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Design Tool for Kinematics of Multibody Systems

Monika Filiposka
University of Windsor

Design tool for kinematics of multibody systems

By

Monika Filiposka

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ABSTRACT

This research provides a methodology and a tool for selection of appropriate robotic system based on the singularities in the workspace of the machines, suitable for both, designers and users. The kinematic problem solutions are managed through design methodology and represented with function modelling language, IDEF0.

This novel approach specifies step by step activities on how to model robotic systems with math and programming tools, like Maple 17 and Matlab 2010. Symbolical and numerical solutions of kinematics, Jacobian matrix, singularities and workspace are successfully obtained for three types of multibody systems; general CNC machine, Mitsubishi MELFA RV-3SDB robot and Yaskawa Motoman DA-20, dual arm collaborative robot. CNC-R Global Reconfigurable Kinematic Model is developed for analyses of different types of manipulators.

The main purpose of this design tool for kinematics of multibody systems is to help in kinematics problem solving, by providing visual representation of the workspace with the singularity locus of the same. It represents a set of iterative methods for kinematic design of manipulators, and so at the end, visual presentation of the effective work region, including singular configurations. The methodology is appropriate for any n-DOF multibody system, even for dual arm collaborative robots.

DEDICATION

To my children,

Frosina and Aleksandar!

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I am expressing my honorable appreciation to all those who provided me the possibility to complete this thesis. A special gratitude I give to my supervisor, the distinguished Dr. Waguih ElMaraghy, whose encouraging suggestions helped me to coordinate my research. I cannot express enough thanks to my other committee members for their continued support and encouragement, Dr. Mitra Mirhassani and Dr. Ana Djuric.

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To my parents and parents in law, my heartfelt thanks. The countless times you kept my children during our stay in Canada will not be forgotten. Thank you for allowing me time away from my kids to research and write.

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Finally, to my kids. You are the most significant achievement in my life and nothing in the world can change that. Remember that nothing worth doing is ever easy!

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NOMENCLATURE

CNC-R GKM - CNC- Robot global kinematic model

D-H - Denavit-Hartenberg parameters

DOF- Degrees of freedom

${}^{i-1}A_i$ - Homogeneous matrices

d_i - Link offset along previous Z to the common normal

θ_i - Joint angle about Z, from old X to new X

a_i - Link length of the common normal

α_i - Twist angle about common normal, from old Z axis to new Z axis

K_{Si} - Twist angles sinus control parameter

K_{Ci} - Twist angles cosines control parameter

${}^{0}R_i$ - Rotational matrix

${}^{i-1}P_i$ - Position matrix

Z_{i-1} - Unit vector

J - Jacobian matrix

$\dot{X}$ - End effector velocity

$\dot{q}$ - Joint velocity

m - Number of independent equations

n - Number of links, number of joint variables

CHAPTER 1

INTRODUCTION

1.1. Robots through the history and their classification

Ancient civilizations through the centuries were thinking of mechanisms and machines, and they are still inspiration for today's biggest design achievements, because they were predecessors of today's intelligent systems. The word "robot" was first mentioned in the Czech play "Rossum's Universal Robots" by Karel Čapek in 1920; it originates from the Slavic group of languages "robota'" which means "work". The term "robotics" refers to the study and use of robots, first adopted by Isaac Asimov (1920-1992), a Russian born author and professor, in 1941 through his short science fiction story, “Runaround”. He proposed the three laws of "robotics" to protect us from intelligent generations of robots. A robot may not injure a human being, must obey orders given it by humans and must protect its own existence (Jazar, 2010).

Robots originate from the CNC machines. The computer numerical control (CNC) of milling machines was developed to increase the precision in machining. During the World War II another technology for handling radioactive material was developed, the teleoperators. As a result of combining these two technologies, the robots appeared. Although, the first robot on the market, Unimate (Fig.1a), was developed by Unimation in 1959, and installed at General Motors Company in 1961. Kuka produced the first industrial robot with 6-axes, FAMULUS (Fig.1b), in 1973, while the first fully electric industrial robot, IRB 6(Fig.1c), was developed in ASEA Sweden (today ABB) in 1974. These robots, termed as industrial robots, were mainly used in automotive industries. In 1998 the world fastest robot was developed by ABB, Flex Picker; it had parallel structure (Fig.1d) and used for pick and place applications. In 2010 Fanuc-Japan lunched the first "Learning Control Robot" (Fig.1e). Since 1990 the research and development in industrial robots is increasing. Aiming to improve the quality of living, the field of service robots is also well explored in the recent years. New robots had to be at low cost and multipurpose capabilities. In 2004 Motoman, Japan introduced NX100, a new controller capable to control 38 axes at the same time, or up to 4 robots. Thanks to this controller, dual arm robot for industrial purposes was presented by Motoman, Japan in 2005; the DA-20 (Fig.1f). This novelty created the collaborative robots, embarked development of multi arm systems, and expanded the robotic applications. Another reason why these collaborative robots are considered as new “boom” in robotics is that they are low-cost, easy to install, reconfigurable manipulators, safe and capable to work hand-by-hand with humans.

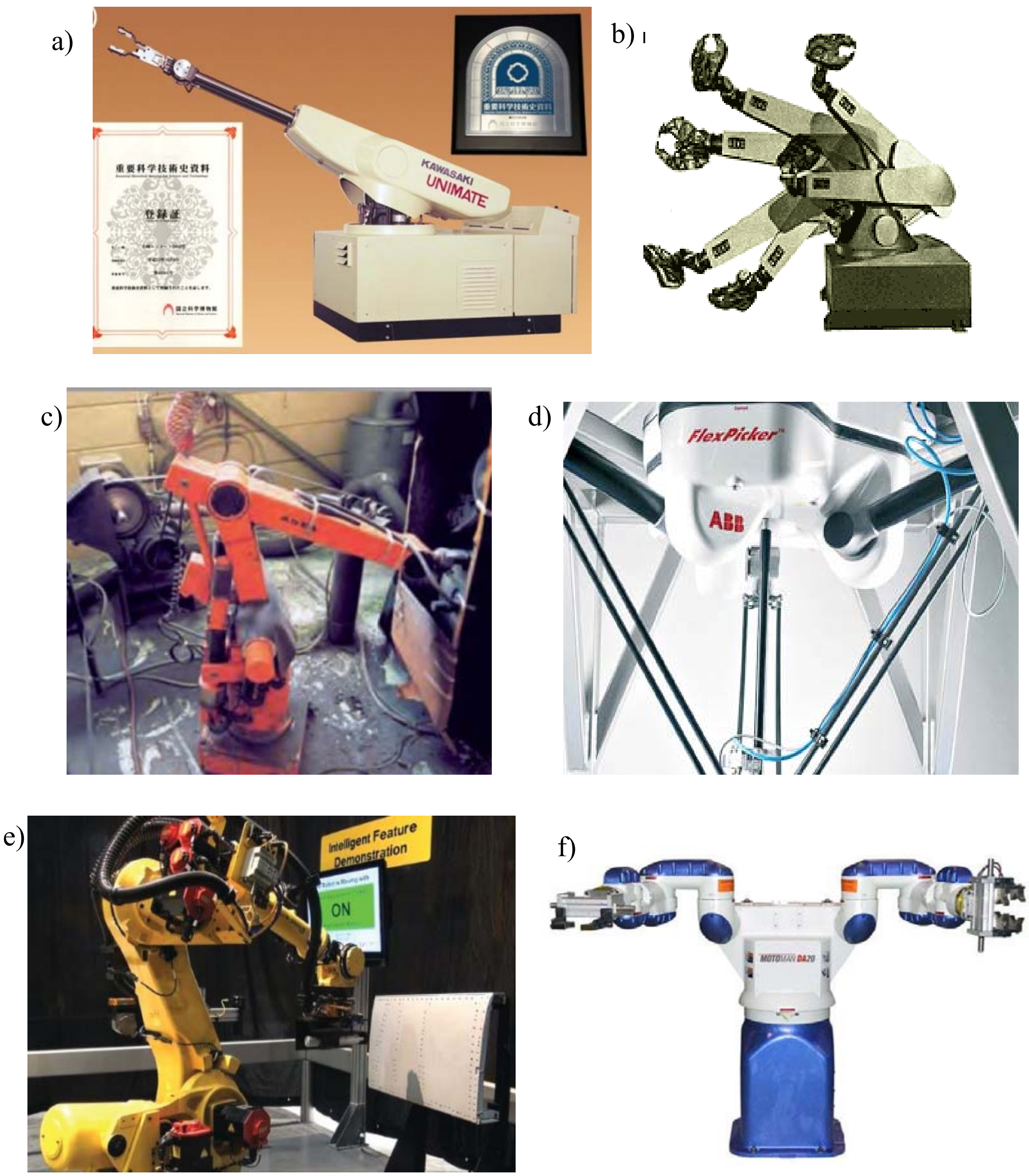


Figure 1 Industrial robots through the history (source: International Federation of Robotics, 2012)

We are witnessing an enormous number of robots with different parameters. Robots can be classified by mechanical characteristics, like number of axis or degrees of freedom DOF, payload and workspace. They can also be classified by application. Fundamental classification begins with separating the robots into two groups: Industrial and Service robots (Table 1).

Table 1 Robot classification by application

Service Robots

- Defense
- Field Logistics
- Medical/Surgery
- Rehabilitation and healthcare
- Exoskeletons
- Cleaning
- Inspection
- Rescue and Security
- Entertainment and leisure
- Household/Domestic
- Telerobots
- Underwater robots
- Aerial robots
- Space robots
- Agricultural and forestry robots
- Construction
- Hazardous applications
- Mining
- Search and rescue
- Intelligent vehicles
- Educational
- Humanoid
- Bio robots
- Social

Industrial Robots

- Arc Welding
- Assembly
- Cutting
  - Plasma
  - Laser
  - Water jet
- Dispensing
- Glazing
- Gluing/Sealing
- Grinding
- Coating
- Enameling
- Material Handling
  - Die Cast
  - Machine Tending
  - Palletizing/ Packing
  - Part Transfer
  - Press/Forming
- Injection molding
- Measuring
- Powdering
- Pre-Machining
- Material Removal
  - Machining
  - Polishing/Finishing
- Painting
- Spot Welding
- Pick and Place
- Cleaning/Spraying

Most frequent robotic systems are the industrial robots. They are often used in tasks where their grippers (end-effectors) are displaced from point to point. In the automobile industry they are usually employed in welding and other material handling applications.

1.1.1. *Statistics of industrial robots*

Robotics is a very large field, which is growing fast, especially in this new millennium, called "Robotic Era". It appears that the interest in the area of robots will never stop, knowing the fact that today's robots are more reliable, safe, accurate, also collaborative and reconfigurable. Big achievements have been made in different fields of robot applications, like medicine, defense, forestry, even entertainment and leisure. The forecast predicts that the sales will reach a peak by 2016, and more than 50% of total robot sales will be in Asia. Projections for the global robotics market grow are at a rate of around 7.5% for the next years till 2016 (Figure 2).

By the International Federation of Robotics (Ifr, 2013) report, robotics is expected to be the major driver for global job creation for the next 5-7 years. In progressing manufacturing markets, the use of machines looks like a good alternative to human labor, which does not mean that robots will replace humans; on the contrary, they are the key factor to stop outsourcing and bring the production back home.

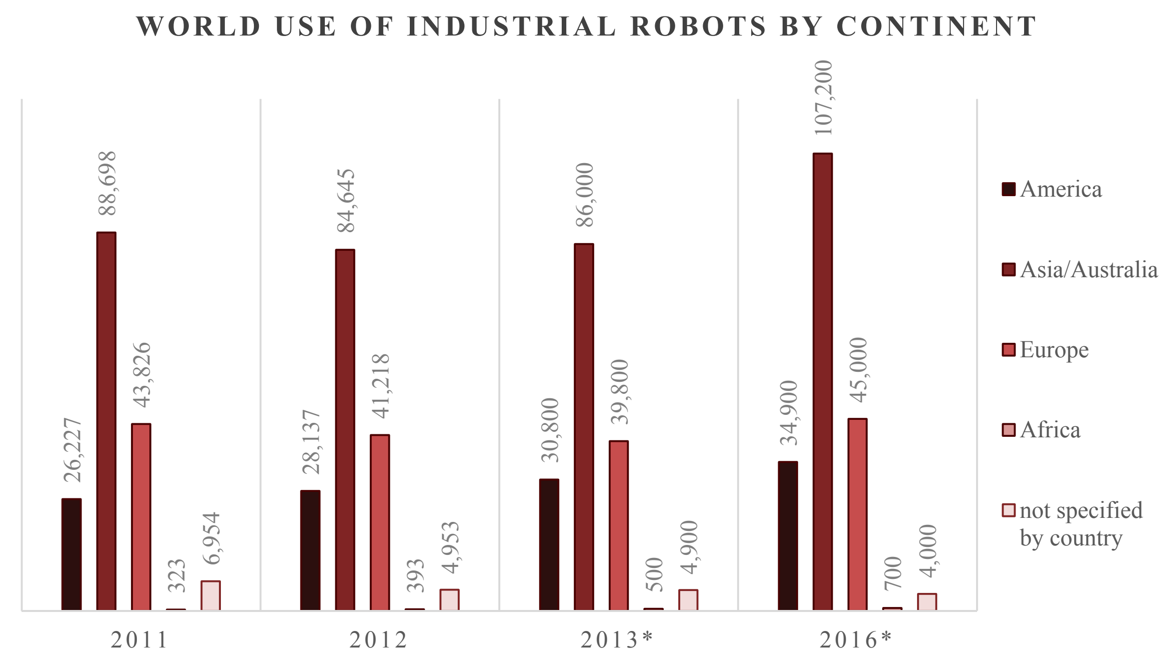


Figure 2 Industrial robot use and forecast by continent (International Federation of Robotics Report, 2013)

In Figure 3 the articulated robots are remaining among the most reliable robots in the industry with 63% installed articulated robots in the world (Gorle and Clive, 2013). Depending on their application, these robots are implemented mostly for material handling

38% and welding 28% (Statista, 2014) (Figure 4). Therefore, the interest in these robots is so far the highest.

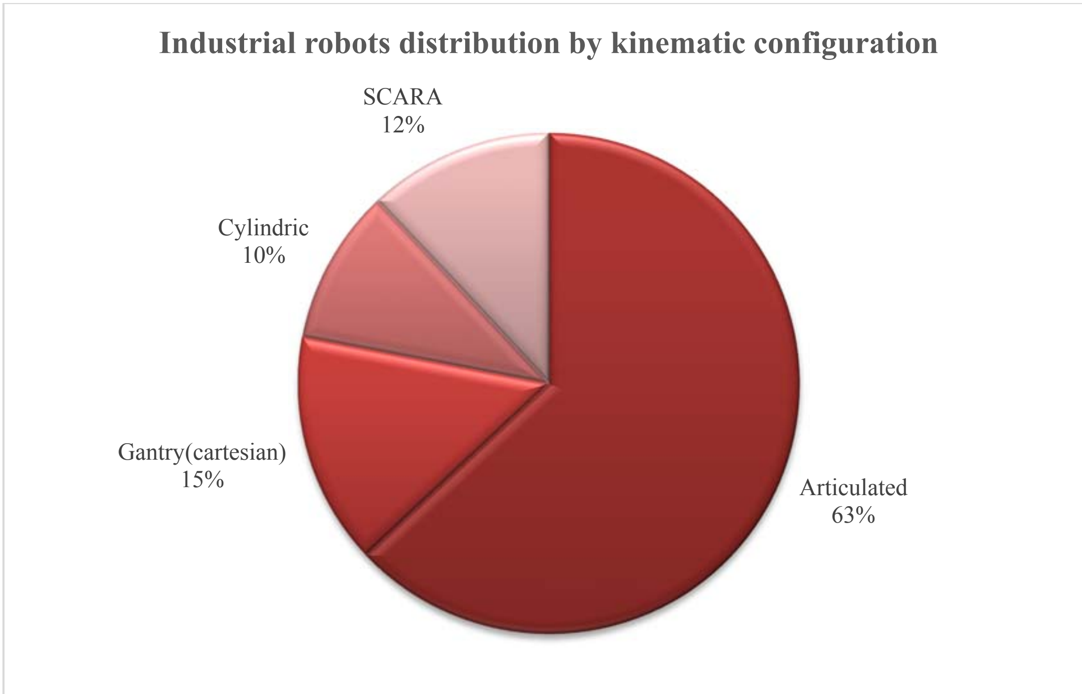


Figure 3 Industrial robot distribution by kinematic configuration

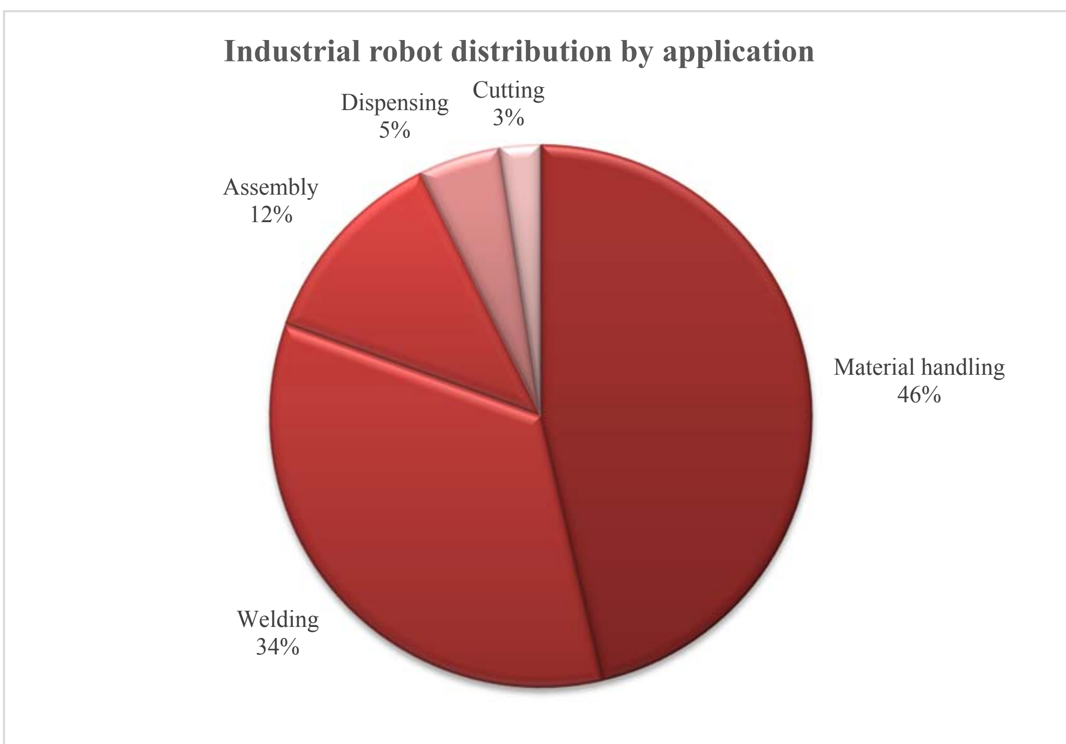


Figure 4 Industrial robot distribution by application

1.2. Robot fundamentals

The definition for robot by the Robotics Institute of America (RIA) is: "A robot is a reprogrammable multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks."

Robots can be defined by their mechanical structure, which consists of a sequence of links interconnected by sets of articulated joints. The anatomy of industrial manipulators is very similar to human arm. It is characterized by an arm that ensures mobility, a wrist that grants dexterity, and an end-effector that performs the task required of the robot. The arm involves shoulder and elbow, and the wrist contains three intersecting joints, creating a spherical joint (Figure 5). The end-effector is the part where different tools can be attached, for performing variety of tasks.

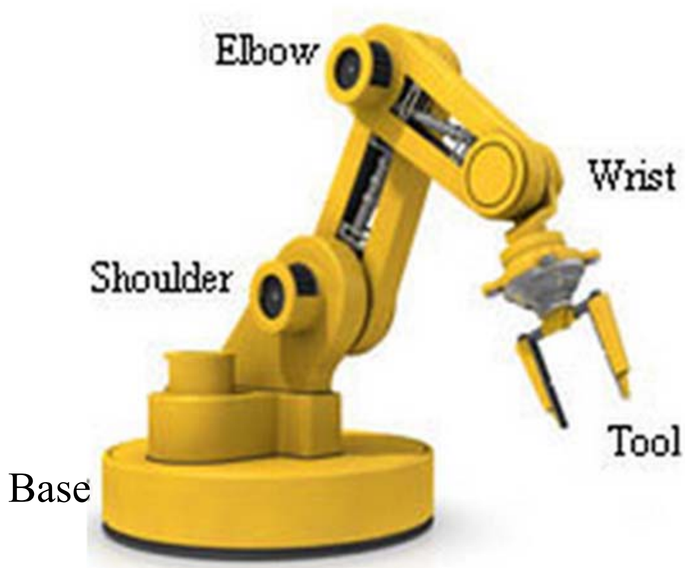


Figure 5 Industrial robot anatomy (source: http://www.bbc.co.uk)

1.2.1. Joints and links

Articulation between two consecutive links can be realized by either a translational or a rotational joint; their presence ensures mobility of the manipulator.

The robot joint is the important element in a robot which helps the links to travel in different kind of movements. There are two major types of joints with one degree of freedom:

- Translational joints - They are usually indicated as T– Joint. This type of joints can perform both translational and sliding movements. For achieving the linear movement, the two links should be in parallel axes (Figure 6 a).
- Rotational joints - Are represented as R – Joint, and allows movements in a rotary motion along the axis, which is vertical to the arm axes (Figure 6 b).

Figure 6 Basic types of robot joints a) translational and b) rotational

All other types of joints can be modeled as combinations of these two fundamental joints. Some other used joints, derived from the basic ones are: twisting joint, orthogonal joint, revolving, screw, cylindrical, spherical and universal joints (Lenarčič et al., 2013). Most industrial robots have only the basic types of joints, in order to avoid complexity in motion planning and control.

1.2.2. Robot geometry

When performing a task, the arm is positioning the wrist which then is required to orient the end-effector. The manipulators are classified by the type of the arm's joints, starting from the base. On the market we can find five different commercially available serial industrial robot manipulators:

- Cartesian,
- Cylindrical,
- Spherical,
- SCARA, and
- Articulated

Cartesian manipulators are described by translational movements, whose axes are jointly orthogonal (Figure 7). The Cartesian structure offers very good mechanical stiffness and constant wrist positioning accuracy, but has low dexterity since at least one joint is translational. Cartesian manipulators are also called Gantry manipulators. Most CNC machines have this kinematic structure, which allows large effective workspace and enables manipulation of large and heavy objects. They are often used for material handling and assembly.

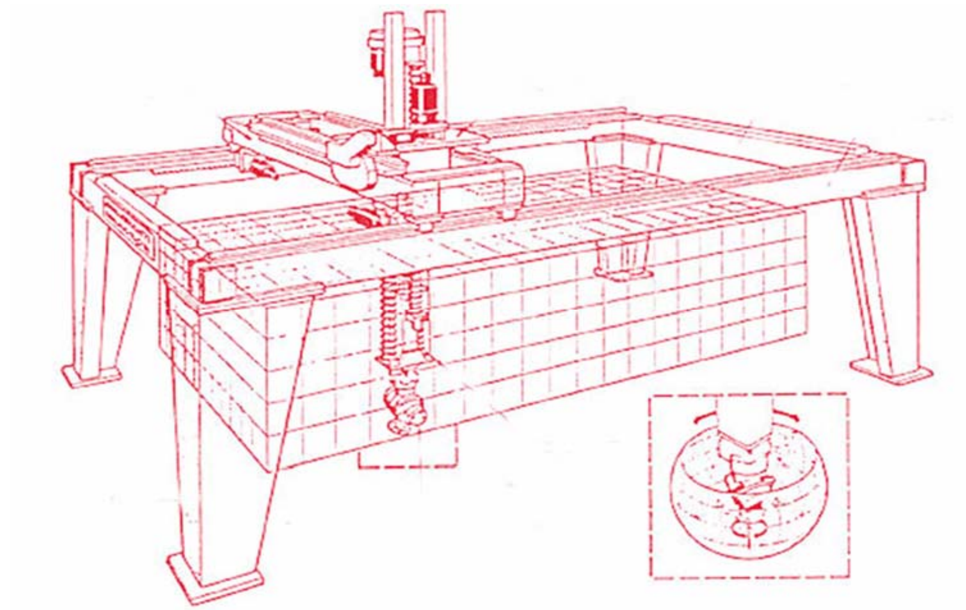


Figure 7 Gantry/Cartesian robot geometry (source: NASA.com)

Cylindrical manipulators are mainly employed for carrying heavy and large objects, while SCARA manipulators are suitable for manipulation of small objects. Spherical manipulators are mainly employed for machining (Siciliano et al., 2009). Their structure and workspace are shown in Figure 8.

Articulated geometry is similar to the human arm (Figure 9). This structure, also called anthropomorphic, is the most dexterous one, since all the joints are revolute, thus the accuracy varies in the workspace. The range of applications of anthropomorphic manipulators is wide and they are the most used manipulators in industry.

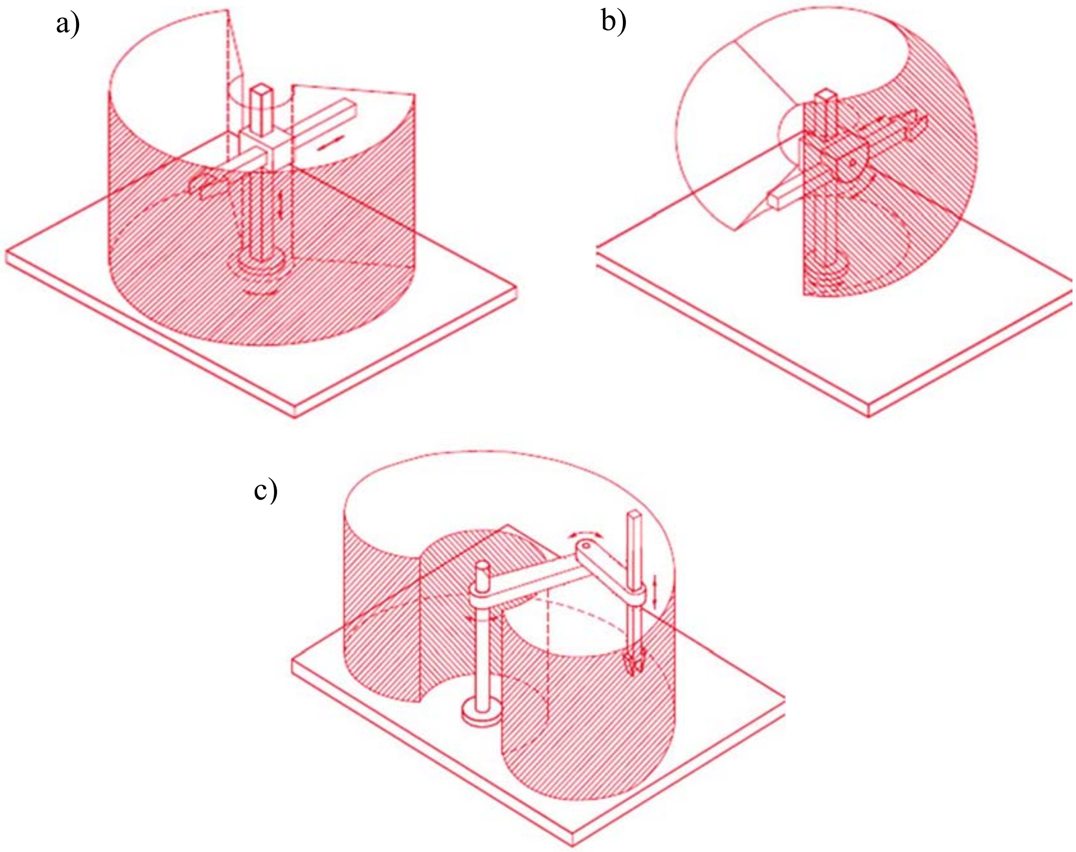


Figure 8 a) Cylindrical, b) Spherical and c) SCARA robot geometry (source: Siciliano et al., 2009)

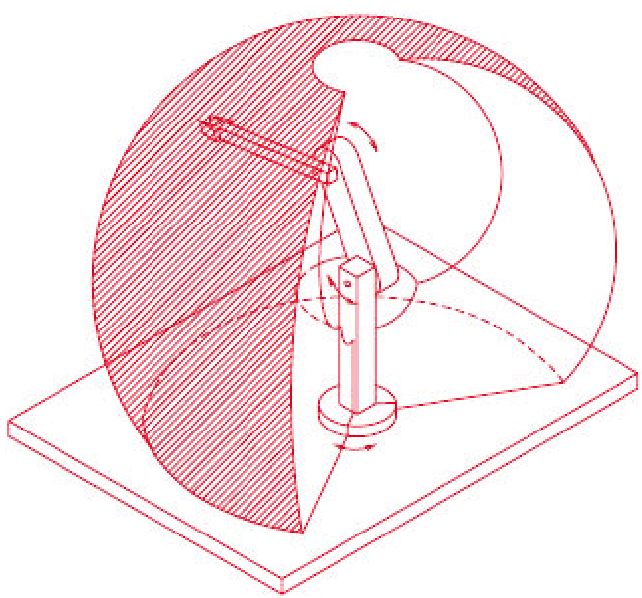


Figure 9 Articulated robot geometry (source: Siciliano et al., 2009)

1.2.3. Workspace of a manipulator

The workspace of a manipulator is defined as the set of all end-effector configurations which can be reached by certain choice of joint angles (Figure 10). For given values of the joint variables, it is important to specify the locations of the links with respect to each other. This is possible by using the manipulator kinematic equations determining the relation between the end-effector and the base joint.

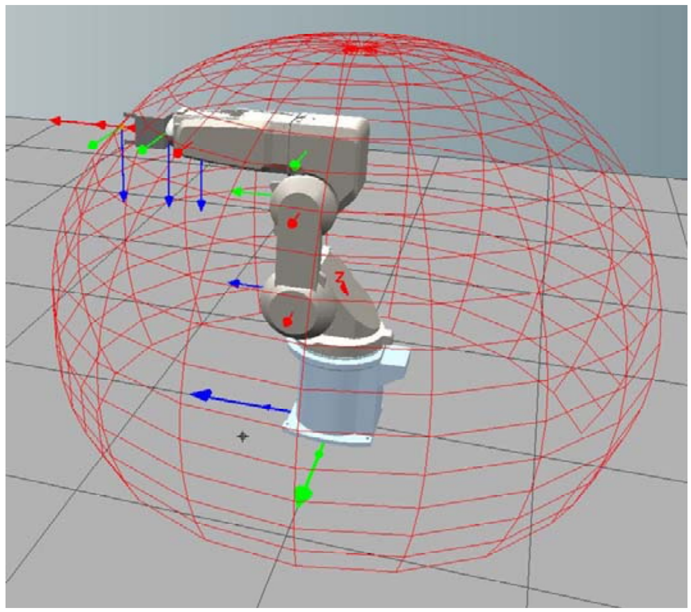


Figure 10 Robot Workspace (source: CIROS® Programming)

Generating workspace for robots can be challenging, due to their functional working area and singularities. In order to make the multibody system move in a desired direction, there is a need to control the position and orientation in various coordinate systems. Path planning and motion control are the most complex problems in robotics, highly dependent of the manipulability and singularities of the manipulators workspace.

1.2.4. Kinematics problem

Kinematics describes the analytical relationship between the joint positions and the end-effector position and orientation. Kinematics of a manipulator represents the basis of a systematic, general derivation of its dynamics. To solve the kinematics problem, there is variety of methods: geometric, trigonometric and algebraic. The formulation of the kinematics relationship allows the study of three key problems of robotics, the direct kinematics problem, the inverse kinematics problem and the differential (velocity) kinematics (Craig, 2005, Spong and Vidyasagar, 2008).

Forward (direct) kinematics is a mapping from joint space to Cartesian space. This mapping is one to one - there is a unique Cartesian robot configuration for a given set of joint variables. It is important to be able to specify the locations of the links with respect to each other and with respect to the base frame.

Differential or Velocity kinematics describes the analytical relationship between the joint motion and the end-effector motion in terms of velocities, through the manipulator Jacobian matrix. Upon solving the forward kinematics for a system, the Jacobian matrix is the next priority, for determining singular configurations. The Jacobian matrix calculation is substantial because it is giving the transformation between velocities in the workspace and the kinematic structure (Merzouki et al., 2012).

The non-linearity of forward kinematics rises complexity in inverse kinematics direct computation (Siciliano et al., 2009). Inverse kinematics represents the mapping from Cartesian space to joint space. The inverse kinematics mapping is typically one to many, but a lot of them may not be physically realizable or the target positions are reachable only with full extension of the links.

1.2.5. Singularities

A singular configuration of a robot manipulator is a configuration at which the manipulator Jacobian matrix drops rank, and they are affecting the size of the end-effector forces that the manipulator can apply. Singularities are remaining as the main problem in robot control and motion planning. They can cause problems in inverse kinematics and design of the robot. When the structure is at a singular configuration, infinite solutions to the inverse kinematics problem may exist. A robot singularity can occur either on the robot's workspace boundary or within the interior of the robot's workspace. Boundary singularities occur when the manipulator is maximum stretched or minimum contracted. From a point of view of dexterity, singularities on workspace boundaries can be avoided by simply fetching the desired operation into the interior of the robot's workspace (Lenarčič et al., 2013). However, singularities in the interior of the workspace are problematic, as they can be encountered anywhere in the reachable workspace. These singularities are caused by the alignment of two or more axes, or by the realization of particular end-effector configurations. Figure 11 represents some examples of common singularities.

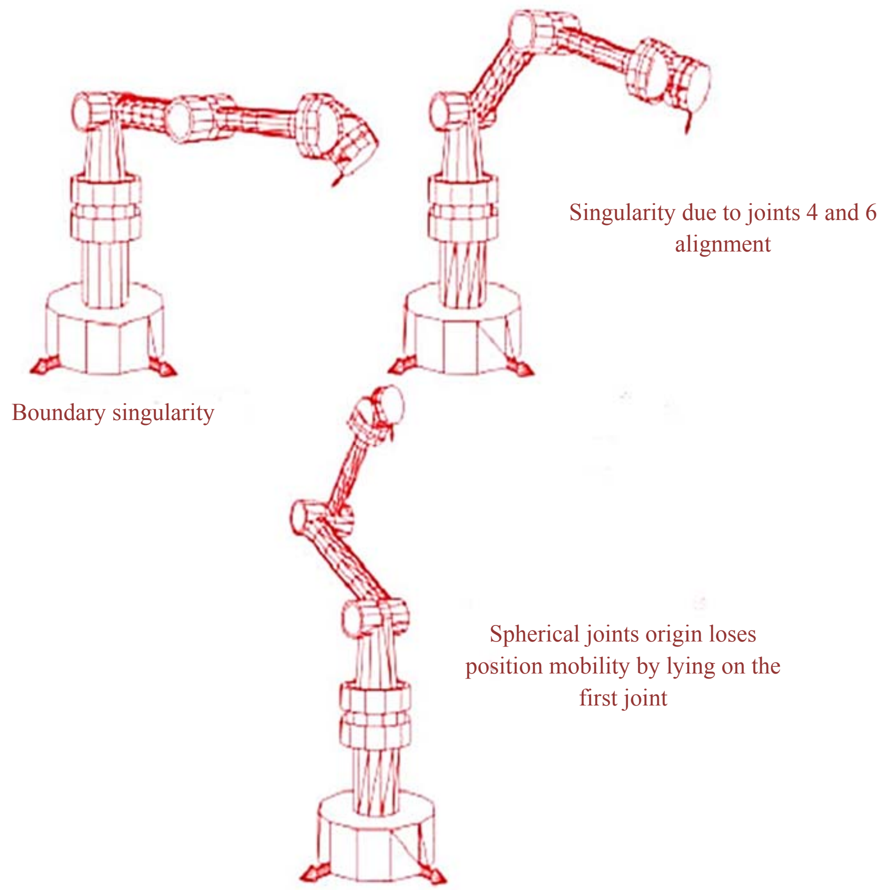


Figure 11 Typical singularities (source: http://www.et.byu.edu)

Robot singularities have negative impact to dexterity and positive to mechanical improvement. At singular configuration mobility of the structure is reduced, which mean it is not possible for the manipulator to impose an arbitrary motion to the end-effector. In the neighborhood of a singularity, the Jacobian matrix determinant has relatively small value which can cause large joint velocities (Bajd et al., 2010). It is known that for almost all manipulator architectures, the theoretical joint space must contain singularities.

1.3. Problem statement and motivation

The mechanical systems defined as a multibodies includes robots, heavy machinery, spacecraft, automobiles, packaging machinery, machine tools, CNC, CMM machines, rapid prototyping machines and others. The manipulability over different workspaces has been increased; single robot can achieve variety of kinematic configurations, thanks to the modularity, reconfigurability and the collaboration with humans and other robots. But, these systems usually have issues with large displacements, and that is why under normal conditions, they endure large variations in geometric configurations. Furthermore, there has been an increase in the operating speed, accelerations and forces, which is resulting in manipulability and singularity issues. The existence of singularities in the effective workspace or the manipulators can intensely affect their performance and control, causing unsupportable torques or forces on the links, loss of stiffness or compliance, and failure of control algorithms. Therefore, kinematic singularities analysis is an essential step in the design of any multibody system. These large forces, later on will lead to the appearance of dynamic problems that must be predicted and controlled.

Currently, when selecting a robot for particular operation, the manufacturers may provide only the maximum reach, but not the functional and effective workspace. Technical manuals offer insufficient information for robot's workspace. Data is restricted, giving rough information about the dimension and the shape of the workspace. Thus, it is impossible to comprehend the manipulability and the real velocity levels of the end effector of the robot in an arbitrary point in the workspace.

The articulated serial 6 DOF robots are the main subject of interest in robotics, and the gantry machines are second most used in the industry. A manipulator with less than six 6 DOFs cannot take any arbitrary position and orientation in space. Usually, the first three joints are representing the robotic arm and they are used to control the position of the robot manipulator and the last three for the so called wrist, used for orientation. The industrial robot arms and CNC machines are having one important shared property; the axes of two neighboring joints are either parallel or perpendicular. If the industrial robot has articulated geometry, another property is the connection between the second and third joint. For workspace evaluation, this property is essential, because if joint two is rotated for certain angle, joint three will move for the same angle, respectively.

Singularities are still the biggest problem in path planning and control of robotic manipulators, thus predicting them in early stage of design is necessary premise for coping with this problem. Depending on the kinematic structure and the required posture for specific task, singularities can appear in different points within the interior of the workspace. Therefore, in addition to the mathematical modelling, visual representation of the singularity locus is necessary.

1.4. Rationale and scope

This research was initiated from the need of a special tool when selecting the best alternative among various robots available on the market, suitable for both designers and users. The main purpose of this design tool for kinematics of multibody systems is to help in kinematics problem solving, by providing visual representation of the workspace with the singularity locus of the same. It represents a set of iterative methods for kinematic design of manipulators, and so at the end, visual presentation of the effective work region, including singular configurations. The methodology is straightforward and it is appropriate for any n-DOF multibody system, even for dual arm collaborative robots.

Having the possibility to visualize the workspace with singularity locus of any robotic system is required. The novelistic approach is represented in a compact comprehendible notion, trough design methodology and IDEF0 modelling language. Its scope is really important because if needed, there is a possibility to combine any CNC-machine and any robot manipulator in one model. Therefore, it is also appropriate for dual or collaborative machines with at least 12 controllable axes. The design methodology outlines the main phases in the modelling, and the IDEF0 summaries the input, output and enablers in the functions insight.

1.5. Outline of the thesis

Firstly, in Chapter 2, a literature survey was conducted on kinematics problem and singularities in general. Then, current research results with respect to the presented problem formulation are examined as the state of the art. In Chapter 3 the novel multibody kinematic design process is explained in terms of a design methodology and IDEF0 modelling language. Chapter 4 contains step-by-step modeling examples, for verification and validation of the method. Complete kinematic models, along with workspace visualization results and singularity conditions are provided for three types of multibody systems; general CNC Machine, Mitsubishi MELFA RV-3SDB and Yaskawa Motoman Dual Arm DA20. Future work on the reconfigurable CNC-R GKM is discussed, as well as developing a capability map and including the workspace boundary singular planes, in order to get the functional work envelope. Chapter 5 contains remarks and conclusions for this design tool.

CHAPTER 2

LITERATURE SURVEY

2.1. State of the art on kinematics problem

The study of robot kinematics is the study of the motion of kinematic structures, using the mathematical tools of linear algebra and screw theory. Kinematics, as the most fundamental aspect of robot design is foundational problem in analysis, control, and simulation. The Denavit- Hartenberg (DH) convention represents the de facto standard in robotics for representing the relationship between the joint angles and the end-effector (Denavit and Hartenberg, 1955). In addition to this relationship, the Jacobian matrix is the link between joint and end-effector velocities.

2.1.1. Jacobian matrix

Whitney, 1955 first introduced the Jacobian matrix, for control needs for manipulators, and since then, this area in instantaneous kinematics of manipulators and their singularities has a number of significant articles. Generally speaking, singularities remain a major problem in manipulator kinematics; therefore modeling a robot manipulator is necessary premise to finding motion control strategies. Kecskeméthy, 1996, presented kinematic design of Jacobian matrix, which is essential for singularities and workspace definition. The evaluation begins with modeling of global Jacobian with generating and implementing several Jacobians from existing ones. The method is suitable for reducing costs, which are difficult to estimate due to inherent nonlinearities. The model is appropriate for reconfigurable and modular systems, and it is suited for rapid prototyping and complex multibody systems. Meldrum et al., 1991 has pointed out the importance of the Jacobian matrix inversion for control purposes. When the manipulator is at singular position the Jacobian is not directly invertible; this issue leads to impossible directions of the end-effector in the workspace.

2.1.2. Workspace evaluation

Geometric optimization of manipulators has origins, back in the 80'es, when Vijaykumar et al., 1986, presented singular configurations for both, rotational and translational joints. The dexterous or functional workspace is graphically presented, inside the boundary points of the work envelope. Singularities are successfully reduces into small areas, and the joint ranges are not affected a lot. Hansen et al., 1983 made the evaluation and generation of the workspace computationally affordable. Their mathematical model has become feasible for analysis of manipulators workspace, thanks to points reduction and refinement, but singular points are not considered in this model.

Nadal et al., 2010, explores new method for determination of the workspace boundary, which is very important in singularity determination. The singularities at the boundary can be easily found with this method, at the same time avoided. The benefits from this method are that all the boundary points can be detected and covered in the computation. The negative side of this method is that is not applicable to manipulators with more than 3DOF.

Ceccarelli, 2012 has established 4 steps for workspace generation in his keynote paper:

1. Kinematic determination,
2. Static performance,
3. Dynamic response, and
4. Final operation-Workspace

These steps are fundamental for designing of manipulators task and for rational use of the mechanical system. Examples of functional workspace with singular and collision areas are provided.

In Djuric et al., 2013 only KUKA robots workspaces have been examined. Three key elements for effective tool frame positioning are outlined; position, orientation and singularities. Excellent examples on functional workspace are presented, with all the singular configurations encountered, besides the boundary singularities. This method can be implemented into design methodology to predefine the feasible or effective workspace evaluation and optimization, especially in RMS environment. With visual representation of the singularities, proper robot manipulator can be selected for specific operation. This is extremely important in operations that require very high precision and accuracy, like in medical field, micro-component assembly or hazardous material operations. In Djuric et al., 2014a, only Fanuc family of robots has been examined. Singularity analyses on 10 different Fanuc robots have been performed. Variations in the workspace are compared for future path planning purposes. Possible reconfiguration is encountered, because the algorithms are reliable for calculating kinematics and Jacobian matrix for any robot. An excessive study on the workspace of Fanuc, ABB and Comau family of robots has been done in Djuric et al., 2014b. They used a reconfigurable modeling approach, where the 2D and 3D boundary workspace is created by using a method identified as the Filtering Boundary Points (FBP) algorithm. The visualizing provides virtually changing the task set or a system reconfiguration, and this method can be employed to determine feasibility prior to physical modification in the manufacturing environment, which results in time and cost savings. Special focus has been given to the end-effector tool position and orientation. Visualization of the singularities is presented in the work window, and evaluation of the effective workspace is conducted.

Another significant work for workspace visualization is by Zacharias, 2012. In this book a novel general representation of the kinematic capabilities of a robot arm is presented. The versatile workspace is introduced to describe in which orientations the end effector attached to a robot arm can reach a position. The author is using manipulability measures to evaluate the velocity at every position of the robot.

2.1.3. Manipulability and singularities

Manipulability is primarily important in optimization and utilization of manipulators workspace and optimal placement determination. Definition of manipulability was given by Yoshikawa, 1985, evaluated with 2DOF, SCARA and Puma 560 robots. The manipulability can be calculated with Jacobian matrix use, and it can be determined in the design stage of the manipulator, with utilizing the best posture of articulated robots with respect to their workspace and singular configurations, considering only the first three joints. These postures are called optimal postures and optimal working positions from a view point of manipulability and they are useful for planning the working positions of robots for various tasks. This algorithm is based on kinematics only, and it depends on the velocity; as higher the velocity is, the manipulability is greater, consequently this indicates that the manipulator is far from singular configurations. Pamanes et al., 1991 have presented a method for optimizing the posture of robotic manipulator, considering constrains inflicted by the workspace and joints limitations. Measure for manipulability is the first step in the implementation of this method, but it does not consider the obstacle and singularities avoidance. Gotlih et al., 2011 have spotted the lack of information in the manufacturer manuals about the robot workspace, especially the manipulability and the real velocity levels. A 3D procedure has been performed for analysis of robot's workspace. The selected robot is used for MAG welding, where the precision plays key role. Changes in the velocity have been noticed, and the variations are higher at the edges of the workspace, because of the low anisotropy parameter; a parameter that is directly dependent of the approaching singular points.

The origins of the study of singularities in mechanism and machine research literature goes back to the 1980s and relate particularly to determination of the degree of mobility via screw theory and the analysis of workspace for serial manipulators (Lipkin and Duffy, 1982). Kieffer, 1994 has provided detailed classification to singularities (Figure 12):

- Turning (ordinary),

- Osculation (ordinary),
- Bifurcation (non-ordinary) and
- Isolated (unsolvable) point's singularities.

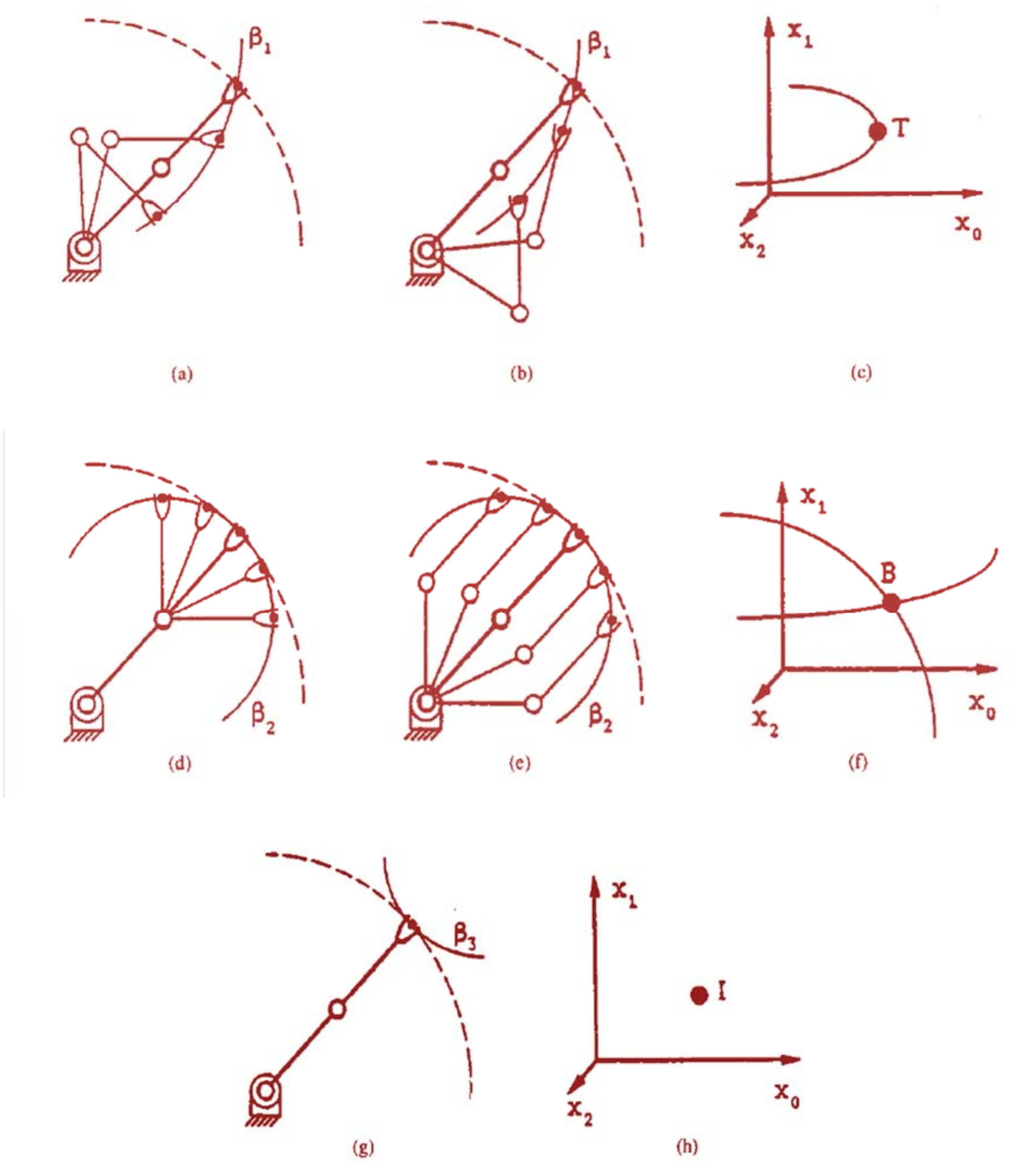


Figure 12 Ordinary singularities (a)-(c), bifurcation (d)-(f) and isolated singularities (g)-(h)

(source: Kieffer, 1994)

The displacement curve is presented in Figure 13, where the singularities are presented at points where they usually occur.

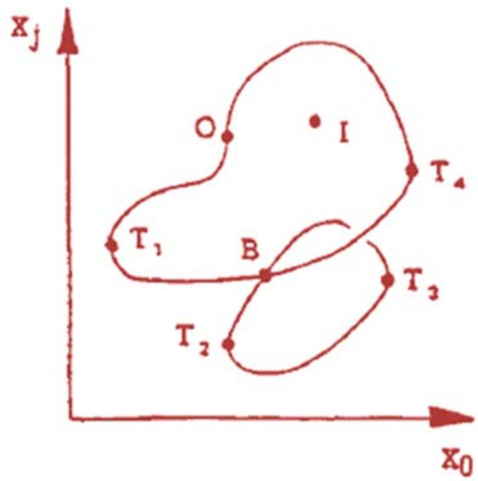


Figure 13 Displacement curve with ordinary singularities ($T_{1\text{-}4}$; O), bifurcation (B) and isolated singularities (I)

(source: Kieffer, 1994)

It has been found that avoiding singularities is difficult and sometimes impossible, by mechanical design or even in the trajectory planning. Even points near singularities, called ill-conditions can be problematic. The damped least square (DLS) method for now, offers the best results in trajectory tracking, with reducing the velocity in singular positions, but if the path needs to be with constant velocity, the problem appears. The author proposed third order differentiation for precise definition of singularities, including velocity, acceleration and jerk motion in the calculations.

Pai and Leu, 1992 have studied the orientational and translational robot singularities. They have outlined that kinematics of robot is important in all areas of robotics; dynamics, control and motion planning. Jacobian-screw theory for calculating singularities has been used for PPP, PPR, PRP, RPP structures (easy to implement), and for RRR, RRP, RPR, PRR structures (difficult to implement).

Duleba and Sasiadek, 2002 has divided techniques for dealing with kinematic singularities into five groups: simple avoiding singular configurations, robust inverses, a normal form approach, extended Jacobian techniques and channeling algorithms. They developed modified Jacobian method of transversal passing through singular configuration. The method, stands for both redundant and non-redundant robots and it is not applicable for hyperbolic singularities, only for quadratic, besides it is computationally affordable and can be implemented in real-time control. Basically, the ill conditioned row of the Jacobian matrix can be deleted and replaced with the differential of the Jacobian determinant. The passing through singularity is achievable with smooth velocities and no stopping or jerking. Kinematic singularities analysis is essential step in manipulator design, so Donelan, 2008, has been working towards applying mathematical singularity theory for exploring robot singularities, bifurcation analysis, singularity locus definition. Mathematical model for testing manipulator genericity and higher order of singularities has been proposed.

2.1.4. Handling singularities

Egeland and Spangelo, 1991, proposed control solution for manipulators in singular configurations. The velocity and acceleration mapping does not constitute a complete model of the manipulator kinematics, so the Jacobian must be included explicitly in the model. To achieve change in Jacobian, null space motion in the singularity and singular value decomposition (SVD) method, based on DLS has been used. Another example for application of DLS method and SVD of the Jacobian is proposed by Kircanski, 1993. Puma 600 and Stanford manipulator examples are presented and numerical complexity is outlined for Jacobian decoupling. When manipulators approach singular configurations they are facing with speed reduction and positioning error. When manipulator is leaving the singular configuration these issues are annulled, and the issues can be successfully overcame. Also, this paper outlines the importance of the singularities and the problems they can cause when high precision is needed. For some applications errors in the positioning are unacceptable. That is why it is better if these singularities are known, so the path planning can be done with avoiding them Chiaverini et al., 1994 proved that it is possible to control a robot manipulator through DLS method, with singularity avoidance. ABB IRB 200 has been selected for validating results. They have also added error feedback, and several errors have been reported, but successfully reduced.

Singularity model including torques in calculations, based on operational space formulation is explored in Oetomo et al., 2001. Unfortunately, besides Jacobian matrix decoupling, jerkiness is a problem when implementing the method into control. Singularities are examined on Puma 560; the most researched robot in the history. Based on reciprocal screw theory, Zhunqing et al., 2002 developed new algorithm for singularity control of manipulators. The Jacobian matrix is rearranged, and the final results are providing straight line following, without sudden joint rates increase in movements. Fang and Tsai, 2003 have presented Puma-type of robot, following a path which is always at singular configuration. There are 5 straight singular lines in the workspace presented, and the robot can deal with all of them.

Singularities, ordinary and non-ordinary, are reviewed in Gracia et al., 2009. Simple avoiding of the singularities can cause reduction of the workspace. Detailed and excessive explanation to ordinary and non-ordinary singularities is given, and how can they be avoided with adjoint Jacobian method, DLS and SVD. These unified views of singularity problems are validated with KUKA KR 15/2 manipulator.

Bok, 2012 has determined new geometric approach for defining singularities, based on two Jacobians, the screw based Jacobian and the reciprocal Jacobian matrix. Acceleration and velocity vectors can be directly obtained. This method is proven in German industry, and general rules have been obtained, for complex spatial mechanisms.

With kinematic decoupling procedure, avoiding singularities and reduce jerk motion caused by tool retraction and cut path connection is accomplished, by Xiao et al., 2011. The observed manipulator is 6R, REIS RV16. In milling operations, redundancy is problem, so for that to be avoided, in the planning phase taking into consideration this characteristic is critical. This approach is successfully implemented in machining application.

2.2. State of the art on kinematics of different types of multibody systems

The design tool for kinematic of multibody systems focuses on 4 main groups of multibody systems: Articulated 6DOF industrial robot arms, 6 axes CNC machines, reconfigurable multibody systems and collaborative of dual arm machines with at least 12 controllable axes. The literature examined in the previous heading is mostly for the serial chain 6DOF industrial robots, and here the other 3 groups are reviewed.

2.2.1. CNC machines singularities and workspace

Lin and Koren, 1996, have presented efficient tool path panning algorithm for 3axis CNC machine, but also applicable in 6axis (6DOF) CNC machines. Two algorithms for singularity avoidance and their successful implementation into milling robots control are presented in Vosniakos and Matsas, 2010. When manipulator is performing a milling operation, usually the used material is with low-strength, and the accuracy is very important. Optimization is needed in order to achieve precise results, along the planned path. The initial position of the end-effector needs to be located at a spot, which will provide constant acceleration and force. With respect to the base frame and singularities, the algorithms provided effective optimization, with 1% errors in the following of the trajectory. Du et al., 2010, presented kinematics of 3 axes CNC machine tool for error modeling. Two key factors are important in CNC machining; geometry and force. The geometry can be predefined; also the force can be stable if there are no singularities encountered. The initial positioning error is the most common error in CNC machining; therefore optimization must be performed before selecting the most appropriate pose of the tool, to ensure fewer errors in the rotational elements. Kunpeng et al., 2011, are presenting singularities analysis for micro milling toll, where high precision is demanded. No evidence of effective workspace or visualization is found in this paper, only mathematical modeling is performed.

2.2.2. Reconfigurable Multibody Systems (RMS) kinematics

Reconfigurability is a set of methodologies and techniques that aid in design, diagnostic and ramp-up of reconfigurable systems and machines that give corporation the engineering tools that they need to be flexible and respond quickly to market opportunities and changes. Koren and Shpitalni, 2010, in their paper explained the necessary premises for designing a RMS. There are 6 main characteristics of RMS:

- Customization
- Convertibility
- Scalability
- Modularity
- Integrability
- Diagnosability

Kelmar and Khosla, 1988, developed a Reconfigurable Modular Manipulator System (RMMS) prototype and an algorithm to determine the forward and inverse solutions for the same, for R and T joints, n-DOF, for any redundant and non-redundant systems. The implementation time was only 35 milliseconds. Special algorithm for singularity avoidance and design parameters selection was developed too.

Reconfigurable Modular Multibody System (RMMS), its workspace and kinematics were explored in Paredis and Khosla, 1993. Numerical model and design examples have been provided. Starting from the fact that for every task there is a need of design methodology that can generate appropriate kinematic and dynamic model of a manipulator, the authors have proposed iterative design procedure applicable into RMS environment. In the numerical solutions obstacle avoidance in the workspace is included, but not singular configurations.

In general robots are reprogrammable machines and they can be reprogrammed for specific task, but each robot is applicable for limited number of operations. Modular robot with uncommon application was developed by Bolmsjo and Olsson, 1999, applicable for assistance to disabled persons. A design approach is presented, taking into consideration the payload, accelerations, velocities, also modularity and reconfigurability. For analysis, Matlab and Maple software have been used. Completely new control and validation system has been constructed, including kinematic parameters for conceptual design solutions.

A novel n-DOF Global Kinematic Model (GKM) for reconfigurable manufacturing systems (RMS), was generated by Djuric et al., 2010. All the possible values and configurations, also forward kinematic reconfigurable solutions are given. N-DOF GKM joints modeling is suitablefor any combination of either rotational or translational type of joints. The total number of supported structures is $48(48-1)^{n-1}$; for a 6DOF system, equals to 11,008,560,336 possible kinematic structures. The mathematical model stands for unified approach for kinematics of reconfigurable multibody system.

2.2.3. Dual arm manipulators kinematics modelling

Koga and Latombe, 1994 have spotted a need on the market for dual arm employment since 1994. The concept was based on collision avoidance, and the two arms can either work independently, either they can collaborate in manipulating one object. Graphical examples of the manipulation planning are provided, both in 2D and 3D space.

In Smith et al., 2012, short, but deep survey in dual arm manipulation was presented. In 2012, the word "collaborative" got written permission to be used in robotics. The main motivation for using two collaborative arms was outlined. Basically, dual arm concept is used for performing bimanual tasks, similar to humans. Even the interactions between the two arms are human-like. Another reason why these collaborative robots are considered as new “boom” in robotics is that they are low-cost, easy to install, reconfigurable manipulators, capable to work hand-by-hand to humans. Their flexibility and combination of stiffness, when operating one part can be easily transferred into the workspace, achieving greater manipulability. Collaborative robots, by ISO-10218 are defined like robots with velocity lower than 0.25m/s and dynamic power less than 80W, with maximum static force of 150N. Haddadin et al., 2007, investigated several collaborative robots for safety concerns. Robots mass and velocity and potential unsafe circumstances were examined. Velocity of 2m/s is safe enough and cannot harm to humans or cause injuries. When a manipulator approaches singularities, even 30cm before, the robot stiffness cannot stop the robot enter the singularity, but can only reduce the impact with the use of collision detection. This approach leads to effective workspace reduction. Besides that, it is concluded that even the biggest force cannot cause life threatening or serious injury. Recommendation to conduct biomechanical analyses is proposed, so the standard can be changed, so more collaborative robots can become available on the market. Caccavale et al., 2000, have proposed mathematical formulation of cooperative task-space formulation. Regulations have been established, also equilibrium and stability analysis have been performed, so the mathematical model is complete, validated and proved.

A dual arm robot has been developed by Park et al., 2008. The robot has 2DOF torso and 6DOF at each arm. The robot main application is aimed to be automotive assembly line. Workspace analyses are provided, with respect to the cooperative workspace of the two arms, but singularity is not considered. Two similar robots are developed by the same authors, one in 2006, with 3DOF torso and 6DOF for both arm, and second with 5DOF per arm in 2012. The same authors, Park et al., 2009, have provided excellent kinematic analyses for the two arms, left and right, of the dual arm robot, designed for automotive assembly. Singularity analyses have not been performed, but only cooperative workspace evaluation, and further dynamic and control validations. Zhai et al., 2012, have conducted kinematic analysis to a dual arm humanoid cooking robot. The kinematic of this robot corresponds to Motoman DA20 robot, but the representation of the DH parameters is weak. The joint representation was reduced, so the calculation is simplified, but not accurate. The kinematic equations and Jacobian matrix are established, providing positioning and orienting of the arm in the workspace. Analyses of the left arm are provided only, and no difference or comparison between the two arms is pointed.

Ding et al., 2013, emphasizes the interest in safe human-robot collaboration, too. Experiments with ABB Dual-arm robot, also known as FRIDA, are performed. Examples on the workspace are given. With this control method, Finite State Automata (FSA), the authors accomplished reduction of the robot stoppages, while human interacting in the shared workspace. Only speed reduction is sufficient. Two groups of researchers, from ABB corporate research in Germany and Sweden, are still working on ABB's FRIDA. This robot was successfully implemented into several European Union funded projects. As suggested by the authors Kock et al., 2011, implementation of this kind of collaborative robot can grant inexpensive and flexible assembly lines. Even the speed is not as high as on the other robots, therefore the minimal cycle time in agile production can be accomplished.

In Basile et al., 2012, a task-oriented motion planning novel approach for general cooperative multi-arm robot work cells or systems is proposed. The main formulation is work piece-oriented, and the motions for single arm can be computed separately, relevant to the corresponding coordinate frames. The goal is to merge the manipulator task planning with the whole system planning. Furthermore, planning strategy for safe human-robot collaboration is proposed. De Luca and Flacco, 2012, were emphasizing the new research field in robotics, safe human-robot collaboration. The control system for collaborative robots proposed in this paper can avoid collision and singularities; third and fourth derivative are included in the calculations for more accurate Jacobian matrix derivation; jerk and snap motion.

