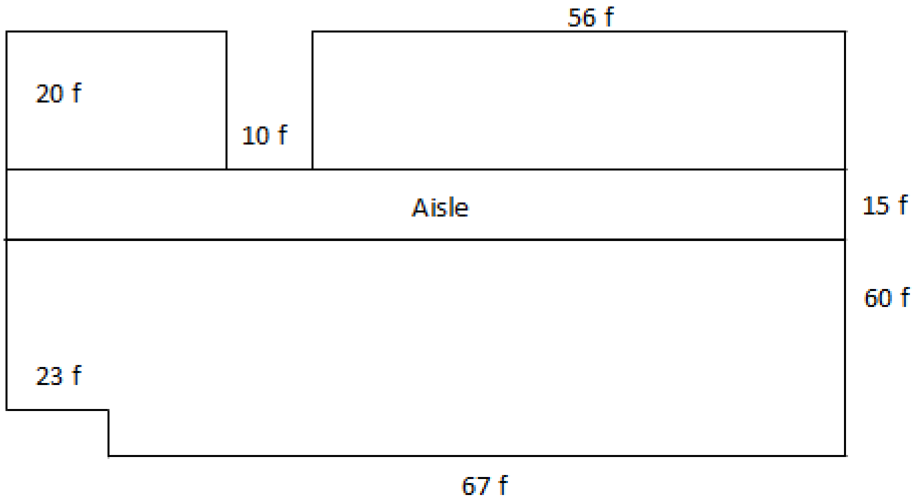
The main operation which is done on inserts is grinding. However, there are different kinds of grinding operations such as surface grinding, top and bottom grinding, periphery grinding and so on. Those operations are processed by variety of grinding machine tools. Totally, there are 12 different kinds of machine tools, both CNC and conventional. Table (15) represents the description of machine tools. Some of the machine tools have identical copies on the shop floor to increase productivity. The part demand is shared between the same machine tools. Moreover, there are three workstations such as inspection, wash, and packaging. These three sites are the final destinations of all products. The operations sequence for each cutting insert tool is different from others. In other words, all the operations are not being processed for each part. The list of operations of each inserts and those machine tools are used for those operation are shown in table (16).

Table (15): Machine tools descriptions

ID	Machine	Dimension	
		Length	Width
M1,M2	Blanchard (2)	6	9.07
M3	Double disk (1)	12.67	5
M4,M5,M6	Wendt (3)	8.5 6.8	6.1 9.45
M7	Polish (1)	6	5
M8, M9,M10	Surface grinding (3)	7	6
M11, M12	Swing fixture (2)	8	6
M13	V-bottom (1)	7	6
M14,M15	Wire-cutting (2)	7.8 7.4	6.7 5.7
M16	Laser M/C (1)	7.6	9.74
M17	Brazing (1)	4	1.8
M18	Ewag (1)		
M19	ETCH (1)	3	4
ST1	Inspection (1)	4	3
ST2	Wash (1)	5	3
ST3	Packing (1)	16	8

5.1.2 Shop Floor

The company’s shop floor does not have complete rectangular shape. There is an inventory in left bottom corner of the shop, a horizontal aisle for material flow and transportation, and a garage door for shipment. Figure (54) represents the available area of the floor with exact dimension.



Figure(54): Scheme of company’s shop floor

Table (16): Parts' specifications and the operations sequence

Part	Demand	Operation Sequence									
Dog Bone	1600	M3	M11, M12	M13	M8, M9	M19	ST2	ST1	ST3		
S Shape	900	M3	M13	M11, M12	M8, M9	M19	ST2	ST1	ST3		
Triangular	300	M3	M7	M4, M5, M6	M8, M9	M19	ST2	ST1	ST3		
Top Notch	300	M1, M2	M4, M5, M6	M8, M9	M19	ST2	ST1	ST3			
Diamond (1)	600	M1, M2	M4, M5, M6	M10	M14, M15	M17	M18	M19	ST2	ST1	ST3
Diamond (2)	600	M1, M2	M4, M5, M6	M10	M14, M15	M17	M16	M19	ST2	ST1	ST3
Diamond (3)	600	M17	M18	M19	ST2	ST1	ST3				

5.1.3 Current Layout:

The current layout they have is job shop layout. Hence, machine tools are operating the same operations have been located in the same locations. For an example, all surface grinding machines grouped together. There is no special material handling device for transforming unfinished products among machine tools. Plant manager expressed different problems such as lots of material movements, delays in lead times, high volume of work in process. Moreover, by considering table (2) it becomes obvious that the number of operations done on each part is many. This is the good enough evidence of huge number of movements taking place every day on the floor.

Plant management group considered CM is the best option for them to overcome those difficulties. The top management group has decided to group machine tools into four cells. Since the family of Diamond has completely different sequence of operations one cell allocated to that family and its corresponding parts. Table (17) represents the GF.

A total of 17 operations are being performed on the five different types of families of products. Each family has different parts with different sequence of operations. For simplification, here we did not consider different variants of inserts except for the Diamond one, which has 3 different types of variants; hence according to the sequence of operations there are 7 different types of products. Moreover, all products do not have the same operations sequence; and also all operations are not being performed on all products.

Table (17): GF results

Cell Name	**Machine tools / Work Station**				
Primary Grinding	Double Disc (1) Surface Grinding (2)	Blanchard (2) Swing Fixture (2)	Polish (1) V-Bottom (1)	Wendt (3)	
Diamond	Wire-cutting (2)	Surface Grinding (1)	EWAG (1)	Brazing (1)	Laser M/c (1)
Final	ETCH (1)	Inspection (1)	Wash (1)	Packing and Shipment (1)	

*The number of units for each machine tools shown in bracket.

FICO Xpress Optimization Suit Software has been used to solve the continuous formulation of this paper. Since the mathematical formulation is nonlinear both

Successive Linear Programing (SLP) and Non-linear Programming (NLP) solver have been used.

5.2 Computational Results

5.2.1 Mathematical Modelling

Both linear and nonlinear model have been applied for leader and follower problems. The intra-cell cost for Dog Bone, S Shape, Triangular, Top Notch, and Diamond family are ¢10, ¢10, ¢15, ¢12, and ¢20 respectively. Additionally, the inter-cell costs are ¢12, ¢12, ¢18, ¢15, and ¢15.

5.2.1.1 Nonlinear Model:

The nonlinear MIP is applied for both leader and follower problem.

Intracellular Layout:

- ✓ **Primary Cell**

The result of NLMIP for primary cell is presented in table (18) and the layout scheme showed in figure (55).

Table (18): Intra-cell layout for Primary Cell- Nonlinear model

Machine Tool	Centroid		Dimension	
	X	*Y*	**Length**	**Width**
Blanchard(1)	8.49	10.49	[5.49,11.49]	[5.95,15.02]
Blanchard (2)	15.29	8.26	[12.29,18.29]	[3.73,12.80]
Double Disc	24.63	17.52	[18.29,30.96]	[15.02,20.02]
Wendt (1)	22.54	11.97	[18.29,26.79]	[8.29,15.02]
Wendt (2)	22.54	5.87	[18.29,26.79]	[2.82,8.92]
Wendt (3)	14.89	17.52	[11.49,18.29]	[12.80,22.25]
Polish	8.49	17.52	[5.49,11.49]	[15.02,20.02]
Cell Dimension: 35 × 25		**MHC:** $ 1,191.550		

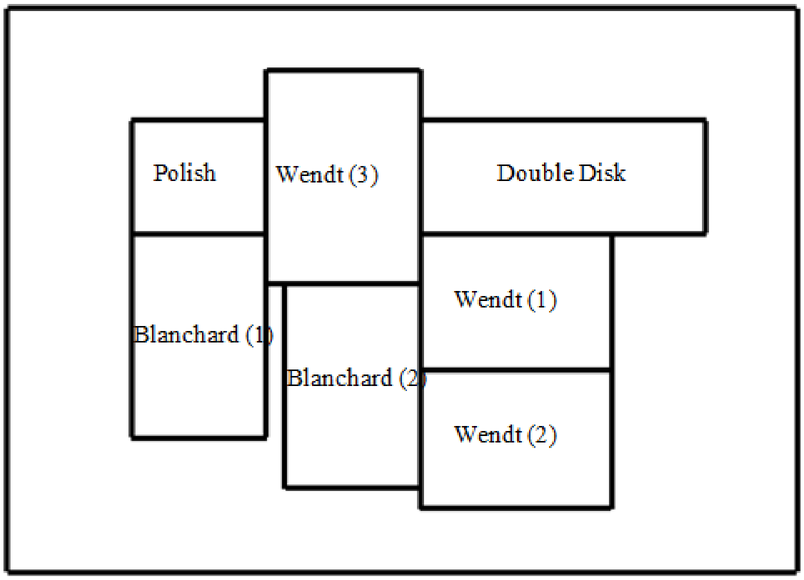


Figure (55): Intra-cell layout of Primary Cell- Nonlinear model

- ✓ **Grinding Cell**

The result of NLMIP for grinding cell is shown in table (19) and the layout scheme is presented in figure (56).

Table (19): Intra-cell layout for Grinding Cell- Nonlinear model

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	21.5	13.32	[18,25]	[10.32,16.32]
Surface Grinding (2)	7.5	13.32	[4,11]	[10.32,16.32]
Swing Fixture (1)	7	7.32	[3,11]	[4.32,10.32]
Swing Fixture (2)	22	7.32	[18,26]	[4.32,10.32]
V-Bottom	14.5	13.30	[11,18]	[10.30,16.30]
Cell Dimension: 26 × 20		**MHC:** $ 520.588		

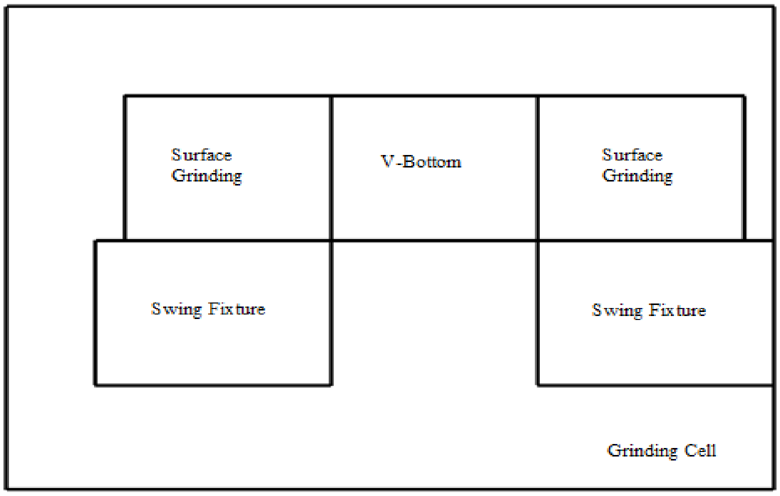


Figure (56): Intra-cell layout of Grinding Cell- Nonlinear model

✓ Diamond Cell

Table (20) represents the result of NLMIP for diamond cell and the layout scheme is shown in figure (57).

Table (20): Intra-cell layout for Diamond Cell- Nonlinear model

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Wire Cutting (1)	4.4	3.77	[0.5,8.3]	[0.42,7.12]
Wire Cutting (2)	20.3	11.99	[16.6,24]	[9.14,14.84]
Surface Grinding	27	3.77	[24,30]	[0,7.54]
Brazing	10.3	3.77	[8.3,12.3]	[2.87,4.67]
Ewag	14.45	3.77	[12.3,16.6]	[0.12,7.42]
Laser M/c	4.5	11.99	[0.7,8.3]	[10.26,16.86]
Cell Dimension: 30 × 20		**MHC:** $764.580		

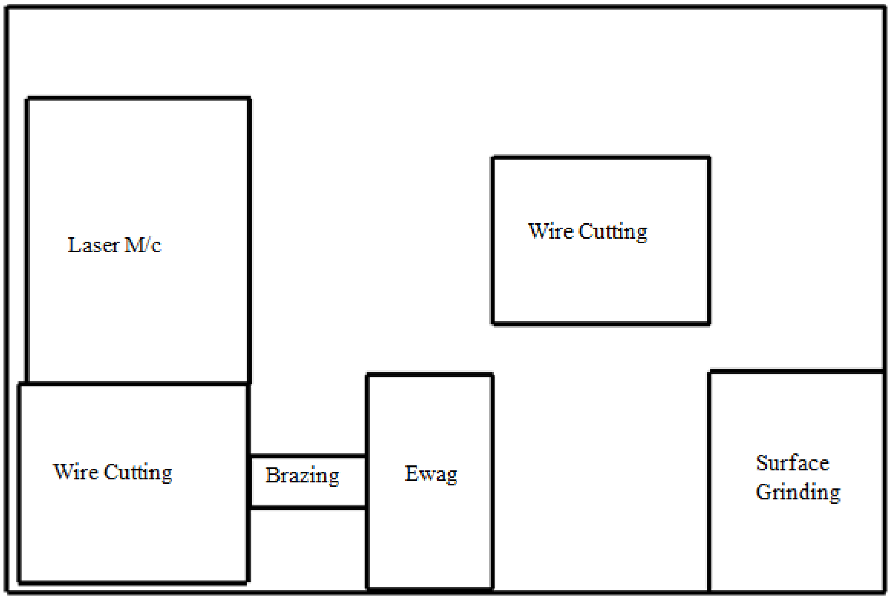


Figure (57): Intra-cell layout for Diamond Cell- Nonlinear model

✓ Final Cell

The result of NLMIP for final cell is shown in table (21) and figure (58) represents the layout scheme for this cell.

Table (21): Intra-cell layout for Final Cell- Nonlinear model

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	1.5	13.29	[0,3]	[13.44,17.44]
Wash	25.5	13.29	[3,8]	[13.44,17.44]
Inspection	21	13.29	[8,12]	[13.44,17.44]
Pack and Shipment	11	13.29	[8,12]	[11.44,19.44]
Cell Dimension: 30 × 20		**MHC:** $1,056.350		

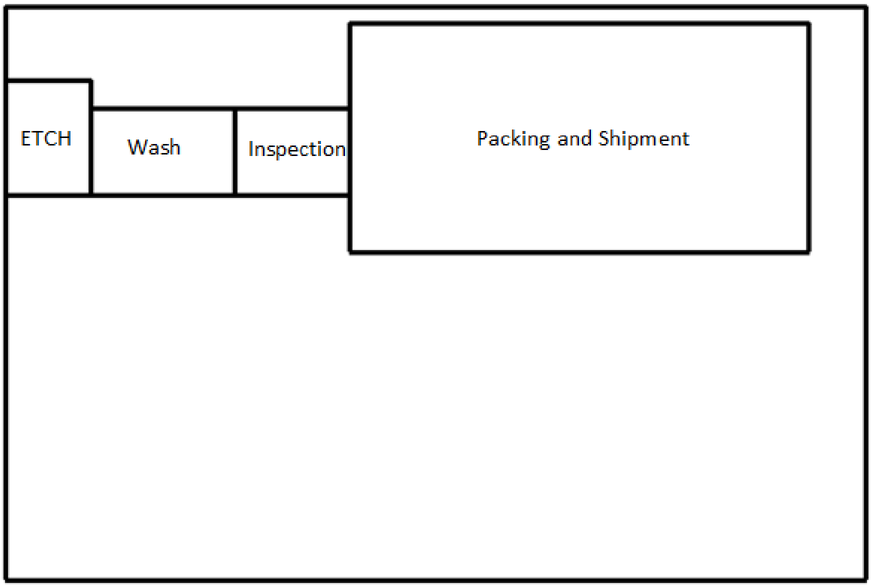


Figure (58): Intra-cell layout for Final Cell- Nonlinear model

Intercellular Layout:

The follower problem, inter-cellular layout was solved by NLMIP and the results are shown in table (22).

Table (22): Inter-cell layout - Nonlinear model

Cells	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	42.5	13.5	[25,60]	[1,26]
Grinding	74	50	[64,84]	[40,60]
Diamond	45	59.22	[33,63]	[44.22,59.22]
Final	75	8	[60,90]	[0,16]
Blocks	***X***	***Y***	**Length**	**Width**
Garage Door	29	50	10	20
Inventory	11.37	3.5	6.5	23
Aisle	45	32.5	90	60
Shop Dimension: 90 × 60		**MHC:** $7,520.420		

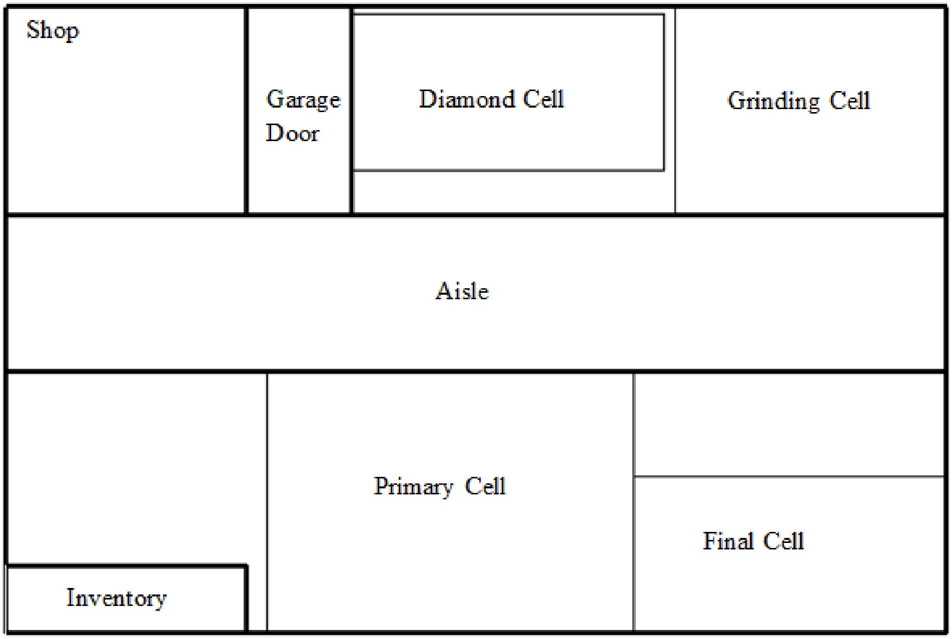


Figure (59): Inter-cell layout design- Nonlinear Model

5.2.1.2 Linear Model

Leader Problem- Intra-cell

- ✓ **Primary Cell**

The result of linear MIP for primary cell is presented in table (23) and the layout scheme is shown in figure (60).

Table (23): Intra-cell layout for Primary Cell- linear model

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Double Disc	3	16.735	[0,6]	[12.2,21.27]
Blanchard(1)	9	16.735	[6,12]	[12.2,21.27]
Blanchard (2)	15.4	9.15	[9.06,21.73]	[6.65,11.65]
Wendt (1)	8.15	3.05	[3.9,12.4]	[0,6.1]
Wendt (2)	4.81	9.15	[0.56,9.06]	[6.1,12.2]
Wendt (3)	15.4	16.735	[12,18.8]	[12.01,21.46]
Polish	15.4	4.15	[13.4,18.4]	[1.65,6.65]
Cell Dimension: 35 × 25		**MHC:** $503.024		

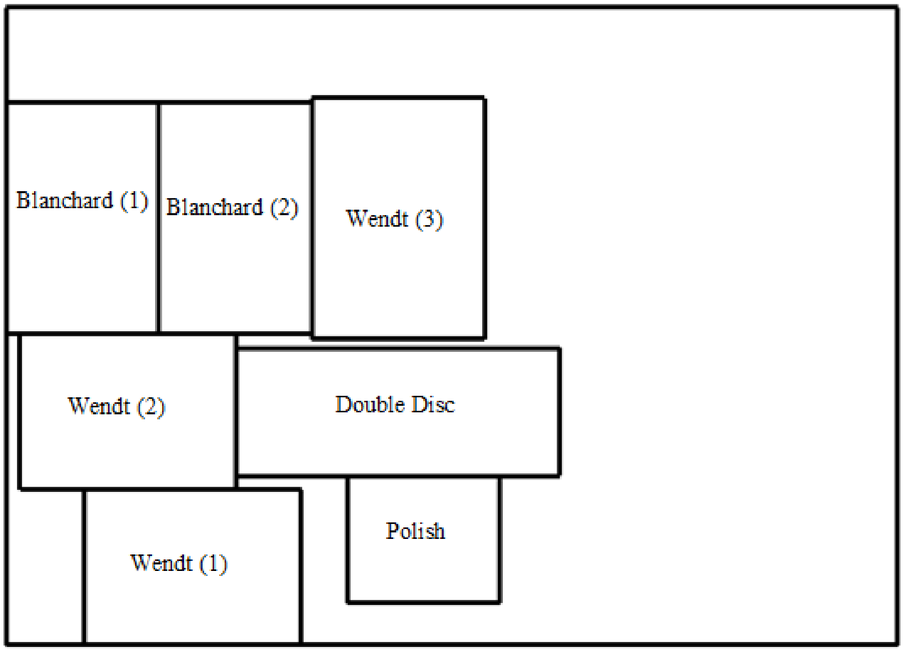


Figure (60): Intra-cell layout of Primary cell- Linear model

✓ Grinding Cell

Table (24) presents the result of linear model for grinding cell and the layout scheme is shown in figure (61).

Table (24): Intra-cell layout for Grinding Cell- Linear model

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	4	15	[0.5,7.5]	[12,18]
Surface Grinding (2)	11	9	[7.5,14.5]	[6,12]
Swing Fixture (1)	11.5	15	[7.5,15.5]	[12,18]
Swing Fixture (2)	4	3	[0,8]	[0,6]
V-Bottom	4	9	[0.5,7.5]	[6,12]
Cell Dimension: 26 × 20		**MHC:** $399.750		

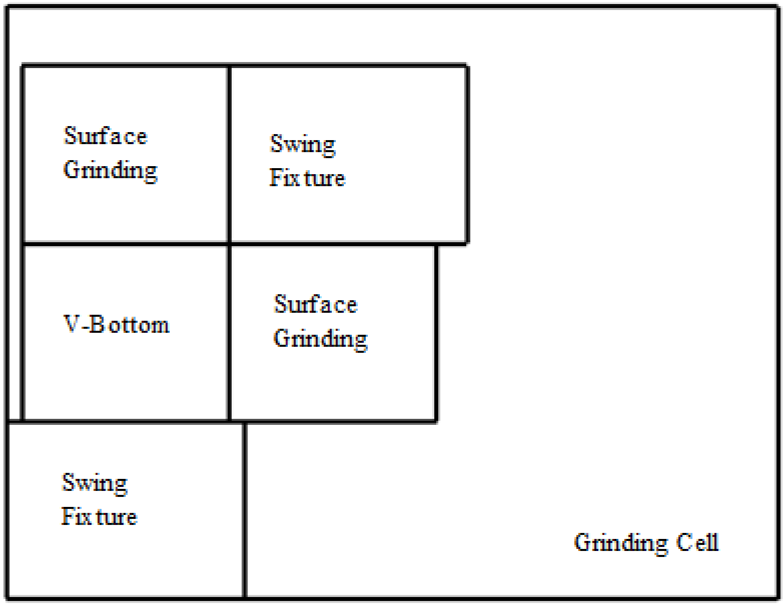


Figure (61): Intra-cell layout of Grinding cell- Linear Model

✓ Diamond Cell

The result of linear model for diamond cell is represented in table (25) and the layout scheme is shown in figure (62).

Table (25): Intra-cell layout for Diamond Cell- Linear model

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Wire Cutting (1)	11.5	9.65	[7.6,15.4]	[6.3,13]
Wire Cutting (2)	18.2	16.5	[14.5,21.9]	[13,20]
Surface Grinding	11.5	16.5	[8.5,14.5]	[13,20]
Brazing	18.2	12.75	[16.2,20.2]	[11.85,13.65]
Ewag	18.2	8.2	[16.05,20.35]	[4.55,11.85]
Laser M/c	3.8	4.87	[0,7.6]	[0,9.74]
Cell Dimension: 30 × 20		**MHC:** $360.800		

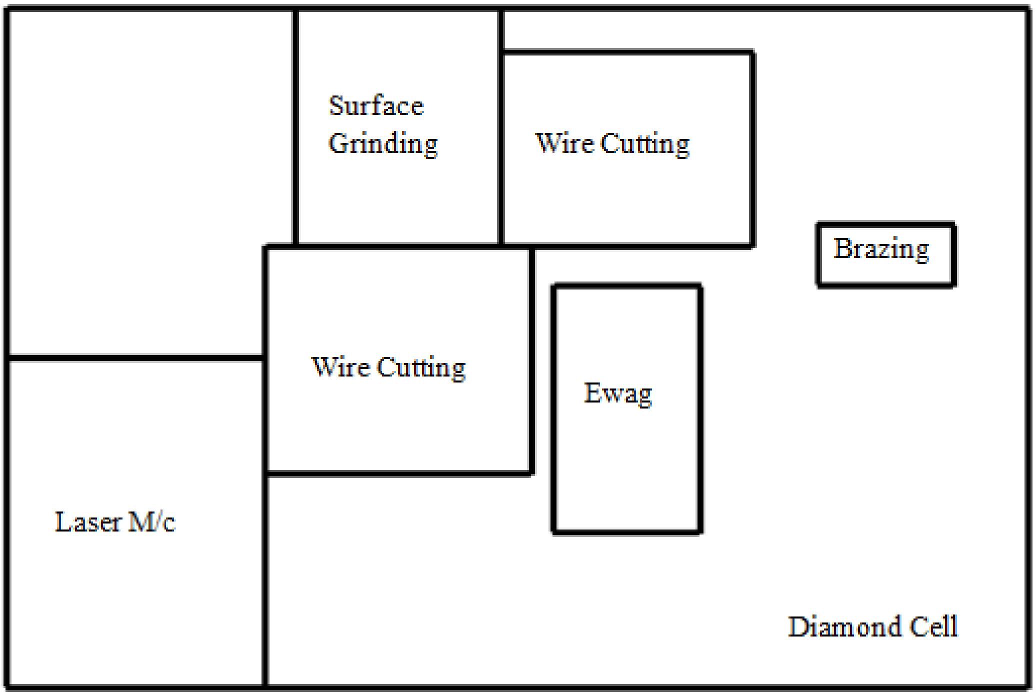


Figure (62): Intra-cell layout of Diamond cell- Linear Model

✓ Final Cell

Table (26) illustrates the result of linear model for final cell and figure (63) shows the scheme of layout this cell.

