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A Linear Programming Approach for Performance Evaluation of Multi-Type Production Lines Applied in Manufacturing Strategies Comparison

by

Shengxuan Wang

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2012

A Linear Programming Approach for Performance Evaluation of Multi-Type Production Lines Applied in Manufacturing Strategies Comparison

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September 19, 2012

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ABSTRACT

This research work develops an analytical approach to calculate the cycle time for each type of products in a multi-type serial production line with finite intermediate buffers and stochastic processing times. I consider the stochastic variables follow a certain distribution with mean and variance. The basic idea is to solve a linear programming approach modeling a production line operating with batch sized arrivals of different types of products and the cycle time can be found based on the batch restriction. A simulation model is created to test the relevance of the analytical approach and validate the proposed method's correctness. Scenarios such as switching processing from one product type to another without setup, with setup, and failure and repair are considered separately and comprehensive experiments combining these scenarios together are conducted as well. The failure and repair situation is stochastic as well. Experiment results are shown to validate the efficiency of the method. Periodic sampling approach is explored and considered while tackling these manufacturing strategies.

DEDICATION

I dedicate this thesis

To my beloved parents, friends, supportive supervisor,

and

To honor our God

ACKNOWLEDGEMENTS

This master program helps me to learn how to dig into finding the blank of researches, addressing a new approach and then applying to solve practical problems. That is quite fantastic experience. It is my great honor to enjoy this adventure.

I am grateful to my supervisor Dr. Walid Abdul-Kader whose insights, kindness, wisdom and encouragement have enlightened me to go through the hardship. A part of this research is based on his contribution and the research principle and procedure is taught by him. Without him, I cannot go so far.

I would like to express my gratitude to my committee members Dr. Leo Oriet and Dr. Ben Chaouch for their constructive criticism and valuable suggestions. Dr. Oriet hinted me to try manufacturing strategies like JIT while introducing the production line history in proposal. Dr. Chaouch helped me improving my methodology when pointing out one formula mistake. Because of their strict, the accuracy and validation of my research shows significantly important.

I also want to acknowledge the following staffs –Mrs. Angela Haskell, Ms Qin Tu, Mrs. Shipin Zhu, Dr. Guoqing Zhang, Mr. Dave McKenzie, Mr. Ram Barakat in the IMSE department.

TABLE OF CONTENTS

CHAPTER

I. INTRODUCTION

II. REVIEW OF LITERATURE

III. DESIGN AND METHODOLOGY

IV. **NUMERICAL RESULTS**

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The first machine assembly line for manufacturing system was created for Ford automobiles in 1913 as one of the greatest technology breakthrough. Before the line was built up, it took 12.5 hours to assemble an auto chassis. After the line came to its final form, each worker worked on a small unit of work and the chassis was moved mechanically. The average labor time for each auto chassis was shortened to 93 minutes. Such a great achievement thrilled the whole industries and this invention turned into a signal for machine age's coming.

In 1950s, Taiichi Ohno of Toyota Motor Company began to develop a new manufacturing system which would make Toyota more competitive with US motor manufacturers. For achieving cost reduction through elimination of waste, Just-in-Time (JIT) manufacturing strategy makes the inventory reduced to minimum levels. JIT is the extended level of Drum-Buffer-Rope (DBR) that considers the pull effect – the bottleneck impacts, on the contrary that Pure-Push (PP) only focuses on the push effect – the production efficiency but ignores the inventory cost.

The production line in my research is different from the general concept of the assembly line. It is a set of sequential operations with buffers allocated between each two machines. The materials are put through the whole operations to produce an end-product.

With the fundamental roles of the production lines in manufacturing system, considerable research has been conducted in the recent decades to estimate and improve their performance. As modern manufacturers cater for different tastes from different levels' customers, more and more production lines try to fit the requirement of flexible manufacturing. The analytical evaluation of multi-type production line becomes an inevitable problem.

This thesis is focused on developing an analytical approach to estimate stochastic multi-type production line with linear programming, then I apply this approach to make comparison of the manufacturing strategies between Just-in-Time, Drum-Buffer-Rope and Pure-Push considering the maximum total profit.

CHAPTER II

REVIEW OF LITERATURE

2.1 Production Line Performance Evaluation

Substantial researches have been conducted to analyze manufacturing systems. Some of the researches are focused on buffer effect towards the whole production line. While a few consider the unreliable situation in the series-parallel production line with approximation approach, other researches try to figure out methods on how to apply linear programming to estimate the performance of production lines. This research refers to their advanced methodology and the scope takes the hint from their scenario description and background information.

Among the early research works on using analytical approach to estimate the performance of unreliable production line, Gershwin and Schick (1983) evaluated the performance of three-stage unreliable production line by applying Markov chain model from upstream machine to the downstream machine repeatedly till to the third machine. But the scope of the model is restricted to the single-type unreliable 3-machine production line, which cannot be extended to longer production lines processing multi-type products because of the limitation of their iteration approach.

Another early research of Bowman (1960) applied linear programming to solve assembly-line balancing program. The author builds two approaches to solve one specific example, but the second method with linear programming because of requiring fewer variables and constraints makes it more advanced than the preceded method. His research is based on a specific problem in assembly-line which is different from production line concept. But the idea of better linear programming model with fewer variables and constraints helps me to judge which linear programming model is better among the following published models: Johri (1987), Schruben (2000), and Helber (2011) on stochastic processing time situation.

Approximation method is applied to evaluate the performance of production line in some researches.

By considering the defectives generated during manufacturing, Pourbabai (1990) presented another perspective to view this situation in production line with inspection. The production rate was estimated by observing the throughput of the bottleneck machine. The approximation in each machine was applied to identify the bottleneck machine. But the cycle time of production line was not discussed in this paper.

Tempelmeier and Burger (2001) presented an approximation method to calculate the production rate in a production line with generally distributed stochastic processing times. In addition, breakdowns and imperfect production are also considered. The scenarios description in my research refers to this paper's categories, reliable production, unreliable production and imperfect production.

Aziz et al. (2010) showed the idea of equivalent processing rate of series-parallel machines machine in flow line with failure and repair situations through the concept of Markov chain. They considered the parallel machines in one stage as one equivalent machine. The equivalent processing rates, the failure rates and the repair rates can be generated by using the Markov chain theory. Then they applied simulation to estimate the production line. This equivalent thought hints me in this research.

As well, in 2011, the thesis of Jarrahi (2011) applied a similar approach to estimate the cycle time in the multi-products scenario. He considered the multi-type unreliable production line as well. But the new arrival type cannot enter the production line until the setup of the whole production line has been completed, and in his research, the processing time follows general distributions.

Han and Park (2002) developed an approximation method for the average steady state throughput of a serial production line with quality inspection machines and based on this method, they proposed an analytical buffer assignment optimization.

The analytical approach by applying linear programming to estimate the performance of flow line has been discussed in several papers below.

Johri (1987) introduced the mathematical bottleneck analysis method and developed the linear programming with the objective of minimizing the cycle time to evaluate the performance of reliable production line. Input side constraints and output constraints are defined according to the machine behaviors with processing arrival items and moving processed items. Batch sized multi-type products production line is considered in his model.

Abdul-Kader (2006) developed an approach to estimate the production line performance by considering random failure and repair of workstations. Failure and repair were assumed to be exponentially distributed. A simulation model was created to prove the accuracy of this approach. The constraint in linear programming cannot be applied in the stochastic program. The fictive product thought to treat the time to repair in this paper is referred in my thesis.

The contribution of Schruben (2000) is based on continuous-time linear programming and considers the relationship among upstream machine, intermediate buffer and downstream machine. However, this approach is not realistic to use in a stochastic situation because the random processing time for each item will generate miscellaneous variables and constraints and makes it hard to set the model and calculate the results.

From the paper of Helber (2011), the performance of single type stochastic production line was evaluated by the discrete time linear programming. In my thesis, the multi-type stochastic production line evaluation approach is extended from his theory. He applied the periodic sampling to identify the quantity and capacity for each machine in each period. Helber’s evaluation criteria are exactly the same as in Hillier et al. (1993).

The literature information for production line evaluation is collected in Table 1. However, among all the literature, no research tries to evaluate the multi-type stochastic production line with the scenarios of unreliable production line. To fill this research blank, a part of my research is focused on developing more practical analytical evaluation method to deal with such manufacturing systems.

Year	Authors	Methodology	Contributed main method	Scenario	Objective
1960	Bowman	Analytical method	Two perspectives in establishing linear programming with the production capacity	One specific assembly line without a buffer between two sequential machines	Solving assembly line balance problem
1983	Gershwin& Schick	Analytical method	Iterating Markov chain	Three-stage unreliable single-type production line	Production rate
1990	Pourbabai	Approximation	Linear programming for each stage	Imperfect production line	Production rate
2010	Aziz	Approximation	View the series-parallel machines as one equivalent machine and the equations to calculate its attribute variables	Series-parallel multi-type unreliable production line	Production rate
2001	Tempelmeier & Burger	Approximation	Decomposition of production line into neighbor sequential machines with the queuing model	Imperfect single-type production line with generally distributed stochastic processing times	Production rate
1993	Hillier et al.	Simulation	Developing the optimal buffers allocation by discrete linear programming and validating it with simulation	Optimal allocation of buffer size in production line	Optimal buffer assignment
1987	Johri	Analytical method	Continuous linear programming with input side and output side constraints	Reliable multi-type & batch size deterministic production line	Cycle time
2006	Abdul-Kader	Analytical method	Insertion of fictive products to solve failure and repair situation with Johri's linear model	Unreliable multi-type & batch size deterministic production line	Cycle time
2000	Schruben	Analytical method	Continuous linear programming with simulated relationships among the upstream machine, downstream machine and intermediate buffer	A single-type production line	Production rate
2011	Helber	Analytical Method	Discrete linear programming with periodic view	A single-type stochastic production line	Production rate

Table 1: Literature Information for Production Line

2.2 Manufacturing Strategies

Manufacturing strategies are intended for the raw material procurement policies. The philosophy of several principle manufacturing strategies, such as Just-in-Time (JIT) production strategy, Drum-Buffer-Rope (DBR) strategy and Pure-Push (PP) strategy are discussed in some researches. Some applied these strategies in supply chain management and the other modified part of module to fit in the production line.

The research of Chakravorty (1996) made a comparative study of the manufacturing strategies in serial production line. The customer demand module was set at the rear of production line and the arrival part is linked with the output, which fits the thought of JIT that when an order arrives, raw materials enter the production line. But the processing times in all machines are all the same in his case.

Chakravorty (2001) discussed the DBR control mechanism in the job shop environment. Multi-type products and different manufacturing steps were considered. But the evaluation is conducted only by simulation and there is only four-machine job shop in the experiment. In a later research of Chakravorty and Atwater (2005), DBR scheduling logic was elaborated.

Hopp and Spearman (2003) demonstrated the essence of the pull and lean manufacturing systems. Pull in simplest terms means that no one upstream workstation should produce a good or service until the customer downstream asks for it.

Sule and Norris (1992) discussed the manpower assignment strategies in serial production line in pull rule. The cost structure is referred in my research to decide the optimal strategy.

Wu, Morris and Gordon (1994) applied DBR in the real furniture business. Job flow was categorized into five departments, in which the whole manufacturing part and transportation module was in the scope. At macroscopic level, the job flow is like a production line without buffers. Better makespan of DBR is found in his example through simulation.

Daniel and Guide Jr. (1997) investigated the impact of DBR towards the remanufacturing operations. The analytical performance measures were demonstrated to test the influence of DBR. Priority dispatch rules were arranged with different combinations.

The thesis of Ng (2007) applied the thought of JIT in the pull side with a specific project example to demonstrate the improvement of his new scheduling approach.

Kadipasaoglu, Xiang, Hurley and Khumawala (1998) discussed the relationship between DBR and JIT while investigating the effect of locations of protective capacity in production line using simulation. Better protective capacity can reach better performance through the less flow time. More experiments were made in Sloan (2001).

Betteron and Cox (2009) investigated the Drum-Buffer-Rope (DBR) scheduling and flow control in serial production line. They compared the production strategies of DBR and push model with exponential raw materials arrivals regarding to their throughput and WIP using simulation. The market customer requirement is not included in their research scope.

Most of the researches on JIT and DBR are based on simulation tools. No research applied analytical approach to evaluate the multi-type stochastic production line with DBR or JIT manufacturing strategy.

CHAPTER III

DESIGN AND METHODOLOGY

3.1 Problem Definition

The three ways concerned about transferring parts between machines as introduced in the review of Papadopoulos (1996) on queuing theory in manufacturing systems are: 1) Synchronous transferring refers to the move between machines simultaneously and it can also be called transfer line; 2) Asynchronous transferring means the processing rates among different stages are not the same; and 3) Continuous transferring allows the parts to move in a constant speed. Based on that theory, my research deals with asynchronous transferring which presents the distinctive processing time in each machine, and intermediate finite buffers are located between two sequential machines.

The serial production line is widely applied in the modern manufacturing system. Each buffer is located between two sequential machines. This layout can help the machines to avoid the starvation and blocking phenomena. When the upstream machine is down or starved, the downstream machine can still process the items from the intermediate buffer between them. Based on the similar logic, when the downstream machine is down or blocked, the upstream machine can still process the items which will be stored in the intermediate buffer.

With slight change of each machine, different types of products can enter the same production line to satisfy the customers' different requirements. In Jarrahi (2011), the multi-type production line structure is presented. I referred to his contribution and describe the multi-type production line in Figure 1. The dashed rectangle represents the arrival section and the arrows inside are the product types in their separate batch sizes, N_j.

Subscript j represents the product type. They enter the production line from type 1 to type j one by one in increasing order. The other rectangles represent the machines, and the triangles represent the buffers. The production line has K machines and K-1 buffers.

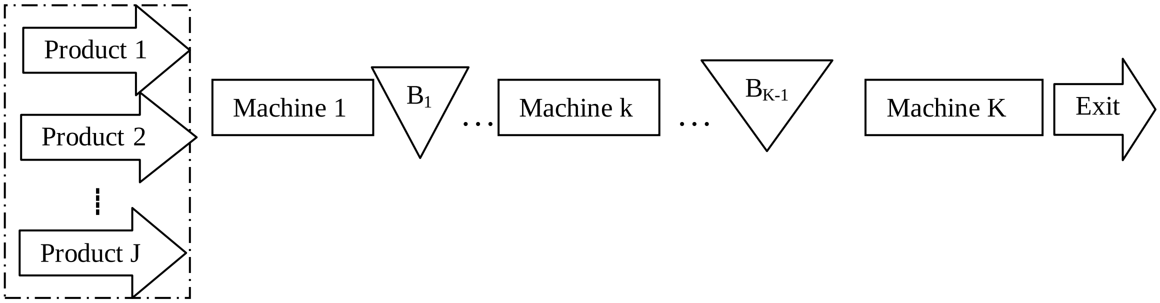

Figure 1: Multi-type Serial Production Line with K Stations and K-1 Buffers

I consider three scenarios in this thesis; these are:

- Scenario 1, multi-type reliable stochastic production line without setup while switching from one product-type to another.
- Scenario 2, multi-type reliable stochastic production line with setup while switching from one product-type to another.
- Scenario 3, multi-type unreliable stochastic production line with setup while switching from one product-type to another.