2.3. *Literature review summary*

Table 2 State of the art in multibody kinematics

Denavit and Hartenberg, 1955

- DH convention

Yoshikawa, 1985

- Manipulability measures

Vijaykumar et al., 1986; Hansen et al., 1983; Fang and Tsai, 2003

- Geometric optimization of reachable and dexterous workspace using numerical techniques

Kieffer, 1994

- Provided detailed classification of singularities

Abdel-Malek, 1997; Caccavale et al., 2000; Ceccarelli, 2012

- Mathematical formulation of the workspace

Djuric and ElMaraghy, 2006

- Reconfigurable Kinematics, Dynamics and Control for multibody systems

Park et al., 2008; Zhai et al., 2012

- Dual Arm Robot kinematics analysis

Gotlih et al., 2011

- Velocity anisotropy visualization

Zacharias, 2012

- Visual representation of the effective workspace for dual and single arm robots

Djuric et al., 2014

- Visualization of the singularity locus within the workspace

In this thesis, basic literature in kinematics of rigid bodies have been examined (Table 2), like the genuine standard in robotics, DH convention (Denavit and Hartenberg, 1955), the Jacobian matrix (Whitney, 1955; Duleba and Sasiadek, 2002), functional workspace (Vijaykumar et al., 1986; Fang and Tsai, 2003; Hansen et al., 1983; Caccavale et al., 2000) and singularities (Kieffer, 1994; Yoshikawa, 1985).

Four major groups of multibody systems were compared in Table 3. The literature is extensive in kinematics and workspace of 6DOF robot manipulators, but not much in their singularities. Also the research in reconfigurable robots and their potential applications is widely explored, but not for their workspace (Kelmar and Khosla, 1988; Djuric et al., 2010; Kecskeméthy, 1996; Koren and Shpitalni, 2010; Paredis and Khosla, 1993). The CNC machines are treated like separate family of robots and their kinematic structure corresponds with the robot's one. Because they are capable of performing similar tasks, they usually work together (Lin and Koren, 1996; Nadal et al., 2010; Du et al., 2010; Kunpeng et al., 2011; Djuric et al., 2014b). The future in robots and machinery in industry are the new, safe, collaborative robots which can operate next to people. Employment of collaborative machinery is the key factor for achieving higher level of efficiency and productivity of robotic work-cells. Very few references are found for kinematic analyses of dual arm robots (Basile et al., 2012; Caccavale et al., 2000; De Luca and Flacco, 2012; Park et al., 2009; Koga and Latombe, 1994; Ding et al., 2013; Zhai et al., 2012; Zacharias, 2012). These system offers safe human-robot interaction, support and cooperative workspace use, with one main goal – to establish better performance and reliability in every industry sector. It has been concluded that the dual arm robots have more advantages than the single ones, but the modeling is more challenging and more difficult.

The research in handling singularities is for inverse kinematics, path planning and control, but not much for design of manipulators. As stated before the singularities are remaining as number one issue in modelling a robotic systems and they need to be predefined in order to avoid inverse and control problems.

From design point of view, with mathematical modeling, visual representation and simulation, design parameters can be attained with respect to the functional requirements. There are a lot of limitations in the designing, but the mathematical modeling, combined with the simulation results, can provide excellent starting point for successful kinematic design, which will encompass all kinematic characteristics of the robot manipulator. These analyses can be used for selecting appropriate robot for specific task, which will help industries to get what they need. The analyses can be considered as a set of iterative steps for kinematic modeling of manipulators with appreciation to Denavit Hartenberg convention and n-DOF GKM.

Table 3 Literature review comparison summary

Kinematics	6DOF family of industrial robots	CNC machines	Reconfigurable robots	Collaborative robots
Jacobian matrix & Singularities	**Ro1**	**Cn1**	**Re1**	**Co1**
Effective Workspace	**Ro2**	**Cn2**	**Re2**	**Co2**
Kinematic Design	**Ro3**	**Cn3**	**Re3**	**Co3**

Ro1 – Egeland and Spangelo, 1991; Meldrum et al., 1991; Kircanski, 1993; Chiaverini et al., 1994; Kieffer, 1994; Zhunqing et al., 2002; Gracia et al., 2009; Duleba and Sasiadek, 2002; Pai and Leu, 1992; Oetomo et al., 2001; Fang and Tsai, 2003; Vosniakos and Matsas, 2010; Kunpeng et al., 2011; Xiao et al., 2011; Bok, 2012; Donelan, 2008; Gotlih et al., 2011; Djuric et al., 2013

Ro2 – Pamanes et al., 1991; Hansen et al., 1983; Nadal et al., 2010;

Ro3 – Bolmsjo and Olsson, 1999; Pettersson, 2008; Suatoni et al., 2012

Cn1 - Lin and Koren, 1996; Du et al., 2010

Cn2 – Nadal et al., 2010; Djuric et al., 2014b

Cn3 – Kunpeng et al., 2011

Re1 - Kelmar and Khosla, 1988; Kecskeméthy, 1996; Djuric et al., 2010

Re2 – Djuric et al., 2014b

Re3 – Paredis and Khosla, 1993; Koren and Shpitalni, 2010

Co1 – Park et al., 2009; Zhai et al., 2012; De Luca and Flacco, 2012;

Co2 – Koga and Latombe, 1994; Caccavale et al., 2000; Basile et al., 2012; Zacharias, 2012; Ding et al., 2013

Co3 - Park et al., 2008; Kock et al., 2011

CHAPTER 3

DESIGN TOOL FOR KINEMATICS OF MULTIBODY SYSTEMS

3.1. Kinematics design process

When designing a robot for specific tasks, some basic requirements should be met, for wider set of potential applications. That is why the robots differ by the payload capacity, number of axes, and workspace volume, for applications like painting, welding, assembly, machining, and wide handling tasks. In general one or more robots are combined in robotic work cells, and thanks to the pre-planning, configuring and integrating they are providing cost savings in production. When planning a motion for manipulation task there are different uncertainties, so the manipulability is subject of constrains, depending of the task. These constraints are imposed mainly by the kinematics of the mechanisms, because the kinematic structure is defining the workspace of the manipulator. For robot designers, dealing with these uncertainties is puzzling, as robots differ by mechanical structure.

Like in any other design process, the goal in robot design is to provide optimal solutions based on the robot's functional requirements. Angeles, 2007, divided the process of robot's kinematic design into 4 main steps: Determining topology, Robot architecture, Structural dimensioning and Static performance. Thus this open ended method is a set of iterative steps, to a large extent the steps are independent of each other. Widely used design methodology, is suggested by Pahl et al., 2007, with respect to the specified task. With merging the two methodologies, a novel design tool for kinematics of any multibody system is developed, starting from conceptual to a detailed design. In Figure 14 all the activities involved in the process are presented.

In concept ideation, basic robot workspace topology is selected. Selection of kinematic structure and estimation of link and joint parameters (DH parameters) are included in the conceptual phase. Third phase, embodiment or parametric design is the stage where the link and joint parameters are established, also the static performance and dynamic response calculations are conducted. The Jacobian matrix and singularity conditions are analysed in this phase. Final dimensioning of axes and defining the base are performed in the detailed design phase. This process is very tedious with regards to the fact that it involves advanced mathematical modeling features and simulation.

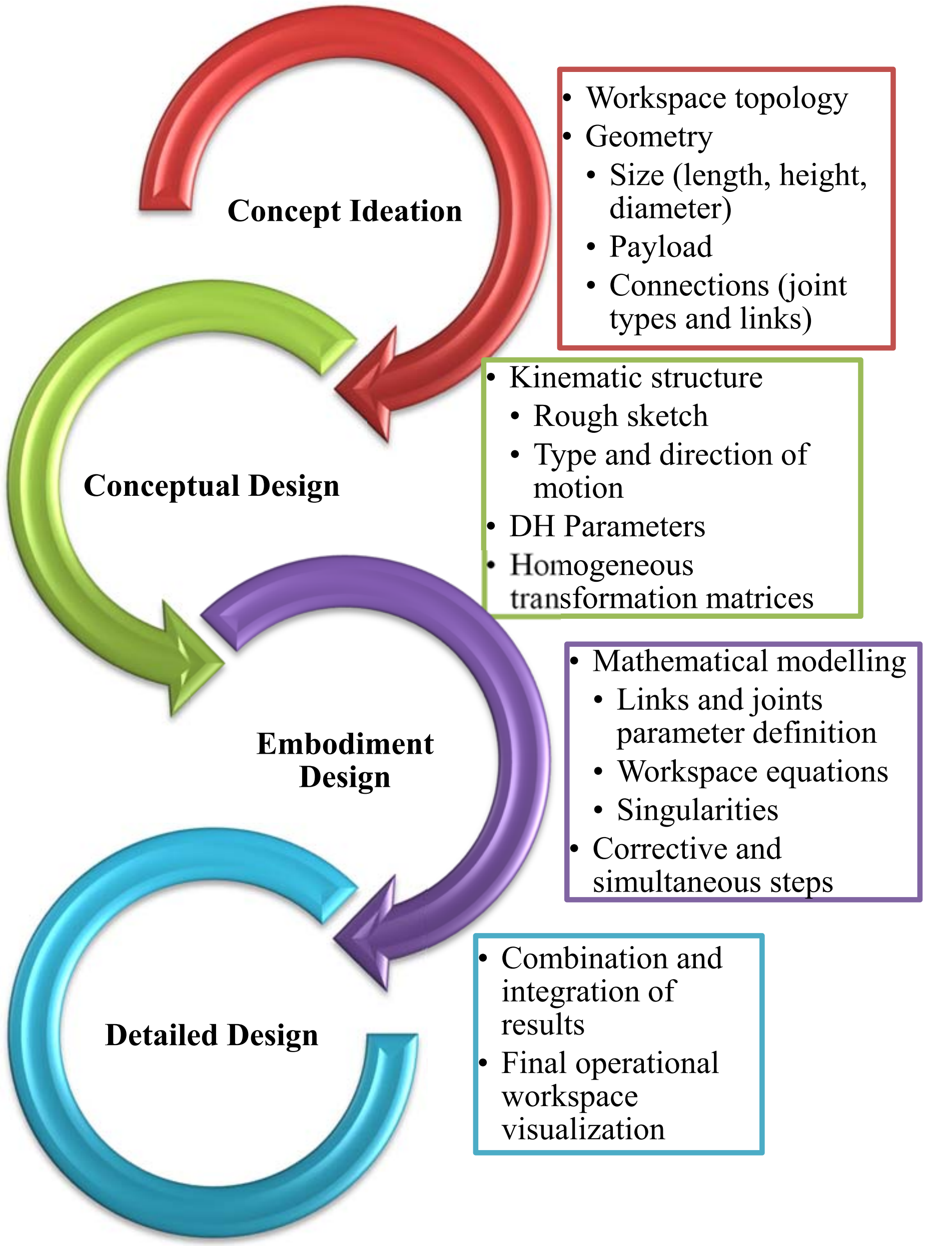


Figure 14 Kinematic design method for multibody systems

3.1.1. Concept Ideation

A fundamental problem in multi degree of freedom kinematic chains design is defining the functional workspace with a set of required characteristics. As noted before in Chapter 1, in the workspace, singularities occur. Based on the shape of the workspace, some design rules can be stated (Angeles and Park, 2008), like:

1. If the workspace is auxiliary symmetric and finite, robot with revolute joints should be used.

2. If the workspace should be prismatic and infinite, gantry robot should be used, with at least one (usually the first) joint prismatic, and with limited larger manipulability to one direction than the others.

3. If axial symmetry is not required, but a workspace with coplanar axes, dual arm robot can be used.

Figure 15 also represents these basic design rules.

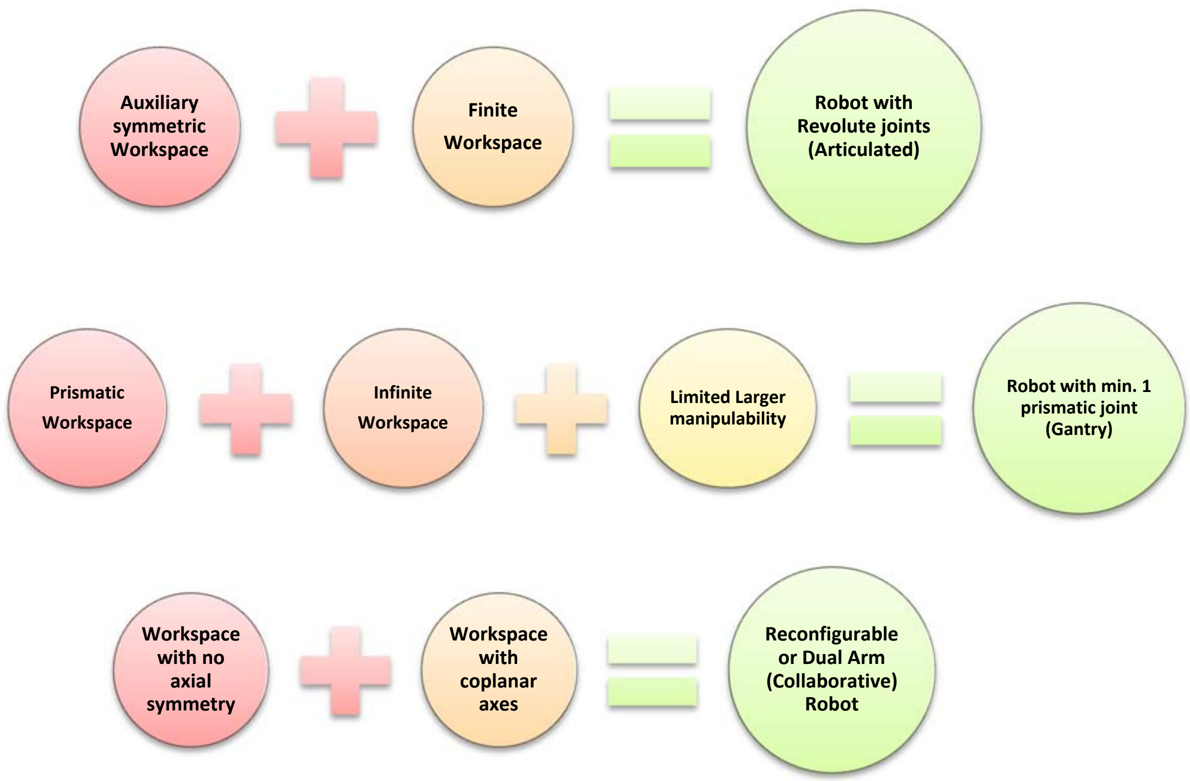


Figure 15 Design rules for determining workspace topology

Concept ideation is related with the basic geometry of the system; size, length, height, diameter, connections, payload etc. Links and joints, through degrees of freedom are determined as well. Because of the fact that each multibody system should be able to perform family of tasks, in order to be reusable, also the fact that each multibody system should have 6 degrees of freedom to be able to take any arbitrary position in space, in most cases the selection falls to 6DOF systems.

3.1.2. Conceptual Design

In the conceptual design stage there is an absence of mathematical model, thus preliminary modeling can be performed. In this modelling phase, three main steps are essential:

1. Kinematic structure
2. DH Parameters definition
3. Homogeneous transformation matrices

In the first step, initial rough drawing is sketched, to represent the structure of the multibody. It is very important to specify the position and preliminary appearance of the system. Usually, simple straight lines are used to represent the links of the system, and at each joining point coordinate frame is assigned, to verify the type of joint (rotational or prismatic) and the type of motion as well. For rotational joints curved arrow, indicating the positive direction of rotation, and for prismatic joints left-right arrows to indicate the sliding directions of the link, noting that the positive direction of a translational motion corresponds with the positive direction of the links frame (Figure 17).

When working with reference coordinate frames, a right-hand convention is used for orienting the coordinate axes (Figure 16 a). Likewise, the direction of rotations about an axis is defined by the same rule (Figure 16 b); when the thumb is pointed in the positive direction of any axis, the fingers are curled in a positive rotation (Goebel, 2014).

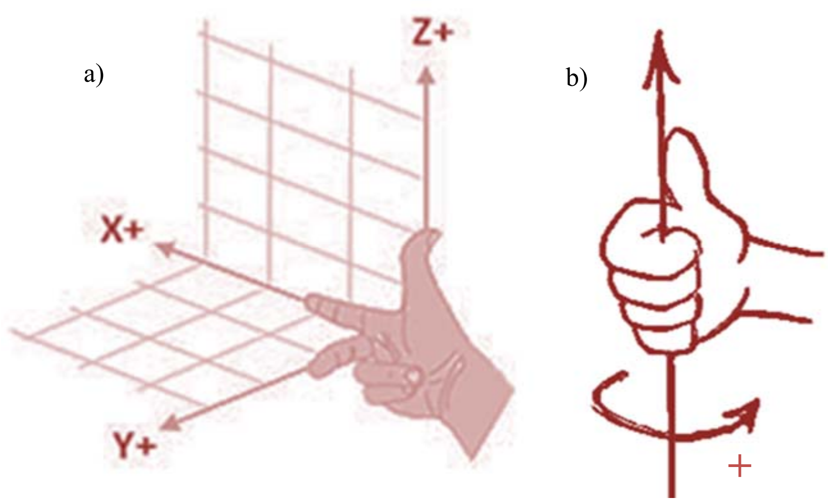


Figure 16 Right hand rule for coordinate frames (source: pirobot.org)

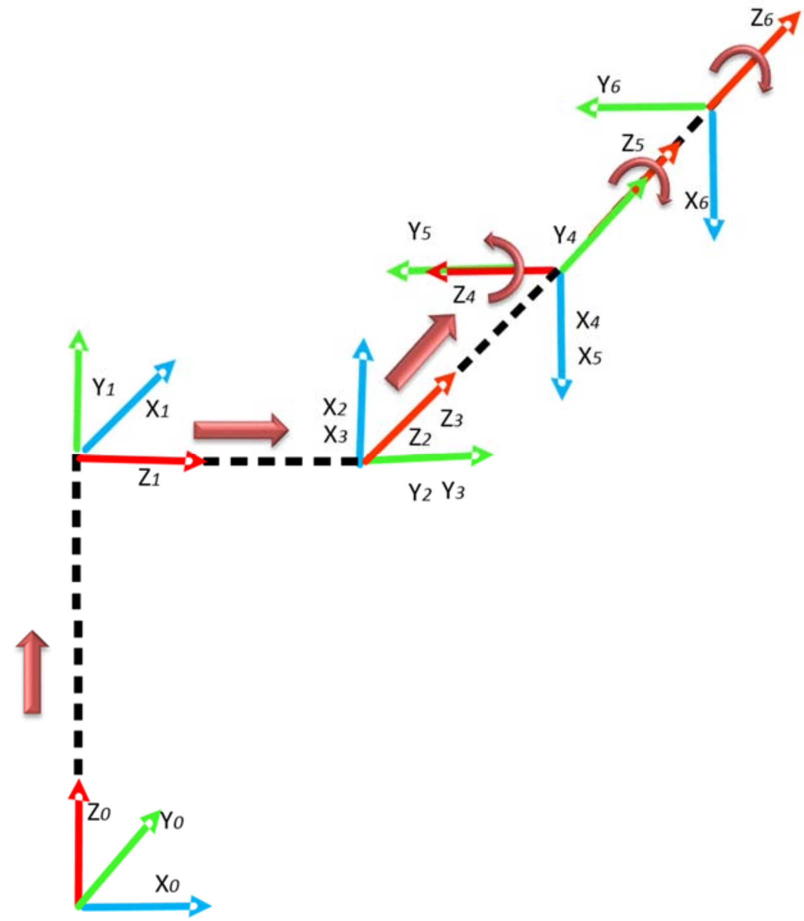


Figure 17 Example on coordinate frames assignment and joint type representation

Reference frames are used to describe the motion of links. In serial, non-mobile robotic systems, the first frame is motionless and fixed to the ground. The usual motion of a robot takes place in this frame, also called the global reference frame. A moving frame is a reference frame that moves with the corresponding link and is called local reference frame. The relation of the links with respect to the fixed, base frame can be explained by the position and orientation of its local reference frame in the global reference frame (Jazar, 2010).

The second step is highly related to the first one; assigning coordinate frames is part of DH convention (Denavit and Hartenberg, 1955). By this convention, four parameters are needed to describe each link (Figure 18):

***θi* -** The *angle* of joint *i,* is defined as the angle about Z axis, between the links

di - The *offset* of link *i,* or displacement, is defined as the distance along the *Z* axis of joint *i,* between the links

ai **-** The *length* of link *i,* is defined as the length along X axis, of the common perpendicular axes

αi **-** The *twist* of link *i,* or link length is defined as the angle between the axis about X axis

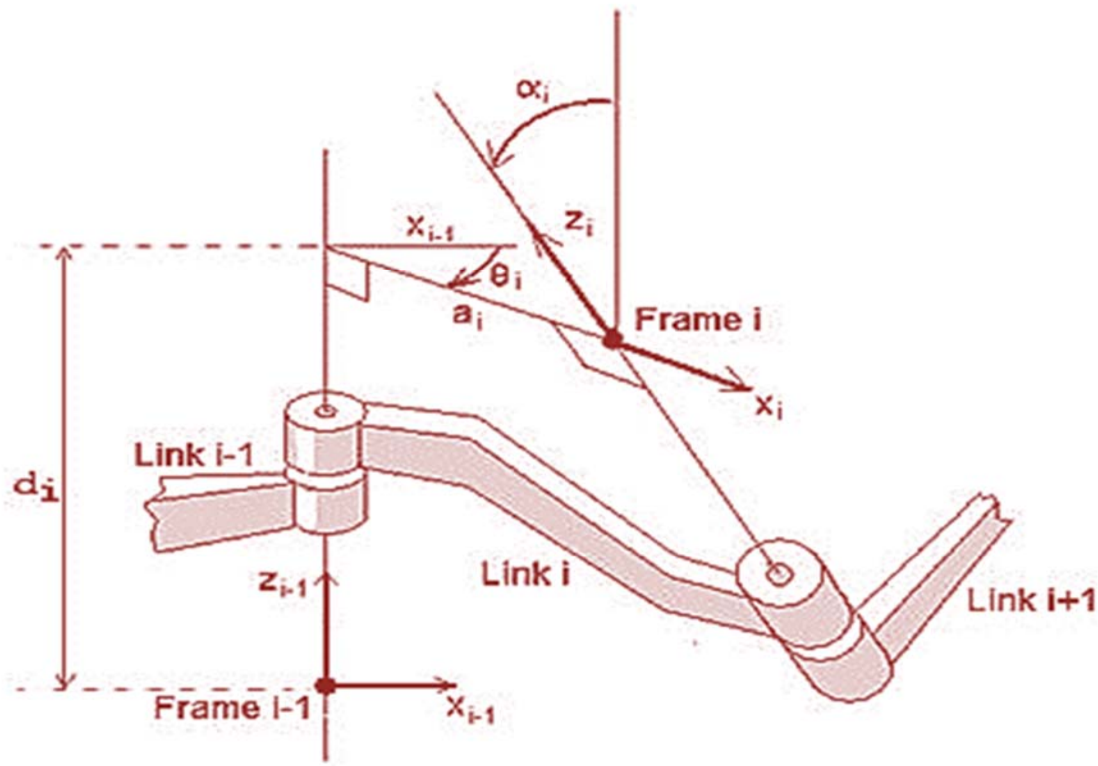


Figure 18 DH Parameters convention (source: uwf.edu)

The link length parameter, ***ai*** is always constant, but the displacement ***di*** and joint angle ***θi,*** depending on the joint type can be constant or variable. If the joint is translational, ***di*** is variable and ***θi*** constant; if the joint is translational, vice versa.

To formulate the DH parameters table, it is customary to associate the coordinate frame with each link. The DH table contains by one row of the four parameters of each link, like in the Table 4 below:

Table 4 Denavit-Hartenberg parameters table example

DH Parameters table				
i	d_i	θ_i	a_i	α_i
1	d_1	θ_1	a_1	α_1
2	d_2	θ_2	a_2	α_2
…	…	…	…	…
n	d_n	θ_n	a_n	α_n

The final step in the conceptual design phase is the homogeneous transformation defining. With linear algebra and use of matrices it is possible to describe any rigid motion of a body. When moving from one link to another, four transformations occur, as determined in the DH convention:

1. Translation along d
2. Rotation about θ
3. Translation along a
4. Rotation about α

Following these steps, the multibody system can be described with linear algebra by using standard notation. Communication among the coordinate frames, which is called transformation of frames, is a fundamental concept in the modeling a rigid body, thus each frame *Fi-1* relative to *Fi* can be described as a homogeneous transformation matrix. If it is requested to perform transformation in three dimension, and rotation and translation with one operation, a homogeneous transformation matrix can be created. This matrix contains 16 elements, and it is unifying the orientation and position or rotation and translation into a single matrix. The transformation from frame $Fi{-}1$ to frame Fi depends only on the joint position of joint *qi.* Once the DH table is filled with parameters, it is easy to determine homogeneous transformation matrices, with respect of two neighboring joints. The matrix general form is given in Eq.1 (Denavit and Hartenberg, 1955).

$$
{}^{i-1}A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; i = 1,2,\ldots,n \tag{1}
$$

The last step in the parametric phase, is to compute each of the link transformations in a matrix form.

3.1.3. Embodiment (Parametric) Kinematic Design

Forward kinematics, workspace evaluation, Jacobian matrix derivation and solving for singularity conditions are part of the embodiment kinematic design phase. At this stage all the mathematical modelling is completed, along with the singularity analyses. In the conceptual design, the robot dimensions are already defined, and types of links and joints as well. The parametric design begins with homogeneous transformation matrices where position vectors and rotation matrices are combined together in a compact notation. Homogeneous transformation matrices include 16 elements; 4 are defined to be 0 or 1, and the remaining elements are composing the rotation matrix and the position vector. Therefore, the main relevant auxiliary relationships are those associated with the rotation matrix.

The homogeneous transformation matrix computed in the conceptual phase consists of two important matrices (Eq.2), ${}^{i-1}R_i$ which is giving the rotation matrix of the robot and ${}^{i-1}P_i$, which represents the position vector matrix.

$$
{}^{i-1}A_i = \left[\begin{array}{ccc|c} & {}^{i-1}R_i & & {}^{i-1}P_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{2}
$$

After computing the rotation and position, the next step is forward kinematics calculation. Direct kinematics gives the position of the end effector, if the joint variables are given. Mathematically this means multiplication of all transformation matrices, as shown in Eq. 3, where ${}^{0}A_n$ is the pose of the end-effector relative to the base frame; ${}^{i-1}A_i$ is the link transformation for the i^{th} joint; and n is the number of links. ${}^{0}A_n = {}^{0}A_1\, {}^{1}A_2\, {}^{2}A_3\, {}^{3}\ldots {}^{n-1}A_n$
(3)

$$
{}^{0}A_n = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

The result matrix is 4 x 4, and it gives the relationship between the base frame *F0* and end effector frame *Fn* (Figure 19).

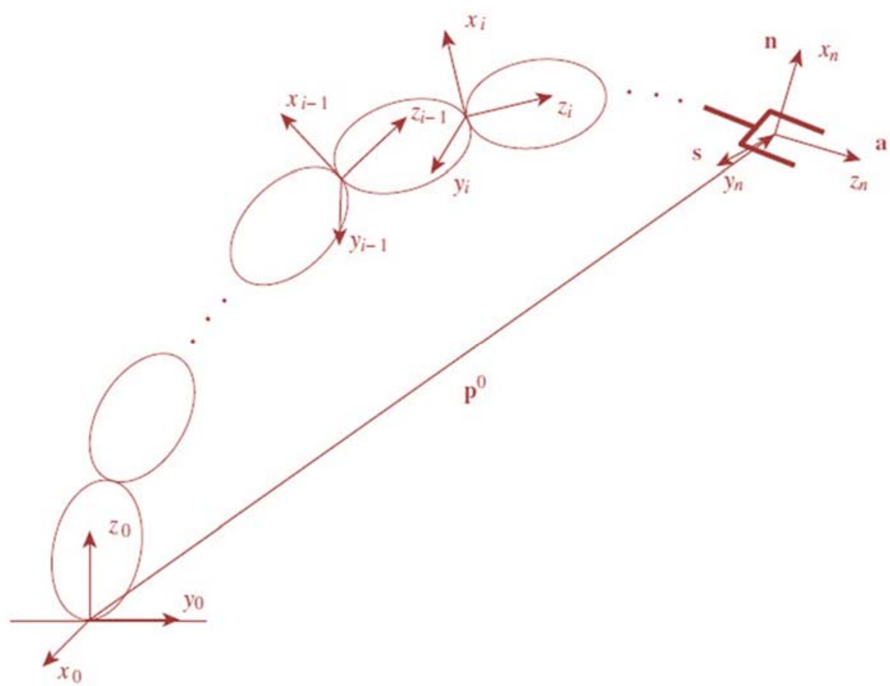


Figure 19 Geometric model of a serial robot (source: Bajd et al., 2013)

With complex configurations, the difficulty in calculation increases, thus beside the use of Maple 17 software, in most cases manual simplification is needed.

The Jacobian of the forward kinematics function determines the velocity relationships of the multibody system (Eq.14). This Jacobian is a matrix-valued function and can be understood as vector version of the ordinary derivative of a scalar function. The Jacobian matrix stands for the most important quantity for analysis and control of the robot motion (Spong and Vidyasagar, 2008) (Figure 20). $J_{(m \times n)}$ can be interpreted as a linear mapping from an *m*-dimensional vector space *X*, to an *n*-dimensional vector space *q,* where *n* is the number of joints, and *m* represents the dimension of the end-effector vector *X* and always equals to 6.

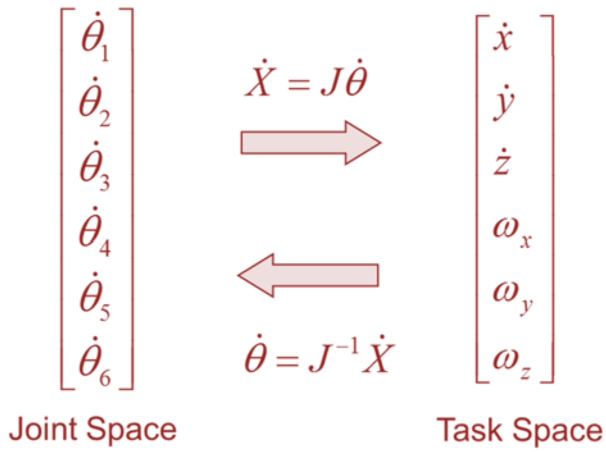


Figure 20 Jacobian matrix relation

The Jacobian matrix is essential for inverse kinematics, manipulability and dexterity measures, singularity analyses, workspace definition, path planning, control… In this methodology it is used for singularity conditions analysis.

For Jacobian calculation, Vector method is recommended by Filiposka et al., 2014, because is simpler to use, compared to Newton Euler method. The method is based on link transformation matrices found in forward (direct) kinematics. If only kinematic solutions are desired, the Vector method is less complex. Furthermore, it does not require computation of linear and angular velocities and it directly gives the Jacobian matrix relative to the base frame. In contrary, the Newton-Euler method gives the Jacobian relative to the end-effector frame, and in order to obtain it in base frame coordinates, additional computation to appropriate rotational transpose matrix is required. Vector method in Maple 17 calculation is more appropriate for calculating Jacobian matrix of complex reconfigurable multibody machinery systems.

Equation 5 represents Jacobian matrix in Vector general form. Depending on the type of joints, a complete Jacobian can be derived.

$$J(q) = \begin{cases} \begin{bmatrix} Z_{i-1} \times {}^{i-1}P_n \\ Z_{i-1} \end{bmatrix} \rightarrow & \text{For rotational joints} \\ \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix} \rightarrow & \text{For translational joints} \end{cases} \qquad i = 1, 2, \ldots n \tag{5}$$

In order to calculate the Jacobian, Z_n unit (generating) vectors computation is needed (Eq.6), along the motion of the joints expressed in base frame coordinates. The unit vectors can be interpreted as the third column of the corresponding rotation matrix, ${}^0R_n$.

$$Z_n = R_n Z_0 \tag{6}$$

For calculation of the unit vectors it is necessary to compute separate rotation matrices and position vectors found in the homogeneous transformation matrices, in terms of two neighboring joints, *i* and *i+1*.

Also position matrix P_n, computation is required (Eq.7).

$$P_n = (R_1(R_2(R_3(R_4(R_5 p_6)) + p_5) + p_4)) + p_3)) + p_2)) + p_1 \qquad n = 6 \tag{7}$$

The full Jacobian matrix (Eq.8) for 6DOF multibody system, derived using the Vector cross multiplication method is relative to the base coordinate frame.

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} \end{bmatrix} \tag{8}$$

In order to find singularity points, the determinant of the 6×6 Jacobian matrix needs to be equal to zero (Eq.9). When it is singular, the Jacobian loses its full rank (one row of it has only 0 values)

$$\det(J) = 0 \tag{9}$$

There are several configurations where this condition can be satisfied. As previously mentioned, by anatomy robots consists of two main parts, an arm with 3DOF and one spherical wrist with 3 joints intersecting in 1 point. Following two research papers for singularity analysis of 6DOF robots, Vaezi et al., 2011 and Kim et al., 1999, the Jacobian is partitioned into two 6×3 parts (Eq.10), as the first J_O will determine so-called arm singularities, which are singularities resulting from motion of the first three joints, while the second J_P will determine the wrist singularities resultant from the motion of the spherical wrist .

$$J = \left[J_O \middle| J_P \right] \tag{10}$$

This characteristic allows decoupling of the determination of singular configurations, knowing that the upper 3 x 6 matrix of the Jacobian, corresponds with the linear velocity and the lower 3 x 6 with the angular velocity of the manipulator. The decoupled Jacobian will have three 3 x 3 segments, as in equation 11:

$$J = \begin{bmatrix} J_V \\ J_\omega \end{bmatrix} = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array} \right] \tag{11}$$

Since the wrist axes intersect at a common point, the determinant of J_{12} will always have value 0 (zero), thus equation 11 can be rewritten as equation 12:

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline J_{12} & J_{22} \end{array} \right] \tag{12}$$

Therefore the set of singular configurations of any multibody with similar anatomy, will be synthesis of the set of configurations satisfying the condition $\det J_{11} = 0$ and another set of configurations satisfying $\det J_{22} = 0$, or (Eq.13)

$$\det J = \det J_{11} \cdot \det J_{22} = 0 \tag{13}$$

If the multibody systems have more than 6DOF, they are redundant and their Jacobian matrix has 6 x 7 elements. This non-quadratic Jacobian is not invertible, so it needs to be multiplied by its transpose in order to solve for singularity conditions. When it is partitioned, the first 4 joints are defining the position, and the last 3 the orientation of the system.

For parametric design the use of a computation and calculation software is necessary, although manual simplifications are compulsory. In this research Maple 17 software has been used.

3.1.4. Detailed Kinematic Design

In this phase 3 main steps are evident:

1. Forward kinematics validation
2. Workspace generation
3. Singularity locus plotting in order to get the functional workspace region

The workspace of every robot can be determined by a set of equations (Ceccarelli, 2012). As mentioned in the introduction part, manipulators with +6 degrees of freedom are capable of taking any arbitrary position in space, therefore they have the most dexterous range of motions.

The reachable workspace should be distinguished from the dexterous workspace; the dexterous one is a subset of the reachable one, because it represents the robot workspace attainable at all orientations of the end-effector, but not at any orientation like the reachable workspace. The reachable workspace is also called translational workspace, because it is determined by the first 3 joints of the manipulator and it is easier to visualize (Spong and Vidyasagar, 2008). In this research it is denoted simply as workspace.

Within the workspace, singularities occur. Usually they correspond with points at the boundary of the workspace, and they are not problematical, because those points are at the maximum reach of the manipulator. The singularities which corresponds with points in the interior of the workspace may be unreachable under small changes of the link parameters, and they are concern for every robot designer and user. The detailed design stage allows computation and visualization of interior singularities (examples are provided in the next chapter). In the singularity locus infinite solutions for inverse kinematics may exist. For optimization purposes it is always better to generate the singularity locus in early stage of the design, because when selecting from set of possible position and orientation points, their velocity level plays crucial role.

Using Matlab tools, there is a possibility to visualize the workspace of different manipulators. Corke, 2011 has developed a robotics toolbox (which is not used in this research), specifically for visualisation and calculation of kinematics, dynamics and control of different types of manipulators. Similar to this, with the help of the Lecture Notes from the Industrial Robots Kinematics, Dynamics and Control course (Djuric, 2013b) and the kinematic equations, a visualisation of the reachable workspace was generated. The plot function used here can generate all the points in the workspace, point-by-point, but then the figure becomes too dense. That is why, deliberately, the selected points in the workspace are presented with bigger step, and its shape is comprehendible.

3.2. IDEF0 for better understanding of the design tool

IDEF is an acronym for Integrated Definition Methodology, and it is extension of the FDD (Functional Decomposition Diagramming) representation scheme. It is often used as a method designed to model decisions, actions, and activities of an organization or a system (Lyons and Duffey, 1995). As a modelling tool, IDEF0 is capable of graphically representing a wide variety of operations to any level of detail, through simple blocks. The IDEF0 process starts with identification of the main function, which defines the scope. From this diagram lower-level diagrams are generated, cross-referenced with text. IDEF0 assists in identifying what functions are performed, what is needed to perform those functions (Kossiakoff et al., 2011). Functions are represented with blocks and the relations between them with arrows (Figure 21).

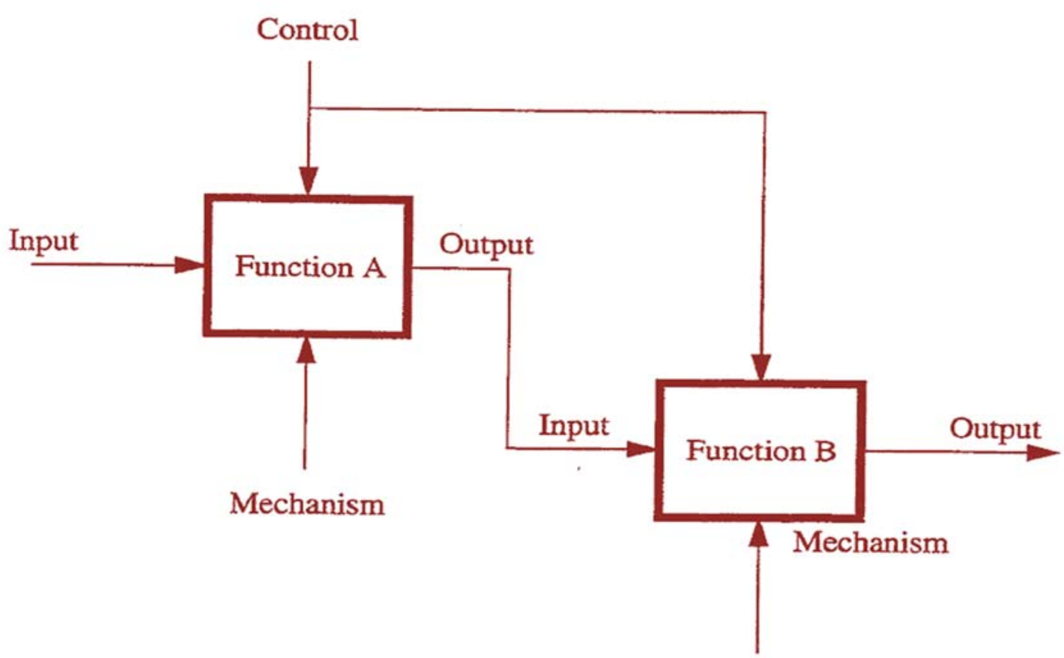


Figure 21 IDEF0 representation (source: Lyons and Duffey, 1995)

Following the Standard for IDEF0 published in Fipspub183, 1993, and Živanović et al., 2009 the design methodology for kinematics of multibody systems is represented with IDEF0, for better understanding of the input and the output from each phase, as well as the enablers and mechanisms used along the modelling. Figure 22 represents the first general node of the design tool. The decoupled node, containing all 4 phases of the tool is given in Figure 23. These two figures can be interpreted as parent-child diagrams. The order of the diagrams is as follow:

A_0 – Design Tool for Kinematics of Multibody Systems (Figure 22)

A0 – Decoupled node of the Design Tool for Kinematics of Multibody Systems (Figure 23)

A1 – Concept Ideation (Figure 24)

A2 – Conceptual Design (Figure 25)

A3 – Parametric Design (Figure 26)

A4 – Detailed Design (Figure 27)

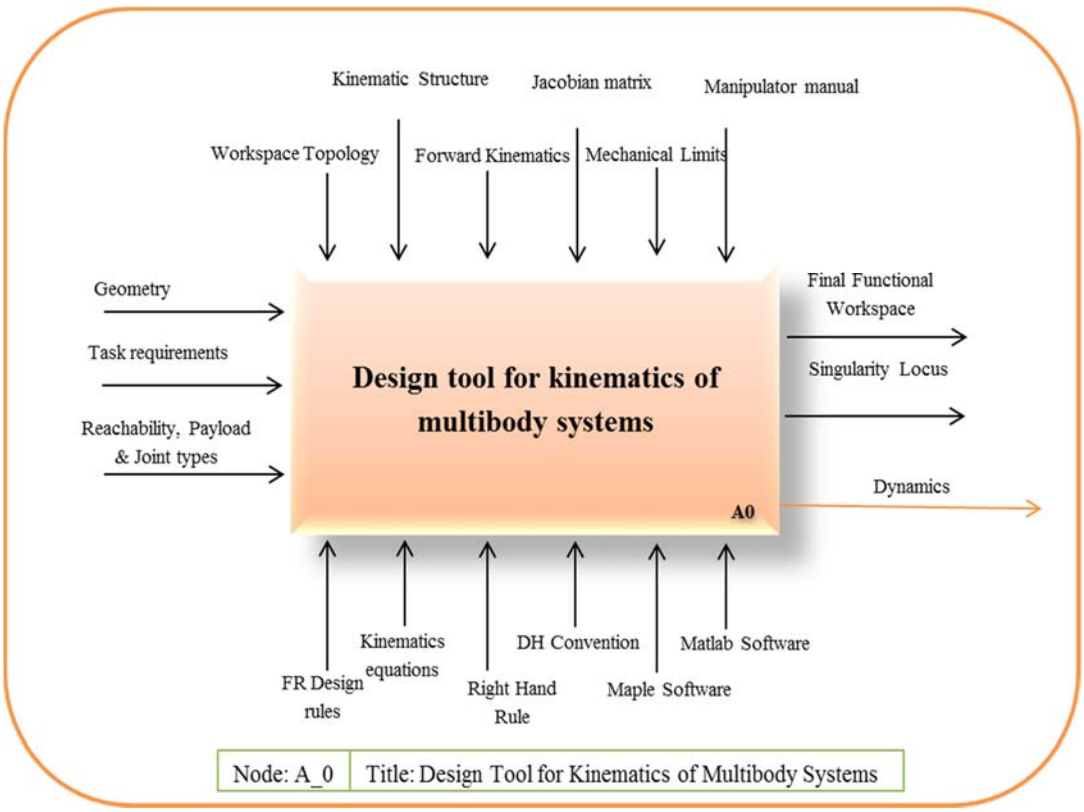


Figure 22 Main function block of the design tool for kinematics of multibody systems

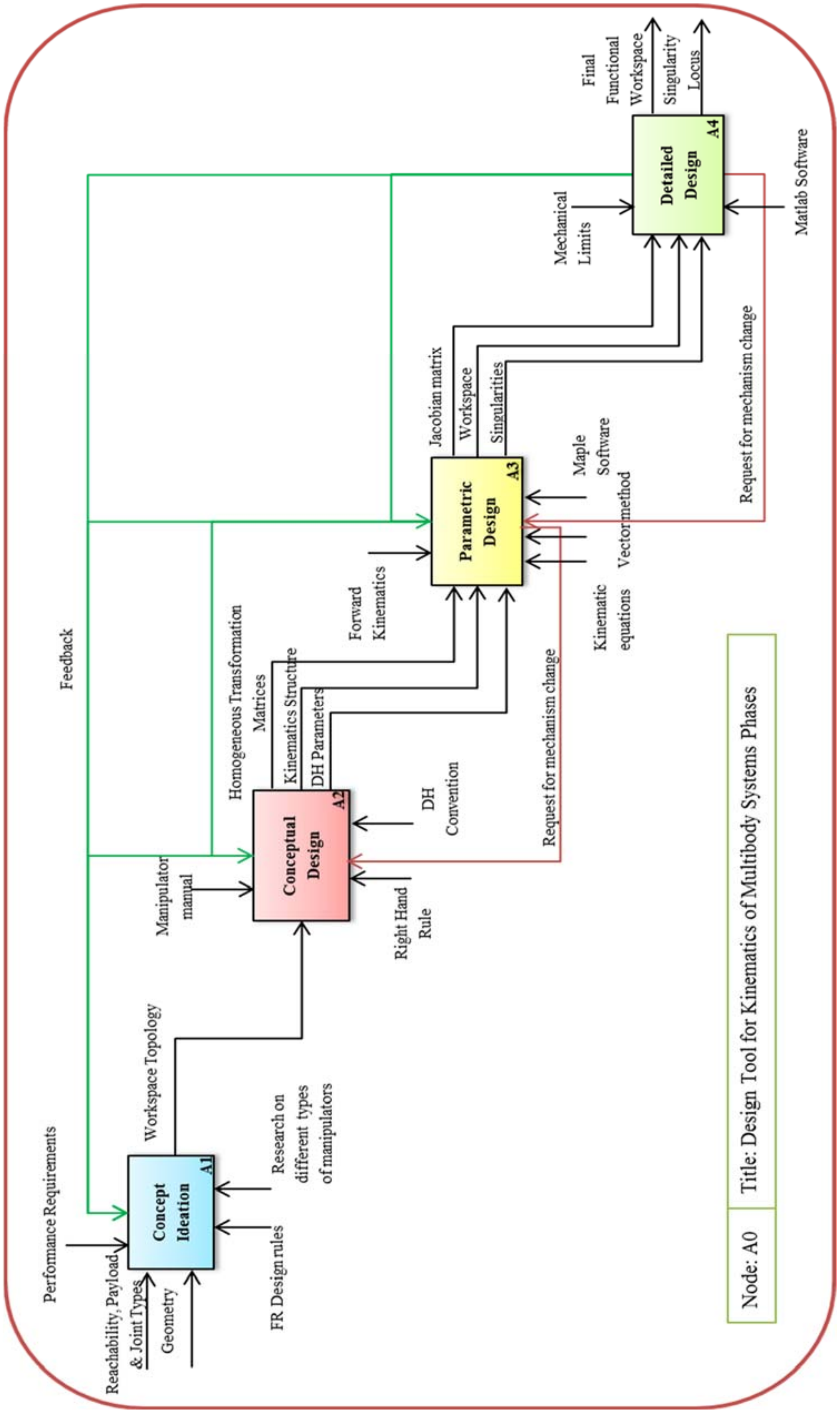


Figure 23 Decoupled node A0, the Design Tool for Kinematics of Multibody Systems

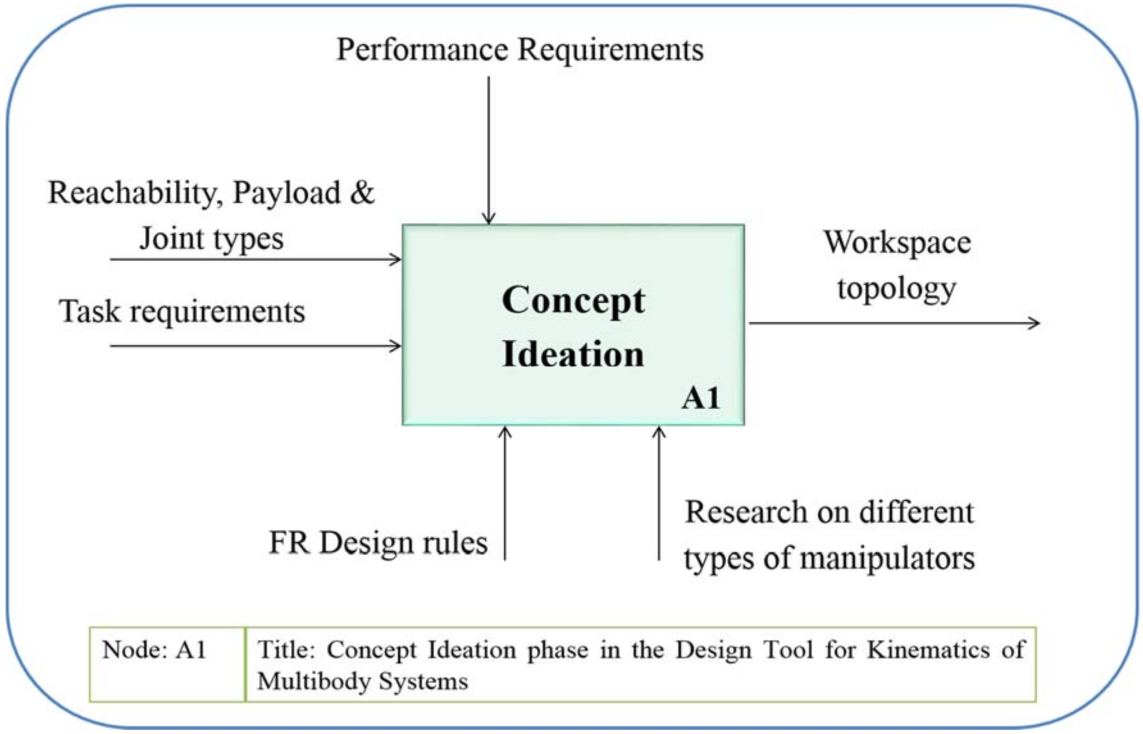


Figure 24 Node A1, Concept Ideation phase

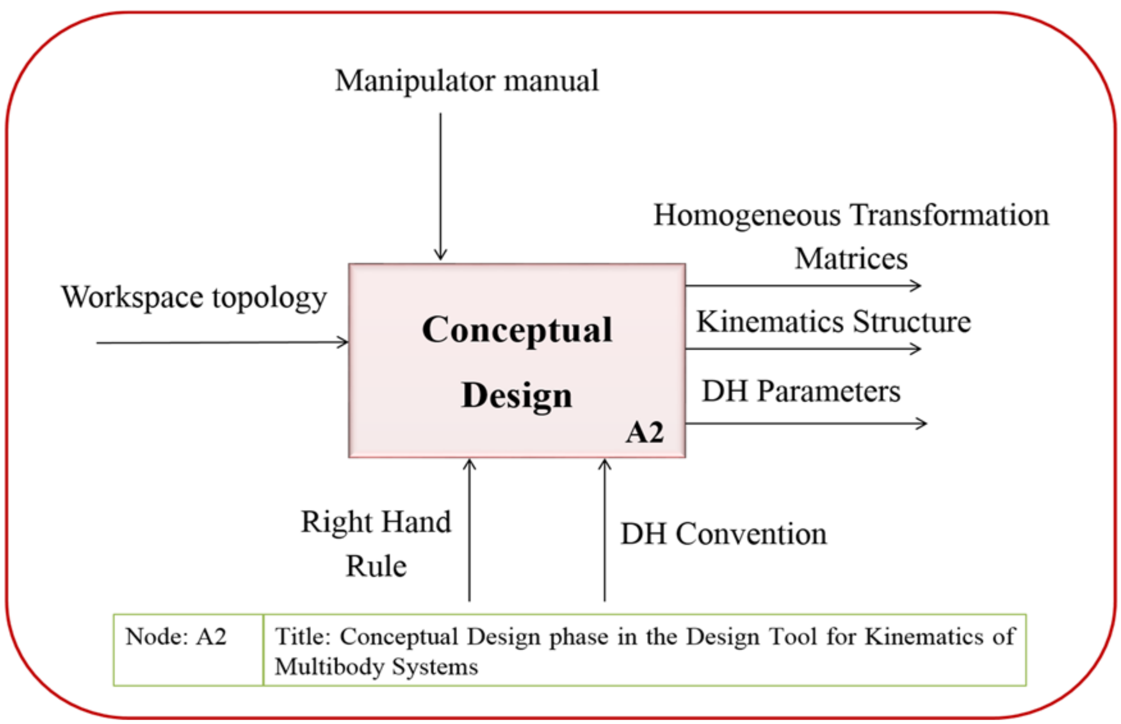


Figure 25 Node A2, Conceptual Design phase

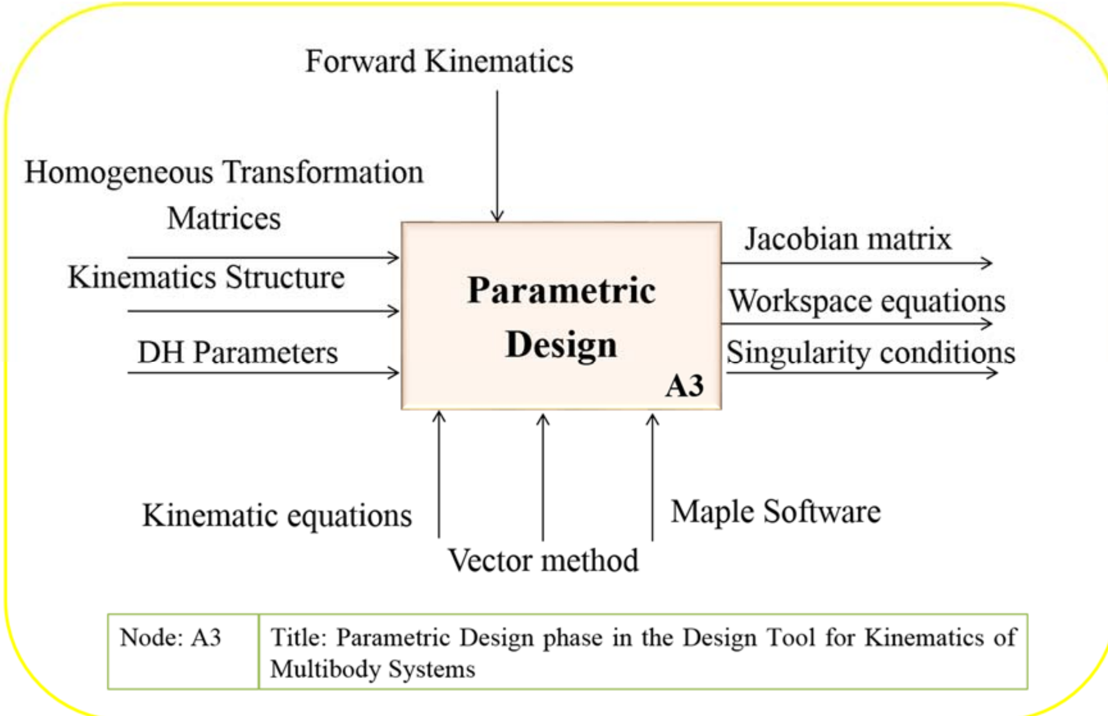


Figure 26 Node A3, Parametric (Embodiment) Design phase

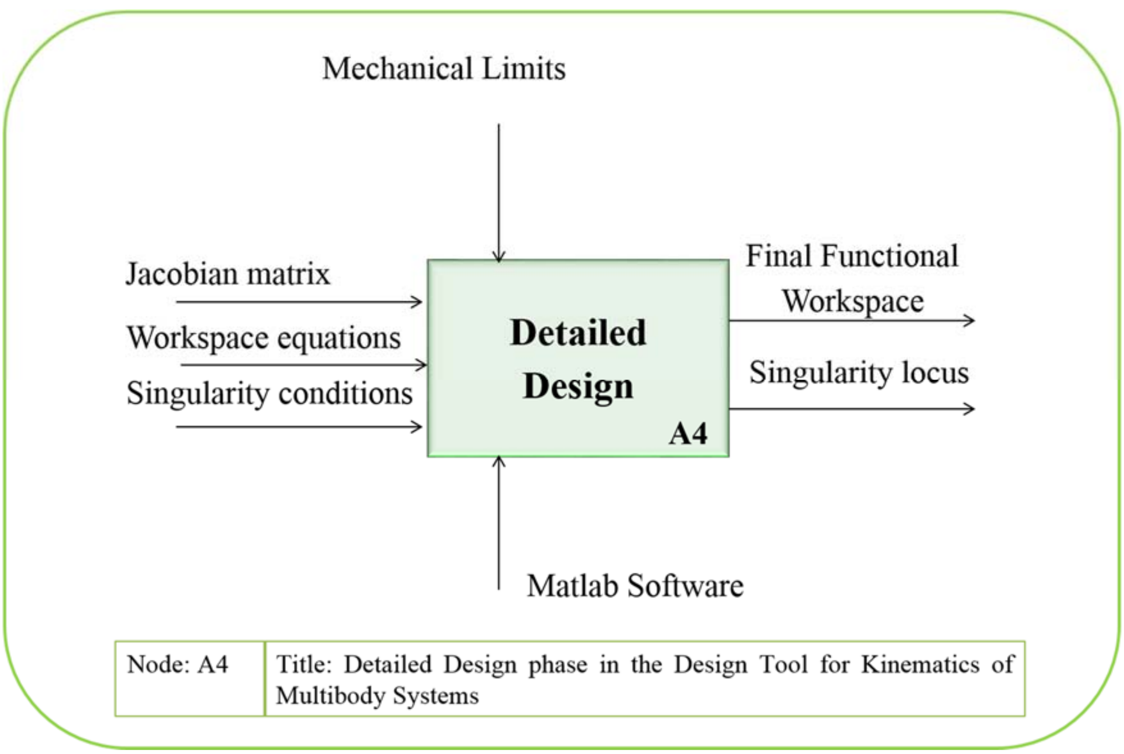


Figure 27 Node A4, Detailed Design phase

CHAPTER 4

DESIGN EXAMPLES, VALIDATION AND RESULTS

4.1. Use of the design tool for kinematics of multibody systems

In order to validate the design tool for kinematics of multibody systems, three random systems with different capabilities are selected. Full kinematics modelling is performed and the results are used as benchmark for modelling systems, belonging to a same family of robots.

4.1.1. General CNC machine kinematic modelling

Cartesian robots are mainly employed where there is a need of large workspaces. By following the second rule of concept ideation, if the workspace should be prismatic and infinite, gantry robot should be used, with at least one (usually the first) joint prismatic, and with limited larger manipulability to one direction than the others. Common CNCs have three translational joints XYZ (3DOF), often with an attached wrist (+3DOF), to allow rotation moves and option to reach any position through linear motions, within the rectangular workspace envelope.

Depending on the task (functional) requirements, in Figure 28 a general CNC machine is selected and represented along with its kinematic structure (Figure 29). The DH parameters that corresponds with this structure are found in the Workspace software, which makes the modelling easier (Table 5).

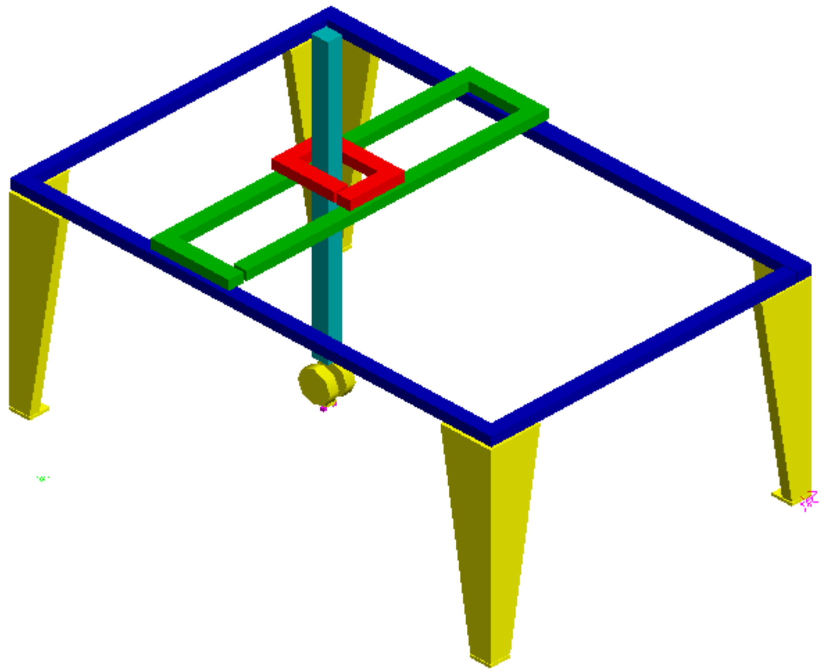


Figure 28 General CNC machine (source: Workspace software, 2013)

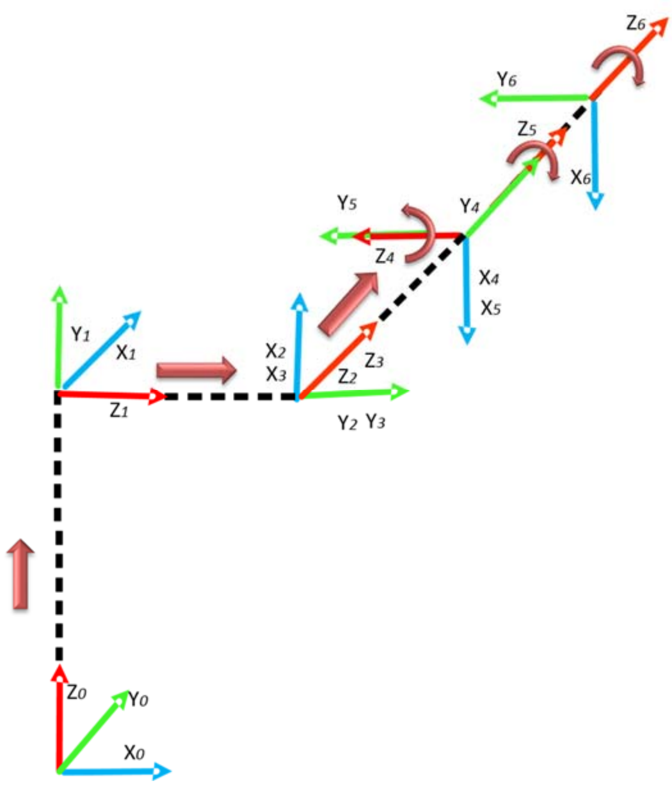


Figure 29 General CNC machine kinematic structure

Table 5 General CNC machine DH parameters and axes limits (Source: Workspace software)

CNC Machine DH Parameters					
Joint	*d*	θ	*a*	α	Mechanical Limit[1]
1	4715.22*[2]	90°	0	90°	-1000, +1000
2	2714.59*	90°	0	90°	-1000, +1000
3	0*	0°	0	0°	-1000, +1000
4	-2664.68	180° *	0	-90°	-150, +150
5	0	0° *	0	90°	-150, +150
6	-281.25	0° *	0	0°	-150, +150

Homogeneous transformation matrices computation is next step in the conceptual design stage. Using Equation 1, by substituting the given parameters, all 6 matrices can be computed in Maple software (detailed computation in Appendix A) as follow (Eq.14-19):

$$^{0}A_1 = \begin{bmatrix} \cos\theta_1 & -\cos\alpha_1 \sin\theta_1 & \sin\alpha_1 \sin\theta_1 & a_1 \cos\theta_1 \\ \sin\theta_1 & \cos\alpha_1 \cos\theta_1 & -\sin\alpha_1 \cos\theta_1 & a_1 \sin\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$^{1}A_2 = \begin{bmatrix} \cos\theta_2 & -\cos\alpha_2 \sin\theta_2 & \sin\alpha_2 \sin\theta_2 & a_2 \cos\theta_2 \\ \sin\theta_2 & \cos\alpha_2 \cos\theta_2 & -\sin\alpha_2 \cos\theta_2 & a_2 \sin\theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$^{2}A_3 = \begin{bmatrix} \cos\theta_3 & -\cos\alpha_3 \sin\theta_3 & \sin\alpha_3 \sin\theta_3 & a_3 \cos\theta_3 \\ \sin\theta_3 & \cos\alpha_3 \cos\theta_3 & -\sin\alpha_3 \cos\theta_3 & a_3 \sin\theta_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

[1] The mechanical limits are specified by the manufacturer
[2] * denotes joint variable parameter.

$$
{}^{3}A_{4}=\begin{bmatrix} \cos\theta_4 & -\cos\alpha_4\sin\theta_4 & \sin\alpha_4\sin\theta_4 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\alpha_4\cos\theta_4 & -\sin\alpha_4\cos\theta_4 & a_4\sin\theta_4 \\ 0 & \sin\alpha_4 & \cos\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}
$$

$$
{}^{4}A_{5}=\begin{bmatrix} \cos\theta_5 & -\cos\alpha_5\sin\theta_5 & \sin\alpha_5\sin\theta_5 & a_5\cos\theta_5 \\ \sin\theta_5 & \cos\alpha_5\cos\theta_5 & -\sin\alpha_5\cos\theta_5 & a_5\sin\theta_5 \\ 0 & \sin\alpha_5 & \cos\alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}
$$

$$
{}^{5}A_{6}=\begin{bmatrix} \cos\theta_6 & -\cos\alpha_5\sin\theta_6 & \sin\alpha_6\sin\theta_6 & a_6\cos\theta_6 \\ \sin\theta_6 & \cos\alpha_5\cos\theta_6 & -\sin\alpha_6\cos\theta_6 & a_6\sin\theta_6 \\ 0 & \sin\alpha_6 & \cos\alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}
$$

When all homogeneous transformation matrices are identified, by multiplying all of them, forward kinematics is calculated (Eq. 20) and so begins the parametric design phase.

$$
{}^{0}A_{6}={}^{0}A_{1}\,{}^{1}A_{2}\,{}^{2}A_{3}\,{}^{3}A_{4}\,{}^{4}A_{5}\,{}^{5}A_{6}=
$$

$$
=\begin{bmatrix} \sin\theta_4\cos\theta_5\cos\theta_6+\cos\theta_4\sin\theta_6 & -\sin\theta_4\cos\theta_5\sin\theta_6+\cos\theta_4\cos\theta_6 & \sin\theta_4\sin\theta_5 & \sin\theta_4\sin\theta_5 d_6+d_2 \\ -\sin\theta_5\cos\theta_6 & \sin\theta_5\sin\theta_6 & \cos\theta_5 & \cos\theta_5 d_6+d_4+d_3 \\ \cos\theta_4\cos\theta_5\cos\theta_6-\sin\theta_4\sin\theta_6 & -\cos\theta_4\cos\theta_5\sin\theta_6-\sin\theta_4\cos\theta_6 & \cos\theta_4\sin\theta_5 & \cos\theta_4\sin\theta_5 d_6+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}
$$

Forward kinematics gives the position and orientation of the end-effector, comparative to the base frame of the manipulator. Mathematically, these equations define a function between the space of Cartesian positions and orientations and the space of joint positions (Craig, 2005).