Table (26): Intra-cell layout for Final Cell- Linear model

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	8	2	[6.5,9.5]	[0,4]
Wash	8	2.5	[5.5,10.5]	[4,7]
Inspection	8	8.5	[6,10]	[7,10]
Pack and Shipment	8	14	[0,16]	[10,18]
Cell Dimension: 30 × 20		**MHC:** $685.200		

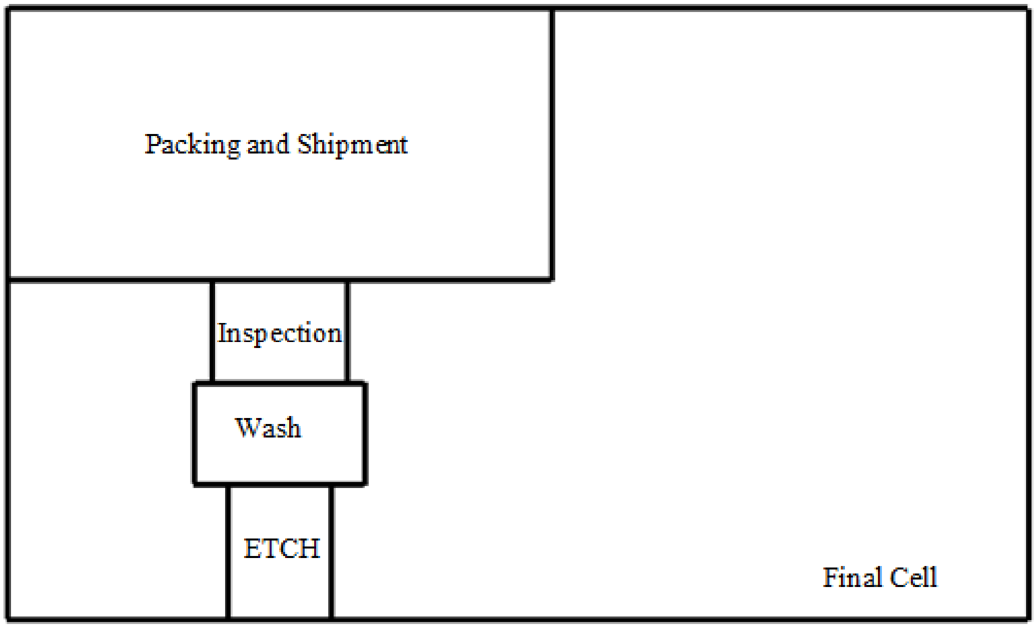


Figure (63): Intra-cell layout of Diamond cell- Linear model

Intercellular Layout Design

The result of linear model for inter cellular is presented in table (27) and the layout scheme is shown in figure (64).

Table (27): Inter-cell layout-Linear model

Cells	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	42.5	13.5	[25,60]	[1,26]
Grinding	74	50	[34,60]	[40,60]
Diamond	45	59.22	[60,90]	[6,26]
Final	75	8	[60,90]	[40,30]
Blocks	***X***	***Y***	**Length**	**Width**
Garage Door	29	50	10	20
Inventory	11.37	3.5	6.5	23
Aisle	45	32.5	90	60
Shop Dimension: 90 × 60		**MHC:** $4,213.900		

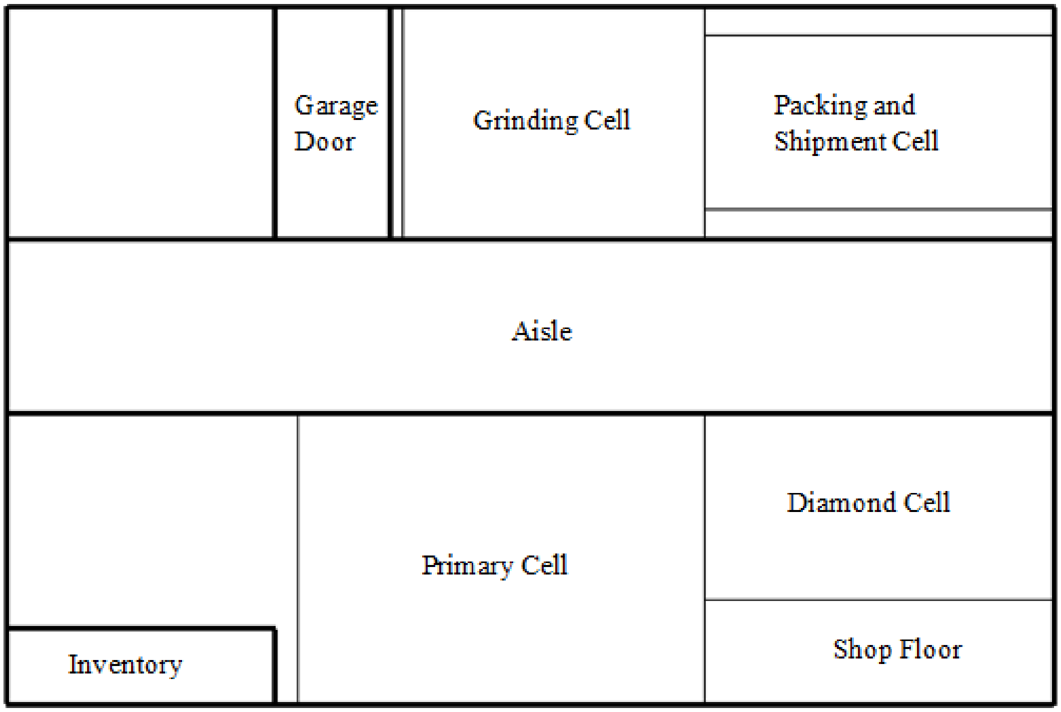


Figure (64): Inter-cell layout design- Linear Model

5.2.2 Heuristics

5.2.2.1. Heuristic

- **Primary Cell**

The result of heuristic algorithm for primary cell is presented in table (28) and the layout scheme is shown in figure (65).

Table (28): Intra-cell layout for Primary Cell- Heuristic algorithm

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard(1)	14.5	18.30	[11.5,17.5]	[13.77,22.85]
Blanchard (2)	21.27	17.16	[18.27,24.27]	[12.62,21.69]
Double Disc	23.83	10	[17.5,30.17]	[7.5,12.5]
Wendt (1)	7.25	17.72	[3,11.5]	[14.67,20.77]
Wendt (2)	23.34	4.45	[19.09,27.59]	[1.4,7.5]
Wendt (3)	6.58	7.64	[3.18,9.98]	[2.91,12.36]
Polish	14.5	5.98	[11.5,17.5]	[3.48,8.48]
Cell Dimension: 35 × 25				

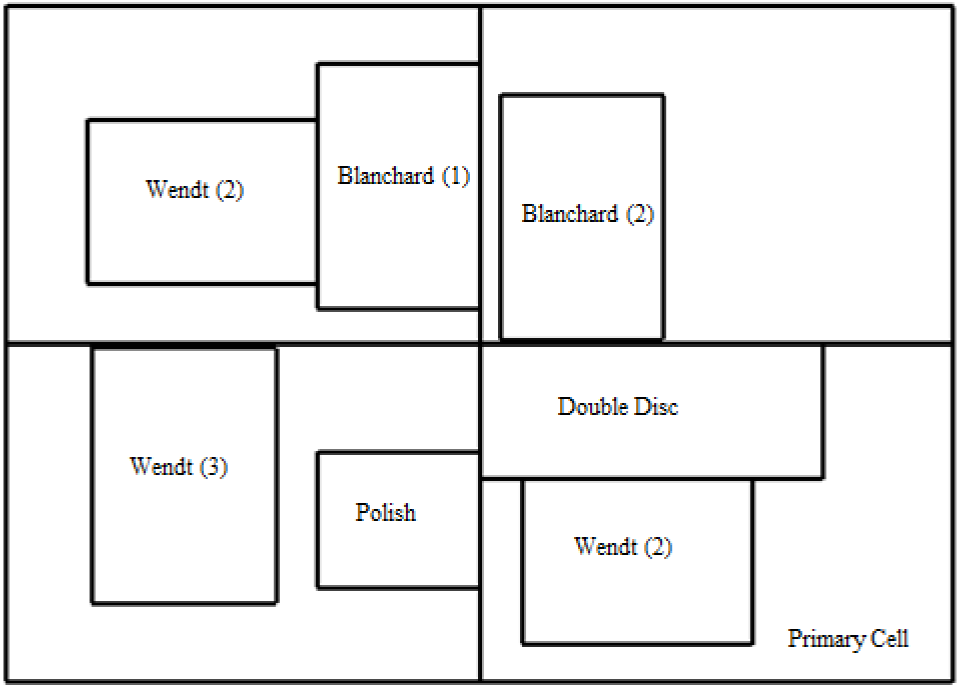


Figure (65): Primary Cell Layout Based on Developed Heuristic

- **Grinding Cell**

The result of heuristic algorithm for grinding cell is presented in table (29) and the layout scheme is shown in figure (66).

Table (29): Intra-cell layout for Grinding Cell- Heuristic algorithm

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	16.79	4	[13.29,20.29]	[1,7]
Surface Grinding (2)	21.05	16	[17.55,24.55]	[13,19]
Swing Fixture (1)	18.97	10	[14.97,22.97]	[7,13]
Swing Fixture (2)	5.72	15.26	[1.75,9.75]	[12.26,18.26]
V-Bottom	8.55	6.76	[5.05,12.05]	[3.76,9.76]
Cell Dimension: 26 × 20				

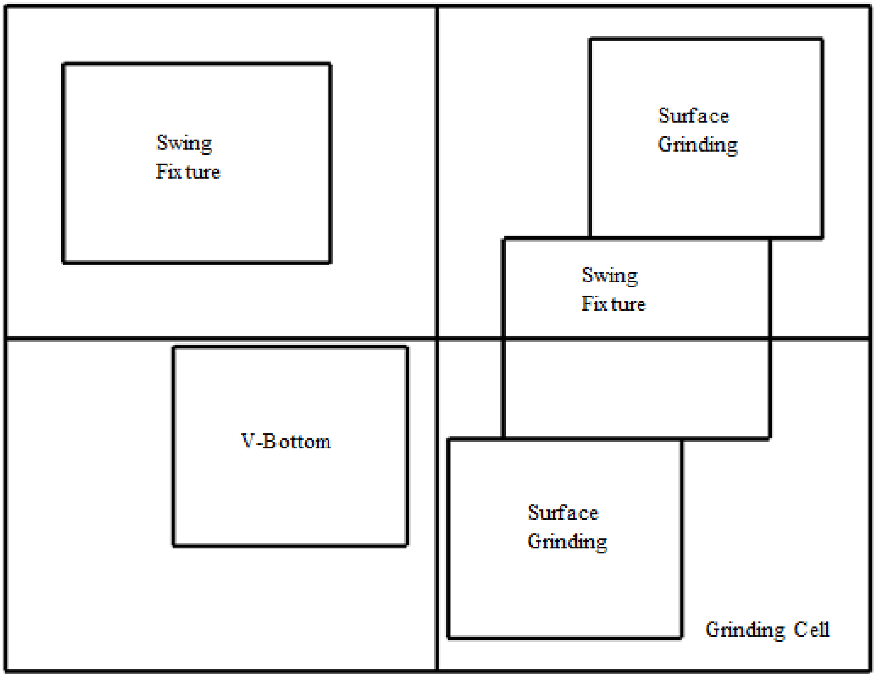


Figure (66): Grinding Cell Layout Based on Developed Heuristic

- **Diamond Cell**

Table (30) represents the result of heuristic algorithm for diamond cell and the layout scheme is shown in figure (67).

Table (30): Intra-cell layout for Diamond Cell- Heuristic algorithm

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Wire Cutting (1)	10.39	3.8	[6.49,14.29]	[0.45,7.15]
Wire Cutting (2)	5.50	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	18	5.64	[15,21]	[1.87,9.41]
Brazing	26	10	[24,28]	[9.1,10.9]
Ewag	11.53	16.31	[9.20,13.50]	[12.66,19.96]
Laser M/c	18.8	14.87	[15,22.6]	[10,19.74]
Cell Dimension: 30 × 20				

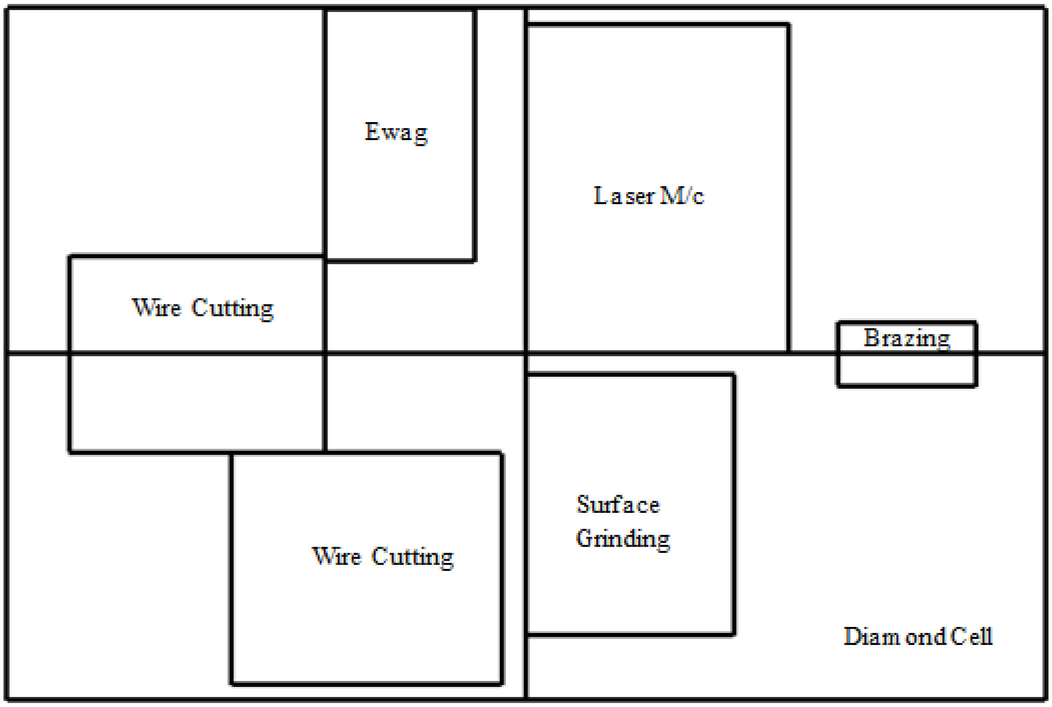


Figure (67): Diamond Cell Layout Based on Developed Heuristic

- **Final Cell**

The result of heuristic algorithm for final cell is illustrated in table (31) and the layout scheme is represented in figure (68).

Table (31): Intra-cell layout for Final Cell- Heuristic model

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	26.19	9	[24.69,27.69]	[7,11]
Wash	15	12.42	[12.5,17.5]	[10.92,13.92]
Inspection	4.89	9	[2.89,6.89]	[7.5,10.5]
Pack and Shipment	15	5	[7,23]	[1,9]
Cell Dimension: 30 × 20				

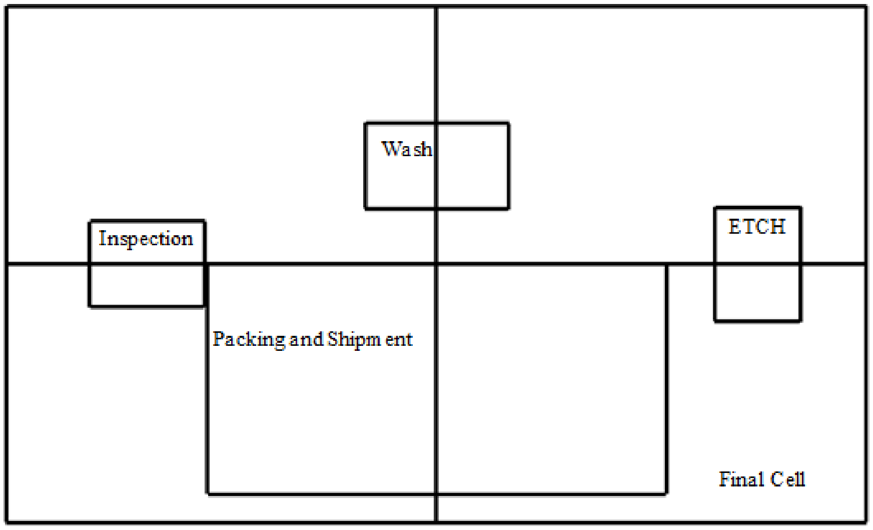


Figure (68): Final Cell Layout Based on Developed Heuristic

5.5.2.2 Initial Solution for SA

Developed Simulated annealing is applied for both leader and follower problems. The developed heuristic algorithm used for initializing the SA algorithm. The summary of the data is provided in tables (32)-(35)

Intra-cellular layout design

Table (32): Initial solution-Primary Cell

Machine Tool	Centroid		Dimension	
	X	***Y***	**Length**	**Width**
Double Disc	14.5	18.30	[11.5,17.5]	[13.77,22.85]
Blanchard(1)	21.27	17.16	[18.27,24.27]	[12.62,21.69]
Blanchard (2)	23.83	10	[17.5,30.17]	[7.5,12.5]
Wendt (1)	7.25	17.72	[3,11.5]	[14.67,20.77]
Wendt (2)	23.34	4.45	[19.09,27.59]	[1.4,7.5]
Wendt (3)	6.58	7.64	[3.18,9.98]	[2.91,12.36]
Polish	14.5	5.98	[11.5,17.5]	[3.48,8.48]
Cell Dimension: 35 × 25				

Table (33): Initial solution-Grinding Cell

Machine Tool	Centroid		Dimension	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	16.79	4	[13.29,20.29]	[1,7]
Surface Grinding (2)	21.05	16	[17.55,24.55]	[13,19]
Swing Fixture (1)	18.97	10	[14.97,22.97]	[7,13]
Swing Fixture (2)	5.72	15.26	[1.75,9.75]	[12.26,18.26]
V-Bottom	8.55	6.76	[5.05,12.05]	[3.76,9.76]
Cell Dimension: 26 × 20				

Table (34): Initial solution-Diamond Cell

Machine Tool	Centroid		Dimension	
	X	***Y***	**Length**	**Width**
Wire Cutting (1)	10.39	3.8	[6.49,14.29]	[0.45,7.15]
Wire Cutting (2)	5.50	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	18	5.64	[15,21]	[1.87,9.41]
Brazing	26	10	[24,28]	[9.1,10.9]
Ewag	11.53	16.31	[9.20,13.50]	[12.66,19.96]
Laser M/c	18.8	14.87	[15,22.6]	[10,19.74]
Cell Dimension: 30 × 20				

Table(35): Initial solution-Final Cell

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	26.19	9	[24.69,27.69]	[7,11]
Wash	15	12.42	[512.5,17.5]	[10.92,13.92]
Inspection	4.89	9	[2.89,6.89]	[7.5,10.5]
Pack and Shipment	15	5	[7,23]	[1,9]
Cell Dimension: 30 × 20				

5.5.2.3 Simulated Annealing

Intra-cellular layout

- **Primary Cell**

The results of inter cellular layout plan for primary cell using SA are shown in table (36).

Table (36): Intra-cell layout for primary Cell- SA algorithm

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	21.27	17.16	[18.27, 24.27]	[12.625, 21.695]
Double Disc	23.83	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.249	16.753	[2.999, 11.499]	[13.703, 19.802]
Wendt (2)	23.340	4.450	[19.09, 27.59]	[1.4, 7.5]
Wendt (3)	8.099	8.977	[4.699, 11.499]	[4.252, 13.702]
Polish	14.500	5.980	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 701.592				

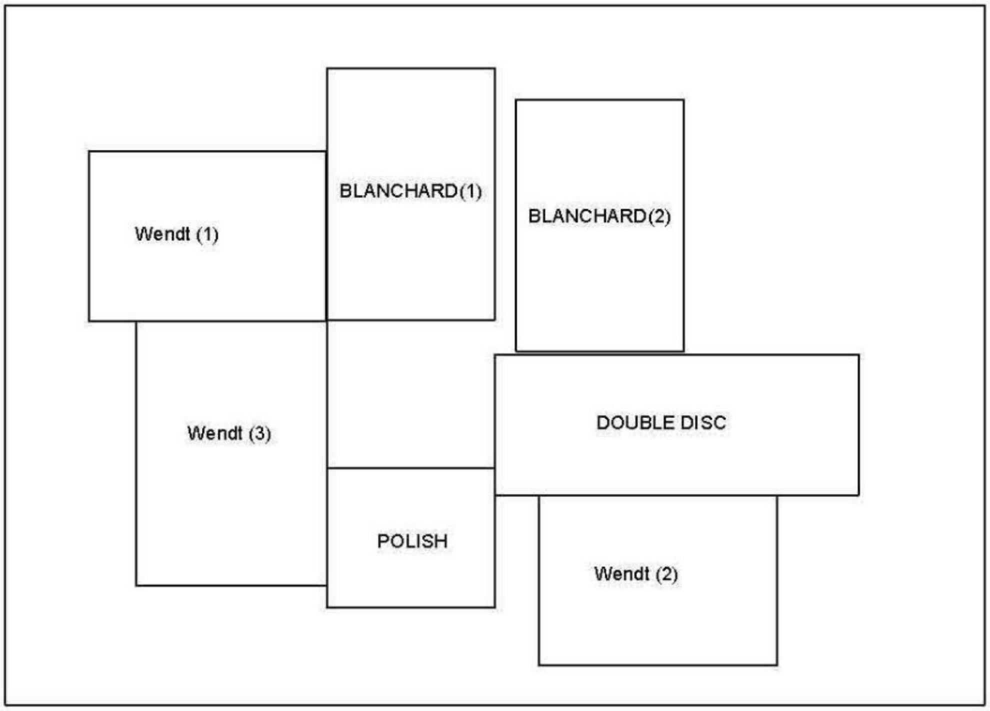


Figure (69): Primary Cell Layout Based on SA

- **Grinding Cell**

The results of inter cellular layout plan for grinding cell using SA are presented in table (37).

Table (37): Intra-cell layout for grinding Cell- SA algorithm

Machine Tool	Centroid		Dimension	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	15.552	4	[12.052, 19.052]	[1, 7]
Surface Grinding (2)	3.5	12.76	[0,7]	[9.76, 15.76]
Swing Fixture (1)	18.97	10	[14.97, 22.97]	[7, 13]
Swing Fixture (2)	10.97	12.76	[6.97, 14.97]	[9.76, 15.76]
V-Bottom	8.553	6.76	[5.053, 12.053]	[3.76, 9.76]
MHC: $526.004				

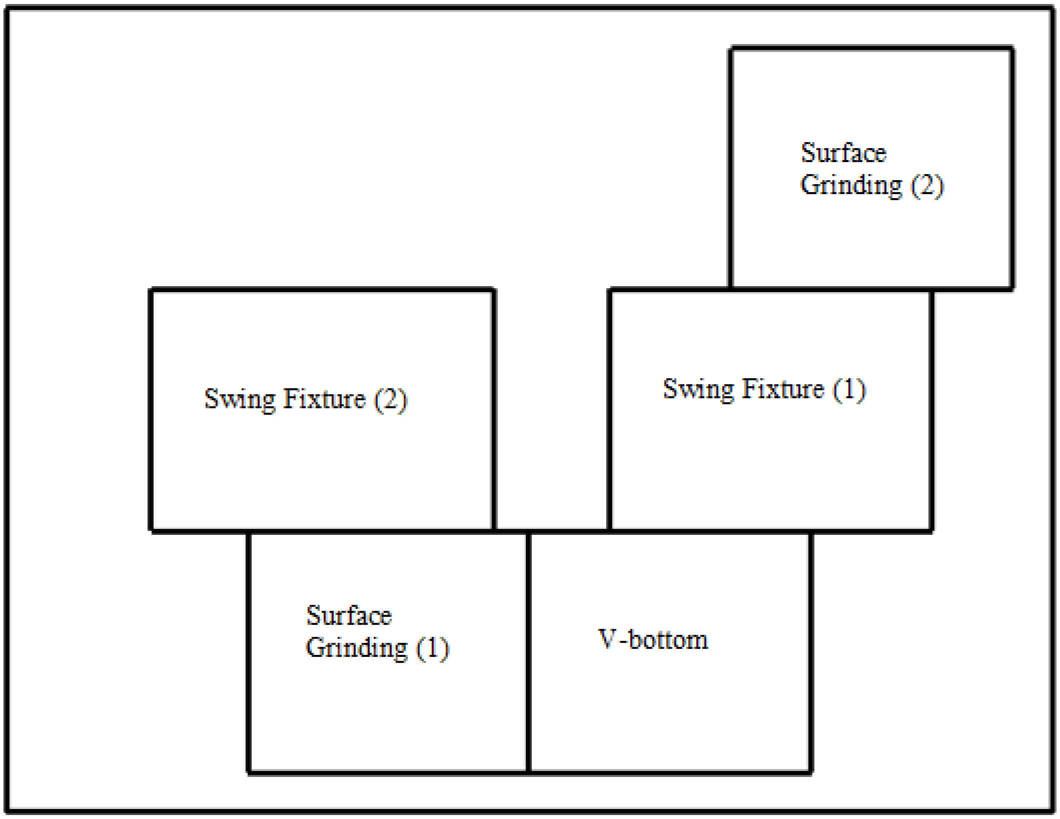


Figure (70): Grinding Cell Layout Based on SA

- **Diamond Cell**

Table (38) represents the results of inter cellular layout plan for diamond cell using SA.