Scenario 1 fits in the circumstance that the difference of product-types only exists in the materials qualitative attribute and all the manufacturing steps in the production line are the same, therefore, setup is not required. In garment industry, like producing the same style of red coats and blue coats, the difference among them is predetermined by the cloth of different colors. The similar situation can be found for the polymer materials, like white plastic fans and black plastic fans. In such a situation, once the former type of products completes, the latter upcoming batch can follow up immediately without the need for any setup.

Scenario 2 describes the circumstance considering setup while changing the different types in the reliable production line context. The preparation time in each machine is required when switching from the last item of former type product to the arrival of the new type product.

Scenario 3 demonstrates that the production line is not reliable and the failure and repair situation will be taken into consideration.

Scenarios 2 and 3 above have been discussed previously in the literature: Johri (1987) introduced a continuous-time model to solve the multi-type reliable production line, and Abdul-Kader (2006) extended the solution to unreliable situation, but the processing time is deterministic in their cases. Jarrahi (2011) considers similar scenarios for the cases of general distribution processing times and his setup occurs when the last item exits the whole production line. Helber (2011) created a discrete-time linear programming to evaluate the production rate and optimal buffer assignment for single-type stochastic reliable production line. In my research, the linear programming will be used to develop the optimal buffer assignment in the multi-type stochastic unreliable production line.

The evaluation criteria in each scenario are the production rate, the cycle time for each product-type and the total cycle time for all product types. The developed analytical approach will be validated and compared with the simulation results.

3.2 Multi-type Production Line Performance Evaluation Methodology

The detailed methodology for solving the three above-indicated scenarios will be addressed in this section. Before the thesis demonstrates the linear programming, the notations and the assumptions used in periodic sampling view and the scenarios considered are described in the following subsection.

3.2.1 Notations

Periodic Sampling:

$k=1,\ldots,K$	machines in the flow line.
$t=1,...,T$	periods
$j=1,\ldots,J$	product-types
Q_{ktj}	production quantity for machine k during period t for product type j
d_{kmj}	duration time for the mth item of product type j completed in machine k
C_{ktj}	production capacity for machine k during period t for product type j
T_{Kj}	cycle time to finish product type j with batch size N_j in last machine K
N_j	batch size for product type j in multi-type production line

Scenario 1:

TE_j	enough periods in objective function consumed for the completion of all the items of batch size N_j for product type j
b_k	capacity of buffer k
b_{tot}	total buffer capacity between all the machines
Y_{ktj}	inventory quantity in buffer k for product type j at beginning of period t
TS_{Kj}	completion period for product type j in the last machine K
TS_{Kj}’	equivalent completion period for product type j in the last machine K

n_{j+1}	n^{th} item of product type j+1
T_{tot}	total cycle time which is the time consumed for the completion of all the batches of all the product types in the last machine
p_{kj}	mean processing time in machine k for product type j
PR_j	production rate for product type j
TB_{kj}	time consumed in the downstream machine k for processing the lowest buffer level from buffer i(shown in Reduced time subsection) to last buffer K and one item remaining in the downstream machine
$NIT_{K,j+1}$	idle time in the last machine K caused by the new arrival of product type j+1
bl_k	buffer inventory in buffer k
$RT_{tot,j}$	overlapped time when switching from product type j-1 to product type j
IT_{Kj}	idle time consumed for the type j in the last machine K which is counted after the last item of type j-1 entering the first buffer
T_j	cycle time for product type j is defined as the duration of time between the first item entering the first machine and the last item of the same type exiting from last machine

Scenario 2:

St_{kj}	time to setup machine k to process product type j

Scenario 3:

TTF_{kij}	consumed time from the end of the $(i-1)^{th}$ repair to the beginning of

	the i^{th} time failure of machine k
$MTTF_{kj}$	mean time to failure of machine k for product type j
TTR_{kij}	consumed time from the beginning of the $(i-1)^{th}$ failure to the end of the i^{th} time repair of machine k
$MTTR_{kj}$	mean time to repair of machine k for product type j
CF_{ktj}	production capacity of machine k during period t for product type j with the fictive products' insertion approach

3.2.2 Assumptions

1. If the buffer on the output side of a machine fills up, then the machine has to temporarily stop production until the buffer has space for more output. It is known as blocking. The last machine is never blocked.

2. A machine may also have to stop if there is no input available when the upstream buffer is empty. This is the starvation phenomenon. It is also assumed that the first machine has always input material to process; and therefore, it cannot starve.

3. In the model, the experiment periods TE_j is defined long enough for the completion of all the items of batch size N_j for product type j.

4. The stochastic processing time follows certain distributions. In the first group of my experiments, it follows normal distribution with a known mean and standard deviation.

5. The failure time and the repair time both follow exponential distributions with a known mean.

6. Cycle time for each product type is defined as the duration of time between the first item entering the first machine and the last item of the same type exiting from last machine.

7. Cycle time of last machine K is defined as the duration of time elapsed between the first item entering the last machine and the last item of the same product type exiting from the last machine.

8. Total cycle time is defined as the duration of time between the first item of the first product type entering the last machine and the last item of the last product type exiting from the last machine.

9. Repairperson can immediately repair a machine when it fails.

10. Each machine can only process one item at a time.

11. The new arrival type waits in line before the last item of the former type exits from the first machine. Therefore, no waiting time for the first machine will be wasted while changing types.

3.2.3 Periodic Sampling

Johri's continuous-time model (1987) considers deterministic processing times for each type of products, but his approach cannot be applied in stochastic production. Stochastic production line represents the processing time is consisted of random values. If applied Johri's approach, many random processing time variables in stochastic situation have to be inputted in the constraints, which will make Johri's model cumbersome to solve.

Even though the processing time is stochastic, in my assumption it follows a type of distribution with a known mean. In order to input the stochastic processing time into discrete-event linear programming, I refer to the contribution of Helber (2011) on the periodic view to get production capacity in certain time unit. But to fit the multi-type situation, the three dimension subscript for the capacity and the quantity is required. Figure 2 describes the definition of one period T_1, .. T_i is composed of n common time units.

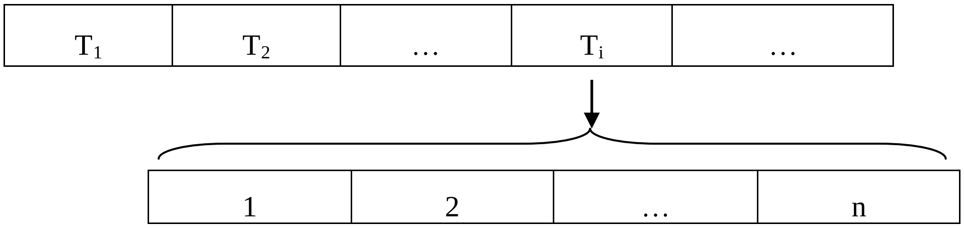

Figure 2: Definition of One Period Composed of n Common Time Unit

Now in Figure 3a below, Q_{ktj} is the production quantity of machine k during period t for product type j. I count the number of products completed or processed in the time period and set it as Q_{ktj}. d_{kmj} represents the duration time for item m of product type j completed on machine k. The first item is completed in duration time d_{k1j}, the following item 2 is processed in duration time d_{k2j}. But the duration time d_{k3j} for the third item has exceeded the range of time period T_1. Thus, the production quantity for product type j in time period 1 is 3. In the following periods, the same approach is applied to calculate the rest of Q_{ktj}.

With the same theory, the production capacity C_{ktj} for machine k during period t for product type j can be determined in Figure 3b. However, the duration time d_{kmj} is replaced by the processing time p_{kmj} to get the capacity for each machine in each period. Figures 3a,b demonstrate the difference in the periodic view of the production quantity Q_{ktj} and production capacity C_{ktj}.

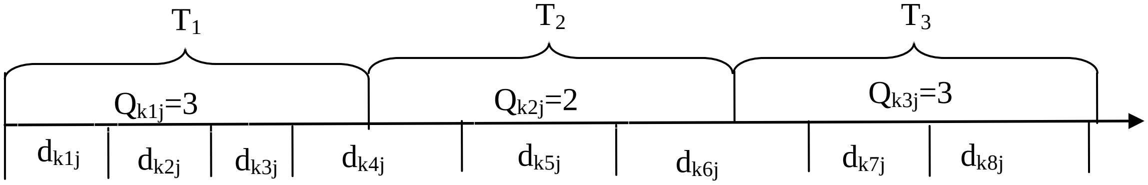

Figure 3a: Periodic View of the j Type Production Quantity of Machine k in 3 Periods

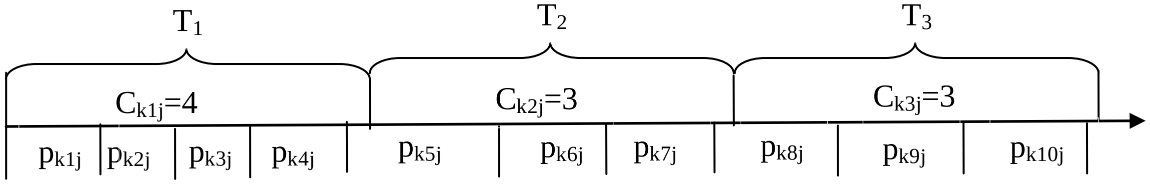

Figure 3b: Periodic View of the j Type Production Capacity of Machine k in 3 Periods

In Figure 4 below, assuming that batch size for type 1 products is 80, for type 2 products is 100 and for last product type j is 13, I set the different types of products on separate time axis and restrict them with assumptive batch sizes to get the cycle time of machine K. The cycle time T_{Kj} represents the duration time between the first item of product type j exiting from last machine K and the last item of the same type exiting from last machine K.

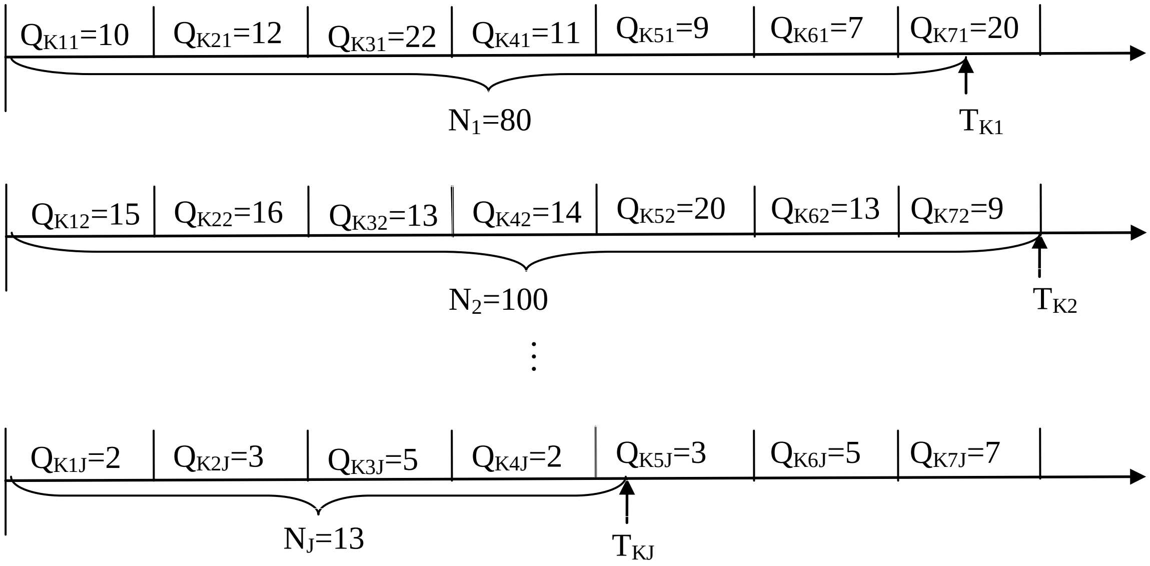

Figure 4: Periodic View of Multi-type Product with Batch Sizes of the Last Machine

The production quantity Q_{Ktj} and the batch size N_j are predefined to show the basic idea about restricting each time axis with each batch size to calculate T_{Kj}. In the following subsections, the linear programming approach will show the detailed procedure to reach T_{Kj}.

3.2.4 Scenario 1, multi-type products in reliable stochastic production line without setup occurring while switching types

In this scenario, the failure and repair situation is not taken into consideration due to the reliability of the production line. The setup procedure is not considered as well because the difference for different product types is predetermined by the raw materials and rest steps in the production line are all the same among all the product types. Thus, there's no necessity for setting up the machines to produce new types. Figure 5 describes the situation while switching from product type j to j+1.

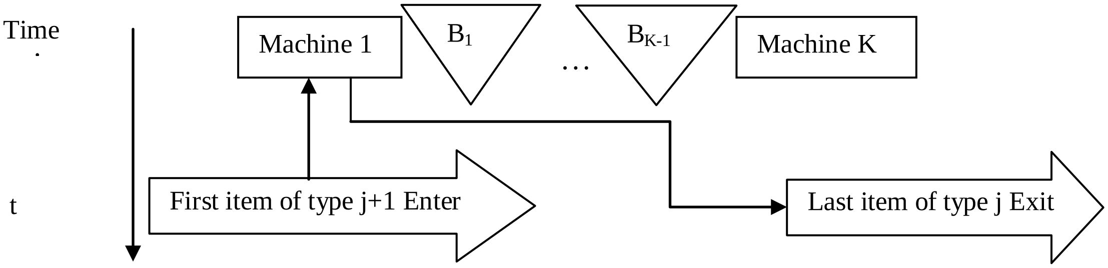

Figure 5: Non-setup Situation while Switching Product type j to Type j+1 Products in Scenario1

The mathematical model is referred to the contribution of Helber et al. (2011). The discrete-time model (Helber et al. 2011) instead of the continuous-time model (Jorhi 1987), is applied to solve the stochastic processing problem.