The separate rotation matrices (Eq. 21-27) and position vectors (Eq. 28-32) found in the homogeneous transformation matrices, in terms of two neighboring joints, *i* and *i+1*, for the current CNC are:

$$ {}^{0}R_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (21) \qquad {}^{1}R_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (22) $$

$$ {}^{2}R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (23) \qquad {}^{3}R_4 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 \\ \sin\theta_4 & 0 & \cos\theta_4 \\ 0 & -1 & 0 \end{bmatrix} \quad (24) $$

$$ {}^{4}R_5 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 \\ \sin\theta_5 & 0 & -\cos\theta_5 \\ 0 & 1 & 0 \end{bmatrix} \quad (25) \qquad {}^{5}R_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (26) $$

$$ {}^{0}P_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix} \quad (27) \qquad {}^{1}P_2 = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix} \quad (28) \qquad {}^{2}P_3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix} \quad (29) $$

$$ {}^{3}P_4 = \begin{bmatrix} 0 \\ 0 \\ d_4 \end{bmatrix} \quad (30) \qquad {}^{4}P_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (31) \qquad {}^{5}P_6 = \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix} \quad (32) $$

also the rotation matrices (Eq.33-38), and position vectors (Eq.39-44), in terms of the base frame, *0* and the corresponding joint *i*, found in the iterative multiplication of the transformation matrices (details are provided in Appendix A):

$$ {}^{0}R_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (33) \qquad {}^{0}R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (34) $$

$$ {}^{0}R_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (35) \qquad {}^{0}R_4 = \begin{bmatrix} \sin\theta_4 & 0 & \cos\theta_4 \\ 0 & -1 & 0 \\ \cos\theta_4 & 0 & -\sin\theta_4 \end{bmatrix} \quad (36) $$

$$ {}^0R_5 = \begin{bmatrix} \sin\theta_4\cos\theta_5 & \cos\theta_4 & \sin\theta_4\sin\theta_5 \\ -\sin\theta_5 & 0 & \cos\theta_5 \\ \cos\theta_4\cos\theta_5 & -\sin\theta_4 & \cos\theta_4\sin\theta_5 \end{bmatrix} \tag{37} $$

$$ {}^0R_6 = \begin{bmatrix} \sin\theta_4\cos\theta_5\cos\theta_6 + \cos\theta_4\sin\theta_6 & -\sin\theta_4\cos\theta_5\sin\theta_6 + \cos\theta_4\cos\theta_6 & \sin\theta_4\sin\theta_5 \\ -\sin\theta_5\cos\theta_6 & \sin\theta_5\sin\theta_6 & \cos\theta_5 \\ \cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\sin\theta_6 & -\cos\theta_4\cos\theta_5\sin\theta_6 - \sin\theta_4\cos\theta_6 & \cos\theta_4\sin\theta_5 \end{bmatrix} \tag{38} $$

$$ {}^0P_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix} \tag{39} $$ $$ {}^0P_2 = \begin{bmatrix} d_2 \\ 0 \\ d_1 \end{bmatrix} \tag{40} $$ $$ {}^0P_3 = \begin{bmatrix} d_2 \\ d_3 \\ d_1 \end{bmatrix} \tag{41} $$

$$ {}^0P_4 = \begin{bmatrix} d_2 \\ d_4 + d_3 \\ d_1 \end{bmatrix} \tag{42} $$ $$ {}^0P_5 = \begin{bmatrix} d_2 \\ d_4 + d_3 \\ d_1 \end{bmatrix} \tag{43} $$ $$ {}^0P_6 = \begin{bmatrix} \sin\theta_4 \sin\theta_5 d_6 + d_2 \\ \cos\theta_5 d_6 + d_4 + d_3 \\ \cos\theta_4 \sin\theta_5 d_6 + d_1 \end{bmatrix} \tag{44} $$

The modelling continues with Zero vectors computation (Eq. 45-50). Each Z_{i-1} unit vector represents the third column of the corresponding rotational ${}^0R_i$ matrix.

$$ Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{45} $$ $$ Z_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{46} $$ $$ Z_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{47} $$

$$ Z_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{48} $$ $$ Z_4 = \begin{bmatrix} \cos\theta_4 \\ 0 \\ -\sin\theta_4 \end{bmatrix} \tag{49} $$ $$ Z_5 = \begin{bmatrix} \sin\theta_4\sin\theta_5 \\ \cos\theta_5 \\ \cos\theta_4\sin\theta_5 \end{bmatrix} \tag{50} $$

Following equation 5, because the CNC machine's first three joints are translational, and the last three rotational, the Jacobian will have the following form (Eq.51):

$$ J = \begin{bmatrix} z_0 & z_1 & z_2 & z_3 \times ({}^0p_6 - {}^0p_3) & z_4 \times ({}^0p_6 - {}^0p_4) & z_5 \times ({}^0p_6 - {}^0p_5) \\ 0 & 0 & 0 & z_3 & z_4 & z_5 \end{bmatrix} \tag{51} $$

After cross multiplication of the position vectors relative to the base coordinate frame, ${}^0P_i$ with the relevant unit vector, z_{i-1} (for the last three joints only) (Eq.52-54), the Jacobian matrix can be assembled, by importing the joint variables and constant parameters.

$$z_3 \times ({}^0P_6 - {}^0P_3) = \begin{bmatrix} \cos\theta_4 \sin\theta_5 d_6 \\ 0 \\ -\sin\theta_4 \sin\theta_5 d_6 \end{bmatrix} \tag{52}$$

$$z_4 \times ({}^0P_6 - {}^0P_4) = \begin{bmatrix} \sin\theta_4 \cos\theta_5 d_6 \\ -\sin\theta_5 d_6 \\ \cos\theta_4 \cos\theta_5 d_6 \end{bmatrix} \tag{53}$$

$$z_5 \times ({}^0P_6 - {}^0P_5) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{54}$$

For the current CNC machine, the full Jacobian, 6 x 6, is given in Eq.55:

$$J = \begin{bmatrix} -\cos\theta_5 d_6 - d_4 - d_3 & 0 & \cos\theta_4 \sin\theta_5 d_6 & \cos\theta_4 \sin\theta_5 d_6 & \sin\theta_4 \cos\theta_5 d_6 & 0 \\ \sin\theta_4 \sin\theta_5 d_6 + d_2 & -\cos\theta_4 \sin\theta_5 d_6 & 0 & 0 & -\sin\theta_5 d_6 & 0 \\ 0 & \cos\theta_5 d_6 + d_4 + d_3 & -\sin\theta_4 \sin\theta_5 d_6 & -\sin\theta_4 \sin\theta_5 d_6 & \cos\theta_4 \cos\theta_5 d_6 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta_4 & \sin\theta_4 \sin\theta_5 \\ 0 & 0 & 0 & 1 & 0 & \cos\theta_5 \\ 0 & 0 & 0 & 0 & -\sin\theta_4 & \cos\theta_4 \sin\theta_5 \end{bmatrix} \tag{55}$$

By rigidly attaching a coordinate frame to each link in a manner as needed, then multiplying them together as desired, Jacobian matrix can be derived for any manipulator. The use of the D-H convention is simple systematic procedure to do this. The resulting equations are dependent on the chosen coordinate frames, but the manipulator configuration itself, is representing geometric quantities, independent of the frames used to describe them (Spong and Vidyasagar, 2008).

The condition for singular configuration is given in Eq.9. As previously mentioned, the selected general CNC machine can be viewed as XYZ machine with attached spherical wrist with 3 joints intersecting in 1 point. This argument allows partitioning of the Jacobian into four 3 × 3 parts (Eq.12). Since the first three joints are translational and there is no angular velocity at this point, the value of J_{21} will be 0, thus the Jacobian can be decoupled as follow:

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline 0 & J_{22} \end{array}\right] \tag{56}$$

In the case of the CNC machine (Eq.57)

$$\det J_{11} = \det \begin{bmatrix} -\cos\theta_5 d_6 - d_4 - d_3 & 0 & \cos\theta_4 \sin\theta_5 d_6 \\ \sin\theta_4 \sin\theta_5 d_6 + d_2 & -\cos\theta_4 \sin\theta_5 d_6 & 0 \\ 0 & \cos\theta_5 d_6 + d_4 + d_3 & -\sin\theta_4 \sin\theta_5 d_6 \end{bmatrix} =$$

$$= \cos\theta_4 \sin\theta_5 d_2 d_6 (\cos\theta_5 d_6 + d_4 + d_3) = 0 \tag{57}$$

The only condition produced by this equation is $\theta_5 = 0^\circ$ and $\theta_4 = 90^\circ$, because the part in the brackets, $(\cos\theta_5 d_6 + d_4 + d_3)$, can never equate to 0 (See Appendix A).

The only singularity of the spherical wrist happens when the joint axes z3 and z5, are collinear, which physically means the 4th and the 6th joint are aligned (Eq.58):

$$\det J_{22} = \det \begin{bmatrix} 0 & \cos\theta_4 & \sin\theta_4 \sin\theta_5 \\ 1 & 0 & \cos\theta_5 \\ 0 & -\sin\theta_4 & \cos\theta_4 \sin\theta_5 \end{bmatrix} = -\sin\theta_5 = 0 \tag{58}$$

This is true only when $\theta_5 = 0$, or $\theta_5 = 180^\circ$, and because the maximum limit of the joint is 150°, 180° can never be reached, so only 0° is taken into consideration.

Upon solving for singularity conditions, the final step in the parametric design is over. Next is proceeding with the detailed design stage where visualization of the workspace with the singularities encountered is completed.

Before proceeding to the final steps of the design, validation of the model needs to be completed. Matlab software provides precise numerical and visual results for forward kinematics of manipulators. The kinematic structure and the end-effector position is computed as shown in Figure 30.

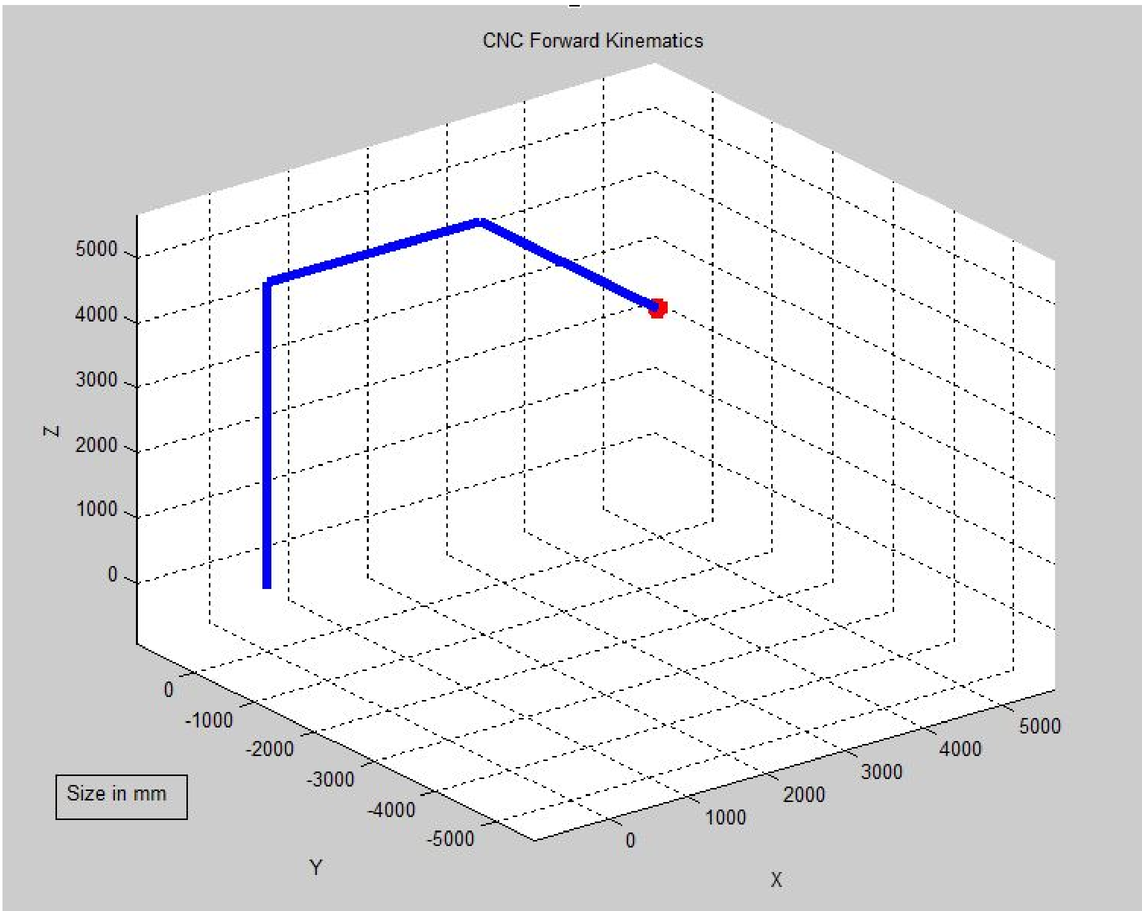


Figure 30 CNC Forward kinematics validation

With the forward kinematics equations, it is possible to plot any position of the end-effector, within the workspace. When plotting the home position of the machine, a dot will appear at the position point of the end-effector. This validates the results in forward kinematics, also allows continuation of the computation of the workspace. CNC's workspace can be computed with a set of equations (Eq.59) which are describing every reachable point of the machine's end-effector. Here, the mechanical limits, specified by the manufacturer, should be considered, as they are already given in Table 5.

$$\begin{cases} X = \sin\theta_4 \sin\theta_5 d_6 + d_2 \\ Y = \cos\theta_5 d_6 + d_4 + d_3 \\ Z = \cos\theta_4 \sin\theta_5 d_6 + d_1 \end{cases} \tag{59}$$

These equations are same as in ${}^0P_6$ position vector in equation 44. By specifying the limits for the joint variables d_1, d_2 and d_3, from minimum -1000 to maximum 1000, the fully reachable workspace is generated. (Appendix E contains the Matlab codes for the CNC machine).

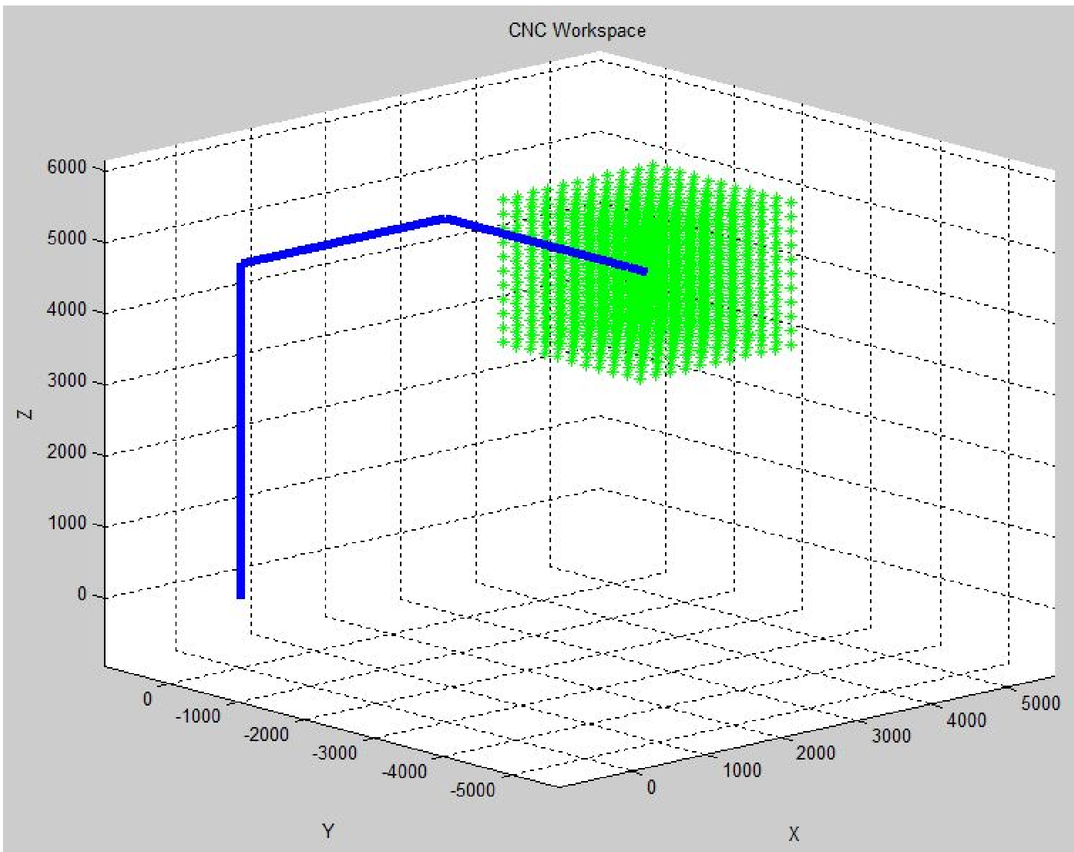


Figure 31 CNC Workspace

As shown in Figure 31 above, this general CNC machine's workspace has cubic shape, with length of 2000mm, and the center of the cube is the end-effector home position.

In the case of the CNC machine the only singularity produced was at joint 5, and because this singularity is considered as common singularity of all manipulators with spherical joint, there is no need of special visualization.

4.1.2. Mitsubishi MELFA RV-3SDB kinematic modelling

The kinematic modelling begins with the concept ideation, where auxiliary symmetric and finite workspace is needed and articulated robot is selected for performing the task requirements. The designated robot for is Mitsubishi MELFA RV-3SDB (Figure 32), with all 6 joints rotational. With maximum speed of 5500mm/s, payload of 3kg, reach of 642mm and repeatability of ±0.02mm, Mitsubishi MELFA RV-3SDB is successfully employed in Festo Didactic iFactory, courtesy of the Intelligent Manufacturing Systems Centre at the University of Windsor. This robot is designed for easy integration in existing work cells, where movements are restricted or the processing points are far apart. It gives the users more flexibility for planning automation, whit the ability to install the robot not only next to the machine, but within the machine itself. Other possible applications are the metal cutting, where the robot can be exposed to oil and fluids, assembly operations and similar (Mitsubishielectric, 2012, Standard Specifications Manual).

Figure 32 Mitsubishi Melfa RV-3SDB (source: Mitsubishi Electric, 2012, Standard Specifications Manual)

In continuation, conceptual phase of the design is following. In the literature, a reference was not found on kinematics analyses of Mitsubishi Melfa RV-3SDB, therefore the only available source of information, and the most specific is the manufacturers manual.

By observing the positive Z rotations of each axis in Figure 33, coordinate frames are assigned to each link and the kinematic structure that corresponds with this robot is sketched as in Figure 35.

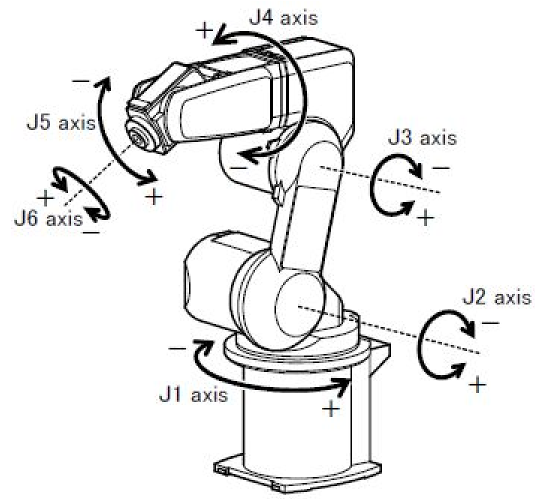


Figure 33 Positive Z directions for each joint of the MELFA RV-3SDB (source: Mitsubishi Electric, 2012, Standard Specifications Manual)

Also, for the DH parameters of the RV-3SDB definition, as a reference are taken the outside dimensions specified in the manual Mitsubishielectric, 2012, (Figure 34)

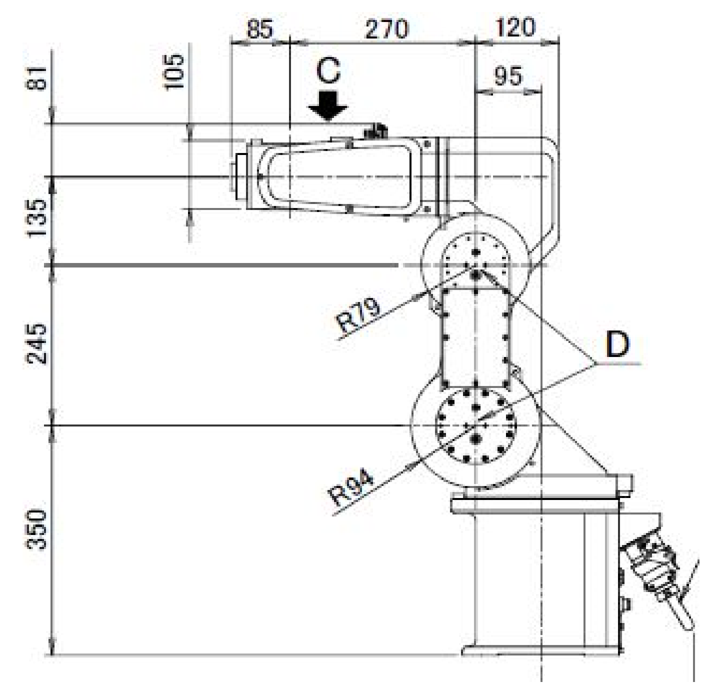


Figure 34 Outside dimensions of RV-3SDB (source: Mitsubishi Electric, 2012, Standard Specifications Manual)

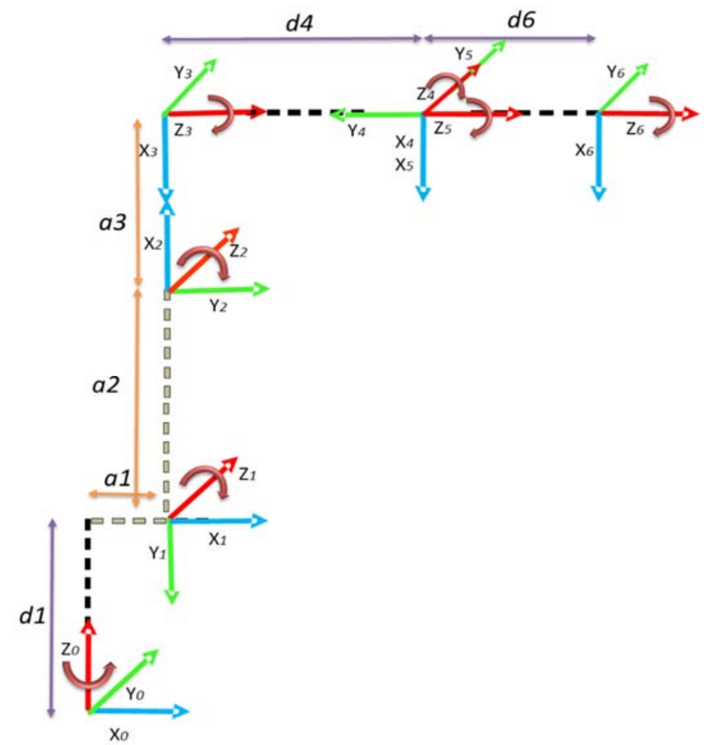


Figure 35 Kinematic structure sketch of Mitsubishi MELFA RV-3SDB

By following the kinematic structure sketch, the DH parameters are presented in Table 6.

Table 6 Mitsubishi MELFA RV-3SDB DH parameters and axis limits

Mitsubishi MELFA RV-3SDB DH Parameters					
Joint	d	θ	a	α	Mechanical Limit
1	350	0° *	95	-90°	±170
2	0	-90° *	245	0°	+135,-90
3	0	±180° *	-135	90°	+171,-20
4	270	0° *	0	-90°	±160
5	0	0° *	0	90°	±120
6	85	0° *	0	0°	±360

After completing the DH table, homogeneous transformation matrices, in terms of every two neighboring joints are computed, by inserting the joint parameters into equation 1. They have the following forms (Eq.60-65):

$$A_{01} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1\sin\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{60}$$

$$A_{12} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{61}$$

$$A_{23} = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & -\cos\theta_3 & a_3\sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{62}$$

$$A_{34} = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{63}$$

$$A_{45} = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{64}$$

$$A_{56} = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{65}$$

The relationship between the base frame and the end-effector is given by the forward kinematics, and because of the size of this equation, it is provided in Appendix B. Forward kinematics gives the position and orientation of the end-effector, comparative to the base frame of the manipulator, thus generation of the position vectors and rotation matrices is done simultaneously in the Maple program.

The Jacobian, as the most important quantity for analysis of the motion of the robots, is calculated next. For its derivation it is necessary to obtain separate rotation matrices (Eq. 66-71) and position vectors (Eq.72-77) found in the homogeneous transformation matrices, in terms of two neighboring joints, *i* and *i+1*:

$$ {}^{0}R_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix} \quad (66) \qquad {}^{1}R_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (67) $$

$$ {}^{2}R_3 = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 \\ \sin\theta_3 & 0 & -\cos\theta_3 \\ 0 & 1 & 0 \end{bmatrix} \quad (68) \qquad {}^{3}R_4 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 \\ \sin\theta_4 & 0 & \cos\theta_4 \\ 0 & -1 & 0 \end{bmatrix} \quad (69) $$

$$ {}^{4}R_5 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 \\ \sin\theta_5 & 0 & -\cos\theta_5 \\ 0 & 1 & 0 \end{bmatrix} \quad (70) \qquad {}^{5}R_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (71) $$

$$ {}^{0}P_1 = \begin{bmatrix} a_1\cos\theta_1 \\ a_1\sin\theta_1 \\ d_1 \end{bmatrix} \quad (72) \qquad {}^{1}P_2 = \begin{bmatrix} a_2\cos\theta_2 \\ a_2\sin\theta_2 \\ 0 \end{bmatrix} \quad (73) \qquad {}^{2}P_3 = \begin{bmatrix} a_3\cos\theta_3 \\ a_3\sin\theta_3 \\ 0 \end{bmatrix} \quad (74) $$

$$ {}^{3}P_4 = \begin{bmatrix} 0 \\ 0 \\ d_4 \end{bmatrix} \quad (75) \qquad {}^{4}P_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (76) \qquad {}^{5}P_6 = \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix} \quad (77) $$

as well as the rotation matrices (Eq.78-83), and position vectors (Eq.84-89), relative to the base frame, *0* and the i^{th} joint found in the iterative multiplication of the transformation matrices (detailed calculation in Appendix B):

$$
{}^{0}R_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix} \quad (78)
\qquad
{}^{0}R_2 = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & \cos\theta_1 \sin\theta_2 & -\sin\theta_1 \\ \sin\theta_1 \cos\theta_2 & -\sin\theta_1 \sin\theta_2 & \cos\theta_1 \\ -\sin\theta_2 & -\cos\theta_2 & 0 \end{bmatrix} \quad (79)
$$

$$
{}^{0}R_3 = \begin{bmatrix} -\cos\theta_1 \cos(\theta_2+\theta_3) & -\sin\theta_1 & \cos\theta_1 \sin(\theta_2+\theta_3) \\ -\sin\theta_1 \cos(\theta_2+\theta_3) & \cos\theta_1 & \sin\theta_1 \sin(\theta_2+\theta_3) \\ -\sin(\theta_2+\theta_3) & 0 & -\cos(\theta_2+\theta_3) \end{bmatrix} \quad (80)
$$

$$
{}^{0}R_4 = \begin{bmatrix} -\cos\theta_4 \cos\theta_1 \cos(\theta_2+\theta_3) - \sin\theta_1 \sin\theta_4 & -\cos\theta_1 \sin(\theta_2+\theta_3) & \sin\theta_4 \cos\theta_1 \cos(\theta_2+\theta_3) - \sin\theta_1 \sin\theta_4 \\ -\cos\theta_4 \cos\theta_1 \cos(\theta_2+\theta_3) + \cos\theta_1 \sin\theta_4 & -\sin\theta_1 \sin(\theta_2+\theta_3) & \sin\theta_4 \sin\theta_1 \cos(\theta_2+\theta_3) + \cos\theta_1 \sin\theta_4 \\ -\cos\theta_4 \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & -\cos\theta_4 \sin(\theta_2+\theta_3) \end{bmatrix} \quad (81)
$$

$$
{}^{0}R_5 = \begin{bmatrix} -\cos\theta_1(\cos\theta_4 \cos\theta_5 \cos(\theta_2+\theta_3) + \sin\theta_5 \sin(\theta_2+\theta_3)) - \cos\theta_5 \sin\theta_1 \sin\theta_4 & \sin\theta_4 \cos\theta_1 \cos(\theta_2+\theta_3) - \sin\theta_1 \sin\theta_4 & -\cos\theta_1(\cos\theta_4 \sin\theta_5 \cos(\theta_2+\theta_3) - \cos\theta_5 \sin(\theta_2+\theta_3)) - \sin\theta_5 \sin\theta_1 \sin\theta_4 \\ -\sin\theta_1(\cos\theta_4 \cos\theta_5 \cos(\theta_2+\theta_3) + \sin\theta_5 \sin(\theta_2+\theta_3)) + \cos\theta_5 \cos\theta_1 \sin\theta_4 & \sin\theta_4 \cos\theta_1 \cos(\theta_2+\theta_3) + \cos\theta_1 \cos\theta_4 & -\sin\theta_1(\cos\theta_4 \sin\theta_5 \cos(\theta_2+\theta_3) - \cos\theta_5 \sin(\theta_2+\theta_3)) + \sin\theta_5 \cos\theta_1 \sin\theta_4 \\ \sin\theta_5 \cos(\theta_2+\theta_3) - \cos\theta_4 \cos\theta_5 \sin(\theta_2+\theta_3) & \sin\theta_4 \sin(\theta_2+\theta_3) & -\cos\theta_5 \cos(\theta_2+\theta_3) - \cos\theta_4 \sin\theta_5 \sin(\theta_2+\theta_3) \end{bmatrix} \quad (82)
$$

$$
{}^{0}R_6 = \begin{bmatrix}
-\cos\theta_1(\cos\theta_6(\cos\theta_4\cos\theta_5\cos(\theta_2+\theta_3)+\sin\theta_5\sin(\theta_2+\theta_3))-\sin\theta_4\sin\theta_6\cos(\theta_2+\theta_3)) -\sin\theta_1(\cos\theta_6\cos\theta_5\sin\theta_4+\sin\theta_6\cos\theta_4) & \cos\theta_1(\sin\theta_6(\cos\theta_4\cos\theta_5\cos(\theta_2+\theta_3)+\sin\theta_5\sin(\theta_2+\theta_3))+\sin\theta_4\cos\theta_6\cos(\theta_2+\theta_3)) -\sin\theta_1(-\sin\theta_6\cos\theta_5\sin\theta_4+\cos\theta_6\cos\theta_4) & -\cos\theta_1(\cos\theta_4\sin\theta_5\cos(\theta_2+\theta_3)-\cos\theta_5\sin(\theta_2+\theta_3))-\sin\theta_5\sin\theta_1\sin\theta_4 \\
-\sin\theta_1(\cos\theta_6(\cos\theta_4\cos\theta_5\cos(\theta_2+\theta_3)+\sin\theta_5\sin(\theta_2+\theta_3))-\sin\theta_4\sin\theta_6\cos(\theta_2+\theta_3)) +\cos\theta_1(\cos\theta_6\cos\theta_5\sin\theta_4+\sin\theta_6\cos\theta_4) & \sin\theta_1(\sin\theta_6(\cos\theta_4\cos\theta_5\cos(\theta_2+\theta_3)+\sin\theta_5\sin(\theta_2+\theta_3))+\sin\theta_4\cos\theta_6\cos(\theta_2+\theta_3)) +\cos\theta_1(-\sin\theta_6\cos\theta_5\sin\theta_4+\cos\theta_6\cos\theta_4) & -\sin\theta_1(\cos\theta_4\sin\theta_5\cos(\theta_2+\theta_3)-\cos\theta_5\sin(\theta_2+\theta_3))+\sin\theta_5\cos\theta_1\sin\theta_4 \\
-\cos\theta_6(\cos\theta_4\cos\theta_5\sin(\theta_2+\theta_3)+\sin\theta_5\cos(\theta_2+\theta_3))+\sin\theta_4\sin\theta_6\sin(\theta_2+\theta_3) & \sin\theta_6(\cos\theta_4\cos\theta_5\sin(\theta_2+\theta_3)+\sin\theta_5\cos(\theta_2+\theta_3))+\sin\theta_4\cos\theta_6\sin(\theta_2+\theta_3) & -\cos\theta_5\cos(\theta_2+\theta_3)-\cos\theta_4\sin\theta_5\sin(\theta_2+\theta_3)
\end{bmatrix} \tag{83}
$$

$$
{}^{0}P_1 = \begin{bmatrix} a_1\cos\theta_1 \\ a_1\sin\theta_1 \\ d_1 \end{bmatrix} \tag{84}
$$

$$
{}^{0}P_2 = \begin{bmatrix} \cos\theta_1 a_2\cos\theta_2 + a_1\cos\theta_1 \\ \sin\theta_1 a_2\cos\theta_2 + a_1\sin\theta_1 \\ -a_2\sin\theta_2 + d_1 \end{bmatrix} \tag{85}
$$

$$
{}^{0}P_3 = \begin{bmatrix} \cos\theta_1\cos\theta_2 a_3\cos\theta_3 - \cos\theta_1\sin\theta_2 a_3\sin\theta_3 + \cos\theta_1 a_2\cos\theta_2 + a_1\cos\theta_1 \\ \sin\theta_1\cos\theta_2 a_3\cos\theta_3 - \sin\theta_1\sin\theta_2 a_3\sin\theta_3 + \sin\theta_1 a_2\cos\theta_2 + a_1\sin\theta_1 \\ -\sin\theta_2 a_3\cos\theta_3 - \cos\theta_2 a_3\sin\theta_3 - a_2\sin\theta_2 + d_1 \end{bmatrix} \tag{86}
$$

$$
{}^{0}P_4 = \begin{bmatrix} \cos\theta_1(\cos\theta_2(\sin\theta_3 d_4 + a_3\cos\theta_3) - \sin\theta_2(-\cos\theta_3 d_4 + a_3\sin\theta_3) + a_2\cos\theta_2) + a_1\cos\theta_1 \\ \sin\theta_1(\cos\theta_2(\sin\theta_3 d_4 + a_3\cos\theta_3) - \sin\theta_2(-\cos\theta_3 d_4 + a_3\sin\theta_3) + a_2\cos\theta_2) + a_1\sin\theta_1 \\ -\sin\theta_2(\sin\theta_3 d_4 + a_3\cos\theta_3) - \cos\theta_2(-\cos\theta_3 d_4 + a_3\sin\theta_3) - a_2\sin\theta_2 + d_1 \end{bmatrix} \tag{87}
$$

$$
{}^{0}P_5 = \begin{bmatrix} \cos\theta_1(\cos\theta_2(\sin\theta_3 d_4 + a_3\cos\theta_3) - \sin\theta_2(-\cos\theta_3 d_4 + a_3\sin\theta_3) + a_2\cos\theta_2) + a_1\cos\theta_1 \\ \sin\theta_1(\cos\theta_2(\sin\theta_3 d_4 + a_3\cos\theta_3) - \sin\theta_2(-\cos\theta_3 d_4 + a_3\sin\theta_3) + a_2\cos\theta_2) + a_1\sin\theta_1 \\ -\sin\theta_2(\sin\theta_3 d_4 + a_3\cos\theta_3) - \cos\theta_2(-\cos\theta_3 d_4 + a_3\sin\theta_3) - a_2\sin\theta_2 + d_1 \end{bmatrix} \tag{88}
$$

$$
{}^{0}P_6 = \begin{bmatrix} \cos\theta_1(-\cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \sin(\theta_2+\theta_3)\cos\theta_5 d_6 - \cos(\theta_2+\theta_3)a_3 + \sin(\theta_2+\theta_3)d_4 \\ + a_2\cos\theta_2 + a_1) - \sin\theta_1\sin\theta_4\sin\theta_5 d_6 \\ \\ \sin\theta_1(-\cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \sin(\theta_2+\theta_3)\cos\theta_5 d_6 - \cos(\theta_2+\theta_3)a_3 + \sin(\theta_2+\theta_3)d_4 \\ + a_2\cos\theta_2 + a_1) + \cos\theta_1\sin\theta_4\sin\theta_5 d_6 \\ \\ -\cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \sin(\theta_2+\theta_3)\cos\theta_5 d_6 - \cos(\theta_2+\theta_3)a_3 + \sin(\theta_2+\theta_3)d_4 + d_1 \end{bmatrix} \tag{89}
$$

The third column of each ${}^{0}R_i$ matrix is representing the relevant Z_{i-1} unit vector, needed for Jacobian matrix calculation (Eq.90-95)

$$
Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{90}
$$

$$
Z_1 = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix} \tag{91}
$$

$$
Z_2 = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix} \tag{92}
$$

$$
Z_3 = \begin{bmatrix} \cos\theta_1\sin(\theta_2+\theta_3) \\ \sin\theta_1\sin(\theta_2+\theta_3) \\ -\cos(\theta_2+\theta_3) \end{bmatrix} \tag{93}
$$

$$
Z_4 = \begin{bmatrix} \cos\theta_1\sin\theta_4\cos(\theta_2+\theta_3) - \sin\theta_1\cos\theta_4 \\ \sin\theta_1\sin\theta_4\cos(\theta_2+\theta_3) + \cos\theta_1\cos\theta_4 \\ \sin\theta_4\sin(\theta_2+\theta_3) \end{bmatrix} \tag{94}
$$

$$
Z_5 = \begin{bmatrix} \cos\theta_1(-\cos\theta_4\sin\theta_5\cos(\theta_2+\theta_3) + \cos\theta_5\sin(\theta_2+\theta_3)) - \sin\theta_1\sin\theta_4\sin\theta_5 \\ \sin\theta_1(-\cos\theta_4\sin\theta_5\cos(\theta_2+\theta_3) + \cos\theta_5\sin(\theta_2+\theta_3)) + \cos\theta_1\sin\theta_4\sin\theta_5 \\ -\cos\theta_4\sin\theta_5\sin(\theta_2+\theta_3) - \cos\theta_5\cos(\theta_2+\theta_3) \end{bmatrix} \tag{95}
$$

Following equation 5, all the joints are rotational joints, thus the Jacobian has form as in equation 96:

$$J = \begin{bmatrix} Z_0 \times {}^0P_6 & Z_1 \times ({}^0P_6 - {}^0P_1) & Z_2 \times ({}^0P_6 - {}^0P_2) & ..... & Z_{6-1} \times ({}^0P_6 - {}^0P_{6-1}) \\ Z_0 & Z_1 & Z_2 & ..... & Z_{6-1} \end{bmatrix} \tag{96}$$

The cross product of the position vectors relative to the base coordinate frame, ${}^0P_i$ and the corresponding unit vector, Z_{i-1}, the Jacobian matrix elements are assembled. For the Mitsubishi MELFA RV-3SDB, the full Jacobian, 6 x 6 elements are given in equations 97-132 respectively:

$$\begin{aligned} J_{11} &= -\sin\theta_1(-\cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \sin(\theta_2+\theta_3)\cos\theta_5 d_6 \\ &- \cos(\theta_2+\theta_3)a_3 + \sin(\theta_2+\theta_3)d_4 + a_2\cos\theta_2 + a_1) - \sin\theta_4\cos\theta_1\sin\theta_5 d_6 \end{aligned} \tag{97}$$

$$\begin{aligned} J_{21} &= \cos\theta_1(-\cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \sin(\theta_2+\theta_3)\cos\theta_5 d_6 \\ &- \cos(\theta_2+\theta_3)a_3 + \sin(\theta_2+\theta_3)d_4 + a_2\cos\theta_2 + a_1) - \sin\theta_1\sin\theta_4\sin\theta_5 d_6 \end{aligned} \tag{98}$$

$$J_{31} = 0 \tag{99}$$

$$J_{41} = 0 \tag{100}$$

$$J_{51} = 0 \tag{101}$$

$$J_{61} = 1 \tag{102}$$

$$\begin{aligned} J_{12} &= -\cos\theta_1(\sin(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \cos(\theta_2+\theta_3)\cos\theta_5 d_6 + \sin(\theta_2+\theta_3)a_3 \\ &+ \cos(\theta_2+\theta_3)d_4 + a_2\sin\theta_2) \end{aligned} \tag{103}$$

$$\begin{aligned} J_{22} &= -\sin\theta_1(\sin(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \cos(\theta_2+\theta_3)\cos\theta_5 d_6 + \sin(\theta_2+\theta_3)a_3 \\ &+ \cos(\theta_2+\theta_3)d_4 + a_2\sin\theta_2) \end{aligned} \tag{104}$$

$$\begin{aligned} J_{32} &= \cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 - \sin(\theta_2+\theta_3)d_4 + \cos(\theta_2+\theta_3)a_3 \\ &- a_2\cos\theta_2 - \sin(\theta_2+\theta_3)\cos\theta_5 d_6 \end{aligned} \tag{105}$$

$$J_{42} = -\sin\theta_1 \tag{106}$$

$$J_{52} = \cos\theta_1 \tag{107}$$

$$J_{62} = 0 \tag{108}$$

$$J_{13} = -\cos\theta_1(\sin(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \cos(\theta_2+\theta_3)\cos\theta_5 d_6 + \sin(\theta_2+\theta_3)a_3 + \cos(\theta_2+\theta_3)d_4) \tag{109}$$

$$J_{23} = -\sin\theta_1(\sin(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 + \cos(\theta_2+\theta_3)\cos\theta_5 d_6 + \sin(\theta_2+\theta_3)a_3 + \cos(\theta_2+\theta_3)d_4) \tag{110}$$

$$J_{33} = \cos(\theta_2+\theta_3)\sin\theta_5\cos\theta_4 d_6 - \sin(\theta_2+\theta_3)d_4 + \cos(\theta_2+\theta_3)a_3 - \sin(\theta_2+\theta_3)\cos\theta_5 d_6 \tag{111}$$

$$J_{43} = -\sin\theta_1 \tag{112}$$

$$J_{53} = \cos\theta_1 \tag{113}$$

$$J_{63} = 0 \tag{114}$$

$$J_{14} = \sin\theta_5(\cos(\theta_2+\theta_3)\cos\theta_1\sin\theta_4 d_6 - \sin\theta_1\cos\theta_4 d_6) \tag{115}$$

$$J_{24} = \sin\theta_5(\cos(\theta_2+\theta_3)\cos\theta_1\sin\theta_4 d_6 + \cos\theta_1\cos\theta_4 d_6) \tag{116}$$

$$J_{34} = \sin\theta_5\sin\theta_4\sin(\theta_2+\theta_3)d_6 \tag{117}$$

$$J_{44} = \cos\theta_1\sin(\theta_2+\theta_3) \tag{118}$$

$$J_{54} = \sin\theta_1\sin(\theta_2+\theta_3) \tag{119}$$

$$J_{64} = -\cos(\theta_2+\theta_3) \tag{120}$$

$$J_{15} = d_6(\cos\theta_1(-\cos\theta_4\cos\theta_5\cos(\theta_2+\theta_3) - \sin\theta_5\sin(\theta_2+\theta_3)) - \cos\theta_5\sin\theta_1\sin\theta_4) \tag{121}$$

$$J_{25} = d_6(\sin\theta_1(-\cos\theta_4 \cos\theta_5 \cos(\theta_2 + \theta_3) - \sin\theta_5 \sin(\theta_2 + \theta_3)) + \cos\theta_5 \cos\theta_1 \sin\theta_4) \tag{122}$$

$$J_{35} = -d_6(\cos\theta_4 \cos\theta_5 \sin(\theta_2 + \theta_3) - \sin\theta_5 \cos(\theta_2 + \theta_3)) \tag{123}$$

$$J_{45} = \cos\theta_1 \sin\theta_4 \cos(\theta_2 + \theta_3) - \sin\theta_1 \cos\theta_4 \tag{124}$$

$$J_{55} = \sin\theta_1 \sin\theta_4 \cos(\theta_2 + \theta_3) + \cos\theta_1 \cos\theta_4 \tag{125}$$

$$J_{65} = \sin\theta_4 \sin(\theta_2 + \theta_3) \tag{126}$$

$$J_{16} = 0 \tag{127}$$

$$J_{26} = 0 \tag{128}$$

$$J_{36} = 0 \tag{129}$$

$$J_{46} = \cos\theta_1(-\cos\theta_4 \sin\theta_5 \cos(\theta_2 + \theta_3) + \cos\theta_5 \sin(\theta_2 + \theta_3)) - \sin\theta_1 \sin\theta_4 \sin\theta_5 \tag{130}$$

$$J_{56} = \sin\theta_1(-\cos\theta_4 \sin\theta_5 \cos(\theta_2 + \theta_3) + \cos\theta_5 \sin(\theta_2 + \theta_3)) + \sin\theta_4 \sin\theta_5 \cos\theta_1 \tag{131}$$

$$J_{66} = -\cos\theta_4 \sin\theta_5 \sin(\theta_2 + \theta_3) - \cos\theta_5 \cos(\theta_2 + \theta_3) \tag{132}$$

As previously mentioned in the introduction, any robot manipulator by anatomy, consists of 3 parts: shoulder, elbow and wrist with 3 joints intersecting in 1 point (Figure 5). The shoulder and the elbow are the representing the arm of the robot and they are giving the position of the same. The wrist is defining the orientation.

Denoting equation 13, there are two conditions produced by this equation; one for boundary singularities and the other for interior singularities. Referring to Abderrahmane et al., 2014 and Djuric and Elmaraghy, 2007, in the case of this particular robot the equation 133 is condition for boundary singularities:

$$C_b = -a_3 \sin\theta_3 + d_4 \cos\theta_3 = 0 \tag{133}$$

and equation 134, condition for interior singularities

$$C_i = a_1 + a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3) + d_4 \sin(\theta_2 + \theta_3) = 0 \tag{134}$$

The outcome from these equations is

$$\theta_3 = a \tan 2\left(\frac{d_4}{-a_3}\right) \tag{135}$$

which gives by two values for angle $\theta_3 = 114.59^\circ$ or -65.41°. Because the mechanical limit of joint 3 is from -20° to 171°, $\theta_3 \neq -65.41^\circ$ and only $\theta_3 = 114.59^\circ$ is considered.

For θ_2 the condition is equation 136.

$$\theta_2 = a \tan 2\left(\frac{a_1 - a_2 - a_3 \cos\theta_3 - d_4 \sin\theta_3}{a_3 \sin\theta_3 - d_4 \cos\theta_3}\right) \tag{136}$$

which again, gives two values for the angle $\theta_2 = 117.56^\circ$ or -83.206°, both reachable as the mechanical limit for joint 2 is from -90° to 135°.

The only singularity of the spherical wrist happens when the joint axes z3 and z5, are collinear, which physically means the 4th and the 6th joint are aligned (Eq.137):

$$\det J_{22} = -\sin\theta_5 = 0 \tag{137}$$

This is true only when $\theta_5 = 0^\circ$, or $\theta_5 = 180^\circ$. Since the mechanical limits for joint 5 are from -120° to 120°, only $\theta_5 = 0^\circ$ is considered. This singularity appears in every industrial robot with spherical wrist, and it is commonly known, so the angle between joint 4 and 6 is always different than 0 (zero).

Upon deriving the conditions for singularity, as final step in the parametric design, next is the detailed design stage where visual representation of the workspace is conducted with the singularities encountered, as well as graphical representation of the singularities.

The kinematic structure and the end-effectors home position is drawn as shown in Figure 36. Figure 37 represents screen capture from Matlab calculation, where the end-effector position in space, by XYZ axes is derived.

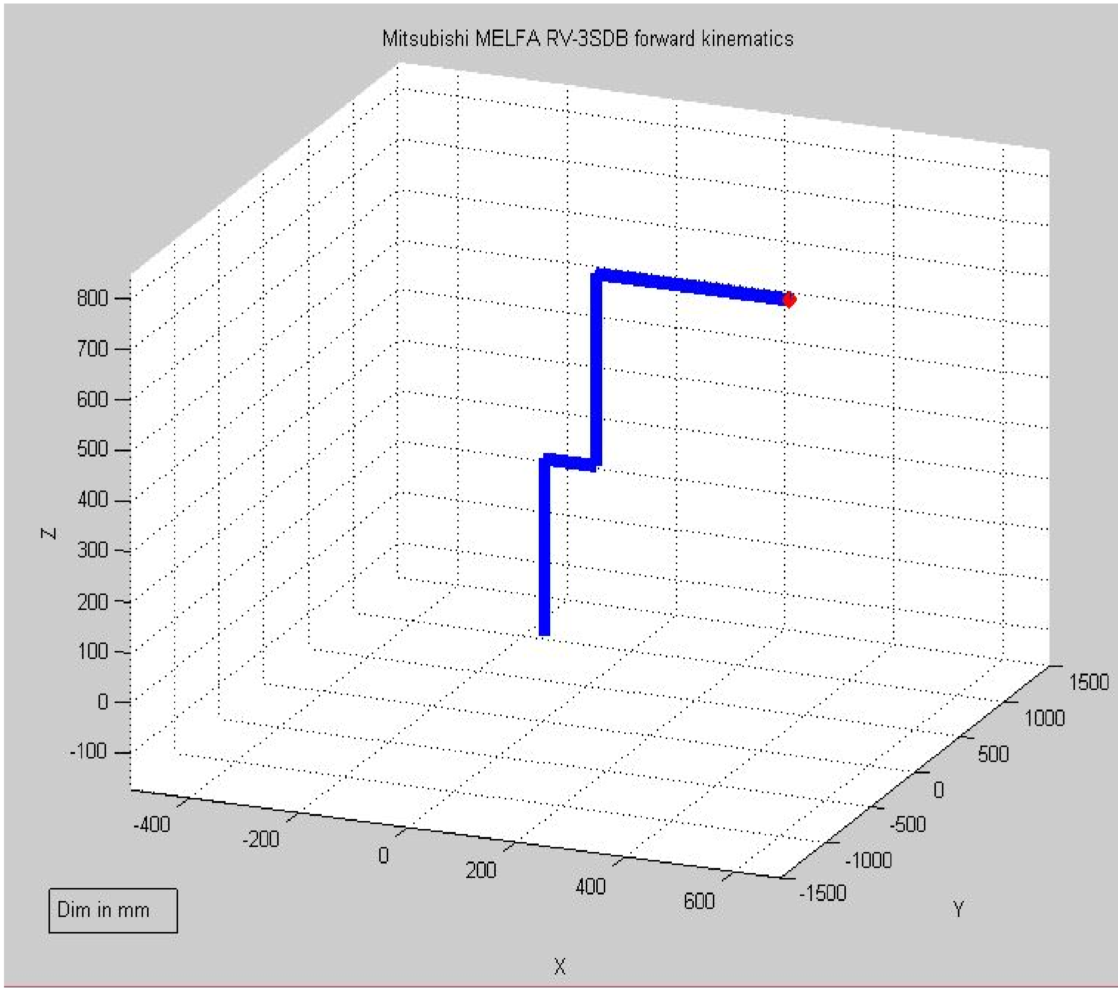


Figure 36 MELFA RV-3SDB Forward kinematics

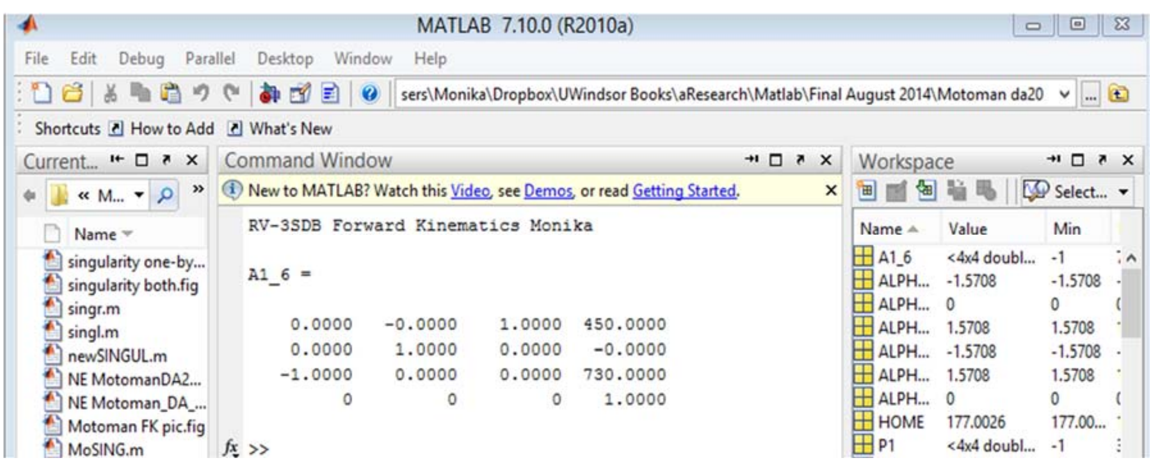


Figure 37 Capture from the forward kinematics validation of the RV-3SDB

The end-effector position is at the place where the last link ends. If the result from the forward kinematics (Figure 37), corresponds with the plot (Figure 36), this matlab code proves the correctness of frame assignment in the conceptual design, and validates the forward kinematics in the parametric design. Having the mechanical limits, provided by the manufacturer (Table 6), RV-3SDB workspace can be computed with a set of equations describing every reachable point of the robot's end-effector (Eq.90). Bellow figures are giving 2D (Figure 38) and 3D (Figure 39) view of the Mitsubishi Melfa RV-3SDB workspace. (Appendix F contains the Matlab codes for Mitsubishi RV-3SDB).

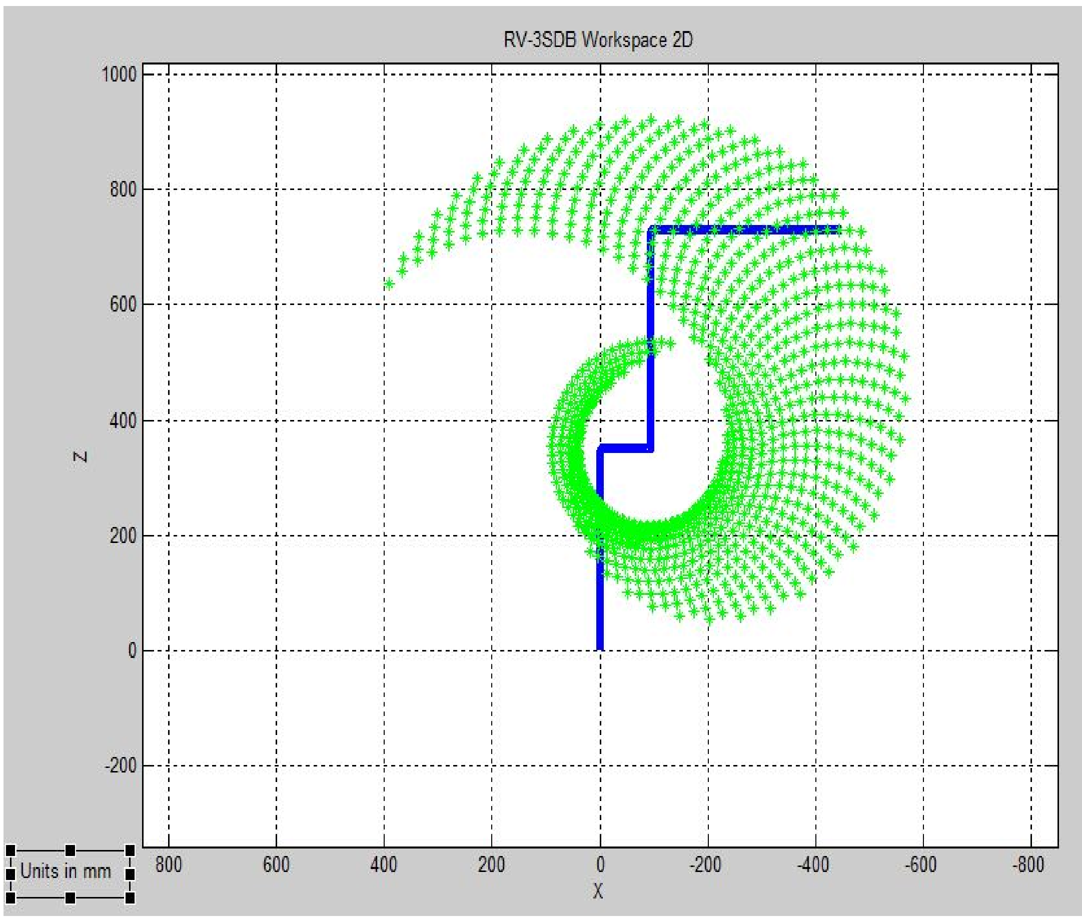


Figure 38 2D view of Mitsubishi RV-3SDB workspace

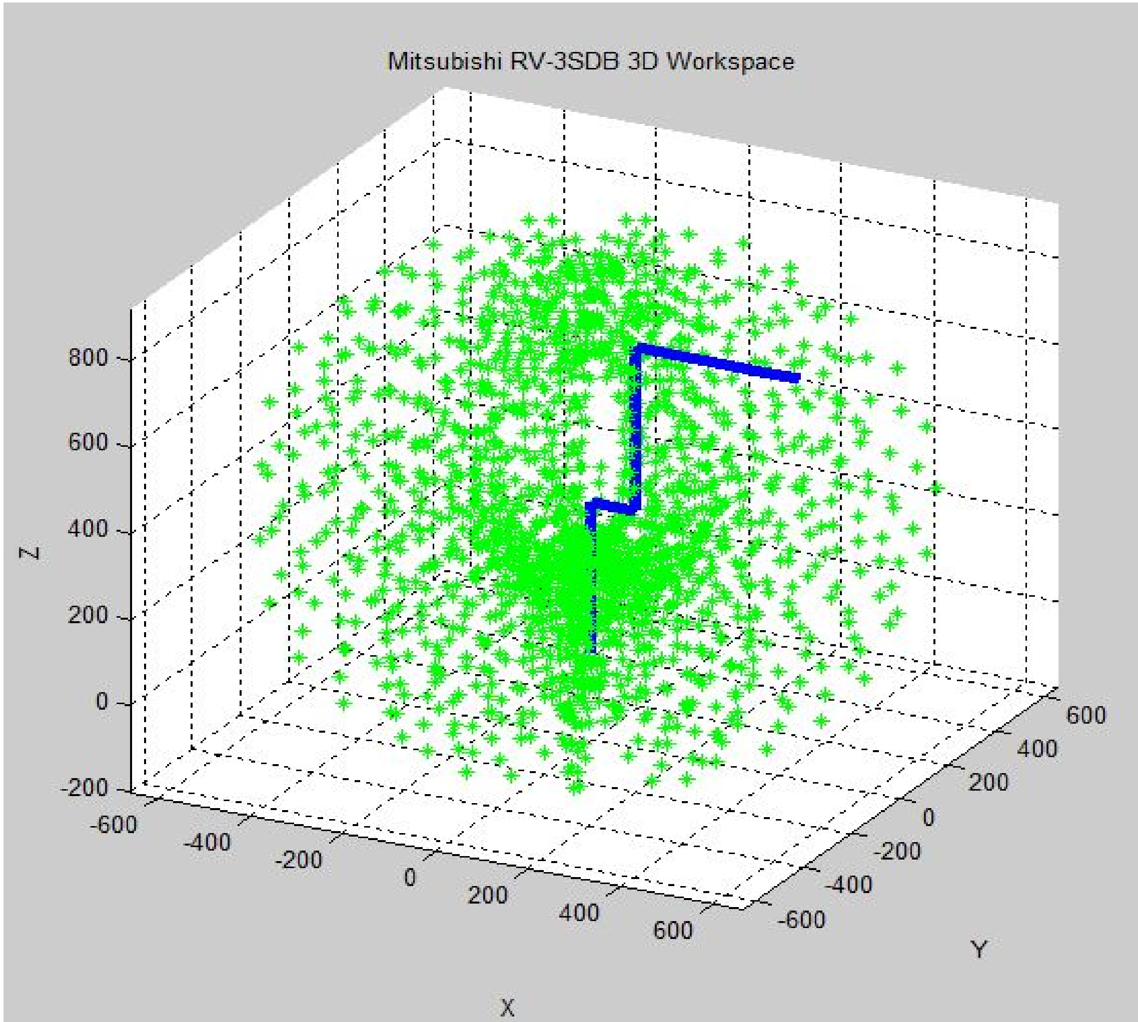


Figure 39 3D view of Mitsubishi RV-3SDB workspace

With inputting the values for singular configuration into the Matlab code it is possible to compute the position of the end-effector at singular configuration (Figure 40). The interior singularity locus is presented in the Figure 41 below (or closer view in Figure 42).