Table (38): Intra-cell layout for diamond Cell- SA algorithm

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	11.03	3.8	[7.13,14.93]	[0.45,7.15]
Wire Cutting (2)	5.5	10	[1.8,9.2]	[7.15,12.85]
Surface Grinding	17.93	5.62	[14.93,20.93]	[2.12,9.12]
Brazing	22.93	8.33	[20.93,24.93]	[7.43,9.23]
Ewag	11.53	16.30	[9.38,13.68]	[12.65,19.95]
Laser M/c	17.48	15.12	[13.68,21.28]	[10.25,19.99]
MHC: $787.940				

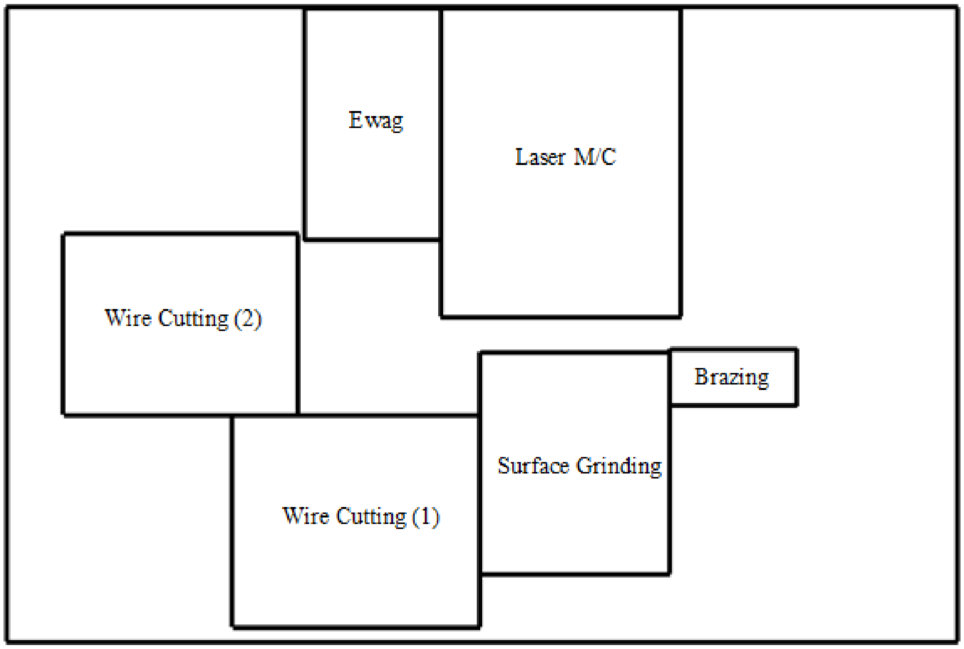


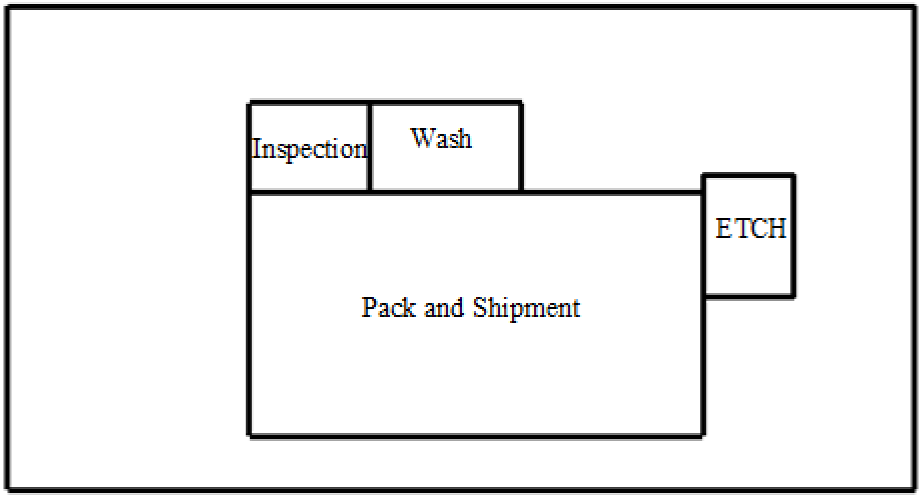
Figure (71): Diamond Cell Layout Based on SA

- **Final Cell**

The results of inter cellular layout plan for final cell using SA are shown in table (39).

Table (39): Intra-cell layout for final Cell- SA algorithm

Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	19.01	11.721	[17.511, 20.51]	[9.722, 13.722]
Wash	15.011	14.221	[12.511, 17.511]	[12.722, 15.722]
Inspection	15.012	11.222	[13.012, 17.012]	[9.722, 12.722]
Pack and Shipment	15.011	5.722	[7.011, 23.011]	[1.722, 9.722]
MHC: $ 856.508				



Figure(72):Final Cell Layout Based on SA

- **Inter-cellular Layout**

It has to be noted, in inter-cell the only block considered is the aisle. The initial solution for SA by using the developed heuristic algorithm is presented in the table (40).

Table (40): Inter-cell initial solution for SA algorithm

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55,90]	[0,25]
Grinding	63	50	[50,76]	[40,60]
Diamond	31.79	10	[16.79,46.79]	[0,20]
Final	15	50	[0,30]	[40,60]
Aisle	45	32.5	90	15
Shop Size: 90×60				

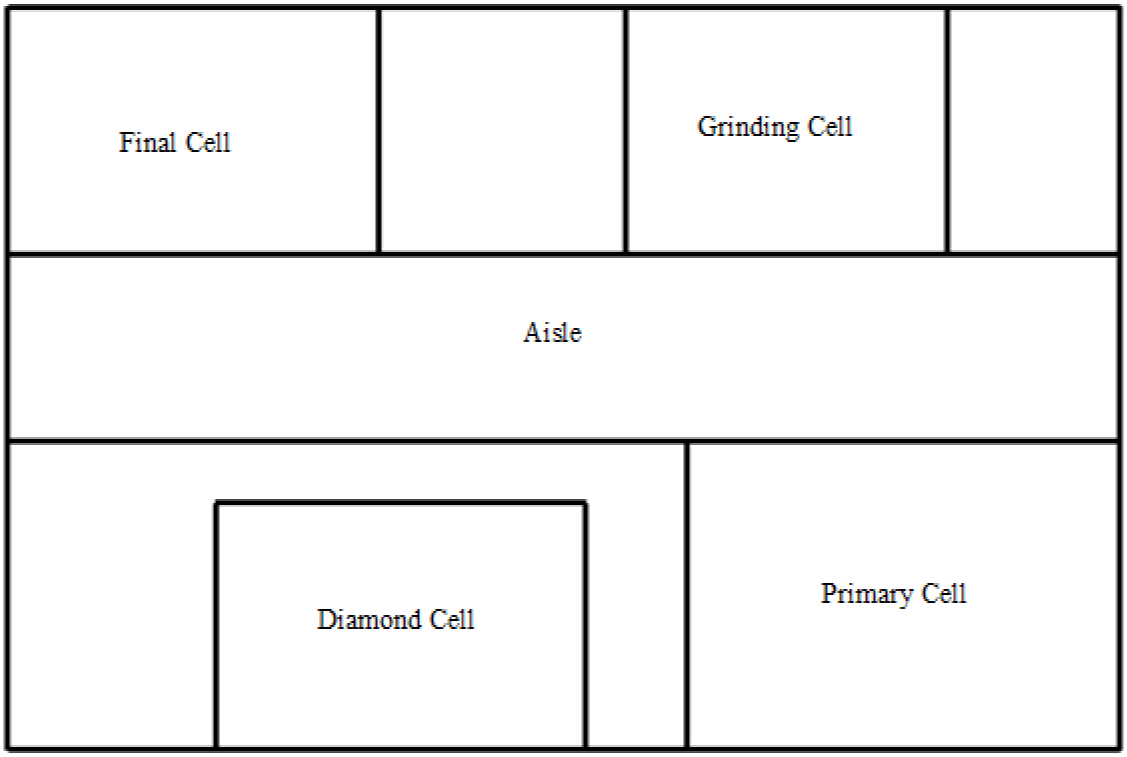


Figure (73): Initial solution for inter-cell layout using heuristic

Table (41): Inter-cellular layout based on SA algorithm

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55,90]	[0,25]
Grinding	42	10	[29,55]	[0,20]
Diamond	15	50	[0,30]	[40,60]
Final	45	50	[30,60]	[40,60]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $6,167.600		

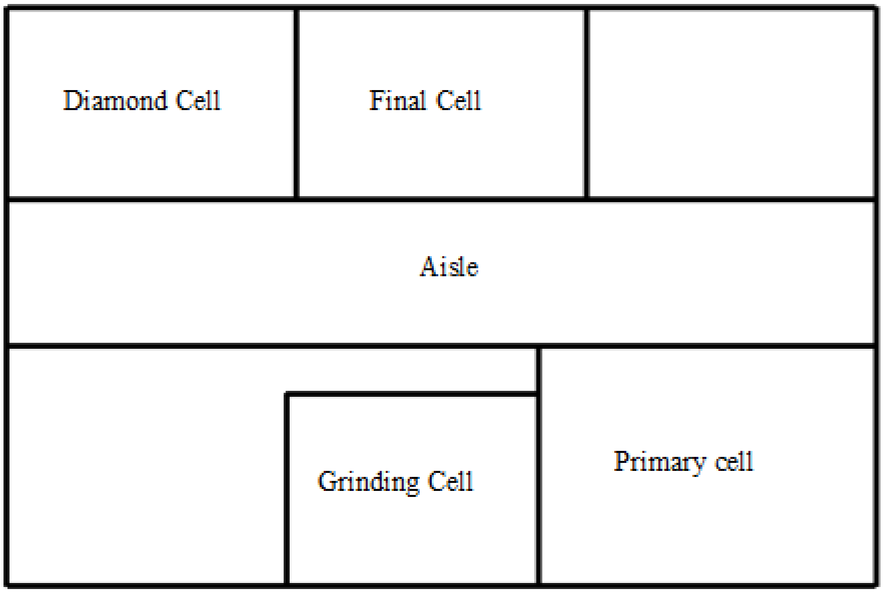


Figure (74): Inter-cell layout by using SA algorithm

5.2.3. Validation of the SA

In order to validate and prove the efficiency of the developed simulated annealing algorithm, the developed SA has been applied 10 runs for each of cells.

- **Primary Cell**

The summary of the solutions for Primary cell layout design are represented in table (42) to (51).

Table (42): 1st run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	20.501	17.041	[17.501, 23.501]	[12.506, 21.576]
Double Disc	23.835	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.25	17.72	[3, 11.5]	[14.67, 20.77]
Wendt (2)	21.75	4.45	[17.5, 26]	[1.4, 7.5]
Wendt (3)	8.099	9.937	[4.699, 11.499]	[5.212, 14.662]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 681.674				

Table (43): 2nd run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.500	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	20.501	17.036	[17.501, 23.501]	[12.501, 21.571]
Double Disc	23.835	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.250	17.72	[3, 11.5]	[14.67, 20.77]
Wendt (2)	22.895	4.45	[18.644, 27.145]	[1.4, 7.5]
Wendt (3)	8.092	8.909	[4.692, 11.492]	[4.184, 13.634]
Polish	14.500	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 698.815				

Table (44): 3rd run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	20.506	17.055	[17.506, 23.506]]	[12.52, 21.59]
Double Disc	23.83	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.25	17.72	[3, 11.5]	[14.67, 20.77]
Wendt (2)	23.44	4.45	[19.19, 27.6]	[1.4, 7.5]
Wendt (3)	8.1	9.945	[4.7, 11.5	[5.22, 14.67]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 690.530				

Table (45): 4th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	21.27	17.16	[18.27, 24.27]	[12.625, 21.695]
Double Disc	23.83	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.25	17.72	[3, 11.5]	[14.67, 20.77]
Wendt (2)	23.34	4.45	[19.09, 27.59]	[1.4, 7.5]
Wendt (3)	8.099	9.944	[4.699, 11.499]	[5.219, 14.669]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 693.485				

Table (46): 5th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	20.501	17.035	[17.501, 23.501]	[12.5, 21.57]
Double Disc	23.83	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.249	17.294	[2.999, 11.499]	[14.244, 20.344]
Wendt (2)	21.75	4.45	[17.5, 26]	[1.4, 7.5]
Wendt (3)	8.092	9.182	[4.692, 11.492]	[4.457, 13.907]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 687.327				

Table (47): 6th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.49	13.08	[11.49, 17.49]	[8.548, 17.618]
Blanchard(2)	20.49	17.04	[17.49, 23.49]	[12.5, 21.57]
Double Disc	23.83	10	[17.49, 30.16]	[7.5, 12.5]
Wendt (1)	7.249	15.42	[2.99, 11.49]	[12.365, 18.465]
Wendt (2)	21.74	4.45	[17.49, 25.99]	[1.401, 7.501]
Wendt (3)	8.09	7.64	[4.69, 11.49]	[2.915, 12.365]
Polish	14.49	6.05	[11.49, 17.49]	[3.547, 8.547]
MHC: $ 661.647				

Table (48): 7th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	15.935	[11.5, 17.5]	[11.4, 20.47]
Blanchard(2)	20.501	17.036	[17.501, 23.501]	[12.501, 21.571]
Double Disc	23.83	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.25	15.933	[3, 11.5]	[12.883, 18.983]
Wendt (2)	22.054	4.45	[17.804, 26.304]	[1.4, 7.5]
Wendt (3)	7.936	7.356	[4.536, 11.336]	[2.631, 12.081]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 683.297				

Table (49): 8th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	15.935	[11.5, 17.5]	[11.4, 20.47]
Blanchard(2)	20.501	17.036	[17.501, 23.501]	[12.501, 21.571]
Double Disc	23.83	10	[17.5, 30.17]	[7.5, 12.5]
Wendt (1)	7.25	15.933	[3, 11.5]	[12.883, 18.983]
Wendt (2)	22.054	4.45	[17.804, 26.304]	[1.4, 7.5]
Wendt (3)	7.936	7.356	[4.536, 11.336]	[2.631, 12.081]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 683.297				

Table (50): 9th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	20.51	17.035	[17.51, 23.51]	[12.5, 21.57]
Blanchard(2)	14.51	17.057	[11.51, 17.51]	[12.522, 21.592]
Double Disc	23.80	10	[17.47, 30.14]	[7.5, 12.5]
Wendt (1)	7.26	17.258	[3.01, 11.51]	[14.208, 20.308]
Wendt (2)	21.72	4.450	[17.47, 25.97]	[1.4, 7.5]
Wendt (3)	8.07	9.483	[4.67, 11.47]	[4.758, 14.208]
Polish	14.47	6.215	[11.47, 17.47]	[3.715, 8.715]
MHC: $ 668.901				

Table (51): 10th run of SA algorithm for Primary Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Blanchard (1)	14.5	18.3	[11.5, 17.5]	[13.765, 22.835]
Blanchard(2)	20.507	17.142	[17.507, 23.507]	[12.607, 21.677]
Double Disc	23.83	10	[17.495, 30.165]	[7.5, 12.5]
Wendt (1)	7.249	16.781	[2.999, 11.499]	[13.731, 19.831]
Wendt (2)	21.769	4.451	[17.519, 26.019]	[1.401, 7.501]
Wendt (3)	8.1	9.005	[4.7, 11.5]	[4.28, 13.73]
Polish	14.5	5.98	[11.5, 17.5]	[3.48, 8.48]
MHC: $ 689.611				

- **Grinding Cell**

The summary of the solutions for grinding cell layout design are represented in table (52) to (61).

Table (52): 1rd run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	9.79	4	[6.29, 13.29]	[1, 7]
Surface Grinding (2)	18.47	16	[14.97, 21.97]	[13, 19]
Swing Fixture (1)	18.97	10	[14.97, 22.97]	[7, 13]
Swing Fixture (2)	10.97	12.76	[6.97, 14.97]	[9.76, 15.76]
V-Bottom	16.79	4	[13.29, 20.29]	[1, 7]
MHC: $ 495.465				

Table (53): 2th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	8.55	4.001	[5.05, 12.05]	[1.001, 7.001]
Surface Grinding (2)	23.741	9.722	[20.241, 27.241]	[6.722, 12.722]
Swing Fixture (1)	16.241	10	[12.241, 20.241]	[7, 13]
Swing Fixture (2)	8.241	10.001	[4.241, 12.241]	[7.001, 13.001]
V-Bottom	15.551	4	[12.051, 19.051]	[1, 7]
MHC: $ 491.189				

Table (54): 3th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	8.55	4	[5.05, 12.05]	[1, 7]
Surface Grinding (2)	16.322	16	[12.822, 19.822]	[13, 19]
Swing Fixture (1)	16.822	10	[12.822, 20.822]	[7, 13]
Swing Fixture (2)	8.822	12.76	[4.822, 12.822]	[9.76, 15.76]
V-Bottom	15.55	4	[12.05, 19.05]	[1, 7]
MHC: $ 478.387				

Table (55): 4th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	9.63	4	[6.13, 13.13]	[1, 7]
Surface Grinding (2)	18.48	16	[14.98, 21.98]	[13, 19]
Swing Fixture (1)	16.69	10	[12.69, 20.69]	[7, 13]
Swing Fixture (2)	8.69	10	[4.69, 12.69]	[7, 13]
V-Bottom	16.69	4	[13.12, 20.12]	[1, 7]
MHC: $ 490.829				

Table (56): 5th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	8.552	4.001	[5.052, 12.052]	[1.001, 7.001]
Surface Grinding (2)	15.992	16.001	[12.492, 19.492]	[13.001, 19.001]
Swing Fixture (1)	16.492	10.001	[12.492, 20.492]	[7.001, 13.001]
Swing Fixture (2)	8.492	10.001	[4.492, 12.492]	[7.001, 13.001]
V-Bottom	15.552	4.001	[12.052, 19.052]	[1.001, 7.001]
MHC: $ 460.381				

Table (57): 6th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	9.634	4	[6.134, 13.134]	[1, 7]
Surface Grinding (2)	16.634	16	[13.134, 20.134]	[13, 19]
Swing Fixture (1)	7.078	10	[3.078, 11.078]	[7, 13]
Swing Fixture (2)	15.129	10	[11.129, 19.129]	[7, 13]
V-Bottom	16.634	4	[13.134, 20.134]	[1, 7]
MHC: $ 486.863				

Table (58): 7th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	16.79	4	[13.29, 20.29]	[1, 7]
Surface Grinding (2)	13.251	16	[9.751, 16.751]	[13, 19]
Swing Fixture (1)	17.29	10	[13.29, 21.29]	[7, 13]
Swing Fixture (2)	5.751	15.261	[1.751, 9.751]	[12.261, 18.261]
V-Bottom	9.79	9.261	[6.29, 13.29]	[6.261, 12.261]
MHC: $ 514.697				

Table (59): 8th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	15.767	4	[12.267, 19.267]	[1, 7]
Surface Grinding (2)	13.22	16	[9.72, 16.72]	[13, 19]
Swing Fixture (1)	16.267	10	[12.267, 20.267]	[7, 13]
Swing Fixture (2)	5.72	15.26	[1.72, 9.72]	[12.26, 18.26]
V-Bottom	8.767	9.26	[5.267, 12.267]	[6.26, 12.26]
MHC: $ 508.014				

Table (60): 9th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	9.79	4	[6.29, 13.29]	[1, 7]
Surface Grinding (2)	18.476	16	[14.976, 21.976]	[13, 19]
Swing Fixture (1)	18.97	10	[14.97, 22.97]	[7, 13]
Swing Fixture (2)	10.97	12.501	[6.97, 14.97]	[9.501, 15.501]
V-Bottom	16.79	4	[13.29, 20.29]	[1, 7]
MHC: $ 492.283				

Table (61): 10th run of SA algorithm for Grinding Cell

Machine Tool	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Surface Grinding (1)	8.55	4	[5.05, 12.05]	[1, 7]
Surface Grinding (2)	18.474	16	[14.974, 21.974]	[13, 19]
Swing Fixture (1)	10.969	10	[6.969, 14.969]	[7, 13]
Swing Fixture (2)	18.969	10	[14.969, 22.969]	[7, 13]
V-Bottom	15.55	4	[12.05, 19.05]	[1, 7]
MHC: $ 506.264				

- **Diamond Cell**

  Table (62) to (71) shows the summary of the solutions for Diamond cell layout design.

Table (62): 1st run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	11.03	3.8	[7.13,14.93]	[0.45,7.15]
Wire Cutting (2)	5.5	10	[1.8,9.2]	[7.15,12.85]
Surface Grinding	17.93	5.62	[14.93,20.93]	[2.12,9.12]
Brazing	22.93	8.33	[20.93,24.93]	[7.43,9.23]
Ewag	11.53	16.30	[9.38,13.68]	[12.65,19.95]
Laser M/c	17.48	15.12	[13.68,21.28]	[10.25,19.99]
MHC: $787.940				

Table (63): 2nd run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	10.39	3.80	[6.49, 14.29]	[0.45, 7.15]
Wire Cutting (2)	5.5	10	[1.80, 9.20]	[7.15, 12.85]
Surface Grinding	17.29	5.468	[14.29, 20.29]	[1.968, 8.968]
Brazing	22.29	8.069	[20.29, 24.29]	[7.169, 8.9697]
Ewag	11.53	16.31	[9.38, 13.68]	[12.66, 19.96]
Laser M/c	17.48	15.13	[13.68, 21.28]	[10.261,20]
MHC: $740				

Table (64): 3rd run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	11.10	3.8	[7.20,15]	[0.45,7.15]
Wire Cutting (2)	5.50	10	[1.8,9.20]	[7.15,12.85]
Surface Grinding	18	5.64	[15,21]	[2.14,9.14]
Brazing	24.60	10.949	[22.6,26.6]	[10.04,11.846]
Ewag	11.53	16.31	[9.38,13.68]	[12.66,19.96]
Laser M/c	18.8	14.86	[15,22.6]	[9.999,19.739]
MHC: $780				

Table (65): 4th run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	11.10	3.8	[7.20,15]	[0.45,7.15]
Wire Cutting (2)	5.50	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	18	5.64	[15,21]	[2.14,9.14]
Brazing	24.6	10.946	[22.6,26.6]	[10.046,11.846]
Ewag	11.53	16.31	[9.38,13.68]	[12.66,19.96]
Laser M/c	18.80	14.87	[15,22.60]	[10,19.74]
MHC: $780,000				

Table (66): 5th run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	10.39	3.80	[6.49, 14.29]	[0.45,7.15]
Wire Cutting (2)	5.5	10	[1.80, 9.20]	[7.15,12.85]
Surface Grinding	17.29	5.468	[14.29, 20.29]	[1.968, 8.968]
Brazing	22.29	8.07	[20.29, 24.29	[7.17, 8.97]
Ewag	11.53	16.31	[9.38, 13.68]	[12.66, 19.96]
Laser M/c	17.48	15.13	[13.68, 21.28]	[10.26,20]
MHC: $740				

Table (67): 6th run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	11.07	3.964	[7.17, 14.971]	[0.61,7.31
Wire Cutting (2)	31.70	10	[28, 35.40]	[7.15,12.85]
Surface Grinding	17.79	5.63	[14.97,20.97]	[2.13,9.13]
Brazing	26	10	[24,28]	[9.10,10.90]
Ewag	12.85	16.04	[10.70,15]	[12.39,19.69]
Laser M/c	18.80	14.87	[15,22.60]	[10,19.74]
MHC: $790				

Table (68): 7th run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	10.39	3.8	[6.49,14.29]	[0.45,7.15]
Wire Cutting (2)	5.50	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	17.29	5.468	[14.29,20.29]	[1.968,8.99]
Brazing	22.29	8.06	[20.29,24.29]	[7.169,9]
Ewag	11.53	16.31	[9.38,13.68]	[12.66,19.96]
Laser M/c	17.48	15.13	[13.68,21.28]	[10.26,20]
MHC: $739.961				

Table (69): 8th run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	10.39	3.8	[6.49,14.29]	[0.45,7.15]
Wire Cutting (2)	5.5	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	17.29	5.468	[14.29,20.29]	[1.968,8.98]
Brazing	22.29	8.06	[20.29,24.29]	[7.169,9.96]
Ewag	11.53	16.31	[9.38,13.68]	[12.66,19.96]
Laser M/c	17.48	15.13	[13.68,21.28]	[10.26,20]
MHC: $740				

Table (70): 9th run of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	10.39	3.8	[6.49,14.29]	[0.45,7.15]
Wire Cutting (2)	5.5	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	17.29	5.468	[14.29,20.29]	[1.968,8.96]
Brazing	23.28	11.67	[21.28,25.28]	[10.77,12.57]
Ewag	11.53	16.31	[9.38,13.68]	[12.66,19.96]
Laser M/c	17.48	15.13	[13.68,21.28]	[10.26,20]
MHC: $760				

Table (71): 11trun of SA algorithm for Diamond Cell

Diamond Cell	**Centroid**		**Dimension**	
	X	**Y**	**Horizontal**	**Vertical**
Wire Cutting (1)	10.48	3.8	[6.58,14.38]	[0.45,7.15]
Wire Cutting (2)	5.5	10	[1.80,9.20]	[7.15,12.85]
Surface Grinding	17.38	5.49	[14.38,20.38]	[1.99,9]
Brazing	23.28	8.10	[20.38,24.38]	[7.20,19.96]
Ewag	11.53	16.31	[9.38,13.68]	[12.66,19.96]
Laser M/c	17.48	15.13	[13.68,21.28]	[10.26,20]
MHC: $740				

- **Final Cell**

  The summary of the solutions for Final cell layout design are represented in table (72) to (81).