Continuous-time model treats the time as a common concept. It does not transfer the time concept into the discrete periods. The objective function of the continuous-time model (Jorhi 1987) is to minimize the time with known capacities; and the constraints are set to restrict the duration time based on the batch sizes of different product types.

Because according to Assumption 4, processing time of each product type is stochastic, if applied continuous-time model, the linear programming have to contain the redundant processing time variables. That makes it impossible to be solved.

With the periodic sampling in the discrete-time model (Helber 2011), it does not rely on the linear programming to restrict the time length; on the contrary that it divides the continuous time into discrete periods.

To solve the multi-type production line, the three dimension matrix is required by adding the subscript of product type j. The objective function (Eq.1) of the discrete-time model is focused on maximizing the quantity while the constraints restrict the quantity based on the capacity. Considering the restriction of the batch sizes makes the estimation of the cycle time for each type of products possible.

$$\text{Max} \sum_{k}^{K} \sum_{t}^{TE_j} \sum_{j}^{J} Q_{kt\mathrm{j}} \qquad (1)$$

Subject to:

$$Y_{ktj} + Q_{ktj} = Y_{k,t+1,j} + Q_{k+1,t,j} \qquad \mathrm{t} = 1,\ldots,\mathrm{T},\ \mathrm{k} = 1,\ldots,\mathrm{K}-1,\ \mathrm{j} = 1,\ldots,\mathrm{J} \qquad (2)$$

$$Q_{ktj} \le C_{ktj} \qquad \mathrm{t} = 1,\ldots,\mathrm{T},\ \mathrm{k} = 1,\ldots,\mathrm{K},\ \mathrm{j} = 1,\ldots,\mathrm{J} \qquad (3)$$

$$Y_{ktj} \le b_k \qquad \mathrm{t} = 1,\ldots,\mathrm{T},\ \mathrm{k} = 1,\ldots,\mathrm{K} \qquad (4)$$

$$Y_{ktj} = 0 \qquad \mathrm{t} < \mathrm{k},\ \mathrm{j} = 1,\ldots,\mathrm{J} \qquad (5)$$

$$Q_{ktj} \ge 0 \qquad \mathrm{t} = 1,\ldots,\mathrm{T},\ \mathrm{k} = 1,\ldots,\mathrm{K},\ \mathrm{j} = 1,\ldots,\mathrm{J} \qquad (6)$$

$$Y_{ktj} \ge 0 \qquad \mathrm{t} = 1,\ldots,\mathrm{T},\ \mathrm{k} = 1,\ldots,\mathrm{K},\ \mathrm{j} = 1,\ldots,\mathrm{J} \qquad (7)$$

$$b_k \ge 0 \qquad \mathrm{k} = 1,\ldots,\mathrm{K} \qquad (8)$$

Constraint 2 represents the balance thought in the production line. No new item is created in the machines and no scrap is found in the reliable production line. Buffer inventory at the beginning of period t+1, should be equal to its inventory at the beginning of previous period t plus the output of upstream machine k in period t minus the output of downstream machine k+1 in period t. That balance thought of two sequential machines in two sequential periods can be shown in Figure 6.

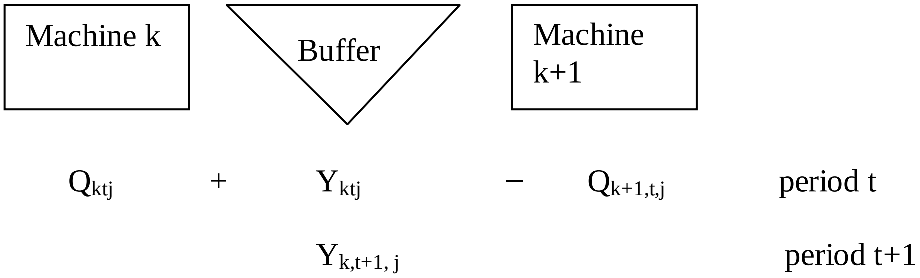

Figure 6: Balance Thought of Two Sequential Machines in Two Sequential Periods

Constraint 3 shows the quantity of product type j produced in period t for machine k cannot exceed the capacity of machine k in period t. I assume the stochastic processing time for each product type follows one type of distribution. I can generate random numbers as the processing time for each item according to its specific distribution. The capacity can be generalized from the method shown in Figure 2b above.

From Constraint 4, the buffer inventory should not exceed its capacity.

Constraint 6 is related to Constraint 2. The listing of constraint 2 for three-machine production line in three periods is shown as following.

$Y_{111}+Q_{111}=Y_{121}+Q_{211}$

$Y_{121}+Q_{121}=Y_{131}+Q_{221}$

$Y_{221}+Q_{221}=Y_{231}+Q_{321}$ Y_{211} is not found.

If there is no Constraints 5, Y_{ktj} that cannot be found in Constraint 2 will be any value which satisfies Constraint 4. Therefore, because of lack of initializing variables in Constraint 2, Constraints 5 is necessary to initialize the buffer inventory in the early periods.

Constraints 6,7,8 represent that there are no negative buffer inventory, buffer capacity and the quantity processed in each machine for each period.

The inputted known variables are capacity C_{ktj} and buffer capacity b_k. All the remaining variables quantity Q_{ktj} and buffer inventory Y_{ktj} are generated through the linear programming model.

To estimate the cycle time for each type in last machine, the exact time point when the last item exiting the production line is extremely important. Therefore, the completion period TS_{Kj} and the equivalent completion period TS_{Kj}' both are contributed to get the estimated time point when the batch finished in the last machine K for product type j.

Equations 9a,b below show the product type j exits the production line in period $TS_{Kj}+1$ when the batch size N_j was finished in the last machine.

$$\sum_{t}^{TS_{Kj}} Q_{Ktj} \le N_j \tag{9a}$$

$$\sum_{t}^{TS_{Kj}+1} Q_{Ktj} \ge N_j \tag{9b}$$

After given the stochastic processing time for each machine, I can find the bottleneck machine which has the longest mean processing time. The periods consumed all items for type j TE_j in object function can be estimated though the equation as

$TE_j = \left\lceil \frac{N_j \cdot \max_k \{p_{kj}\}}{n} \right\rceil$. p_{kj} is the mean processing time in machine k for product type j.

But TE_j here is only roughly estimated and it may be still not enough periods for completing the batch size N_j. That means the completion period TS_{Kj} cannot be found because Equations 9ab will be never satisfied without the long enough period TE_j in objective function. If so, TE_j needs to be extended by incrementing one extra period till it can be long enough to find completion period TS_{Kj}. Through periodic sampling, C_{ktj} in the extra periods needs be generated to input in Constraint 3.

Equivalent completion period TS_{Kj}' is defined to estimate the exact time point when the whole batch size of type j is completed in last machine K and it shows as following Equation 10.

$$\frac{N_j - \sum_{t}^{TS_{Kj}} Q_{Ktj}}{TS_{Kj}'} = \frac{\sum_{t}^{TS_{Kj}} Q_{Ktj}}{TS_{Kj}} \qquad (10)$$

The relationship between the completion period TS_{Kj} and the equivalent completion period TS_{Kj}' in last machine K for product type j can be described in Figure 7.

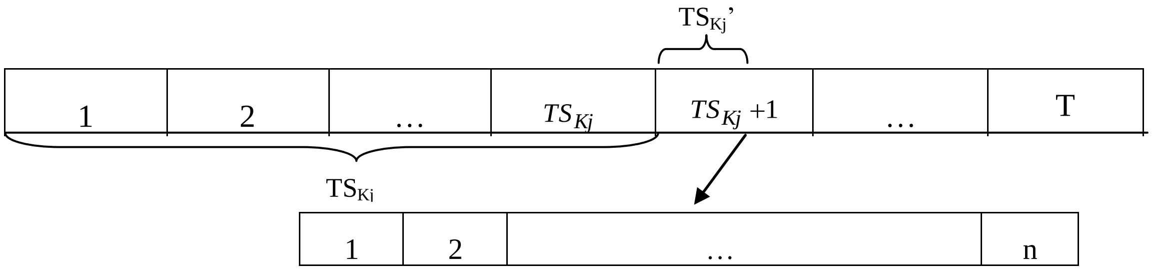

Figure 7: Relationship between Completion Period TS_{Kj} and Equivalent Completion Period TS_{Kj}'

After calculating TS_{Kj} and TS_{Kj}', T_{Kj} can be shown as following Equation 11 which is the cycle time for last machine which is the duration time from the first item of type j enters to the last item of type j exits in last machine K. n is the time units composed in each period.

$$T_{Kj} = (TS_{Kj} + TS_{Kj}')*n \qquad (11)$$

However, T_{Kj} is not the cycle time in Assumption 6. The reduced time is required to be considered to generate the cycle time for product type j.

The following Figures 8ab show the relationship among the total reduced time for type j+1 products $RT_{tot, j+1}$, the cycle time for type j+1 products in last machine K $T_{K,j+1}$ and the cycle time for type j+1 products T_{j+1} in a 4-machine production line while changing from product type j to type j+1. n_{j+1} represents the n^{th} item of type j+1.

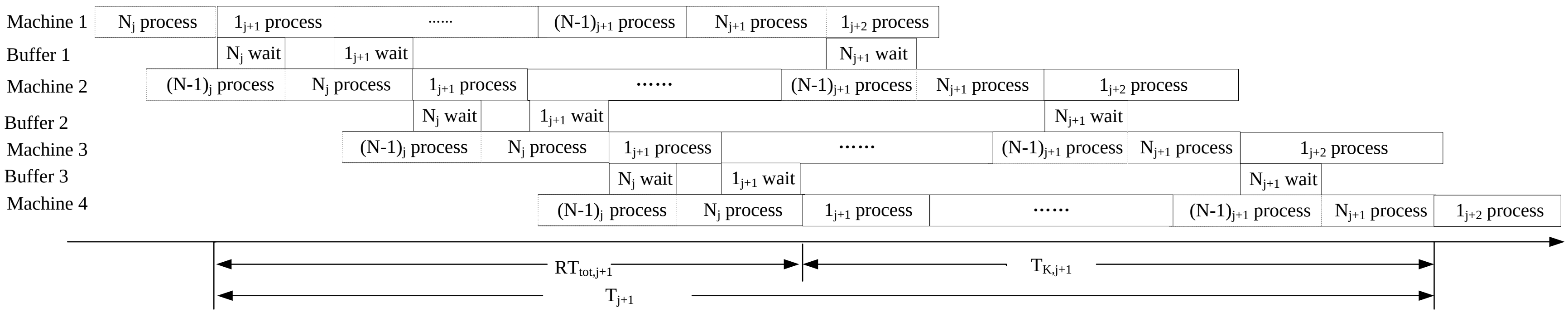

Figure 8a: Reduced Time for type j+1 products $RT_{tot,j+1}$, Cycle time for last machine K $T_{K,j+1}$ and Total Cycle time for type j+1 products T_{j+1} in a 4-machine Production Line

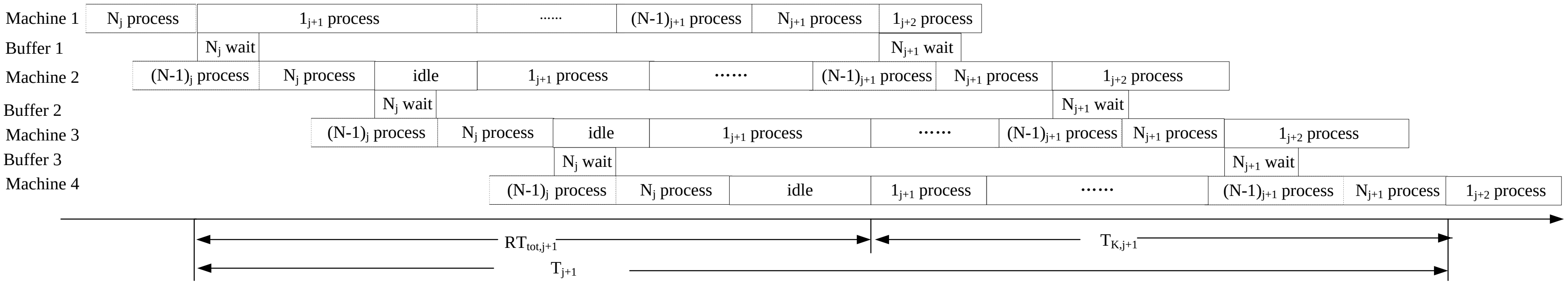

Figure 8b: Reduced Time for type j+1 products $RT_{tot,j+1}$, Cycle time for last machine K $T_{K,j+1}$ and Total Cycle time for type j+1 products T_{j+1} in a 4-machine Production Line

3.2.4.1 Reduced Time

The reduced time from Figures 8ab can be described as the duration between the last item of former type exiting from the first machine and the first item of new arrival type entering the last machine. In Figure 8a, there is no idle time in the last machine created by the new type postponed arrival. But in Figure 8b, when the processing time for the new type lasts long enough, there is the idle time existed in the last machine due to the late arrival of the first item of the new type.

After solving the linear program (Eq.1-8), I can get the inventory in buffer k at the beginning of the completion period TS_{Kj} (Eq.9ab) by substituting period t for period $TS_{Kj}+1$.

I assume $Y_{k,TS_{Kj}+1,j}$ is the inventory in buffer k for product type j when the last item of batch N_j exits from the first machine because $TS_{Kj}+1$ means the beginning of completion period plus 1 which is close to the equivalent completion period.

All the items of product type j remaining in the production line will be produced in the last machine K after the last item of batch N_j exiting from the first machine. The total reduced time can be treated as the consumed time for processing those remaining items of the former type and waiting for the first item of new type product. According to Assumption 1, the block cannot exist in the last machine. The idle time in last machine K is consisted of the idle time caused by the remaining former type products and the idle time caused by the first item of the new type products. Both of them are composed in the total reduced time.

The total reduced time is described as the following Equation 12. p_{kj} is the mean processing time in machine k for product type j, $IT_{K,j}$ is the idle time in the last machine for product type j and K-1 means the item number in the downstream machines according Assumption 10. $NIT_{K,j+1}$ is the idle time in the last machine K caused by the new arrival type j+1 products.

$$RT_{tot,\,j+1} = (\sum_{k}^{K-1} Y_{k,TS_{Kj}+1,\,j} + (\mathrm{K}-1)) \cdot p_{Kj} + IT_{K,\,j} + NIT_{K,\,j+1} \qquad (12)$$

TB_{kj} is the consumed time in the downstream machine k for processing the lowest buffer inventory from buffer i to last buffer K and one item remaining in the following machine. TB_{kj} can be calculated as the following Equations 13 and bl_k is the buffer inventory in buffer k.

$$TB_{kj} = (MIN_i^K \{bl_k\} + 1) \cdot p_{kj} \qquad (13)$$

In order to calculate the IT_{Kj} and NIT_{Kj}, I need to compare with each buffer with recursion to trace down the idle time part in the last machine K.