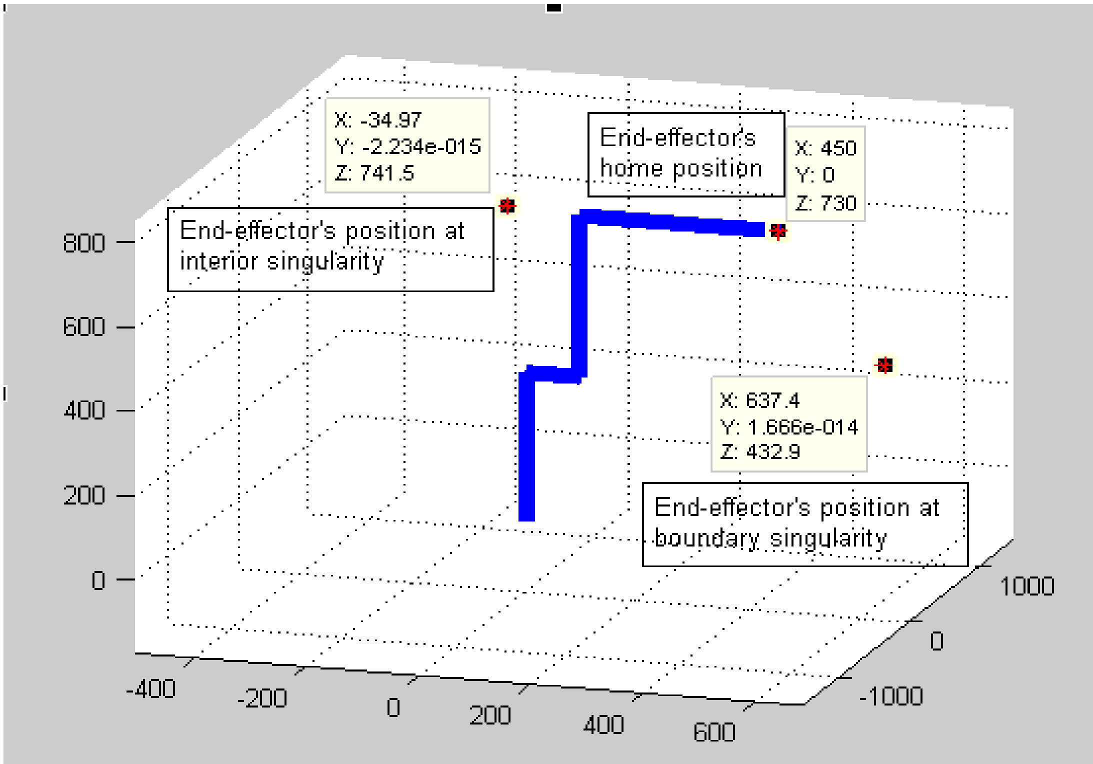


Figure 40 End-effector positions in home configuration, interior and boundary singular configuration

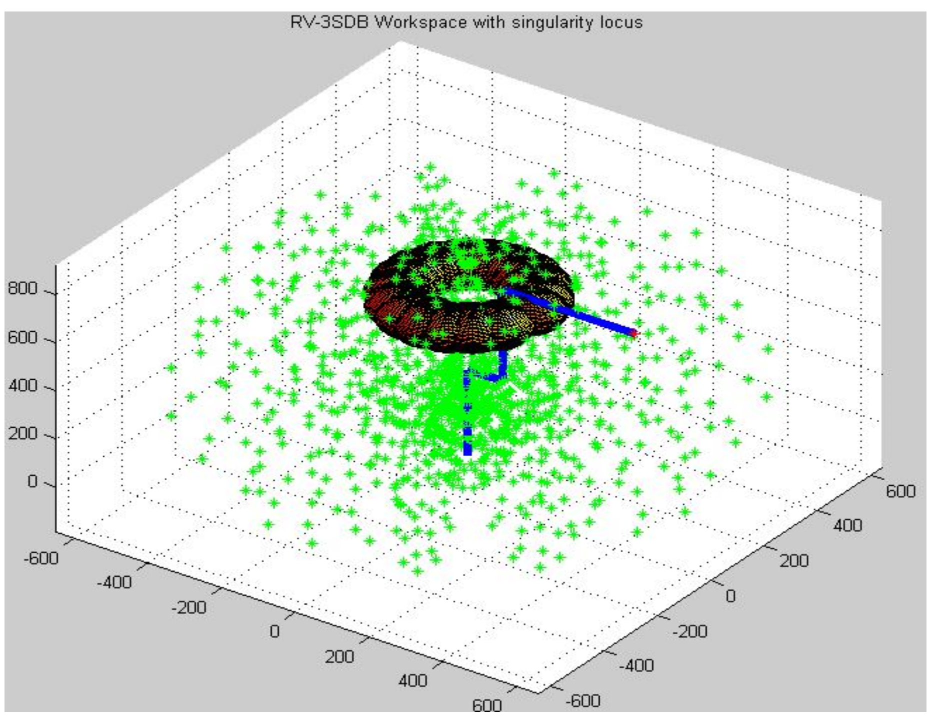


Figure 41 Singularity locus in the workspace of RV-3SDB

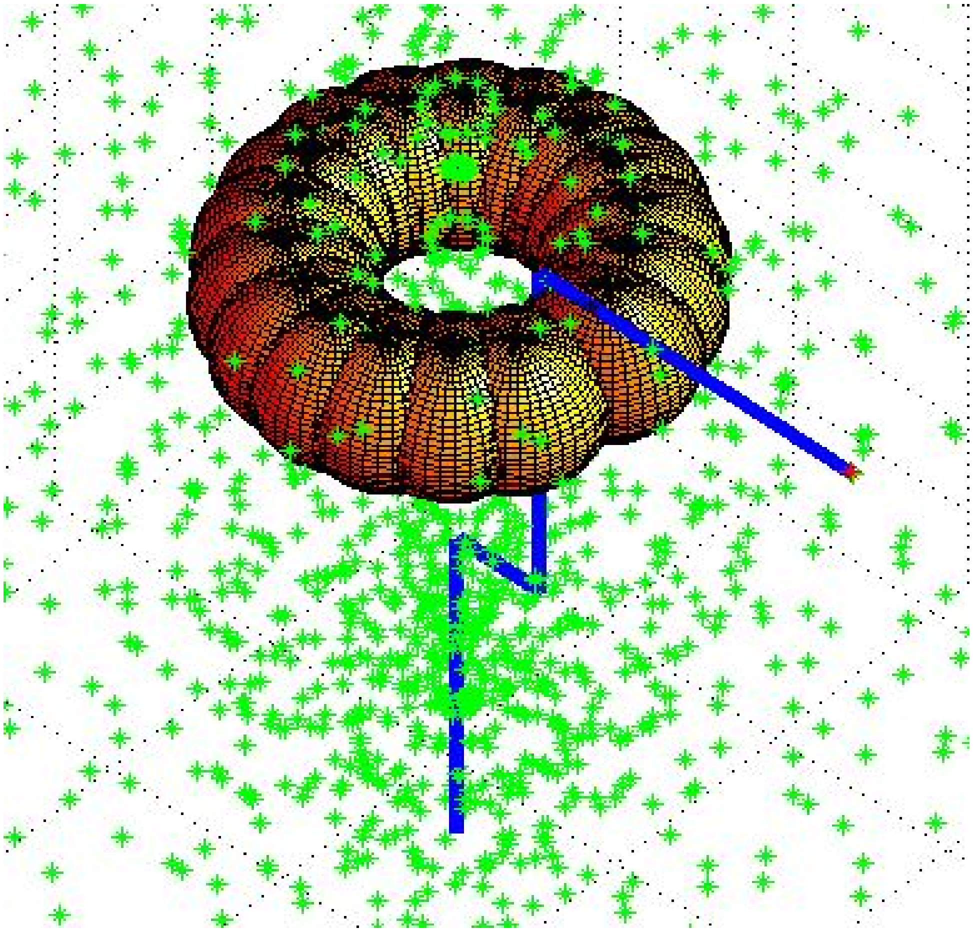


Figure 42 Singularity locus RV-3SDB closer view

4.1.3. Motoman Yaskawa DA20 kinematic modelling

Motoman's revolutionary dual-arm DA20 robot (Figure 43) provides superior human manipulability, due to its six axes per arm, plus a single axis for base rotation, totaling 13 axes. This collaborative manipulation and the innovative design makes the DA20 robot ideally suited for a wide variety of tasks that formerly could only be done by people. The DA20 robot can be used to transfer parts from one to the other of his arms with no re-griping the part, allowing one robot arm to hold the part while the other performs the required operation.

The DA20 robot has a 20 kg payload and 765 mm reach per arm. Both robot arms can work together to double the payload or accomplish complex tasks. In addition, both robot arms can perform tasks independently without affecting the output.

Figure 43 Motoman Yaskawa DA20 (source: Yaskawa Motoman, 2006, DA20 Manipulator Manual)

By the general design rules in concept ideation, dual arm robot is selected if axial symmetry is not required, but a workspace with coplanar axes. Modeling a dual arm robot is challenging, mostly because both arms are same, but they have different mechanical limits, as this particular Yaskawa Motoman robot, DA-20. By the specifications provided in the robot's manual (Figure 44), the conceptual design can begin.

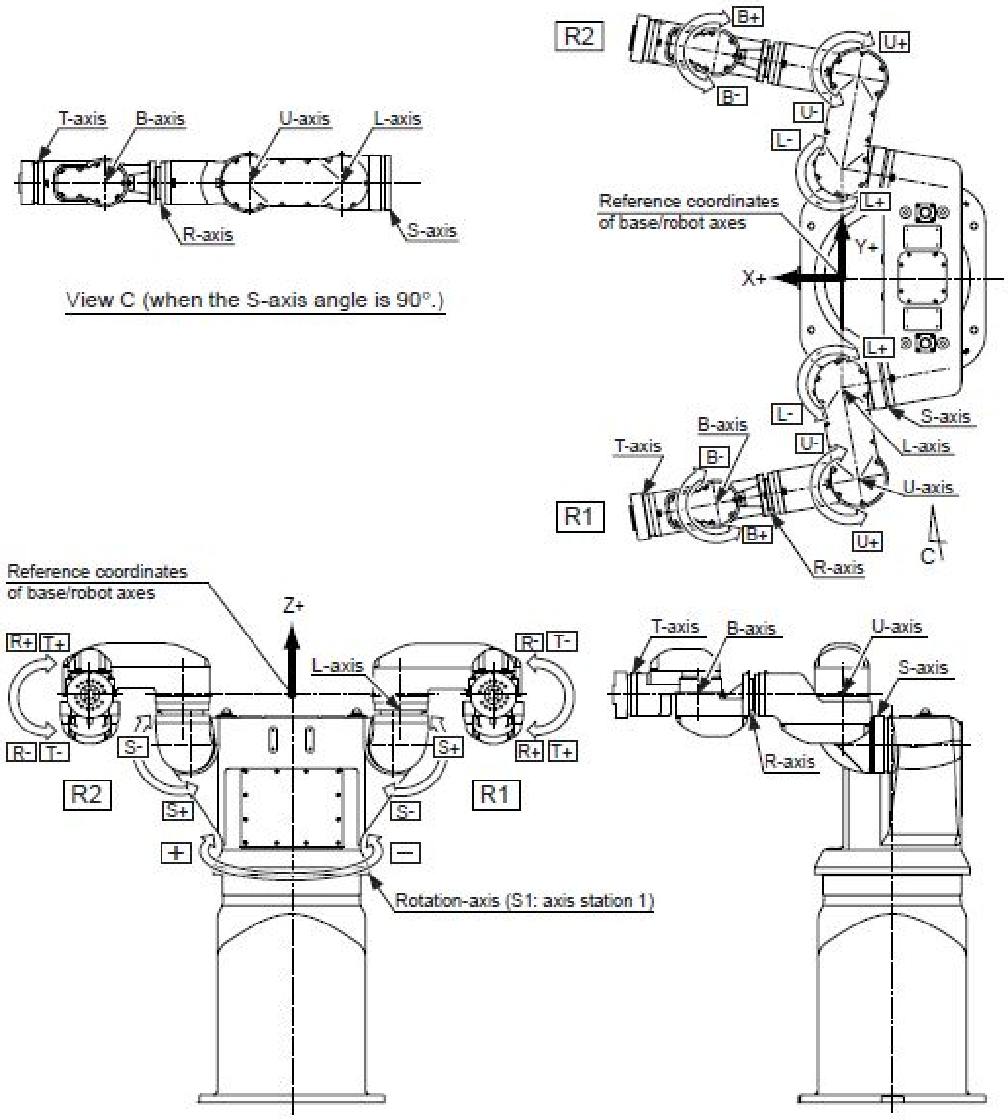


Figure 44 Parts and working axes of DA-20 (source: Yaskawa Motoman, 2006, DA20 Manipulator Manual)

In the Figure 44 above, the 13 working axes of the manipulator are given, along with their positive rotation, and in Figure 45 the dimensions are provided.

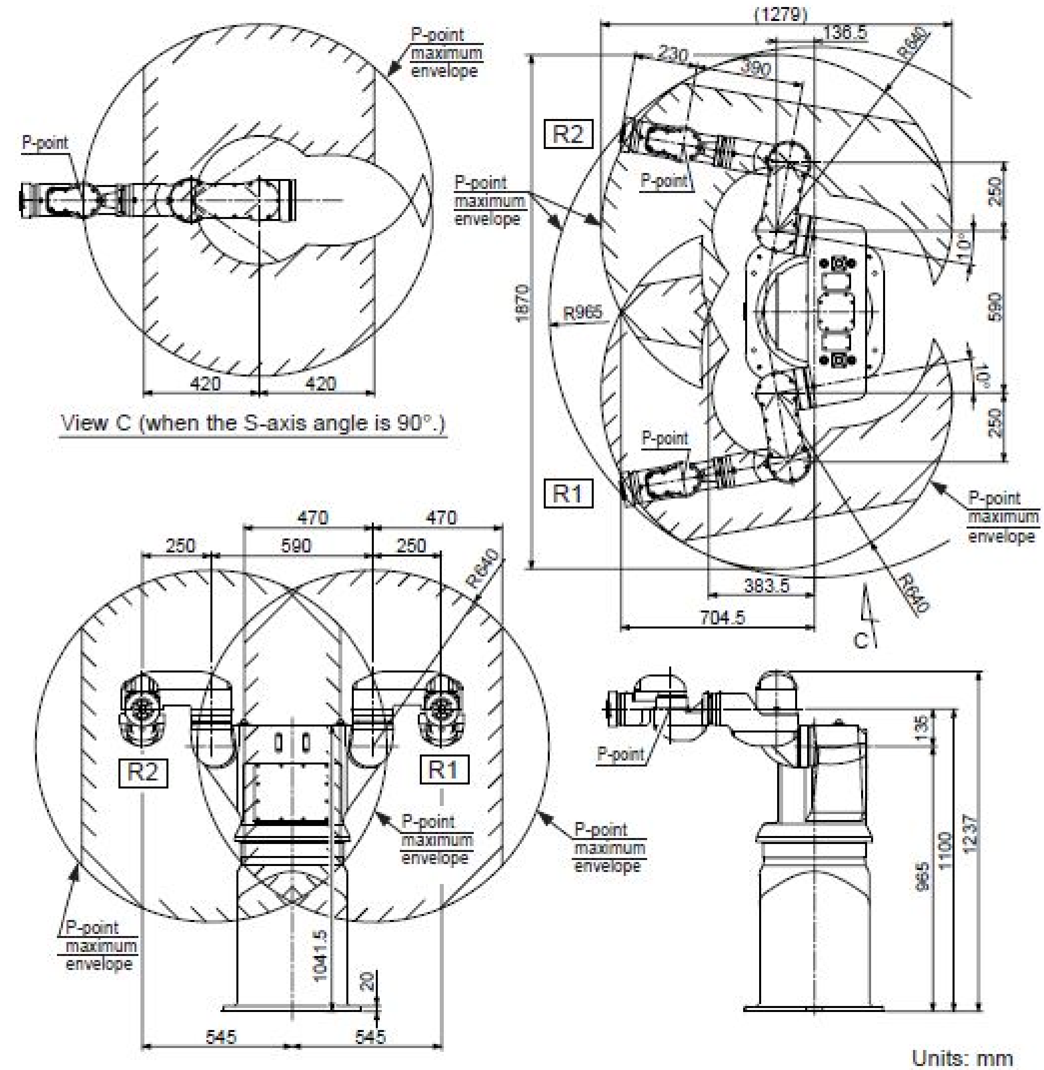


Figure 45 DA-20 dimensions (source: Yaskawa Motoman, 2006 DA20 Manipulator Manual)

From Figure 44 and Figure 45 the kinematic structure is sketched (Figure 46) and the DH parameters are determined (Table 7). It is notable that the both arms are identical, but rotated for 180° degrees when related one to the other. This means that they differ only by their base coordinate frame, which is the 13th axis, located at the body of the robot, denoted as S1 rotation axis in Figure 44. This robot structure is not typical industrial structure, thus the modelling is complex even more, signifying the humble information found in the literature for dual arm manipulators kinematics.

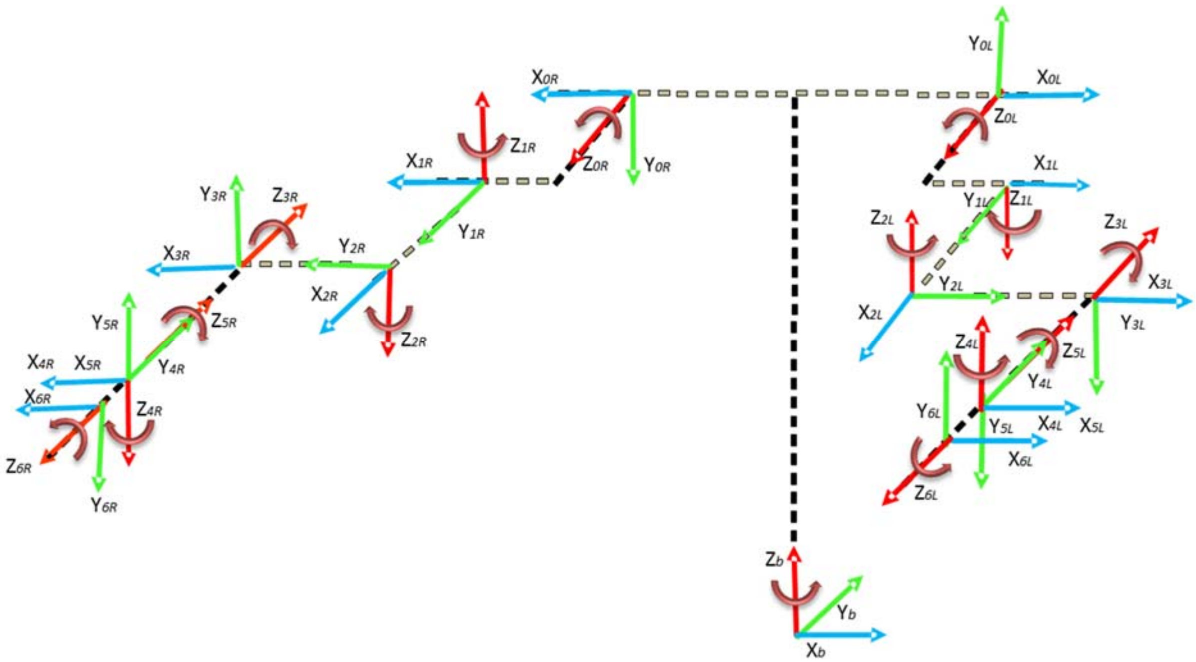


Figure 46 Kinematic structure of Motoman DA20

Table 7 Yaskawa Motoman DA20 DH parameters and axes limits

Motoman DA20 DH Parameters								
Joint	*d*	θL	θR	*a*	α L	α R	Mechanical Limit L	Mechanical Limit R
b	965	0°*	180°*	295	90°	-90°	±180	
1	136.5	0°*		135	90°		+80; -190	+190; -80
2	0	90°*		250	180°		+220; -40	
3	0	90°*		390	-90°		+215; -35	
4	-230	0°*		0	90°		±180	
5	0	0°*		0	-90°		±120	
6	-65	0°*		0	180°		±180	

The conceptual design ends with generating the homogeneous transformation matrices. Here, both arms should be examined, not with respect to their base (joint 0), but with respect to the base of the whole system (joint b). Although the arms are identical, their forward kinematics is diverse because their position and orientation in the space is different. Only the transformation matrix A_{0b} is different for the left and right arm, and contributes into the difference in forward kinematics. Equations from 138-145 are homogeneous transformation matrices, and equations for forward kinematic of the left and the right arm are given in Appendix C.

$$A_{0bL} = \begin{bmatrix} \cos\theta_{bL} & 0 & \sin\theta_{bL} & a_{bl}\cos\theta_{bL} \\ \sin\theta_{bL} & 0 & -\cos\theta_{bL} & a_{bl}\sin\theta_{bL} \\ 0 & 1 & 0 & d_{bL} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{138}$$

$$A_{0bR} = \begin{bmatrix} \cos\theta_{bR} & 0 & -\sin\theta_{bR} & a_1\cos\theta_{bR} \\ \sin\theta_{bR} & 0 & \cos\theta_{bR} & a_1\sin\theta_{bR} \\ 0 & -1 & 0 & d_{bR} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{139}$$

$$A_{01} = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & a_1\sin\theta_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{140}$$

$$A_{12} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & -\cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{141}$$

$$A_{23} = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{142}$$

$$A_{34} = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{143}$$

$$A_{45} = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{144}$$

$$A_{56} = \begin{bmatrix} \cos\theta_6 & \sin\theta_6 & 0 & 0 \\ \sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & -1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{145}$$

The Jacobian matrices of the both arms are examined separately. Here, both arms with respect to their base (joint 0) are examined. The Jacobian matrices are identical, thus only one is represented Appendix C. Joint b is not considered in the calculation of the Jacobian matrix.

The singularity conditions produced for both arms are the same, as derived in equations 146 and 147.

$$C_b = -a_3 \sin\theta_3 - d_4 \cos\theta_3 = 0 \tag{146}$$

$$C_i = a_1 + a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3) - d_4 \sin(\theta_2 + \theta_3) = 0 \tag{147}$$

The outcome from these equations is

$$\theta_3 = -a\tan 2\left(\frac{d_4}{a_3}\right) \tag{148}$$

which gives two values for angle $\theta_3 = 30.53^\circ$ or -149.47°. Because the mechanical limit of joint 3 is from -35° to 215°, $\theta_3 \neq -149.47^\circ$ and only $\theta_3 = 30.53^\circ$ is considered.

The solution for θ_2 is equation 149.

$$\theta_2 = a\tan 2\left(\frac{-a_1 + a_2 + a_3\cos\theta_3 - d_4\sin\theta_3}{a_3\sin\theta_3 + d_4\cos\theta_3}\right) \tag{149}$$

, which again, gives two values for the angle $\theta_2 = 158.78^\circ$ or -49.669°, and only the first one attainable as the mechanical limit for joint 2 is from -40° to 220°.

The only singularity of the spherical wrist is (Eq.150):

$$\det J_{22} = -\sin\theta_5 = 0 \tag{150}$$

, which is true only when $\theta_5 = 0^\circ$, or $\theta_5 = 180^\circ$, but because of the limits in joint 5 from -120° to 120°, only $\theta_5 = 0^\circ$ is considered.

The detailed design begins with plotting the kinematic structure in Matlab software, and validating the forward kinematics as in Figure 47. (Appendix G contains the Matlab codes for the Motoman DA-20)

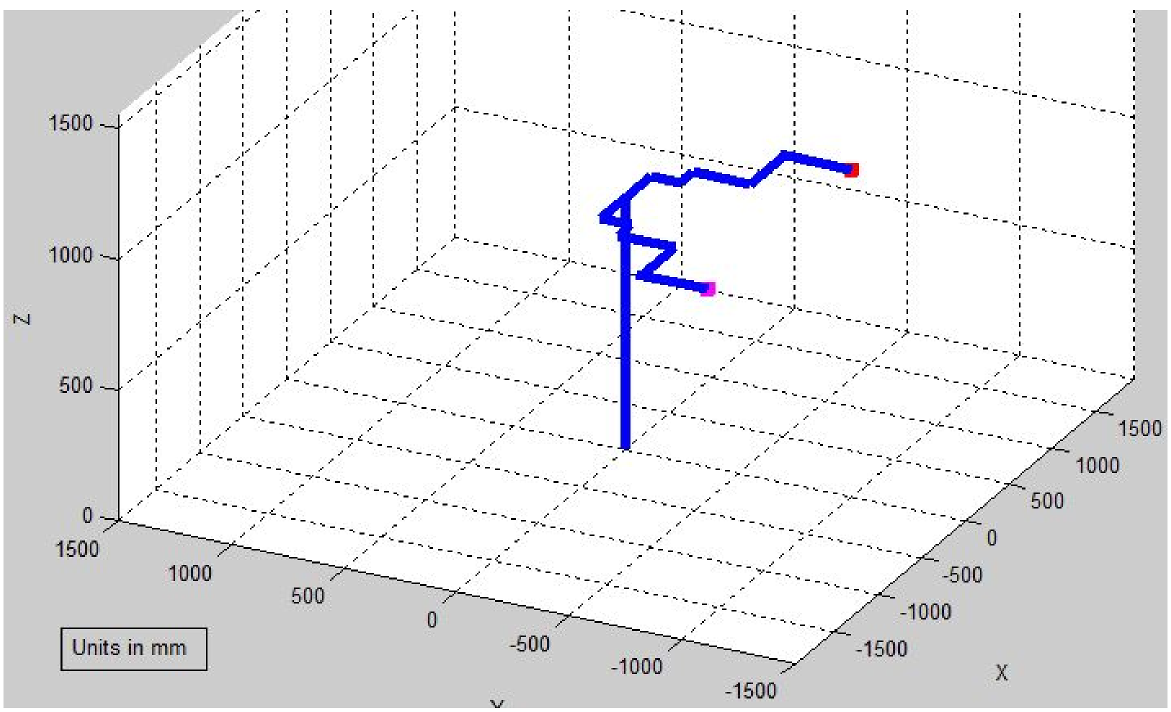


Figure 47 DA-20 Forward Kinematics validation

Then the workspace is computed in order to attain the reachable points by both arms. In Figure 48 the workspace of the both arms is plotted with different colours. In Figure 49 a closer view of the workspaces of the both arms is given, where the overlapping points can be seen.

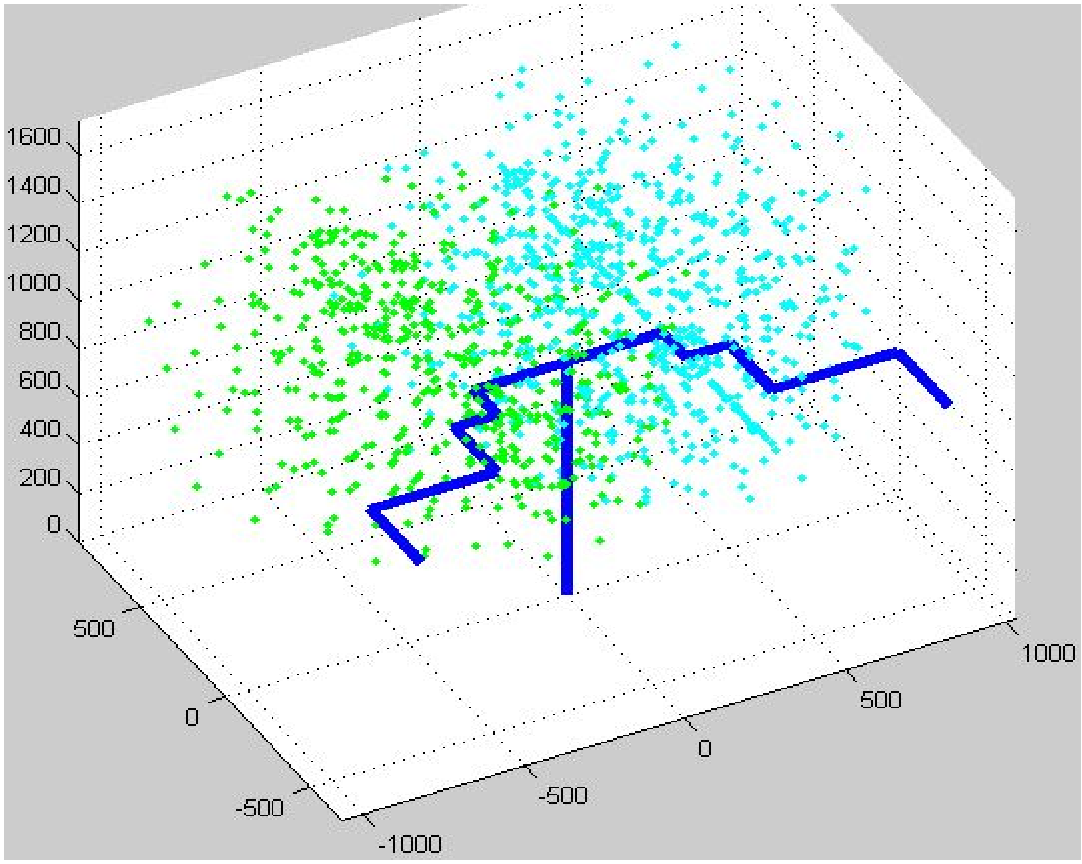


Figure 48 DA-20 Workspace

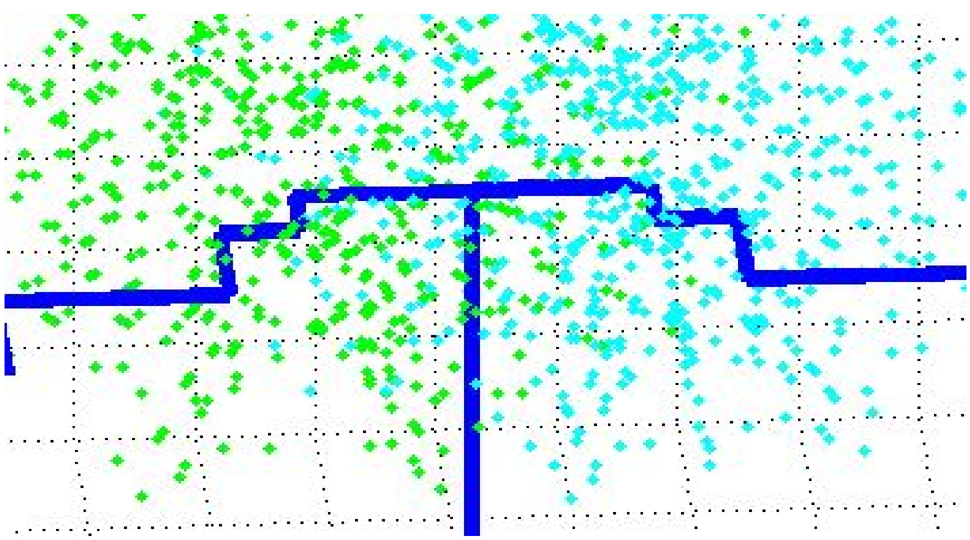


Figure 49 DA-20 closer view of the overlapping workspaces of the both arms

The position of the end-effector in home configuration and interior singularity configuration for the both arms is provided in Figure 50. At the end the singularity conditions are plotted in order to get visual representation of the same.

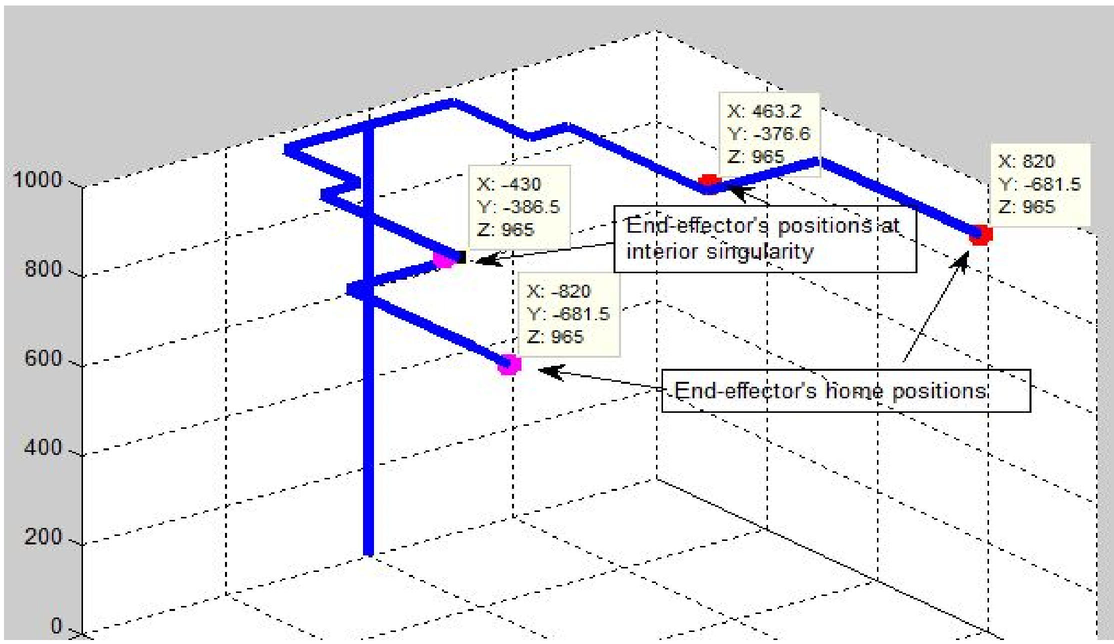


Figure 50 End-effector's position in home configuration and interior singularity configuration for the both arms

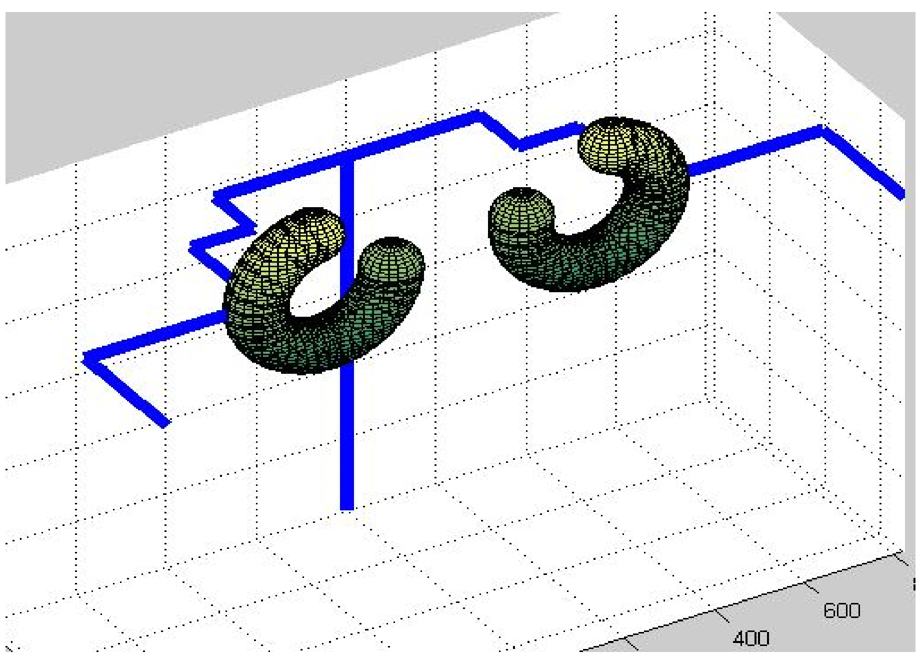


Figure 51 Motoman DA20 Singularity locus of each arm

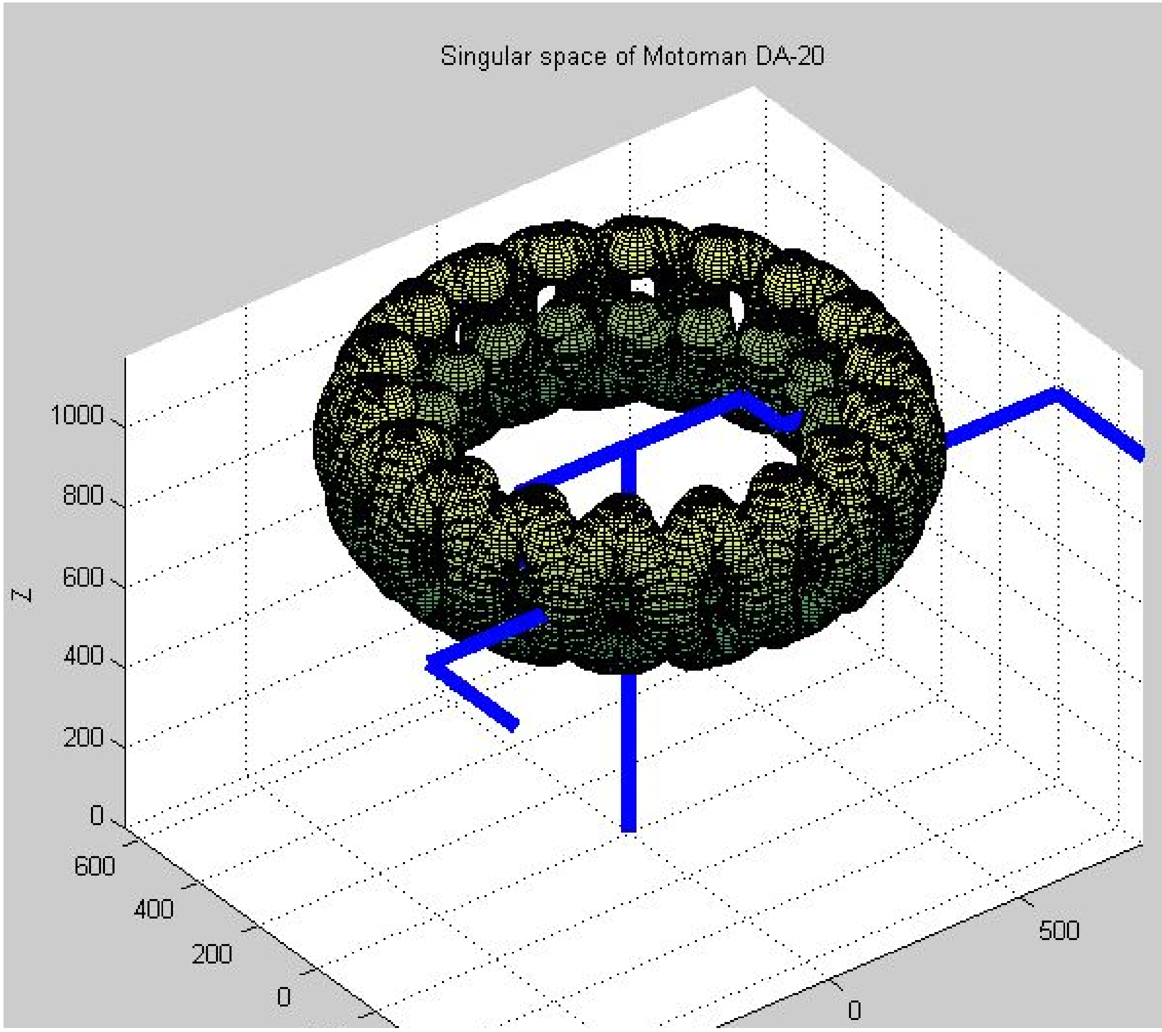


Figure 52 Motoman DA20 singularity locus overlapping

Figure 51 represents the interior singularity locus shape of each arm, and the Figure 52 represents the singularity locus when joint b is rotated form -180° to 180° degrees. The singular spaces of both arms are overlapping, same as the workspace.

4.1.4. Reconfigurable multibody system kinematic modelling

The need of being able to combine any robot manipulator and any CNC machine DH parameters has resulted in development of CNC-R Global Kinematic Model, graphically presented in Figure 53 (Filiposka et al., 2014). Combined joint types increase the model complexity, but provide the knowledge of many machines kinematics problem, which can be used as benchmark in the design tool for new machine kinematic structure, and much more. CNC-R GKM was selected for analysis and the emphasis was on the significance of kinematics for designing and building reconfigurable multibody machinery system for the industry. The kinematic model is combination of 6DOF robot manipulator and general CNC-machine.

Each mathematical modeling (parametric design stage) begins with previously determined DH parameters in the conceptual phase. But, when having such a complex system to model, DH parameters needs to be amended. Depending on the joint type, reconfigurable control parameters are added. This reconfigurable joint modelling is introduced by Djuric and Elmaraghy, 2006. Because each joint has six different positive directions for rotations or translations, any joint's vector can be in positive or negative direction in Cartesian space. R_i and T_i are used to control the selection of joint type (rotational and/or translational)(Eq.151,152).

Rotational Joints: $$R_i = 1, T_i = 0 \tag{151}$$

Translational Joints: $$R_i = 0, T_i = 1 \tag{152}$$

The link twist angle α_i, in the full reconfigurable kinematic model Djuric and Elmaraghy, 2006 usually has five different values, and remains perpendicularity to the joint's coordinate frames; however for the current CNC-R GKM the DH parameters are presented in Table 8.

Table 8 DH parameters for 6-DOF CNC-R GKM

i	θ_i	d_i	a_i	α_i
1	$R_1\theta_1 + T_1\theta_{DH\,1}$	$R_1 d_{DH\,1} + T_1 d_1$	a_1	$\pm 90°$
2	$R_2\theta_2 + T_2\theta_{DH\,2}$	$R_2 d_{DH\,2} + T_2 d_2$	a_2	$\pm 180°;\pm 90°;0$
3	$R_3\theta_3 + T_3\theta_{DH\,3}$	$R_3 d_{DH\,3} + T_3 d_3$	a_3	$\pm 180°;\pm 90°;0$
4	$\theta_{DH\,4}$	$d_{DH\,4}$	a_4	$\pm 90°$
5	$\theta_{DH\,5}$	$d_{DH\,5}$	a_5	$\pm 90°$
6	$\theta_{DH\,6}$	$d_{DH\,6}$	a_6	$\pm 90°;0°$

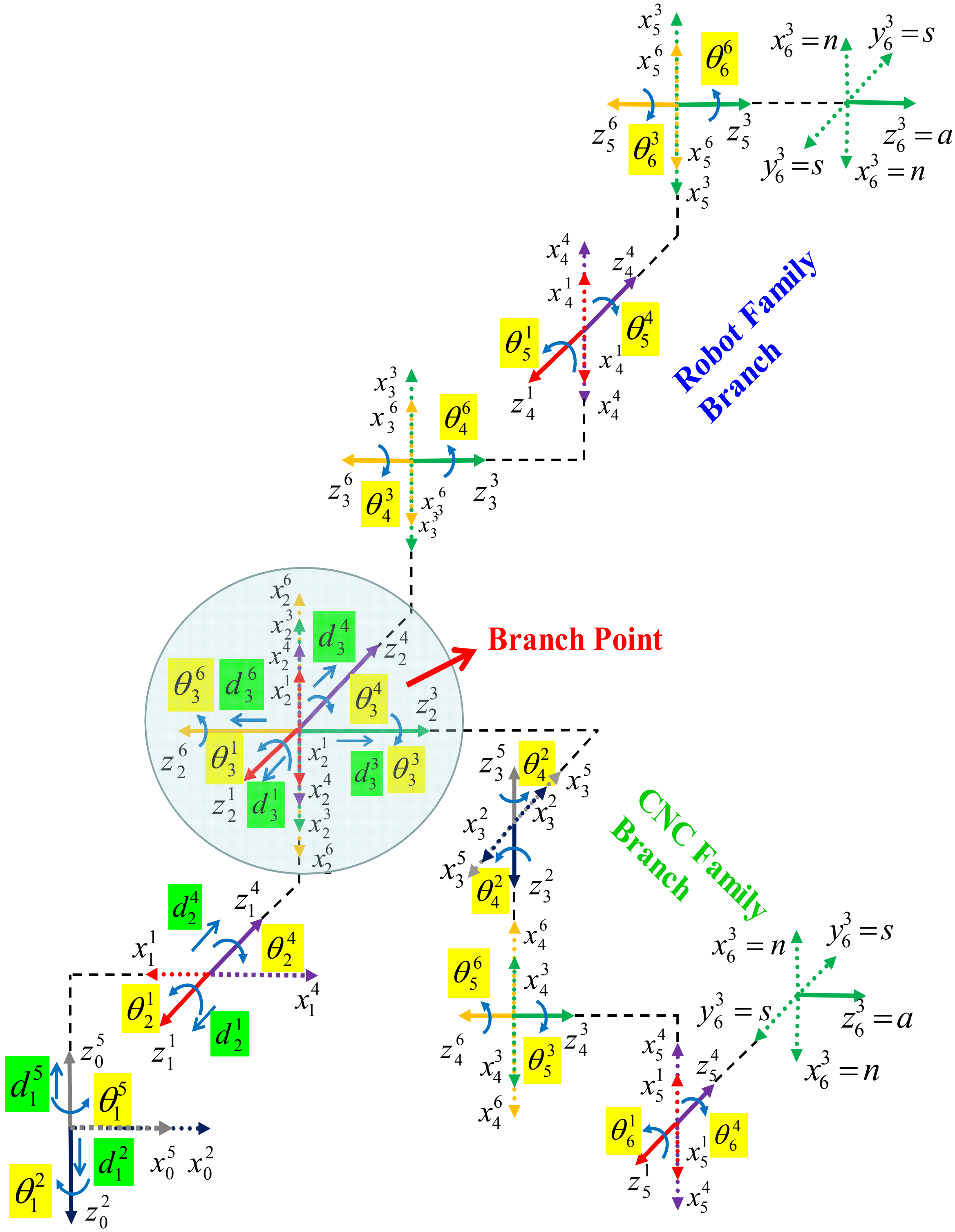


Figure 53 CNC-R Global Kinematic Model

α_i has five possible values only at second and third joint. This condition makes joint 3 the most complex joint in the model, which is named Branch Point. From joint 3 two branches are formed, one for group of robots named Robot Family Branch, and one for group of CNC machines named CNC Family Branch (Figure 53). This issue implies increase of the complexity in the system. It is extremely difficult to simplify or decouple the results in order to get optimal solutions.

The homogeneous transformation matrix ${}^{i-1}A_i$ in the n-DOF GKM (Djuric and Elmaraghy, 2007) is giving the relationship between two joints in Cartesian coordinate frame, and i represents the number of joints, in this case, 6 (Eq.155). Their sine and cosines of the twist angle are defined as the joint's reconfigurable parameters and expressed in equations 153 and 154.

$$K_{si} = \sin \alpha_i \tag{153}$$

$$K_{ci} = \cos \alpha_i \tag{154}$$

$$ {}^{i-1}A_i = \begin{bmatrix} \cos(R_i\theta_i + T_i\theta_{DHi}) & -K_{Ci}\sin(R_i\theta_i + T_i\theta_{DHi}) & K_{Si}\sin(R_i\theta_i + T_i\theta_{DHi}) & a_i\cos(R_i\theta_i + T_i\theta_{DHi}) \\ \sin(R_i\theta_i + T_i\theta_{DHi}) & K_{Ci}\cos(R_i\theta_i + T_i\theta_{DHi}) & -K_{Si}\cos(R_i\theta_i + T_i\theta_{DHi}) & a_i\sin(R_i\theta_i + T_i\theta_{DHi}) \\ 0 & K_{Si} & K_{Ci} & R_i d_{DHi} + T_i d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$i = 1,2,3,4,5,6 \tag{155}$$

Rotational matrices, ${}^{i-1}R_i$ are defined by the upper left [3x3] sub matrices from the homogenous transformation matrices ${}^{i-1}A_i$ (Eq.1), correlated with each joint (Eq.156)

$$ {}^{i-1}R_i = \begin{bmatrix} \cos(R_i\theta_i + T_i\theta_{DHi}) & -K_{Ci}\sin(R_i\theta_i + T_i\theta_{DHi}) & K_{Si}\sin(R_i\theta_i + T_i\theta_{DHi}) \\ \sin(R_i\theta_i + T_i\theta_{DHi}) & K_{Ci}\cos(R_i\theta_i + T_i\theta_{DHi}) & -K_{Si}\cos(R_i\theta_i + T_i\theta_{DHi}) \\ 0 & K_{Si} & K_{Ci} \end{bmatrix} \tag{156}$$

The position vectors ${}^{i-1}P_i$, are defined by the upper right [3x1] sub matrices from the homogenous transformation matrices ${}^{i-1}A_i$ (Eq.5), correlated with each joint (Eq. 157).

$$
{}^{i-1}P_i = \begin{bmatrix} a_i \cos(R_i\theta_i + T_i\theta_{DHi}) \\ a_i \sin(R_i\theta_i + T_i\theta_{DHi}) \\ R_i d_{DHi} + T_i d_i \end{bmatrix} \tag{157}
$$

The control values for CNC-R reconfigurable Global Kinematic Model parameters are presented in Table 9.

Table 9 CNC-R GKM reconfigurable parameters

Control Values					
Joint	Sine	Cosine	Joint	Sine	Cosine
1	$K_{S1} = \pm 1$	$K_{C1} = 0$	**4**	$K_{S4} = \pm 1$	$K_{C4} = 0$
2	K_{S2}	K_{C2}	**5**	$K_{S5} = \pm 1$	$K_{C5} = 0$
3	K_{S3}	K_{C3}	**6**	K_{S6}	K_{C6}

In serial link manipulators there are series of links, connecting the end-effector to the base, by actuated joints. The CNC-R GKM covers $36(36-1)^2(18-1)^3$ possible kinematic configurations, and each configuration can be modelled with one set of reconfigurable parameters, presented in Table 8 and 9. Compared to the initial 6-DOF GKM, which can have $48(48-1)^{n-1}$ configurations (Djuric and Elmaraghy, 2006), this kinematic model cannot encompass all the 11,008,560,336 possible structures, but includes 446,071,500 kinematic configurations.

With complex configurations, the difficulty in calculation increases, thus beside the use of Maple 17 Software, in most cases manual simplification is needed. Detailed symbolic calculation is provided in Appendix D.

For Jacobian matrix computation, with the use of Vector cross multiplication method in equation 158 the full Jacobian matrix relative to the base coordinate frame is derived.

$$
J_V = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ 0 & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ 0 & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ 0 & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & 0 & J_{63} & J_{64} & 0 & 0 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{bmatrix} + \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ 0 & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ 0 & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ 0 & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & 0 & J_{63} & J_{64} & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \tag{158}
$$

The Jacobian matrix constitutes of two parts; one related to the rotational joints and the other related to the translational joints. The full Jacobian can be derived with summing the two parts, after determining the type of joints. This Jacobian increases the complexity in determining singularity conditions of reconfigurable multibody system.

4.2. Discussions for future work

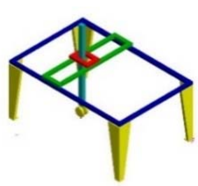
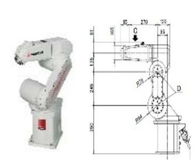
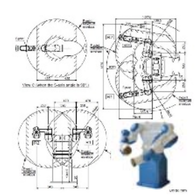
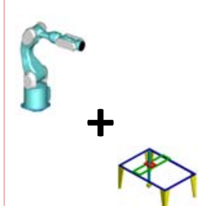
The results derived from the kinematics models of the 4 types of multibody systems examined in this research are provided and compared in Table 10 below.

It is notable that the most challenging system is the reconfigurable one. By increasing the number of reconfigurable joints, the complexity rises. A system like the one proposed in this thesis has not been designed yet, but in this research the structure is used for examining the kinematics of different configuration of joints.

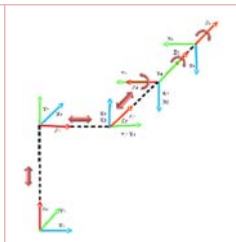
This work could be considered as preliminary design in complete robot design, where all other characteristics are considered, like dynamics modelling, actuators, sensors, payload, repeatability, accuracy, control… Task planning and path generation are areas where the real velocities at the end-effector pose are primarily important. At singular configuration the velocity level of these points approaches infinity, and they need to be avoided.

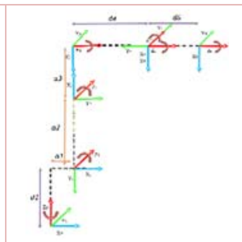
An important constraint which is not fully examined in this research is the inverse kinematics. At singular configuration there could be infinite number of possible solutions (industrial robots usually 16 or 8). Optimization of the posture is an important feature, depending on the task requirements.

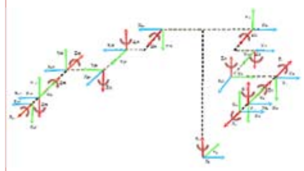
Table 10 Comparison of results for different kinematics structures

Type of multibody system	CNC machine	Mitsubishi RV-3SDB	Motoman DA-20	CNC-R GKM
Robot picture				

Frame assignment







DH parameters

CNC Machine DH Parameters				
Joint	D	Theta	A	Alpha
1	4715.22	90	0	90
2	2714.59	90	0	90
3	0	0	0	0
4	-2664.68	180	0	-90
5	0	0	0	90
6	-281.25	0	0	0

Mitsubishi MELFA RV-3SDB DH Parameters					
Joint	d	θ	a	α	Mechanical Limit
1	350	0	95	-90	±170
2	0	-90	245	0	+135,-90
3	0	±180	-135	90	+171,-20
4	270	0	0	-90	±160
5	0	0	0	90	±120
6	85	0	0	0	±360

Motoman DA20 DH Parameters								
Joint	d	θL	θR	a	αL	αR	Mechanical Limit L	Mechanical Limit R
b	965	90	-90	295	90	-90	±180	
1	-136.5	0		135	-90	90	+80, -100	+100, -80
2	0	0		270	0		+220, -40	
3	0	0		190	90	-90	+215, -35	
4	250	0		0	-90	90	±180	
5	0	0		0	90	-90	±120	
6	85	0		0	0		±180	

CNC-R GKM DH reconfigurable parameters				
i	θ_i	d_i	a_i	α_i
1	$R_1\theta_1 + T_1\theta_{DH1}$	$R_1 d_{DH1} + T_1 d_1$	a_1	±90°
2	$R_2\theta_2 + T_2\theta_{DH2}$	$R_2 d_{DH2} + T_2 d_2$	a_2	±180°,±90°,0
3	$R_3\theta_3 + T_3\theta_{DH3}$	$R_3 d_{DH3} + T_3 d_3$	a_3	±180°,±90°,0
4	θ_{DH4}	d_{DH4}	a_4	±90°
5	θ_{DH5}	d_{DH5}	a_5	±90°
6	θ_{DH6}	d_{DH6}	a_6	±90°,0

Jacobian matrix

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline 0 & J_{22} \end{array}\right]$$

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline J_{12} & J_{22} \end{array}\right]$$

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline J_{12} & J_{22} \end{array}\right]$$

$$J_V = JR_i + JT_i$$

Singularity conditions

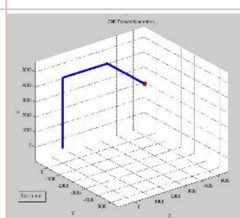
$$(Determinant(J_{11})) = \sin(\theta_5)\cos(\theta_4)\, d_2 d_3 (\cos(\theta_5)\, d_6 + d_4)$$
$$(Determinant(J_{22})) = -\sin(\theta_5)$$
$$\theta_5 = 0^\circ$$
$$\theta_4 = 90^\circ$$

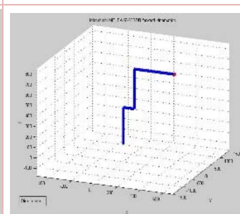
$$C_b = -a_3 \sin\theta_3 + d_4 \cos\theta_3 = 0$$
$$C_i = a_1 + a_2\cos\theta_2 + a_3\cos(\theta_2+\theta_3) + d_4\sin(\theta_2+\theta_3) = 0$$
$$\theta_3 = a\tan 2\left(\frac{d_4}{-a_3}\right)$$
$$\theta_2 = a\tan 2\left(\frac{a_1 - a_2 - a_3\cos\theta_3 - d_4\sin\theta_3}{a_3\sin\theta_3 - d_4\cos\theta_3}\right)$$
$$\theta_3 = 114.59^\circ$$
$$\theta_2 = 117.56^\circ \text{ or } -83.206$$

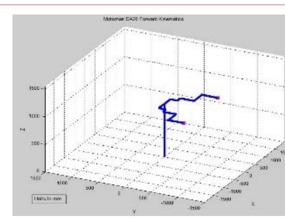
$$C_b = -a_3 \sin\theta_3 - d_4 \cos\theta_3 = 0$$
$$C_i = a_1 + a_2\cos\theta_2 + a_3\cos(\theta_2+\theta_3) - d_4\sin(\theta_2+\theta_3) = 0$$
$$\theta_3 = -a\tan 2\left(\frac{d_4}{a_3}\right)$$
$$\theta_2 = a\tan 2\left(\frac{a_1 - a_2 - a_3\cos\theta_3 + d_4\sin\theta_3}{a_3\sin\theta_3 + d_4\cos\theta_3}\right)$$
$$\theta_3 = 30.53^\circ$$
$$\theta_2 = 158.78^\circ$$

Future work

Forward kinematics validation

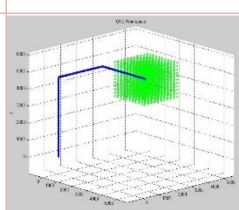


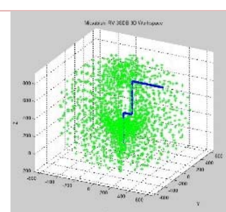


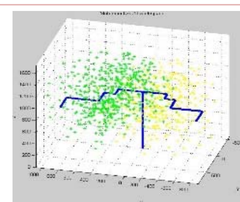


Future work

Workspace



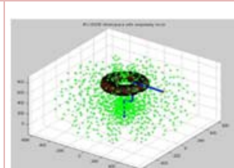


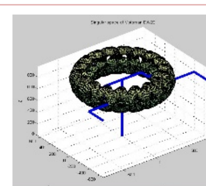


Future work

Singularity locus

N/A





Future work

The direction for continuing this research is getting the functional working envelope, also called workspace envelope, where all singular planes in the workspace are considered, boundary and interior voids. This formulation is presented by Abdel-Malek et al., 1997 and gives formulation for determining the exact workspace of serial manipulators. Every point in the workspace is obtained as a vector function with joint limits considered. In order to visualize the workspace, it is necessary to identify the boundary of this volume by computing the varieties associated with the rank-deficiency condition of the Jacobian. The obtained space is collision free work envelope (Figure 54).

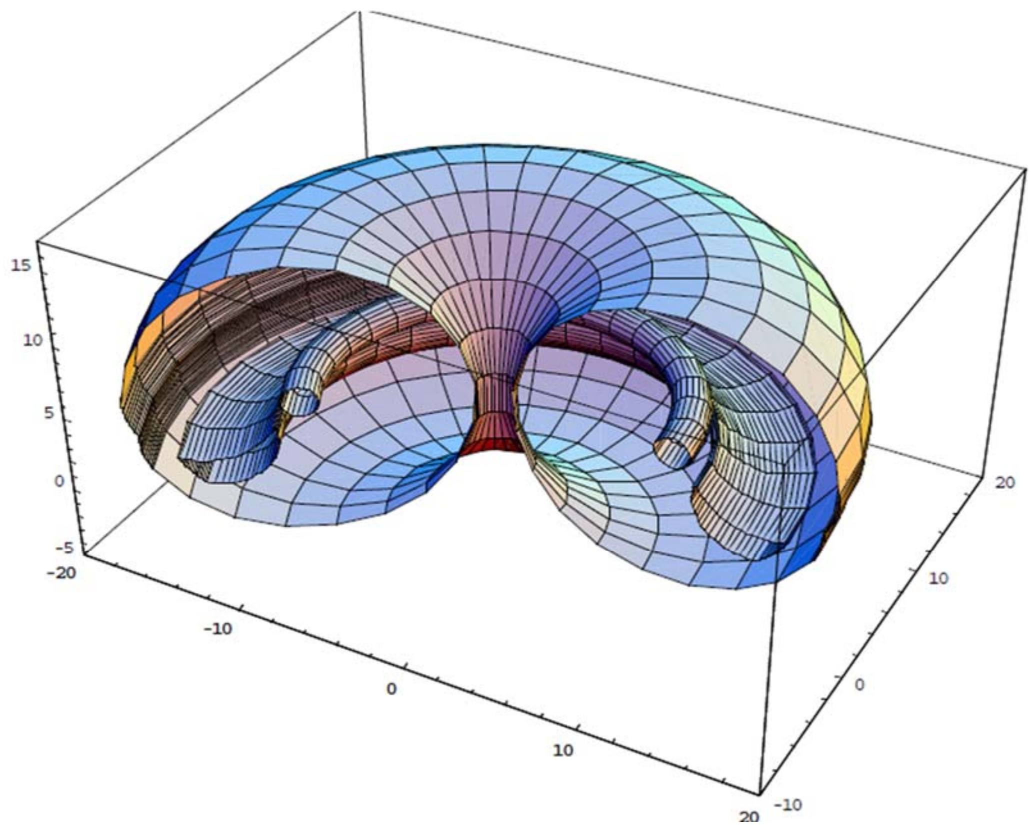


Figure 54 Section view of work envelope of 4DOF manipulator (source: Abdel-Malek, 2000)

The next goal is to merge this work envelope into the Zacharias et al., 2013 work (Figure 55). Having the work envelope with velocity level for each point in the workspace, is something that will give new outlook in the design and use of multibody systems and will open the gates into reducing the kinematics problem of future complex systems. Having a complete kinematics workspace of any system, makes easier to calculate the dynamics of the mutibody systems and the dynamics of the actuators which are used for control purposes. The kinematic and dynamic models can be used as a design tool for multisystem robotic cells and the whole manufacturing systems.