Table (72): 1st run of SA algorithm for Final Cell

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	13.391	10.999	[11.891, 14.891]	[8.999, 12.999]
Wash	9.391	10.5	[6.891, 11.891]	[9, 12]
Inspection	4.891	9	[2.891, 6.891]	[7.5, 10.5]
Pack and Shipment	14.891	4.999	[6.891, 22.891]	[0.999, 8.999]
MHC: $ 930.368				

Table (73): 2nd run of SA algorithm for Final Cell

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	18.5	11.703	[17.001, 20.001]	[9.703, 13.703]
Wash	14.5	14.205	[12.001, 17.001]	[12.705, 15.705]
Inspection	15	11.205	[13.001, 17.001]	[9.705, 12.705]
Pack and Shipment	15	5.703	[7, 23]	[1.703, 9.703]
MHC: $ 885.196				

Table (74): 3rd run of SA algorithm for Final Cell

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	19.552	12.521	[18.052, 21.052]	[10.521, 14.521]
Wash	15.552	12.194	[13.052, 18.052]	[10.694, 13.694]
Inspection	15.966	15.194	[13.966, 17.966]	[13.694, 16.694]
Pack and Shipment	15.553	6.521	[7.553, 23.553]	[2.521,10.521]
MHC: $ 960.729				

Table (75): 4th run of SA algorithm for Final Cell

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	26.185	9.345	[24.685, 27.685]	[7.345, 11.345]
Wash	22.185	10.5	[19.685, 24.685]	[9, 12]
Inspection	16.720	10.5	[14.720, 18.720]	[9, 12]
Pack and Shipment	16.685	5	[8.685, 24.685]	[1, 9]
MHC: $ 922.512				

Table (76): 5th run of SA algorithm for Final Cell

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
ETCH	19	11.197	[17.5, 20.5]	[9.197, 13.197]
Wash	15	13.697	[12.5, 17.5]	[12.197, 15.197]
Inspection	15.250	10.697	[13.25, 17.25]	[9.197, 12.197]
Pack and Shipment	15.001	5.197	[7.001, 23.001]	[1.197, 9.197]
MHC: $ 884.798				

Table (77): 6th run of SA algorithm for Final Cell

Machine Tool/ Station	Centroid		Dimension	
	X	*Y*	Length	Width
ETCH	19	11.197	[17.5, 20.5]	[9.197, 13.197]
Wash	15	13.697	[12.5, 17.5]	[12.197, 15.197]
Inspection	15.229	10.697	[13.229, 17.229]	[9.197, 12.197]
Pack and Shipment	15.061	5.197	[7.061, 23.061]	[1.197, 9.197]
MHC: $ 879.006				

Table (78): 7th run of SA algorithm for Final Cell

Machine Tool/ Station	Centroid		Dimension	
	X	*Y*	Length	Width
ETCH	19.03	11	[17.53, 20.53]	[9,13]
Wash	15.03	10.5	[12.53, 17.53]	[9, 12]
Inspection	15.128	13.5	[13.128, 17.128]	[12, 15]
Pack and Shipment	15.003	5	[7.003, 23.003]	[1, 9]
MHC: $ 926.465				

Table (79): 8th run of SA algorithm for Final Cell

Machine Tool/ Station	Centroid		Dimension	
	X	*Y*	Length	Width
ETCH	4.812	12.917	[3.312, 6.312]	[10.917, 14.917]
Wash	8.956	13.082	[6.456, 11.456]	[11.582, 14.582]
Inspection	13.456	13.082	[11.456, 15.456]	[11.582, 14.582]
Pack and Shipment	14.312	7.582	[6.312, 22.312]	[3.582, 11.582]
MHC: $ 865.914				

Table (80): 9th run of SA algorithm for Final Cell

Machine Tool/ Station	Centroid		Dimension	
	X	*Y*	Length	Width
ETCH	18.979	28.672	[17.479, 20.479]	[26.672, 30.672]
Wash	14.979	31.172	[12.479, 17.479]	[29.672, 32.672]
Inspection	15.026	28.172	[13.026, 17.026]	[26.672, 29.672]
Pack and Shipment	14.979	22.672	[6.979, 22.979]	[18.672, 26.672]
MHC: $ 861.831				

Table (81): 10th run of SA algorithm for Final Cell

Machine Tool/ Station	Centroid		Dimension	
	X	*Y*	Length	Width
ETCH	14.057	11.236	[12.557, 15.557]	[9.236, 13.236]
Wash	15.057	10.941	[12.557, 17.557]	[9.441, 12.441]
Inspection	10.557	10.736	[8.557, 12.557]	[9.236, 12.236]
Pack and Shipment	14.986	5.236	[6.986, 22.986]	[1.236, 9.236]
MHC: $ 909.336				

- **Inter-Cell**

Table (82): 1st run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.43	12.5	[54.93, 89.93]	[0,25]
Grinding	15.6	14.58	[2.6, 28.6]	[4.58, 24.58]
Diamond	63	50	[48,78]	[40, 60]
Final	15.6	50	[0.6,30.6]	[39.03, 59.03]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5,945.76		

Table (83): 2nd run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.43	12.5	[54.93, 89.93]	[0, 25]
Grinding	63	50	[50, 76]	[40, 60]
Diamond	39.93	13.57	[24.93, 54.93]	[3.57, 23.57]
Final	63.98	70	[48.98, 78.98]	[60, 80]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5,577.62		

Table (84): 3rd run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	31.79	10	[18.79, 44.79]	[0, 20]
Diamond	63	50	[48, 78]	[40, 60]
Final	30.77	30	[15.77, 45.77]	[20, 40]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5,032.35		

Table (85): 4th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	67.28	12	[49.78, 84.78]	[0, 24]
Grinding	63	50	[50, 76]	[40, 60]
Diamond	34.78	10.17	[19.78, 49.78]	[0.17, 20.17]
Final	15	50	[0, 30]	[40, 60]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5798.61		

Table (86): 5th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	63	50	[50, 76]	[40, 60]
Diamond	39.99	10.50	[24.99, 54.99]	[0.50, 20.50]
Final	15	50	[0, 30]	[40, 60]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $6,105.83		

Table (87): 6th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	63	50	[50, 76]	[40, 60]
Diamond	31.79	10	[16.79, 46.79]	[0, 20]
Final	30.63	30	[15.63, 45.63]	[20, 40]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5,289.68		

Table (88): 7th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	40	14.8	[27, 53]	[4, 24]
Diamond	63	50	[48, 78]	[40, 60]
Final	15	50	[0, 30]	[40, 60]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5,994.18		

Table (89): 8th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	31.79	10	[18.79, 44.79]	[0, 20]
Diamond	63	50	[48, 78]	[40, 60]
Final	31.05	30	[16.05, 46.05]	[20, 40]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5,014.99		

Table (90): 9th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	31.79	10	[18.79, 44.79]	[0, 20]
Diamond	63	50	[48, 78]	[40, 60]
Final	31.50	30	[16.50, 46.50]	[20, 40]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $4,986.35		

Table (91): 10th run of SA algorithm for Inter-cellular layout

Machine Tool/ Station	**Centroid**		**Dimension**	
	X	***Y***	**Length**	**Width**
Primary	72.5	12.5	[55, 90]	[0, 25]
Grinding	31.79	10	[18.79, 44.79]	[0, 20]
Diamond	63	50	[48, 78]	[40, 60]
Final	19.75	30	[12.50, 27]	[20, 40]
Aisle	45	32.5	90	15
Shop Size: 90×60		**MHC:** $5237.55		

5.3. Discussion

The comparison between the solution provided nonlinear, linear model and simulated annealing represented in the table (82). Linear model gives the exact optimum solution, however simulates annealing provides near optimum solution. The results also prove this fact. In both leader and follower problem; *i.e.*, intra- and inter-cell respectively, the total material handling cost is less than costs provided by nonlinear mixed integer programming and simulated annealing.

Table (92): Comparisons between mathematical modeling and simulate annealing

	Leader Problem				**Follower problem**
Method	*Primary Cell*	*Grinding Cell*	*Diamond Cell*	*Final Cell*	*Shop*
NLMIP	$ 1,191.550	$520.588	$764.580	$1,056.350	$7,520.420
LMIP	$503.024	$399.750	$360.800	$685.200	$2,838.6
SA	$701.592	$526.004	$787.940	$856.508	$6,167.6

The follower problem solved by simulated annealing has just assumed aisle.

Generally speaking the linearized model obviously has yielded exact optimal results which proved to be better than those obtained by both simulated annealing and the original nonlinear model. This was quite expected; in most cases simulated annealing resulted in better solutions than the nonlinear model, however there were cases where the nonlinear model results was slightly better than those obtained by simulated annealing. The exception was for grinding cell and diamond cell where the nonlinear model outperformed slightly than simulated annealing.

Table (93): Mean and standard deviation of SA solutions

Cell	**Average**	**SDV**
Primary	$633.86	$11.19
Grinding	$492.44	$15.63
Diamond	$759.790	$22.315
Final	$902.62	$32.23
Inter-Cell	$5,474.61	$423.97

Table (93) summarizes the results from both leader and follower problems. Both mean and SDV from the performed 10 runs are being provided. Standard deviation is good except for inter-cell layout problem. For inter-cell we believe the algorithm is yet to be improved, variance as indicated by table (93) is relatively high.

CHAPTER SIX:
CONCLUSION AND FUTURE WORK

Cellular manufacturing system (CMS) layout has recently begun to receive heightened attention worldwide. The design of a CMS includes (1) cell formation (CF), (2) group layout, (3) group, and (4) resource allocation . An effective CMS implementation help any company improve machine utilization and quality; it also makes reduction in setup time, work-in-process inventory, material handling cost, part makespan, and expediting costs .

There are two main approaches to FLP such as discrete and continuous approach. Discrete approach holds two main assumptions: one is all facilities are equal size and shape; the other one is predetermined locations of facilities. However, these kinds of assumptions are not realistic. Discrete approach is not suited to represent the exact locations of facilities. Moreover, this approach is not applicable for FLP with unequal size and shape facilities. The appropriate approach to this kind of FLP is continuous representation.

Generally speaking, the design of layout cannot be efficient if manufacturing attributes are not being considered init. To illustrate, operations sequencing and parts' demand are the two factors which have significant impacts on the flow rate which minimizing that is the main objective of FLP. The majority of literatures have not considered these factors in the design of layout plan. Besides those manufacturing attributes, the available area of the shop that can be used for locating facilities are the other factor that has to be considered.

The facility layout problem for cellular manufacturing system in both inter and intra cellular levels is considered in this thesis. The problem is to arrange facilities that are cells in the leader problem and machine tools in the follower problem in the continual planar site. Operation sequence and parts' demand are the two main manufacturing attributes considered in the developed model. The MIP has been presented for both leader and follower problem. The novel aisle constraints have been presented in the mathematical formulation. Since the model is nonlinear, the linearized model has been developed. Additionally, a novel mathematical modelling has been developed for considering block constraints such as fixed departments and facilities. Since the FLP is a NP-hard problem novel heuristics presented in this thesis.

A novel heuristic model developed for finding feasible initial solution for designed metaheuristic algorithm, simulated annealing. The initial solution is based on the radial movement. In other words the algorithm placed facilities along specific radius with certain angle within site. The algorithm starts with dividing site into 4 equal sized quadrants, start placing facilities into first quadrant to the fourth one. After placing any new facility, the overlap's possibility between facilities and between facility and site boundaries is being checked. The different repair functions have been designed for different cases.

The SA algorithm developed for both inter and intra cellular problem. The results of heuristic have used to initialize the developed SA algorithm. However, in order to have more efficient SA, the cell size used in heuristic algorithm is assumed two times of the original size of the cells. The two main operators used are move and swap operator. Move operator decrease distance between facilities by moving the target facility towards the closest facility to it. Furthermore, the swap operator developed by defining the concept of the free zone.

As the future work, the improved heuristic as well as improved metheuristic are under consideration. Moreover, applying other manufacturing attributes like as machine relocation cost, setup cost and so on would be a potential field of study. Based on the literature there are very few work tried to make continuous one dynamic; *i.e.,* have more multiple time period and design different layout plan for each of period. Therefore, making continuous problem dynamic will be another potential field.

References

ALFA, A. S., CHEN, M. & HERAGU, S. S. 1992. Integrating the grouping and layout problems in cellular manufacturing systems. *Computers & Industrial Engineering,* 23**,** 55-58.

ARIAFAR, S., ISMAIL, N., TANG, S. H., ARIFFIN, M. K. A. M. & FIROOZI, Z. 2011. Design of a facility layout model in hybrid cellular manufacturing systems under variable demand. *International Journal of Industrial and Systems Engineering,* 9**,** 373-387.

ARIAFAR, S., ISMAIL, N., TANG, S. H., ARIFFIN, M. K. A. M. & FIROOZI, Z. 2012. The reconfiguration issue of stochastic facility layout design in cellular manufacturing systems. *International Journal of Services and Operations Management,* 11**,** 255-266.

ARKAT, J., FARAHANI, M. H. & AHMADIZAR, F. 2012a. Multi-objective genetic algorithm for cell formation problem considering cellular layout and operations scheduling. *International Journal of Computer Integrated Manufacturing,* 25**,** 625-635.

ARKAT, J., HOSSEINABADI FARAHANI, M. & HOSSEINI, L. 2012b. Integrating cell formation with cellular layout and operations scheduling. *International Journal of Advanced Manufacturing Technology,* 61**,** 637-47.

BALAKRISHNAN, J. & CHENG, C. H. 2000. Genetic search and the dynamic layout problem. *Computers & Operations Research,* 27**,** 587-593.

BAYKASOĞLU, A. & GINDY, N. N. Z. 2001. A simulated annealing algorithm for dynamic layout problem. *Computers & Operations Research,* 28**,** 1403-1426.

BAZARGAN-LARI, M. 1999. Layout designs in cellular manufacturing. *European Journal of Operational Research,* 112**,** 258-272.

BAZARGAN-LARI, M. & KAEBERNICK, H. 1997. An approachto the machine layout problem in a cellular manufacturing environment. *Production Planning & Control,* 8**,** 41-55.

BAZARGAN-LARI, M., KAEBERNICK, H. & HARRAF, A. 2000. Cell formation and layout designs in a cellular manufacturing environment - a case study. *International Journal of Production Research,* 38**,** 1689-1709.

CONWAY, D. G. & VENKATARAMANAN, M. A. 1994. Genetic search and the dynamic facility layout problem. *Computers & Operations Research,* 21**,** 955-960.

DEFERSHA, F. M. & CHEN, M. 2006. A comprehensive mathematical model for the design of cellular manufacturing systems. *International Journal of Production Economics,* 103**,** 767-783.

DRIRA, A., PIERREVAL, H. & HAJRI-GABOUJ, S. 2007. Facility layout problems: a survey. *Annual Reviews in Control,* 31**,** 255-267.

GREEN, T. J. & SADOWSKI, R. P. 1984. A review of cellular manufacturing assumptions, advantages and desing techniques. *Journal of Operations Management*, 4, 85-97.

GOU, L. 2012. Two-stage design for cellular manufacturing system. *Applied mechanics and materials,* 220**,** 137-140.

HERAGU, S. S. & KUSIAK, A. 1991. Efficient models for the facility layout problem. *European Journal of Operational Research,* 53**,** 1-13.

IMAM, M. H. & MIR, M. 1993. Automated layout of facilities of unequal areas. *Computers & Industrial Engineering,* 24**,** 355-366.

KIA, R., BABOLI, A., JAVADIAN, N., TAVAKKOLI-MOGHADDAM, R., KAZEMI, M. & KHORRAMI, J. 2012. Solving a group layout design model of a dynamic cellular manufacturing system with alternative process routings, lot splitting and flexible reconfiguration by simulated annealing. *Computers & Operations Research,* 39**,** 2642-2658.

KAEBERNICK, H., BAZARGAN-LARI, M. & ARNDT, G. 1996. An Integrated Approach to the Design of Cellular Manufacturing. *CIRP Annals - Manufacturing Technology,* 45**,** 421-425.

MELLER, R. D. & Gau, K-Y. 1996. The facility layout problem: recent and emerging trends and perspectives. *Journal of Manufacturing Systems*, 15**,** 351-66.

MELLER, R. D., NARAYANAN, V. & PAMELA. H. V. 1999. Optimal facility layout design. *Operations Research Letters*, 23, 117-127.

MIR, M. & IMAM, M. H. 2001. A hybrid optimization approach for layout design of unequal-area facilities. *Computers & Industrial Engineering,* 39**,** 49-63.

MONTREUIL, B. 1990. A Modelling Framework for Integrating Layout Design and flow Network Design. *Material Handling '90.* Springer Berlin Heidelberg.

Press, W., Teukolsky, S. A., Vetterling, W. T. & Flanner, B. P. 2007. *Numerical Recipes The Art of Sientific Computing*. 549- 555

ROSENBLATT, M. J. 1986. The dynamics of plant layout. *Management. Science.,* 32**,** 76-86.

SAFAEI, N., SAIDI-MEHRABAD, M. & JABAL-AMELI, M. S. 2008. A hybrid simulated annealing for solving an extended model of dynamic cellular manufacturing system. *European Journal of Operational Research,* 185**,** 563-592.

TAVAKKOLI-MOGHADDAM, R., ARYANEZHAD, M. B., SAFAEI, N. & AZARON, A. 2005. Solving a dynamic cell formation problem using metaheuristics. *Applied Mathematics and Computation*, 170, 761-780.

TAVAKKOLI-MOGHADDAM, R., JAVADIAN, N., JAVADI, B. & SAFAEI, N. 2007. Design of a facility layout problem in cellular manufacturing systems with stochastic demands. *Applied Mathematics and Computation,* 184**,** 721-728.

TIAN, Z., LI, H. & ZHAO, Y. Optimization of continuous dynamic facility layout problem with budget constraints. 2nd International Conference on Information Science and Engineering, ICISE 2010, Hangzhou, China. IEEE Computer Society, 387-390.

WEMMERLOV, U. & JOHNSON, D. J. 1997. Cellular manufacturing at 46 user plants: implementation experiences and performance improvements. *International Journal of Production Research,* 35**,** 29-49.

WILHELM, M. R. & WARD, T. L. 1987. Solving quadratic assignment problems by simulated annealing. *IIE Transactions,* 107-119.

WU, X., CHU, C.-H., WANG, Y. & DIANMIN, Y. 2007b. Genetic algorithm for integrating cell formation with machine layout and scheduling. *Computers and Industrial Engineering,* 53**,** 277-289.

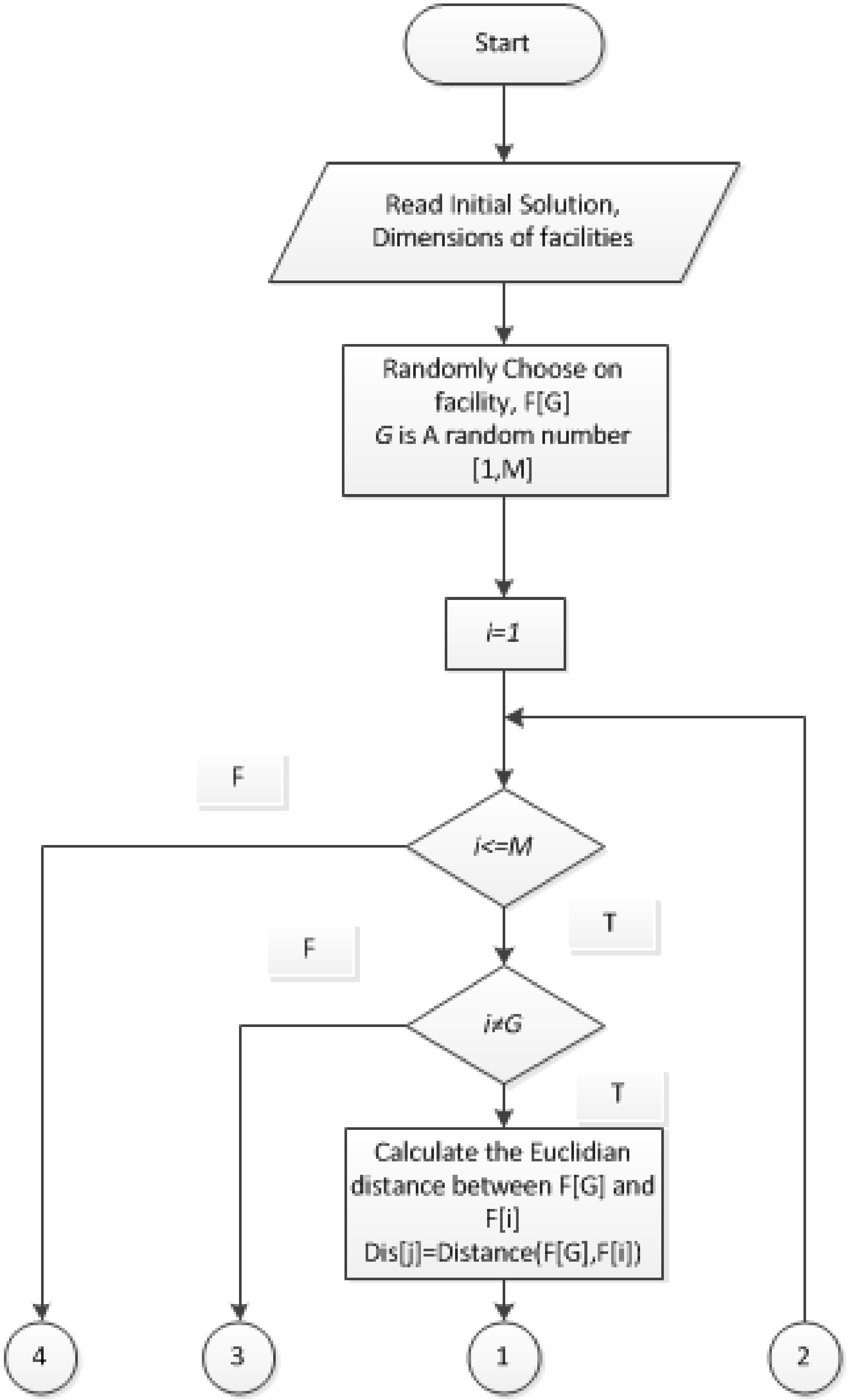
WU, X., CHU, C.-H., WANG, Y. & YAN, W. 2007a. A genetic algorithm for cellular manufacturing design and layout. *European Journal of Operational Research,* 181**,** 156-167.