For product type j:

At the beginning, set the idle times in all the buffers are 0. $IT_{k,j}=0$ and $NIT_{k,j+1}=0$. Initialize the buffer inventory $bl_k = Y_{k,TS_{Kj}+1,\,j}$.

In stage 1, based on Equation 11, find $Max_k\{TB_{kj}\}$, then $IT_{1,j} = IT_{1,j} + Max_k\{TB_{kj}\} - TB_{Kj}$, re-evaluate the inventory level in buffer 1, $bl_1 = bl_1 - Min\{bl_k\}$. Repeat procedure above till $bl_1 \leq 0$, which represents no items left in buffer 1. If $p_{1,\,j+1} > (Y_{1,TS_{Kj}+1,\,j} + 1) \cdot p_{2,\,j}$, that means there is idle time caused by the first item of the new type, which can be represented as following:

$NIT_{2,\,j+1} = p_{1,\,j+1} - (Y_{1,TS_{Kj}+1,\,j} + 1) \cdot p_{2,\,j}$, then move to stage 2.

In stage 2, move to buffer 2, find $\mathrm{Max}_k\{TB_{kj}\}$, then $IT_{2,j} = IT_{2,j} + \mathrm{Max}_k\{TB_{kj}\} - TB_{Kj}$, re-evaluate the inventory level in buffer 1, $bl_2 = bl_2 - \mathrm{Min}\{bl_k\}$. Repeat procedure above till $bl_2 \leq 0$, which represents no items left in buffer 1. If $NIT_{2,j+1} + p_{2,j+1} > (Y_{2,TS_{Kj}+1,j} + 1) \cdot p_{3,j}$, that means there is idle time caused by the first item of the new type, which can be represented as following:

$NIT_{3,j+1} = NIT_{2,j+1} + p_{2,j+1} - (Y_{2,TS_{Kj}+1,j} + 1) \cdot p_{3,j}$, then move to stage 3.

In stage i, move to buffer i-1, find $\mathrm{Max}_k\{TB_{kj}\}$, then $IT_{i,j} = IT_{i,j} + \mathrm{Max}_k\{TB_{kj}\} - TB_{Kj}$, re-evaluate the inventory level in buffer 1, $bl_i = bl_i - \mathrm{Min}\{bl_k\}$. Repeat procedure above till $bl_i \leq 0$, which represents there is no items left in buffer 1. If $NIT_{i,j+1} + p_{i,j+1} > (Y_{i,TS_{Kj}+1,j} + 1) \cdot p_{i+1,j}$, that means there is idle time caused by the first item of the new type, which can be represented as following:

$NIT_{i+1,j+1} = NIT_{i,j+1} + p_{i,j+1} - (Y_{i,TS_{Kj}+1,j} + 1) \cdot p_{i+1,j}$, then move to stage i+1.

Repeat the stage above till to the last buffer K-1, the idle time for the last machine can be described as per Equation 14:

$$IT_{K,j} = \sum_{k}^{K-1} IT_{kj} \tag{14}$$

The following Figure 9 demonstrates the procedure to get the reduced time after solving the linear programming. Y means it satisfies the condition and N means it does not satisfy the condition.

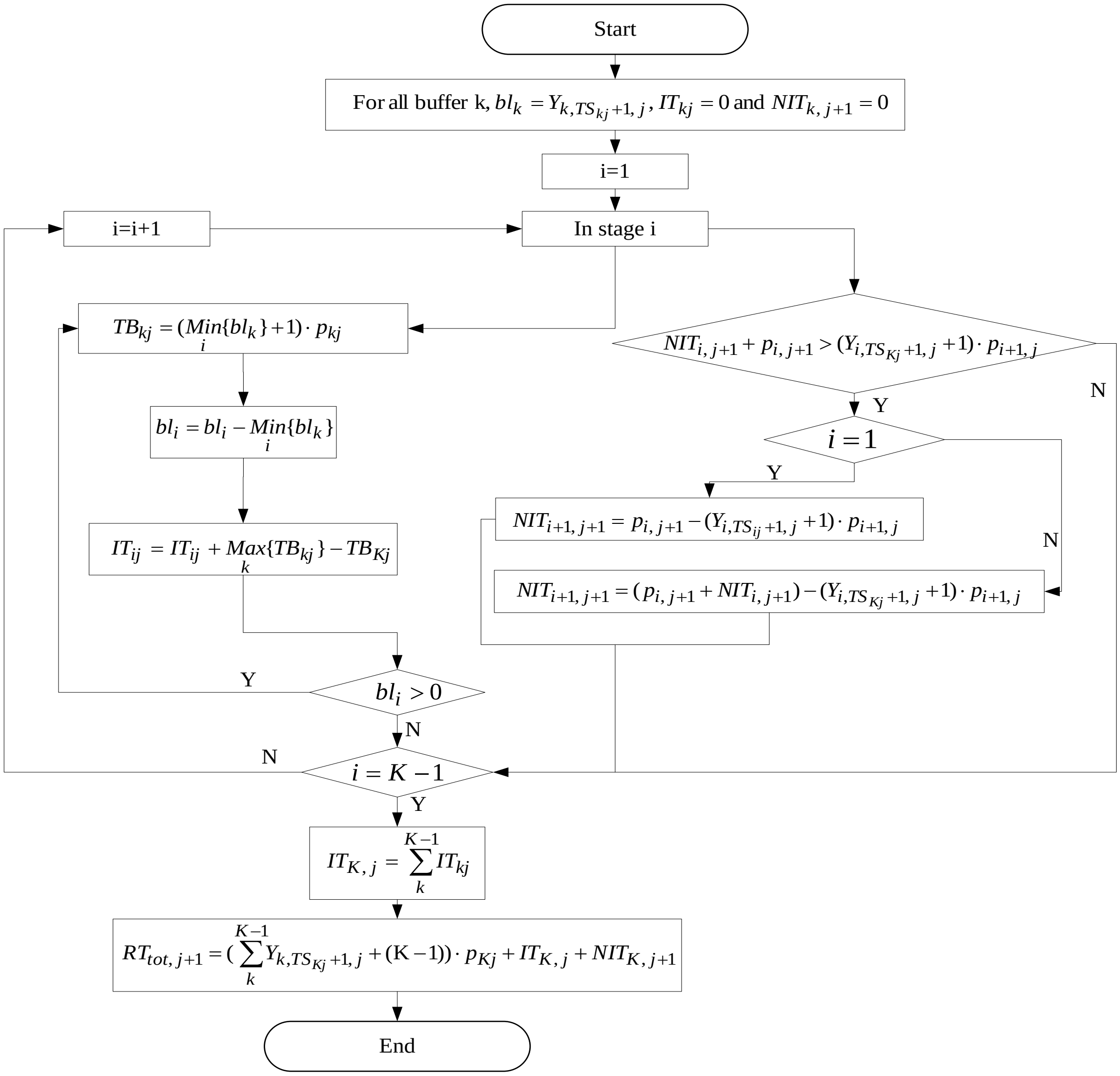

Figure 9: Procedure on Calculating the Reduced Time

3.2.4.2 Cycle Time and Production Rate

By considering the reduced time, the cycle time for product type j should be calculated exclude the warm up period as shown in Equation 15.

$$T_j = (TS_{Kj} + TS_{Kj}')n + RT_{tot,j} \tag{15}$$

Idle time caused by the new arrival type should be taken into consideration for total cycle time. Because T_{Kj} is the cycle time in last machine K between the first item for type j entering the last machine K and the last item exiting from the last machine K. The idle time caused by the new arrival type NIT_{Kj} is not included in Figure 8b.

Total cycle time is the duration time for the completion of all the batches of all the types' products in the last machine, which can be shown in Equation 16. According to Equations 8 and 10, the cumulative quantity for type j during the cycle time is actually its batch size. The production rate for type j is the quantity of product type j exit per time unit, which is described as in Equations 17 ab.

$$T_{tot} = \sum_{j}^{J} (T_{Kj} + NIT_{Kj}) \tag{16}$$

$$PR_j = \frac{\sum_{t}^{TS_{Kj}+TS_{Kj}'} Q_{Ktj}}{T_j} \tag{17a}$$

$$PR_j = \frac{N_j}{(TS_{Kj} + TS_{Kj}') \cdot n + RT_{tot,j}} \tag{17b}$$

The following Figure 10 is the flow chart to show the steps in the analytical approach in scenario 1.

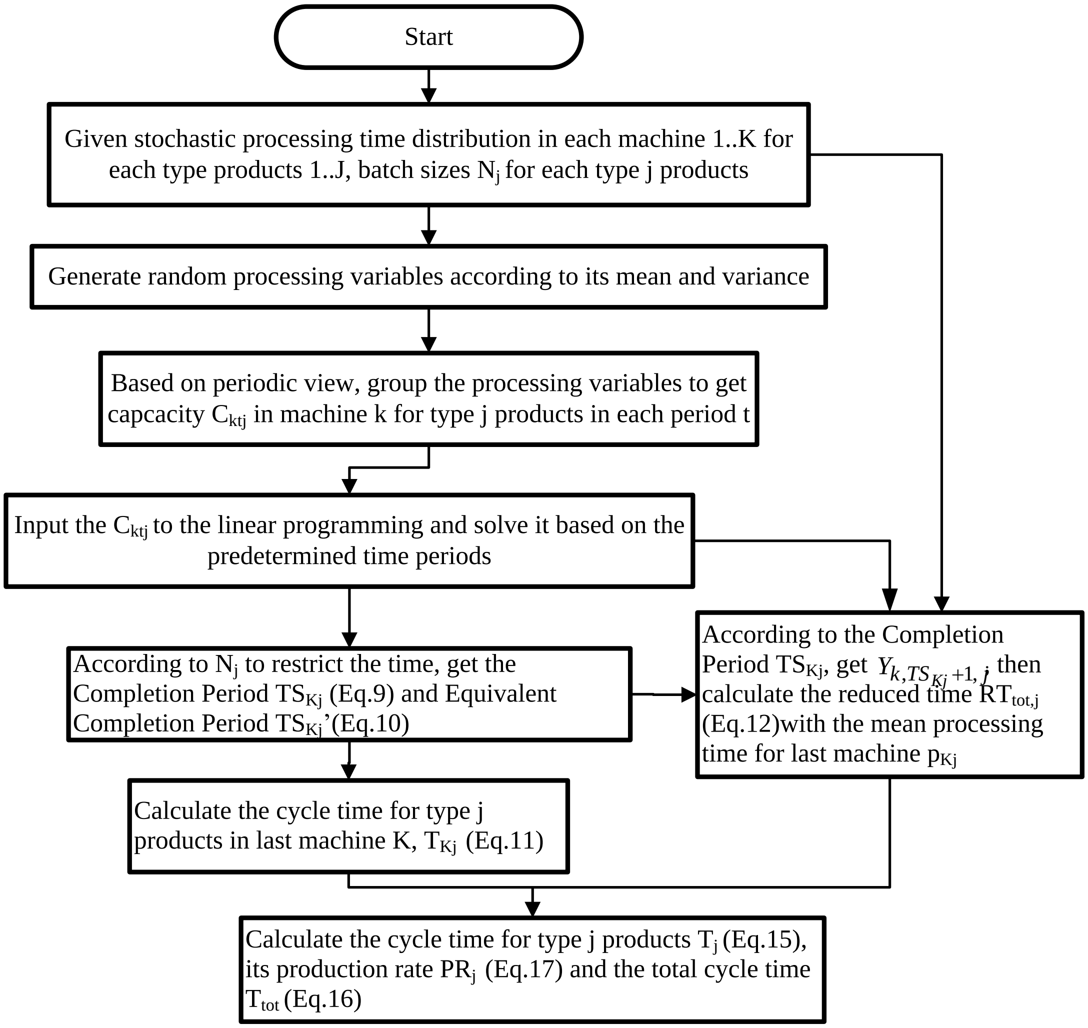

Figure 10: Flow Chart of the Steps in Analytical Approach in Scenario 1

3.2.5 Scenario 2, multi-type products in reliable stochastic production line with setup occurring while switching types

In this scenario, the reliable stochastic production line means there is no failure and repair but the processing time follows some stochastic distribution, which is the same as the scenario 1. However, the setup in each machine is necessary to be taken into consideration, which occurs for the arrival product type in each machine before the first item of new arrival product type enters this machine. The following Figure 11 describes the setup situation in this scenario.

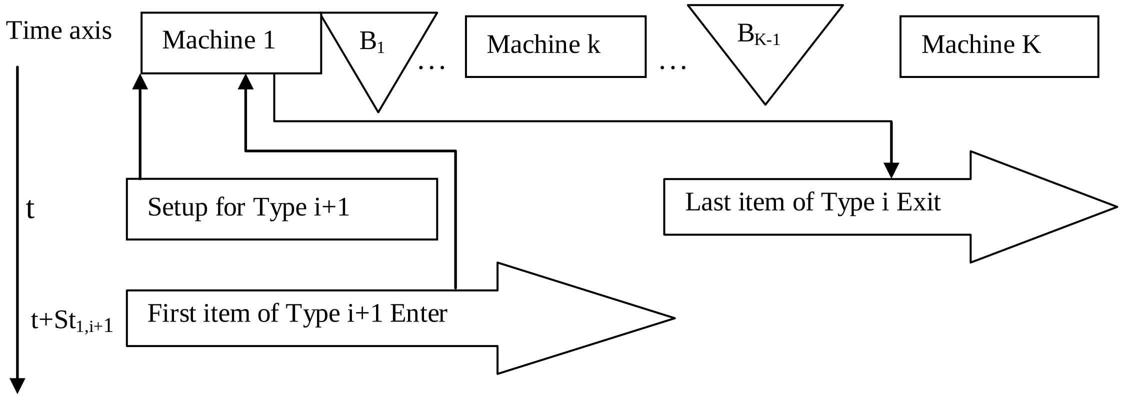

Figure 11: Setup Situation while Switching Product Type i to Product Type i+1 in Machine 1 in Scenario 2

In Scenario 2, the linear model to estimate the cycle time in last machine for each product type is the same as the objective function (Eq.1) and the Constraints (Eq.2-8) in Scenario 1. However, the way to calculate the cycle time and production rate should be modified to fit in this setup situation and the total cycle time should be added the overlapped setup time in the last machine.