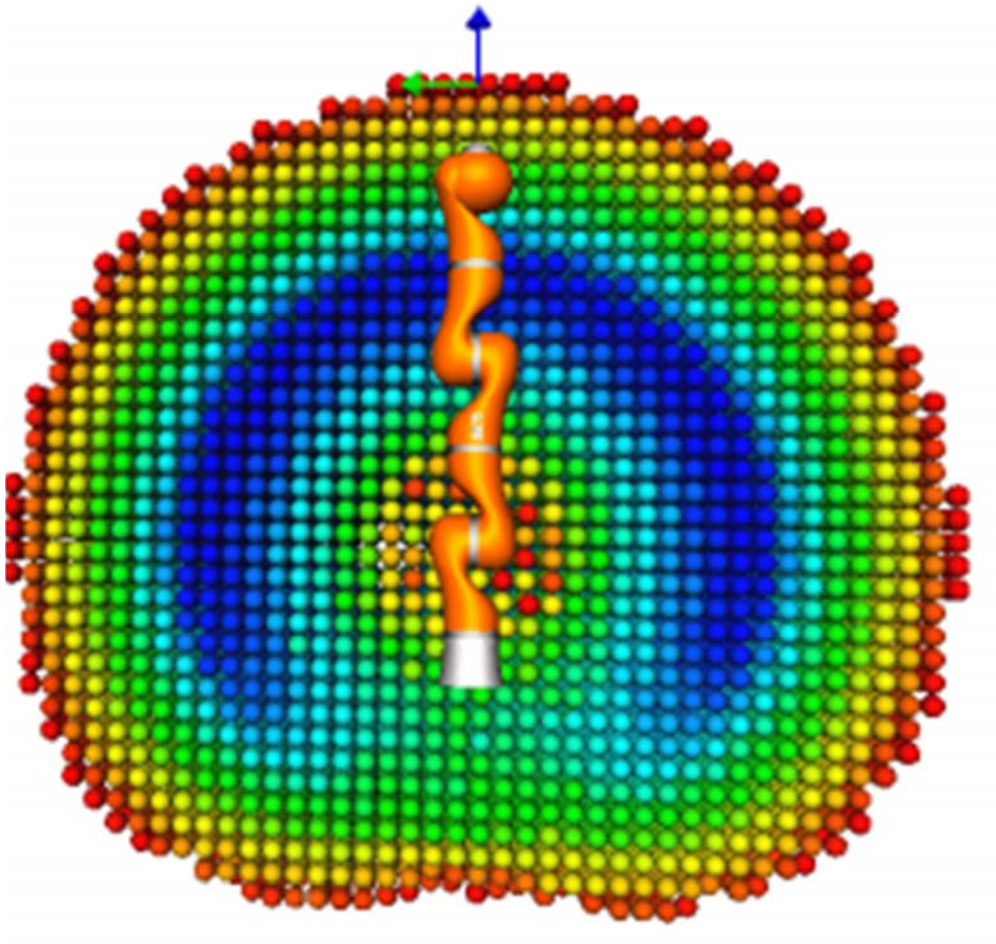


Figure 55 Capability map of KUKA LRW (source: Zacharias, 2008)

CHAPTER 5

REMARKS AND CONCLUSIONS

5.1. Remarks

This research has presented a design tool for kinematics of any multibody system. Its scope is truly important because of the possibility to combine any CNC-machine and any robot manipulator in one model. Therefore, it is also appropriate for dual or collaborative machines with at least 12 controllable axes.

Symbolical and numerical solutions on kinematics, Jacobian matrix, singularities and workspace are successfully obtained for three types of multibody systems:

- General CNC machine
- Mitsubishi MELFA RV-3SDB robot
- Yaskawa Motoman DA-20 dual arm collaborative robot

CNC-R Global Reconfigurable Kinematic Model is developed for analyses of different types of manipulators.

The mathematical modelling results, extended to manipulator's architecture for effective workspace evaluation with singularity locus graphical representation represents a benchmark for future innovative design of machines ideally suited for a wide variety of tasks. This work can easily be extended to dynamic and control for future collaborative robotic cells.

The importance of the Jacobian matrix is pointed out several times in this thesis, because of the possibility to get and visualize the real velocity levels in every point within the workspace of a manipulator.

5.2. Conclusions

This design tool for kinematics of multibody systems is suitable for innovative design for reconfigurable machines preferably suited for a wide variety of tasks. It represents a unified approach, constituted of iterative methods. The emphasis is on the significance of DH parameters for designing and building multibody system for industry. The solutions can be applied to any industrial robot, CNC, CMM or rapid prototyping machine, for calculating singular conditions, consequently optimizing the same, and granting sustainable solution for the machinery control.

The main purpose of the design tool for kinematics of multibody systems is to help in kinematics problem solving, by providing visual representation of the workspace with the singularity locus of the same. The kinematic problem solutions are managed in an uncommon notion, through design methodology and represented with function modelling language, IDEF0. This novel approach specifies step by step activities on how to model robotic system with math and programming tools, like Maple 17 and Matlab 2010.

It can be used by designers, for designing of new robotic systems and by users, for selection appropriate systems for their needs. It can also be used in path generation for providing a singularity-free path. It provides full kinematics problem solutions and singularity conditions.

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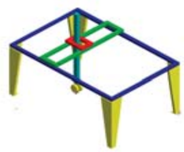
Appendix A
Maple program for CNC Machine FK, Jacobian Matrix and Singularity Conditions

CNC Machine DH Parameters				
Joint	D	Theta	A	Alpha
1	4715.22	90	0	90
2	2714.59	90	0	90
3	0	0	0	0
4	-2664.68	180	0	-90
5	0	0	0	90
6	-281.25	0	0	0



$with(LinearAlgebra):$

#D-H parameters

$\#d_1 := 4715.22$
$\#d_2 := 2714.59$
$d_3 := 0$
$\#d_4 := -2664.68$
$d_5 := 0$
$\#d_6 := -281.25$

$\theta_1 := \frac{\pi}{2} = \frac{1}{2}\pi$

$\theta_2 := \frac{\pi}{2}\ \frac{1}{2}\pi$

$\theta_3 := 0 = 0$
$\#\theta_4 := 0$
$\#\theta_5 := \pi$
$\#\theta_6 := 0$

$a_1 := 0 = 0$
$a_2 := 0 = 0$
$a_3 := 0 = 0$
$a_4 := 0 = 0$
$a_5 := 0 = 0$
$a_6 := 0 = 0$

$\alpha_1 := \frac{\pi}{2}$

$\alpha_2 := \frac{\pi}{2} = \frac{1}{2}\pi$

$\alpha_3 := 0 = 0$

$\alpha_4 := -\frac{\pi}{2} = -\frac{1}{2}\pi$

$\alpha_5 := \frac{\pi}{2} = \frac{1}{2}\pi$

$\alpha_6 := 0 = 0$

#A Matrices

#Homogenious Transformation Matrix A_{01}:

$$A_{01} := \begin{bmatrix} \cos(\theta_1) & -\cos(\alpha_1)\cdot\sin(\theta_1) & \sin(\alpha_1)\cdot\sin(\theta_1) & a_1\cdot\cos(\theta_1) \\ \sin(\theta_1) & \cos(\alpha_1)\cdot\cos(\theta_1) & -\sin(\alpha_1)\cdot\cos(\theta_1) & a_1\cdot\sin(\theta_1) \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{01}

$$R_{01} := \begin{bmatrix} A_{01}[1,1] & A_{01}[1,2] & A_{01}[1,3] \\ A_{01}[2,1] & A_{01}[2,2] & A_{01}[2,3] \\ A_{01}[3,1] & A_{01}[3,2] & A_{01}[3,3] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

#Position Matrix P_{01} :

$$P_{01} := \begin{bmatrix} A_{01}[1,4] \\ A_{01}[2,4] \\ A_{01}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{12}:

$$A_{12} := \begin{bmatrix} \cos(\theta_2) & -\cos(\alpha_2)\cdot\sin(\theta_2) & \sin(\alpha_2)\cdot\sin(\theta_2) & a_2\cdot\cos(\theta_2) \\ \sin(\theta_2) & \cos(\alpha_2)\cdot\cos(\theta_2) & -\sin(\alpha_2)\cdot\cos(\theta_2) & a_2\cdot\sin(\theta_2) \\ 0 & \sin(\alpha_2) & \cos(\alpha_2) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_{02} := Multiply(A_{01}, A_{12}) =$

$$\begin{bmatrix} 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{12}

$$R_{12} := \begin{bmatrix} A_{12}[1,1] & A_{12}[1,2] & A_{12}[1,3] \\ A_{12}[2,1] & A_{12}[2,2] & A_{12}[2,3] \\ A_{12}[3,1] & A_{12}[3,2] & A_{12}[3,3] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

#Rotation Matrix R_{02}

$$R_{02} := Multiply(R_{01}, R_{12}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

#Position Matrix P_{12} :

$$P_{12} := \begin{bmatrix} A_{12}[1,4] \\ A_{12}[2,4] \\ A_{12}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix}$$

#Position Matrix P_{02}

$$P_{02} := \begin{bmatrix} A_{02}[1,4] \\ A_{02}[2,4] \\ A_{02}[3,4] \end{bmatrix} = \begin{bmatrix} d_2 \\ 0 \\ d_1 \end{bmatrix}$$

$$P_{02} := Multiply(R_{01}, P_{12}) + P_{01} = \begin{bmatrix} d_2 \\ 0 \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{23} :

$$A_{23} := \begin{bmatrix} \cos(\theta_3) & -\cos(\alpha_3)\cdot\sin(\theta_3) & \sin(\alpha_3)\cdot\sin(\theta_3) & a_3\cdot\cos(\theta_3) \\ \sin(\theta_3) & \cos(\alpha_3)\cdot\cos(\theta_3) & -\sin(\alpha_3)\cdot\cos(\theta_3) & a_3\cdot\sin(\theta_3) \\ 0 & \sin(\alpha_3) & \cos(\alpha_3) & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{03} := Multiply(A_{02}, A_{23}) = \begin{bmatrix} 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{23}

$$R_{23} := \begin{bmatrix} A_{23}[1,1] & A_{23}[1,2] & A_{23}[1,3] \\ A_{23}[2,1] & A_{23}[2,2] & A_{23}[2,3] \\ A_{23}[3,1] & A_{23}[3,2] & A_{23}[3,3] \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{03}

$$R_{03} := simplify(Multiply(R_{02}, R_{23})) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

#Position Matrix P_{23}

$$P_{23} := \begin{bmatrix} A_{23}[1,4] \\ A_{23}[2,4] \\ A_{23}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#Position Matrix P_{03}

$$P_{03} := \begin{bmatrix} A_{03}[1,4] \\ A_{03}[2,4] \\ A_{03}[3,4] \end{bmatrix} = \begin{bmatrix} d_2 \\ 0 \\ d_1 \end{bmatrix}$$

$$P_{03} := Multiply(R_{01}, (Multiply(R_{12}, P_{23})) + P_{12}) + P_{01} = \begin{bmatrix} d_2 \\ 0 \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{34} :

$$A_{34} := \begin{bmatrix} \cos(\theta_4) & -\cos(\alpha_4)\cdot\sin(\theta_4) & \sin(\alpha_4)\cdot\sin(\theta_4) & a_4\cdot\cos(\theta_4) \\ \sin(\theta_4) & \cos(\alpha_4)\cdot\cos(\theta_4) & -\sin(\alpha_4)\cdot\cos(\theta_4) & a_4\cdot\sin(\theta_4) \\ 0 & \sin(\alpha_4) & \cos(\alpha_4) & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{04} := Multiply(A_{03}, A_{34}) = \begin{bmatrix} \sin(\theta_4) & 0 & \cos(\theta_4) & d_2 \\ 0 & -1 & 0 & d_4 \\ \cos(\theta_4) & 0 & -\sin(\theta_4) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{34}

$$R_{34} := \begin{bmatrix} A_{34}[1,1] & A_{34}[1,2] & A_{34}[1,3] \\ A_{34}[2,1] & A_{34}[2,2] & A_{34}[2,3] \\ A_{34}[3,1] & A_{34}[3,2] & A_{34}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) \\ \sin(\theta_4) & 0 & \cos(\theta_4) \\ 0 & -1 & 0 \end{bmatrix}$$

#Rotation Matrix R_{04}

$$R_{04} := simplify(Multiply(R_{03}, R_{34})) = \begin{bmatrix} \sin(\theta_4) & 0 & \cos(\theta_4) \\ 0 & -1 & 0 \\ \cos(\theta_4) & 0 & -\sin(\theta_4) \end{bmatrix}$$

#Position Matrix P_{34}

$$P_{34} := \begin{bmatrix} A_{34}[1,4] \\ A_{34}[2,4] \\ A_{34}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_4 \end{bmatrix}$$

#Position Matrix P_{04}

$$P_{04} := \begin{bmatrix} A_{04}[1,4] \\ A_{04}[2,4] \\ A_{04}[3,4] \end{bmatrix} = \begin{bmatrix} d_2 \\ d_4 \\ d_1 \end{bmatrix}$$

$$P_{04} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, P_{34})) + P_{23}) + P_{12})) + P_{01} = \begin{bmatrix} d_2 \\ d_4 \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{45} :

$$A_{45} := \begin{bmatrix} \cos(\theta_5) & -\cos(\alpha_5)\cdot\sin(\theta_5) & \sin(\alpha_5)\cdot\sin(\theta_5) & a_5\cdot\cos(\theta_5) \\ \sin(\theta_5) & \cos(\alpha_5)\cdot\cos(\theta_5) & -\sin(\alpha_5)\cdot\cos(\theta_5) & a_5\cdot\sin(\theta_5) \\ 0 & \sin(\alpha_5) & \cos(\alpha_5) & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{05} := Multiply(A_{04}, A_{45}) = \begin{bmatrix} \sin(\theta_4)\cos(\theta_5) & \cos(\theta_4) & \sin(\theta_4)\sin(\theta_5) & d_2 \\ -\sin(\theta_5) & 0 & \cos(\theta_5) & d_4 \\ \cos(\theta_4)\cos(\theta_5) & -\sin(\theta_4) & \cos(\theta_4)\sin(\theta_5) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{45}

$$R_{45} := \begin{bmatrix} A_{45}[1,1] & A_{45}[1,2] & A_{45}[1,3] \\ A_{45}[2,1] & A_{45}[2,2] & A_{45}[2,3] \\ A_{45}[3,1] & A_{45}[3,2] & A_{45}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) \\ \sin(\theta_5) & 0 & -\cos(\theta_5) \\ 0 & 1 & 0 \end{bmatrix}$$

#Rotation Matrix R_{05}

$$R_{05} := simplify(Multiply(R_{04}, R_{45})) = \begin{bmatrix} \sin(\theta_4)\cos(\theta_5) & \cos(\theta_4) & \sin(\theta_4)\sin(\theta_5) \\ -\sin(\theta_5) & 0 & \cos(\theta_5) \\ \cos(\theta_4)\cos(\theta_5) & -\sin(\theta_4) & \cos(\theta_4)\sin(\theta_5) \end{bmatrix}$$

#Position Matrix P₄₅

$$P_{45} := \begin{bmatrix} A_{45}[1,4] \\ A_{45}[2,4] \\ A_{45}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#Position Matrix P_{05}

$$P_{05} := \begin{bmatrix} A_{05}[1,4] \\ A_{05}[2,4] \\ A_{05}[3,4] \end{bmatrix} = \begin{bmatrix} d_2 \\ d_4 \\ d_1 \end{bmatrix}$$

$$P_{05} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, (Multiply(R_{34}, P_{45})) + P_{34})) + P_{23})) + P_{12})) + P_{01}$$

$$= \begin{bmatrix} d_2 \\ d_4 \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{56} :

$$A_{56} := \begin{bmatrix} \cos(\theta_6) & -\cos(\alpha_6)\cdot\sin(\theta_6) & \sin(\alpha_6)\cdot\sin(\theta_6) & a_6\cdot\cos(\theta_6) \\ \sin(\theta_6) & \cos(\alpha_6)\cdot\cos(\theta_6) & -\sin(\alpha_6)\cdot\cos(\theta_6) & a_6\cdot\sin(\theta_6) \\ 0 & \sin(\alpha_6) & \cos(\alpha_6) & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#FORWARD KINEMATICS

$A_{06} := Multiply(A_{05}, A_{56}) =$

$[[\sin(\theta_4)\cos(\theta_5)\cos(\theta_6) + \cos(\theta_4)\sin(\theta_6), -\sin(\theta_4)\cos(\theta_5)\sin(\theta_6) + \cos(\theta_4)\cos(\theta_6),$

$\sin(\theta_4)\sin(\theta_5), \sin(\theta_4)\sin(\theta_5)\,d_6 + d_2],$
$[-\sin(\theta_5)\cos(\theta_6), \sin(\theta_5)\sin(\theta_6), \cos(\theta_5), \cos(\theta_5)\,d_6 + d_4],$
$[\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_4)\sin(\theta_6), -\cos(\theta_4)\cos(\theta_5)\sin(\theta_6) - \sin(\theta_4)\cos(\theta_6),$
$\cos(\theta_4)\sin(\theta_5), \cos(\theta_4)\sin(\theta_5)\,d_6 + d_1],$
$[0, 0, 0, 1]]$

#Rotation Matrix R_{56}

$$R_{56} := \begin{bmatrix} A_{56}[1,1] & A_{56}[1,2] & A_{56}[1,3] \\ A_{56}[2,1] & A_{56}[2,2] & A_{56}[2,3] \\ A_{56}[3,1] & A_{56}[3,2] & A_{56}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{06}

$R_{06} := simplify(Multiply(R_{05}, R_{56})) =$
$[[\sin(\theta_4)\cos(\theta_5)\cos(\theta_6) + \cos(\theta_4)\sin(\theta_6), -\sin(\theta_4)\cos(\theta_5)\sin(\theta_6) + \cos(\theta_4)\cos(\theta_6),$
$\sin(\theta_4)\sin(\theta_5)],$
$[-\sin(\theta_5)\cos(\theta_6), \sin(\theta_5)\sin(\theta_6), \cos(\theta_5)],$
$[\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_4)\sin(\theta_6), -\cos(\theta_4)\cos(\theta_5)\sin(\theta_6) - \sin(\theta_4)\cos(\theta_6),$
$\cos(\theta_4)\sin(\theta_5)]]$

#Position Matrix P_{56}

$$P_{56} := \begin{bmatrix} A_{56}[1,4] \\ A_{56}[2,4] \\ A_{56}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$$

#Position Matrix P_{06}

$$P_{06} := \begin{bmatrix} A_{06}[1,4] \\ A_{06}[2,4] \\ A_{06}[3,4] \end{bmatrix} = \begin{bmatrix} \sin(\theta_4)\sin(\theta_5)\,d_6 + d_2 \\ \cos(\theta_5)\,d_6 + d_4 \\ \cos(\theta_4)\sin(\theta_5)\,d_6 + d_1 \end{bmatrix}$$

$$P_{06} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, (Multiply(R_{34}, (Multiply(R_{45}, P_{56})) + P_{45})) + P_{34})) + P_{23})) + P_{12})) + P_{01}$$

$$= \begin{bmatrix} \sin(\theta_4)\sin(\theta_5)\, d_6 + d_2 \\ \cos(\theta_5)\, d_6 + d_4 \\ \cos(\theta_4)\sin(\theta_5)\, d_6 + d_1 \end{bmatrix}$$

#Calculting J_B by using Vector cross-multiplication method :

 # Zero Vectors

$$Z_0 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 := Multiply(R_{01}, Z_0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$Z_2 := Multiply(R_{02}, Z_0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$Z_3 := Multiply(R_{03}, Z_0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$Z_4 := Multiply(R_{04}, Z_0) = \begin{bmatrix} \cos(\theta_4) \\ 0 \\ -\sin(\theta_4) \end{bmatrix}$$

$$Z_5 := Multiply(R_{05}, Z_0) = \begin{bmatrix} \sin(\theta_4)\sin(\theta_5) \\ \cos(\theta_5) \\ \cos(\theta_4)\sin(\theta_5) \end{bmatrix}$$

#Calculating Jacobian matrix:

$$J_{B1} := simplify(CrossProduct(Z_0, P_{06})) = \begin{bmatrix} -\cos(\theta_5)\, d_6 - d_4 \\ \sin(\theta_4)\sin(\theta_5)\, d_6 + d_2 \\ 0 \end{bmatrix}$$

$$J_{B2} := CrossProduct(Z_1, (P_{06} - P_{01})) = \begin{bmatrix} 0 \\ -\cos(\theta_4)\sin(\theta_5)\, d_6 \\ \cos(\theta_5)\, d_6 + d_4 \end{bmatrix}$$

$$J_{B3} := simplify(CrossProduct(Z_2, (P_{06} - P_{02}))) = \begin{bmatrix} \cos(\theta_4)\sin(\theta_5)\, d_6 \\ 0 \\ -\sin(\theta_4)\sin(\theta_5)\, d_6 \end{bmatrix}$$

$$J_{B4} := simplify(CrossProduct(Z_3, (P_{06} - P_{03}))) = \begin{bmatrix} \cos(\theta_4)\sin(\theta_5)\, d_6 \\ 0 \\ -\sin(\theta_4)\sin(\theta_5)\, d_6 \end{bmatrix}$$

$$J_{B5} := simplify(CrossProduct(Z_4, (P_{06} - P_{04}))) = \begin{bmatrix} \sin(\theta_4)\cos(\theta_5)\, d_6 \\ -\sin(\theta_5)\, d_6 \\ \cos(\theta_4)\cos(\theta_5)\, d_6 \end{bmatrix}$$

$$J_{B6} := simplify(CrossProduct(Z_5, (P_{06} - P_{05}))) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#Assembling Jacobian matrix

$$J_B := \begin{bmatrix} J_{B1}[1] & J_{B2}[1] & J_{B3}[1] & 0 & 0 & 0 \\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] & 0 & 0 & 0 \\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_3[1] & Z_4[1] & Z_5[1] \\ 0 & 0 & 0 & Z_3[2] & Z_4[2] & Z_5[2] \\ 0 & 0 & 0 & Z_3[3] & Z_4[3] & Z_5[3] \end{bmatrix} =$$

$$\begin{aligned}&[[-\cos(\theta_5)\, d_6 - d_4, 0, \cos(\theta_4)\, \sin(\theta_5)\, d_6, 0, 0, 0],\\ &[\sin(\theta_4)\, \sin(\theta_5)\, d_6 + d_2, -\cos(\theta_4)\, \sin(\theta_5)\, d_6, 0, 0, 0, 0],\\ &[0, \cos(\theta_5)\, d_6 + d_4, -\sin(\theta_4)\, \sin(\theta_5)\, d_6, 0, 0, 0],\\ &[0, 0, 0, 0, \cos(\theta_4), \sin(\theta_4)\, \sin(\theta_5)],\\ &[0, 0, 0, 1, 0, \cos(\theta_5)],\\ &[0, 0, 0, 0, -\sin(\theta_4), \cos(\theta_4)\, \sin(\theta_5)]]\end{aligned}$$

#Jacobian matrix decoupling

$$\#J := \begin{bmatrix} J_{11} & J_{21} \\ J_{12} & J_{22} \end{bmatrix}$$

$$J_{11} := \begin{bmatrix} J_{B1}[1] & J_{B2}[1] & J_{B3}[1] \\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] \\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] \end{bmatrix} =$$

$$\begin{bmatrix} -\cos(\theta_5)\, d_6 - d_4 & 0 & \cos(\theta_4)\, \sin(\theta_5)\, d_6 \\ \sin(\theta_4)\, \sin(\theta_5)\, d_6 + d_2 & -\cos(\theta_4)\, \sin(\theta_5)\, d_6 & 0 \\ 0 & \cos(\theta_5)\, d_6 + d_4 & -\sin(\theta_4)\, \sin(\theta_5)\, d_6 \end{bmatrix}$$

$$simplify(Determinant(J_{11})) = \cos(\theta_4)\, \sin(\theta_5)\, d_2\, d_6\, (\cos(\theta_5)\, d_6 + d_4)$$

$$J_{22} := \begin{bmatrix} Z_3[1] & Z_4[1] & Z_5[1] \\ Z_3[2] & Z_4[2] & Z_5[2] \\ Z_3[3] & Z_4[3] & Z_5[3] \end{bmatrix} = \begin{bmatrix} 0 & \cos(\theta_4) & \sin(\theta_4)\, \sin(\theta_5) \\ 1 & 0 & \cos(\theta_5) \\ 0 & -\sin(\theta_4) & \cos(\theta_4)\, \sin(\theta_5) \end{bmatrix}$$

$$simplify(Determinant(J_{22})) = -\sin(\theta_5)$$

#Determining singularities (Manipulability measures)

$$simplify(Determinant(J_B)) = -\cos(\theta_4)\, \sin(\theta_5)^2\, (\cos(\theta_5)\, d_6 + d_4)\, d_2\, d_6 \xrightarrow{\text{equate to 0}}$$

$$-\cos(\theta_4)\, \sin(\theta_5)^2\, (\cos(\theta_5)\, d_6 + d_4)\, d_2\, d_6 = 0 \xrightarrow{\text{solve for theta[4]}} \left[\left[\theta_4 = \frac{1}{2}\pi\right]\right] \xrightarrow{\text{solve for theta[5]}} \left[\left[\theta_5 = \pi - \arccos\left(\frac{d_4}{d_6}\right)\right], [\theta_5 = 0]\right]$$

$$\Theta_4 := \frac{1.570796327 \cdot 180}{\pi} = \frac{282.7433389}{\pi} \xrightarrow{\text{at 5 digits}} 89.999$$

Appendix B
Maple program for Mitsubishi MELFA RV-3 SDB FK, Jacobian Matrix and Singularity Conditions

Mitsubishi MELFA RV-3SDB DH Parameters					
Joint	d	θ	a	α	Mechanical Limit
1	350	0	95	-90	±170
2	0	-90	245	0	+135,-90
3	0	±180	-135	90	+171,-20
4	270	0	0	-90	±160
5	0	0	0	90	±120
6	85	0	0	0	±360

with(*LinearAlgebra*) :

#D-H parameters for Mitsubishi RV-3SDB

$\#d_1 := 350$
$d_2 := 0 = 0$
$d_3 := 0 = 0$
$\#d_4 := 270$
$d_5 := 0 = 0$
$\#d_6 := 85$

$\#\theta_1 := 0$

$\#\theta_2 := -\frac{\pi}{2}$
$\#\theta_3 := \pi$
$\#\theta_4 := 0$
$\#\theta_5 := 0$
$\#\theta_6 := 0$

$\#a_1 := 95$
$\#a_2 := 245$
$\#a_3 := -135$
$a_4 := 0 = 0$
$a_5 := 0 = 0$
$a_6 := 0 = 0$

$\alpha_1 := -\frac{\pi}{2} = -\frac{1}{2}\pi$
$\alpha_2 := 0 = 0$

$\alpha_3 := \frac{\pi}{2} = \frac{1}{2}\pi$
$\alpha_4 := -\frac{\pi}{2} = -\frac{1}{2}\pi$
$\alpha_5 := \frac{\pi}{2} = \frac{1}{2}\pi$
$\alpha_6 := 0 = 0$

#A Matrices

#Homogenious Transformation Matrix A_{01} **:**

$$A_{01} := \begin{bmatrix} \cos(\theta_1) & -\cos(\alpha_1)\cdot\sin(\theta_1) & \sin(\alpha_1)\cdot\sin(\theta_1) & a_1\cdot\cos(\theta_1) \\ \sin(\theta_1) & \cos(\alpha_1)\cdot\cos(\theta_1) & -\sin(\alpha_1)\cdot\cos(\theta_1) & a_1\cdot\sin(\theta_1) \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & a_1\cos(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & a_1\sin(\theta_1) \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{01}

$$R_{01} := \begin{bmatrix} A_{01}[1,1] & A_{01}[1,2] & A_{01}[1,3] \\ A_{01}[2,1] & A_{01}[2,2] & A_{01}[2,3] \\ A_{01}[3,1] & A_{01}[3,2] & A_{01}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) \\ 0 & -1 & 0 \end{bmatrix}$$

#Position Matrix P_{01} :

$$P_{01} := \begin{bmatrix} A_{01}[1,4] \\ A_{01}[2,4] \\ A_{01}[3,4] \end{bmatrix} = \begin{bmatrix} a_1\cos(\theta_1) \\ a_1\sin(\theta_1) \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{12} **:**

$$A_{12} := \begin{bmatrix} \cos(\theta_2) & -\cos(\alpha_2)\cdot\sin(\theta_2) & \sin(\alpha_2)\cdot\sin(\theta_2) & a_2\cdot\cos(\theta_2) \\ \sin(\theta_2) & \cos(\alpha_2)\cdot\cos(\theta_2) & -\sin(\alpha_2)\cdot\cos(\theta_2) & a_2\cdot\sin(\theta_2) \\ 0 & \sin(\alpha_2) & \cos(\alpha_2) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{02} := Multiply(A_{01}, A_{12}) =$$

$$\begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) & \cos(\theta_1)\, a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & \sin(\theta_1)\, a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & -a_2\sin(\theta_2) + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{12}

$$R_{12} := \begin{bmatrix} A_{12}[1,1] & A_{12}[1,2] & A_{12}[1,3] \\ A_{12}[2,1] & A_{12}[2,2] & A_{12}[2,3] \\ A_{12}[3,1] & A_{12}[3,2] & A_{12}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{02}

$$R_{02} := Multiply(R_{01}, R_{12}) = \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 \end{bmatrix}$$

#Position Matrix P_{12} :

$$P_{12} := \begin{bmatrix} A_{12}[1,4] \\ A_{12}[2,4] \\ A_{12}[3,4] \end{bmatrix} = \begin{bmatrix} a_2\cos(\theta_2) \\ a_2\sin(\theta_2) \\ 0 \end{bmatrix}$$

#Position Matrix P_{02}

$$P_{02} := \begin{bmatrix} A_{02}[1,4] \\ A_{02}[2,4] \\ A_{02}[3,4] \end{bmatrix} = \begin{bmatrix} \cos(\theta_1)\, a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\, a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ -a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

$$P_{02} := Multiply(R_{01}, P_{12}) + P_{01} = \begin{bmatrix} \cos(\theta_1)\, a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\, a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ -a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{23} :

$$A_{23} := \begin{bmatrix} \cos(\theta_3) & -\cos(\alpha_3)\cdot\sin(\theta_3) & \sin(\alpha_3)\cdot\sin(\theta_3) & a_3\cdot\cos(\theta_3) \\ \sin(\theta_3) & \cos(\alpha_3)\cdot\cos(\theta_3) & -\sin(\alpha_3)\cdot\cos(\theta_3) & a_3\cdot\sin(\theta_3) \\ 0 & \sin(\alpha_3) & \cos(\alpha_3) & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & a_3\cos(\theta_3) \\ \sin(\theta_3) & 0 & -\cos(\theta_3) & a_3\sin(\theta_3) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{03} := Multiply(A_{02}, A_{23}) =$$

$$\begin{aligned} &[[\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3), -\sin(\theta_1), \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) \\ &\quad + \cos(\theta_1)\sin(\theta_2)\cos(\theta_3), \cos(\theta_1)\cos(\theta_2)\, a_3\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\, a_3\sin(\theta_3) \\ &\quad + \cos(\theta_1)\, a_2\cos(\theta_2) + a_1\cos(\theta_1)], \\ &[\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3), \cos(\theta_1), \sin(\theta_1)\cos(\theta_2)\sin(\theta_3) \\ &\quad + \sin(\theta_1)\sin(\theta_2)\cos(\theta_3), \sin(\theta_1)\cos(\theta_2)\, a_3\cos(\theta_3) - \sin(\theta_1)\sin(\theta_2)\, a_3\sin(\theta_3) \\ &\quad + \sin(\theta_1)\, a_2\cos(\theta_2) + a_1\sin(\theta_1)], \\ &[-\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3), 0, -\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3), \\ &-\sin(\theta_2)\, a_3\cos(\theta_3) - \cos(\theta_2)\, a_3\sin(\theta_3) - a_2\sin(\theta_2) + d_1], \\ &[0, 0, 0, 1]] \end{aligned}$$

#Rotation Matrix R_{23}

$$R_{23} := \begin{bmatrix} A_{23}[1,1] & A_{23}[1,2] & A_{23}[1,3] \\ A_{23}[2,1] & A_{23}[2,2] & A_{23}[2,3] \\ A_{23}[3,1] & A_{23}[3,2] & A_{23}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) \\ \sin(\theta_3) & 0 & -\cos(\theta_3) \\ 0 & 1 & 0 \end{bmatrix}$$

#Rotation Matrix $\boldsymbol{R_{03}}$

$R_{03} := simplify(Multiply(R_{02}, R_{23})) =$

$[[-\cos(\theta_1)\,(\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)), -\sin(\theta_1), \cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)$
$+ \cos(\theta_2)\sin(\theta_3))],$
$[-\sin(\theta_1)\,(\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)), \cos(\theta_1), \sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)$
$+ \cos(\theta_2)\sin(\theta_3))],$
$[-\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3), 0, -\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)]]$

#Position Matrix $\boldsymbol{P_{23}}$

$$P_{23} := \begin{bmatrix} A_{23}[1,4] \\ A_{23}[2,4] \\ A_{23}[3,4] \end{bmatrix} = \begin{bmatrix} a_3\cos(\theta_3) \\ a_3\sin(\theta_3) \\ 0 \end{bmatrix}$$

#Position Matrix $\boldsymbol{P_{03}}$

$$P_{03} := \begin{bmatrix} A_{03}[1,4] \\ A_{03}[2,4] \\ A_{03}[3,4] \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) + \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) - \sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) + \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ -\sin(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_2)\,a_3\sin(\theta_3) - a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

$\boldsymbol{P_{03} := Multiply(R_{01}, (Multiply(R_{12}, P_{23})) + P_{12}) + P_{01}} =$

$$\begin{bmatrix} \cos(\theta_1)\,(\cos(\theta_2)\,a_3\cos(\theta_3) - \sin(\theta_2)\,a_3\sin(\theta_3) + a_2\cos(\theta_2)) + a_1\cos(\theta_1) \\ \sin(\theta_1)\,(\cos(\theta_2)\,a_3\cos(\theta_3) - \sin(\theta_2)\,a_3\sin(\theta_3) + a_2\cos(\theta_2)) + a_1\sin(\theta_1) \\ -\sin(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_2)\,a_3\sin(\theta_3) - a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix $\boldsymbol{A_{34}}$ ***:***

$$A_{34} := \begin{bmatrix} \cos(\theta_4) & -\cos(\alpha_4)\cdot\sin(\theta_4) & \sin(\alpha_4)\cdot\sin(\theta_4) & a_4\cdot\cos(\theta_4) \\ \sin(\theta_4) & \cos(\alpha_4)\cdot\cos(\theta_4) & -\sin(\alpha_4)\cdot\cos(\theta_4) & a_4\cdot\sin(\theta_4) \\ 0 & \sin(\alpha_4) & \cos(\alpha_4) & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_{04} := Multiply(A_{03}, A_{34}) =$

$[[(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4) - \sin(\theta_1)\sin(\theta_4),$
$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) - \cos(\theta_1)\sin(\theta_2)\cos(\theta_3), -(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$
$- \cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4) - \sin(\theta_1)\cos(\theta_4), (\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+ \cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4 + \cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$
$+ \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1)],$
$[(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4) + \cos(\theta_1)\sin(\theta_4),$
$-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3) - \sin(\theta_1)\sin(\theta_2)\cos(\theta_3), -(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$
$- \sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4) + \cos(\theta_1)\cos(\theta_4), (\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+ \sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4 + \sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) - \sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$
$+ \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1)],$
$[(-\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\cos(\theta_4), \sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3), -($
$-\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\sin(\theta_4), (-\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3))\,d_4$
$- \sin(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_2)\,a_3\sin(\theta_3) - a_2\sin(\theta_2) + d_1],$
$[0, 0, 0, 1]]$

#Rotation Matrix $\boldsymbol{R_{34}}$

$$R_{34} := \begin{bmatrix} A_{34}[1,1] & A_{34}[1,2] & A_{34}[1,3] \\ A_{34}[2,1] & A_{34}[2,2] & A_{34}[2,3] \\ A_{34}[3,1] & A_{34}[3,2] & A_{34}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) \\ \sin(\theta_4) & 0 & \cos(\theta_4) \\ 0 & -1 & 0 \end{bmatrix}$$

#Rotation Matrix $\boldsymbol{R_{04}}$

$R_{04} := simplify(Multiply(R_{03}, R_{34})) =$

$[[-\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_4),$

$$-\cos(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right),\ \cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)$$
$$-\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4)\Big],$$
$$\Big[-\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_4),$$
$$-\sin(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right),\ \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)$$
$$-\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4)\Big],$$
$$\Big[-\left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right)\cos(\theta_4),\ \sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3),$$
$$\left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right)\sin(\theta_4)\Big]\Big]$$

#Position Matrix P_{34}

$$P_{34} := \begin{bmatrix} A_{34}[1,4] \\ A_{34}[2,4] \\ A_{34}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_4 \end{bmatrix}$$

#Position Matrix P_{04}

$$P_{04} := \begin{bmatrix} A_{04}[1,4] \\ A_{04}[2,4] \\ A_{04}[3,4] \end{bmatrix} =$$

$$\Big[\Big[\left(\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$-\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)\Big],$$
$$\Big[\left(\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$-\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)\Big],$$
$$\Big[\left(-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3)\right)d_4-\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)$$
$$-a_2\sin(\theta_2)+d_1\Big]\Big]$$

$$P_{04} := Multiply\left(R_{01}, \left(Multiply\left(R_{12}, \left(Multiply\left(R_{23}, P_{34}\right)\right) + P_{23}\right) + P_{12}\right)\right) + P_{01} =$$
$$\Big[\Big[\cos(\theta_1)\left(\cos(\theta_2)\left(\sin(\theta_3)\,d_4+a_3\cos(\theta_3)\right)-\sin(\theta_2)\left(-\cos(\theta_3)\,d_4+a_3\sin(\theta_3)\right)\right.$$
$$\left.+a_2\cos(\theta_2)\right)+a_1\cos(\theta_1)\Big],$$
$$\Big[\sin(\theta_1)\left(\cos(\theta_2)\left(\sin(\theta_3)\,d_4+a_3\cos(\theta_3)\right)-\sin(\theta_2)\left(-\cos(\theta_3)\,d_4+a_3\sin(\theta_3)\right)\right.$$
$$\left.+a_2\cos(\theta_2)\right)+a_1\sin(\theta_1)\Big],$$
$$\Big[-\sin(\theta_2)\left(\sin(\theta_3)\,d_4+a_3\cos(\theta_3)\right)-\cos(\theta_2)\left(-\cos(\theta_3)\,d_4+a_3\sin(\theta_3)\right)-a_2\sin(\theta_2)+d_1$$
$$\Big]\Big]$$

#Homogenious Transformation Matrix A_{45} :

$$A_{45} := \begin{bmatrix} \cos(\theta_5) & -\cos(\alpha_5)\cdot\sin(\theta_5) & \sin(\alpha_5)\cdot\sin(\theta_5) & a_5\cdot\cos(\theta_5) \\ \sin(\theta_5) & \cos(\alpha_5)\cdot\cos(\theta_5) & -\sin(\alpha_5)\cdot\cos(\theta_5) & a_5\cdot\sin(\theta_5) \\ 0 & \sin(\alpha_5) & \cos(\alpha_5) & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{05} := Multiply(A_{04}, A_{45}) =$$
$$\Big[\Big[\left(\left(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4)\right)\cos(\theta_5)+\big($$
$$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\big)\sin(\theta_5),\ -\big(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\big)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4),\ \big(\big(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\big)\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4)\big)\sin(\theta_5)-\big(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\big)\cos(\theta_5),\ \left(\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)d_4$$
$$+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)$$
$$\Big],$$
$$\Big[\left(\left(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4)\right)\cos(\theta_5)$$
$$+\left(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)\sin(\theta_5),\ -\big(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\big)\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4),\ \big(\big(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\big)\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4)\big)\sin(\theta_5)-\big(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\big)\cos(\theta_5),\ \left(\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)d_4$$
$$+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)\Big],$$
$$\Big[\left(-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)\cos(\theta_5)+\big(\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3)\big)\sin(\theta_5),\ -\left(-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\sin(\theta_4),\ \big($$
$$-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\big)\cos(\theta_4)\sin(\theta_5)-\big(\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3)\big)\cos(\theta_5),\ \left(-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3)\right)d_4-\sin(\theta_2)\,a_3\cos(\theta_3)$$
$$-\cos(\theta_2)\,a_3\sin(\theta_3)-a_2\sin(\theta_2)+d_1\Big],$$
$$\Big[0, 0, 0, 1\Big]\Big]$$

#Rotation Matrix R_{45}

$$R_{45} := \begin{bmatrix} A_{45}[1,1] & A_{45}[1,2] & A_{45}[1,3] \\ A_{45}[2,1] & A_{45}[2,2] & A_{45}[2,3] \\ A_{45}[3,1] & A_{45}[3,2] & A_{45}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) \\ \sin(\theta_5) & 0 & -\cos(\theta_5) \\ 0 & 1 & 0 \end{bmatrix}$$

#Rotation Matrix R_{05}

$$R_{05} := simplify(Multiply(R_{04}, R_{45})) =$$

$$\begin{aligned}
&[[-\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2) \\
&\quad -\cos(\theta_5)\sin(\theta_1)\sin(\theta_4)-\sin(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3) \\
&\quad -\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3), \cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) \\
&\quad -\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4), \\
&\quad -\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2) \\
&\quad -\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)+\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3) \\
&\quad +\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)], \\
&\quad [-\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2) \\
&\quad +\cos(\theta_5)\cos(\theta_1)\sin(\theta_4)-\sin(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) \\
&\quad -\sin(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3), \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) \\
&\quad -\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4), \\
&\quad -\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2) \\
&\quad +\sin(\theta_5)\cos(\theta_1)\sin(\theta_4)+\cos(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) \\
&\quad +\cos(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)], \\
&\quad [-\cos(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)-\cos(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3) \\
&\quad +\sin(\theta_5)\sin(\theta_2)\sin(\theta_3)-\sin(\theta_5)\cos(\theta_3)\cos(\theta_2), (\sin(\theta_2)\cos(\theta_3) \\
&\quad +\cos(\theta_2)\sin(\theta_3))\sin(\theta_4), -\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3) \\
&\quad -\sin(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_5)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\cos(\theta_3)\cos(\theta_2)]]
\end{aligned}$$

#Position Matrix P_{45}

$$P_{45} := \begin{bmatrix} A_{45}[1,4] \\ A_{45}[2,4] \\ A_{45}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#Position Matrix P_{05}

$$P_{05} := \begin{bmatrix} A_{05}[1,4] \\ A_{05}[2,4] \\ A_{05}[3,4] \end{bmatrix} =$$

$$\begin{aligned}
&[[(\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) \\
&\quad -\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)], \\
&\quad [(\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) \\
&\quad -\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)], \\
&\quad [(-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3))\,d_4-\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3) \\
&\quad -a_2\sin(\theta_2)+d_1]]
\end{aligned}$$

$$P_{05} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, (Multiply(R_{34}, P_{45}))+\mathrm{P}_{34})+P_{23})) + P_{12})) + P_{01}$$

$$=$$

$$\begin{aligned}
&[[\cos(\theta_1)\,(\cos(\theta_2)\,(\sin(\theta_3)\,d_4+a_3\cos(\theta_3))-\sin(\theta_2)\,(-\cos(\theta_3)\,d_4+a_3\sin(\theta_3)) \\
&\quad +a_2\cos(\theta_2))+a_1\cos(\theta_1)], \\
&\quad [\sin(\theta_1)\,(\cos(\theta_2)\,(\sin(\theta_3)\,d_4+a_3\cos(\theta_3))-\sin(\theta_2)\,(-\cos(\theta_3)\,d_4+a_3\sin(\theta_3)) \\
&\quad +a_2\cos(\theta_2))+a_1\sin(\theta_1)], \\
&\quad [-\sin(\theta_2)\,(\sin(\theta_3)\,d_4+a_3\cos(\theta_3))-\cos(\theta_2)\,(-\cos(\theta_3)\,d_4+a_3\sin(\theta_3))-a_2\sin(\theta_2)+d_1 \\
&\quad]]
\end{aligned}$$

#Homogenious Transformation Matrix A_{56}:

$$A_{56} := \begin{bmatrix} \cos(\theta_6) & -\cos(\alpha_6)\cdot\sin(\theta_6) & \sin(\alpha_6)\cdot\sin(\theta_6) & a_6\cdot\cos(\theta_6) \\ \sin(\theta_6) & \cos(\alpha_6)\cdot\cos(\theta_6) & -\sin(\alpha_6)\cdot\cos(\theta_6) & a_6\cdot\sin(\theta_6) \\ 0 & \sin(\alpha_6) & \cos(\alpha_6) & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#FORWARD KINEMATICS

$$A_{06} := Multiply(A_{05}, A_{56}) =$$

$$\begin{aligned}
&[[(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4))\cos(\theta_5) \\
&\quad +(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)+(
\end{aligned}$$

$-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4))\sin(\theta_6),$
$-(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)$
$+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)+($
$-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4))\cos(\theta_6),$
$((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)$
$-(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),$
$(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)$
$-(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6$
$+(\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$
$-\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)],$
$[(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)$
$+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)+($
$-(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4))\sin(\theta_6),$
$-(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)$
$+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)+($
$-(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4))\cos(\theta_6),$
$((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)$
$-(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),$
$(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)$
$-(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6$
$+(\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$
$-\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$
$[((-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(\sin(\theta_2)\sin(\theta_3)$
$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)-(-\sin(\theta_2)\cos(\theta_3)$
$-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\sin(\theta_6),\ -((-\sin(\theta_2)\cos(\theta_3)$
$-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)$
$-(-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\cos(\theta_6),\ (-\sin(\theta_2)\cos(\theta_3)$
$-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)-(\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5),\ (($
$-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)-(\sin(\theta_2)\sin(\theta_3)$
$-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+(-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3))\,d_4$
$-\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)-a_2\sin(\theta_2)+d_1],$
$[0, 0, 0, 1]]$

#Rotation Matrix R_{56}

$$R_{56} := \begin{bmatrix} A_{56}[1,1] & A_{56}[1,2] & A_{56}[1,3] \\ A_{56}[2,1] & A_{56}[2,2] & A_{56}[2,3] \\ A_{56}[3,1] & A_{56}[3,2] & A_{56}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{06}

$R_{06} := simplify(Multiply(R_{05}, R_{56})) =$
$[[-\cos(\theta_6)\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$+\cos(\theta_6)\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)-\cos(\theta_6)\cos(\theta_5)\sin(\theta_1)\sin(\theta_4)$
$-\cos(\theta_6)\sin(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)-\cos(\theta_6)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+\cos(\theta_1)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\sin(\theta_6)-\cos(\theta_1)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\sin(\theta_6)$
$-\sin(\theta_1)\cos(\theta_4)\sin(\theta_6),\ \sin(\theta_6)\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$-\sin(\theta_6)\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)+\sin(\theta_6)\cos(\theta_5)\sin(\theta_1)\sin(\theta_4)$
$+\sin(\theta_6)\sin(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)+\sin(\theta_6)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+\cos(\theta_1)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\cos(\theta_6)-\cos(\theta_1)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\cos(\theta_6)$
$-\sin(\theta_1)\cos(\theta_4)\cos(\theta_6),\ -\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$+\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)-\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)$
$+\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)+\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)],$
$[-\cos(\theta_6)\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$+\cos(\theta_6)\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)+\cos(\theta_6)\cos(\theta_5)\cos(\theta_1)\sin(\theta_4)$
$-\cos(\theta_6)\sin(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)-\cos(\theta_6)\sin(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+\sin(\theta_1)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\sin(\theta_6)-\sin(\theta_1)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\sin(\theta_6)$
$+\cos(\theta_1)\cos(\theta_4)\sin(\theta_6),\ \sin(\theta_6)\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$-\sin(\theta_6)\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)-\sin(\theta_6)\cos(\theta_5)\cos(\theta_1)\sin(\theta_4)$
$+\sin(\theta_6)\sin(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)+\sin(\theta_6)\sin(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+\sin(\theta_1)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\cos(\theta_6)-\sin(\theta_1)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\cos(\theta_6)$
$+\cos(\theta_1)\cos(\theta_4)\cos(\theta_6),\ -\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$+\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)+\sin(\theta_5)\cos(\theta_1)\sin(\theta_4)$
$+\cos(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)+\cos(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)],$
$[-\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6)-\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)\cos(\theta_6)$
$+\sin(\theta_5)\sin(\theta_2)\sin(\theta_3)\cos(\theta_6)-\sin(\theta_5)\cos(\theta_3)\cos(\theta_2)\cos(\theta_6)$
$+\sin(\theta_4)\sin(\theta_2)\cos(\theta_3)\sin(\theta_6)+\sin(\theta_4)\cos(\theta_2)\sin(\theta_3)\sin(\theta_6),$
$\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)\sin(\theta_6)+\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)\sin(\theta_6)$
$-\sin(\theta_5)\sin(\theta_2)\sin(\theta_3)\sin(\theta_6)+\sin(\theta_5)\cos(\theta_3)\cos(\theta_2)\sin(\theta_6)$

$$+\sin(\theta_4)\sin(\theta_2)\cos(\theta_3)\cos(\theta_6)+\sin(\theta_4)\cos(\theta_2)\sin(\theta_3)\cos(\theta_6),$$
$$-\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)-\sin(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_5)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\cos(\theta_3)\cos(\theta_2)]]$$

#Position Matrix P_{56}

$$P_{56} := \begin{bmatrix} A_{56}[1,4] \\ A_{56}[2,4] \\ A_{56}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$$

#Position Matrix P_{06}

$$P_{06} := \begin{bmatrix} A_{06}[1,4] \\ A_{06}[2,4] \\ A_{06}[3,4] \end{bmatrix} =$$

$$[[(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)$$
$$-(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6$$
$$+(\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$-\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)],$$
$$[(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)$$
$$-(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6$$
$$+(\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$-\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$$
$$[((-\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)-(\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+(-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3))\,d_4$$
$$-\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)-a_2\sin(\theta_2)+d_1]]$$

$$\boldsymbol{P_{06} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, (Multiply(R_{34}, (Multiply(R_{45}, P_{56}))+P_{45})}$$
$$\boldsymbol{+P_{34}))+P_{23}))+P_{12}))+P_{01}}$$

$=$

$$[[\cos(\theta_1)\,(\cos(\theta_2)\,(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\,(\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))$$
$$-\sin(\theta_2)\,(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\,(\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6+a_1\cos(\theta_1)],$$
$$[\sin(\theta_1)\,(\cos(\theta_2)\,(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\,(\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))$$
$$-\sin(\theta_2)\,(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\,(\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6+a_1\sin(\theta_1)],$$

$$[-\sin(\theta_2)\,(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\,(\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))$$
$$-\cos(\theta_2)\,(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\,(\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))$$
$$-a_2\sin(\theta_2)+d_1]]$$

#Calculting J_B by using Vector cross-multiplication method :

 # Zero Vectors

$$Z_0 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 := Multiply(R_{01}, Z_0) = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix}$$

$$Z_2 := Multiply(R_{02}, Z_0) = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix}$$

$$Z_3 := Multiply(R_{03}, Z_0) = \begin{bmatrix} \cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)) \\ \sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)) \\ -\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3) \end{bmatrix}$$

$$Z_4 := Multiply(R_{04}, Z_0) =$$
$$\begin{bmatrix} \cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)-\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4) \\ \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4) \\ (\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))\sin(\theta_4) \end{bmatrix}$$

$$Z_5 := Multiply(R_{05}, Z_0) =$$
$$[[-\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)$$
$$-\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)+\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)],$$
$$[-\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)$$

$$
\begin{aligned}
&+\sin(\theta_5)\cos(\theta_1)\sin(\theta_4)+\cos(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\\
&+\cos(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)],\\
&[-\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)-\sin(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)\\
&-\cos(\theta_5)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\cos(\theta_3)\cos(\theta_2)]]
\end{aligned}
$$

#Calculating Jacobian matrix:

$J_{B1} := CrossProduct(Z_0, P_{06}) =$

$$
\begin{aligned}
&[[-\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))\\
&\quad-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))\\
&\quad+a_2\cos(\theta_2))-\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-a_1\sin(\theta_1)],\\
&\quad[\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))\\
&\quad-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))\\
&\quad+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6+a_1\cos(\theta_1)],\\
&\quad[0]]
\end{aligned}
$$

$J_{B2} := CrossProduct(Z_1, (P_{06}-P_{01})) =$

$$
\begin{aligned}
&[[\cos(\theta_1)(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))\\
&\quad-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))\\
&\quad-a_2\sin(\theta_2))],\\
&\quad[\sin(\theta_1)(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))\\
&\quad-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))\\
&\quad-a_2\sin(\theta_2))],\\
&\quad[-\sin(\theta_1)(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&\quad+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&\quad+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6)\\
&\quad-\cos(\theta_1)(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&\quad+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&\quad+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6)]]
\end{aligned}
$$

$J_{B3} := CrossProduct(Z_2, (P_{06}-P_{02})) =$

$$
\begin{aligned}
&[[\cos(\theta_1)(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))\\
&\quad-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3)))],\\
&\quad[\sin(\theta_1)(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))\\
&\quad-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3)))],
\end{aligned}
$$

$$
\begin{aligned}
&[-\sin(\theta_1)(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\sin(\theta_1)a_2\cos(\theta_2))\\
&-\cos(\theta_1)(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\cos(\theta_1)a_2\cos(\theta_2))]]
\end{aligned}
$$

$J_{B4} := CrossProduct(Z_3, (P_{06}-P_{03})) =$

$$
\begin{aligned}
&[[\sin(\theta_1)(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))+\sin(\theta_2)a_3\cos(\theta_3)+\cos(\theta_2)a_3\sin(\theta_3))-(\\
&\quad-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3))(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6\\
&\quad-\sin(\theta_1)(\cos(\theta_2)a_3\cos(\theta_3)-\sin(\theta_2)a_3\sin(\theta_3)+a_2\cos(\theta_2)))],\\
&\quad[-\cos(\theta_1)(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))+\sin(\theta_2)a_3\cos(\theta_3)+\cos(\theta_2)a_3\sin(\theta_3))+(\\
&\quad-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3))(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6\\
&\quad-\cos(\theta_1)(\cos(\theta_2)a_3\cos(\theta_3)-\sin(\theta_2)a_3\sin(\theta_3)+a_2\cos(\theta_2)))],\\
&\quad[\cos(\theta_1)(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6\\
&\quad-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6\\
&\quad-\sin(\theta_1)(\cos(\theta_2)a_3\cos(\theta_3)-\sin(\theta_2)a_3\sin(\theta_3)+a_2\cos(\theta_2)))\\
&\quad-\sin(\theta_1)(\sin(\theta_2)\cos(\theta_3)\\
&\quad+\cos(\theta_2)\sin(\theta_3))(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6\\
&\quad+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)\\
&\quad+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\cos(\theta_1)(\cos(\theta_2)a_3\cos(\theta_3)\\
&\quad-\sin(\theta_2)a_3\sin(\theta_3)+a_2\cos(\theta_2)))]]
\end{aligned}
$$

$J_{B5} := simplify(CrossProduct(Z_4, (P_{06}-P_{04}))) =$

$$[[-d_6(-\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)+\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$$

$+\cos(\theta_5)\sin(\theta_1)\sin(\theta_4)+\sin(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)$
$+\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3))],$
$[-d_6(-\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)+\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)$
$-\cos(\theta_5)\cos(\theta_1)\sin(\theta_4)+\sin(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)$
$+\sin(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3))],$
$[d_6(-\cos(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)-\cos(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)$
$+\sin(\theta_5)\sin(\theta_2)\sin(\theta_3)-\sin(\theta_5)\cos(\theta_3)\cos(\theta_2))]]$

$$J_{B6} := simplify(CrossProduct(Z_5, (P_{06}-P_{05}))) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#Assembling Jacobian matrix

$$J_B := \begin{bmatrix} J_{B1}[1] & J_{B2}[1] & J_{B3}[1] & J_{B4}[1] & J_{B5}[1] & J_{B6}[1] \\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] & J_{B4}[2] & J_{B5}[2] & J_{B6}[2] \\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] & J_{B4}[3] & J_{B5}[3] & J_{B6}[3] \\ Z_0[1] & Z_1[1] & Z_2[1] & Z_3[1] & Z_4[1] & Z_5[1] \\ Z_0[2] & Z_1[2] & Z_2[2] & Z_3[2] & Z_4[2] & Z_5[2] \\ Z_0[3] & Z_1[3] & Z_2[3] & Z_3[3] & Z_4[3] & Z_5[3] \end{bmatrix} =$$

$[[-\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))$
$-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))$
$+a_2\cos(\theta_2))-\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-a_1\sin(\theta_1),\cos(\theta_1)($
$-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))$
$-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))$
$-a_2\sin(\theta_2)),\cos(\theta_1)(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\cos(\theta_3))-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))),\sin(\theta_1)(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))($
$-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))$
$-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))$
$+\sin(\theta_2)a_3\cos(\theta_3)+\cos(\theta_2)a_3\sin(\theta_3))-(-\sin(\theta_2)\sin(\theta_3)$
$+\cos(\theta_2)\cos(\theta_3))(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6$
$+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\sin(\theta_1)(\cos(\theta_2)a_3\cos(\theta_3)$

$-\sin(\theta_2)a_3\sin(\theta_3)+a_2\cos(\theta_2))),-d_6(-\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)$
$+\cos(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\sin(\theta_1)\sin(\theta_4)$
$+\sin(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)+\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)),0],$
$[\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))$
$-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))$
$+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6+a_1\cos(\theta_1),\sin(\theta_1)($
$-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))$
$-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))$
$-a_2\sin(\theta_2)),\sin(\theta_1)(-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\cos(\theta_3))-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))),-\cos(\theta_1)(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))($
$-\sin(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))$
$-\cos(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\sin(\theta_3))$
$+\sin(\theta_2)a_3\cos(\theta_3)+\cos(\theta_2)a_3\sin(\theta_3))+(-\sin(\theta_2)\sin(\theta_3)$
$+\cos(\theta_2)\cos(\theta_3))(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6$
$+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\cos(\theta_1)(\cos(\theta_2)a_3\cos(\theta_3)$
$-\sin(\theta_2)a_3\sin(\theta_3)+a_2\cos(\theta_2))),-d_6(-\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)$
$+\cos(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)-\cos(\theta_5)\cos(\theta_1)\sin(\theta_4)$
$+\sin(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)+\sin(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)),0],$
$[0,-\sin(\theta_1)(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6)$
$-\cos(\theta_1)(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6),$
$-\sin(\theta_1)(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\sin(\theta_1)a_2\cos(\theta_2))$
$-\cos(\theta_1)(\cos(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6-\cos(\theta_3)(\cos(\theta_5)d_6+d_4)$
$+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)d_6-\cos(\theta_1)a_2\cos(\theta_2)),$
$\cos(\theta_1)(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))(\sin(\theta_1)(\cos(\theta_2)(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6$
$+\sin(\theta_3)(\cos(\theta_5)d_6+d_4)+a_3\cos(\theta_3))-\sin(\theta_2)(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)d_6$

$$-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\Big)+a_2\cos(\theta_2)\Big)+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6$$
$$-\sin(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3)-\sin(\theta_2)\,a_3\sin(\theta_3)+a_2\cos(\theta_2)\right)\Big)$$
$$-\sin(\theta_1)\Big(\sin(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_2)\sin(\theta_3)\Big)\Big(\cos(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\big(\cos(\theta_5)\,d_6$$
$$+d_4\big)+a_3\cos(\theta_3)\Big)-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)+a_2\cos(\theta_2)\Big)-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_1)\Big(\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$-\sin(\theta_2)\,a_3\sin(\theta_3)+a_2\cos(\theta_2)\Big)\Big),\ d_6\Big(-\cos(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_5)\sin(\theta_2)\sin(\theta_3)-\sin(\theta_5)\cos(\theta_3)\cos(\theta_2)\Big),\ 0\Big],$$
$$\Big[0,\ -\sin(\theta_1),\ -\sin(\theta_1),\ \cos(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right),$$
$$\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)-\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4),$$
$$-\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)$$
$$-\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)+\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\Big],$$
$$\Big[0,\ \cos(\theta_1),\ \cos(\theta_1),\ \sin(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right),$$
$$\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4),$$
$$-\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)$$
$$+\sin(\theta_5)\cos(\theta_1)\sin(\theta_4)+\cos(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\Big],$$
$$\Big[1,\ 0,\ 0,\ -\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3),\ \left(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)\right)\sin(\theta_4),$$
$$-\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)-\sin(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_5)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\cos(\theta_3)\cos(\theta_2)\Big]\Big]$$