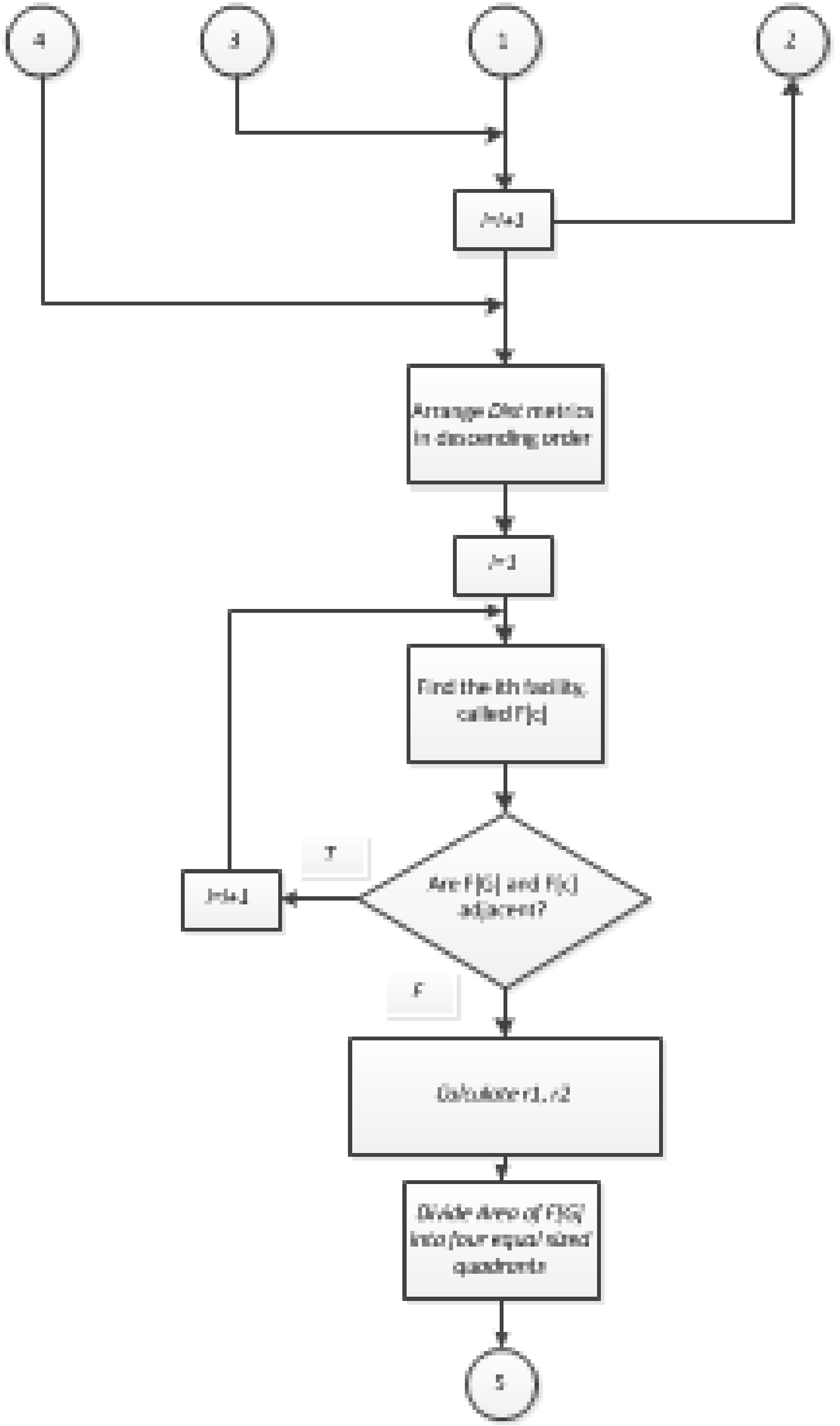
WU, X., CHU, C. H., WANG, Y. & YAN, W. 2006. Concurrent design of cellular manufacturing systems: A genetic algorithm approach. *International Journal of Production Research,* 44**,** 1217-1241.

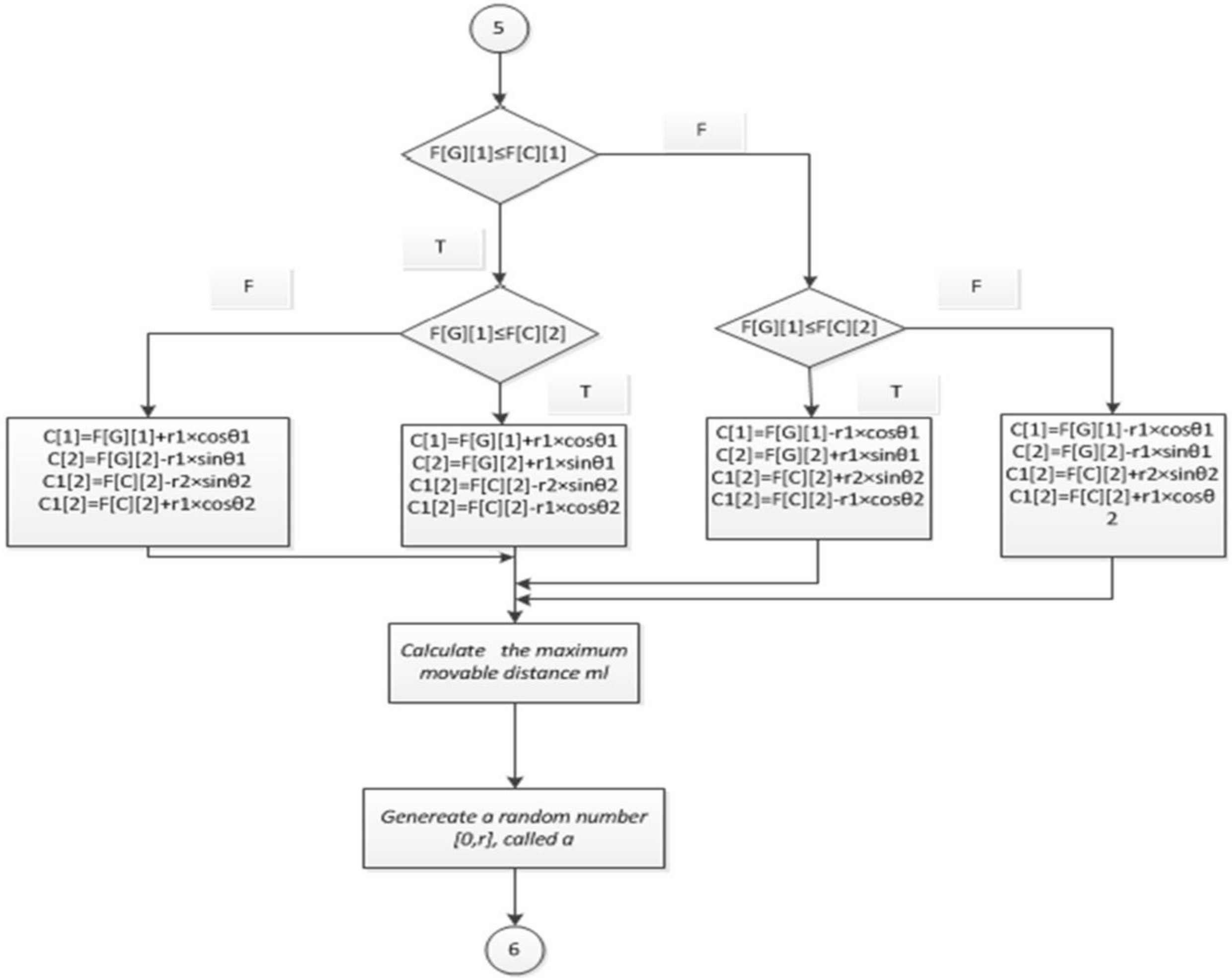
YAMAN, R., GETHIN, D. T. & CLARKE, M. J. 1993. An effective sorting method for facility layout construction. *International Journal of Production Research,* 31**,** 413-27.

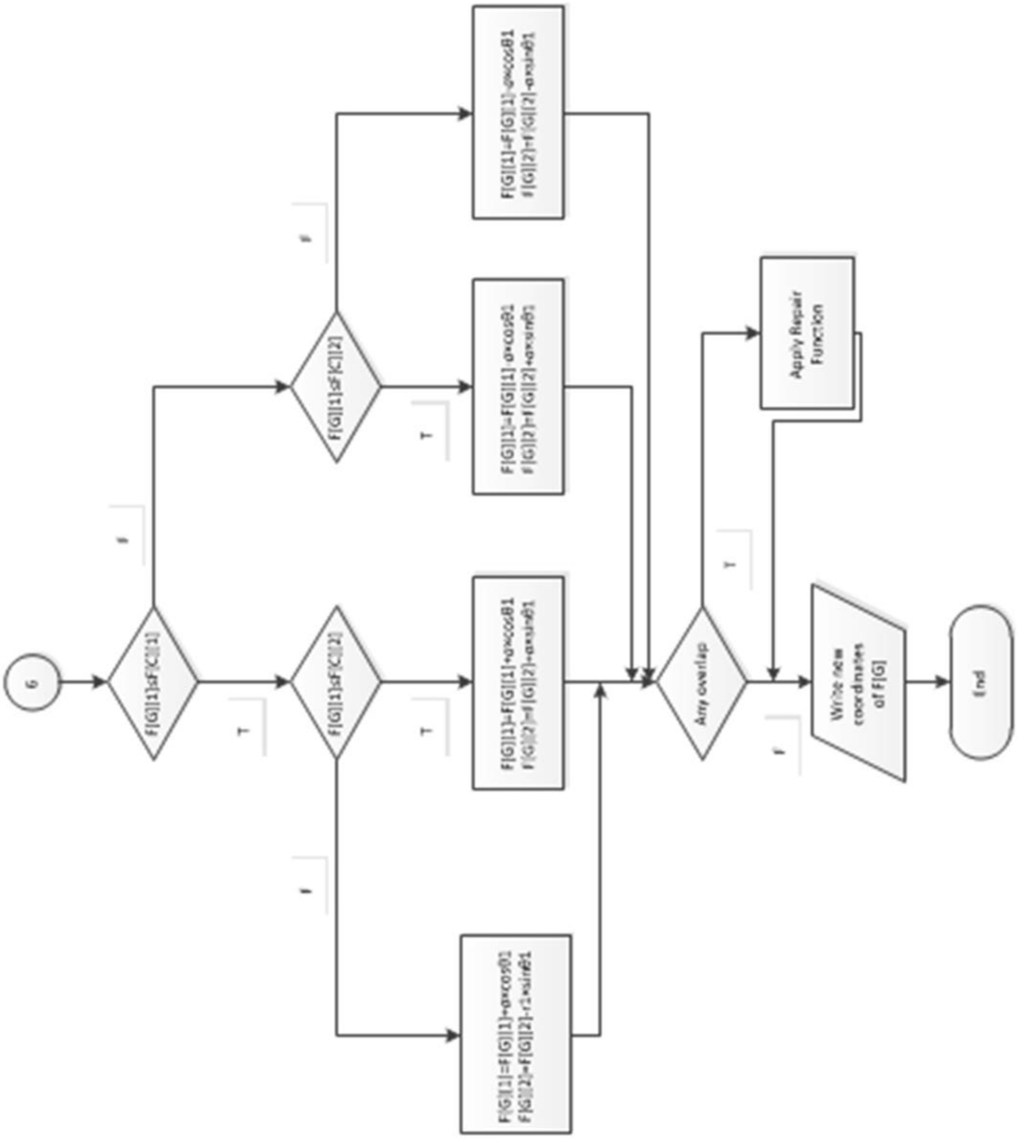
APPENDIX ONE

Move Operator


Start
Read Initial Solution, Dimensions of facilities
Randomly Choose on facility, F[G]
G is A random number [1,M]
i=1
F
i<=M
T
F
i≠G
T
Calculate the Euclidian distance between F[G] and F[i]
Dis[j]=Distance(F[G],F[i])
4
3
1
2

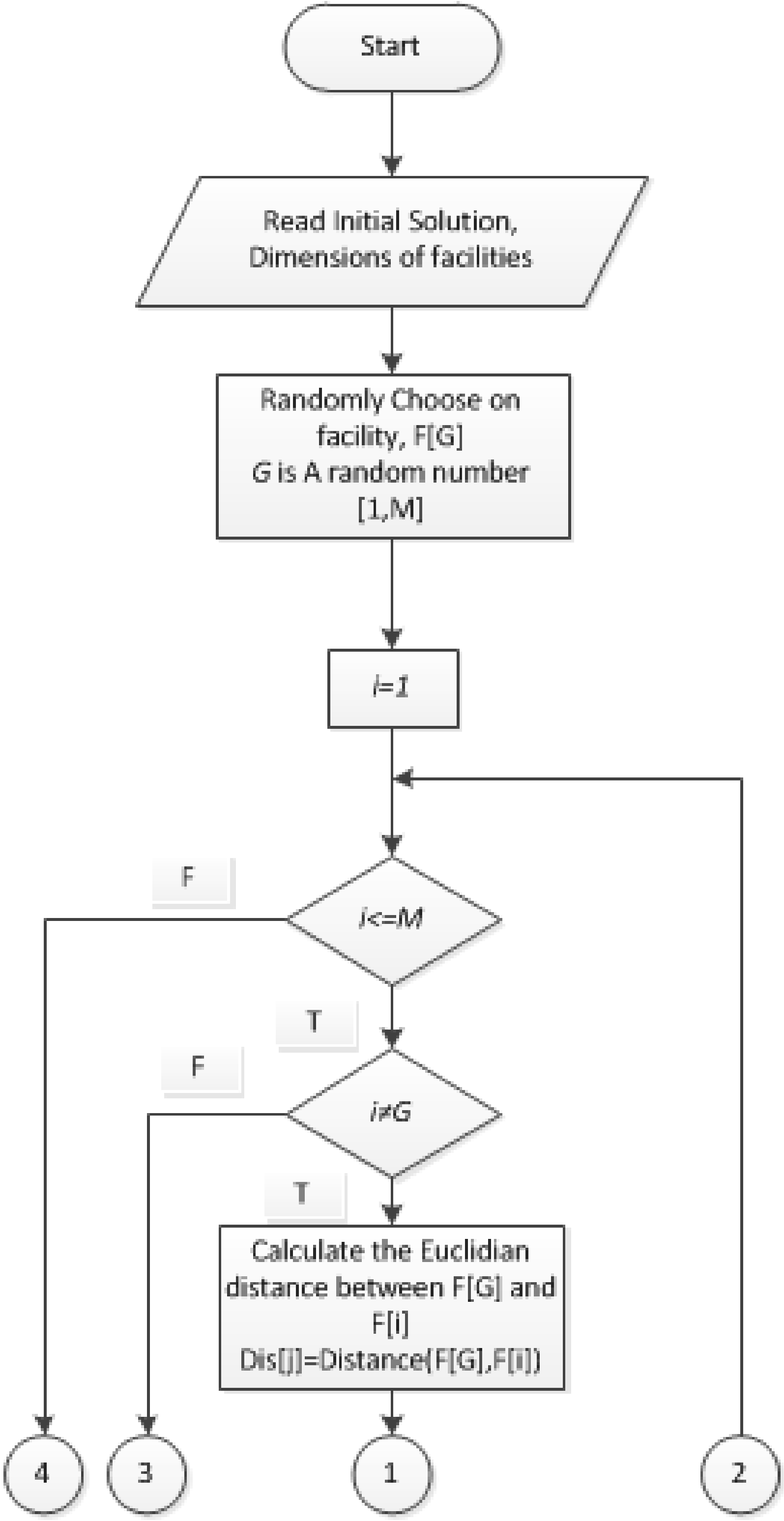

4
3
1
2
i=i+1
Arrange *Dist* metrics in descending order
i=1
Find the ith facility, called F[c]
T
i=i+1
Are F[G] and F[c] adjacent?
F
Calculate r1, r2
Divide Area of F[G] into four equal sized quadrants
5

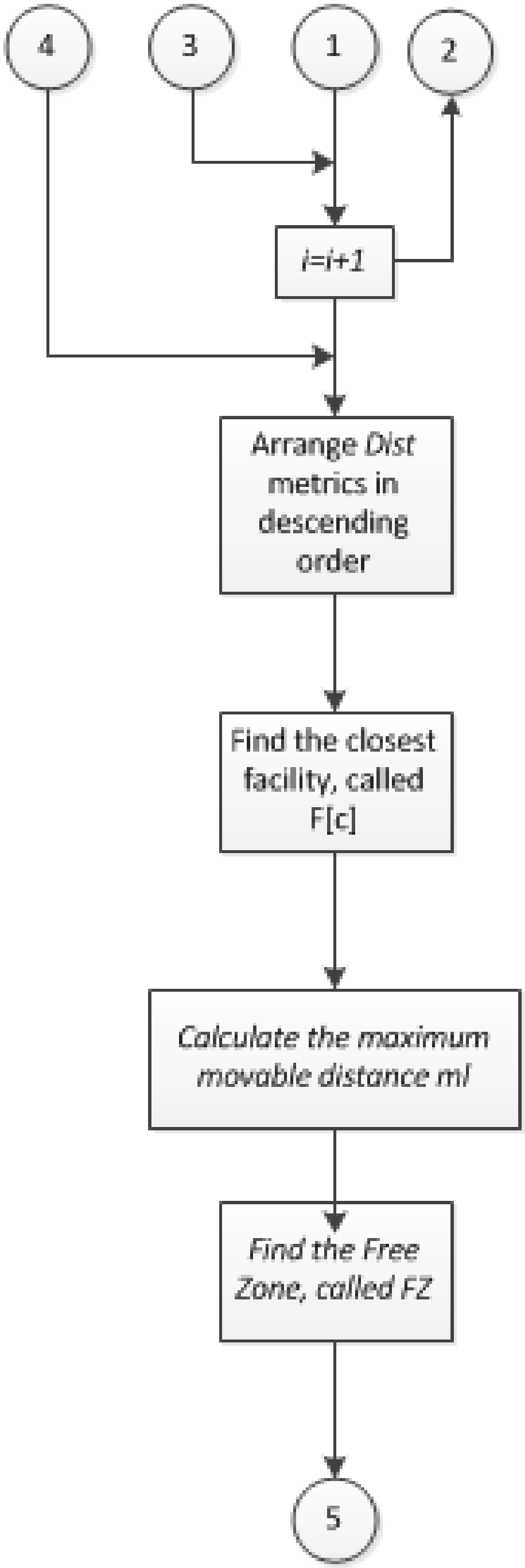

5
F[G][1]≤F[C][1]
T
F
F[G][1]≤F[C][2]
F
T
F[G][1]≤F[C][2]
T
F
C[1]=F[G][1]+r1×cosθ1
C[2]=F[G][2]-r1×sinθ1
C1[2]=F[C][2]-r2×sinθ2
C1[2]=F[C][2]+r1×cosθ2
C[1]=F[G][1]+r1×cosθ1
C[2]=F[G][2]+r1×sinθ1
C1[2]=F[C][2]-r2×sinθ2
C1[2]=F[C][2]-r1×cosθ2
C[1]=F[G][1]-r1×cosθ1
C[2]=F[G][2]+r1×sinθ1
C1[2]=F[C][2]+r2×sinθ2
C1[2]=F[C][2]-r1×cosθ2
C[1]=F[G][1]-r1×cosθ1
C[2]=F[G][2]-r1×sinθ1
C1[2]=F[C][2]+r2×sinθ2
C1[2]=F[C][2]+r1×cosθ
2
Calculate the maximum movable distance ml
Genereate a random number [0,r], called a
6

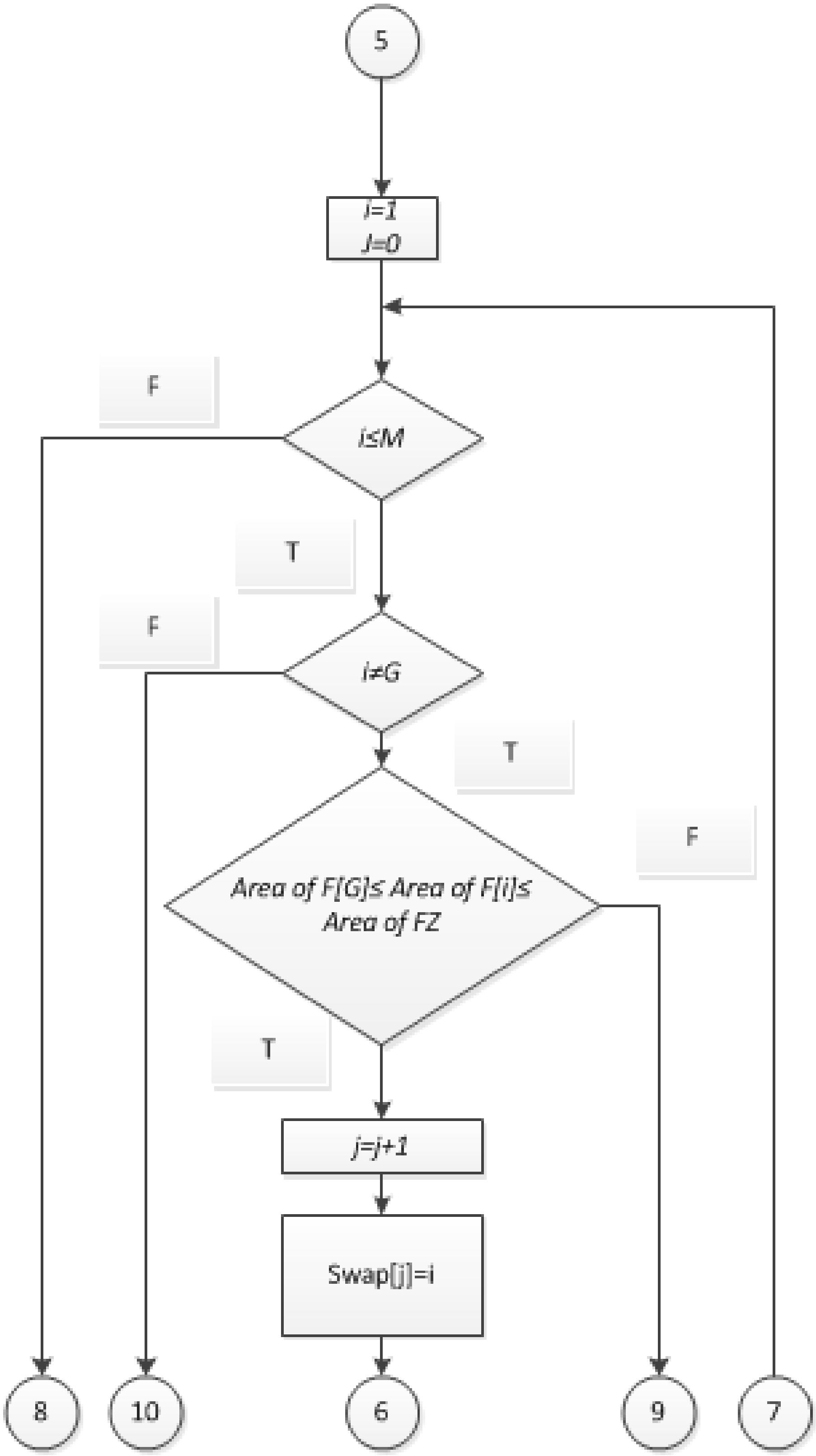

6
F[G][1]≤F[C][1]
T
F
F[G][1]≤F[C][2]
T
F
F[G][1]≤F[C][2]
T
F
F[G][1]=F[G][1]+α×cosθ1
F[G][2]=F[G][2]−α×sinθ1
F[G][1]=F[G][1]−α×cosθ1
F[G][2]=F[G][2]+α×sinθ1
F[G][1]=F[G][1]−α×cosθ1
F[G][2]=F[G][2]−α×sinθ1
F[G][1]=F[G][1]+α×cosθ1
F[G][2]=F[G][2]−r1×sinθ1
Any overlap
T
F
Apply Repair Function
Write new coordinates of F[G]
End

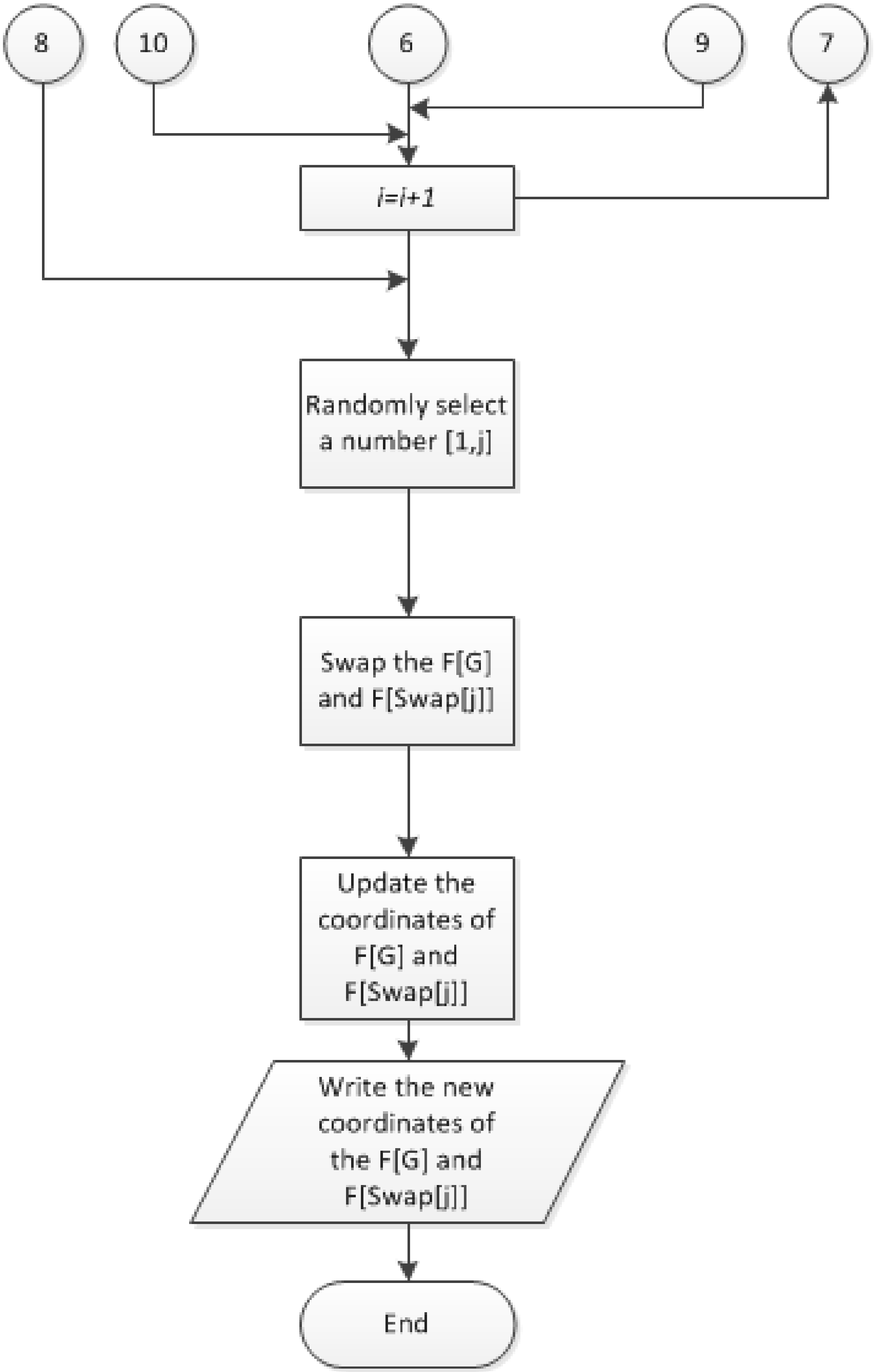
APPENDIX TWO

Swap Operator


Start
Read Initial Solution,
Dimensions of facilities
Randomly Choose on facility, F[G]
G is A random number [1,M]
i=1
i<=M
F
T
i≠G
F
T
Calculate the Euclidian distance between F[G] and F[i]
Dis[j]=Distance(F[G],F[i])
4
3
1
2