3.2.5.1 Reduced Time

As mentioned in Scenario 1, reduced time is the duration time between the last item of former product type exiting from the first machine and the first item of the new arrival product type entering into the last machine, and it will be applied to calculate the cycle time for the latter arrival type.

The duration time to process the same inventory of last buffer K-1 and the item remaining in the downstream machine with adding the setup time is changed as per Equation 18:

$$TB_{kj} = (MIN_i^K \{bl_k\} + 1) \cdot p_{kj} + St_{k,j+1} \qquad (18)$$

Like TB_{kj} is adjusted to fit the setup situation, the total reduced time should consider the setup time in the last machine, because the first item of the new product type gets processed only after the setup is done. Therefore, the total reduced time can be described as per Equation 19:

$$RT_{tot,j+1} = (\sum_{k}^{K-1} Y_{k,TS_{Kj}+1,j} + (\mathrm{K}-1)) \cdot p_{Kj} + IT_{K,j+1} + St_{K,j+1} \qquad (19)$$

The idle time is calculated through the same stages in Scenario 1.

3.2.5.2 Cycle Time and Production Rate

The cycle time in Scenario 2 is defined as the period from the beginning of the setup in the first machine to the last item of the product type exiting from the last machine, which is given by Equation 20.

$$T_j = (TS_{Kj} + TS_{Kj}')n + RT_{tot,\,j} \tag{20}$$

The total cycle time for all the product types with setup situation can be shown in Equation 21. The concept of total cycle time in Scenario 2 remains the same as in Scenario 1. But the setup in the last machine needs to be considered. The production rate for product type j is the quantity of product type j exit per time unit, which is described as in Equation 22.

$$T_{tot} = \sum_{j}^{J}((TS_{Kj} + TS_{Kj}') \cdot n + \mathrm{St}_{\mathrm{Kj}} + NIT_{Kj}) \tag{21}$$

$$PR_j = \frac{N_j}{(TS_{Kj} + TS_{Kj}') \cdot n + RT_{tot,\,j}} \tag{22}$$

The following Figure 12 shows the steps I apply in the scenario 2.

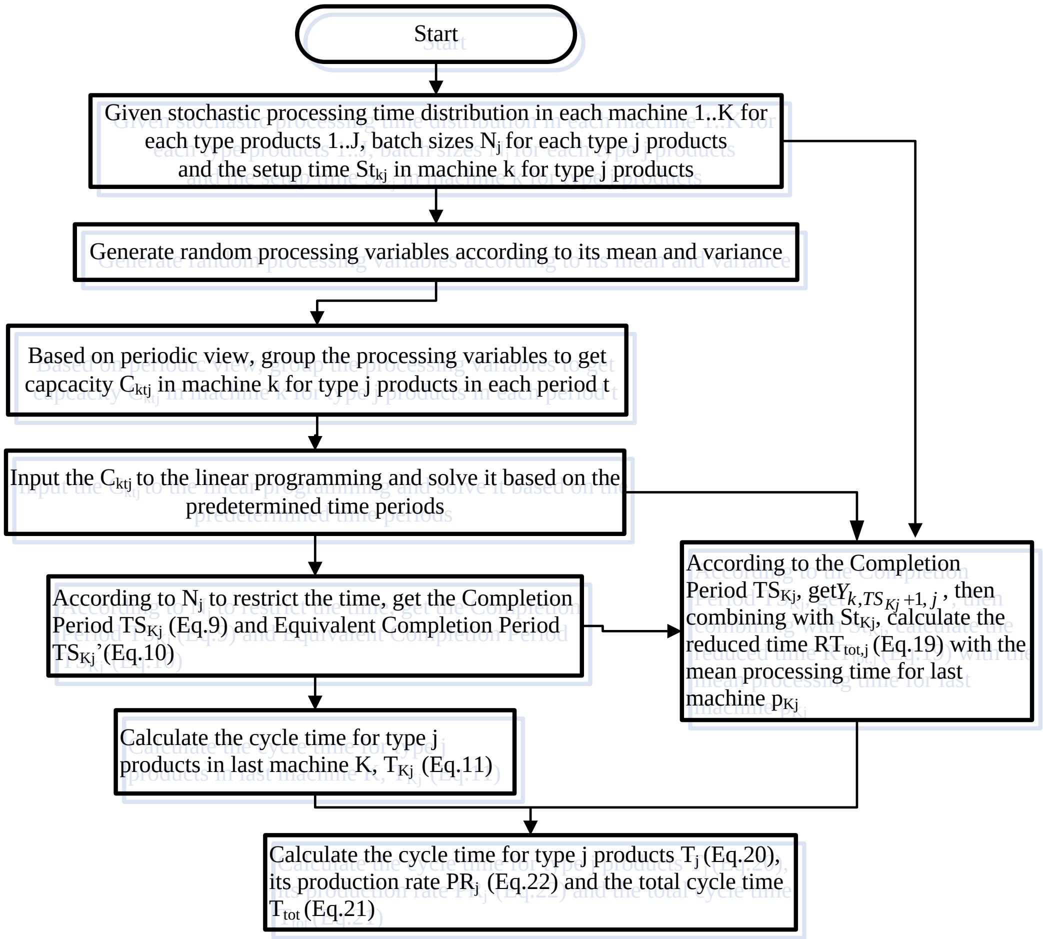

Figure 12: Flow Chart of the Steps in Analytical Approach in Scenario 2

3.2.6 Scenario 3: multi-type unreliable stochastic production line with setup while switching from Types

The stochastic failure and repair situation needs to be taken into consideration in the unreliable production line. Enough resources for repairing are assumed in the manufacturing system, which represents that once any machine fails, the immediate repair will be conducted without any delay. Once the machine is fixed back to work, the item remaining in the machine will continue to be processed. No scrap will be produced and the rework for the item remaining in the failed machine is not required.

The time to failure TTF means the time between two sequential failures. Due to the processing of different product types; the intensity for machine tooling head will be altered. Therefore, the TTF is distinctive for each particular type of products. The same consequence happens to the time to repair TTR, which represents the time consumed while fixing the failed machine.

I assume the time to failure for machine k in the i^{th} time for product type j $TTF_{k,i,j}$ to follow an exponential distribution with the mean time to failure for machine k for product type j as $MTTF_{k,j}$. The time to repair for machine k in the i^{th} time for product type j $TTR_{k,i,j}$ follows an exponential distribution with mean time to failure for machine k for product type j as $MTTR_{k,j}$. Since the assumption of enough resource for repairing, $TTR_{k,m,j}$ can be also treated as the time consumed between the machine m^{th} failure and its reoperation.

The insertion of fictive product idea in the research of Abdul-Kader (2006) provides some hints on the performance evaluation in the failure and repair situation. The repair time after the machine's failure can be treated as one fictive product insertion and the TTR becomes the processing time of the fictive product.

The following Figure 13 demonstrates the relationship between $TTF_{k,i,j}$ and $TTR_{k,i,j}$ with the thought of inserting fictive product while producing product type j. One cell represents one item and number a+1, a+2,…,a+b mean the item number during the $TTF_{k,i+1,j}$. The length of each cell represents the duration time for each item.

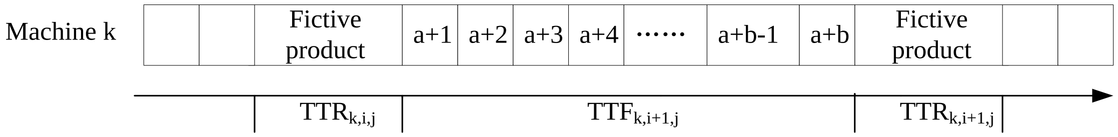

Figure 13: Relationship between $TTF_{k,i,j}$ and $TTR_{k,i,j}$ with Insertion of Fictive Products in the Sequence

The linear programming model in this scenario is similar to that in scenario 1. But the capacity for each period C_{ktj} needs to be modified into CF_{ktj} when the fictive product is considered. The following Figures 14a,b show the difference of the capacities between the reliable production line and the unreliable production line. Two arrows in Figure 14a represent the failure occurring time point in Figure 14b.

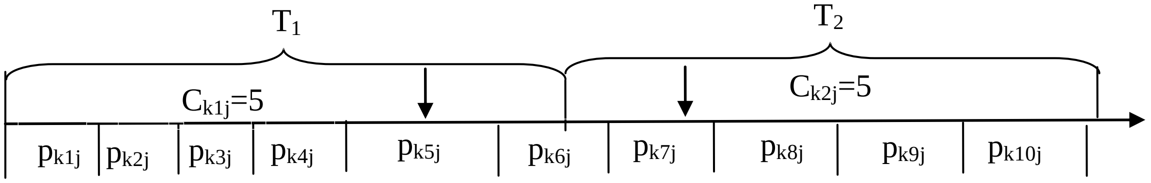

Figure 14a: Periodic Sampling of the j Type Production Capacity in Reliable Production Line in the First Two Periods

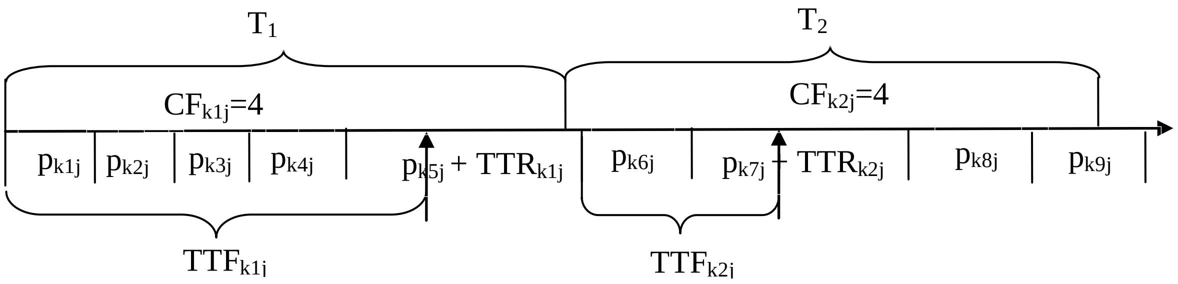

Figure 14b: Periodic Sampling of the j Type Production Capacity with Failure and Repair Consideration in the First Two Periods

Constraint 3 in Scenario1 restricts the production quantity in machine k is required to be changed to fit Scenario 3 as shown in Constraint 23:

$$Q_{ktj} \le CF_{ktj} \qquad \mathrm{t} = 1,\ldots,\mathrm{T},\ \mathrm{k} = 1,\ldots,\mathrm{K},\ \mathrm{j} = 1,\ldots,\mathrm{J} \qquad (23)$$

Objective function and the left constraints remain the same. According to the unreliable production line, the way to estimate the long enough periods TE_j for product type j in objective function needs to be as $\mathrm{TE}_{\mathrm{j}} = \left\lceil \frac{\mathrm{N}_{\mathrm{j}} \cdot \max\limits_{k}\{p_{kj}\} \cdot (\frac{\mathrm{MTTR}}{\mathrm{MTTF}} + 1)}{n} \right\rceil$. p_{kj} is the mean processing time in machine k for product type j. According to the same theory in Scenario 1, TE_j is only roughly estimated and there may not be enough periods for completing the batch size N_j. If so, TE_j needs to be extended by incrementing one extra period till it can be long enough to find completion period TS_{Kj}. Through periodic sampling, CF_{ktj} in the extra added periods needs be generated to input in Constraint 23.

The total reduced time, the cycle time for each type products, the total cycle time and the production rate can be calculate as the same as Equation 19, Equation 20, Equation 21 and Equation 22 in scenario 2.

The following Figure 15 shows the steps I apply in the scenario 3.

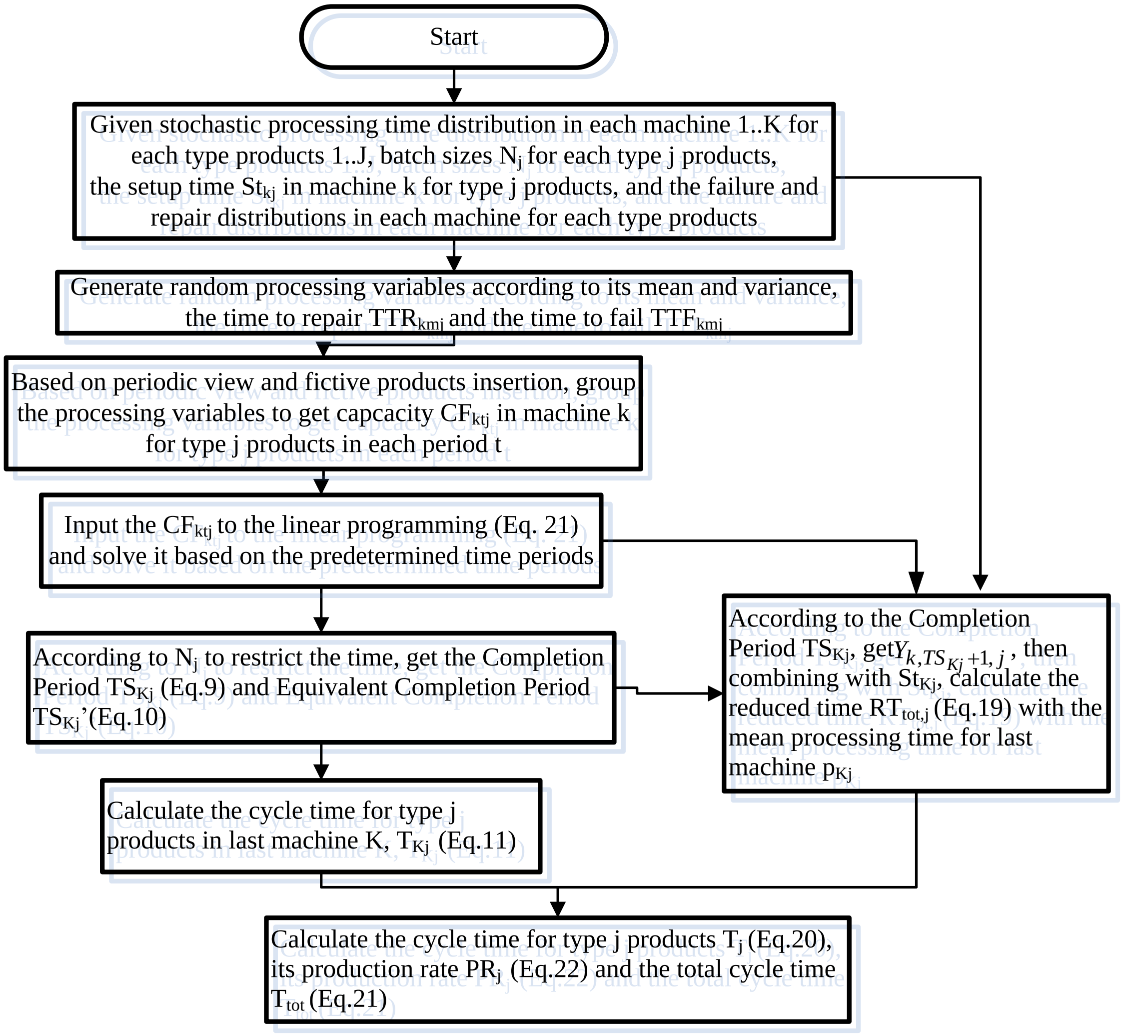

Figure 15: Flow Chart Shows the Steps in Scenario 3

3.2.7 Deterministic Production Line with Periodic Sampling

The processing times in serial production line are stochastic in the subsections above. But the periodic sampling analytical method is not limited to the stochastic production line. In the deterministic production line, the capacities are determined by the processing times which are not required to be generated and arranged like in Figure 3b. They can be calculated by the Equation 24 as follows:

$$C_{ktj} = \left\lfloor \frac{n}{p_{kj}} \right\rfloor \qquad t=1,\ldots,T \qquad (24)$$

The rest steps follow the stochastic production line evaluation approach according to the production line scenarios.