#Jacobian matrix decoupling

$$\#J := \begin{bmatrix} J_{11} & J_{21} \\ J_{12} & J_{22} \end{bmatrix}$$

$$J_{11} := \begin{bmatrix} J_{B1}[1] & J_{B2}[1] & J_{B3}[1] \\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] \\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] \end{bmatrix} =$$

$$\Big[\Big[-\sin(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\Big)$$
$$-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\Big)$$
$$+a_2\cos(\theta_2)\Big)-\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-a_1\sin(\theta_1),\ \cos(\theta_1)\Big($$
$$-\sin(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\Big)$$

$$-\cos(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\Big)$$
$$-a_2\sin(\theta_2)\Big),\ \cos(\theta_1)\Big(-\sin(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\cos(\theta_3)\Big)-\cos(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)\Big)\Big],$$
$$\Big[\cos(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\Big)$$
$$-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\Big)$$
$$+a_2\cos(\theta_2)\Big)-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6+a_1\cos(\theta_1),\ \sin(\theta_1)\Big($$
$$-\sin(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\Big)$$
$$-\cos(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\Big)$$
$$-a_2\sin(\theta_2)\Big),\ \sin(\theta_1)\Big(-\sin(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\cos(\theta_3)\Big)-\cos(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)\Big)\Big],$$
$$\Big[0,\ -\sin(\theta_1)\Big(\sin(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\cos(\theta_3)\Big)-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)+a_2\cos(\theta_2)\Big)+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6\Big)$$
$$-\cos(\theta_1)\Big(\cos(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\cos(\theta_3)\Big)-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)+a_2\cos(\theta_2)\Big)-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6\Big),$$
$$-\sin(\theta_1)\Big(\sin(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\cos(\theta_3)\Big)-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)+a_2\cos(\theta_2)\Big)+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_1)\,a_2\cos(\theta_2)\Big)$$
$$-\cos(\theta_1)\Big(\cos(\theta_1)\Big(\cos(\theta_2)\Big(\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\cos(\theta_3)\Big)-\sin(\theta_2)\Big(\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)$$
$$+a_3\sin(\theta_3)\Big)+a_2\cos(\theta_2)\Big)-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_1)\,a_2\cos(\theta_2)\Big)\Big]\Big]$$

$$\mathit{simplify}(\mathit{Determinant}(J_{11}))=$$
$$a_2\Big(\cos(\theta_4)^2\sin(\theta_2)\cos(\theta_3)^2\,d_6^2+\cos(\theta_4)^2\sin(\theta_2)\cos(\theta_5)^2\,d_6^2-\cos(\theta_2)\sin(\theta_3)\,d_4^2\cos(\theta_3)$$
$$-2\cos(\theta_3)^2\cos(\theta_2)\,a_3\,d_4-\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_5)^2\,d_6^2-a_2\cos(\theta_2)\cos(\theta_3)\,d_4$$
$$-a_1\cos(\theta_3)\cos(\theta_5)\,d_6+\cos(\theta_3)\cos(\theta_2)\,a_3^2\sin(\theta_3)+a_2\cos(\theta_2)\sin(\theta_3)\,a_3$$
$$+\cos(\theta_2)\cos(\theta_5)\,d_6\,a_3-\cos(\theta_4)^2\cos(\theta_3)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)^2\,d_6^2$$
$$+2\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\,d_6\,a_3\cos(\theta_3)^2-2\sin(\theta_5)\cos(\theta_4)\cos(\theta_3)^2\cos(\theta_2)\,d_6^2\cos(\theta_5)$$
$$-2\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)\,d_6\cos(\theta_3)\,d_4+a_2\cos(\theta_2)\sin(\theta_5)\cos(\theta_4)\sin(\theta_3)\,d_6$$
$$+2\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)\,d_6\sin(\theta_3)\,a_3-2\sin(\theta_5)\cos(\theta_4)\cos(\theta_3)^2\cos(\theta_2)\,d_6\,d_4$$

$$
\begin{aligned}
&+2\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)\,d_6^2\cos(\theta_3)\cos(\theta_5)\\
&+2\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)\,d_6\cos(\theta_3)\,d_4\\
&+2\sin(\theta_5)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)\,d_6\sin(\theta_3)\,a_3+\cos(\theta_4)^2\cos(\theta_3)\cos(\theta_2)\,d_6^2\sin(\theta_3)\\
&+\cos(\theta_2)\cos(\theta_5)\,d_6^2\sin(\theta_5)\cos(\theta_4)+\cos(\theta_2)\,d_4\sin(\theta_5)\cos(\theta_4)\,d_6\\
&-2\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\,d_6\,a_3-a_2\cos(\theta_2)\cos(\theta_3)\cos(\theta_5)\,d_6-\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)^2\\
&d_6^2\cos(\theta_3)-2\cos(\theta_3)^2\cos(\theta_2)\,a_3\cos(\theta_5)\,d_6+a_1\sin(\theta_5)\cos(\theta_4)\sin(\theta_3)\,d_6\\
&+2\sin(\theta_2)\cos(\theta_3)\,d_4\sin(\theta_3)\,a_3-2\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_5)\,d_6\,d_4\\
&-\cos(\theta_4)^2\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_5)^2\,d_6^2+\cos(\theta_2)\,d_4\,a_3-\cos(\theta_4)^2\sin(\theta_2)\,d_6^2-a_1\cos(\theta_3)\,d_4\\
&+a_1\sin(\theta_3)\,a_3-\sin(\theta_2)\cos(\theta_3)^2\,d_4^2+\sin(\theta_2)\cos(\theta_3)^2\,a_3^2-\sin(\theta_2)\,a_3^2\big)
\end{aligned}
$$

$$
J_{22} := \begin{bmatrix} Z_3[1] & Z_4[1] & Z_5[1] \\ Z_3[2] & Z_4[2] & Z_5[2] \\ Z_3[3] & Z_4[3] & Z_5[3] \end{bmatrix} =
$$

$$
\begin{aligned}
&\big[\big[\cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)),\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\\
&\quad-\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)-\sin(\theta_1)\cos(\theta_4),\\
&\quad-\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\cos(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)\\
&\quad-\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)+\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\\
&\quad+\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\big],\\
&\quad\big[\sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3)),\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\\
&\quad-\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\sin(\theta_4)+\cos(\theta_1)\cos(\theta_4),\\
&\quad-\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\sin(\theta_2)\sin(\theta_3)+\sin(\theta_5)\sin(\theta_1)\cos(\theta_4)\cos(\theta_3)\cos(\theta_2)\\
&\quad+\sin(\theta_5)\cos(\theta_1)\sin(\theta_4)+\cos(\theta_5)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\\
&\quad+\cos(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\big],\\
&\quad\big[-\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3),(\sin(\theta_2)\cos(\theta_3)+\cos(\theta_2)\sin(\theta_3))\sin(\theta_4),\\
&\quad-\sin(\theta_5)\cos(\theta_4)\sin(\theta_2)\cos(\theta_3)-\sin(\theta_5)\cos(\theta_4)\cos(\theta_2)\sin(\theta_3)\\
&\quad-\cos(\theta_5)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_5)\cos(\theta_3)\cos(\theta_2)\big]\big]
\end{aligned}
$$

$simplify(Determinant(J_{22})) = -\sin(\theta_5)$

#Determining singularities (Manipulability measures)

$simplify(Determinant(J_B)) =$

$-\sin(\theta_5)\,(\sin(\theta_2)\cos(\theta_3)^2 a_3^2-\sin(\theta_2)\cos(\theta_3)^2 d_4^2+2\sin(\theta_2)\cos(\theta_3)\,d_4\sin(\theta_3)\,a_3$

$$
\begin{aligned}
&-2\cos(\theta_3)^2\cos(\theta_2)\,a_3\,d_4+\cos(\theta_3)\cos(\theta_2)\,a_3^2\sin(\theta_3)-\cos(\theta_2)\sin(\theta_3)\,d_4^2\cos(\theta_3)\\
&-a_2\cos(\theta_2)\cos(\theta_3)\,d_4+a_2\cos(\theta_2)\sin(\theta_3)\,a_3-\sin(\theta_2)\,a_3^2-a_1\cos(\theta_3)\,d_4+a_1\sin(\theta_3)\,a_3\\
&+\cos(\theta_2)\,d_4\,a_3)\,a_2
\end{aligned}
$$

$$
\xrightarrow{\text{equate to 0}}
$$

$$
\begin{aligned}
&-\sin(\theta_5)\,(\sin(\theta_2)\cos(\theta_3)^2 a_3^2-\sin(\theta_2)\cos(\theta_3)^2 d_4^2+2\sin(\theta_2)\cos(\theta_3)\,d_4\sin(\theta_3)\,a_3\\
&-2\cos(\theta_3)^2\cos(\theta_2)\,a_3\,d_4+\cos(\theta_3)\cos(\theta_2)\,a_3^2\sin(\theta_3)-\cos(\theta_2)\sin(\theta_3)\,d_4^2\cos(\theta_3)\\
&-a_2\cos(\theta_2)\cos(\theta_3)\,d_4+a_2\cos(\theta_2)\sin(\theta_3)\,a_3-\sin(\theta_2)\,a_3^2-a_1\cos(\theta_3)\,d_4+a_1\sin(\theta_3)\,a_3\\
&+\cos(\theta_2)\,d_4\,a_3)\,a_2=0
\end{aligned}
$$

$$
\xrightarrow{\text{solve for theta[5]}} [[\theta_5=0]]
$$

#Boundary Singularity condition

$Cb := -a_3\sin(\theta_3)+d_4\cos(\theta_3) = -a_3\sin(\theta_3)+\cos(\theta_3)\,d_4 \xrightarrow{\text{equate to 0}} -a_3\sin(\theta_3)+\cos(\theta_3)\,d_4=0$

$\xrightarrow{\text{solve for theta[3]}} \left[\left[\theta_3=\arctan\left(\frac{d_4}{a_3}\right)\right]\right]$

$(\Theta_3) := \frac{270}{135}\cdot\frac{180}{\pi} = \frac{360}{\pi} \xrightarrow{\text{at 5 digits}} 114.59$

#Interior Singularity condition

$Ca := a_1+a_2\cos(\theta_2)+a_3\cos(\theta_2+\theta_3)+d_4\sin(\theta_2+\theta_3) =$
$a_1+a_2\cos(\theta_2)+a_3\cos(\theta_2+\theta_3)+d_4\sin(\theta_2+\theta_3)$

$95+245\cos(\theta_2)-135\cos(\theta_2+114.59)+270\sin(\theta_2+114.59)=0 \xrightarrow{\text{solve for theta[2]}}$
$[[\theta_2=2.051763536],[\theta_2=-1.452240440]]$

$\Theta_{2.1} := 2.051763536\cdot\frac{180}{\pi} = \frac{369.3174365}{\pi} \xrightarrow{\text{at 5 digits}} 117.56$

$\Theta_{2.2} := -1.452240440\cdot\frac{180}{\pi} = -\frac{261.4032792}{\pi} \xrightarrow{\text{at 5 digits}} -83.206$

Appendix C
Maple program for Yaskawa Motoman DA $-$ 20 FK, Jacobian Matrix and Singularity Conditions

Motoman DA20 DH Parameters								
Joint	d	θL	θR	a	α L	α R	Mechanical Limit L	Mechanical Limit R
b	965	0	180	295	90	-90	±180	
1	136.5	0		135	90		+80; -190	+190; -80
2	0	90		250	180		+220; -40	
3	0	90		390	-90		+215; -35	
4	-230	0		0	90		±180	
5	0	0		0	-90		±120	
6	-65	0		0	180		±180	

$with(LinearAlgebra)$:

#D-H parameters for Motoman DA20

$\#d_b := 965$
$\#d_1 := 136.5$
$d_2 := 0 = 0$
$d_3 := 0 = 0$
$\#d_4 := -230$
$d_5 := 0 = 0$
$\#d_6 := -65$

$\#\theta_{bL} := 0$
$\#\theta_{bR} := \pi$
$\#\theta_1 := 0$
$\#\theta_2 := \frac{\pi}{2}$
$\#\theta_3 := \frac{\pi}{2}$
$\#\theta_4 := 0$
$\#\theta_5 := 0$
$\#\theta_6 := 0$

$\#a_b := 295$
$\#a_1 := 135$
$\#a_2 := 250$
$\#a_3 := 390$
$a_4 := 0 = 0$
$a_5 := 0 = 0$
$a_6 := 0 = 0$

$\alpha_{bL} := \frac{\pi}{2} = \frac{1}{2}\pi$
$\alpha_{bR} := -\frac{\pi}{2} = -\frac{1}{2}\pi$
$\alpha_1 := \frac{\pi}{2} = \frac{1}{2}\pi$
$\alpha_2 := \pi = \pi$
$\alpha_3 := -\frac{\pi}{2} = -\frac{1}{2}\pi$
$\alpha_4 := \frac{\pi}{2} = \frac{1}{2}\pi$
$\alpha_5 := -\frac{\pi}{2} = -\frac{1}{2}\pi$
$\alpha_6 := \pi = \pi$

#A Matrices

#Homogenious Transformation Matrix A_{0bL} :

$$A_{0bL} := \begin{bmatrix} \cos(\theta_{bL}) & -\cos(\alpha_{bL})\cdot\sin(\theta_{bL}) & \sin(\alpha_{bL})\cdot\sin(\theta_{bL}) & a_b\cdot\cos(\theta_{bL}) \\ \sin(\theta_{bL}) & \cos(\alpha_{bL})\cdot\cos(\theta_{bL}) & -\sin(\alpha_{bL})\cdot\cos(\theta_{bL}) & a_b\cdot\sin(\theta_{bL}) \\ 0 & \sin(\alpha_{bL}) & \cos(\alpha_{bL}) & d_b \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_{bL}) & 0 & \sin(\theta_{bL}) & a_b\cos(\theta_{bL}) \\ \sin(\theta_{bL}) & 0 & -\cos(\theta_{bL}) & a_b\sin(\theta_{bL}) \\ 0 & 1 & 0 & d_b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{0bL}

$$R_{0bL} := \begin{bmatrix} A_{0bL}[1,1] & A_{0bL}[1,2] & A_{0bL}[1,3] \\ A_{0bL}[2,1] & A_{0bL}[2,2] & A_{0bL}[2,3] \\ A_{0bL}[3,1] & A_{0bL}[3,2] & A_{0bL}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_{bL}) & 0 & \sin(\theta_{bL}) \\ \sin(\theta_{bL}) & 0 & -\cos(\theta_{bL}) \\ 0 & 1 & 0 \end{bmatrix}$$

#Position Matrix P_{0bL} :

$$P_{0bL} := \begin{bmatrix} A_{0bL}[1,4] \\ A_{0bL}[2,4] \\ A_{0bL}[3,4] \end{bmatrix} = \begin{bmatrix} a_b \cos(\theta_{bL}) \\ a_b \sin(\theta_{bL}) \\ d_b \end{bmatrix}$$

#Homogenious Transformation Matrix A_{0bR}:

$$A_{0bR} := \begin{bmatrix} \cos(\theta_{bR}) & -\cos(\alpha_{bR})\cdot\sin(\theta_{bR}) & \sin(\alpha_{bR})\cdot\sin(\theta_{bR}) & a_b\cdot\cos(\theta_{bR}) \\ \sin(\theta_{bR}) & \cos(\alpha_{bR})\cdot\cos(\theta_{bR}) & -\sin(\alpha_{bR})\cdot\cos(\theta_{bR}) & a_b\cdot\sin(\theta_{bR}) \\ 0 & \sin(\alpha_{bR}) & \cos(\alpha_{bR}) & d_b \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_{bR}) & 0 & -\sin(\theta_{bR}) & a_b \cos(\theta_{bR}) \\ \sin(\theta_{bR}) & 0 & \cos(\theta_{bR}) & a_b \sin(\theta_{bR}) \\ 0 & -1 & 0 & d_b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{0bR}

$$R_{0bR} := \begin{bmatrix} A_{0bR}[1,1] & A_{0bR}[1,2] & A_{0bR}[1,3] \\ A_{0bR}[2,1] & A_{0bR}[2,2] & A_{0bR}[2,3] \\ A_{0bR}[3,1] & A_{0bR}[3,2] & A_{0bR}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_{bR}) & 0 & -\sin(\theta_{bR}) \\ \sin(\theta_{bR}) & 0 & \cos(\theta_{bR}) \\ 0 & -1 & 0 \end{bmatrix}$$

#Position Matrix P_{0bR} :

$$P_{0bR} := \begin{bmatrix} A_{0bR}[1,4] \\ A_{0bR}[2,4] \\ A_{0bR}[3,4] \end{bmatrix} = \begin{bmatrix} a_b \cos(\theta_{bR}) \\ a_b \sin(\theta_{bR}) \\ d_b \end{bmatrix}$$

#Homogenious Transformation Matrix A_{01}:

$$A_{01} := \begin{bmatrix} \cos(\theta_1) & -\cos(\alpha_1)\cdot\sin(\theta_1) & \sin(\alpha_1)\cdot\sin(\theta_1) & a_1\cdot\cos(\theta_1) \\ \sin(\theta_1) & \cos(\alpha_1)\cdot\cos(\theta_1) & -\sin(\alpha_1)\cdot\cos(\theta_1) & a_1\cdot\sin(\theta_1) \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & a_1 \cos(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & a_1 \sin(\theta_1) \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{01}

$$R_{01} := \begin{bmatrix} A_{01}[1,1] & A_{01}[1,2] & A_{01}[1,3] \\ A_{01}[2,1] & A_{01}[2,2] & A_{01}[2,3] \\ A_{01}[3,1] & A_{01}[3,2] & A_{01}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) \\ 0 & 1 & 0 \end{bmatrix}$$

#Position Matrix P_{01} :

$$P_{01} := \begin{bmatrix} A_{01}[1,4] \\ A_{01}[2,4] \\ A_{01}[3,4] \end{bmatrix} = \begin{bmatrix} a_1 \cos(\theta_1) \\ a_1 \sin(\theta_1) \\ d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{12}:

$$A_{12} := \begin{bmatrix} \cos(\theta_2) & -\cos(\alpha_2)\cdot\sin(\theta_2) & \sin(\alpha_2)\cdot\sin(\theta_2) & a_2\cdot\cos(\theta_2) \\ \sin(\theta_2) & \cos(\alpha_2)\cdot\cos(\theta_2) & -\sin(\alpha_2)\cdot\cos(\theta_2) & a_2\cdot\sin(\theta_2) \\ 0 & \sin(\alpha_2) & \cos(\alpha_2) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 & a_2 \cos(\theta_2) \\ \sin(\theta_2) & -\cos(\theta_2) & 0 & a_2 \sin(\theta_2) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{02} := Multiply(A_{01}, A_{12}) =$$

$$\begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) & \cos(\theta_1)\, a_2 \cos(\theta_2) + a_1 \cos(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & \sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & \sin(\theta_1)\, a_2 \cos(\theta_2) + a_1 \sin(\theta_1) \\ \sin(\theta_2) & -\cos(\theta_2) & 0 & a_2 \sin(\theta_2) + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#Rotation Matrix R_{12}

$$R_{12} := \begin{bmatrix} A_{12}[1,1] & A_{12}[1,2] & A_{12}[1,3] \\ A_{12}[2,1] & A_{12}[2,2] & A_{12}[2,3] \\ A_{12}[3,1] & A_{12}[3,2] & A_{12}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ \sin(\theta_2) & -\cos(\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

#Rotation Matrix R_{02}

$$R_{02} := Multiply(R_{01}, R_{12}) = \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & \sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) \\ \sin(\theta_2) & -\cos(\theta_2) & 0 \end{bmatrix}$$

#Position Matrix P_{12} :

$$P_{12} := \begin{bmatrix} A_{12}[1,4] \\ A_{12}[2,4] \\ A_{12}[3,4] \end{bmatrix} = \begin{bmatrix} a_2\cos(\theta_2) \\ a_2\sin(\theta_2) \\ 0 \end{bmatrix}$$

#Position Matrix P_{02}

$$P_{02} := \begin{bmatrix} A_{02}[1,4] \\ A_{02}[2,4] \\ A_{02}[3,4] \end{bmatrix} = \begin{bmatrix} \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

$$P_{02} := Multiply(R_{01}, P_{12}) + P_{01} = \begin{bmatrix} \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{23} :

$$A_{23} := \begin{bmatrix} \cos(\theta_3) & -\cos(\alpha_3)\cdot\sin(\theta_3) & \sin(\alpha_3)\cdot\sin(\theta_3) & a_3\cdot\cos(\theta_3) \\ \sin(\theta_3) & \cos(\alpha_3)\cdot\cos(\theta_3) & -\sin(\alpha_3)\cdot\cos(\theta_3) & a_3\cdot\sin(\theta_3) \\ 0 & \sin(\alpha_3) & \cos(\alpha_3) & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_3) & 0 & -\sin(\theta_3) & a_3\cos(\theta_3) \\ \sin(\theta_3) & 0 & \cos(\theta_3) & a_3\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_{03} := Multiply(A_{02}, A_{23}) =$

$$\begin{aligned}
&[[\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + \cos(\theta_1)\sin(\theta_2)\sin(\theta_3), \sin(\theta_1), -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) \\
&\quad + \cos(\theta_1)\sin(\theta_2)\cos(\theta_3), \cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) + \cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) \\
&\quad + \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1)], \\
&[\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_3), -\cos(\theta_1), -\sin(\theta_1)\cos(\theta_2)\sin(\theta_3) \\
&\quad + \sin(\theta_1)\sin(\theta_2)\cos(\theta_3), \sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) \\
&\quad + \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1)], \\
&[\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3), 0, -\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3), \sin(\theta_2)\,a_3\cos(\theta_3) \\
&\quad - \cos(\theta_2)\,a_3\sin(\theta_3) + a_2\sin(\theta_2) + d_1], \\
&[0, 0, 0, 1]]
\end{aligned}$$

#Rotation Matrix R_{23}

$$R_{23} := \begin{bmatrix} A_{23}[1,1] & A_{23}[1,2] & A_{23}[1,3] \\ A_{23}[2,1] & A_{23}[2,2] & A_{23}[2,3] \\ A_{23}[3,1] & A_{23}[3,2] & A_{23}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_3) & 0 & -\sin(\theta_3) \\ \sin(\theta_3) & 0 & \cos(\theta_3) \\ 0 & -1 & 0 \end{bmatrix}$$

#Rotation Matrix R_{03}

$R_{03} := simplify(Multiply(R_{02}, R_{23})) =$

$$\begin{aligned}
&[[\cos(\theta_1)\,(\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)), \sin(\theta_1), \cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3) \\
&\quad - \cos(\theta_2)\sin(\theta_3))], \\
&[\sin(\theta_1)\,(\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)), -\cos(\theta_1), \sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3) \\
&\quad - \cos(\theta_2)\sin(\theta_3))], \\
&[\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3), 0, -\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)]]
\end{aligned}$$

#Position Matrix P_{23}

$$P_{23} := \begin{bmatrix} A_{23}[1,4] \\ A_{23}[2,4] \\ A_{23}[3,4] \end{bmatrix} = \begin{bmatrix} a_3\cos(\theta_3) \\ a_3\sin(\theta_3) \\ 0 \end{bmatrix}$$

#Position Matrix P_{03}

$$P_{03} := \begin{bmatrix} A_{03}[1,4] \\ A_{03}[2,4] \\ A_{03}[3,4] \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) + \cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) + \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1) \\ \sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) + \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1) \\ \sin(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_2)\,a_3\sin(\theta_3) + a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

$$P_{03} := Multiply(R_{01}, (Multiply(R_{12}, P_{23})) + P_{12}) + P_{01} =$$

$$\begin{bmatrix} \cos(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3) + \sin(\theta_2)\,a_3\sin(\theta_3) + a_2\cos(\theta_2)\right) + a_1\cos(\theta_1) \\ \sin(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3) + \sin(\theta_2)\,a_3\sin(\theta_3) + a_2\cos(\theta_2)\right) + a_1\sin(\theta_1) \\ \sin(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_2)\,a_3\sin(\theta_3) + a_2\sin(\theta_2) + d_1 \end{bmatrix}$$

#Homogenious Transformation Matrix A_{34} :

$$A_{34} := \begin{bmatrix} \cos(\theta_4) & -\cos(\alpha_4)\cdot\sin(\theta_4) & \sin(\alpha_4)\cdot\sin(\theta_4) & a_4\cdot\cos(\theta_4) \\ \sin(\theta_4) & \cos(\alpha_4)\cdot\cos(\theta_4) & -\sin(\alpha_4)\cdot\cos(\theta_4) & a_4\cdot\sin(\theta_4) \\ 0 & \sin(\alpha_4) & \cos(\alpha_4) & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & -\cos(\theta_4) & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{04} := Multiply(A_{03}, A_{34}) =$$

$[[(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + \cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4) + \sin(\theta_1)\sin(\theta_4),$
$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\sin(\theta_2)\cos(\theta_3), (\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$
$+ \cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4) - \sin(\theta_1)\cos(\theta_4), (-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+ \cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4 + \cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) + \cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$
$+ \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1)],$
$[(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4) - \cos(\theta_1)\sin(\theta_4),$
$-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3) + \sin(\theta_1)\sin(\theta_2)\cos(\theta_3), (\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$
$+ \sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4) + \cos(\theta_1)\cos(\theta_4), (-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$
$+ \sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4 + \sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$

$+ \sin(\theta_1)\,a_2\cos(\theta_2) + a_1\sin(\theta_1)],$
$[(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\cos(\theta_4), -\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3),$
$(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\sin(\theta_4), (-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\,d_4$
$+ \sin(\theta_2)\,a_3\cos(\theta_3) - \cos(\theta_2)\,a_3\sin(\theta_3) + a_2\sin(\theta_2) + d_1],$
$[0, 0, 0, 1]]$

#Rotation Matrix R_{34}

$$R_{34} := \begin{bmatrix} A_{34}[1,1] & A_{34}[1,2] & A_{34}[1,3] \\ A_{34}[2,1] & A_{34}[2,2] & A_{34}[2,3] \\ A_{34}[3,1] & A_{34}[3,2] & A_{34}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) \\ \sin(\theta_4) & 0 & -\cos(\theta_4) \\ 0 & 1 & 0 \end{bmatrix}$$

#Rotation Matrix R_{04}

$$R_{04} := simplify(Multiply(R_{03}, R_{34})) =$$

$[[\cos(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_4),$
$\cos(\theta_1)(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3)), \sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$
$+ \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \sin(\theta_1)\cos(\theta_4)],$
$[\cos(\theta_4)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_4),$
$\sin(\theta_1)(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3)), \sin(\theta_4)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)$
$+ \sin(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + \cos(\theta_1)\cos(\theta_4)],$
$[(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\cos(\theta_4), -\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3),$
$(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\sin(\theta_4)]]$

#Position Matrix P_{34}

$$P_{34} := \begin{bmatrix} A_{34}[1,4] \\ A_{34}[2,4] \\ A_{34}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_4 \end{bmatrix}$$

#Position Matrix P_{04}

$$P_{04} := \begin{bmatrix} A_{04}[1,4] \\ A_{04}[2,4] \\ A_{04}[3,4] \end{bmatrix} =$$

$[[(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4 + \cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$
$+ \cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3) + \cos(\theta_1)\,a_2\cos(\theta_2) + a_1\cos(\theta_1)],$

$$[(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$$
$$[(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)$$
$$+a_2\sin(\theta_2)+d_1]]$$

$$\boldsymbol{P_{04} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, P_{34})) + P_{23})) + P_{12})) + P_{01}} =$$
$$[[\cos(\theta_1)\,(\cos(\theta_2)\,(-\sin(\theta_3)\,d_4+a_3\cos(\theta_3))+\sin(\theta_2)\,(\cos(\theta_3)\,d_4+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))+a_1\cos(\theta_1)],$$
$$[\sin(\theta_1)\,(\cos(\theta_2)\,(-\sin(\theta_3)\,d_4+a_3\cos(\theta_3))+\sin(\theta_2)\,(\cos(\theta_3)\,d_4+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))+a_1\sin(\theta_1)],$$
$$[\sin(\theta_2)\,(-\sin(\theta_3)\,d_4+a_3\cos(\theta_3))-\cos(\theta_2)\,(\cos(\theta_3)\,d_4+a_3\sin(\theta_3))+a_2\sin(\theta_2)+d_1]]$$

#Homogenious Transformation Matrix $\boldsymbol{A_{45}}$ **:**

$$A_{45} := \begin{bmatrix} \cos(\theta_5) & -\cos(\alpha_5)\cdot\sin(\theta_5) & \sin(\alpha_5)\cdot\sin(\theta_5) & a_5\cdot\cos(\theta_5) \\ \sin(\theta_5) & \cos(\alpha_5)\cdot\cos(\theta_5) & -\sin(\alpha_5)\cdot\cos(\theta_5) & a_5\cdot\sin(\theta_5) \\ 0 & \sin(\alpha_5) & \cos(\alpha_5) & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} \cos(\theta_5) & 0 & -\sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & \cos(\theta_5) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{05} := Multiply(A_{04}, A_{45}) =$$
$$[[((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)+($$
$$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5),\ -(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4),\ -((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),\ (-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$$
$$+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)],$$
$$[((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)$$
$$+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5),\ -(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\cos(\theta_1)\cos(\theta_4),\ -((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),\ (-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4$$
$$+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$$
$$[(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5),\ -(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4),$$
$$-(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5),\ (-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_2)\,a_3\cos(\theta_3)$$
$$-\cos(\theta_2)\,a_3\sin(\theta_3)+a_2\sin(\theta_2)+d_1],$$
$$[0, 0, 0, 1]]$$

#Rotation Matrix $\boldsymbol{R_{45}}$

$$R_{45} := \begin{bmatrix} A_{45}[1,1] & A_{45}[1,2] & A_{45}[1,3] \\ A_{45}[2,1] & A_{45}[2,2] & A_{45}[2,3] \\ A_{45}[3,1] & A_{45}[3,2] & A_{45}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_5) & 0 & -\sin(\theta_5) \\ \sin(\theta_5) & 0 & \cos(\theta_5) \\ 0 & -1 & 0 \end{bmatrix}$$

#Rotation Matrix $\boldsymbol{R_{05}}$

$$\boldsymbol{R_{05}} := simplify(Multiply(R_{04}, R_{45})) =$$
$$[[\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)+\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)$$
$$+\sin(\theta_1)\sin(\theta_4)\cos(\theta_5)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)$$
$$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5),\ -\sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$-\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\cos(\theta_4),$$
$$-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)-\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)$$
$$-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)$$
$$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],$$
$$[\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)+\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)$$
$$-\cos(\theta_1)\sin(\theta_4)\cos(\theta_5)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)$$
$$-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5),\ -\sin(\theta_4)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$-\sin(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\cos(\theta_4),$$
$$-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)-\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)$$
$$+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)$$
$$-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],$$
$$[\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)-\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)-\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)$$
$$-\cos(\theta_3)\cos(\theta_2)\sin(\theta_5),\ -(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4),$$

$$-\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)+\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)$$
$$-\sin(\theta_2)\sin(\theta_3)\cos(\theta_5)-\cos(\theta_3)\cos(\theta_2)\cos(\theta_5)]]$$

#Position Matrix P_{45}

$$P_{45} := \begin{bmatrix} A_{45}[1,4] \\ A_{45}[2,4] \\ A_{45}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#Position Matrix P_{05}

$$P_{05} := \begin{bmatrix} A_{05}[1,4] \\ A_{05}[2,4] \\ A_{05}[3,4] \end{bmatrix} =$$

$$[[(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)],$$
$$[(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$$
$$[(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)$$
$$+a_2\sin(\theta_2)+d_1]]$$

$$\boldsymbol{P_{05} := Multiply(R_{01}, (Multiply(R_{12}, (Multiply(R_{23}, (Multiply(R_{34}, P_{45}))+\mathrm{P}_{34})+P_{23}))+P_{12}))+P_{01}}$$

$$=$$

$$[[\cos(\theta_1)\,(\cos(\theta_2)\,(-\sin(\theta_3)\,d_4+a_3\cos(\theta_3))+\sin(\theta_2)\,(\cos(\theta_3)\,d_4+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))+a_1\cos(\theta_1)],$$
$$[\sin(\theta_1)\,(\cos(\theta_2)\,(-\sin(\theta_3)\,d_4+a_3\cos(\theta_3))+\sin(\theta_2)\,(\cos(\theta_3)\,d_4+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))+a_1\sin(\theta_1)],$$
$$[\sin(\theta_2)\,(-\sin(\theta_3)\,d_4+a_3\cos(\theta_3))-\cos(\theta_2)\,(\cos(\theta_3)\,d_4+a_3\sin(\theta_3))+a_2\sin(\theta_2)+d_1]]$$

#Homogenious Transformation Matrix A_{56} :

$$A_{56} := \begin{bmatrix} \cos(\theta_6) & -\cos(\alpha_6)\cdot\sin(\theta_6) & \sin(\alpha_6)\cdot\sin(\theta_6) & a_6\cdot\cos(\theta_6) \\ \sin(\theta_6) & \cos(\alpha_6)\cdot\cos(\theta_6) & -\sin(\alpha_6)\cdot\cos(\theta_6) & a_6\cdot\sin(\theta_6) \\ 0 & \sin(\alpha_6) & \cos(\alpha_6) & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_6) & \sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & -1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#FORWARD KINEMATICS for the arms only

$$A_{06} := Multiply(A_{05}, A_{56}) =$$

$$[[(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)$$
$$+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)+($$
$$-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4))\sin(\theta_6),$$
$$(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)$$
$$+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)-($$
$$-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4))\cos(\theta_6),$$
$$((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)$$
$$-(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),\ ($$
$$-((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)$$
$$+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+($$
$$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)],$$
$$[(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)$$
$$+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)+($$
$$-(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\cos(\theta_1)\cos(\theta_4))\sin(\theta_6),$$
$$(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)$$
$$+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)-($$
$$-(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\cos(\theta_1)\cos(\theta_4))\cos(\theta_6),$$
$$((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)$$
$$-(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),\ ($$
$$-((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)$$
$$+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+($$
$$-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$$
$$[((\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)-(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\sin(\theta_6),\ ((\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)$$

$$+\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\sin(\theta_5)\big)\sin(\theta_6)+\big(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3)\big)\sin(\theta_4)\cos(\theta_6),\ \left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)\sin(\theta_5)$$
$$-\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\cos(\theta_5),\ \big(-\big(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3)\big)\cos(\theta_4)\sin(\theta_5)+\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\cos(\theta_5)\big)\,d_6+\big($$
$$-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\big)\,d_4+\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)$$
$$+a_2\sin(\theta_2)+d_1],$$
$$[0, 0, 0, 1]]$$

#Rotation Matrix R_{56}

$$R_{56} := \begin{bmatrix} A_{56}[1,1] & A_{56}[1,2] & A_{56}[1,3] \\ A_{56}[2,1] & A_{56}[2,2] & A_{56}[2,3] \\ A_{56}[3,1] & A_{56}[3,2] & A_{56}[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_6) & \sin(\theta_6) & 0 \\ \sin(\theta_6) & -\cos(\theta_6) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

#Rotation Matrix R_{06}

$$R_{06} := simplify(Multiply(R_{05}, R_{56})) =$$
$$[[\cos(\theta_6)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)$$
$$+\cos(\theta_6)\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)+\cos(\theta_6)\sin(\theta_1)\sin(\theta_4)\cos(\theta_5)$$
$$+\cos(\theta_6)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)-\cos(\theta_6)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5)$$
$$-\sin(\theta_6)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)-\sin(\theta_6)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\cos(\theta_1)$$
$$+\sin(\theta_6)\cos(\theta_4)\sin(\theta_1),\ \sin(\theta_6)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)$$
$$+\sin(\theta_6)\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)+\sin(\theta_6)\sin(\theta_1)\sin(\theta_4)\cos(\theta_5)$$
$$+\sin(\theta_6)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)-\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5)$$
$$+\cos(\theta_6)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)+\cos(\theta_6)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\cos(\theta_1)$$
$$-\cos(\theta_6)\cos(\theta_4)\sin(\theta_1),\ \cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)$$
$$+\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)+\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)$$
$$-\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)+\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],$$
$$[\cos(\theta_6)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)$$
$$+\cos(\theta_6)\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)-\cos(\theta_6)\cos(\theta_1)\sin(\theta_4)\cos(\theta_5)$$
$$+\cos(\theta_6)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)-\cos(\theta_6)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5)$$
$$-\sin(\theta_6)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\sin(\theta_1)-\sin(\theta_6)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\sin(\theta_1)$$
$$-\sin(\theta_6)\cos(\theta_4)\cos(\theta_1),\ \sin(\theta_6)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)$$
$$+\sin(\theta_6)\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)-\sin(\theta_6)\cos(\theta_1)\sin(\theta_4)\cos(\theta_5)$$
$$+\sin(\theta_6)\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)-\sin(\theta_6)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5)$$

$$+\cos(\theta_6)\sin(\theta_4)\sin(\theta_2)\sin(\theta_3)\sin(\theta_1)+\cos(\theta_6)\sin(\theta_4)\cos(\theta_3)\cos(\theta_2)\sin(\theta_1)$$
$$+\cos(\theta_6)\cos(\theta_4)\cos(\theta_1),\ \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)$$
$$+\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)-\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)$$
$$-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)+\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],$$
$$[\cos(\theta_6)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)-\cos(\theta_6)\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)$$
$$-\cos(\theta_6)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)-\cos(\theta_6)\cos(\theta_3)\cos(\theta_2)\sin(\theta_5)$$
$$-\sin(\theta_6)\sin(\theta_4)\sin(\theta_2)\cos(\theta_3)+\sin(\theta_6)\sin(\theta_4)\cos(\theta_2)\sin(\theta_3),$$
$$\sin(\theta_6)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)-\sin(\theta_6)\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)$$
$$-\sin(\theta_6)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)-\sin(\theta_6)\cos(\theta_3)\cos(\theta_2)\sin(\theta_5)$$
$$+\cos(\theta_6)\sin(\theta_4)\sin(\theta_2)\cos(\theta_3)-\cos(\theta_6)\sin(\theta_4)\cos(\theta_2)\sin(\theta_3),$$
$$\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)-\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)+\sin(\theta_2)\sin(\theta_3)\cos(\theta_5)$$
$$+\cos(\theta_3)\cos(\theta_2)\cos(\theta_5)]]$$

#Position Matrix P_{56}

$$P_{56} := \begin{bmatrix} A_{56}[1,4] \\ A_{56}[2,4] \\ A_{56}[3,4] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$$

#Position Matrix P_{06}

$$P_{06} := \begin{bmatrix} A_{06}[1,4] \\ A_{06}[2,4] \\ A_{06}[3,4] \end{bmatrix} =$$
$$[[\big(-\big(\left(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4)\big)\sin(\theta_5)$$
$$+\left(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)\cos(\theta_5)\big)\,d_6+\big($$
$$-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\big)\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1)],$$
$$[\big(-\big(\left(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4)\big)\sin(\theta_5)$$
$$+\left(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\right)\cos(\theta_5)\big)\,d_6+\big($$
$$-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\big)\,d_4+\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)+\sin(\theta_1)\,a_2\cos(\theta_2)+a_1\sin(\theta_1)],$$
$$[\big(-\left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\cos(\theta_4)\sin(\theta_5)+\big(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3)\big)\cos(\theta_5)\big)\,d_6+\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)d_4$$
$$+\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)+a_2\sin(\theta_2)+d_1]]$$

$$\boldsymbol{P_{06} := Multiply\big(R_{01}, \big(Multiply\big(R_{12}, \big(Multiply\big(R_{23}, \big(Multiply\big(R_{34}, \big(Multiply\big(R_{45}, P_{56}\big)\big) + P_{45}\big)}$$
$$\boldsymbol{+ P_{34}\big)\big) + P_{23}\big)\big) + P_{12}\big)\big) + P_{01}}$$
$$=$$
$$\big[\big[\cos(\theta_1)\ \big(\cos(\theta_2)\ \big(-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6 - \sin(\theta_3)\ \big(\cos(\theta_5)\ d_6 + d_4\big) + a_3 \cos(\theta_3)\big)$$
$$+ \sin(\theta_2)\ \big(-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6 + \cos(\theta_3)\ \big(\cos(\theta_5)\ d_6 + d_4\big) + a_3 \sin(\theta_3)\big)$$
$$+ a_2 \cos(\theta_2)\big) - \sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6 + a_1 \cos(\theta_1)\big],$$
$$\big[\sin(\theta_1)\ \big(\cos(\theta_2)\ \big(-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6 - \sin(\theta_3)\ \big(\cos(\theta_5)\ d_6 + d_4\big) + a_3 \cos(\theta_3)\big)$$
$$+ \sin(\theta_2)\ \big(-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6 + \cos(\theta_3)\ \big(\cos(\theta_5)\ d_6 + d_4\big) + a_3 \sin(\theta_3)\big)$$
$$+ a_2 \cos(\theta_2)\big) + \cos(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6 + a_1 \sin(\theta_1)\big],$$
$$\big[\sin(\theta_2)\ \big(-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6 - \sin(\theta_3)\ \big(\cos(\theta_5)\ d_6 + d_4\big) + a_3 \cos(\theta_3)\big) - \cos(\theta_2)\ \big($$
$$-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6 + \cos(\theta_3)\ \big(\cos(\theta_5)\ d_6 + d_4\big) + a_3 \sin(\theta_3)\big) + a_2 \sin(\theta_2) + d_1\big]\big]$$

#FORWARD KINEMATICS for the arms whole system

$$A_{b6L} := Multiply(A_{0bL}, A_{06}) =$$
$$\big[\big[\big(\big(\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4) + \sin(\theta_1)\ \sin(\theta_4)\big)\ \cos(\theta_5)$$
$$+ \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \cos(\theta_6) + \big($$
$$-\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4) + \sin(\theta_1)\ \cos(\theta_4)\big)\ \sin(\theta_6)\big)$$
$$\cos(\theta_{bR}) - \big(\big(\big(\sin(\theta_2)\ \cos(\theta_3) - \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \cos(\theta_5) + \big(-\sin(\theta_2)\ \sin(\theta_3)$$
$$- \cos(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \cos(\theta_6) - \big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4)\ \sin(\theta_6)\big)\ \sin(\theta_{bR}),\ \big(\big(\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4) + \sin(\theta_1)\ \sin(\theta_4)\big)\ \cos(\theta_5) + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \sin(\theta_6) - \big(-\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4) + \sin(\theta_1)\ \cos(\theta_4)\big)\ \cos(\theta_6)\big)\ \cos(\theta_{bR})$$
$$- \big(\big(\big(\sin(\theta_2)\ \cos(\theta_3) - \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \cos(\theta_5) + \big(-\sin(\theta_2)\ \sin(\theta_3)$$
$$- \cos(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \sin(\theta_6) + \big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4)\ \cos(\theta_6)\big)\ \sin(\theta_{bR}),\ \cos(\theta_{bR})\ \big(\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4) + \sin(\theta_1)\ \sin(\theta_4)\big)\ \sin(\theta_5) - \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big) - \sin(\theta_{bR})\ \big(\big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \sin(\theta_5) - \big(-\sin(\theta_2)\ \sin(\theta_3) - \cos(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big),$$
$$\cos(\theta_{bR})\ \big(\big(-\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)$$
$$+ \sin(\theta_1)\ \sin(\theta_4)\big)\ \sin(\theta_5) + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big)\ d_6 + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ d_4 + \cos(\theta_1)\ \cos(\theta_2)\ a_3 \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ a_3 \sin(\theta_3)$$
$$+ \cos(\theta_1)\ a_2 \cos(\theta_2) + a_1 \cos(\theta_1)\big) - \sin(\theta_{bR})\ \big(\big(-\big(\sin(\theta_2)\ \cos(\theta_3)$$

$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \sin(\theta_5) + \big(-\sin(\theta_2)\ \sin(\theta_3) - \cos(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big)\ d_6 + \big($$
$$-\sin(\theta_2)\ \sin(\theta_3) - \cos(\theta_2)\ \cos(\theta_3)\big)\ d_4 + \sin(\theta_2)\ a_3 \cos(\theta_3) - \cos(\theta_2)\ a_3 \sin(\theta_3)$$
$$+ a_2 \sin(\theta_2) + d_1\big) + a_b \cos(\theta_{bR})\big],$$
$$\big[\big(\big(\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)$$
$$+ \sin(\theta_1)\ \sin(\theta_4)\big)\ \cos(\theta_5) + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \cos(\theta_6) + \big(-\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4) + \sin(\theta_1)\ \cos(\theta_4)\big)\ \sin(\theta_6)\big)\ \sin(\theta_{bR})$$
$$+ \big(\big(\big(\sin(\theta_2)\ \cos(\theta_3) - \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \cos(\theta_5) + \big(-\sin(\theta_2)\ \sin(\theta_3)$$
$$- \cos(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \cos(\theta_6) - \big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4)\ \sin(\theta_6)\big)\ \cos(\theta_{bR}),\ \big(\big(\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4) + \sin(\theta_1)\ \sin(\theta_4)\big)\ \cos(\theta_5) + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \sin(\theta_6) - \big(-\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4) + \sin(\theta_1)\ \cos(\theta_4)\big)\ \cos(\theta_6)\big)\ \sin(\theta_{bR})$$
$$+ \big(\big(\big(\sin(\theta_2)\ \cos(\theta_3) - \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \cos(\theta_5) + \big(-\sin(\theta_2)\ \sin(\theta_3)$$
$$- \cos(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \sin(\theta_6) + \big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4)\ \cos(\theta_6)\big)\ \cos(\theta_{bR}),\ \sin(\theta_{bR})\ \big(\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4) + \sin(\theta_1)\ \sin(\theta_4)\big)\ \sin(\theta_5) - \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big) + \cos(\theta_{bR})\ \big(\big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \sin(\theta_5) - \big(-\sin(\theta_2)\ \sin(\theta_3) - \cos(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big),$$
$$\sin(\theta_{bR})\ \big(\big(-\big(\big(\cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)$$
$$+ \sin(\theta_1)\ \sin(\theta_4)\big)\ \sin(\theta_5) + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big)\ d_6 + \big(-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ d_4 + \cos(\theta_1)\ \cos(\theta_2)\ a_3 \cos(\theta_3) + \cos(\theta_1)\ \sin(\theta_2)\ a_3 \sin(\theta_3)$$
$$+ \cos(\theta_1)\ a_2 \cos(\theta_2) + a_1 \cos(\theta_1)\big) + \cos(\theta_{bR})\ \big(\big(-\big(\sin(\theta_2)\ \cos(\theta_3)$$
$$- \cos(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)\ \sin(\theta_5) + \big(-\sin(\theta_2)\ \sin(\theta_3) - \cos(\theta_2)\ \cos(\theta_3)\big)\ \cos(\theta_5)\big)\ d_6 + \big($$
$$-\sin(\theta_2)\ \sin(\theta_3) - \cos(\theta_2)\ \cos(\theta_3)\big)\ d_4 + \sin(\theta_2)\ a_3 \cos(\theta_3) - \cos(\theta_2)\ a_3 \sin(\theta_3)$$
$$+ a_2 \sin(\theta_2) + d_1\big) + a_b \sin(\theta_{bR})\big],$$
$$\big[-\big(\big(\big(\sin(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3) + \sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4)$$
$$- \cos(\theta_1)\ \sin(\theta_4)\big)\ \cos(\theta_5) + \big(-\sin(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \sin(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \cos(\theta_6) - \big(-\big(\sin(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4) - \cos(\theta_1)\ \cos(\theta_4)\big)\ \sin(\theta_6),\ -\big(\big(\big(\sin(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \cos(\theta_4) - \cos(\theta_1)\ \sin(\theta_4)\big)\ \cos(\theta_5) + \big(-\sin(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)$$
$$+ \sin(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\big)\ \sin(\theta_5)\big)\ \sin(\theta_6) + \big(-\big(\sin(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$
$$+ \sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\big)\ \sin(\theta_4) - \cos(\theta_1)\ \cos(\theta_4)\big)\ \cos(\theta_6),\ -\big(\big(\sin(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)$$

$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),-(-((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6-(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4-\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$$
$$-\sin(\theta_1)\,a_2\cos(\theta_2)-a_1\sin(\theta_1)+d_b],$$
$$[0,0,0,1]]$$

$$A_{b6R} := \mathit{Multiply}(A_{0bR}, A_{06}) =$$
$$[[((((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)$$
$$+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)+($$
$$-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4))\sin(\theta_6))$$
$$\cos(\theta_{bR})-(((\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)-(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\sin(\theta_6))\sin(\theta_{bR}),((((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)-(-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4))\cos(\theta_6))\cos(\theta_{bR})$$
$$-(((\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)+(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\cos(\theta_6))\sin(\theta_{bR}),\cos(\theta_{bR})(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)-(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))-\sin(\theta_{bR})((\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)-(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5)),$$
$$\cos(\theta_{bR})((-((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)$$
$$+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$$
$$+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1))-\sin(\theta_{bR})((-(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+($$
$$-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)$$
$$+a_2\sin(\theta_2)+d_1)+a_b\cos(\theta_{bR})],$$
$$[((((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)$$
$$+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$

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$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)+(-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4))\sin(\theta_6))\sin(\theta_{bR})$$
$$+(((\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)-(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\sin(\theta_6))\cos(\theta_{bR}),((((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\cos(\theta_5)+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)-(-(\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)+\sin(\theta_1)\cos(\theta_4))\cos(\theta_6))\sin(\theta_{bR})$$
$$+(((\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\cos(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)$$
$$-\cos(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)+(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4)\cos(\theta_6))\cos(\theta_{bR}),\sin(\theta_{bR})(((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)-(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))+\cos(\theta_{bR})((\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)-(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5)),$$
$$\sin(\theta_{bR})((-((\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)$$
$$+\sin(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+(-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4+\cos(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)+\cos(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$$
$$+\cos(\theta_1)\,a_2\cos(\theta_2)+a_1\cos(\theta_1))+\cos(\theta_{bR})((-(\sin(\theta_2)\cos(\theta_3)$$
$$-\cos(\theta_2)\sin(\theta_3))\cos(\theta_4)\sin(\theta_5)+(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6+($$
$$-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3))\,d_4+\sin(\theta_2)\,a_3\cos(\theta_3)-\cos(\theta_2)\,a_3\sin(\theta_3)$$
$$+a_2\sin(\theta_2)+d_1)+a_b\sin(\theta_{bR})],$$
$$[-(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)$$
$$-\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\cos(\theta_6)-(-(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\cos(\theta_1)\cos(\theta_4))\sin(\theta_6),-(((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\cos(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\sin(\theta_5))\sin(\theta_6)+(-(\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\sin(\theta_4)-\cos(\theta_1)\cos(\theta_4))\cos(\theta_6),-((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5),-(-((\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\sin(\theta_3))\cos(\theta_4)-\cos(\theta_1)\sin(\theta_4))\sin(\theta_5)+(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\cos(\theta_5))\,d_6-(-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3))\,d_4-\sin(\theta_1)\cos(\theta_2)\,a_3\cos(\theta_3)-\sin(\theta_1)\sin(\theta_2)\,a_3\sin(\theta_3)$$

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$$-\sin(\theta_1)\, a_2 \cos(\theta_2) - a_1 \sin(\theta_1) + d_b],$$
$$[0, 0, 0, 1]]$$

#Calculting J_B by using Vector cross- multiplication method :

 # Zero Vectors

$$Z_0 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 := Multiply(R_{01}, Z_0) = \begin{bmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{bmatrix}$$

$$Z_2 := Multiply(R_{02}, Z_0) = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix}$$

$$Z_3 := Multiply(R_{03}, Z_0) = \begin{bmatrix} \cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3)) \\ \sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3)) \\ -\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3) \end{bmatrix}$$

$$Z_4 := Multiply(R_{04}, Z_0) =$$
$$\begin{bmatrix} \sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \sin(\theta_1)\cos(\theta_4) \\ \sin(\theta_4)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + \cos(\theta_1)\cos(\theta_4) \\ (\sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3))\sin(\theta_4) \end{bmatrix}$$

$$Z_5 := Multiply(R_{05}, Z_0) =$$
$$[[-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5) - \cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)$$
$$- \sin(\theta_1)\sin(\theta_4)\sin(\theta_5) + \cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)$$
$$- \cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],$$
$$[-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5) - \sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)$$

$$+ \cos(\theta_1)\sin(\theta_4)\sin(\theta_5) + \sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)$$
$$- \sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],$$
$$[-\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\sin(\theta_5) + \cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)$$
$$- \sin(\theta_2)\sin(\theta_3)\cos(\theta_5) - \cos(\theta_3)\cos(\theta_2)\cos(\theta_5)]]$$

#Calculating Jacobian matrix:

$$J_{B1} := CrossProduct(Z_0, P_{06}) =$$
$$[[-\sin(\theta_1)\,(\cos(\theta_2)\,(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\cos(\theta_3))$$
$$+ \sin(\theta_2)\,(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\sin(\theta_3))$$
$$+ a_2\cos(\theta_2)) - \cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\, d_6 - a_1\sin(\theta_1)],$$
$$[\cos(\theta_1)\,(\cos(\theta_2)\,(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\cos(\theta_3))$$
$$+ \sin(\theta_2)\,(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\sin(\theta_3))$$
$$+ a_2\cos(\theta_2)) - \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\, d_6 + a_1\cos(\theta_1)],$$
$$[0]]$$

$$J_{B2} := CrossProduct(Z_1, (P_{06} - P_{01})) =$$
$$[[-\cos(\theta_1)\,(\sin(\theta_2)\,(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\cos(\theta_3))$$
$$- \cos(\theta_2)\,(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\sin(\theta_3))$$
$$+ a_2\sin(\theta_2))],$$
$$[-\sin(\theta_1)\,(\sin(\theta_2)\,(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\cos(\theta_3))$$
$$- \cos(\theta_2)\,(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\sin(\theta_3))$$
$$+ a_2\sin(\theta_2))],$$
$$[\sin(\theta_1)\,(\sin(\theta_1)\,(\cos(\theta_2)\,(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4)$$
$$+ a_3\cos(\theta_3)) + \sin(\theta_2)\,(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4)$$
$$+ a_3\sin(\theta_3)) + a_2\cos(\theta_2)) + \cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\, d_6) + \cos(\theta_1)\,(\cos(\theta_1)\,(\cos(\theta_2)\,($$
$$-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\cos(\theta_3)) + \sin(\theta_2)\,($$
$$-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\sin(\theta_3)) + a_2\cos(\theta_2))$$
$$- \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\, d_6)]]$$

$$J_{B3} := CrossProduct(Z_2, (P_{06} - P_{02})) =$$
$$[[\cos(\theta_1)\,(\sin(\theta_2)\,(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 - \sin(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\cos(\theta_3))$$
$$- \cos(\theta_2)\,(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\, d_6 + \cos(\theta_3)\,(\cos(\theta_5)\, d_6 + d_4) + a_3\sin(\theta_3)))],$$

$$\left[\sin(\theta_1)\left(\sin(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\right)\right.\right.$$
$$\left.\left.-\cos(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\right)\right)\right],$$
$$\left[-\sin(\theta_1)\left(\sin(\theta_1)\left(\cos(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)\right.\right.\right.$$
$$+a_3\cos(\theta_3)\big)+\sin(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)\right.$$
$$+a_3\sin(\theta_3)\big)+a_2\cos(\theta_2)\big)+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_1)\,a_2\cos(\theta_2)\big)$$
$$-\cos(\theta_1)\left(\cos(\theta_1)\left(\cos(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)\right.\right.\right.$$
$$+a_3\cos(\theta_3)\big)+\sin(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)\right.$$
$$\left.+a_3\sin(\theta_3)\big)+a_2\cos(\theta_2)\big)-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_1)\,a_2\cos(\theta_2)\big)\right]\Big]$$

$$J_{B4}:=CrossProduct\left(Z_3,\left(P_{06}-P_{03}\right)\right)=$$
$$\Big[\Big[\sin(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\left(\sin(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.\right.$$
$$-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)-\cos(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.$$
$$+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)-\sin(\theta_2)\,a_3\cos(\theta_3)+\cos(\theta_2)\,a_3\sin(\theta_3)\big)-\big($$
$$-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\big)\left(\sin(\theta_1)\left(\cos(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.\right.\right.$$
$$-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)+\sin(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.$$
$$+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)+a_2\cos(\theta_2)\big)+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6$$
$$-\sin(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3)+\sin(\theta_2)\,a_3\sin(\theta_3)+a_2\cos(\theta_2)\right)\big)\Big],$$
$$\Big[-\cos(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\left(\sin(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.\right.$$
$$-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)-\cos(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.$$
$$+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)-\sin(\theta_2)\,a_3\cos(\theta_3)+\cos(\theta_2)\,a_3\sin(\theta_3)\big)+\big($$
$$-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\big)\left(\cos(\theta_1)\left(\cos(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.\right.\right.$$
$$-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)+\sin(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.$$
$$+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)+a_2\cos(\theta_2)\big)-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6$$
$$-\cos(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3)+\sin(\theta_2)\,a_3\sin(\theta_3)+a_2\cos(\theta_2)\right)\big)\Big],$$
$$\Big[\cos(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\left(\sin(\theta_1)\left(\cos(\theta_2)\right.\right.\big($$
$$-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)+\sin(\theta_2)\big($$
$$-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)+a_2\cos(\theta_2)\big)$$
$$+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3)+\sin(\theta_2)\,a_3\sin(\theta_3)\right.$$
$$+a_2\cos(\theta_2)\big)\big)-\sin(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\left(\cos(\theta_1)\left(\cos(\theta_2)\right.\right.\big($$
$$-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)+\sin(\theta_2)\big($$
$$-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)+a_2\cos(\theta_2)\big)$$
$$-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-\cos(\theta_1)\left(\cos(\theta_2)\,a_3\cos(\theta_3)+\sin(\theta_2)\,a_3\sin(\theta_3)\right.$$
$$\left.+a_2\cos(\theta_2)\big)\big)\Big]\Big]$$

$$J_{B5}:=simplify\left(CrossProduct\left(Z_4,\left(P_{06}-P_{04}\right)\right)\right)=$$
$$\Big[\Big[-d_6\left(\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)+\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)\right.$$
$$+\sin(\theta_1)\sin(\theta_4)\cos(\theta_5)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)$$
$$\left.-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5)\right)\Big],$$
$$\Big[-d_6\left(\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)+\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)\right.$$
$$-\cos(\theta_1)\sin(\theta_4)\cos(\theta_5)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\sin(\theta_5)$$
$$\left.-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\sin(\theta_5)\right)\Big],$$
$$\Big[-d_6\left(\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)-\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)\right.$$
$$\left.-\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)-\cos(\theta_3)\cos(\theta_2)\sin(\theta_5)\right)\Big]\Big]$$

$$J_{B6}:=simplify\left(CrossProduct\left(Z_5,\left(P_{06}-P_{05}\right)\right)\right)=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

#Assembling Jacobian matrix

$$J_B:=\begin{bmatrix}J_{B1}[1] & J_{B2}[1] & J_{B3}[1] & J_{B4}[1] & J_{B5}[1] & J_{B6}[1]\\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] & J_{B4}[2] & J_{B5}[2] & J_{B6}[2]\\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] & J_{B4}[3] & J_{B5}[3] & J_{B6}[3]\\ Z_0[1] & Z_1[1] & Z_2[1] & Z_3[1] & Z_4[1] & Z_5[1]\\ Z_0[2] & Z_1[2] & Z_2[2] & Z_3[2] & Z_4[2] & Z_5[2]\\ Z_0[3] & Z_1[3] & Z_2[3] & Z_3[3] & Z_4[3] & Z_5[3]\end{bmatrix}=$$
$$\Big[\Big[-\sin(\theta_1)\left(\cos(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\right)\right.$$
$$+\sin(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\right)$$
$$+a_2\cos(\theta_2)\big)-\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-a_1\sin(\theta_1),\ -\cos(\theta_1)\left(\sin(\theta_2)\right.\big($$
$$-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)-\cos(\theta_2)\big($$
$$-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)+a_2\sin(\theta_2)\big),$$
$$\cos(\theta_1)\left(\sin(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\right)\right.$$
$$\left.-\cos(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\right)\right),$$
$$\sin(\theta_1)\left(\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\right)\left(\sin(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.\right.$$
$$-\sin(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\cos(\theta_3)\big)-\cos(\theta_2)\left(-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.$$
$$+\cos(\theta_3)\left(\cos(\theta_5)\,d_6+d_4\right)+a_3\sin(\theta_3)\big)-\sin(\theta_2)\,a_3\cos(\theta_3)+\cos(\theta_2)\,a_3\sin(\theta_3)\big)-\big($$
$$-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\big)\left(\sin(\theta_1)\left(\cos(\theta_2)\left(-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\right.\right.\right.$$

$$-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\sin(\theta_1)\ (\cos(\theta_2)\ a_3\cos(\theta_3)+\sin(\theta_2)\ a_3\sin(\theta_3)+a_2\cos(\theta_2))),$$
$$-d_6\ (\cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\ \cos(\theta_4)\ \cos(\theta_5)+\cos(\theta_1)\ \cos(\theta_3)\ \cos(\theta_2)\ \cos(\theta_4)\ \cos(\theta_5)$$
$$+\sin(\theta_1)\ \sin(\theta_4)\ \cos(\theta_5)+\cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\ \sin(\theta_5)$$
$$-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)\ \sin(\theta_5)),0],$$
$$[\cos(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))$$
$$+\sin(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))$$
$$+a_2\cos(\theta_2))-\sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6+a_1\cos(\theta_1),-\sin(\theta_1)\ (\sin(\theta_2)\ ($$
$$-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))-\cos(\theta_2)\ ($$
$$-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\sin(\theta_2)),$$
$$\sin(\theta_1)\ (\sin(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))$$
$$-\cos(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))),$$
$$-\cos(\theta_1)\ (\sin(\theta_2)\ \cos(\theta_3)-\cos(\theta_2)\ \sin(\theta_3))\ (\sin(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))-\cos(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))-\sin(\theta_2)\ a_3\cos(\theta_3)+\cos(\theta_2)\ a_3\sin(\theta_3))+($$
$$-\sin(\theta_2)\ \sin(\theta_3)-\cos(\theta_2)\ \cos(\theta_3))\ (\cos(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\cos(\theta_1)\ (\cos(\theta_2)\ a_3\cos(\theta_3)+\sin(\theta_2)\ a_3\sin(\theta_3)+a_2\cos(\theta_2))),$$
$$-d_6\ (\sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\ \cos(\theta_4)\ \cos(\theta_5)+\sin(\theta_1)\ \cos(\theta_3)\ \cos(\theta_2)\ \cos(\theta_4)\ \cos(\theta_5)$$
$$-\cos(\theta_1)\ \sin(\theta_4)\ \cos(\theta_5)+\sin(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\ \sin(\theta_5)$$
$$-\sin(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)\ \sin(\theta_5)),0],$$
$$[0,\sin(\theta_1)\ (\sin(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)$$
$$+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)$$
$$+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6)+\cos(\theta_1)\ (\cos(\theta_1)\ (\cos(\theta_2)\ ($$
$$-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ ($$
$$-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))$$
$$-\sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6),-\sin(\theta_1)\ (\sin(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\sin(\theta_1)\ a_2\cos(\theta_2))-\cos(\theta_1)\ (\cos(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6$$
$$+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6$$

$$-\cos(\theta_1)\ a_2\cos(\theta_2)),\cos(\theta_1)\ (\sin(\theta_2)\ \cos(\theta_3)-\cos(\theta_2)\ \sin(\theta_3))\ (\sin(\theta_1)\ (\cos(\theta_2)\ ($$
$$-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ ($$
$$-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))$$
$$+\cos(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_1)\ (\cos(\theta_2)\ a_3\cos(\theta_3)+\sin(\theta_2)\ a_3\sin(\theta_3)$$
$$+a_2\cos(\theta_2)))-\sin(\theta_1)\ (\sin(\theta_2)\ \cos(\theta_3)-\cos(\theta_2)\ \sin(\theta_3))\ (\cos(\theta_1)\ (\cos(\theta_2)\ ($$
$$-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ ($$
$$-\sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6+\cos(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))$$
$$-\sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)\ d_6-\cos(\theta_1)\ (\cos(\theta_2)\ a_3\cos(\theta_3)+\sin(\theta_2)\ a_3\sin(\theta_3)$$
$$+a_2\cos(\theta_2))),-d_6\ (\sin(\theta_2)\ \cos(\theta_3)\ \cos(\theta_4)\ \cos(\theta_5)-\cos(\theta_2)\ \sin(\theta_3)\ \cos(\theta_4)\ \cos(\theta_5)$$
$$-\sin(\theta_2)\ \sin(\theta_3)\ \sin(\theta_5)-\cos(\theta_3)\ \cos(\theta_2)\ \sin(\theta_5)),0],$$
$$[0,\sin(\theta_1),-\sin(\theta_1),\cos(\theta_1)\ (\sin(\theta_2)\ \cos(\theta_3)-\cos(\theta_2)\ \sin(\theta_3)),$$
$$\sin(\theta_4)\ \cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)+\sin(\theta_4)\ \cos(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)-\sin(\theta_1)\ \cos(\theta_4),$$
$$-\cos(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)-\cos(\theta_1)\ \cos(\theta_3)\ \cos(\theta_2)\ \cos(\theta_4)\ \sin(\theta_5)$$
$$-\sin(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)+\cos(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\ \cos(\theta_5)$$
$$-\cos(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)\ \cos(\theta_5)],$$
$$[0,-\cos(\theta_1),\cos(\theta_1),\sin(\theta_1)\ (\sin(\theta_2)\ \cos(\theta_3)-\cos(\theta_2)\ \sin(\theta_3)),$$
$$\sin(\theta_4)\ \sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)+\sin(\theta_4)\ \sin(\theta_1)\ \cos(\theta_2)\ \cos(\theta_3)+\cos(\theta_1)\ \cos(\theta_4),$$
$$-\sin(\theta_1)\ \sin(\theta_2)\ \sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)-\sin(\theta_1)\ \cos(\theta_3)\ \cos(\theta_2)\ \cos(\theta_4)\ \sin(\theta_5)$$
$$+\cos(\theta_1)\ \sin(\theta_4)\ \sin(\theta_5)+\sin(\theta_1)\ \sin(\theta_2)\ \cos(\theta_3)\ \cos(\theta_5)$$
$$-\sin(\theta_1)\ \cos(\theta_2)\ \sin(\theta_3)\ \cos(\theta_5)],$$
$$[1,0,0,-\sin(\theta_2)\ \sin(\theta_3)-\cos(\theta_2)\ \cos(\theta_3),(\sin(\theta_2)\ \cos(\theta_3)-\cos(\theta_2)\ \sin(\theta_3))\ \sin(\theta_4),$$
$$-\sin(\theta_2)\ \cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)+\cos(\theta_2)\ \sin(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)$$
$$-\sin(\theta_2)\ \sin(\theta_3)\ \cos(\theta_5)-\cos(\theta_3)\ \cos(\theta_2)\ \cos(\theta_5)]]$$