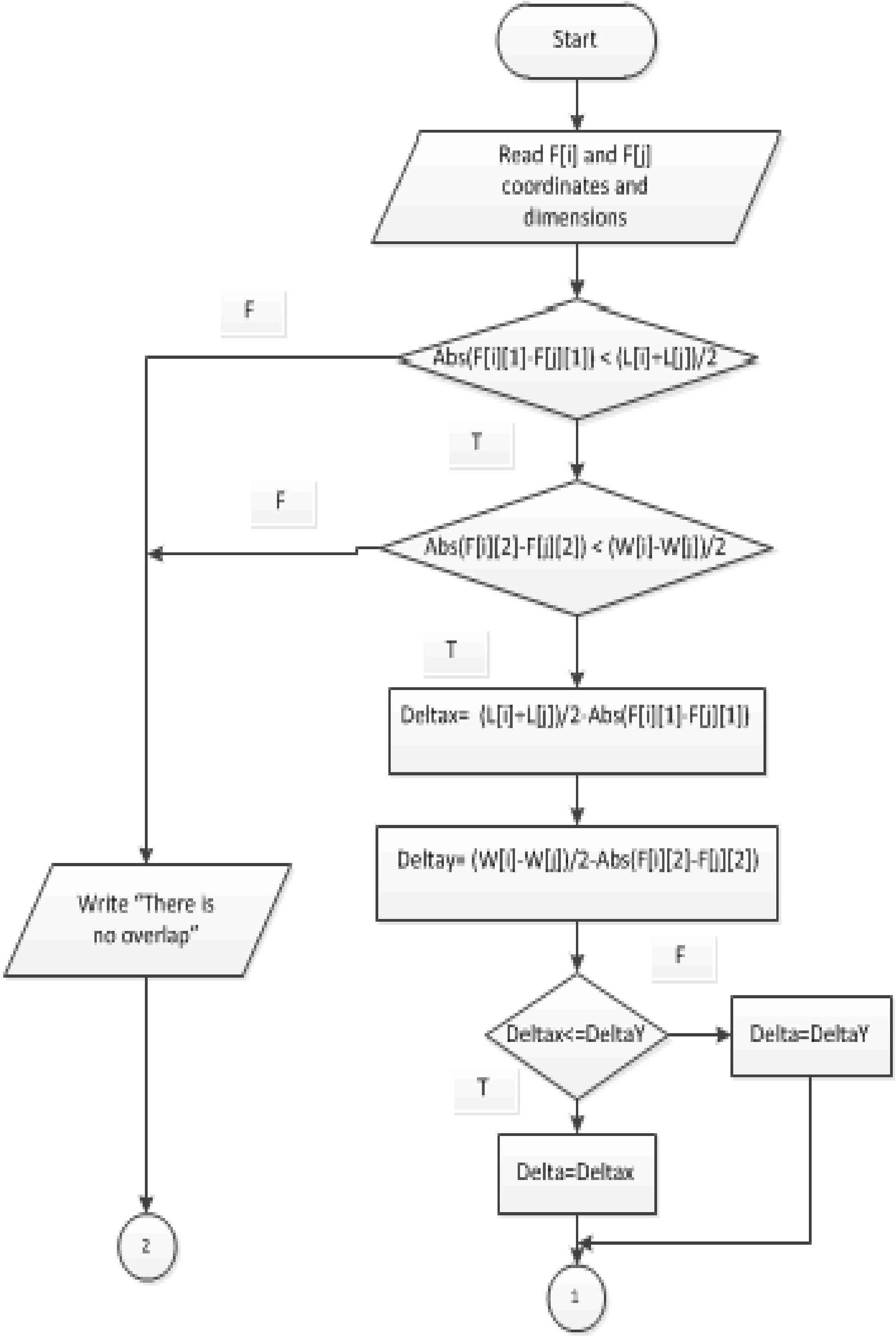

4
3
1
2
i=i+1
Arrange Dist metrics in descending order
Find the closest facility, called F[c]
Calculate the maximum movable distance ml
Find the Free Zone, called FZ
5

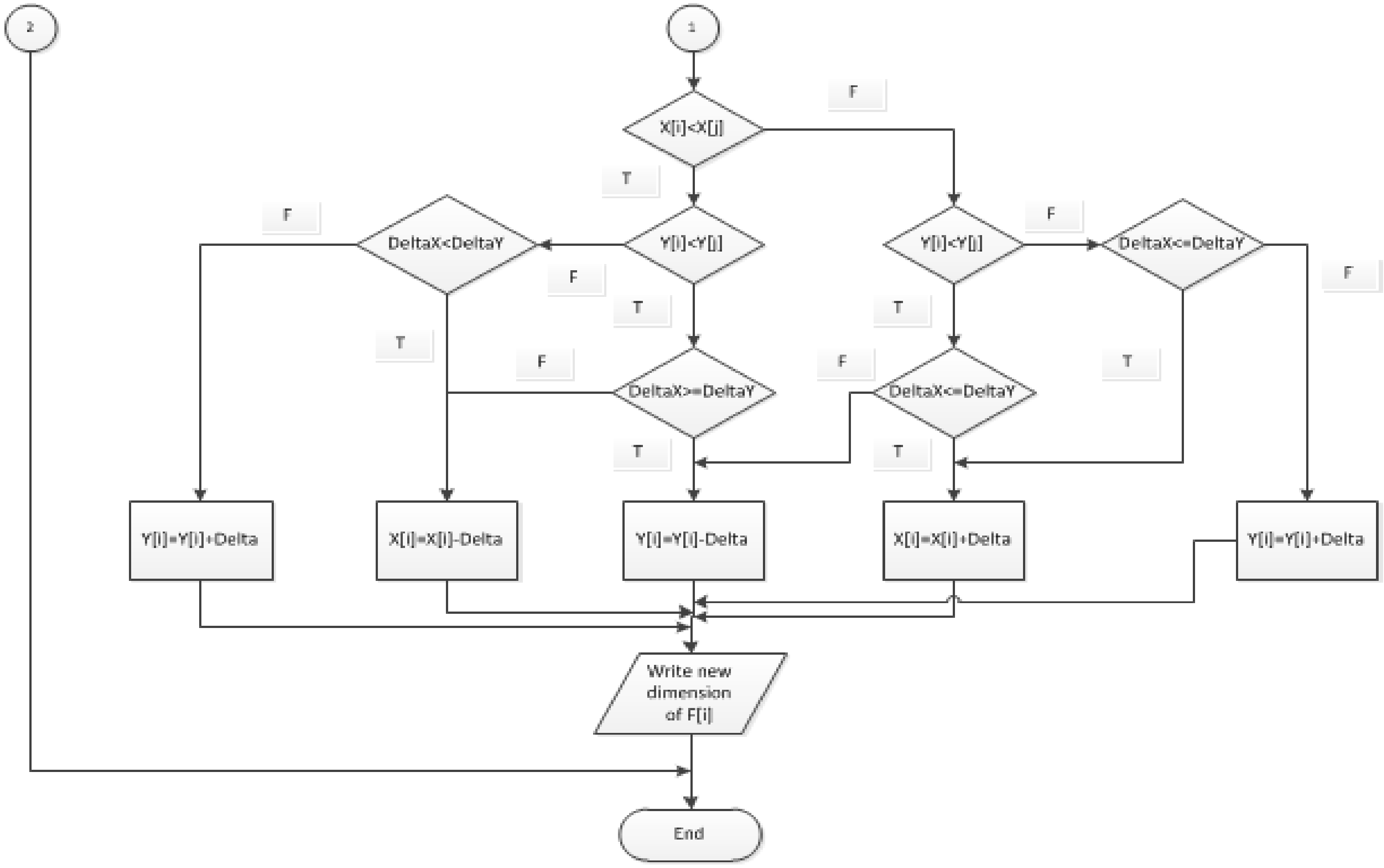

5
i=1
J=0
F
i≤M
T
F
i≠G
T
F
Area of F[G]≤ Area of F[i]≤ Area of FZ
T
j=j+1
Swap[j]=i
8
10
6
9
7


8
10
6
9
7
i=i+1
Randomly select a number [1,j]
Swap the F[G] and F[Swap[j]]
Update the coordinates of F[G] and F[Swap[j]]
Write the new coordinates of the F[G] and F[Swap[j]]
End

APPENDIX THREE

Overlap Checking Function

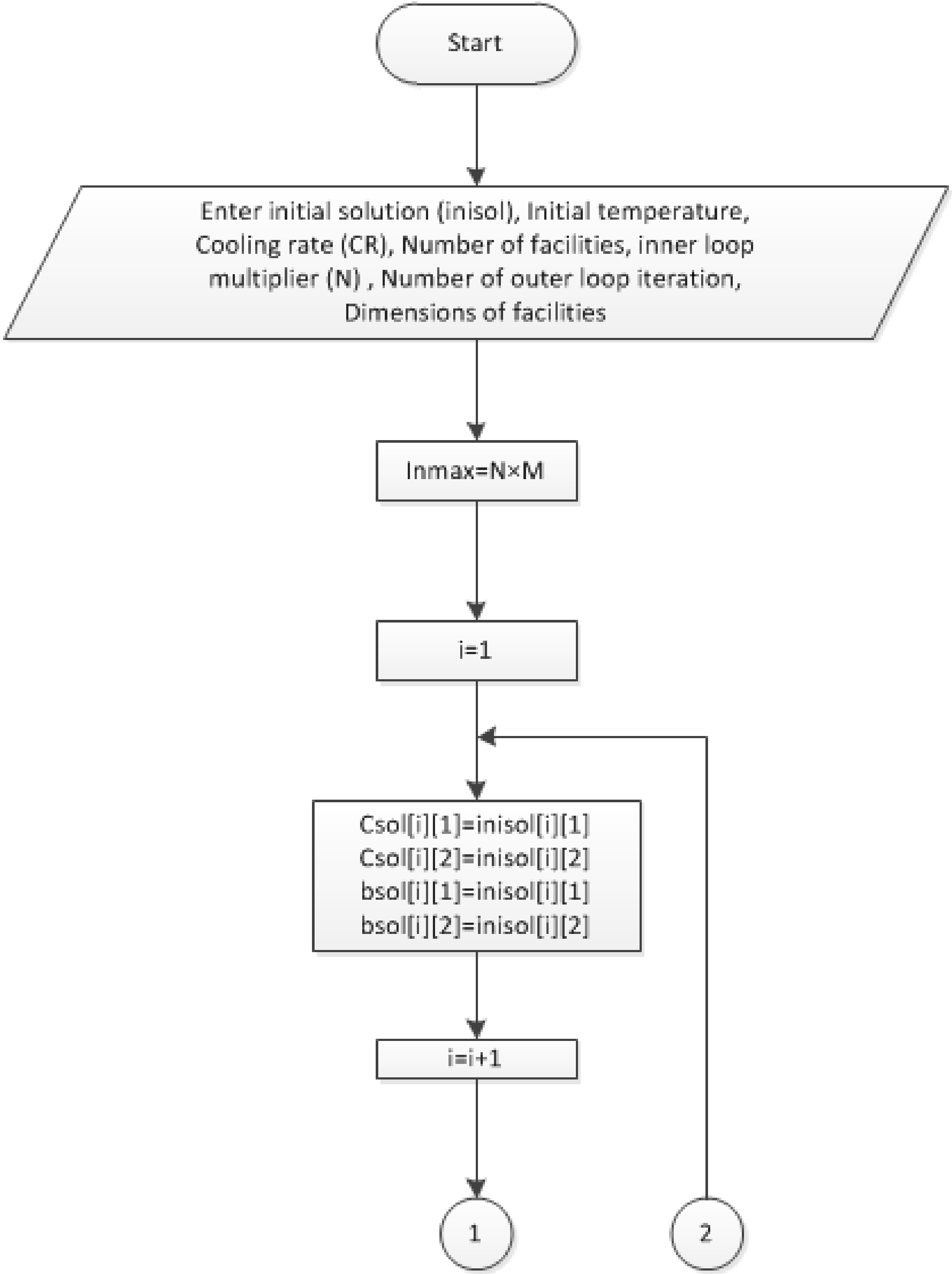

Start
Read F[i] and F[j] coordinates and dimensions
F
Abs(F[i][1]-F[j][1]) < (L[i]+L[j])/2
T
F
Abs(F[i][2]-F[j][2]) < (W[i]-W[j])/2
T
Deltax= (L[i]+L[j])/2-Abs(F[i][1]-F[j][1])
Deltay= (W[i]-W[j])/2-Abs(F[i][2]-F[j][2])
Write "There is no overlap"
F
Deltax<=DeltaY
Delta=DeltaY
T
Delta=Deltax
2
1

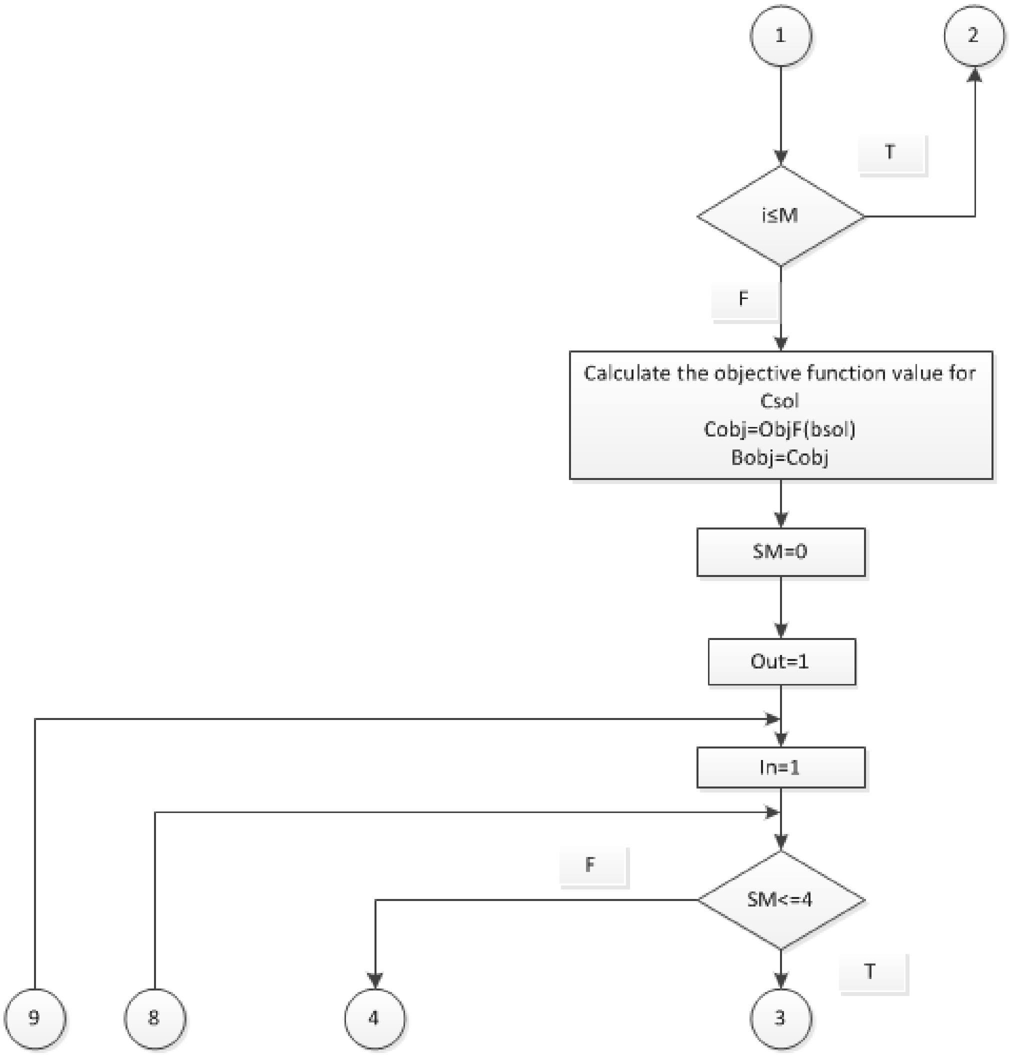

2
1
X[i]<X[j]
F
T
Y[i]<Y[j]
F
Y[i]<Y[j]
F
DeltaX<DeltaY
F
DeltaX<=DeltaY
F
T
T
T
F
F
DeltaX>=DeltaY
DeltaX<=DeltaY
T
T
T
Y[i]=Y[i]+Delta
X[i]=X[i]-Delta
Y[i]=Y[i]-Delta
X[i]=X[i]+Delta
Y[i]=Y[i]+Delta
Write new
dimension
of F[i]
End

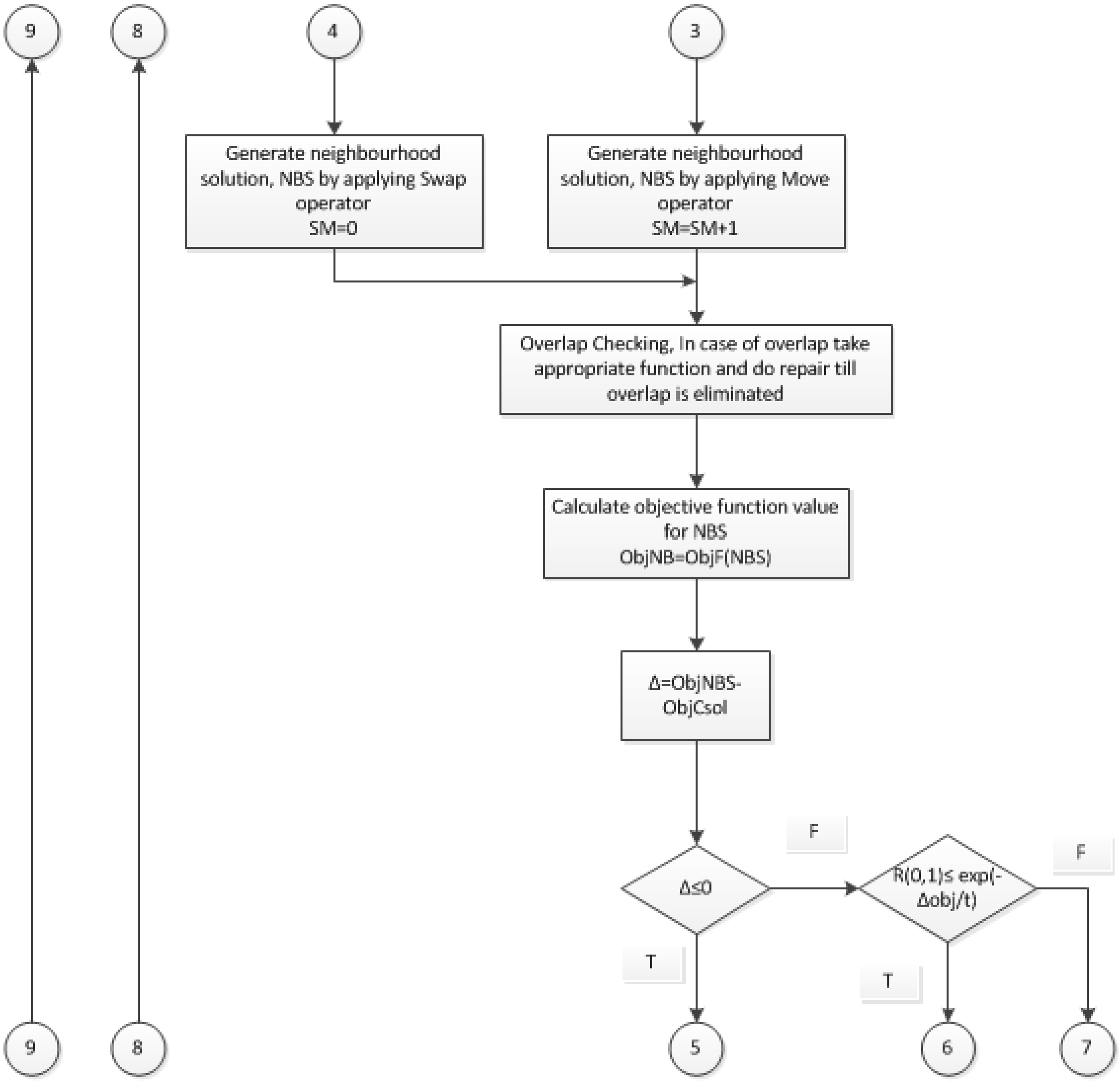
APPENDIX FOUR

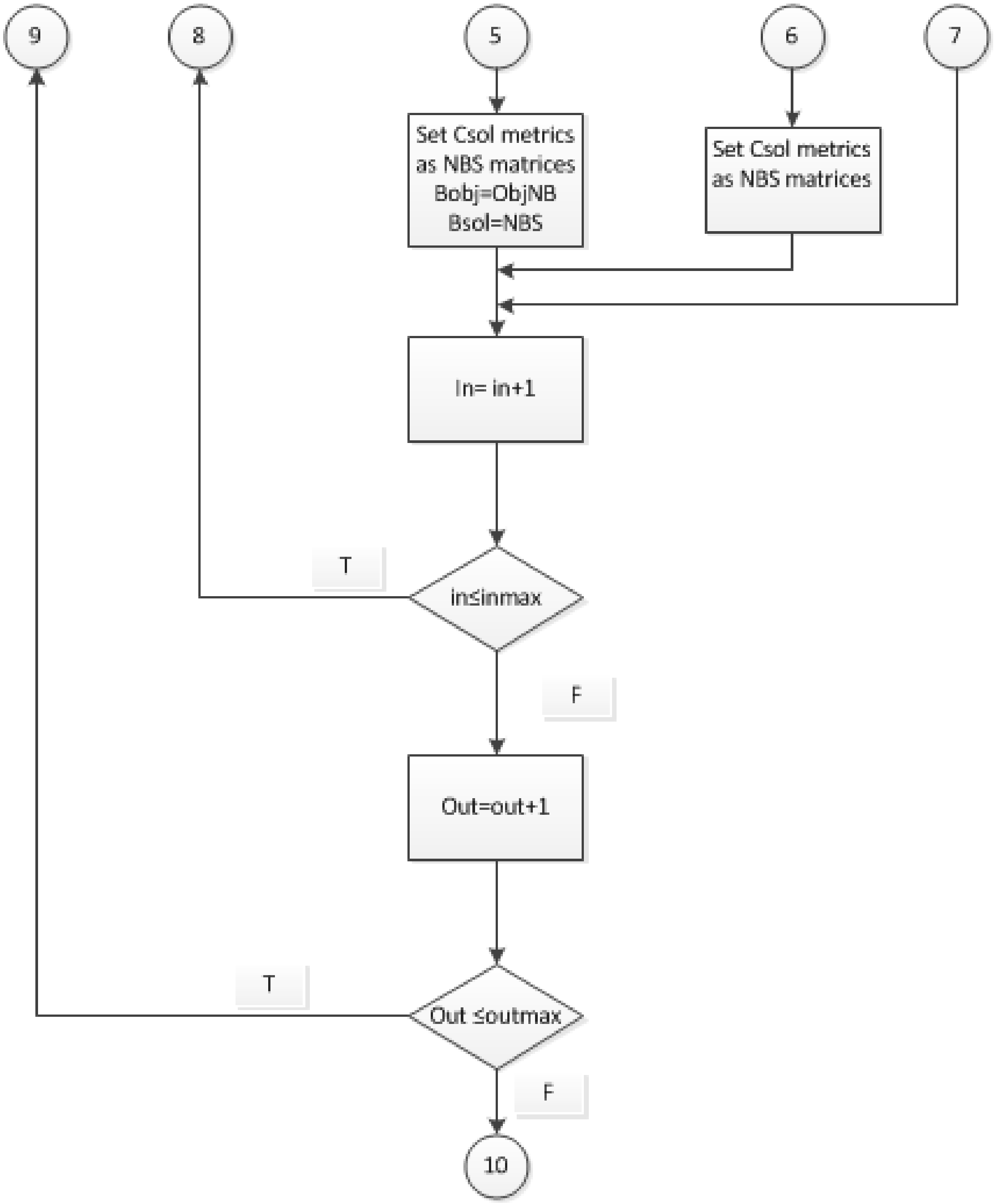
SA Algorithm

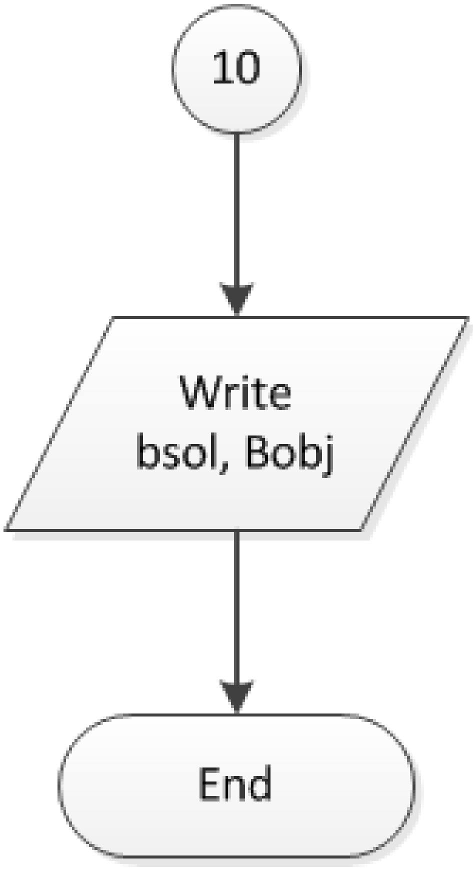
Move and Swap Operators


Start
Enter initial solution (inisol), Initial temperature, Cooling rate (CR), Number of facilities, inner loop multiplier (N) , Number of outer loop iteration, Dimensions of facilities
Inmax=N×M
i=1
Csol[i][1]=inisol[i][1]
Csol[i][2]=inisol[i][2]
bsol[i][1]=inisol[i][1]
bsol[i][2]=inisol[i][2]
i=i+1
1
2