3.3 Manufacturing Strategies Comparison

3.3.1 Introduction

Pure-Push strategy (PP) is widely discussed in the production line, which does not concern about the pull effect (Market demands). The analytical approaches for evaluating cycle times in PP have been introduced in the subsections above. For this strategy, one of its assumptions is either the endless materials arrival in single production line or the batch sized materials input in multi-type production line. In this thesis, considering the expected sale volume, Pure-Push only orders the batch sized volume for each type of products waiting in raw materials inventory after the last item of former type entering.

Drum-Buffer-Rope (DBR) in this thesis is defined as a manufacturing strategy which relies on the output of bottleneck machine (Drum) in serial production line. The arrival raw material order depends on the completion signal (Rope) of one item from the bottleneck machine. That manufacturing strategy can balance the buffer inventory between the first machine and the bottleneck machine, which means that the buffer inventory will remain stable after the completion signal is sent. Figure 16a is referred from Betterton and Cox III (2009) to demonstrate the loop section in DBR.

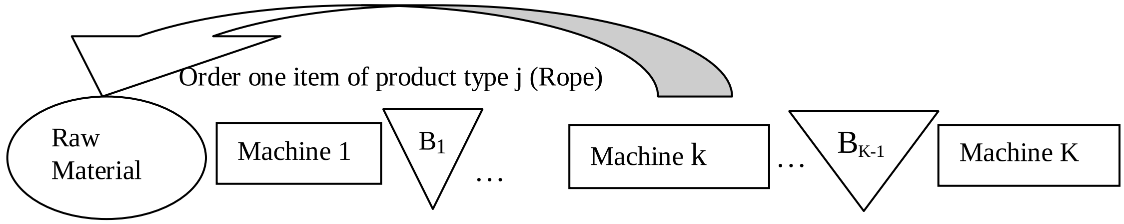

Figure 16a: Drum-Buffer-Rope (DBR) in serial production line

Just-in-Time (JIT) is the extended scope for DBR. This strategy aim is focused on the minimized the buffer inventory in serial production line and the inventory for storing the finished products. Only the order arrivals from the demanding market, the raw materials can be transported to the waiting section. Chakravorty and Atwater (1996) made a similar line design with the market pull system at the rear, but the line is not serial production line. From Figure 16b, the market module at the rear is the drum to send the signal for delivering the raw material in the waiting section.

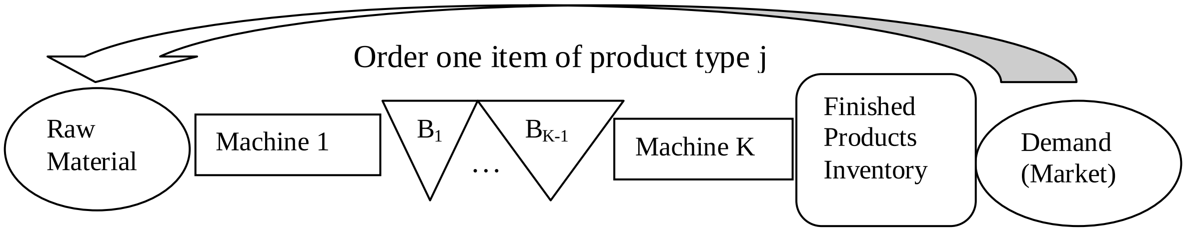

Figure 16b: Just-in-Time (JIT) in serial production line

3.3.2 Notations

WC_j	workforce expense for product type j
MC_j	materials cost for product type j
FC_j	finished products inventory cost for product type j
RC_j	raw materials inventory cost for product type j
BC_j	buffer levels cost for product type j
MT_j	machine wear & tear cost for product type j
IP_j	per item price for product type j
V_j	sale volume for product type j
OR	ordering rate (the cost per ordering)
M_j	material cost per item for product type j
PV_j	accumulative pre-ordered volume for product type j
BR	buffer carrying rate (the cost per item per period for storing in buffer)
RR	raw materials carrying rate (the cost per item per period for storing in raw materials inventory)
FR	finished products inventory carrying rate (the cost per item per period for storing in finished products inventory)
RI_{tj}	raw materials inventory at the beginning of period t for product type j
FI_{tj}	finished products inventory at the beginning of period t for type j products
RE_j	revenue for product type j
CO_j	cost for product type j
PRO_j	profit for product type j
PRO_{tot}	total profit for all types of products

3.3.3 Assumptions

1. The sale volumes V_j for all types of products are predictable. The ordering rate OR_j for product type j can be assumed as well.

2. The workers receive their wages by the time they work. Therefore the workforce expense WC_j is related to the work rate WR and the work time consumed.

3. Machine wear and tear cost MT_j is relevant to machine wear and tear rate MR and the work time consumed.

4. Material costs MC_j can be calculated with the sale volume V_j and material cost per item M_j for product type j.

5. Finished products inventory costs are carrying costs. Raw materials inventory costs are composed of carrying cost and ordering cost. The fixed cost component of these two inventories is added in the total cost for all types of products.

6. The sale department can receive the orders of all types of products. However, the manufacturing department (serial production line) will not produce other types of products without completing the current undergoing one. Therefore, at the beginning of producing product type j, the accumulative pre-ordered volume PV_j will be first ordered into the raw materials inventory. The rest orders in JIT policy are corresponding to the demand market. In DBR policy, the rest orders are regarding to the bottleneck machine.

7. Pre-ordered volume PV_j for product type j can be reached by the ordering rate OR and the rest types of products cycle times.

8. Ordering rate OR, Buffer level carrying rate BR, raw materials inventory carrying rate RR and finished products inventory carrying rate FR are irrelevant to the different types of products.

9. The transportation is not considered in the manufacturing strategies comparison.

10. The manufacturing pace always satisfies and exceeds the market demands pace.

3.3.4 Comparison among PP, DBR and JIT

Raw materials inventory, the finished products inventory and buffer levels are related to the total expense. The profit issue is taken into consideration to compare among the three manufacturing strategies among Pure-Push, DBR and JIT.

As all known, the profit is the revenue minus the cost. The aim of any manufacturing systems is to maximize the profit. In this thesis, the serial production line is not excluded as well.

For product type j, the cost is composed of workforce expense WC_j, materials cost MC_j, finished products inventory costs FC_j, raw materials inventory costs RC_j, buffer levels cost BC_j and machine wear and tear cost MT_j. Meanwhile the revenue depends on the price of products per item IP_j and its sale volume V_j.

The ways to calculate the costs, the profits and the revenues will be discussed in the following subsections.

3.3.4.1 Performance Criteria

The cost CO_j for product type j can be calculated as the Equation 25.

$$CO_j=(WR+MR)\times T_j+RC_j+BC_j+FC_j+MC_j \tag{25}$$

Material cost is equal to the each item material cost MI_j multiplied by the sale volume V_j, which can be shown in Equation 26.

$$MC_j=MI_j\times V_j \tag{26}$$

Buffer levels cost BC_j for product type j depends on the average level for each buffer in each period for product type j and the buffer carrying rate BR, which is shown in Equation 27 as follows:

$$BC_j=\sum_{k}^{K-1}\sum_{t}^{TS_{Kj}+1} Y_{ktj}\times BR \tag{27}$$

Finished products inventory cost FC_j should be considered this inventory carrying rate FR and average inventory storage, FI_{tj} is finished products inventory storage for product type j in period t. Equations 28 presents the way to calculate finished products inventory cost FC_j.

$$FC_j=\sum_{t}^{TS_{Kj}+1}(FI_{tj}-PV_j)\times FR \qquad FI_{tj}\geq PV_j \tag{28}$$

Raw materials inventory cost RC_j is composed of carrying cost and ordering cost, which can be shown in Equations 28ab. Carrying cost is calculated as the multiplication of this inventory carrying rate RR and the average raw materials inventory. RI_{tj} is raw materials inventory storage for product type j in period t. The ordering cost should be the ordering rate OR_j multiplied by the ordering times, which is 1 for Pure Push policy and V_j-PV_j+1 for JIT policy or DBR policy.

For JIT or DBR $$RC_j = \sum_{t}^{TS_{Kj}+1} RI_{tj} \times RR + OR \times (V_j - PV_j + 1) \quad (28a)$$

For Pure Push $$RC_j = \sum_{t}^{TS_{Kj}+1} RI_{tj} \times RR + OR_j \quad (28b)$$

Revenue is calculated by the multiplication of the predicated volume Vj and the price per item IPj for product type j, which is shown as Equation 29.

$$RE_j = V_j \times IP_j \quad (29)$$

Profit is revenue subtracting cost, for product type j, which can be presented as Equation 30.

$$PRO_j = RE_j - CO_j \quad (30)$$

Total profit for multi-type serial production line can be shown as Equation 31. That is the only criteria to judge which strategy is optimal.

$$PRO_{tot} = \sum_{j}^{J} RE_j - \sum_{j}^{J} CO_j \quad (31)$$

3.3.4.2 Extended Variables Initialization

Finished products inventory FI_{tj} for product type j at the beginning of period t should be quantity produced $Q_{K,t-1,j}$ in last machine K in previous period t-1 subtract the market demand $D_{t-1,j}$ in previous period t-1, which can be shown in Equation 34.

$$FI_{tj}= Q_{K,t-1,j} +FI_{t-1,j}-D_{t-1,j} \qquad t=2,\ldots,T \qquad (32)$$

Raw materials inventory at the beginning of the first period for product type j RI_{1j} can be initialized as the pre-ordered volume PV_j. Equation 33 can demonstrate raw materials inventory in the first period.

$$RI_{1j}=PV_j \qquad (33)$$

For DBR strategy,

Raw materials inventory in the rest period should equal to the quantity in the bottleneck machine k' in the rest periods $Q_{k'tj}$. That can be shown in Equation 34a.

$$RI_{tj}= Q_{k'tj}+RI_{t-1,j}-Q_{1,t-1,j} \qquad t=2,\ldots,T \qquad (34a)$$

For JIT strategy,

Raw materials inventory in the rest period should equal to the market demands in rest periods D_{tj}, which can be shown in Equation 34b.

$$RI_{tj}=D_{tj}+RI_{t-1,j}-Q_{1,t-1,j} \qquad t=2,\ldots,T \qquad (34b)$$

CHAPTER IV

NUMERICAL RESULTS

4.1 Experiments on Performance Evaluation of Multi-type Serial Production Line

Experiments 1-6 are conducted for the performance evaluation in the multi-type stochastic production line and deterministic production line in the three scenarios. The comparison between the analytical approach and simulation results are shown in following subsections.

4.1.1 Experiment 1

Two types of products with batch sizes N_j are processed in two-machine production line with the stochastic processing times following normal distribution.

4.1.1.1 Scenario 1

The following Table 1.1.1 shows the information of the production line, N represents normal distribution.

Product Type	Machine No.	Processing Time	Batch Size	Buffer Capacity
Product type 1	M1	N(4,1)	160	10
	M2	N(5,1)		
Product type 2	M1	N(3,1)	200	
	M2	N(4,1)		

Table 2.1.1: 2-Type 2-Machine Stochastic Production Line in Scenario 1

The simulation results in the following Table 1.1.2 are the mean value of cycle time for each type after running 10 republications using ProModel.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product 1 cycle time	853.36	854.13	-0.000902316
Product 2 cycle time	845.32	872.70	-0.032390101
Total cycle time	1608.38	1647.83	-0.024527786

Table 1.1.3: Comparison between the Simulation Results and Analytical Methods on Cycle time in Scenario 1

4.1.1.2 Scenario 2

With considering the setup situation while switching different types, Table 1.2.1 shows the given information in two-type two-machine stochastic production line in scenario 2.

Product Type	Machine No.	Processing Time	Setup Time	Batch Size	Buffer Capacity
Product 1	M1	N(4,1)	20	160	10
	M2	N(5,1)	30		
Product 2	M1	N(3,1)	30	200	
	M2	N(4,1)	20		

Table 1.2.1: 2-Type 2-Machine Stochastic Production Line in Scenario 2

The following Table 1.2.2 shows the comparison between the simulation results and analytical methods in scenario 2. The simulation results are the mean of the ten republications in Promodel.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product 1 cycle time	885.15	874.13	0.012449867
Product 2 cycle time	869.96	902.70	-0.037633914
Total cycle time	1667.32	1697.83	-0.018298827

Table 1.2.2: Comparison between the Simulation Results and Analytical Methods on Cycle time in Scenario 2

4.1.1.3 Scenario 3

The failure and repair situation is added in this scenario. The following Table 1.3.1 shows the description for 2-type 2-machine. E represents the exponential distribution and N means the normal distribution. Although the processing time, setup time, batch size and buffer capacity remain the same as before, the cycle time for last machine is changed due to the fictive product insertion. C_{ktj} have to transfer to CF_{ktj} according to Figures 13a,b.