#Jacobian matrix decoupling

$$\#J:=\begin{bmatrix} J_{11} & J_{21} \\ J_{12} & J_{22} \end{bmatrix}$$

$$J_{11}:=\begin{bmatrix} J_{B1}[1] & J_{B2}[1] & J_{B3}[1] \\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] \\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] \end{bmatrix}=$$

$$[[-\sin(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\ \cos(\theta_4)\ \sin(\theta_5)\ d_6-\sin(\theta_3)\ (\cos(\theta_5)\ d_6+d_4)+a_3\cos(\theta_3))$$

$$
\begin{aligned}
&+\sin(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))\\
&+a_2\cos(\theta_2))-\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6-a_1\sin(\theta_1),\ -\cos(\theta_1)\ (\sin(\theta_2)\ (\\
&-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))-\cos(\theta_2)\ (\\
&-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))+a_2\sin(\theta_2)),\\
&\cos(\theta_1)\ (\sin(\theta_2)\ (-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))\\
&-\cos(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))))],\\
&[\cos(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))\\
&+\sin(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))\\
&+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6+a_1\cos(\theta_1),\ -\sin(\theta_1)\ (\sin(\theta_2)\ (\\
&-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))-\cos(\theta_2)\ (\\
&-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))+a_2\sin(\theta_2)),\\
&\sin(\theta_1)\ (\sin(\theta_2)\ (-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))\\
&-\cos(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))))],\\
&[0,\ \sin(\theta_1)\ (\sin(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)\\
&+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)\\
&+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6)+\cos(\theta_1)\ (\cos(\theta_1)\ (\cos(\theta_2)\ (\\
&-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (\\
&-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))\\
&-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6),\ -\sin(\theta_1)\ (\sin(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\\
&-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\\
&+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6\\
&-\sin(\theta_1)\,a_2\cos(\theta_2))-\cos(\theta_1)\ (\cos(\theta_1)\ (\cos(\theta_2)\ (-\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\\
&-\sin(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\cos(\theta_3))+\sin(\theta_2)\ (-\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\\
&+\cos(\theta_3)\ (\cos(\theta_5)\,d_6+d_4)+a_3\sin(\theta_3))+a_2\cos(\theta_2))-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\,d_6\\
&-\cos(\theta_1)\,a_2\cos(\theta_2))]]
\end{aligned}
$$

=

simplify(*Determinant*(J_{11})) =

$$
\begin{aligned}
&a_2\ (-\sin(\theta_2)\,a_3^2-2\sin(\theta_2)\cos(\theta_4)\sin(\theta_5)\,d_6\,a_3\cos(\theta_3)^2\\
&+\sin(\theta_3)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)^2\cos(\theta_5)^2\,d_6^2-2\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)\,d_6\sin(\theta_3)\,a_3\\
&+a_2\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6+2\cos(\theta_2)\cos(\theta_3)^2\cos(\theta_4)\sin(\theta_5)\,d_6^2\cos(\theta_5)\\
&+2\cos(\theta_2)\cos(\theta_3)^2\cos(\theta_4)\sin(\theta_5)\,d_6\,d_4+2\sin(\theta_3)\cos(\theta_2)\cos(\theta_5)\,d_6\cos(\theta_3)\,d_4\\
&+\cos(\theta_2)\cos(\theta_5)\,d_6\,a_3-\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_5)^2\,d_6^2-\cos(\theta_2)\cos(\theta_3)\,a_3^2\sin(\theta_3)\\
&-2\cos(\theta_2)\cos(\theta_3)^2\,a_3\,d_4-a_2\cos(\theta_2)\sin(\theta_3)\,a_3-a_2\cos(\theta_2)\cos(\theta_3)\,d_4
\end{aligned}
$$

$$
\begin{aligned}
&-a_1\cos(\theta_3)\cos(\theta_5)\,d_6+\sin(\theta_3)\cos(\theta_2)\,d_4^2\cos(\theta_3)+\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_4)^2\,d_6^2\\
&+\sin(\theta_2)\cos(\theta_4)^2\cos(\theta_5)^2\,d_6^2-2\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_5)\,d_6\,d_4\\
&-2\sin(\theta_2)\sin(\theta_3)\,a_3\cos(\theta_3)\,d_4+\sin(\theta_3)\cos(\theta_2)\cos(\theta_5)^2\,d_6^2\cos(\theta_3)\\
&+a_1\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6-a_2\cos(\theta_2)\cos(\theta_3)\cos(\theta_5)\,d_6\\
&-2\cos(\theta_2)\cos(\theta_3)^2\,a_3\cos(\theta_5)\,d_6-\sin(\theta_2)\cos(\theta_3)^2\cos(\theta_4)^2\cos(\theta_5)^2\,d_6^2\\
&+2\sin(\theta_2)\cos(\theta_4)\sin(\theta_5)\,d_6\,a_3-\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)^2\,d_6^2\sin(\theta_3)-\cos(\theta_2)\cos(\theta_5)\\
&d_6^2\cos(\theta_4)\sin(\theta_5)-\cos(\theta_2)\,d_4\cos(\theta_4)\sin(\theta_5)\,d_6+2\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\\
&d_6^2\cos(\theta_3)\cos(\theta_5)+2\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\cos(\theta_3)\,d_4\\
&+2\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)\,d_6\sin(\theta_3)\,a_3-\sin(\theta_2)\cos(\theta_4)^2\,d_6^2+\cos(\theta_2)\,d_4\,a_3\\
&-a_1\sin(\theta_3)\,a_3-a_1\cos(\theta_3)\,d_4-\sin(\theta_2)\cos(\theta_3)^2\,d_4^2+\sin(\theta_2)\cos(\theta_3)^2\,a_3^2)
\end{aligned}
$$

$$
J_{22} := \begin{bmatrix} Z_3[1] & Z_4[1] & Z_5[1] \\ Z_3[2] & Z_4[2] & Z_5[2] \\ Z_3[3] & Z_4[3] & Z_5[3] \end{bmatrix} =
$$

$$
\begin{aligned}
&[[\cos(\theta_1)\ (\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)),\ \sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\\
&\quad+\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\sin(\theta_1)\cos(\theta_4),\\
&\quad-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)-\cos(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)\\
&\quad-\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)+\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)\\
&\quad-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],\\
&\quad[\sin(\theta_1)\ (\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)),\ \sin(\theta_4)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\\
&\quad+\sin(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cos(\theta_1)\cos(\theta_4),\\
&\quad-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)-\sin(\theta_1)\cos(\theta_3)\cos(\theta_2)\cos(\theta_4)\sin(\theta_5)\\
&\quad+\cos(\theta_1)\sin(\theta_4)\sin(\theta_5)+\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_5)\\
&\quad-\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)\cos(\theta_5)],\\
&\quad[-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3),\ (\sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3))\sin(\theta_4),\\
&\quad-\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\sin(\theta_5)+\cos(\theta_2)\sin(\theta_3)\cos(\theta_4)\sin(\theta_5)\\
&\quad-\sin(\theta_2)\sin(\theta_3)\cos(\theta_5)-\cos(\theta_3)\cos(\theta_2)\cos(\theta_5)]]
\end{aligned}
$$

simplify(*Determinant*(J_{22})) = $-\sin(\theta_5)$

#Determining singularities (Manipulability measures)

simplify(*Determinant*(J_B)) =

$$\sin(\theta_5)\left(2\sin(\theta_2)\sin(\theta_3)\,a_3\cos(\theta_3)\,d_4-\sin(\theta_2)\cos(\theta_3)^2a_3^2+\sin(\theta_2)\cos(\theta_3)^2d_4^2+\cos(\theta_2)\cos(\theta_3)\,a_3^2\sin(\theta_3)-\sin(\theta_3)\cos(\theta_2)\,d_4^2\cos(\theta_3)+2\cos(\theta_2)\cos(\theta_3)^2a_3\,d_4+a_2\cos(\theta_2)\sin(\theta_3)\,a_3+a_2\cos(\theta_2)\cos(\theta_3)\,d_4+\sin(\theta_2)\,a_3^2+a_1\sin(\theta_3)\,a_3-\cos(\theta_2)\,d_4\,a_3+a_1\cos(\theta_3)\,d_4\right)a_2$$

$$\xrightarrow{\text{equate to 0}}$$

$$\sin(\theta_5)\left(2\sin(\theta_2)\sin(\theta_3)\,a_3\cos(\theta_3)\,d_4-\sin(\theta_2)\cos(\theta_3)^2a_3^2+\sin(\theta_2)\cos(\theta_3)^2d_4^2+\cos(\theta_2)\cos(\theta_3)\,a_3^2\sin(\theta_3)-\sin(\theta_3)\cos(\theta_2)\,d_4^2\cos(\theta_3)+2\cos(\theta_2)\cos(\theta_3)^2a_3\,d_4+a_2\cos(\theta_2)\sin(\theta_3)\,a_3+a_2\cos(\theta_2)\cos(\theta_3)\,d_4+\sin(\theta_2)\,a_3^2+a_1\sin(\theta_3)\,a_3-\cos(\theta_2)\,d_4\,a_3+a_1\cos(\theta_3)\,d_4\right)a_2=0$$

$$\xrightarrow{\text{solve for theta[5]}}[[\theta_5=0]]$$

#Boundary Singularity condition

$$Cb := -a_3\sin(\theta_3)-d_4\cos(\theta_3) = -a_3\sin(\theta_3)-\cos(\theta_3)\,d_4 \xrightarrow{\text{equate to 0}} -a_3\sin(\theta_3)-\cos(\theta_3)\,d_4=0$$

$$\xrightarrow{\text{solve for theta[3]}}\left[\left[\theta_3=-\arctan\left(\frac{d_4}{a_3}\right)\right]\right]$$

$$(\Theta_3) := \left(-\arctan\left(\frac{-230}{390}\right)\cdot\frac{180}{\pi}\right) = \frac{180\arctan\left(\frac{23}{39}\right)}{\pi} \xrightarrow{\text{at 5 digits}} 30.530$$

#Interior Singularity condition

$$Ca := a_1+a_2\cos(\theta_2)+a_3\cos(\theta_2+\theta_3)+d_4\sin(\theta_2+\theta_3) =$$
$$a_1+a_2\cos(\theta_2)+a_3\cos(\theta_2+\theta_3)+d_4\sin(\theta_2+\theta_3)$$

$$135+250\cos(\theta_2)+390\cos(\theta_2+30.53)+230\sin(\theta_2+30.53)=0$$
$$135+250\cos(\theta_2)+390\cos(\theta_2+30.53)+230\sin(\theta_2+30.53)=0 \xrightarrow{\text{solve for theta[2]}} [[\theta_2=2.771274271],[\theta_2=-0.8668959665]]$$

$$\Theta_{2.1} := 2.771274271\cdot\frac{180}{\pi} = \frac{498.8293688}{\pi} \xrightarrow{\text{at 5 digits}} 158.78$$
$$\Theta_{2.2} := -0.8668959665\cdot\frac{180}{\pi} = -\frac{156.0412740}{\pi} \xrightarrow{\text{at 5 digits}} -49.669$$

Appendix D
CNC − R GKM 6 DOF Reconfigurable FK and Jacobian Matrix

CNC-R GKM DH reconfigurable parameters

i	θ_i	d_i	a_i	α_i
1	$R_1\theta_1+T_1\theta_{DH1}$	$R_1d_{DH1}+T_1d_1$	a_1	±90°
2	$R_2\theta_2+T_2\theta_{DH2}$	$R_2d_{DH2}+T_2d_2$	a_2	±180°;±90°;0
3	$R_3\theta_3+T_3\theta_{DH3}$	$R_3d_{DH3}+T_3d_3$	a_3	±180°;±90°;0
4	θ_{DH4}	d_{DH4}	a_4	±90°
5	θ_{DH5}	d_{DH5}	a_5	±90°
6	θ_{DH6}	d_{DH6}	a_6	±90°;0°

$with(LinearAlgebra)$:

#D-H parameters

$\#d_{DH1} := 0$
$\#d_{DH2} := 0$
$\#d_{DH3} := 0$
$\#d_{DH4} := 0$
$\#d_{DH5} := 0$
$\#d_{DH6} := 0$

$\#\theta_{DH1} := 0$
$\#\theta_{DH2} := 0$
$\#\theta_{DH3} := 0$
$\#\theta_{DH4} := 0$
$\#\theta_{DH5} := 0$
$\#\theta_{DH6} := 0$

$\#a_1 := 0$
$\#a_2 := 0$
$\#a_3 := 0$
$\#a_4 := 0$
$\#a_5 := 0$
$\#a_6 := 0$

$\#\alpha_1 := \pm\frac{\pi}{2}$

$\#\alpha_2 := \pm\pi; \pm\frac{\pi}{2}; 0$

$\#\alpha_3 := \pm\pi; \pm\frac{\pi}{2}; 0$

$\#\alpha_4 := \pm\frac{\pi}{2}$

$\#\alpha_5 := \pm\frac{\pi}{2}$

$\#\alpha_6 := \pm\frac{\pi}{2}$

#Determination of the Reconfigurable parameters

$\#R_1 := 1$
$\#R_2 := 1$
$\#R_3 := 1$

$\boldsymbol{R_4 := 1} = 1$
$\boldsymbol{R_5 := 1} = 1$
$\boldsymbol{R_6 := 1} = 1$

$\#T_1 := 0$
$\#T_2 := 0$
$\#T_3 := 0$

$\boldsymbol{T_4 := 0} = 0$
$\boldsymbol{T_5 := 0} = 0$
$\boldsymbol{T_6 := 0} = 0$

$\#K_{s1} := \sin(\alpha_1)$
$\#K_{s2} := \sin(\alpha_2)$
$\#K_{s3} := \sin(\alpha_3)$
$\#K_{s4} := \sin(\alpha_4)$
$\#K_{s5} := \sin(\alpha_5)$
$\#K_{s6} := \sin(\alpha_6)$

$\#K_{c1} := \cos(\alpha_1)$
$\#K_{c2} := \cos(\alpha_2)$
$\#K_{c3} := \cos(\alpha_3)$
$\#K_{c4} := \cos(\alpha_4)$

$\#K_{c5} := \cos(\alpha_5)$
$\#K_{c6} := \cos(\alpha_6)$

#Twist angles are limited and some reconfigurable parameters are equal to zero

$\boldsymbol{\#K_{s2} := 0} =$ **(when add CNC config)**
$\boldsymbol{K_{c1} := 0} = 0$
$\boldsymbol{\#K_{s3} := 0}$
$\boldsymbol{K_{c4} := 0} = 0$
$\boldsymbol{K_{c5} := 0} = 0$
$\boldsymbol{K_{c6} := 0} = 0$

#A Matrices

$$A_1 := [[\cos(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1}), -K_{c1}\cdot\sin(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1}), K_{s1}\cdot\sin(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1}), a_1\cdot\cos(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1})],$$
$$[\sin(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1}), K_{c1}\cdot\cos(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1}), -K_{s1}\cdot\cos(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1}), a_1\cdot\sin(R_1\cdot\theta_1 + T_1\cdot\theta_{DH1})],$$
$$[0, K_{s1}, K_{c1}, R_1\cdot d_{DH1} + T_1\cdot d_1],$$
$$[0, 0, 0, 1]]$$
$$= \begin{bmatrix} \cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) & 0 & K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) & a_1\cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) \\ \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) & 0 & -K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) & a_1\sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \\ 0 & K_{s1} & 0 & R_1\,d_{DH1} + T_1\,d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RO_{01} := \begin{bmatrix} A_1[1,1] & A_1[1,2] & A_1[1,3] \\ A_1[2,1] & A_1[2,2] & A_1[2,3] \\ A_1[3,1] & A_1[3,2] & A_1[3,3] \end{bmatrix} = \begin{bmatrix} \cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) & 0 & K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \\ \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) & 0 & -K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) \\ 0 & K_{s1} & 0 \end{bmatrix}$$

$$A_2 := [[\cos(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2}), -K_{c2}\cdot\sin(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2}), K_{s2}\cdot\sin(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2}), a_2\cdot\cos(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2})],$$
$$[\sin(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2}), K_{c2}\cdot\cos(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2}), -K_{s2}\cdot\cos(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2}), a_2\cdot\sin(R_2\cdot\theta_2 + T_2\cdot\theta_{DH2})],$$

$$\left[0, K_{s2}, K_{c2}, R_2 \cdot d_{DH2} + T_2 \cdot d_2\right],\ \left[0, 0, 0, 1\right]\right]$$

$$= \left[\left[\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right), -K_{c2}\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right), K_{s2}\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right), a_2\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\right], \left[\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right), K_{c2}\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right), -K_{s2}\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right), a_2\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\right], \left[0, K_{s2}, K_{c2}, R_2\,d_{DH2} + T_2\,d_2\right], \left[0, 0, 0, 1\right]\right]$$

$$RO_{12} := \begin{bmatrix} A_2[1,1] & A_2[1,2] & A_2[1,3] \\ A_2[2,1] & A_2[2,2] & A_2[2,3] \\ A_2[3,1] & A_2[3,2] & A_2[3,3] \end{bmatrix} = \begin{bmatrix} \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) & -K_{c2}\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) & K_{s2}\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) \\ \sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) & K_{c2}\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) & -K_{s2}\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) \\ 0 & K_{s2} & K_{c2} \end{bmatrix}$$

$$A_3 := \left[\left[\cos\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right), -K_{c3} \cdot \sin\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right), K_{s3} \cdot \sin\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right), a_3 \cdot \cos\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right)\right], \left[\sin\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right), K_{c3} \cdot \cos\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right), -K_{s3} \cdot \cos\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right), a_3 \cdot \sin\left(R_3 \cdot \theta_3 + T_3 \cdot \theta_{DH3}\right)\right], \left[0, K_{s3}, K_{c3}, R_3 \cdot d_{DH3} + T_3 \cdot d_3\right], \left[0, 0, 0, 1\right]\right]$$

$$= \left[\left[\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right), -K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right), K_{s3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right), a_3\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\right], \left[\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right), K_{c3}\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right), -K_{s3}\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right), a_3\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\right], \left[0, K_{s3}, K_{c3}, R_3\,d_{DH3} + T_3\,d_3\right], \left[0, 0, 0, 1\right]\right]$$

$$RO_{23} := \begin{bmatrix} A_3[1,1] & A_3[1,2] & A_3[1,3] \\ A_3[2,1] & A_3[2,2] & A_3[2,3] \\ A_3[3,1] & A_3[3,2] & A_3[3,3] \end{bmatrix} = \begin{bmatrix} \cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) & -K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) & K_{s3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) \\ \sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) & K_{c3}\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) & -K_{s3}\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) \\ 0 & K_{s3} & K_{c3} \end{bmatrix}$$

$$A_4 := \left[\left[\cos\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right), -K_{c4} \cdot \sin\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right), K_{s4} \cdot \sin\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right), a_4 \cdot \cos\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right)\right], \left[\sin\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right), K_{c4} \cdot \cos\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right), -K_{s4} \cdot \cos\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right), a_4 \cdot \sin\left(R_4 \cdot \theta_4 + T_4 \cdot \theta_{DH4}\right)\right], \left[0, K_{s4}, K_{c4}, R_4 \cdot d_{DH4} + T_4 \cdot d_4\right], \left[0, 0, 0, 1\right]\right]$$

$$= \begin{bmatrix} \cos(\theta_4) & 0 & K_{s4}\sin(\theta_4) & a_4\cos(\theta_4) \\ \sin(\theta_4) & 0 & -K_{s4}\cos(\theta_4) & a_4\sin(\theta_4) \\ 0 & K_{s4} & 0 & d_{DH4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RO_{34} := \begin{bmatrix} A_4[1,1] & A_4[1,2] & A_4[1,3] \\ A_4[2,1] & A_4[2,2] & A_4[2,3] \\ A_4[3,1] & A_4[3,2] & A_4[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_4) & 0 & K_{s4}\sin(\theta_4) \\ \sin(\theta_4) & 0 & -K_{s4}\cos(\theta_4) \\ 0 & K_{s4} & 0 \end{bmatrix}$$

$$A_5 := \left[\left[\cos\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right), -K_{c5} \cdot \sin\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right), K_{s5} \cdot \sin\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right), a_5 \cdot \cos\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right)\right], \left[\sin\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right), K_{c5} \cdot \cos\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right), -K_{s5} \cdot \cos\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right), a_5 \cdot \sin\left(R_5 \cdot \theta_5 + T_5 \cdot \theta_{DH5}\right)\right], \left[0, K_{s5}, K_{c5}, R_5 \cdot d_{DH5} + T_5 \cdot d_5\right], \left[0, 0, 0, 1\right]\right]$$

$$= \begin{bmatrix} \cos(\theta_5) & 0 & K_{s5}\sin(\theta_5) & a_5\cos(\theta_5) \\ \sin(\theta_5) & 0 & -K_{s5}\cos(\theta_5) & a_5\sin(\theta_5) \\ 0 & K_{s5} & 0 & d_{DH5} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$=$$

$$RO_{45} := \begin{bmatrix} A_5[1,1] & A_5[1,2] & A_5[1,3] \\ A_5[2,1] & A_5[2,2] & A_5[2,3] \\ A_5[3,1] & A_5[3,2] & A_5[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_5) & 0 & K_{s5}\sin(\theta_5) \\ \sin(\theta_5) & 0 & -K_{s5}\cos(\theta_5) \\ 0 & K_{s5} & 0 \end{bmatrix}$$

$$A_6 := \Big[\big[\cos(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6}), -K_{c6}\cdot\sin(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6}), K_{s6}\cdot\sin(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6}), a_6\cdot\cos(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6})\big],$$
$$\big[\sin(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6}), K_{c6}\cdot\cos(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6}), -K_{s6}\cdot\cos(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6}), a_6\cdot\sin(R_6\cdot\theta_6 + T_6\cdot\theta_{DH6})\big],$$
$$\big[0, K_{s6}, K_{c6}, R_6\cdot d_{DH6} + T_6\cdot d_6\big],$$
$$\big[0, 0, 0, 1\big]\Big]$$

$$= \begin{bmatrix} \cos(\theta_6) & 0 & K_{s6}\sin(\theta_6) & a_6\cos(\theta_6) \\ \sin(\theta_6) & 0 & -K_{s6}\cos(\theta_6) & a_6\sin(\theta_6) \\ 0 & K_{s6} & 0 & d_{DH6} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RO_{56} := \begin{bmatrix} A_6[1,1] & A_6[1,2] & A_6[1,3] \\ A_6[2,1] & A_6[2,2] & A_6[2,3] \\ A_6[3,1] & A_6[3,2] & A_6[3,3] \end{bmatrix} = \begin{bmatrix} \cos(\theta_6) & 0 & K_{s6}\sin(\theta_6) \\ \sin(\theta_6) & 0 & -K_{s6}\cos(\theta_6) \\ 0 & K_{s6} & 0 \end{bmatrix}$$

#Position Matrices

$$P_{01} := \begin{bmatrix} A_1[1,4] \\ A_1[2,4] \\ A_1[3,4] \end{bmatrix} = \begin{bmatrix} a_1\cos(R_1\theta_1 + T_1\theta_{DH1}) \\ a_1\sin(R_1\theta_1 + T_1\theta_{DH1}) \\ R_1 d_{DH1} + T_1 d_1 \end{bmatrix}$$

$$P_{12} := \begin{bmatrix} A_2[1,4] \\ A_2[2,4] \\ A_2[3,4] \end{bmatrix} = \begin{bmatrix} a_2\cos(R_2\theta_2 + T_2\theta_{DH2}) \\ a_2\sin(R_2\theta_2 + T_2\theta_{DH2}) \\ R_2 d_{DH2} + T_2 d_2 \end{bmatrix}$$

$$P_{23} := \begin{bmatrix} A_3[1,4] \\ A_3[2,4] \\ A_3[3,4] \end{bmatrix} = \begin{bmatrix} a_3\cos(R_3\theta_3 + T_3\theta_{DH3}) \\ a_3\sin(R_3\theta_3 + T_3\theta_{DH3}) \\ R_3 d_{DH3} + T_3 d_3 \end{bmatrix}$$

$$P_{34} := \begin{bmatrix} A_4[1,4] \\ A_4[2,4] \\ A_4[3,4] \end{bmatrix} = \begin{bmatrix} a_4\cos(\theta_4) \\ a_4\sin(\theta_4) \\ d_{DH4} \end{bmatrix}$$

$$P_{45} := \begin{bmatrix} A_5[1,4] \\ A_5[2,4] \\ A_5[3,4] \end{bmatrix} = \begin{bmatrix} a_5\cos(\theta_5) \\ a_5\sin(\theta_5) \\ d_{DH5} \end{bmatrix}$$

$$P_{56} := \begin{bmatrix} A_6[1,4] \\ A_6[2,4] \\ A_6[3,4] \end{bmatrix} = \begin{bmatrix} a_6\cos(\theta_6) \\ a_6\sin(\theta_6) \\ d_{DH6} \end{bmatrix}$$

#Separate ROTATION MAtrix

$$RO_{01} := \begin{bmatrix} A_1[1,1] & A_1[1,2] & A_1[1,3] \\ A_1[2,1] & A_1[2,2] & A_1[2,3] \\ A_1[3,1] & A_1[3,2] & A_1[3,3] \end{bmatrix} = \begin{bmatrix} \cos(R_1\theta_1 + T_1\theta_{DH1}) & 0 & K_{s1}\sin(R_1\theta_1 + T_1\theta_{DH1}) \\ \sin(R_1\theta_1 + T_1\theta_{DH1}) & 0 & -K_{s1}\cos(R_1\theta_1 + T_1\theta_{DH1}) \\ 0 & K_{s1} & 0 \end{bmatrix}$$

$$RO_{02} := Multiply(RO_{01}, RO_{12}) =$$
$$\Big[\big[\cos(R_1\theta_1 + T_1\theta_{DH1})\cos(R_2\theta_2 + T_2\theta_{DH2}), -\cos(R_1\theta_1 + T_1\theta_{DH1})K_{c2}\sin(R_2\theta_2 + T_2\theta_{DH2}) + K_{s1}\sin(R_1\theta_1 + T_1\theta_{DH1})K_{s2}, \cos(R_1\theta_1 + T_1\theta_{DH1})K_{s2}\sin(R_2\theta_2 + T_2\theta_{DH2}) + K_{s1}\sin(R_1\theta_1 + T_1\theta_{DH1})K_{c2}\big],$$
$$\big[\sin(R_1\theta_1 + T_1\theta_{DH1})\cos(R_2\theta_2 + T_2\theta_{DH2}), -\sin(R_1\theta_1 + T_1\theta_{DH1})K_{c2}\sin(R_2\theta_2 + T_2\theta_{DH2}) - K_{s1}\cos(R_1\theta_1 + T_1\theta_{DH1})K_{s2}, \sin(R_1\theta_1 + T_1\theta_{DH1})K_{s2}\sin(R_2\theta_2 + T_2\theta_{DH2}) - K_{s1}\cos(R_1\theta_1 + T_1\theta_{DH1})K_{c2}\big],$$
$$\big[K_{s1}\sin(R_2\theta_2 + T_2\theta_{DH2}), K_{s1}K_{c2}\cos(R_2\theta_2 + T_2\theta_{DH2}), -K_{s1}K_{s2}\cos(R_2\theta_2 + T_2\theta_{DH2})\big]\Big]$$

$$RO_{03} := simplify\left(Multiply\left(RO_{02}, RO_{23}\right)\right) =$$

$$\left[\left[\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2} - \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3\right.\right.\right.$$
$$\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2} + \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right), -\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) + \sin\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c3}\,K_{s1}\,K_{s2} - \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{c3} + \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{s1}\,K_{s3} + \cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3}, \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{s3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2}\,K_{s3}$$
$$+ \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{s3} + \sin\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)K_{c2}\,K_{c3}\,K_{s1} + \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\,K_{s2}\Big],$$
$$\Big[-\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2} - \cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2} + \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right), -\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \sin\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{c3} - \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)K_{c3}\,K_{s1}\,K_{s2} + \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3} - \cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)K_{c2}\,K_{s1}\,K_{s3}, \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)$$
$$+ \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{s3} + \cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2}\,K_{s3} + \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\,K_{s2}$$
$$- \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{c3}\,K_{s1}\Big],$$
$$\Big[K_{s1}\left(\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2} + \cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2\right.\right.$$
$$\left.\left. + T_2\,\theta_{DH2}\right)\right), -K_{s1}\left(-\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2}\,K_{c3} + \sin\left(R_3\,\theta_3\right.\right.$$
$$\left.\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3} + \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3}\right), K_{s1}\left(-\cos\left(R_2\,\theta_2\right.\right.$$
$$\left. + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2}\,K_{s3} + \sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s3}$$
$$\left.\left. - \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\,K_{s2}\right)\Big]\right]$$

$$RO_{04} := simplify\left(Multiply\left(RO_{03}, RO_{34}\right)\right) =$$

$$\left[\left[\cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2} - \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1\right.\right.\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2} + \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) + \sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)K_{c3}\,K_{s1}\,K_{s2} - \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c2}\,K_{c3} + \sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{s1}\,K_{s3} + \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3}, -\left(-\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{s3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) + \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2}\,K_{s3}$$

$$- \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{s3} - \sin\left(R_1\,\theta_1\right.$$
$$\left.\left. + T_1\,\theta_{DH1}\right)K_{c2}\,K_{c3}\,K_{s1} - \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\,K_{s2}\right)K_{s4},$$
$$K_{s4}\left(\sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2} - \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1\right.\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2} + \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) + \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)K_{c3}\,K_{s1}\,K_{s2} + \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c2}\,K_{c3} - \cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{s1}\,K_{s3} - \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1\right.$$
$$\left.\left. + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3}\right)\Big],$$
$$\Big[-\cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}$$
$$- \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2} + \cos\left(\theta_4\right)\sin\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{c3} - \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)K_{c3}\,K_{s1}\,K_{s2} + \sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3}$$
$$- \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{s1}\,K_{s3}, \left(\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{s3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) + \sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c2}\,K_{s3} + \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2}\,K_{s3} + \sin\left(R_1\,\theta_1\right.$$
$$\left.\left. + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\,K_{s2} - \cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{c3}\,K_{s1}\right)K_{s4},$$
$$-K_{s4}\left(\sin\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\right.$$
$$+ \sin\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{s1}\,K_{s2} - \sin\left(\theta_4\right)\sin\left(R_1\,\theta_1\right.$$
$$\left. + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_2\,\theta_2\right.$$
$$\left. + T_2\,\theta_{DH2}\right)K_{c3}\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right) - \cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c2}\,K_{c3} - \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\cos\left(R_3\,\theta_3\right.$$
$$\left. + T_3\,\theta_{DH3}\right)K_{c3}\,K_{s1}\,K_{s2} + \cos\left(\theta_4\right)\sin\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s2}\,K_{s3}$$
$$\left. - \cos\left(\theta_4\right)\cos\left(R_1\,\theta_1 + T_1\,\theta_{DH1}\right)K_{c2}\,K_{s1}\,K_{s3}\right)\Big],$$
$$\Big[-K_{s1}\left(-\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(\theta_4\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2} - \cos\left(\theta_4\right)\cos\left(R_3\,\theta_3\right.\right.$$
$$\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) - \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\sin\left(\theta_4\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2}\,K_{c3}$$
$$+ \sin\left(\theta_4\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3} + \cos\left(R_2\,\theta_2\right.$$
$$\left.\left. + T_2\,\theta_{DH2}\right)\sin\left(\theta_4\right)K_{s2}\,K_{s3}\right), K_{s1}\left(-\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2}\,K_{s3}\right.$$
$$\left. + \sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{s3} - \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)K_{c3}\,K_{s2}\right)K_{s4},$$
$$K_{s1}\,K_{s4}\left(\cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\sin\left(\theta_4\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2} + \sin\left(\theta_4\right)\cos\left(R_3\,\theta_3\right.\right.$$
$$\left. + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right) - \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(\theta_4\right)\cos\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)K_{c2}\,K_{c3}$$
$$+ \cos\left(R_2\,\theta_2 + T_2\,\theta_{DH2}\right)\cos\left(\theta_4\right)K_{s2}\,K_{s3} + \cos\left(\theta_4\right)\sin\left(R_3\,\theta_3 + T_3\,\theta_{DH3}\right)\sin\left(R_2\,\theta_2\right.$$

$$+ T_2\,\theta_{DH2})\,K_{c3})]]$$

$$RO_{05} := simplify(Multiply(RO_{04}, RO_{45})) =$$

$$[[\cos(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} - \cos(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} + \cos(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \cos(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} - \cos(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} + \cos(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3}$$
$$+ \cos(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{s4}\,\sin(\theta_5)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1}$$
$$+ K_{s4}\,\sin(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2}, K_{s4}\,(\sin(\theta_4)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} - \sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} + \sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} + \cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} - \cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3} - \cos(\theta_4)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3})\,K_{s5}, K_{s5}\,(\sin(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} - \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} + \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} - \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3}$$
$$+ \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1}$$
$$- K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2})],$$
$$[-\cos(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}$$
$$- \cos(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}$$

$$+ \cos(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$- \cos(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$- \cos(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{c3}$$
$$- \cos(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2}$$
$$+ \cos(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3}$$
$$- \cos(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4}\,\sin(\theta_5)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,K_{s2} - K_{s4}\,\sin(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1}, -K_{s4}\,(\sin(\theta_4)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} + \sin(\theta_4)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} - \sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} - \cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} + \cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3}$$
$$- \cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3})\,K_{s5}, -K_{s5}\,(\sin(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} + \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} - \sin(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} + \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} - \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3}$$
$$+ \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,K_{s2} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1})],$$
$$[K_{s1}\,(\cos(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(\theta_4)\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c2}$$
$$+ \cos(\theta_5)\,\cos(\theta_4)\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) + \cos(\theta_5)\,\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,\sin(\theta_4)\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c2}\,K_{c3} - \cos(\theta_5)\,\sin(\theta_4)\,\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3} - \cos(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\sin(\theta_4)\,K_{s2}\,K_{s3}$$
$$- K_{s4}\,\sin(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c2}\,K_{s3} + K_{s4}\,\sin(\theta_5)\,\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3} - K_{s4}\,\sin(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2}),$$

$$K_{s1} K_{s4} \left(\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} + \sin(\theta_4) \cos(R_3 \theta_3\right.$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{c3}$$
$$+ \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) K_{s2} K_{s3} + \cos(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$\left.+ T_2 \theta_{DH2}) K_{c3}\right) K_{s5}, -K_{s1} K_{s5} \left(-\sin(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2}\right.$$
$$- \sin(\theta_5) \cos(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - \sin(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{c3} + \sin(\theta_5) \sin(\theta_4) \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} + \sin(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) K_{s2} K_{s3}$$
$$- K_{s4} \cos(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_3 \theta_3$$
$$\left.+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}\right)]]$$

$RO_{06} := simplify(Multiply(RO_{05}, RO_{56})) =$

$$[[\cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$- \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{c3} + \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3}$$
$$+ \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}$$
$$+ \cos(\theta_6) K_{s4} \sin(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- \cos(\theta_6) K_{s4} \sin(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3}$$
$$+ \cos(\theta_6) K_{s4} \sin(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3}$$
$$+ \cos(\theta_6) K_{s4} \sin(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} + \cos(\theta_6) K_{s4} \sin(\theta_5) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2} + K_{s4} K_{s5} \sin(\theta_6) \sin(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - K_{s4} K_{s5} \sin(\theta_6) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} + K_{s4} K_{s5} \sin(\theta_6) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} K_{s5} \sin(\theta_6) \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - K_{s4} K_{s5} \sin(\theta_6) \cos(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + K_{s4} K_{s5} \sin(\theta_6) \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$- K_{s4} K_{s5} \sin(\theta_6) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} - K_{s4} K_{s5} \sin(\theta_6) \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}, K_{s5} \left(\sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3\right.$$
$$+ T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} + \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) - \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) + \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$- \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} + \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{s3} - K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1$$
$$\left.+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}\right) K_{s6}, -K_{s6} \left(-\sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1\right.$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} + \sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} - \sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \sin(\theta_6) \cos(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \sin(\theta_6) \cos(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \sin(\theta_6) \cos(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$- \sin(\theta_6) \cos(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} - \sin(\theta_6) \cos(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3} - \sin(\theta_6) K_{s4} \sin(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + \sin(\theta_6) K_{s4} \sin(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} - \sin(\theta_6) K_{s4} \sin(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} - \sin(\theta_6) K_{s4} \sin(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1}$$
$$- \sin(\theta_6) K_{s4} \sin(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}$$
$$+ K_{s4} K_{s5} \cos(\theta_6) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- K_{s4} K_{s5} \cos(\theta_6) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ K_{s4} K_{s5} \cos(\theta_6) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ K_{s4} K_{s5} \cos(\theta_6) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- K_{s4} K_{s5} \cos(\theta_6) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$+ K_{s4} K_{s5} \cos(\theta_6) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{c3} - K_{s4} K_{s5} \cos(\theta_6) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3}$$
$$\left.- K_{s4} K_{s5} \cos(\theta_6) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}\right)],$$
$$[-\cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$- \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$+ \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- \cos(\theta_6) \cos(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$

$$+ T_2\,\theta_{DH2})\, K_{c2} K_{c3} - \cos(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c3} K_{s1} K_{s2} + \cos(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, K_{s2} K_{s3} - \cos(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{s1} K_{s3}$$
$$+ \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$+ \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} K_{s3}$$
$$+ \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2} K_{s3}$$
$$+ \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} K_{s2}$$
$$- \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{c3} K_{s1} - K_{s4} K_{s5} \sin(\theta_6)\, \sin(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} - K_{s4} K_{s5} \sin(\theta_6)\, \sin(\theta_4)\, \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2} + K_{s4} K_{s5} \sin(\theta_6)\, \sin(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4} K_{s5} \sin(\theta_6)\, \cos(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4} K_{s5} \sin(\theta_6)\, \cos(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} K_{c3}$$
$$+ K_{s4} K_{s5} \sin(\theta_6)\, \cos(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c3} K_{s1} K_{s2}$$
$$- K_{s4} K_{s5} \sin(\theta_6)\, \cos(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s2} K_{s3}$$
$$+ K_{s4} K_{s5} \sin(\theta_6)\, \cos(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{s1} K_{s3},\ -K_{s5}\, (\sin(\theta_5)\, \cos(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} + \sin(\theta_5)\, \cos(\theta_4)\, \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2} - \sin(\theta_5)\, \cos(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, K_{c3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} K_{c3} + \sin(\theta_5)\, \sin(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c3} K_{s1} K_{s2} - \sin(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s2} K_{s3}$$
$$+ \sin(\theta_5)\, \sin(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{s1} K_{s3} + K_{s4} \cos(\theta_5)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4} \cos(\theta_5)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} K_{s3} + K_{s4} \cos(\theta_5)\, \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2} K_{s3} + K_{s4} \cos(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, K_{c3} K_{s2} - K_{s4} \cos(\theta_5)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{c3} K_{s1})\, K_{s6},\ K_{s6}\, ($$
$$-\sin(\theta_6)\, \cos(\theta_5)\, \cos(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2}$$
$$- \sin(\theta_6)\, \cos(\theta_5)\, \cos(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2}$$
$$+ \sin(\theta_6)\, \cos(\theta_5)\, \cos(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$- \sin(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$- \sin(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, K_{c2} K_{c3} - \sin(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c3} K_{s1} K_{s2} + \sin(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2$$

$$+ T_2\,\theta_{DH2})\, K_{s2} K_{s3} - \sin(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{s1} K_{s3}$$
$$+ \sin(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$+ \sin(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} K_{s3}$$
$$+ \sin(\theta_6)\, K_{s4} \sin(\theta_5)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2} K_{s3}$$
$$+ \sin(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} K_{s2}$$
$$- \sin(\theta_6)\, K_{s4} \sin(\theta_5)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{c3} K_{s1} + K_{s4} K_{s5} \cos(\theta_6)\, \sin(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} + K_{s4} K_{s5} \cos(\theta_6)\, \sin(\theta_4)\, \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1} K_{s2} - K_{s4} K_{s5} \cos(\theta_6)\, \sin(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{s4} K_{s5} \cos(\theta_6)\, \cos(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{s4} K_{s5} \cos(\theta_6)\, \cos(\theta_4)\, \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2} K_{c3}$$
$$- K_{s4} K_{s5} \cos(\theta_6)\, \cos(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c3} K_{s1} K_{s2}$$
$$+ K_{s4} K_{s5} \cos(\theta_6)\, \cos(\theta_4)\, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s2} K_{s3}$$
$$- K_{s4} K_{s5} \cos(\theta_6)\, \cos(\theta_4)\, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2} K_{s1} K_{s3})\,],$$
$$[K_{s1}\, (\cos(\theta_6)\, \cos(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2}$$
$$+ \cos(\theta_6)\, \cos(\theta_5)\, \cos(\theta_4)\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})$$
$$+ \cos(\theta_6)\, \cos(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \sin(\theta_4)\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2} K_{c3}$$
$$- \cos(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3}$$
$$- \cos(\theta_6)\, \cos(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \sin(\theta_4)\, K_{s2} K_{s3} - \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2} K_{s3} + \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, K_{s3} - \cos(\theta_6)\, K_{s4} \sin(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} K_{s2} + K_{s4} K_{s5} \sin(\theta_6)\, \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, \sin(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2} + K_{s4} K_{s5} \sin(\theta_6)\, \sin(\theta_4)\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) - K_{s4} K_{s5} \sin(\theta_6)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(\theta_4)\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c2} K_{c3} + K_{s4} K_{s5} \sin(\theta_6)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(\theta_4)\, K_{s2} K_{s3}$$
$$+ K_{s4} K_{s5} \sin(\theta_6)\, \cos(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3}),\ -K_{s1} K_{s5}\, ($$
$$-\sin(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2} - \sin(\theta_5)\, \cos(\theta_4)\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) - \sin(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \sin(\theta_4)\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c2} K_{c3} + \sin(\theta_5)\, \sin(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3}$$
$$+ \sin(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \sin(\theta_4)\, K_{s2} K_{s3} - K_{s4} \cos(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c2} K_{s3} + K_{s4} \cos(\theta_5)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s3}$$
$$- K_{s4} \cos(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3} K_{s2})\, K_{s6},\ K_{s1} K_{s6}\, (\sin(\theta_6)\, \cos(\theta_5)\, \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, \cos(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2} + \sin(\theta_6)\, \cos(\theta_5)\, \cos(\theta_4)\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) + \sin(\theta_6)\, \cos(\theta_5)\, \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, \sin(\theta_4)\, \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c2} K_{c3} - \sin(\theta_6)\, \cos(\theta_5)\, \sin(\theta_4)\, \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3}$$

$$-\sin(\theta_6)\cos(\theta_5)\cos(R_2\theta_2+T_2\theta_{DH2})\sin(\theta_4)K_{s2}K_{s3}-\sin(\theta_6)K_{s4}\sin(\theta_5)\cos(R_2\theta_2$$
$$+T_2\theta_{DH2})\cos(R_3\theta_3+T_3\theta_{DH3})K_{c2}K_{s3}+\sin(\theta_6)K_{s4}\sin(\theta_5)\sin(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2$$
$$+T_2\theta_{DH2})K_{s3}-\sin(\theta_6)K_{s4}\sin(\theta_5)\cos(R_2\theta_2+T_2\theta_{DH2})K_{c3}K_{s2}-K_{s4}K_{s5}\cos(\theta_6)\cos(R_2\theta_2$$
$$+T_2\theta_{DH2})\sin(\theta_4)\sin(R_3\theta_3+T_3\theta_{DH3})K_{c2}-K_{s4}K_{s5}\cos(\theta_6)\sin(\theta_4)\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\sin(R_2\theta_2+T_2\theta_{DH2})+K_{s4}K_{s5}\cos(\theta_6)\cos(R_2\theta_2+T_2\theta_{DH2})\cos(\theta_4)\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})K_{c2}K_{c3}-K_{s4}K_{s5}\cos(\theta_6)\cos(R_2\theta_2+T_2\theta_{DH2})\cos(\theta_4)K_{s2}K_{s3}$$
$$-K_{s4}K_{s5}\cos(\theta_6)\cos(\theta_4)\sin(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2+T_2\theta_{DH2})K_{c3})]]$$

#Separate Position Vector Matrix

$$P_{01} := \begin{bmatrix} A_1[1,4] \\ A_1[2,4] \\ A_1[3,4] \end{bmatrix} = \begin{bmatrix} a_1\cos(R_1\theta_1+T_1\theta_{DH1}) \\ a_1\sin(R_1\theta_1+T_1\theta_{DH1}) \\ R_1 d_{DH1}+T_1 d_1 \end{bmatrix}$$

$$P_{02} := Multiply(RO_{01}, P_{12}) + P_{01} =$$
$$[[\cos(R_1\theta_1+T_1\theta_{DH1})\,a_2\cos(R_2\theta_2+T_2\theta_{DH2})+K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1})\,(R_2 d_{DH2}+T_2 d_2)$$
$$+a_1\cos(R_1\theta_1+T_1\theta_{DH1})],$$
$$[\sin(R_1\theta_1+T_1\theta_{DH1})\,a_2\cos(R_2\theta_2+T_2\theta_{DH2})-K_{s1}\cos(R_1\theta_1+T_1\theta_{DH1})\,(R_2 d_{DH2}+T_2 d_2)$$
$$+a_1\sin(R_1\theta_1+T_1\theta_{DH1})],$$
$$[K_{s1}a_2\sin(R_2\theta_2+T_2\theta_{DH2})+R_1 d_{DH1}+T_1 d_1]]$$

$$P_{03} := Multiply(RO_{01}, (Multiply(RO_{12}, P_{23}))+P_{12})+P_{01} =$$
$$[[\cos(R_1\theta_1+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,a_3\cos(R_3\theta_3+T_3\theta_{DH3})-K_{c2}\sin(R_2\theta_2$$
$$+T_2\theta_{DH2})\,a_3\sin(R_3\theta_3+T_3\theta_{DH3})+K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(R_3 d_{DH3}+T_3 d_3)+a_2\cos(R_2\theta_2$$
$$+T_2\theta_{DH2}))+K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1})\,(K_{s2}a_3\sin(R_3\theta_3+T_3\theta_{DH3})+K_{c2}(R_3 d_{DH3}+T_3 d_3)$$
$$+R_2 d_{DH2}+T_2 d_2)+a_1\cos(R_1\theta_1+T_1\theta_{DH1})],$$
$$[\sin(R_1\theta_1+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,a_3\cos(R_3\theta_3+T_3\theta_{DH3})-K_{c2}\sin(R_2\theta_2$$
$$+T_2\theta_{DH2})\,a_3\sin(R_3\theta_3+T_3\theta_{DH3})+K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(R_3 d_{DH3}+T_3 d_3)+a_2\cos(R_2\theta_2$$
$$+T_2\theta_{DH2}))-K_{s1}\cos(R_1\theta_1+T_1\theta_{DH1})\,(K_{s2}a_3\sin(R_3\theta_3+T_3\theta_{DH3})+K_{c2}(R_3 d_{DH3}+T_3 d_3)$$
$$+R_2 d_{DH2}+T_2 d_2)+a_1\sin(R_1\theta_1+T_1\theta_{DH1})],$$
$$[K_{s1}(\sin(R_2\theta_2+T_2\theta_{DH2})\,a_3\cos(R_3\theta_3+T_3\theta_{DH3})+K_{c2}\cos(R_2\theta_2+T_2\theta_{DH2})\,a_3\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})-K_{s2}\cos(R_2\theta_2+T_2\theta_{DH2})\,(R_3 d_{DH3}+T_3 d_3)+a_2\sin(R_2\theta_2+T_2\theta_{DH2}))+R_1 d_{DH1}$$
$$+T_1 d_1]]$$

$$P_{04} := Multiply(RO_{01}, (Multiply(RO_{12}, (Multiply(RO_{23}, P_{34}))+P_{23})+P_{12}))+P_{01} =$$
$$[[\cos(R_1\theta_1+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)-K_{c3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)+K_{s3}\sin(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\cos(R_3\theta_3+T_3\theta_{DH3}))$$

$$-K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)-K_{s3}\cos(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\sin(R_3\theta_3+T_3\theta_{DH3}))$$
$$+K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(K_{s3}a_4\sin(\theta_4)+K_{c3}d_{DH4}+R_3 d_{DH3}+T_3 d_3)+a_2\cos(R_2\theta_2$$
$$+T_2\theta_{DH2}))+K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1})\,(K_{s2}(\sin(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)-K_{s3}\cos(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\sin(R_3\theta_3+T_3\theta_{DH3}))$$
$$+K_{c2}(K_{s3}a_4\sin(\theta_4)+K_{c3}d_{DH4}+R_3 d_{DH3}+T_3 d_3)+R_2 d_{DH2}+T_2 d_2)+a_1\cos(R_1\theta_1$$
$$+T_1\theta_{DH1})],$$
$$[\sin(R_1\theta_1+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)-K_{c3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)+K_{s3}\sin(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\cos(R_3\theta_3+T_3\theta_{DH3}))$$
$$-K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)-K_{s3}\cos(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\sin(R_3\theta_3+T_3\theta_{DH3}))$$
$$+K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(K_{s3}a_4\sin(\theta_4)+K_{c3}d_{DH4}+R_3 d_{DH3}+T_3 d_3)+a_2\cos(R_2\theta_2$$
$$+T_2\theta_{DH2}))-K_{s1}\cos(R_1\theta_1+T_1\theta_{DH1})\,(K_{s2}(\sin(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)-K_{s3}\cos(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\sin(R_3\theta_3+T_3\theta_{DH3}))$$
$$+K_{c2}(K_{s3}a_4\sin(\theta_4)+K_{c3}d_{DH4}+R_3 d_{DH3}+T_3 d_3)+R_2 d_{DH2}+T_2 d_2)+a_1\sin(R_1\theta_1$$
$$+T_1\theta_{DH1})],$$
$$[K_{s1}(\sin(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)-K_{c3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)+K_{s3}\sin(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\cos(R_3\theta_3+T_3\theta_{DH3}))$$
$$+K_{c2}\cos(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3+T_3\theta_{DH3})\,a_4\cos(\theta_4)+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,a_4\sin(\theta_4)-K_{s3}\cos(R_3\theta_3+T_3\theta_{DH3})\,d_{DH4}+a_3\sin(R_3\theta_3+T_3\theta_{DH3}))$$
$$-K_{s2}\cos(R_2\theta_2+T_2\theta_{DH2})\,(K_{s3}a_4\sin(\theta_4)+K_{c3}d_{DH4}+R_3 d_{DH3}+T_3 d_3)+a_2\sin(R_2\theta_2$$
$$+T_2\theta_{DH2}))+R_1 d_{DH1}+T_1 d_1]]$$

$$P_{05} := Multiply(RO_{01}, (Multiply(RO_{12}, (Multiply(RO_{23}, (Multiply(RO_{34}, P_{45}))+P_{34})+P_{23}))$$
$$+P_{12}))+P_{01}$$

$$[[\cos(R_1\theta_1+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3+T_3\theta_{DH3})\,(\cos(\theta_4)\,a_5\cos(\theta_5)$$
$$+K_{s4}\sin(\theta_4)\,d_{DH5}+a_4\cos(\theta_4))-K_{c3}\sin(R_3\theta_3+T_3\theta_{DH3})\,(\sin(\theta_4)\,a_5\cos(\theta_5)$$
$$-K_{s4}\cos(\theta_4)\,d_{DH5}+a_4\sin(\theta_4))+K_{s3}\sin(R_3\theta_3+T_3\theta_{DH3})\,(K_{s4}a_5\sin(\theta_5)+d_{DH4})$$
$$+a_3\cos(R_3\theta_3+T_3\theta_{DH3}))-K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\cos(\theta_4)\,a_5\cos(\theta_5)+K_{s4}\sin(\theta_4)\,d_{DH5}+a_4\cos(\theta_4))+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\sin(\theta_4)\,a_5\cos(\theta_5)-K_{s4}\cos(\theta_4)\,d_{DH5}+a_4\sin(\theta_4))-K_{s3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}a_5\sin(\theta_5)+d_{DH4})+a_3\sin(R_3\theta_3+T_3\theta_{DH3}))+K_{s2}\sin(R_2\theta_2$$
$$+T_2\theta_{DH2})\,(K_{s3}(\sin(\theta_4)\,a_5\cos(\theta_5)-K_{s4}\cos(\theta_4)\,d_{DH5}+a_4\sin(\theta_4))+K_{c3}(K_{s4}a_5\sin(\theta_5)$$
$$+d_{DH4})+R_3 d_{DH3}+T_3 d_3)+a_2\cos(R_2\theta_2+T_2\theta_{DH2}))+K_{s1}\sin(R_1\theta_1$$

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$$+ T_1\,\theta_{DH1}\big)\,\big(K_{s2}\,\big(\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\,d_{DH5} + a_4\cos(\theta_4)\big)$$
$$+ K_{c3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big)$$
$$- K_{s3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\big)$$
$$+ K_{c2}\,\big(K_{s3}\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big) + K_{c3}\,\big(K_{s4}\,a_5\sin(\theta_5)$$
$$+ d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) + R_2\,d_{DH2} + T_2\,d_2\big) + a_1\cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\big],$$
$$\big[\sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\,\big(\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,a_5\cos(\theta_5)$$
$$+ K_{s4}\sin(\theta_4)\,d_{DH5} + a_4\cos(\theta_4)\big) - K_{c3}\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,a_5\cos(\theta_5)$$
$$- K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big) + K_{s3}\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big)$$
$$+ a_3\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\big) - K_{c2}\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\,d_{DH5} + a_4\cos(\theta_4)\big) + K_{c3}\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big) - K_{s3}\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\big) + K_{s2}\sin\big(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2}\big)\,\big(K_{s3}\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big) + K_{c3}\,\big(K_{s4}\,a_5\sin(\theta_5)$$
$$+ d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) + a_2\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\big) - K_{s1}\cos\big(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1}\big)\,\big(K_{s2}\,\big(\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\,d_{DH5} + a_4\cos(\theta_4)\big)$$
$$+ K_{c3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big)$$
$$- K_{s3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\big)$$
$$+ K_{c2}\,\big(K_{s3}\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big) + K_{c3}\,\big(K_{s4}\,a_5\sin(\theta_5)$$
$$+ d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) + R_2\,d_{DH2} + T_2\,d_2\big) + a_1\sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\big],$$
$$\big[K_{s1}\,\big(\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\,d_{DH5}$$
$$+ a_4\cos(\theta_4)\big) - K_{c3}\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5}$$
$$+ a_4\sin(\theta_4)\big) + K_{s3}\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big) + a_3\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\big)$$
$$+ K_{c2}\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\,d_{DH5}$$
$$+ a_4\cos(\theta_4)\big) + K_{c3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5}$$
$$+ a_4\sin(\theta_4)\big) - K_{s3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\big)$$
$$- K_{s2}\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(K_{s3}\,\big(\sin(\theta_4)\,a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\,d_{DH5} + a_4\sin(\theta_4)\big)$$
$$+ K_{c3}\,\big(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) + a_2\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\big) + R_1\,d_{DH1} + T_1\,d_1$$
$$\big]\big]$$

$$P_{06} := Multiply\big(RO_{01}, \big(Multiply\big(RO_{12}, \big(Multiply\big(RO_{23}, \big(Multiply\big(RO_{34}, \big(Multiply\big(RO_{45}, P_{56}\big)\big) + P_{45}\big)\big) + P_{34}\big)\big) + P_{23}\big)\big) + P_{12}\big)\big) + P_{01}$$
$$=$$
$$\big[\big[\cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\,\big(\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big)$$
$$- K_{c3}\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$

$$- K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) + K_{s3}\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\big) - K_{c2}\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big)$$
$$+ K_{c3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$- K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) - K_{s3}\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\big) + K_{s2}\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(K_{s3}\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+ a_5\cos(\theta_5)\big) - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big)$$
$$+ K_{c3}\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big)$$
$$+ a_2\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\big) + K_{s1}\sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\,\big(K_{s2}\,\big(\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$+ K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) + K_{c3}\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$- K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) - K_{s3}\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\big) + K_{c2}\,\big(K_{s3}\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$- K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) + K_{c3}\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) + R_2\,d_{DH2} + T_2\,d_2\big)$$
$$+ a_1\cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\big],$$
$$\big[\sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\,\big(\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$+ K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) - K_{c3}\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$- K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) + K_{s3}\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\big) - K_{c2}\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(\sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big)$$
$$+ K_{c3}\cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)$$
$$- K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) - K_{s3}\cos\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\sin\big(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}\big)\big) + K_{s2}\sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\,\big(K_{s3}\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+ a_5\cos(\theta_5)\big) - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big)$$
$$+ K_{c3}\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big)$$

$$
\begin{aligned}
&+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 \\
&+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) \\
&+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3 \\
&+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) \\
&- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3 \\
&+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3 \\
&+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) \\
&- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) \\
&- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) + a_1 \sin(R_1 \theta_1 \\
&+ T_1 \theta_{DH1})], \\
&[K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) \\
&+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) \\
&- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) \\
&- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3 \\
&+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3 \\
&+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) \\
&+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) \\
&+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) \\
&- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3 \\
&+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3 \\
&+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} \\
&+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) \\
&+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) \\
&+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) + R_1 d_{DH1} + T_1 d_1]]
\end{aligned}
$$

#Zero vectors

$$Z_0 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 := Multiply(RO_{01}, Z_0) = \begin{bmatrix} K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \\ -K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \\ 0 \end{bmatrix}$$

$$Z_2 := Multiply(RO_{02}, Z_0) = \begin{bmatrix} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} \\ \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} \\ -K_{s1} K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \end{bmatrix}$$

$$
\begin{aligned}
&Z_3 := Multiply(RO_{03}, Z_0) = \\
&[[\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \sin(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 \\
&\quad + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} + \cos(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}], \\
&\quad [\sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + \sin(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 \\
&\quad + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2} - \cos(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1}], \\
&\quad [K_{s1} (-\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3} + \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 \\
&\quad + T_2 \theta_{DH2}) K_{s3} - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2})]]
\end{aligned}
$$

$$
\begin{aligned}
&Z_4 := Multiply(RO_{04}, Z_0) = \\
&[[K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_4) \cos(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} + \sin(\theta_4) \cos(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 \\
&\quad + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 \\
&\quad + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 \\
&\quad + T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} - \cos(\theta_4) \cos(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3})], \\
&\quad [-K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} \\
&\quad + \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_4) \sin(R_1 \theta_1 \\
&\quad + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 \\
&\quad + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 \\
&\quad + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 \\
&\quad + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3} \\
&\quad - \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3})], \\
&\quad [K_{s1} K_{s4} (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} + \sin(\theta_4) \cos(R_3 \theta_3 \\
&\quad + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{c3} \\
&\quad + \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) K_{s2} K_{s3} + \cos(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2
\end{aligned}
$$

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$+ T_2\,\theta_{DH2}\big)\,K_{c3}\big)\big]\big]$

$Z_5 := Multiply\big(RO_{05}, Z_0\big) =$

$$
\begin{aligned}
&\big[\big[K_{s5}\,\big(\sin(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} \\
&\quad - \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} \\
&\quad + \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) \\
&\quad - \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) \\
&\quad + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} \\
&\quad - \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} \\
&\quad + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3} + \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 \\
&\quad + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 \\
&\quad + T_2\,\theta_{DH2})\,K_{s3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 \\
&\quad + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 \\
&\quad + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2}\big)\big], \\
&\quad \big[-K_{s5}\,\big(\sin(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2} \\
&\quad + \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2} \\
&\quad - \sin(\theta_5)\,\cos(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) \\
&\quad + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) \\
&\quad + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{c3} \\
&\quad + \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c3}\,K_{s1}\,K_{s2} \\
&\quad - \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s2}\,K_{s3} \\
&\quad + \sin(\theta_5)\,\sin(\theta_4)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{s1}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 \\
&\quad + T_1\,\theta_{DH1})\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 \\
&\quad + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 \\
&\quad + T_1\,\theta_{DH1})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\sin(R_2\,\theta_2 \\
&\quad + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2} - K_{s4}\,\cos(\theta_5)\,\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1}\big)\big], \\
&\quad \big[-K_{s1}\,K_{s5}\,\big(-\sin(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(\theta_4)\,\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c2} \\
&\quad - \sin(\theta_5)\,\cos(\theta_4)\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) - \sin(\theta_5)\,\cos(R_2\,\theta_2 \\
&\quad + T_2\,\theta_{DH2})\,\sin(\theta_4)\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c2}\,K_{c3} + \sin(\theta_5)\,\sin(\theta_4)\,\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3} + \sin(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\sin(\theta_4)\,K_{s2}\,K_{s3} \\
&\quad - K_{s4}\,\cos(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{c2}\,K_{s3} + K_{s4}\,\cos(\theta_5)\,\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3} - K_{s4}\,\cos(\theta_5)\,\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2}\big)\big]\big]
\end{aligned}
$$