1
2
T
i≤M
F
Calculate the objective function value for Csol
Cobj=ObjF(bsol)
Bobj=Cobj
SM=0
Out=1
In=1
F
SM<=4
T
9
8
4
3


9
8
4
3
Generate neighbourhood solution, NBS by applying Swap operator
SM=0
Generate neighbourhood solution, NBS by applying Move operator
SM=SM+1
Overlap Checking, In case of overlap take appropriate function and do repair till overlap is eliminated
Calculate objective function value for NBS
ObjNB=ObjF(NBS)
Δ=ObjNBS-ObjCsol
Δ≤0
F
R(0,1)≤ exp(-Δobj/t)
F
T
T
9
8
5
6
7

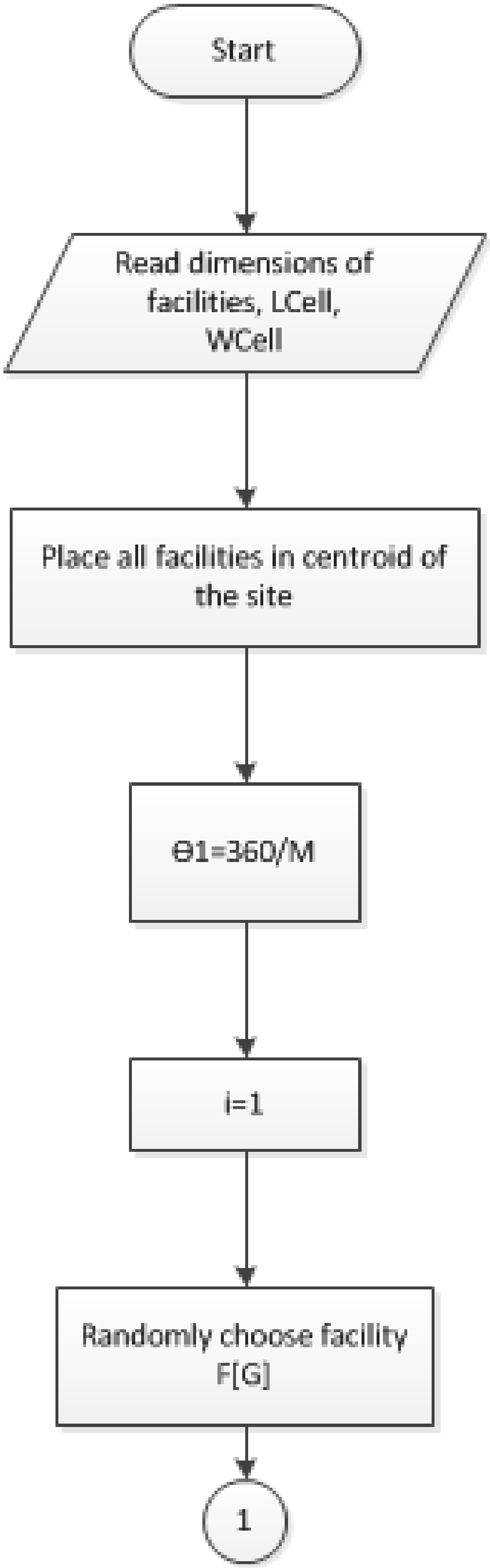

9
8
5
6
7
Set Csol metrics as NBS matrices Bobj=ObjNB Bsol=NBS
Set Csol metrics as NBS matrices
In= in+1
T
in≤inmax
F
Out=out+1
T
Out ≤outmax
F
10

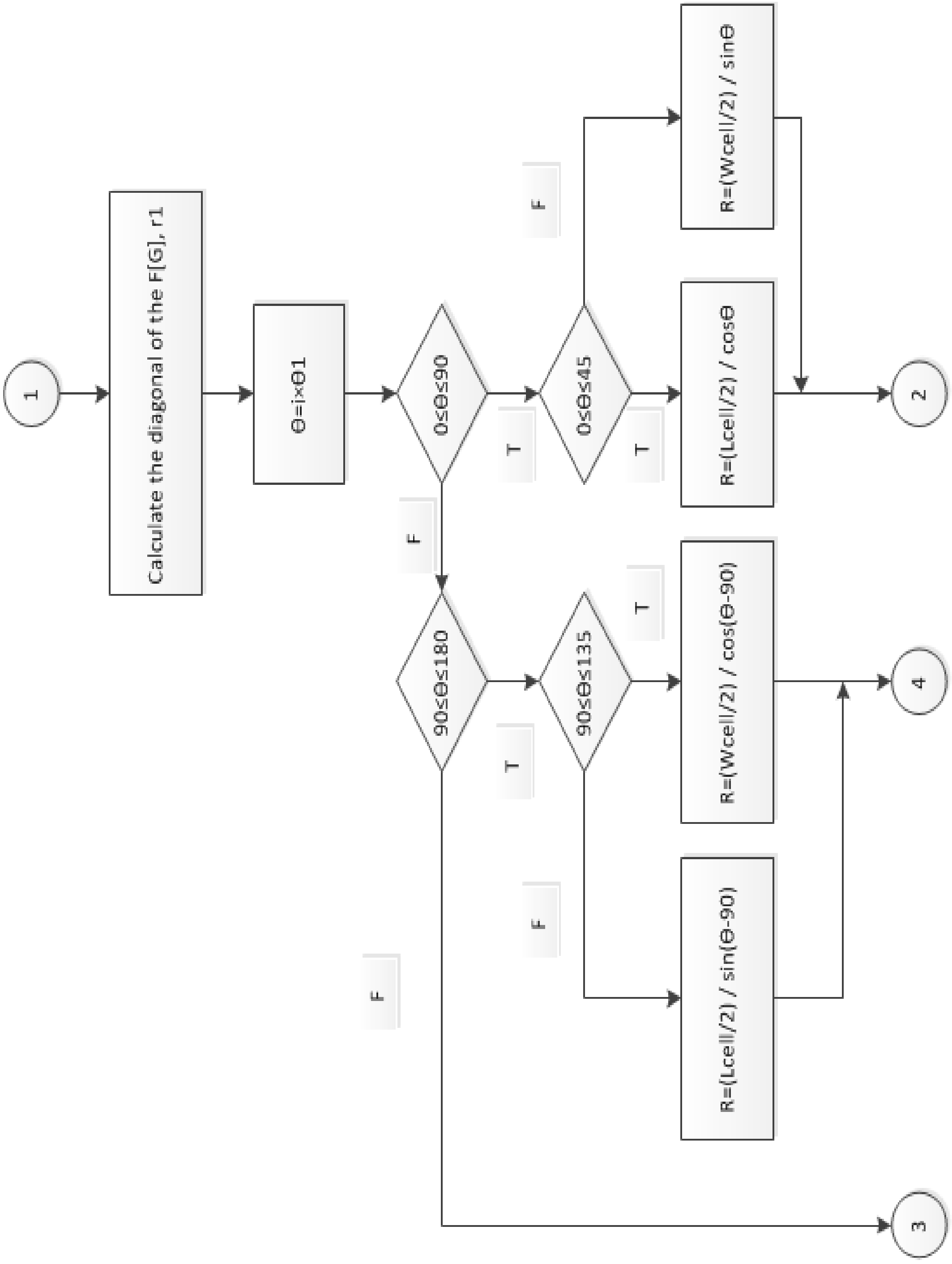

10
Write
bsol, Bobj
End

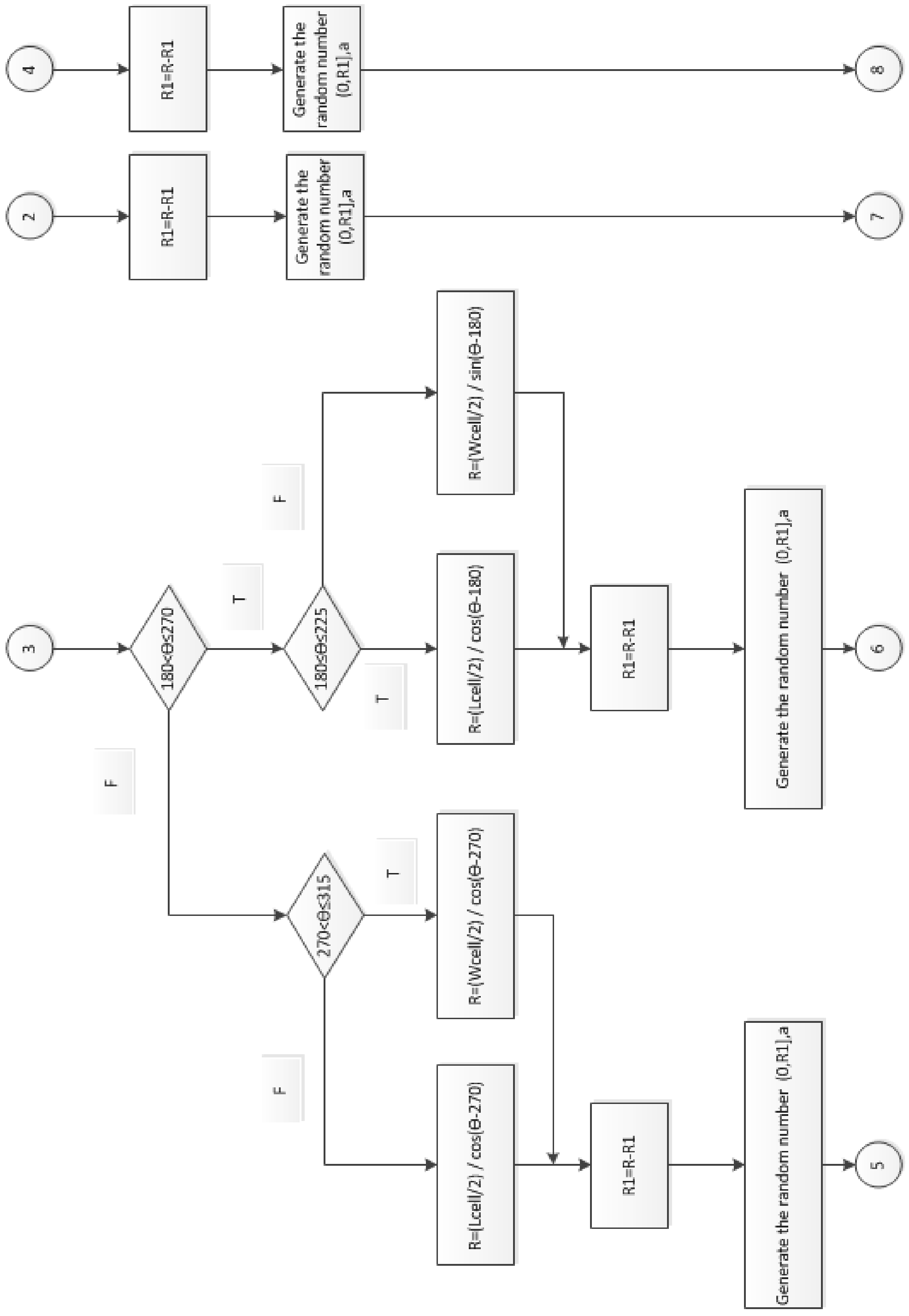
APPENDIX FIVE

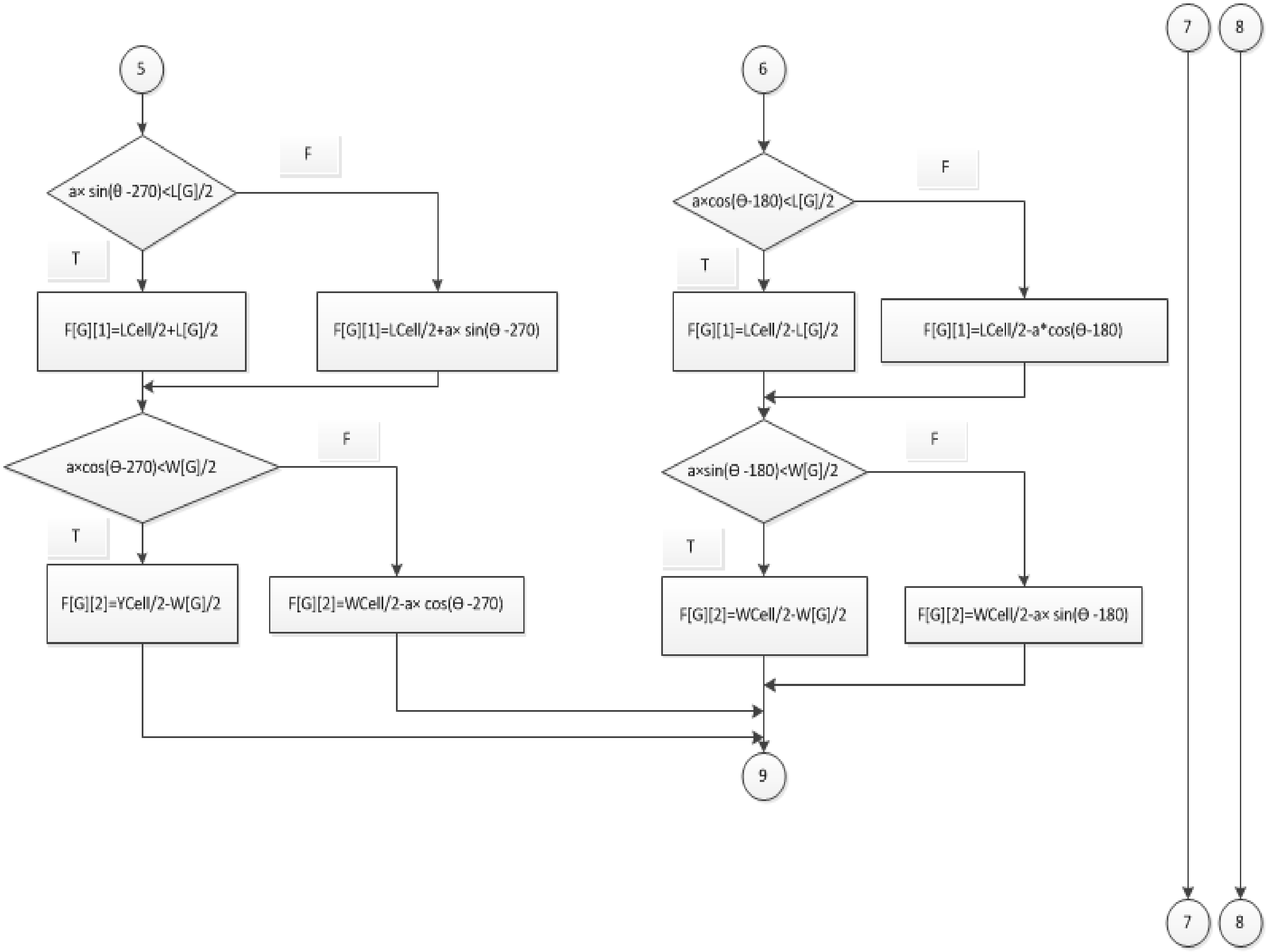
Heuristic Algorithm

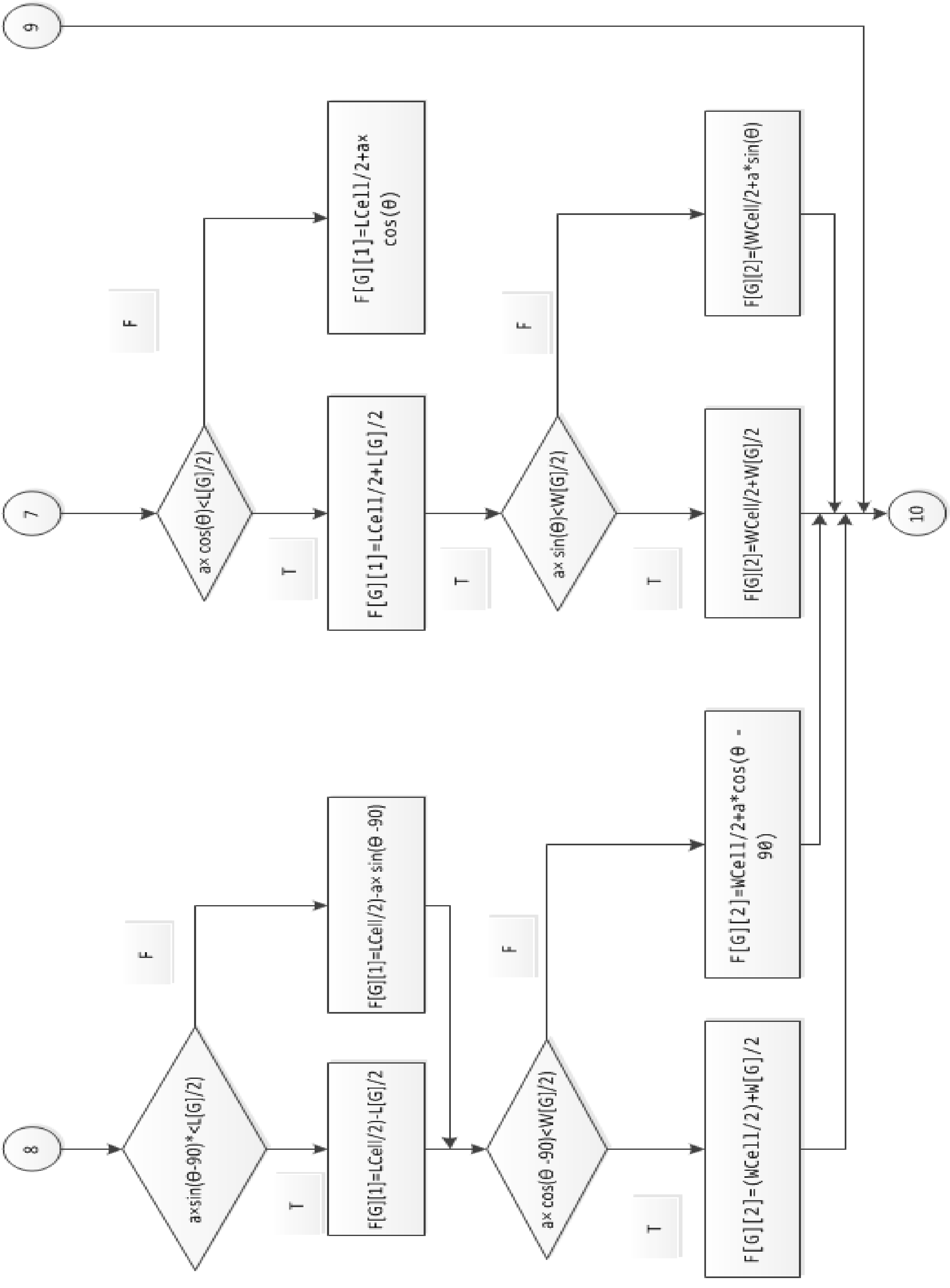
Since there are 126 repair functions designed for heuristic algorithm, only 2 functions here have been shown in flowchart.

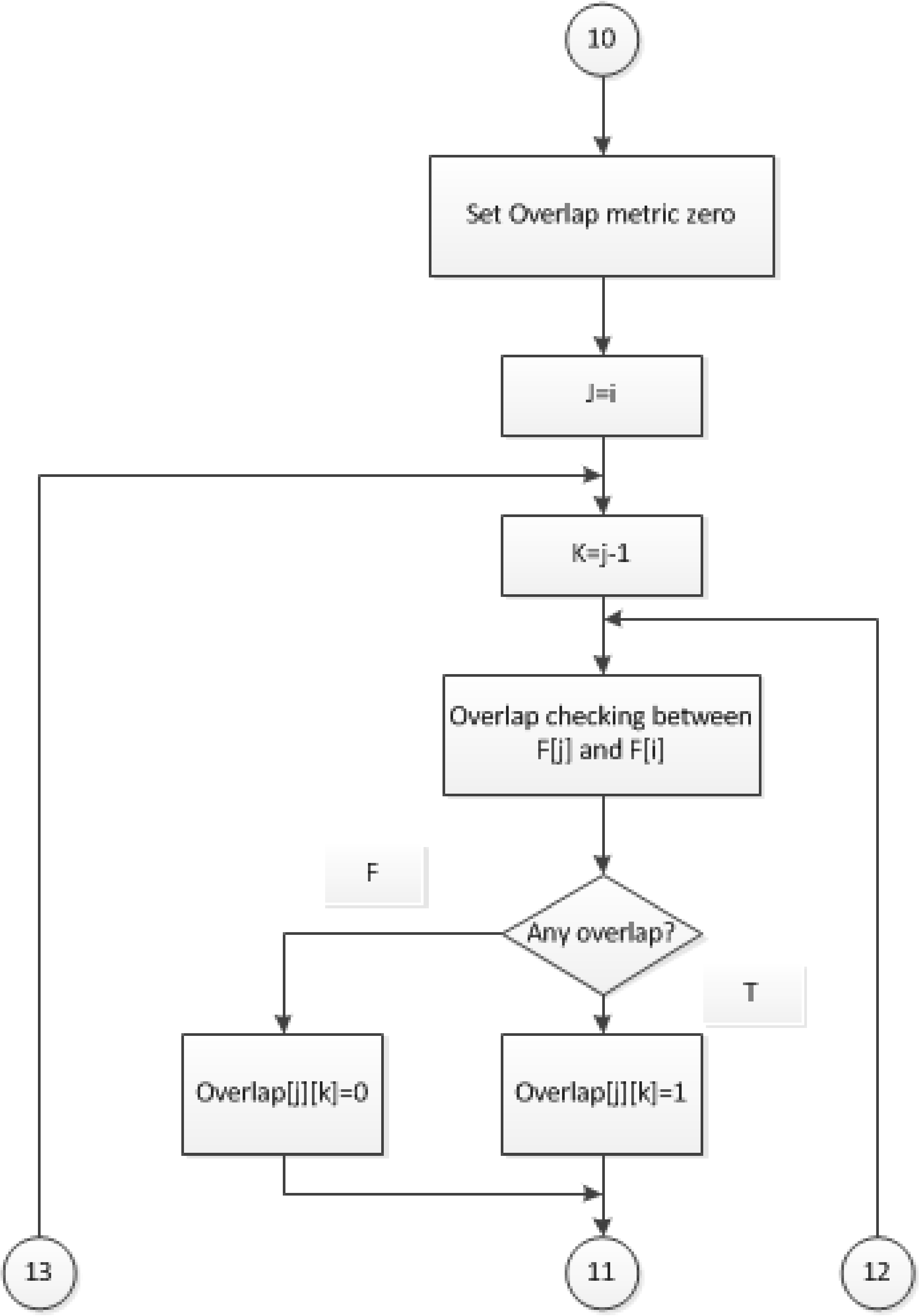

Start
Read dimensions of facilities, LCell, WCell
Place all facilities in centroid of the site
Ɵ1=360/M
i=1
Randomly choose facility F[G]
1