Product Type	Machine	Processing Time	TTF	TTR	Setup Time	Batch size	Buffer Capacity
Product A	M1	N(4,1)	E(100)	E(5)	20	160	10
	M2	N(5,1)	E(200)	E(10)	30		
Product B	M1	N(3,1)	E(100)	E(5)	30	200	
	M2	N(4,1)	E(200)	E(10)	20		

Table 1.3.1: 2-Type 2-Machine Stochastic Production Line in Scenario 3

The following Table 1.3.2 shows the comparison between the simulation results and analytical methods in scenario 3. The simulation results are the mean of the ten republications in Promodel.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product 1 cycle time	920.58	911.682	0.009665646
Product 2 cycle time	917.04	934.562	-0.019107127
Total cycle time	1760.66	1767.244	-0.003739507

Table 1.3.2: Comparison between the Simulation Results and Analytical Methods in Scenario 3

4.1.2 Experiment 2

Two types of products with batch sizes are processed in 5-machine stochastic production line.

4.1.2.1 Scenario 1

The information of a five-machine stochastic production line without setup is shown in Table 2.1.1 as follows:

Product Type	Machine No.	Processing time	Batch size	Buffer capacity
Product A	M1	N(4,1)	150	3
	M2	N(5,1)		
	M3	N(3,0.5)		
	M4	N(4.5,1)		
	M5	N(6,1)		
Product B	M1	N(3,1)	200	
	M2	N(4,1)		
	M3	N(3,0.5)		
	M4	N(4,0.5)		
	M5	N(3.5,0.5)		

Table 2.1.1: 2-Type 5-Machine Stochastic Production Line in Scenario 1

The results can be shown in Table 2.1.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	943.07	937.867	0.005517088
Product B cycle time	884.23	866.105	0.02049806
Total cycle time	1685.50	1760.472	-0.04448057

Table 2.1.2: Comparison between the Simulation Results and Analytical Methods in Scenario 1

4.1.2.2 Scenario 2

The information of a five-machine stochastic production line with setup is shown in Table 2.2.1 as follows:

Product Type	Machine No.	Processing time	Setup time	Batch size	Buffer capacity
Product A	M1	N(4,1)	20	150	3
	M2	N(5,1)	30		
	M3	N(3,0.5)	30		
	M4	N(4.5,1)	20		
	M5	N(6,1)	20		
Product B	M1	N(3,1)	20	200	
	M2	N(4,1)	30		
	M3	N(3,0.5)	30		
	M4	N(4,0.5)	20		
	M5	N(3.5,0.5)	20		

Table 2.2.1: 2-Type 5-Machine Stochastic Production Line in Scenario 2

The results can be shown in Table 2.2.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1039.6281	1054.867	-0.01465803
Product B cycle time	928.6043	976.105	-0.051152789
Total cycle time	1826.11949	1800.472	0.014044804

Table 2.2.2: Comparison between the Simulation Results and Analytical Methods in Scenario 2

4.1.2.3 Scenario 3

The information of a five-machine stochastic production line with the failure and repair situation is shown in Table 2.3.1 as follows:

Product Type	Machine No.	Processing time	Setup time	TTF	TTR	Batch capacity	Buffer capacity
Product A	M1	N(4,1)	20	E(200)	E(5)	150	3
	M2	N(5,1)	30	E(200)	E(5)		
	M3	N(3,0.5)	30	E(200)	E(5)		
	M4	N(4.5,1)	20	E(200)	E(5)		
	M5	N(6,1)	20	E(200)	E(5)		
Product B	M1	N(3,1)	20	E(200)	E(10)	200	
	M2	N(4,1)	30	E(200)	E(10)		
	M3	N(3,0.5)	30	E(200)	E(10)		
	M4	N(4,0.5)	20	E(200)	E(10)		
	M5	N(3.5,0.5)	20	E(200)	E(10)		

Table 2.3.1: 2-Type 5-Machine Stochastic Production Line in Scenario 3

The results can be shown in Table 2.3.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1068.7308	1036.162	0.030474278
Product B cycle time	1018.4266	993.621	0.024356787
Total cycle time	1949	1911.283	0.019351975

Table 2.3.2: Comparison between the Simulation Results and Analytical Methods in Scenario 3

4.1.3 Experiment 3

Two types of products with batch sizes are processed in 5-machine deterministic production line.

4.1.3.1 Scenario 1

The information of a five-machine deterministic production line without setup is shown in Table 3.1.1 as follow:

Product Type	Machine No.	Processing time	Batch size	Buffer capacity
Product A	M1	4	150	3
	M2	5		
	M3	3		
	M4	4.5		
	M5	6		
Product B	M1	3	200	
	M2	4		
	M3	3		
	M4	4		
	M5	3.5		

Table 3.1.1: 2-Type 5-Machine Deterministic Production Line in Scenario 1

From Equation 24, for type A products: $C_{1t1}=\left\lfloor \frac{100}{4} \right\rfloor = 25$, $C_{2t1}=20$, $C_{3t1}=33$, $C_{4t1}=22$, $C_{5t1}=16$.

For type B products: $C_{1t2}=33$, $C_{2t2}=25$, $C_{3t2}=33$, $C_{4t2}=25$, $C_{5t2}=28$.

After solving the linear programming model (Eq.1-8) and following the flow chart in the first scenario (Fig. 10), I can find the results as shown in Table 3.1.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	956.62	965	-0.008760009
Product B cycle time	889.5	878	0.012928612
Total cycle time	1689	1737.5	-0.028715216

Table 3.1.2: Comparison between the Simulation Results and Analytical Methods in Scenario 1

4.1.3.2 Scenario 2

The information of a 5-machine deterministic production line with setup is shown in Table 3.2.1 as follows:

Product Type	Machine No.	Processing time	Setup time	Batch size	Buffer capacity
Product A	M1	4	20	150	3
	M2	5	30		
	M3	3	30		
	M4	4.5	20		
	M5	6	20		
Product B	M1	3	20	200	
	M2	4	30		
	M3	3	30		
	M4	4	20		
	M5	3.5	20		

Table 3.2.1: 2-Type 5-Machine Deterministic Production Line in Scenario 2

The results can be shown in Table 3.2.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1040.5	1037.5	0.002883229
Product B cycle time	925	920	0.005405405
Total cycle time	1818	1777.5	0.022277228

Table 3.2.2: Comparison between the Simulation Results and Analytical Methods in Scenario 2

4.1.3.2 Scenario 3

The information of a 5-machine stochastic production line with the failure and repair situation is shown in Table 3.3.1 as follows:

Product Type	Machine No.	Processing time	Setup time	TTF	TTR	Batch capacity	Buffer capacity
Product A	M1	4	20	E(200)	E(5)	150	3
	M2	5	30	E(200)	E(5)		
	M3	3	30	E(200)	E(5)		
	M4	4.5	20	E(200)	E(5)		
	M5	6	20	E(200)	E(5)		
Product B	M1	3	20	E(200)	E(10)	200	
	M2	4	30	E(200)	E(10)		
	M3	3	30	E(200)	E(10)		
	M4	4	20	E(200)	E(10)		
	M5	3.5	20	E(200)	E(10)		

Table 3.3.1: 2-Type 5-Machine Deterministic Production Line in Scenario 3.

The results can be shown in Table 3.3.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1059.60	1055.414	0.003950547
Product B cycle time	1018.79	1009.091	0.009520117
Total cycle time	1913.60	1904.505	0.004752822

Table 3.3.2: Comparison between the Simulation Results and Analytical Methods in Scenario 3.

4.1.4 Experiment 4

Two types of products with batch sizes are processed in 5-machine stochastic production line with exponential distribution.

4.1.4.1 Scenario 1

The information of a five-machine stochastic production line without setup is shown in Table 4.1.1 as follows:

Product Type	Machine No.	Processing time	Batch size	Buffer capacity
Product A	M1	E(4)	150	3
	M2	E(5)		
	M3	E(3)		
	M4	E(4.5)		
	M5	E(6)		
Product B	M1	E(3)	200	
	M2	E(4)		
	M3	E(3)		
	M4	E(4)		
	M5	E(3.5)		

Table 4.1.1: 2-Type 5-Machine Stochastic Production Line in Scenario 1

The results can be shown in Table 4.1.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	966.24	921.853	0.045937862
Product B cycle time	1078.939	1096.269	-0.016062076
Total cycle time	1839.00	1918.622	-0.043296357

Table 4.1.2: Comparison between the Simulation Results and Analytical Methods in Scenario 1

4.1.4.2 Scenario 2

The information of a 5-machine stochastic production line with setup is shown in Table 4.2.1 as follows:

Product Type	Machine No.	Processing time	Setup time	Batch size	Buffer capacity
Product A	M1	E(4)	20	150	3
	M2	E(5)	30		
	M3	E(3)	30		
	M4	E(4.5)	20		
	M5	E(6)	20		
Product B	M1	E(3)	20	200	
	M2	E(4)	30		
	M3	E(3)	30		
	M4	E(4)	20		
	M5	E(3.5)	20		

Table 4.2.1: 2-Type 5-Machine Stochastic Production Line in Scenario 2.

The results can be shown in Table 4.2.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1054.78	1002.353	0.049704204
Product B cycle time	1087.70	1156.269	-0.06304036
Total cycle time	2055.00	1958.622	0.04689927

Table 4.2.2: Comparison between the Simulation Results and Analytical Methods in Scenario 2.

4.1.4.3 Scenario 3

The information of a 5-machine stochastic production line with the failure and repair situation is shown in Table 4.3.1 as follows:

Product Type	Machine No.	Processing time	Setup time	TTF	TTR	Batch capacity	Buffer capacity
Product A	M1	E(4)	20	E(200)	E(5)	150	3
	M2	E(5)	30	E(200)	E(5)		
	M3	E(3)	30	E(200)	E(5)		
	M4	E(4.5)	20	E(200)	E(5)		
	M5	E(6)	20	E(200)	E(5)		
Product B	M1	E(3)	20	E(200)	E(10)	200	
	M2	E(4)	30	E(200)	E(10)		
	M3	E(3)	30	E(200)	E(10)		
	M4	E(4)	20	E(200)	E(10)		
	M5	E(3.5)	20	E(200)	E(10)		

Table .3.1: 2-Type 5-Machine Stochastic Production Line in Scenario 3.

The results can be shown in Table 4.3.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1131.51	1078.056	0.047241297
Product B cycle time	1244.49	1253.692	-0.007394194
Total cycle time	2215.00	2121.748	0.042100226

Table 4.3.2: Comparison between the Simulation Results and Analytical Methods in Scenario 3.

4.1.5 Experiment 5

Two types of products with batch sizes are processed in 10-machine stochastic production line.

4.1.5.1 Scenario 1

The information of a five-machine stochastic production line without setup is shown in Table 5.1.1 as follows:

Product Type	Machine No.	Processing time	Batch size	Buffer capacity
Product A	M1	N(4,1)	150	3
	M2	N(5,1)		
	M3	N(3,0.5)		
	M4	N(4.5,1)		
	M5	N(6,1)		
	M6	N(4,1)		
	M7	N(5,1)		
	M8	N(3,0.5)		
	M9	N(4.5,1)		
	M10	N(6,1)		
Product B	M1	N(3,1)	200	
	M2	N(4,1)		
	M3	N(3,0.5)		
	M4	N(4,0.5)		
	M5	N(3.5,0.5)		
	M6	N(3,1)		
	M7	N(4,1)		
	M8	N(3,0.5)		
	M9	N(4,0.5)		
	M10	N(3.5,0.5)		

Table 5.1.1: 2-Type 10-Machine Stochastic Production Line in Scenario 1.

The results can be shown in Table 5.1.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	995.5966	989.867	0.005754941
Product B cycle time	939.5229	943.081	-0.003787135
Total cycle time	1693.4	1771.448	-0.046089524

Table 5.1.2: Comparison between the Simulation Results and Analytical Methods in Scenario 1.

4.1.5.2 Scenario 2

The information of a 10-machine stochastic production line with setup is shown in Table 5.2.1 as follows:

Product Type	Machine No.	Processing time	Setup time	Batch size	Buffer capacity
Product A	M1	N(4,1)	20	150	3
	M2	N(5,1)	30		
	M3	N(3,0.5)	30		
	M4	N(4.5,1)	20		
	M5	N(6,1)	20		
	M6	N(4,1)	20		
	M7	N(5,1)	30		
	M8	N(3,0.5)	30		
	M9	N(4.5,1)	20		
	M10	N(6,1)	20		
Product B	M1	N(3,1)	20	200	
	M2	N(4,1)	30		
	M3	N(3,0.5)	30		
	M4	N(4,0.5)	20		
	M5	N(3.5,0.5)	20		
	M6	N(3,1)	20		
	M7	N(4,1)	30		
	M8	N(3,0.5)	30		
	M9	N(4,0.5)	20		
	M10	N(3.5,0.5)	20		

Table 5.2.1: 2-Type 10-Machine Stochastic Production Line in Scenario 2

The results can be shown in Table 5.2.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1183.0338	1138.367	0.037756149
Product B cycle time	1064.0245	1073.081	-0.008511552
Total cycle time	1935	1991.448	-0.029172093

Table 5.2.2: Comparison between the Simulation Results and Analytical Methods in Scenario 2

4.1.5.3 Scenario 3

The information of a 10-machine stochastic production line with the failure and repair situation is shown in Table 5.3.1 as follows:

Product Type	Machine No.	Processing time	Setup time	TTF	TTR	Batch size	Buffer capacity
Product A	M1	N(4,1)	20	E(200)	E(5)	150	3
	M2	N(5,1)	30	E(200)	E(5)		
	M3	N(3,0.5)	30	E(200)	E(5)		
	M4	N(4.5,1)	20	E(200)	E(5)		
	M5	N(6,1)	20	E(200)	E(5)		
	M6	N(4,1)	20	E(200)	E(5)		
	M7	N(5,1)	30	E(200)	E(5)		
	M8	N(3,0.5)	30	E(200)	E(5)		
	M9	N(4.5,1)	20	E(200)	E(5)		
	M10	N(6,1)	20	E(200)	E(5)		
Product B	M1	N(3,1)	20	E(200)	E(10)	200	
	M2	N(4,1)	30	E(200)	E(10)		
	M3	N(3,0.5)	30	E(200)	E(10)		
	M4	N(4,0.5)	20	E(200)	E(10)		
	M5	N(3.5,0.5)	20	E(200)	E(10)		
	M6	N(3,1)	20	E(200)	E(10)		
	M7	N(4,1)	30	E(200)	E(10)		
	M8	N(3,0.5)	30	E(200)	E(10)		
	M9	N(4,0.5)	20	E(200)	E(10)		
	M10	N(3.5,0.5)	20	E(200)	E(10)		

Table 5.3.1: 2-Type 10-Machine Stochastic Production Line in Scenario 3

The results can be shown in Table 5.3.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1217.2169	1152.162	0.05344561
Product B cycle time	1156.8979	1149.091	0.006748132
Total cycle time	2072.2	2061.253	0.005282791

Table 5.3.2: Comparison between the Simulation Results and Analytical Methods in Scenario 3

4.1.6 Experiment 6

Two types of products with batch sizes are processed in 10-machine deterministic production line.