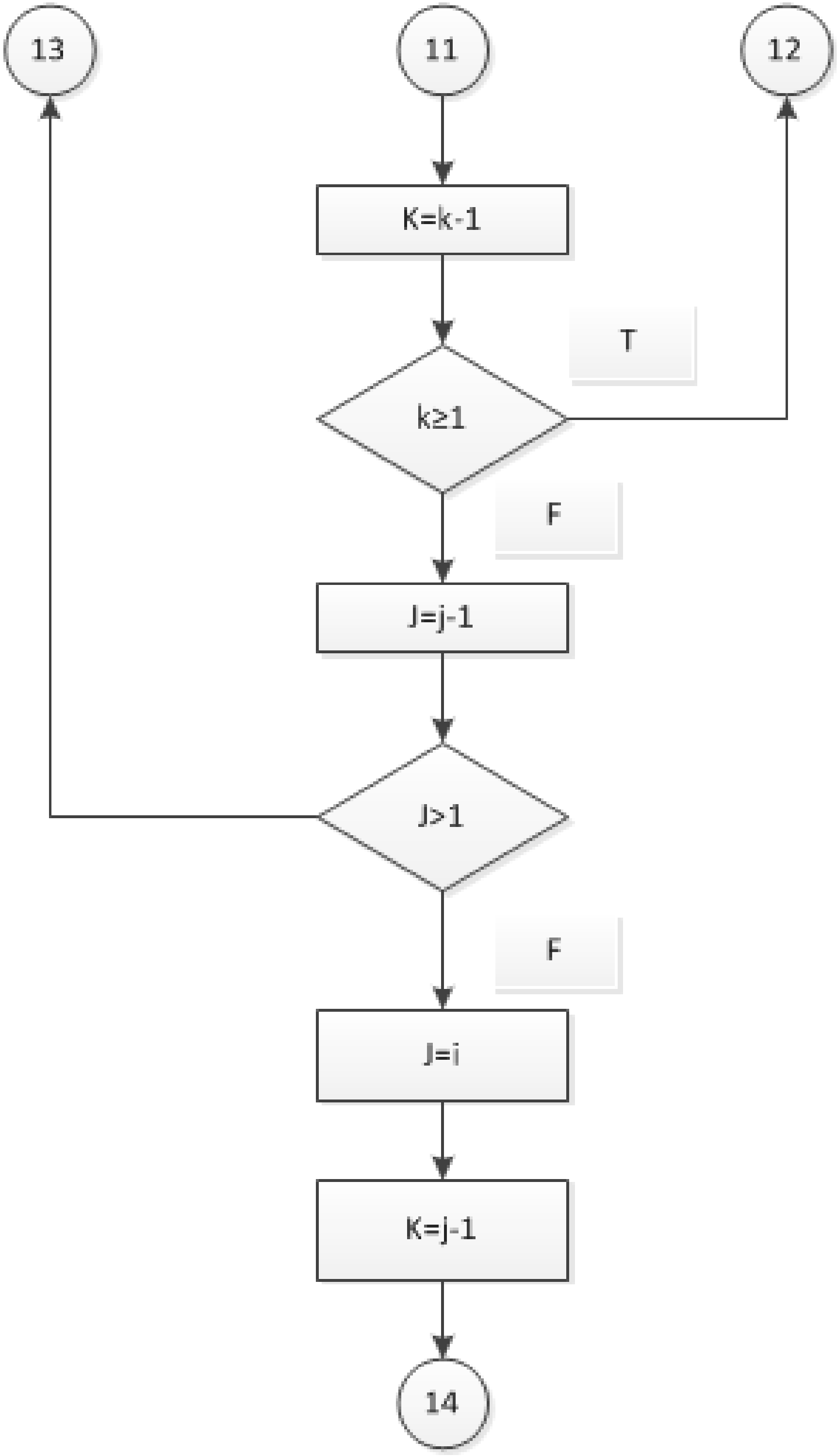

1
Calculate the diagonal of the F[G], r1
Θ=i×Θ1
0≤Θ≤90
T
F
0≤Θ≤45
T
F
R=(Lcell/2) / cosΘ
R=(Wcell/2) / sinΘ
2
90≤Θ≤180
T
F
90≤Θ≤135
T
F
R=(Wcell/2) / cos(Θ-90)
R=(Lcell/2) / sin(Θ-90)
4
3

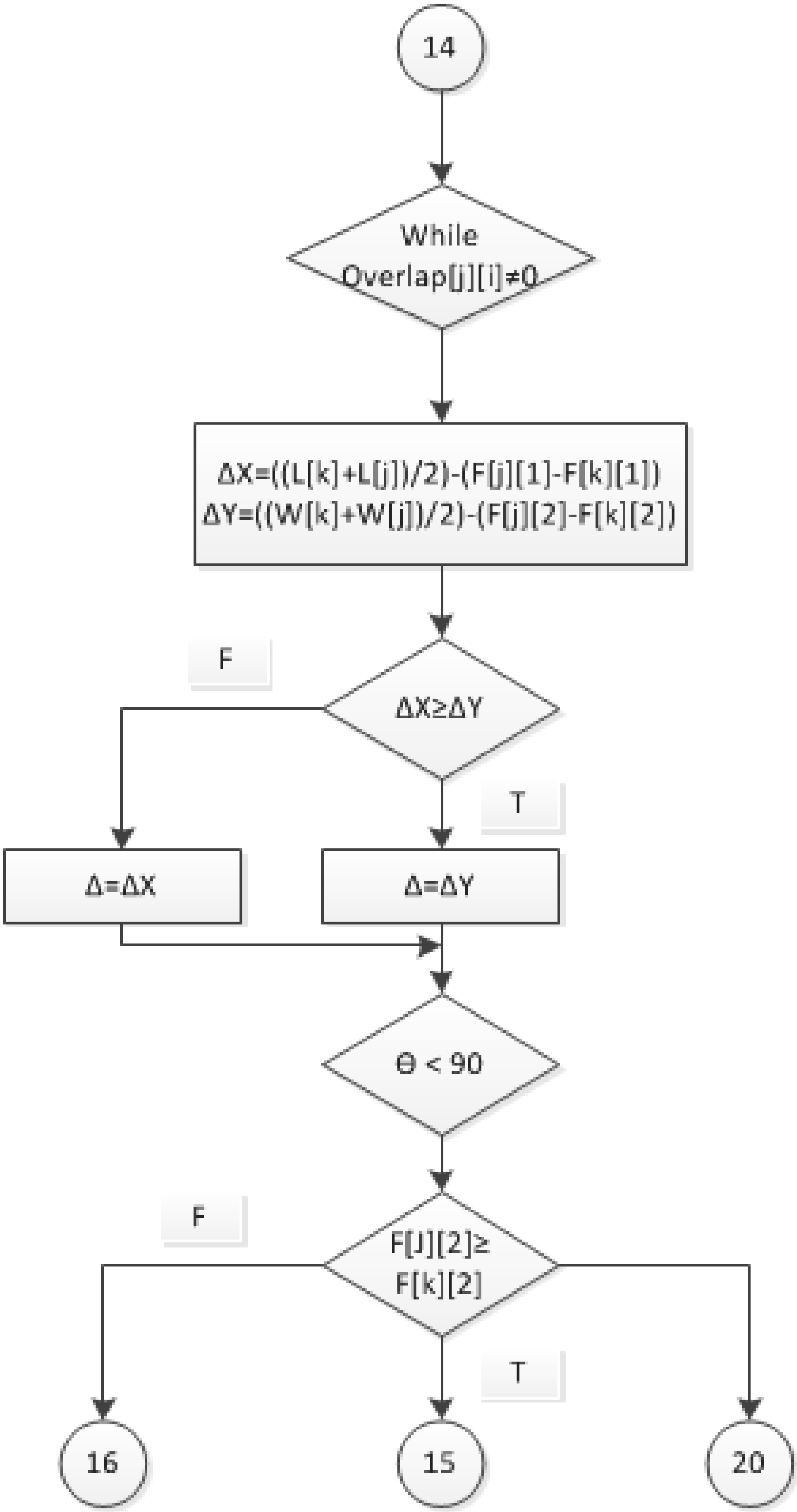

4
R1=R-R1
Generate the random number (0,R1],a
8
2
R1=R-R1
Generate the random number (0,R1],a
7
3
180<Θ≤270
T
180≤Θ≤225
T
R=(Lcell/2) / cos(Θ-180)
F
R=(Wcell/2) / sin(Θ-180)
R1=R-R1
Generate the random number (0,R1],a
6
F
270<Θ≤315
T
R=(Wcell/2) / cos(Θ-270)
F
R=(Lcell/2) / cos(Θ-270)
R1=R-R1
Generate the random number (0,R1],a
5

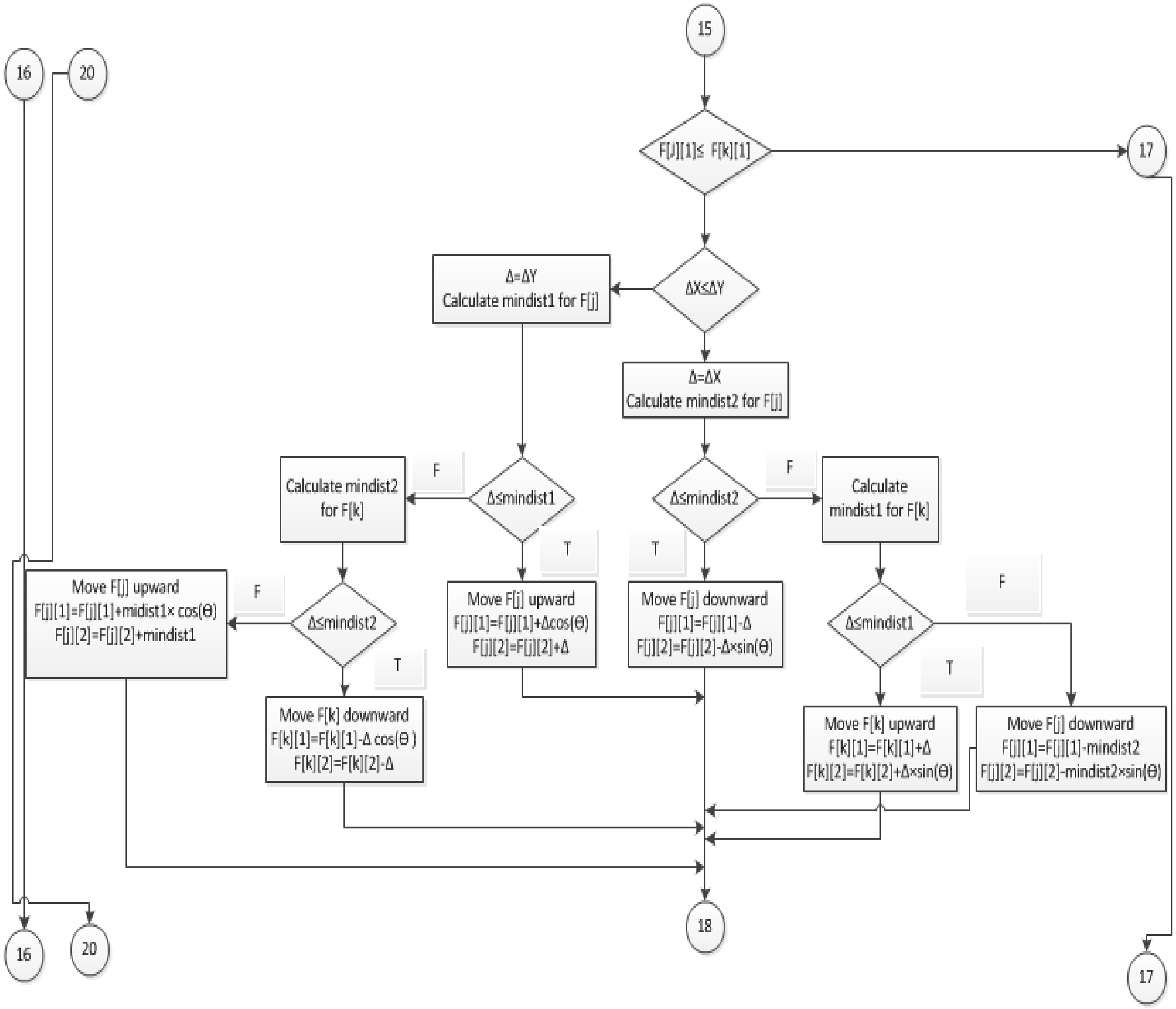

5
a× sin(θ -270)<L[G]/2
F
T
F[G][1]=LCell/2+L[G]/2
F[G][1]=LCell/2+a× sin(Θ -270)
a×cos(Θ-270)<W[G]/2
F
T
F[G][2]=YCell/2-W[G]/2
F[G][2]=WCell/2-a× cos(Θ -270)
6
a×cos(Θ-180)<L[G]/2
F
T
F[G][1]=LCell/2-L[G]/2
F[G][1]=LCell/2-a*cos(Θ-180)
a×sin(Θ -180)<W[G]/2
F
T
F[G][2]=WCell/2-W[G]/2
F[G][2]=WCell/2-a× sin(Θ -180)
9
7
8
7
8

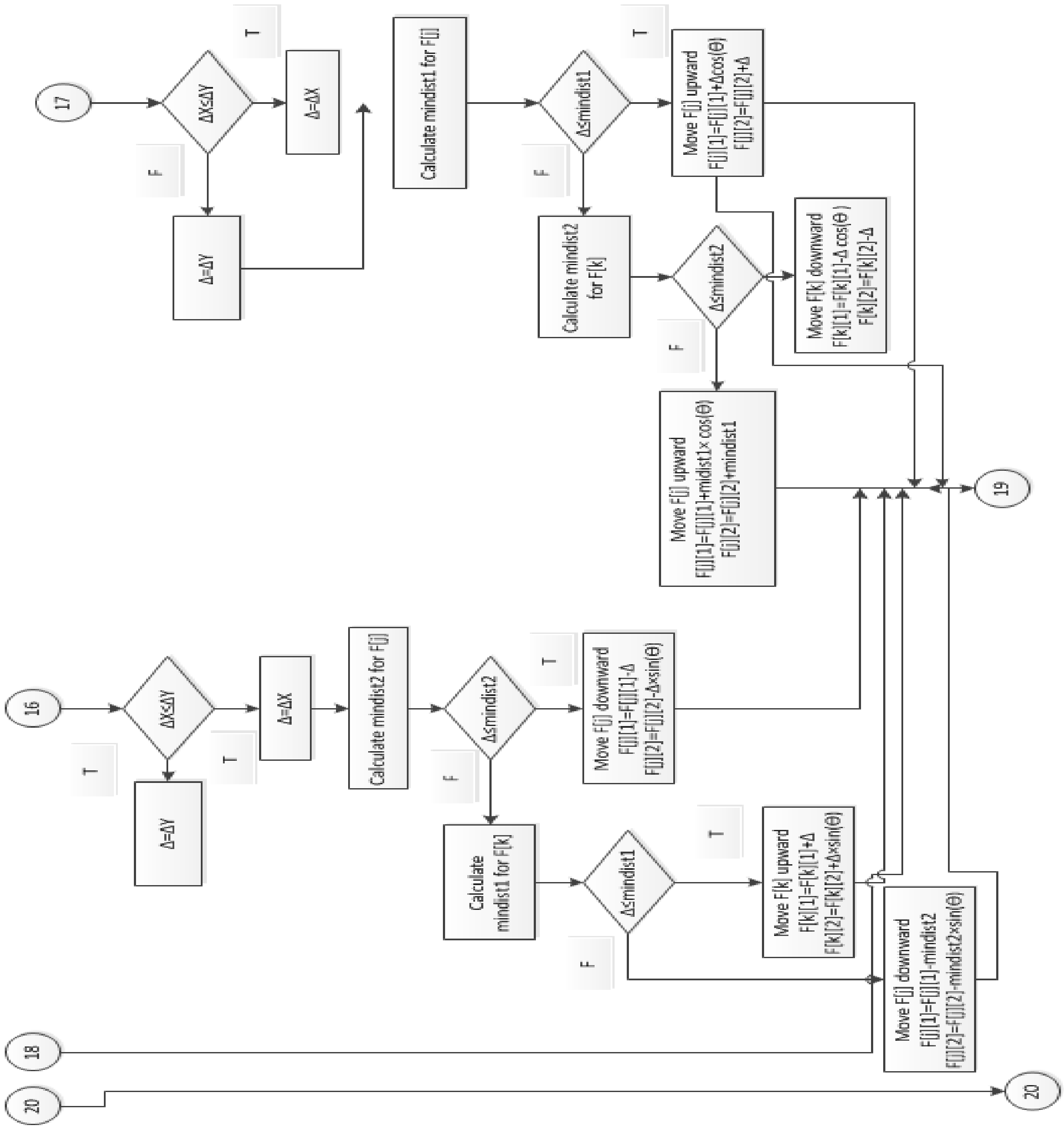

9
7
a× cos(θ)<L[G]/2)
T
F
F[G][1]=LCell/2+L[G]/2
F[G][1]=LCell/2+a× cos(θ)
a× sin(θ)<W[G]/2)
T
F
F[G][2]=WCell/2+W[G]/2
F[G][2]=(WCell/2+a*sin(θ)
8
a×sin(θ-90)*<L[G]/2)
T
F
F[G][1]=LCell/2)-L[G]/2
F[G][1]=LCell/2)-a× sin(θ -90)
a× cos(θ -90)<W[G]/2)
T
F
F[G][2]=(WCell/2)+W[G]/2
F[G][2]=WCell/2+a*cos(θ - 90)
10


10
Set Overlap metric zero
J=i
K=j-1
Overlap checking between F[j] and F[i]
F
Any overlap?
T
Overlap[j][k]=0
Overlap[j][k]=1
13
11
12


13
11
12
K=k-1
T
k≥1
F
J=j-1
J>1
F
J=i
K=j-1
14

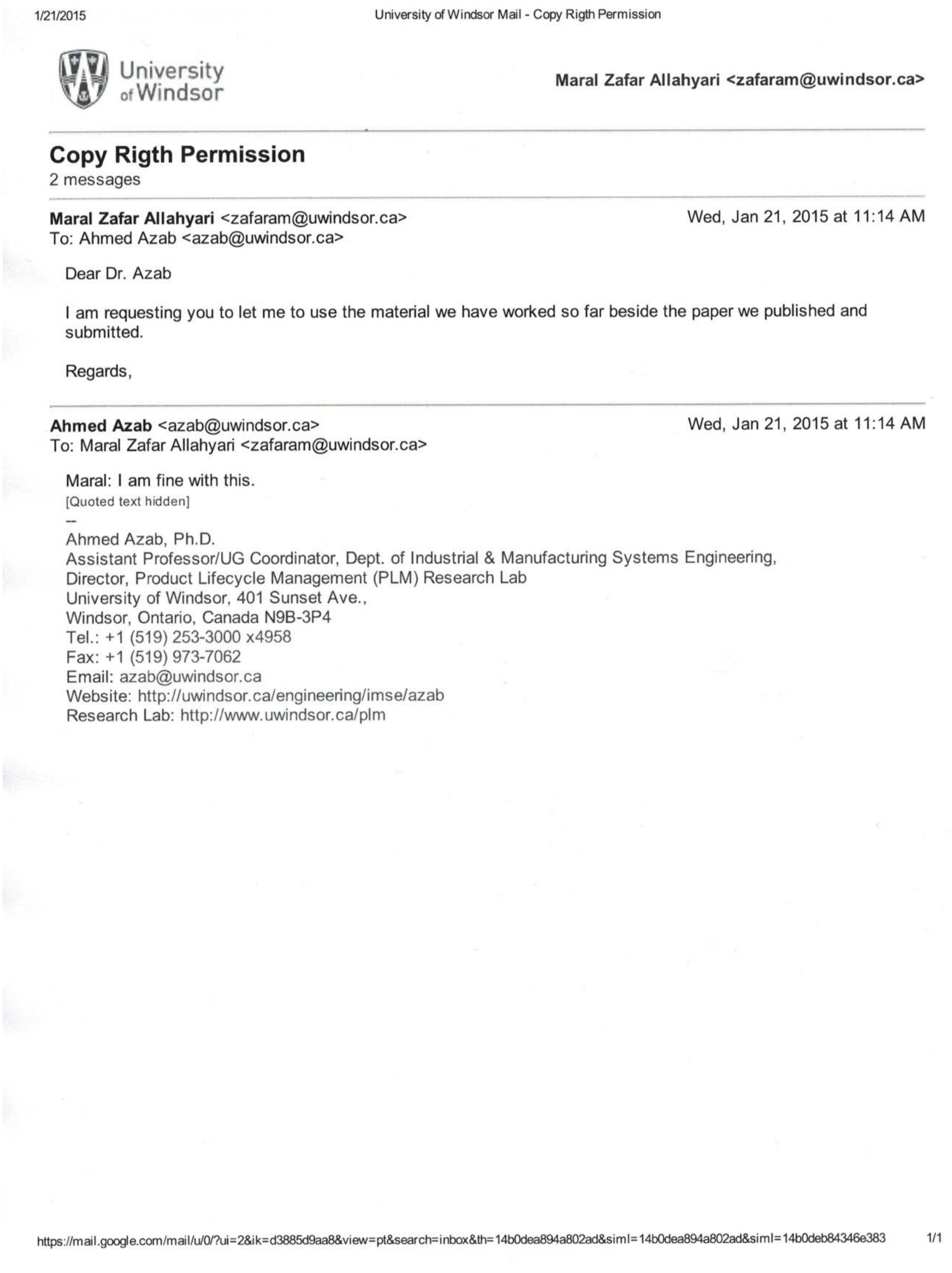

14
While Overlap[j][i]≠0
ΔX=((L[k]+L[j])/2)-(F[j][1]-F[k][1])
ΔY=((W[k]+W[j])/2)-(F[j][2]-F[k][2])
F
ΔX≥ΔY
T
Δ=ΔX
Δ=ΔY
Θ < 90
F
F[J][2]≥ F[k][2]
T
16
15
20


15
F[J][1]≤ F[k][1]
17
ΔX≤ΔY
Δ=ΔY
Calculate mindist1 for F[j]
Δ=ΔX
Calculate mindist2 for F[j]
Δ≤mindist1
F
T
Calculate mindist2 for F[k]
Move F[j] upward
F[j][1]=F[j][1]+Δcos(Θ)
F[j][2]=F[j][2]+Δ
Δ≤mindist2
F
T
Move F[j] upward
F[j][1]=F[j][1]+midist1× cos(Θ)
F[j][2]=F[j][2]+mindist1
Move F[k] downward
F[k][1]=F[k][1]-Δ cos(Θ)
F[k][2]=F[k][2]-Δ
Δ≤mindist2
F
T
Calculate mindist1 for F[k]
Move F[j] downward
F[j][1]=F[j][1]-Δ
F[j][2]=F[j][2]-Δ×sin(Θ)
Δ≤mindist1
F
T
Move F[k] upward
F[k][1]=F[k][1]+Δ
F[k][2]=F[k][2]+Δ×sin(Θ)
Move F[j] downward
F[j][1]=F[j][1]-mindist2
F[j][2]=F[j][2]-mindist2×sin(Θ)
16
20
16
20
18
17


17
ΔX≤ΔY
T
F
Δ=ΔX
Δ=ΔY
Calculate mindist1 for F[j]
Δ≤mindist1
T
F
Move F[j] upward
F[j][1]=F[j][1]+Δcos(Θ)
F[j][2]=F[j][2]+Δ
Calculate mindist2 for F[k]
Δ≤mindist2
F
Move F[k] downward
F[k][1]=F[k][1]-Δ cos(Θ)
F[k][2]=F[k][2]-Δ
Move F[j] upward
F[j][1]=F[j][1]+midist1× cos(Θ)
F[j][2]=F[j][2]+mindist1
19
16
ΔX≤ΔY
T
T
Δ=ΔX
Δ=ΔY
Calculate mindist2 for F[j]
Δ≤mindist2
T
F
Move F[j] downward
F[j][1]=F[j][1]-Δ
F[j][2]=F[j][2]-Δxsin(Θ)
Calculate mindist1 for F[k]
Δ≤mindist1
T
F
Move F[k] upward
F[k][1]=F[k][1]+Δ
F[k][2]=F[k][2]+Δxsin(Θ)
Move F[j] downward
F[j][1]=F[j][1]-mindist2
F[j][2]=F[j][2]-mindist2×sin(Θ)
18
20
20

APPENDIX SIX

CopyRight Permission



1/21/2015 University of Windsor Mail - Copy Rigth Permission

University of Windsor

Maral Zafar Allahyari <zafaram@uwindsor.ca>

Copy Rigth Permission

2 messages

Maral Zafar Allahyari <zafaram@uwindsor.ca> Wed, Jan 21, 2015 at 11:14 AM
To: Ahmed Azab <azab@uwindsor.ca>

Dear Dr. Azab

I am requesting you to let me to use the material we have worked so far beside the paper we published and submitted.

Regards,

Ahmed Azab <azab@uwindsor.ca> Wed, Jan 21, 2015 at 11:14 AM
To: Maral Zafar Allahyari <zafaram@uwindsor.ca>

Maral: I am fine with this.

[Quoted text hidden]

--
Ahmed Azab, Ph.D.
Assistant Professor/UG Coordinator, Dept. of Industrial & Manufacturing Systems Engineering,
Director, Product Lifecycle Management (PLM) Research Lab
University of Windsor, 401 Sunset Ave.,
Windsor, Ontario, Canada N9B-3P4
Tel.: +1 (519) 253-3000 x4958
Fax: +1 (519) 973-7062
Email: azab@uwindsor.ca
Website: http://uwindsor.ca/engineering/imse/azab
Research Lab: http://www.uwindsor.ca/plm

https://mail.google.com/mail/u/0/?ui=2&ik=d3885d9aa8&view=pt&search=inbox&th=14b0dea894a802ad&siml=14b0dea894a802ad&siml=14b0deb84346e383 1/1

VITA AUCTORIS

NAME:	Maral Zafar Allahyari
PLACE OF BIRTH:	Mashhad, Iran
YEAR OF BIRTH:	1983
EDUCATION:	MASc. Industrial and Manufacturing System Engineering, University of Windsor, Windsor, ON 2014 MASc. Industrial Management, Guilan University, Rasht, Iran, 2008 BASc. Applied Mathematics, Ferdowsi University, Mashhad, Iran, 2001