4.1.6.1 Scenario 1

The information of a five-machine deterministic production line without setup is shown in Table 6.1.1 as follows:

Product Type	Machine No.	Processing time	Batch size	Buffer capacity
Product A	M1	4	150	3
	M2	5		
	M3	3		
	M4	4.5		
	M5	6		
	M6	4		
	M7	5		
	M8	3		
	M9	4.5		
	M10	6		
Product B	M1	3	200	
	M2	4		
	M3	3		
	M4	4		
	M5	3.5		
	M6	3		
	M7	4		
	M8	3		
	M9	4		
	M10	3.5		

Table 6.1.1: 2-Type 10-Machine Stochastic Production Line in Scenario 1.

The results can be shown in Table 6.1.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	989.50	985	0.004547751
Product B cycle time	922.50	926	-0.003794038
Total cycle time	1688.00	1737.5	-0.029324645

Table 6.1.2: Comparison between the Simulation Results and Analytical Methods in Scenario 1.

4.1.6.2 Scenario 2

The information of a 10-machine deterministic production line with setup is shown in Table 6.2.1 as follows:

Product Type	Machine No.	Processing time	Setup time	Batch size	Buffer capacity
Product A	M1	4	20	150	3
	M2	5	30		
	M3	3	30		
	M4	4.5	20		
	M5	6	20		
	M6	4	20		
	M7	5	30		
	M8	3	30		
	M9	4.5	20		
	M10	6	20		
Product B	M1	3	20	200	
	M2	4	30		
	M3	3	30		
	M4	4	20		
	M5	3.5	20		
	M6	3	20		
	M7	4	30		
	M8	3	30		
	M9	4	20		
	M10	3.5	20		

Table 6.2.1: 2-Type 10-Machine Stochastic Production Line in Scenario 2.

The results can be shown in Table 6.2.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1183	1177.5	0.004649197
Product B cycle time	1062.5	1040	0.021176471
Total cycle time	1896	1851.5	0.023470464

Table 6.2.2: Comparison between the Simulation Results and Analytical Methods in Scenario 2.

4.1.6.3 Scenario 3

The information of a 10-machine deterministic production line with the failure and repair situation is shown in Table 6.3.1 as follows:

Product Type	Machine No.	Processing time	Setup time	TTF	TTR	Batch size	Buffer capacity
Product A	M1	4	20	E(200)	E(5)	150	3
	M2	5	30	E(200)	E(5)		
	M3	3	30	E(200)	E(5)		
	M4	4.5	20	E(200)	E(5)		
	M5	6	20	E(200)	E(5)		
	M6	4	20	E(200)	E(5)		
	M7	5	30	E(200)	E(5)		
	M8	3	30	E(200)	E(5)		
	M9	4.5	20	E(200)	E(5)		
	M10	6	20	E(200)	E(5)		
Product B	M1	3	20	E(200)	E(10)	200	
	M2	4	30	E(200)	E(10)		
	M3	3	30	E(200)	E(10)		
	M4	4	20	E(200)	E(10)		
	M5	3.5	20	E(200)	E(10)		
	M6	3	20	E(200)	E(10)		
	M7	4	30	E(200)	E(10)		
	M8	3	30	E(200)	E(10)		
	M9	4	20	E(200)	E(10)		
	M10	3.5	20	E(200)	E(10)		

Table 6.3.1: 2-Type 10-Machine Stochastic Production Line in Scenario 3.

The results can be shown in Table 6.3.2.

	Simulation	Analytical method	Error((Sim-Ana)/Sim)
Product A cycle time	1220.502	1214.026	0.005306013
Product B cycle time	1167.2288	1215.61	-0.041449628
Total cycle time	2075.5	2189.636	-0.05499205

Table 6.3.2: Comparison between the Simulation Results and Analytical Methods in Scenario 3.

4.2 Experiments on Manufacturing Strategies Comparison

Manufacturing strategies are taken into consideration in this section and the systems are extended with adding the demand market, finished products inventory and raw materials inventory.

The serial production lines in Experiment 5 Scenario 2 and Experiment 6 Scenario 2 are replicated as the production line part in Experiment 7 and Experiment 8 separately.

The batch sized arrival paces of raw materials are changed according to the different manufacturing strategies. But the expected sale volumes V_j in Experiment 7 and Experiment 8 are set as the same quantity as batch sizes N_j in Experiment 5 and Experiment 6 separately.

New added conditions for the extended part are shown as follows:

Sale price per item PI_1 for type A products is 20 dollars. Sale volume for type A products is 150.

Sale price per item PI_2 for type B products is 15 dollars. Sale volume for type B products is 200.

Each worker receives 0.25 dollars per minute, so workforce expense rate WR is 0.25.

Machines tear and wear rate MR is 0.05 dollars per minute.

Buffer carrying rate BR is 1 dollar per item per 100 minutes (1 dollar per item per period).

Raw material per item cost MI_1 for type A products is 5 dollars.

Raw material per item cost MI_2 for type B products is 3 dollars.

Finished products inventory carrying rate FR is 1 dollar per item per 100 minutes (1 dollar per item per period).

Raw materials inventory carrying rate RR is 0.5 dollars per item per 100 minutes (0.5 dollars per item per period).

Pre-ordered volume PV_j for type both types of products is 50.

Ordering rate for both types of products is 0.3 dollars per order.

4.2.1 Experiment 7

The basic information of multi-type production line is shown in Table 6.2.1 in the Experiment 6 Scenario 2. Market demands D_{tj} for each period is 10 items for both types of products.

4.2.1.1 Pure-Push strategy

As Experiment 6 Scenario 2 shows the analytical cycle time with the periodic sampling cycle time evaluation and batch sized raw materials arrival, the results can be shown in cost and cycle time Table 7.1.1 and profit Table 7.2.2. The analytical cycle time for each product type is replicated from Table 5.2.2.

Cost	Materials	Machine wear& tear	Buffer inventory	Raw material inventory	Finished products inventory	Workforce expense	Analytical cycle time
Product A	750	58.875	84	356.3	4	294.375	1177.5
Product B	600	52	21	439.3	410	260	1040
Total	1350	110.875	105	795.6	414	554.375	

Table 7.1.1: Cost and Cycle Time Table for PP Strategy in Experiment 7.

	Revenue	Cost	Profit
Product A	3000	1547.55	1452.45
Product B	3000	1782.3	1217.7
Total	6000	3329.85	2670.15

Table 7.1.2: Profit Table for PP Strategy in Experiment 7.

4.2.1.2 DBR Strategy

The bottleneck machine in serial production line for type A products is M5 and for type B products is M2 in Table 6.2.1.

After applying the periodic sampling cycle time evaluation approach with adding equations (Eq. 32, Eq. 33 and Eq. 34a), the results can be shown in cost and cycle time Table 7.2.1 and profit Table 7.2.2.

Cost	Materials	Machine wear& Tear	Buffer inventory	Raw material inventory	Finished products inventory	Workforce expense	Analytical cycle time
Product A	750	58.875	84	196.3	4	294.375	1177.5
Product B	600	52	21	222.3	410	260	1040
Total	1350	110.875	105	418.6	414	554.375	

Table 7.2.1: Cost and Cycle Time Table for DBR Strategy in Experiment 7.

	Revenue	Cost	Profit
Product A	3000	1387.55	1612.45
Product B	3000	1565.3	1434.7
Total	6000	2952.85	3047.15

Table 7.2.2: Profit Table for DBR Strategy in Experiment 7.

4.2.1.3 JIT Strategy

After applying the periodic sampling cycle time evaluation approach with adding equations (Eq. 32, Eq. 33 and Eq. 34b), the results can be shown in cost and cycle time Table 7.3.1 and profit Table 7.3.2.

Cost	Materials	Machine wear& tear	Buffer inventory	Raw materials inventory	Finished products inventory	Workforce expense	Analytical cycle time
Product A	750	62	64	115.3	0	310	1240
Product B	600	87	9	95.3	0	435	1740
Total	1350	149	73	210.6	0	745	

Table 7.3.1: Cost and Cycle Time Table for JIT Strategy in Experiment 7.

	Revenue	Cost	Profit
Product A	3000	1301.3	1698.7
Product B	3000	1226.3	1773.7
Total	6000	2527.6	3472.4

Table 7.3.2: Profit Table for JIT Strategy in Experiment 7.

4.2.2 Experiment 8

The basic information of multi-type production line is shown in Table 5.2.1 in the Experiment 5 Scenario 2.

Market demands in each period for different types of products are predicted as random variables and that can be shown in Table 8.1.0.

Periods	1	2	3	4	5	6	7	8	9	10	Preordered	Total
Product A Demands	13	4	7	8	13	8	9	11	12	15	50	150
Product B Demands	13	16	17	18	13	18	11	13	12	19	50	200

Table 8.1.0: Market Demands in 10 Periods for Both Types of Products

4.2.2.1 Pure-Push Strategy

As Experiment 5 Scenario 2 shows the analytical cycle time with the periodic sampling evaluation, the results of the costs and cycle time can be shown in Table 8.1.1 and the profits are described as in Table 8.2.2.

Cost	Materials	Machine wear& tear	Buffer inventory	Raw material inventory	Finished products inventory	Workforce expense	Analytical cycle time
Product A	750	56.918	80	342.3	21	284.592	1138.367
Product B	600	53.654	31	458.3	48	268.270	1073.081
Total	1350	110.572	111	800.6	69	552.862	

Table 8.1.1: Cost and Cycle Time Table for PP Strategy in Experiment 8

	Revenue	Cost	Profit
Product A	3000	1534.810	1465.19
Product B	3000	1459.224	1540.776
Total	6000	2994.034	3005.966

Table 8.1.2: Profit Table for PP Strategy in Experiment 8

4.2.2.2 DBR Strategy

The bottleneck machine in serial production line for type A products is M5 and for type B products is M2 in Table 5.2.1.

After applying the periodic sampling cycle time evaluation approach with adding equations (Eq. 32, Eq. 33 and Eq. 34a), the results can be shown in cost and cycle time Table 8.2.1 and profit Table 8.2.2.

Cost	Materials	Machine wear& tear	Buffer inventory	Raw materials inventory	Finished products inventory	Workforce expense	Analytical cycle time
Product A	750	56.918	80	194.8	21	284.592	1138.367
Product B	600	53.654	31	229.8	48	268.270	1073.081
Total	1350	110.572	111	424.6	69	552.862	

Table 8.2.1: Cost and Cycle Time Table for DBR Strategy in Experiment 8.

	Revenue	Cost	Profit
Product A	3000	1387.310	1612.69
Product B	3000	1230.724	1769.276
Total	6000	2618.034	3381.966

Table 8.2.2: Profit Table for DBR Strategy in Experiment 8.

4.2.2.3 JIT Strategy

After applying the periodic sampling cycle time evaluation approach with adding equations (Eq. 32, Eq. 33 and Eq. 34b), the results can be shown in cost and cycle time Table 8.3.1 and profit Table 8.3.2.

Cost	Materials	Machine wear& tear	Buffer inventory	Raw materials inventory	Finished products inventory	Workforce expense	Analytical cycle time
Product A	750	62	46	101.8	0	310	1240
Product B	600	62	22	121.8	0	310	1240
Total	1350	124	68	223.6	0	620	

Table 8.3.1: Cost and Cycle Time Table for JIT Strategy in Experiment 8.

	Revenue	Cost	Profit
Product A	3000	1245.8	1754.2
Product B	3000	1139.8	1860.2
Total	6000	2385.6	3614.4

Table 8.3.2: Profit Table for JIT Strategy in Experiment 8.

4.3 Analysis of Results

In Experiments 1, 2, 4, 5, simulation results are the means of ten republications simulated by ProModel. Because the processing time in each machine is stochastic, cycle times for different replications are different. The analytical results are resulted from one incidence. Errors, also can be called difference between the analytical results and simulation results, are all within 6%. Therefore, the conclusion can be made that the experiments validate the proposed analytical performance evaluation.

With the setup and unreliable situation considered, the trend of cycle time is increasing. The cycle time in Scenario 3 is longer than that in Scenario 2 and Scenario 1 in each experiment. Buffer capacity does not impact on the percentage of analytical error, like no obvious errors difference between buffer capacity 10 in Experiment 1 and buffer capacity 3 in the following Experiments.

Stable demands per period are shown in Experiment 7 and unstable market demands are shown in Experiment 8. From both experiments, applying Pure-Push (PP) strategy can lead the shortest cycle time but the most finished products inventory under the same circumstance.

JIT manufacturing strategy can make most profit in Experiments 7-8 compared with the other two strategies.

Cycle times for DBR and PP strategy are shown to be the same in Experiments 7-8. Because of different ordering strategy, DBR shows advantage on the raw material inventory holding cost.

JIT has the minimum raw inventory, the finished products inventory and buffer inventory, but because of the longest cycle time caused by JIT, it causes the most workforce expense and machine wear & tear cost.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This thesis provides a new perspective to deal with the performance evaluation of a multi-type production line. The periodic sampling changed the stochastic processing time into deterministic production quantity sequence to input the discrete linear programming. The objective function is to maximize production quantity. With the restriction of batch sizes for different types, the cycle time in last machine can be generated. With the help of reduced time, cycle time for each type of products.

Experiments are undertaken from the production line of two-machine to ten-machine and from stochastic processing time to deterministic processing time. Stochastic processing times are categorized as normal distribution and exponential distribution. The comparisons between this analytical approach and the simulation results are applied to validate the approach's accuracy.

The comparison of manufacturing strategies is actually an application of this analytical approach. With the extended demand market, raw materials inventory and finished products inventory, a longer 'production line' is built up. PP, DBR and JIT are the manufacturing strategies with different raw materials ordering control mechanisms. Using the similar approach validated, the profits for each strategy can be generated.

5.2 Recommendations

The proposed method and consequent numerical study in this thesis established a new way to evaluate the manufacturing performance. The comparison of manufacturing strategies shows a possible path to apply it into extended area.

The raw materials ordering control mechanism in this thesis only applies the existed strategies. The analytical approach for reduced time is a little cumbersome. The cycle time for each type is not generated through the linear programming. In the future, if there is an improvement on simplifying the reduced time, maybe the optimized ordering mechanism control can be generated by applying the objective function with maximum profit.

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