#Vector method

$J_{B1} := CrossProduct\big(Z_0, P_{06}\big) =$

$$
\begin{aligned}
&\big[\big[-\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\big(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\big(\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) \\
&\quad + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) \\
&\quad - K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) + K_{s3}\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\big) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\big(\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) \\
&\quad + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) \\
&\quad + K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) - K_{s3}\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\big) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\big(K_{s3}\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} \\
&\quad + a_5\cos(\theta_5)\big) - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) \\
&\quad + K_{c3}\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) \\
&\quad + a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\big) + K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\big(K_{s2}\,\big(\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) + K_{c3}\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) - K_{s3}\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\big) + K_{c2}\,\big(K_{s3}\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) + K_{c3}\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) \\
&\quad - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + R_3\,d_{DH3} + T_3\,d_3\big) + R_2\,d_{DH2} + T_2\,d_2\big) - a_1\sin(R_1\,\theta_1 \\
&\quad + T_1\,\theta_{DH1})\big], \\
&\quad \big[\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,\big(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\big(\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) - K_{c3}\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) \\
&\quad - K_{s4}\cos(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\sin(\theta_4)\big) + K_{s3}\sin(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\,\big(K_{s4}\,\big(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)\big) + d_{DH4}\big) + a_3\cos(R_3\,\theta_3 \\
&\quad + T_3\,\theta_{DH3})\big) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,\big(\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\big(\cos(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) \\
&\quad + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big) + K_{s4}\sin(\theta_4)\,\big(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}\big) + a_4\cos(\theta_4)\big) \\
&\quad + K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,\big(\sin(\theta_4)\,\big(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)\big)
\end{aligned}
$$

$$
\begin{aligned}
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) + K_{s2} \sin(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6}\\
&+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))\\
&+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3)\\
&+ a_2 \cos(R_2\, \theta_2 + T_2\, \theta_{DH2})) + K_{s1} \sin(R_1\, \theta_1 + T_1\, \theta_{DH1})\ (K_{s2}\ (\sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&+ K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) + K_{c2}\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6)\\
&- K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3) + R_2\, d_{DH2} + T_2\, d_2)\\
&+ a_1 \cos(R_1\, \theta_1 + T_1\, \theta_{DH1})],\\
&[0]]
\end{aligned}
$$

$$
\begin{aligned}
&J_{B2} := CrossProduct(Z_1, (P_{06} - P_{01})) =\\
&[[-K_{s1}^2 \cos(R_1\, \theta_1 + T_1\, \theta_{DH1})\ (\sin(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (\cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&+ K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) + K_{c2} \cos(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (\sin(R_3\, \theta_3 + T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6)\\
&+ K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))\\
&+ K_{c3} \cos(R_3\, \theta_3 + T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) - K_{s2} \cos(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6}\\
&+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))\\
&+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3)\\
&+ a_2 \sin(R_2\, \theta_2 + T_2\, \theta_{DH2}))],
\end{aligned}
$$

$$
\begin{aligned}
&[-K_{s1}^2 \sin(R_1\, \theta_1 + T_1\, \theta_{DH1})\ (\sin(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (\cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&+ K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) + K_{c2} \cos(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (\sin(R_3\, \theta_3 + T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6)\\
&+ K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))\\
&+ K_{c3} \cos(R_3\, \theta_3 + T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) - K_{s2} \cos(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6}\\
&+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))\\
&+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3)\\
&+ a_2 \sin(R_2\, \theta_2 + T_2\, \theta_{DH2}))],\\
&[K_{s1} \sin(R_1\, \theta_1 + T_1\, \theta_{DH1})\ (\sin(R_1\, \theta_1 + T_1\, \theta_{DH1})\ (\cos(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (\cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&+ K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) - K_{c2} \sin(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (\sin(R_3\, \theta_3 + T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6)\\
&+ K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))\\
&+ K_{c3} \cos(R_3\, \theta_3 + T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})) + K_{s2} \sin(R_2\, \theta_2 + T_2\, \theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6}\\
&+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))\\
&+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3)\\
&+ a_2 \cos(R_2\, \theta_2 + T_2\, \theta_{DH2})) - K_{s1} \cos(R_1\, \theta_1 + T_1\, \theta_{DH1})\ (K_{s2}\ (\sin(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&+ K_{s4} \sin(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3\, \theta_3\\
&+ T_3\, \theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\ d_{DH6} + a_5 \cos(\theta_5))\\
&- K_{s4} \cos(\theta_4)\ (K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\, \theta_3
\end{aligned}
$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))$$
$$+ K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))]]$$

$$J_{B3} := CrossProduct(Z_2, (P_{06} - P_{02})) =$$
$$[[(\sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2}) (K_{s1} (\sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$

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$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} K_{s2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2}) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (R_2 d_{DH2} + T_2 d_2))],$$

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$$[-(\cos(R_1\theta_1+T_1\theta_{DH1})\,K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})+K_{s1}\sin(R_1\theta_1$$
$$+T_1\theta_{DH1})\,K_{c2})\,(K_{s1}\,(\sin(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))-K_{c3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{s3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\cos(R_3\theta_3$$
$$+T_3\theta_{DH3}))+K_{c2}\cos(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$+K_{c3}\cos(R_3\theta_3+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\theta_3$$
$$+T_3\theta_{DH3}))-K_{s2}\cos(R_2\theta_2+T_2\theta_{DH2})\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+a_5\cos(\theta_5))-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))$$
$$+K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)$$
$$+a_2\sin(R_2\theta_2+T_2\theta_{DH2}))-K_{s1}\,a_2\sin(R_2\theta_2+T_2\theta_{DH2}))-K_{s1}\,K_{s2}\cos(R_2\theta_2$$
$$+T_2\theta_{DH2})\,(\cos(R_1\theta_1+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))-K_{c3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{s3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\cos(R_3\theta_3$$
$$+T_3\theta_{DH3}))-K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$+K_{c3}\cos(R_3\theta_3+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\theta_3$$
$$+T_3\theta_{DH3}))+K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+a_5\cos(\theta_5))-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))$$
$$+K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)$$
$$+a_2\cos(R_2\theta_2+T_2\theta_{DH2}))+K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1})\,(K_{s2}\,(\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$

$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\theta_3$$
$$+T_3\theta_{DH3}))+K_{c2}\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)$$
$$-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)+R_2\,d_{DH2}+T_2\,d_2)-\cos(R_1\theta_1$$
$$+T_1\theta_{DH1})\,a_2\cos(R_2\theta_2+T_2\theta_{DH2})-K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1})\,(R_2\,d_{DH2}+T_2\,d_2))],$$
$$[(\cos(R_1\theta_1+T_1\theta_{DH1})\,K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})+K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1})\,K_{c2})\,(\sin(R_1\theta_1$$
$$+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$-K_{c3}\sin(R_3\theta_3+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{s3}\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\cos(R_3\theta_3$$
$$+T_3\theta_{DH3}))-K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(\sin(R_3\theta_3+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$+K_{c3}\cos(R_3\theta_3+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\theta_3$$
$$+T_3\theta_{DH3}))+K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+a_5\cos(\theta_5))-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))$$
$$+K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)$$
$$+a_2\cos(R_2\theta_2+T_2\theta_{DH2}))-K_{s1}\cos(R_1\theta_1+T_1\theta_{DH1})\,(K_{s2}\,(\sin(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))+K_{c3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\theta_3$$
$$+T_3\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\theta_3$$
$$+T_3\theta_{DH3}))+K_{c2}\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)$$
$$-K_{s5}\cos(\theta_5)\,d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)+R_2\,d_{DH2}+T_2\,d_2)-\sin(R_1\theta_1$$
$$+T_1\theta_{DH1})\,a_2\cos(R_2\theta_2+T_2\theta_{DH2})+K_{s1}\cos(R_1\theta_1+T_1\theta_{DH1})\,(R_2\,d_{DH2}+T_2\,d_2))-(\sin(R_1\theta_1$$
$$+T_1\theta_{DH1})\,K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2})-K_{s1}\cos(R_1\theta_1+T_1\theta_{DH1})\,K_{c2})\,(\cos(R_1\theta_1$$
$$+T_1\theta_{DH1})\,(\cos(R_2\theta_2+T_2\theta_{DH2})\,(\cos(R_3\theta_3+T_3\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$-K_{c3}\sin(R_3\theta_3+T_3\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\,d_{DH6}+a_5\cos(\theta_5))$$

$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{s3}\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))-K_{c2}\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ (\sin(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$+K_{c3}\cos(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))+K_{s2}\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}$$
$$+a_5\cos(\theta_5))-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))$$
$$+K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)$$
$$+a_2\cos(R_2\,\theta_2+T_2\,\theta_{DH2}))+K_{s1}\sin(R_1\,\theta_1+T_1\,\theta_{DH1})\ (K_{s2}\ (\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))+K_{c3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))+K_{c2}\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)$$
$$-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)+R_2\,d_{DH2}+T_2\,d_2)-\cos(R_1\,\theta_1$$
$$+T_1\,\theta_{DH1})\ a_2\cos(R_2\,\theta_2+T_2\,\theta_{DH2})-K_{s1}\sin(R_1\,\theta_1+T_1\,\theta_{DH1})\ (R_2\,d_{DH2}+T_2\,d_2))]]$$

$$J_{B4} := CrossProduct(Z_3,\ (P_{06}-P_{03}))=$$
$$[[(\sin(R_1\,\theta_1+T_1\,\theta_{DH1})\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ K_{s3}\sin(R_3\,\theta_3+T_3\,\theta_{DH3})+\sin(R_1\,\theta_1$$
$$+T_1\,\theta_{DH1})\cos(R_3\,\theta_3+T_3\,\theta_{DH3})\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ K_{c2}\,K_{s3}+\cos(R_1\,\theta_1+T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ K_{s1}\,K_{s2}\,K_{s3}+\sin(R_1\,\theta_1+T_1\,\theta_{DH1})\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ K_{c3}\,K_{s2}-\cos(R_1\,\theta_1$$
$$+T_1\,\theta_{DH1})\ K_{c2}\,K_{c3}\,K_{s1})\ (K_{s1}\ (\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ (\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))-K_{c3}\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{s3}\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))+K_{c2}\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ (\sin(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$+K_{c3}\cos(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$

$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))-K_{s2}\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}$$
$$+a_5\cos(\theta_5))-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))$$
$$+K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)$$
$$+a_2\sin(R_2\,\theta_2+T_2\,\theta_{DH2}))-K_{s1}\ (\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ a_3\cos(R_3\,\theta_3+T_3\,\theta_{DH3})+K_{c2}\cos(R_2\,\theta_2$$
$$+T_2\,\theta_{DH2})\ a_3\sin(R_3\,\theta_3+T_3\,\theta_{DH3})-K_{s2}\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ (R_3\,d_{DH3}+T_3\,d_3)+a_2\sin(R_2\,\theta_2$$
$$+T_2\,\theta_{DH2})))-K_{s1}\ (-\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\cos(R_3\,\theta_3+T_3\,\theta_{DH3})\ K_{c2}\,K_{s3}+\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ K_{s3}-\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ K_{c3}\,K_{s2})\ (\sin(R_1\,\theta_1$$
$$+T_1\,\theta_{DH1})\ (\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ (\cos(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$-K_{c3}\sin(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{s3}\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))-K_{c2}\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ (\sin(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))$$
$$+K_{c3}\cos(R_3\,\theta_3+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))+K_{s2}\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}$$
$$+a_5\cos(\theta_5))-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))$$
$$+K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)$$
$$+a_2\cos(R_2\,\theta_2+T_2\,\theta_{DH2}))-K_{s1}\cos(R_1\,\theta_1+T_1\,\theta_{DH1})\ (K_{s2}\ (\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$+K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\cos(\theta_4))+K_{c3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))-K_{s3}\cos(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+a_3\sin(R_3\,\theta_3$$
$$+T_3\,\theta_{DH3}))+K_{c2}\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)+K_{s5}\sin(\theta_5)\ d_{DH6}+a_5\cos(\theta_5))$$
$$-K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6)+d_{DH5})+a_4\sin(\theta_4))+K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)$$
$$-K_{s5}\cos(\theta_5)\ d_{DH6}+a_5\sin(\theta_5))+d_{DH4})+R_3\,d_{DH3}+T_3\,d_3)+R_2\,d_{DH2}+T_2\,d_2)-\sin(R_1\,\theta_1$$
$$+T_1\,\theta_{DH1})\ (\cos(R_2\,\theta_2+T_2\,\theta_{DH2})\ a_3\cos(R_3\,\theta_3+T_3\,\theta_{DH3})-K_{c2}\sin(R_2\,\theta_2$$
$$+T_2\,\theta_{DH2})\ a_3\sin(R_3\,\theta_3+T_3\,\theta_{DH3})+K_{s2}\sin(R_2\,\theta_2+T_2\,\theta_{DH2})\ (R_3\,d_{DH3}+T_3\,d_3)+a_2\cos(R_2\,\theta_2$$
$$+T_2\,\theta_{DH2}))+K_{s1}\cos(R_1\,\theta_1+T_1\,\theta_{DH1})\ (K_{s2}\,a_3\sin(R_3\,\theta_3+T_3\,\theta_{DH3})+K_{c2}\ (R_3\,d_{DH3}+T_3\,d_3)$$

$$+ R_2 d_{DH2} + T_2 d_2))],$$
$$[- (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} + \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}) (K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{c2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}))) + K_{s1} (- \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3} + \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}) (\cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$

$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - K_{c2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{c2} (R_3 d_{DH3} + T_3 d_3)$$
$$+ R_2 d_{DH2} + T_2 d_2))],$$
$$[(\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} + \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$

$$+ T_3\,\theta_{DH3}) \left(\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)\right.$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + a_3 \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5)\, a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3) + R_2\, d_{DH2} + T_2\, d_2) - \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1}) (\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, a_3 \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{c2} \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, a_3 \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s2} \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) \left(R_3\, d_{DH3} + T_3\, d_3\right) + a_2 \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})) + K_{s1} \cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) (K_{s2}\, a_3 \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{c2} (R_3\, d_{DH3} + T_3\, d_3)$$
$$+ R_2\, d_{DH2} + T_2\, d_2)) - (\sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$+ \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2}\, K_{s3} + \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1}) \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1}\, K_{s2}\, K_{s3} + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c3}\, K_{s2}$$
$$- \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2}\, K_{c3}\, K_{s1}) (\cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) (\cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) (\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) (\cos(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$+ K_{s4} \sin(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + a_3 \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{c2} \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) (\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5)\, a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + a_3 \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{s2} \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + R_3\, d_{DH3} + T_3\, d_3)$$
$$+ a_2 \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) + K_{s1} \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) (K_{s2} (\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) (\cos(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$+ K_{s4} \sin(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + a_3 \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5)\, a_6 \cos(\theta_6)$$

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$$- K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3\, d_{DH3} + T_3\, d_3) + R_2\, d_{DH2} + T_2\, d_2) - \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1}) (\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\, a_3 \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{c2} \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, a_3 \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s2} \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) \left(R_3\, d_{DH3} + T_3\, d_3\right) + a_2 \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})) - K_{s1} \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) (K_{s2}\, a_3 \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{c2} (R_3\, d_{DH3} + T_3\, d_3)$$
$$+ R_2\, d_{DH2} + T_2\, d_2))]]$$

$$J_{B5} := CrossProduct(Z_4, (P_{06} - P_{04})) =$$

$$[[-K_{s4} (\sin(\theta_4) \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2}$$
$$+ \sin(\theta_4) \cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{s1}\, K_{s2} - \sin(\theta_4) \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1}) \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4) \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\, K_{c3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4) \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{c2}\, K_{c3} - \cos(\theta_4) \cos(R_1\,\theta_1 + T_1\,\theta_{DH1}) \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c3}\, K_{s1}\, K_{s2} + \cos(\theta_4) \sin(R_1\,\theta_1 + T_1\,\theta_{DH1}) \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\, K_{s2}\, K_{s3}$$
$$- \cos(\theta_4) \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\, K_{c2}\, K_{s1}\, K_{s3}) (K_{s1} (\sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) (\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) (\cos(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$+ K_{s4} \sin(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + a_3 \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2} \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) (\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5)\, a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) (\sin(\theta_4) \left(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\right)$$
$$- K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3}) \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + a_3 \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{s2} \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \left(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\right) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \left(K_{s4} \left(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + R_3\, d_{DH3} + T_3\, d_3)$$
$$+ a_2 \sin(R_2\,\theta_2 + T_2\,\theta_{DH2})) - K_{s1} (\sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) (\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, a_4 \cos(\theta_4)$$
$$- K_{c3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, a_4 \sin(\theta_4) + K_{s3} \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, d_{DH4} + a_3 \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2} \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) (\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, a_4 \cos(\theta_4) + K_{c3} \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, a_4 \sin(\theta_4) - K_{s3} \cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\, d_{DH4} + a_3 \sin(R_3\,\theta_3 + T_3\,\theta_{DH3}))$$
$$- K_{s2} \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) \left(K_{s3}\, a_4 \sin(\theta_4) + K_{c3}\, d_{DH4} + R_3\, d_{DH3} + T_3\, d_3\right) + a_2 \sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2}))) - K_{s1}\, K_{s4} (\cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) \sin(\theta_4) \sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\, K_{c2}$$
$$+ \sin(\theta_4) \cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) \sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) - \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) \cos(\theta_4) \cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\, K_{c2}\, K_{c3} + \cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) \cos(\theta_4)\, K_{s2}\, K_{s3} + \cos(\theta_4) \sin(R_3\,\theta_3$$

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$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))],$$
$$[-K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} + \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} - \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}) (K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4)$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}))) + K_{s1} K_{s4} (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2}$$
$$+ \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c2} K_{c3} + \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) K_{s2} K_{s3} + \cos(\theta_4) \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3}) (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))],$$
$$[K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} + \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} - \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))$$
$$+ K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}$$
$$- \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3}) (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))]]$$

$$J_{B6} := CrossProduct(Z_5, (P_{06} - P_{05})) =$$
$$[[-K_{s5} (\sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- \sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$

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$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$- \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + K_{s4} \cos(\theta_5) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} K_{s2} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1}) (K_{s1} (\sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2}))) + K_{s1} K_{s5} (-\sin(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \cos(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} - \sin(\theta_5) \cos(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) - \sin(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} + \sin(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \sin(\theta_4) K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3}$$

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$$+ K_{s4} \cos(\theta_5) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} K_{s2}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5)$$
$$+ K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5)$$
$$- K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4})$$
$$+ a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$

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$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))],$$
$$[-K_{s5} (\sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$- \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} + \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{s3} - K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}) (K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$

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$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3\theta_3 + T_3\theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3\theta_3+T_3\theta_{DH3})) - K_{s2}\cos(R_2\theta_2$$
$$+ T_2\theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5\cos(\theta_5) - K_{s4}\cos(\theta_4) d_{DH5} + a_4\sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2\theta_2 + T_2\theta_{DH2}))) - K_{s1} K_{s5} (-\sin(\theta_5)\cos(R_2\theta_2$$
$$+ T_2\theta_{DH2}) \cos(\theta_4)\sin(R_3\theta_3+T_3\theta_{DH3}) K_{c2} - \sin(\theta_5)\cos(\theta_4)\cos(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2$$
$$+ T_2\theta_{DH2}) - \sin(\theta_5)\cos(R_2\theta_2+T_2\theta_{DH2})\sin(\theta_4)\cos(R_3\theta_3+T_3\theta_{DH3}) K_{c2}K_{c3}$$
$$+ \sin(\theta_5)\sin(\theta_4)\sin(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2+T_2\theta_{DH2}) K_{c3} + \sin(\theta_5)\cos(R_2\theta_2$$
$$+ T_2\theta_{DH2})\sin(\theta_4) K_{s2}K_{s3} - K_{s4}\cos(\theta_5)\cos(R_2\theta_2+T_2\theta_{DH2})\cos(R_3\theta_3+T_3\theta_{DH3}) K_{c2}K_{s3}$$
$$+ K_{s4}\cos(\theta_5)\sin(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2+T_2\theta_{DH2}) K_{s3} - K_{s4}\cos(\theta_5)\cos(R_2\theta_2$$
$$+ T_2\theta_{DH2}) K_{c3}K_{s2}) (\cos(R_1\theta_1+T_1\theta_{DH1}) (\cos(R_2\theta_2+T_2\theta_{DH2}) (\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6) - K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3})) - K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2}) (\sin(R_3\theta_3+T_3\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\theta_3+T_3\theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6) - K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3})) + K_{s2}\sin(R_2\theta_2+T_2\theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6) - K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2\cos(R_2\theta_2+T_2\theta_{DH2})) + K_{s1}\sin(R_1\theta_1+T_1\theta_{DH1}) (K_{s2} (\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6) - K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1\theta_1$$
$$+ T_1\theta_{DH1}) (\cos(R_2\theta_2+T_2\theta_{DH2}) (\cos(R_3\theta_3+T_3\theta_{DH3}) (\cos(\theta_4) a_5\cos(\theta_5)$$
$$+ K_{s4}\sin(\theta_4) d_{DH5} + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\theta_3+T_3\theta_{DH3}) (\sin(\theta_4) a_5\cos(\theta_5)$$

$$- K_{s4}\cos(\theta_4) d_{DH5} + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\theta_3+T_3\theta_{DH3}) (K_{s4} a_5\sin(\theta_5) + d_{DH4})$$
$$+ a_3\cos(R_3\theta_3+T_3\theta_{DH3})) - K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2}) (\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\cos(\theta_4) a_5\cos(\theta_5) + K_{s4}\sin(\theta_4) d_{DH5} + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\sin(\theta_4) a_5\cos(\theta_5) - K_{s4}\cos(\theta_4) d_{DH5} + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (K_{s4} a_5\sin(\theta_5) + d_{DH4}) + a_3\sin(R_3\theta_3+T_3\theta_{DH3})) + K_{s2}\sin(R_2\theta_2$$
$$+ T_2\theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5\cos(\theta_5) - K_{s4}\cos(\theta_4) d_{DH5} + a_4\sin(\theta_4)) + K_{c3} (K_{s4} a_5\sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2\cos(R_2\theta_2+T_2\theta_{DH2})) - K_{s1}\sin(R_1\theta_1$$
$$+ T_1\theta_{DH1}) (K_{s2} (\sin(R_3\theta_3+T_3\theta_{DH3}) (\cos(\theta_4) a_5\cos(\theta_5) + K_{s4}\sin(\theta_4) d_{DH5} + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\theta_3+T_3\theta_{DH3}) (\sin(\theta_4) a_5\cos(\theta_5) - K_{s4}\cos(\theta_4) d_{DH5} + a_4\sin(\theta_4))$$
$$- K_{s3}\cos(R_3\theta_3+T_3\theta_{DH3}) (K_{s4} a_5\sin(\theta_5) + d_{DH4}) + a_3\sin(R_3\theta_3+T_3\theta_{DH3}))$$
$$+ K_{c2} (K_{s3} (\sin(\theta_4) a_5\cos(\theta_5) - K_{s4}\cos(\theta_4) d_{DH5} + a_4\sin(\theta_4)) + K_{c3} (K_{s4} a_5\sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))],$$
$$[K_{s5} (\sin(\theta_5)\cos(\theta_4)\sin(R_1\theta_1+T_1\theta_{DH1})\sin(R_3\theta_3+T_3\theta_{DH3}) K_{s1}K_{s2}$$
$$- \sin(\theta_5)\cos(\theta_4)\cos(R_1\theta_1+T_1\theta_{DH1})\sin(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2+T_2\theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_5)\cos(\theta_4)\cos(R_1\theta_1+T_1\theta_{DH1})\cos(R_2\theta_2+T_2\theta_{DH2})\cos(R_3\theta_3+T_3\theta_{DH3})$$
$$- \sin(\theta_5)\sin(\theta_4)\cos(R_1\theta_1+T_1\theta_{DH1})\cos(R_2\theta_2+T_2\theta_{DH2}) K_{c3}\sin(R_3\theta_3+T_3\theta_{DH3})$$
$$+ \sin(\theta_5)\sin(\theta_4)\sin(R_1\theta_1+T_1\theta_{DH1})\cos(R_3\theta_3+T_3\theta_{DH3}) K_{c3}K_{s1}K_{s2}$$
$$- \sin(\theta_5)\sin(\theta_4)\cos(R_1\theta_1+T_1\theta_{DH1})\cos(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2+T_2\theta_{DH2}) K_{c2}K_{c3}$$
$$+ \sin(\theta_5)\sin(\theta_4)\sin(R_1\theta_1+T_1\theta_{DH1}) K_{c2}K_{s1}K_{s3} + \sin(\theta_5)\sin(\theta_4)\cos(R_1\theta_1$$
$$+ T_1\theta_{DH1})\sin(R_2\theta_2+T_2\theta_{DH2}) K_{s2}K_{s3} - K_{s4}\cos(\theta_5)\cos(R_1\theta_1+T_1\theta_{DH1})\cos(R_2\theta_2$$
$$+ T_2\theta_{DH2}) K_{s3}\sin(R_3\theta_3+T_3\theta_{DH3}) + K_{s4}\cos(\theta_5)\sin(R_1\theta_1+T_1\theta_{DH1})\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) K_{s1}K_{s2}K_{s3} - K_{s4}\cos(\theta_5)\cos(R_1\theta_1+T_1\theta_{DH1})\cos(R_3\theta_3+T_3\theta_{DH3})\sin(R_2\theta_2$$
$$+ T_2\theta_{DH2}) K_{c2}K_{s3} - K_{s4}\cos(\theta_5)\sin(R_1\theta_1+T_1\theta_{DH1}) K_{c2}K_{c3}K_{s1} - K_{s4}\cos(\theta_5)\cos(R_1\theta_1$$
$$+ T_1\theta_{DH1})\sin(R_2\theta_2+T_2\theta_{DH2}) K_{c3}K_{s2}) (\sin(R_1\theta_1+T_1\theta_{DH1}) (\cos(R_2\theta_2$$
$$+ T_2\theta_{DH2}) (\cos(R_3\theta_3+T_3\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6}$$
$$+ a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6) - K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3})) - K_{c2}\sin(R_2\theta_2+T_2\theta_{DH2}) (\sin(R_3\theta_3+T_3\theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\theta_3+T_3\theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6\cos(\theta_6) + K_{s5}\sin(\theta_5) d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4) (K_{s5} a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\theta_3$$
$$+ T_3\theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6\cos(\theta_6) - K_{s5}\cos(\theta_5) d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\theta_3$$

$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5)$$
$$+ K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5)$$
$$- K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4})$$
$$+ a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2)) + K_{s5} (\sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} + \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3} + \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} - \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1$$

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$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + K_{s4} \cos(\theta_5) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} K_{s2} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1}) (\cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5)$$
$$+ K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5)$$
$$- K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4})$$
$$+ a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1$$

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$$+ T_1 \theta_{DH1}) \ (K_{s2} \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) \ d_{DH5} + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) \ d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (K_{s4} \ a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} \ (K_{s3} \ (\sin(\theta_4) \ a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) \ d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} \ (K_{s4} \ a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 \ d_{DH3} + T_3 \ d_3) + R_2 \ d_{DH2} + T_2 \ d_2))]]$$

#Assembling Jacobian matrix

$$J_B := \begin{bmatrix} J_{B1}[1] & J_{B2}[1] & J_{B3}[1] & J_{B4}[1] & J_{B5}[1] & J_{B6}[1] \\ J_{B1}[2] & J_{B2}[2] & J_{B3}[2] & J_{B4}[2] & J_{B5}[2] & J_{B6}[2] \\ J_{B1}[3] & J_{B2}[3] & J_{B3}[3] & J_{B4}[3] & J_{B5}[3] & J_{B6}[3] \\ Z_0[1] & Z_1[1] & Z_2[1] & Z_3[1] & Z_4[1] & Z_5[1] \\ Z_0[2] & Z_1[2] & Z_2[2] & Z_3[2] & Z_4[2] & Z_5[2] \\ Z_0[3] & Z_1[3] & Z_2[3] & Z_3[3] & Z_4[3] & Z_5[3] \end{bmatrix} =$$

$$[[-\sin(R_1 \theta_1 + T_1 \theta_{DH1}) \ (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 \ d_{DH3} + T_3 \ d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \ (K_{s2} \ (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$

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$$+ T_3 \theta_{DH3})) + K_{c2} \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 \ d_{DH3} + T_3 \ d_3) + R_2 \ d_{DH2} + T_2 \ d_2) - a_1 \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}), \ -K_{s1}^2 \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \ (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 \ d_{DH3} + T_3 \ d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})), \ (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) \ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - K_{s1} \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \ K_{c2}) \ (K_{s1} \ (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} \ a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 \ d_{DH3} + T_3 \ d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \ K_{s2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \ (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) \ (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$

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$$+ T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) - K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\ (K_{s2}\ (\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2) - \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) + K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\ (R_2\,d_{DH2} + T_2\,d_2)),\ (\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ K_{c2}\,K_{s3} + \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ K_{s1}\,K_{s2}\,K_{s3} + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ K_{c3}\,K_{s2} - \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\ K_{c2}\,K_{c3}\,K_{s1})\ (K_{s1}\ (\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$

$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{s2}\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})) - K_{s1}\ (\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ a_3\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{c2}\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\ a_3\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{s2}\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (R_3\,d_{DH3} + T_3\,d_3) + a_2\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2}))) - K_{s1}\ (-\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\ K_{c2}\,K_{s3} + \sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ K_{s3} - \cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ K_{c3}\,K_{s2})\ (\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\ (\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$- K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) - K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\ (K_{s2}\ (\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\cos(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\ (K_{s3}\ (\sin(\theta_4)\ (\cos(\theta_5)\ a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\ d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\ (K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3}\ (K_{s4}\ (\sin(\theta_5)\ a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\ d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2) - \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\ (\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\ a_3\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{c2}\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\ a_3\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\ (R_3\,d_{DH3} + T_3\,d_3) + a_2\cos(R_2\,\theta_2$$

$$+ T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{c2} (R_3 d_{DH3} + T_3 d_3)$$
$$+ R_2 d_{DH2} + T_2 d_2)), -K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} + \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} - \sin(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}$$
$$- \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3}) (K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4)$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}))) - K_{s1} K_{s4} (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2}$$
$$+ \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c2} K_{c3} + \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) K_{s2} K_{s3} + \cos(\theta_4) \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$

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$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2)),$$
$$-K_{s5} (\sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- \sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$- \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1$$

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$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1$$

$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{s3} + K_{s4} \cos(\theta_5) \cos(R_1 \theta_1$$

$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) K_{c3} K_{s2} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1}) (K_{s1} (\sin(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$

$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$

$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$

$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$

$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$

$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$

$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$

$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$

$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$

$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2}))) + K_{s1} K_{s5} (-\sin(\theta_5) \cos(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) \cos(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} - \sin(\theta_5) \cos(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) - \sin(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{c3}$$

$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} + \sin(\theta_5) \cos(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) \sin(\theta_4) K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3}$$

$$+ K_{s4} \cos(\theta_5) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) K_{c3} K_{s2}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$

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$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$

$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$

$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$

$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$

$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$

$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$

$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$

$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5)$$

$$+ K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5)$$

$$- K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4})$$

$$+ a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2$$

$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$

$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1$$

$$+ T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$

$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$

$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$

$$+ K_{c2} (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$

$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))],$$

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$$[\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \ (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \ (K_{s2} \ (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2)$$
$$+ a_1 \cos(R_1 \theta_1 + T_1 \theta_{DH1}), \ -K_{s1}^2 \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \ (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})), \ -(\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) + K_{s1} \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \ K_{c2}) \ (K_{s1} \ (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} K_{s2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \ (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \ (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\cos(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \ (K_{s3} \ (\sin(\theta_4) \ (\cos(\theta_5) \ a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \ d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \ (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \ (K_{s4} \ (\sin(\theta_5) \ a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \ d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$

$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2}) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (R_2 d_{DH2} + T_2 d_2)), - (\cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} + \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{s3} + \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} + \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} K_{s2}) (K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{c2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}))) + K_{s1} (-\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3} + \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}) (\cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

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$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - K_{c2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{c2} (R_3 d_{DH3} + T_3 d_3)$$
$$+ R_2 d_{DH2} + T_2 d_2)), -K_{s4} (\sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \cos(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3} - \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3}$$
$$- \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3}) (K_{s1} (\sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$

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$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4)$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}))) + K_{s1} K_{s4} (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2}$$
$$+ \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{c2} K_{c3} + \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(\theta_4) K_{s2} K_{s3} + \cos(\theta_4) \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3}) (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2)),$$
$$-K_{s5} (\sin(\theta_5) \cos(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{s1} K_{s2}$$
$$- \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2}$$
$$+ \sin(\theta_5) \cos(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$- \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3})$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c3} K_{s1} K_{s2}$$
$$- \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{s1} K_{s3} + \sin(\theta_5) \sin(\theta_4) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) K_{s1} K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c2} K_{s3} - K_{s4} \cos(\theta_5) \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2} K_{c3} K_{s1} - K_{s4} \cos(\theta_5) \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} K_{s2}) (K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$

$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} (\sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{c2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{s2} \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \sin(R_2 \theta_2 + T_2 \theta_{DH2}))) - K_{s1} K_{s5} (-\sin(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \cos(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} - \sin(\theta_5) \cos(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) - \sin(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \sin(\theta_4) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{c3}$$
$$+ \sin(\theta_5) \sin(\theta_4) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{c3} + \sin(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \sin(\theta_4) K_{s2} K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \cos(R_3 \theta_3 + T_3 \theta_{DH3}) K_{c2} K_{s3}$$
$$+ K_{s4} \cos(\theta_5) \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \sin(R_2 \theta_2 + T_2 \theta_{DH2}) K_{s3} - K_{s4} \cos(\theta_5) \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) K_{c3} K_{s2}) (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$

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$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5)$$
$$+ K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5)$$
$$- K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4})$$
$$+ a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4) d_{DH5} + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4))$$
$$- K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (K_{s4} a_5 \sin(\theta_5) + d_{DH4}) + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} (K_{s3} (\sin(\theta_4) a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4) d_{DH5} + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} a_5 \sin(\theta_5)$$
$$+ d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))],$$
$$[0, K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$

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$$+ K_{c3} \left(K_{s4} \left(\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)\right) + d_{DH4}\right) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))$$
$$+ K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2)), (\cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2}) (\sin(R_1 \theta_1$$

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$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (K_{s2} (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} (K_{s3} (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2}) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) (R_2 d_{DH2} + T_2 d_2)) - (\sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) K_{c2}) (\cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$- K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) (\cos(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) (\sin(\theta_4) (\cos(\theta_5) a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) (K_{s4} (\sin(\theta_5) a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$

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$$+ T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) + K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(K_{s2}\,(\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2) - \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2}) - K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(R_2\,d_{DH2} + T_2\,d_2)),\ (\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} + \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1} + \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,K_{c3}\,K_{s2})\,(\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) - K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(K_{s2}\,(\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$

$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2) - \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\,(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,a_3\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - K_{c2}\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\,a_3\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(R_3\,d_{DH3} + T_3\,d_3) + a_2\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})) + K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(K_{s2}\,a_3\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{c2}\,(R_3\,d_{DH3} + T_3\,d_3)$$
$$+ R_2\,d_{DH2} + T_2\,d_2)) - (\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$+ \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c2}\,K_{s3} + \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,K_{s1}\,K_{s2}\,K_{s3} + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,K_{c3}\,K_{s2}$$
$$- \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,K_{c2}\,K_{c3}\,K_{s1})\,(\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) + K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\,(K_{s2}\,(\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\cos(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\,(K_{s3}\,(\sin(\theta_4)\,(\cos(\theta_5)\,a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\,d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\,(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3}\,(K_{s4}\,(\sin(\theta_5)\,a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\,d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2) - \cos(R_1\,\theta_1$$

$$+ T_1 \theta_{DH1}) \, (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \, a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - K_{c2} \sin(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \, a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) - K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, (K_{s2} a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}) + K_{c2} \, (R_3 d_{DH3} + T_3 d_3)$$
$$+ R_2 d_{DH2} + T_2 d_2)), K_{s4} \, (\sin(\theta_4) \, \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, K_{s1} K_{s2}$$
$$- \sin(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{c2}$$
$$+ \sin(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \, \cos(R_3 \theta_3 + T_3 \theta_{DH3}) + \cos(\theta_4) \, \cos(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \, \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \, \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \, \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{c2} K_{c3} - \cos(\theta_4) \, \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, K_{c2} K_{s1} K_{s3}$$
$$- \cos(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{s2} K_{s3}) \, (\sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, (\cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \, (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, (\cos(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, (\cos(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (K_{s3} \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) - K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, (K_{s2} \, (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (\cos(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \, (K_{s3} \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6)$$
$$- K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2) - \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \, (\cos(R_2 \theta_2 + T_2 \theta_{DH2}) \, (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, a_4 \cos(\theta_4) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, a_4 \sin(\theta_4) + K_{s3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, d_{DH4} + a_3 \cos(R_3 \theta_3 + T_3 \theta_{DH3}))$$

$$- K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + a_2 \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2})) + K_{s1} \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, (K_{s2} \, (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, a_4 \cos(\theta_4) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, a_4 \sin(\theta_4) - K_{s3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, d_{DH4} + a_3 \sin(R_3 \theta_3 + T_3 \theta_{DH3}))$$
$$+ K_{c2} \, (K_{s3} a_4 \sin(\theta_4) + K_{c3} d_{DH4} + R_3 d_{DH3} + T_3 d_3) + R_2 d_{DH2} + T_2 d_2))$$
$$+ K_{s4} \, (\sin(\theta_4) \, \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{c2}$$
$$+ \sin(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, \sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, K_{s1} K_{s2} - \sin(\theta_4) \, \sin(R_1 \theta_1$$
$$+ T_1 \theta_{DH1}) \, \cos(R_2 \theta_2 + T_2 \theta_{DH2}) \, \cos(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \, \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, \cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \, K_{c3} \sin(R_3 \theta_3 + T_3 \theta_{DH3}) - \cos(\theta_4) \, \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{c2} K_{c3} - \cos(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, K_{c3} K_{s1} K_{s2} + \cos(\theta_4) \, \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, K_{s2} K_{s3}$$
$$- \cos(\theta_4) \, \cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, K_{c2} K_{s1} K_{s3}) \, (\cos(R_1 \theta_1 + T_1 \theta_{DH1}) \, (\cos(R_2 \theta_2$$
$$+ T_2 \theta_{DH2}) \, (\cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, (\cos(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6}$$
$$+ a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) - K_{c3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{s3} \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) - K_{c2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (\sin(R_3 \theta_3 + T_3 \theta_{DH3}) \, (\cos(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6)$$
$$+ K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5)) + K_{s4} \sin(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4))$$
$$+ K_{c3} \cos(R_3 \theta_3 + T_3 \theta_{DH3}) \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{s2} \sin(R_2 \theta_2 + T_2 \theta_{DH2}) \, (K_{s3} \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6}$$
$$+ a_5 \cos(\theta_5)) - K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4))$$
$$+ K_{c3} \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + R_3 d_{DH3} + T_3 d_3)$$
$$+ a_2 \cos(R_2 \theta_2 + T_2 \theta_{DH2})) + K_{s1} \sin(R_1 \theta_1 + T_1 \theta_{DH1}) \, (K_{s2} \, (\sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (\cos(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$+ K_{s4} \sin(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \cos(\theta_4)) + K_{c3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) - K_{s3} \cos(R_3 \theta_3$$
$$+ T_3 \theta_{DH3}) \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5) \, d_{DH6} + a_5 \sin(\theta_5)) + d_{DH4}) + a_3 \sin(R_3 \theta_3$$
$$+ T_3 \theta_{DH3})) + K_{c2} \, (K_{s3} \, (\sin(\theta_4) \, (\cos(\theta_5) \, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5) \, d_{DH6} + a_5 \cos(\theta_5))$$
$$- K_{s4} \cos(\theta_4) \, (K_{s5} a_6 \sin(\theta_6) + d_{DH5}) + a_4 \sin(\theta_4)) + K_{c3} \, (K_{s4} \, (\sin(\theta_5) \, a_6 \cos(\theta_6)$$

$$
\begin{aligned}
&- K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\big) + d_{DH4}\big) + R_3\, d_{DH3} + T_3\, d_3\big) + R_2\, d_{DH2} + T_2\, d_2\big) - \cos\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big)\, \big(\cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(\cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, a_4 \cos(\theta_4) - K_{c3} \sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, a_4 \sin(\theta_4) + K_{s3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, d_{DH4} + a_3 \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\big) \\
&- K_{c2} \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(\sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, a_4 \cos(\theta_4) + K_{c3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, a_4 \sin(\theta_4) - K_{s3} \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, d_{DH4} + a_3 \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\big) \\
&+ K_{s2} \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(K_{s3}\, a_4 \sin(\theta_4) + K_{c3}\, d_{DH4} + R_3\, d_{DH3} + T_3\, d_3\big) + a_2 \cos\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\big) - K_{s1} \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, \big(K_{s2}\, \big(\sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, a_4 \cos(\theta_4) + K_{c3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, a_4 \sin(\theta_4) - K_{s3} \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, d_{DH4} + a_3 \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\big) \\
&+ K_{c2}\, \big(K_{s3}\, a_4 \sin(\theta_4) + K_{c3}\, d_{DH4} + R_3\, d_{DH3} + T_3\, d_3\big) + R_2\, d_{DH2} + T_2\, d_2\big)\big), \\
&K_{s5}\, \big(\sin(\theta_5) \cos(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, K_{s1}\, K_{s2} \\
&- \sin(\theta_5) \cos(\theta_4) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c2} \\
&+ \sin(\theta_5) \cos(\theta_4) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \\
&- \sin(\theta_5) \sin(\theta_4) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \\
&+ \sin(\theta_5) \sin(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, K_{c3}\, K_{s1}\, K_{s2} \\
&- \sin(\theta_5) \sin(\theta_4) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c2}\, K_{c3} \\
&+ \sin(\theta_5) \sin(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, K_{c2}\, K_{s1}\, K_{s3} + \sin(\theta_5) \sin(\theta_4) \cos\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{s2}\, K_{s3} - K_{s4} \cos(\theta_5) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\, K_{s3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) + K_{s4} \cos(\theta_5) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, K_{s1}\, K_{s2}\, K_{s3} - K_{s4} \cos(\theta_5) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \sin\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\, K_{c2}\, K_{s3} - K_{s4} \cos(\theta_5) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, K_{c2}\, K_{c3}\, K_{s1} - K_{s4} \cos(\theta_5) \cos\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c3}\, K_{s2}\big)\, \big(\sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, \big(\cos\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\, \big(\cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\cos(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} \\
&+ a_5 \cos(\theta_5)\big) + K_{s4} \sin(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \cos(\theta_4)\big) - K_{c3} \sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(\sin(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\big) \\
&- K_{s4} \cos(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \sin(\theta_4)\big) + K_{s3} \sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(K_{s4}\, \big(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\big) + d_{DH4}\big) + a_3 \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\big) - K_{c2} \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(\sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\cos(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) \\
&+ K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\big) + K_{s4} \sin(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \cos(\theta_4)\big) \\
&+ K_{c3} \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\sin(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\big) \\
&- K_{s4} \cos(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \sin(\theta_4)\big) - K_{s3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(K_{s4}\, \big(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\big) + d_{DH4}\big) + a_3 \sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\big) + K_{s2} \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(K_{s3}\, \big(\sin(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} \\
&+ a_5 \cos(\theta_5)\big) - K_{s4} \cos(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \sin(\theta_4)\big) \\
&+ K_{c3}\, \big(K_{s4}\, \big(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\big) + d_{DH4}\big) + R_3\, d_{DH3} + T_3\, d_3\big)
\end{aligned}
$$

$$
\begin{aligned}
&+ a_2 \cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\big) - K_{s1} \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, \big(K_{s2}\, \big(\sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(\cos(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\big) \\
&+ K_{s4} \sin(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \cos(\theta_4)\big) + K_{c3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(\sin(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\big) \\
&- K_{s4} \cos(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \sin(\theta_4)\big) - K_{s3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(K_{s4}\, \big(\sin(\theta_5)\, a_6 \cos(\theta_6) - K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\big) + d_{DH4}\big) + a_3 \sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\big) + K_{c2}\, \big(K_{s3}\, \big(\sin(\theta_4)\, \big(\cos(\theta_5)\, a_6 \cos(\theta_6) + K_{s5} \sin(\theta_5)\, d_{DH6} + a_5 \cos(\theta_5)\big) \\
&- K_{s4} \cos(\theta_4)\, \big(K_{s5}\, a_6 \sin(\theta_6) + d_{DH5}\big) + a_4 \sin(\theta_4)\big) + K_{c3}\, \big(K_{s4}\, \big(\sin(\theta_5)\, a_6 \cos(\theta_6) \\
&- K_{s5} \cos(\theta_5)\, d_{DH6} + a_5 \sin(\theta_5)\big) + d_{DH4}\big) + R_3\, d_{DH3} + T_3\, d_3\big) + R_2\, d_{DH2} + T_2\, d_2\big) - \sin\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big)\, \big(\cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(\cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\cos(\theta_4)\, a_5 \cos(\theta_5) \\
&+ K_{s4} \sin(\theta_4)\, d_{DH5} + a_4 \cos(\theta_4)\big) - K_{c3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\sin(\theta_4)\, a_5 \cos(\theta_5) \\
&- K_{s4} \cos(\theta_4)\, d_{DH5} + a_4 \sin(\theta_4)\big) + K_{s3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(K_{s4}\, a_5 \sin(\theta_5) + d_{DH4}\big) \\
&+ a_3 \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\big) - K_{c2} \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, \big(\sin\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(\cos(\theta_4)\, a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4)\, d_{DH5} + a_4 \cos(\theta_4)\big) + K_{c3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(\sin(\theta_4)\, a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4)\, d_{DH5} + a_4 \sin(\theta_4)\big) - K_{s3} \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, \big(K_{s4}\, a_5 \sin(\theta_5) + d_{DH4}\big) + a_3 \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\big) + K_{s2} \sin\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\, \big(K_{s3}\, \big(\sin(\theta_4)\, a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4)\, d_{DH5} + a_4 \sin(\theta_4)\big) + K_{c3}\, \big(K_{s4}\, a_5 \sin(\theta_5) \\
&+ d_{DH4}\big) + R_3\, d_{DH3} + T_3\, d_3\big) + a_2 \cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\big) + K_{s1} \cos\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big)\, \big(K_{s2}\, \big(\sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\cos(\theta_4)\, a_5 \cos(\theta_5) + K_{s4} \sin(\theta_4)\, d_{DH5} + a_4 \cos(\theta_4)\big) \\
&+ K_{c3} \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(\sin(\theta_4)\, a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4)\, d_{DH5} + a_4 \sin(\theta_4)\big) \\
&- K_{s3} \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, \big(K_{s4}\, a_5 \sin(\theta_5) + d_{DH4}\big) + a_3 \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\big) \\
&+ K_{c2}\, \big(K_{s3}\, \big(\sin(\theta_4)\, a_5 \cos(\theta_5) - K_{s4} \cos(\theta_4)\, d_{DH5} + a_4 \sin(\theta_4)\big) + K_{c3}\, \big(K_{s4}\, a_5 \sin(\theta_5) \\
&+ d_{DH4}\big) + R_3\, d_{DH3} + T_3\, d_3\big) + R_2\, d_{DH2} + T_2\, d_2\big)\big) + K_{s5}\, \big(\sin(\theta_5) \cos(\theta_4) \sin\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c2} + \sin(\theta_5) \cos(\theta_4) \cos\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, K_{s1}\, K_{s2} - \sin(\theta_5) \cos(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) + \sin(\theta_5) \sin(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\, K_{c3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) + \sin(\theta_5) \sin(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c2}\, K_{c3} + \sin(\theta_5) \sin(\theta_4) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 \\
&+ T_3\, \theta_{DH3}\big)\, K_{c3}\, K_{s1}\, K_{s2} - \sin(\theta_5) \sin(\theta_4) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{s2}\, K_{s3} \\
&+ \sin(\theta_5) \sin(\theta_4) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, K_{c2}\, K_{s1}\, K_{s3} + K_{s4} \cos(\theta_5) \sin\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \cos\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{s3} \sin\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) + K_{s4} \cos(\theta_5) \sin\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big) \sin\big(R_2\, \theta_2 + T_2\, \theta_{DH2}\big)\, K_{c2}\, K_{s3} + K_{s4} \cos(\theta_5) \cos\big(R_1\, \theta_1 \\
&+ T_1\, \theta_{DH1}\big) \cos\big(R_3\, \theta_3 + T_3\, \theta_{DH3}\big)\, K_{s1}\, K_{s2}\, K_{s3} + K_{s4} \cos(\theta_5) \sin\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big) \sin\big(R_2\, \theta_2 \\
&+ T_2\, \theta_{DH2}\big)\, K_{c3}\, K_{s2} - K_{s4} \cos(\theta_5) \cos\big(R_1\, \theta_1 + T_1\, \theta_{DH1}\big)\, K_{c2}\, K_{c3}\, K_{s1}\big)\, \big(\cos\big(R_1\, \theta_1
\end{aligned}
$$

$$+ T_1\,\theta_{DH1})\;(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\;(\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\cos(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$- K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\sin(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(K_{s4}\;(\sin(\theta_5)\;a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\;d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;(\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\cos(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6)$$
$$+ K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5)) + K_{s4}\sin(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\sin(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(K_{s4}\;(\sin(\theta_5)\;a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\;d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;(K_{s3}\;(\sin(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\;d_{DH6}$$
$$+ a_5\cos(\theta_5)) - K_{s4}\cos(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4))$$
$$+ K_{c3}\;(K_{s4}\;(\sin(\theta_5)\;a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\;d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3)$$
$$+ a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) + K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\;(K_{s2}\;(\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(\cos(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5))$$
$$+ K_{s4}\sin(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(\sin(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(K_{s4}\;(\sin(\theta_5)\;a_6\cos(\theta_6) - K_{s5}\cos(\theta_5)\;d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + a_3\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})) + K_{c2}\;(K_{s3}\;(\sin(\theta_4)\;(\cos(\theta_5)\;a_6\cos(\theta_6) + K_{s5}\sin(\theta_5)\;d_{DH6} + a_5\cos(\theta_5))$$
$$- K_{s4}\cos(\theta_4)\;(K_{s5}\,a_6\sin(\theta_6) + d_{DH5}) + a_4\sin(\theta_4)) + K_{c3}\;(K_{s4}\;(\sin(\theta_5)\;a_6\cos(\theta_6)$$
$$- K_{s5}\cos(\theta_5)\;d_{DH6} + a_5\sin(\theta_5)) + d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2) - \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\;(\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\;(\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\cos(\theta_4)\;a_5\cos(\theta_5)$$
$$+ K_{s4}\sin(\theta_4)\;d_{DH5} + a_4\cos(\theta_4)) - K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\sin(\theta_4)\;a_5\cos(\theta_5)$$
$$- K_{s4}\cos(\theta_4)\;d_{DH5} + a_4\sin(\theta_4)) + K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(K_{s4}\,a_5\sin(\theta_5) + d_{DH4})$$
$$+ a_3\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})) - K_{c2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;(\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(\cos(\theta_4)\;a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\;d_{DH5} + a_4\cos(\theta_4)) + K_{c3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(\sin(\theta_4)\;a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\;d_{DH5} + a_4\sin(\theta_4)) - K_{s3}\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}) + a_3\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})) + K_{s2}\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\;(K_{s3}\;(\sin(\theta_4)\;a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\;d_{DH5} + a_4\sin(\theta_4)) + K_{c3}\;(K_{s4}\,a_5\sin(\theta_5)$$
$$+ d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + a_2\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})) - K_{s1}\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\;(K_{s2}\;(\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\cos(\theta_4)\;a_5\cos(\theta_5) + K_{s4}\sin(\theta_4)\;d_{DH5} + a_4\cos(\theta_4))$$
$$+ K_{c3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(\sin(\theta_4)\;a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\;d_{DH5} + a_4\sin(\theta_4))$$
$$- K_{s3}\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;(K_{s4}\,a_5\sin(\theta_5) + d_{DH4}) + a_3\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}))$$

$$+ K_{c2}\;(K_{s3}\;(\sin(\theta_4)\;a_5\cos(\theta_5) - K_{s4}\cos(\theta_4)\;d_{DH5} + a_4\sin(\theta_4)) + K_{c3}\;(K_{s4}\,a_5\sin(\theta_5)$$
$$+ d_{DH4}) + R_3\,d_{DH3} + T_3\,d_3) + R_2\,d_{DH2} + T_2\,d_2))],$$
$$[0, K_{s1}\sin(R_1\,\theta_1 + T_1\,\theta_{DH1}), \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\;K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) + K_{s1}\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\;K_{c2}, \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;K_{s1}\,K_{s2}\,K_{s3} + \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c2}\,K_{s3} + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\;K_{c2}\,K_{c3}\,K_{s1} + \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c3}\,K_{s2}, K_{s4}\;(\sin(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;K_{s1}\,K_{s2} - \sin(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\;K_{c2} + \sin(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$+ \cos(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$- \cos(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;K_{c3}\,K_{s1}\,K_{s2} + \cos(\theta_4)\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c2}\,K_{c3} - \cos(\theta_4)\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\;K_{c2}\,K_{s1}\,K_{s3} - \cos(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{s2}\,K_{s3}),$$
$$K_{s5}\;(\sin(\theta_5)\cos(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;K_{s1}\,K_{s2}$$
$$- \sin(\theta_5)\cos(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c2}$$
$$+ \sin(\theta_5)\cos(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$- \sin(\theta_5)\sin(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})$$
$$+ \sin(\theta_5)\sin(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\;K_{c3}\,K_{s1}\,K_{s2}$$
$$- \sin(\theta_5)\sin(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c2}\,K_{c3}$$
$$+ \sin(\theta_5)\sin(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\;K_{c2}\,K_{s1}\,K_{s3} + \sin(\theta_5)\sin(\theta_4)\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{s2}\,K_{s3} - K_{s4}\cos(\theta_5)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\;K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + K_{s4}\cos(\theta_5)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;K_{s1}\,K_{s2}\,K_{s3} - K_{s4}\cos(\theta_5)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\;K_{c2}\,K_{s3} - K_{s4}\cos(\theta_5)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\;K_{c2}\,K_{c3}\,K_{s1} - K_{s4}\cos(\theta_5)\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c3}\,K_{s2})],$$
$$[0, -K_{s1}\cos(R_1\,\theta_1 + T_1\,\theta_{DH1}), \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\;K_{s2}\sin(R_2\,\theta_2 + T_2\,\theta_{DH2}) - K_{s1}\cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\;K_{c2}, \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{s3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) + \sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c2}\,K_{s3} + \cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\;K_{s1}\,K_{s2}\,K_{s3} + \sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c3}\,K_{s2} - \cos(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\;K_{c2}\,K_{c3}\,K_{s1}, -K_{s4}\;(\sin(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\sin(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\;K_{c2} + \sin(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\sin(R_3\,\theta_3 + T_3\,\theta_{DH3})\;K_{s1}\,K_{s2} - \sin(\theta_4)\sin(R_1\,\theta_1$$
$$+ T_1\,\theta_{DH1})\cos(R_2\,\theta_2 + T_2\,\theta_{DH2})\cos(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_2\,\theta_2$$
$$+ T_2\,\theta_{DH2})\;K_{c3}\sin(R_3\,\theta_3 + T_3\,\theta_{DH3}) - \cos(\theta_4)\sin(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$
$$+ T_3\,\theta_{DH3})\sin(R_2\,\theta_2 + T_2\,\theta_{DH2})\;K_{c2}\,K_{c3} - \cos(\theta_4)\cos(R_1\,\theta_1 + T_1\,\theta_{DH1})\cos(R_3\,\theta_3$$

$$
\begin{aligned}
&+ T_3\,\theta_{DH3}\big)\, K_{c3}\, K_{s1}\, K_{s2} + \cos\big(\theta_4\big)\, \sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{s2}\, K_{s3} \\
&- \cos\big(\theta_4\big)\, \cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, K_{c2}\, K_{s1}\, K_{s3}\big),\ -K_{s5}\,\big(\sin\big(\theta_5\big)\, \cos\big(\theta_4\big)\, \sin\big(R_1\,\theta_1 \\
&+ T_1\,\theta_{DH1}\big)\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{c2} + \sin\big(\theta_5\big)\, \cos\big(\theta_4\big)\, \cos\big(R_1\,\theta_1 \\
&+ T_1\,\theta_{DH1}\big)\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{s1}\, K_{s2} - \sin\big(\theta_5\big)\, \cos\big(\theta_4\big)\, \sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \cos\big(R_2\,\theta_2 \\
&+ T_2\,\theta_{DH2}\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big) + \sin\big(\theta_5\big)\, \sin\big(\theta_4\big)\, \sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \cos\big(R_2\,\theta_2 \\
&+ T_2\,\theta_{DH2}\big)\, K_{c3}\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big) + \sin\big(\theta_5\big)\, \sin\big(\theta_4\big)\, \sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \cos\big(R_3\,\theta_3 \\
&+ T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{c2}\, K_{c3} + \sin\big(\theta_5\big)\, \sin\big(\theta_4\big)\, \cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \cos\big(R_3\,\theta_3 \\
&+ T_3\,\theta_{DH3}\big)\, K_{c3}\, K_{s1}\, K_{s2} - \sin\big(\theta_5\big)\, \sin\big(\theta_4\big)\, \sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{s2}\, K_{s3} \\
&+ \sin\big(\theta_5\big)\, \sin\big(\theta_4\big)\, \cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, K_{c2}\, K_{s1}\, K_{s3} + K_{s4}\, \cos\big(\theta_5\big)\, \sin\big(R_1\,\theta_1 \\
&+ T_1\,\theta_{DH1}\big)\, \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{s3}\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big) + K_{s4}\, \cos\big(\theta_5\big)\, \sin\big(R_1\,\theta_1 \\
&+ T_1\,\theta_{DH1}\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{c2}\, K_{s3} + K_{s4}\, \cos\big(\theta_5\big)\, \cos\big(R_1\,\theta_1 \\
&+ T_1\,\theta_{DH1}\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{s1}\, K_{s2}\, K_{s3} + K_{s4}\, \cos\big(\theta_5\big)\, \sin\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, \sin\big(R_2\,\theta_2 \\
&+ T_2\,\theta_{DH2}\big)\, K_{c3}\, K_{s2} - K_{s4}\, \cos\big(\theta_5\big)\, \cos\big(R_1\,\theta_1 + T_1\,\theta_{DH1}\big)\, K_{c2}\, K_{c3}\, K_{s1}\big)\big], \\
&\big[1,\ 0,\ -K_{s1}\, K_{s2}\, \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big),\ K_{s1}\,\big(-\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{c2}\, K_{s3} \\
&+ \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{s3} - \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{c3}\, K_{s2}\big), \\
&K_{s1}\, K_{s4}\,\big(\cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \sin\big(\theta_4\big)\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{c2} + \sin\big(\theta_4\big)\, \cos\big(R_3\,\theta_3 \\
&+ T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big) - \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \cos\big(\theta_4\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{c2}\, K_{c3} \\
&+ \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \cos\big(\theta_4\big)\, K_{s2}\, K_{s3} + \cos\big(\theta_4\big)\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 \\
&+ T_2\,\theta_{DH2}\big)\, K_{c3}\big),\ -K_{s1}\, K_{s5}\,\big(-\sin\big(\theta_5\big)\, \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \cos\big(\theta_4\big)\, \sin\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{c2} \\
&- \sin\big(\theta_5\big)\, \cos\big(\theta_4\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big) - \sin\big(\theta_5\big)\, \cos\big(R_2\,\theta_2 \\
&+ T_2\,\theta_{DH2}\big)\, \sin\big(\theta_4\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{c2}\, K_{c3} + \sin\big(\theta_5\big)\, \sin\big(\theta_4\big)\, \sin\big(R_3\,\theta_3 \\
&+ T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{c3} + \sin\big(\theta_5\big)\, \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \sin\big(\theta_4\big)\, K_{s2}\, K_{s3} \\
&- K_{s4}\, \cos\big(\theta_5\big)\, \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, \cos\big(R_3\,\theta_3 + T_3\,\theta_{DH3}\big)\, K_{c2}\, K_{s3} + K_{s4}\, \cos\big(\theta_5\big)\, \sin\big(R_3\,\theta_3 \\
&+ T_3\,\theta_{DH3}\big)\, \sin\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{s3} - K_{s4}\, \cos\big(\theta_5\big)\, \cos\big(R_2\,\theta_2 + T_2\,\theta_{DH2}\big)\, K_{c3}\, K_{s2}\big)\big]\big]
\end{aligned}
$$

Appendix E

Matlab codes for FK (E1) and Workspace (E2) of the CNC machine

E1

```
%Forward Kinematics _General CNC Machine
clc
clear all
% D-H Parameters
a1 = 0;    d1 = 4715.22;    alpha1 = (90*pi)/180;    theta1 = (90*pi)/180;
a2 = 0;    d2 = 2714.59;    alpha2 = (90*pi)/180;    theta2 = (90*pi)/180;
a3 = 0;    d3 = 0;          alpha3 = (0*pi)/180;     theta3 = (0*pi)/180;
a4 = 0;    d4 = -2664.68;   alpha4 = (-90*pi)/180;   theta4 = (180*pi)/180;
a5 = 0;    d5 = 0;          alpha5 = (90*pi)/180;    theta5 = (0*pi)/180;
a6 = 0;    d6 = -281.25;    alpha6 = (0*pi)/180;     theta6 = (0*pi)/180;
% Axis Properties
X=[0 0 d2 d2 d2 d2 d2];
Z=[0 d1 d1 d1 d1 d1 d1];
Y=[0 0 0 0 d4 d4 d4+d6];
Tool = plot3(X,Y,Z,'b','LineWidth',5);
axis([-0.2*(d1) 1.2*(d1) -1.2*(d1) 0.2*(d1) -0.2*(d1) 1.2*(d1)]);
disp ('CNC FK-Monika')
grid on
hold('all')

% Homogeneous transformation matrices
A01 = [cos(theta1), -cos(alpha1)*sin(theta1), sin(alpha1)*sin(theta1),
a1*cos(theta1);       sin(theta1),       cos(alpha1)*cos(theta1),       -
sin(alpha1)*cos(theta1), a1*sin(theta1); 0, sin(alpha1), cos(alpha1), d1;
0, 0, 0, 1 ];
A12 = [cos(theta2), -cos(alpha2)*sin(theta2), sin(alpha2)*sin(theta2),
a2*cos(theta2);       sin(theta2),       cos(alpha2)*cos(theta2),       -
sin(alpha2)*cos(theta2), a2*sin(theta2); 0, sin(alpha2), cos(alpha2), d2;
0, 0, 0, 1 ];
A23 = [cos(theta3), -cos(alpha3)*sin(theta3), sin(alpha3)*sin(theta3),
a3*cos(theta3);       sin(theta3),       cos(alpha3)*cos(theta3),       -
sin(alpha3)*cos(theta3), a3*sin(theta3); 0, sin(alpha3), cos(alpha3), d3;
0, 0, 0, 1 ];
A34 = [cos(theta4), -cos(alpha4)*sin(theta4), sin(alpha4)*sin(theta4),
a4*cos(theta4);       sin(theta4),       cos(alpha4)*cos(theta4),       -
sin(alpha4)*cos(theta4), a4*sin(theta4); 0, sin(alpha4), cos(alpha4), d4;
0, 0, 0, 1 ];
A45 = [cos(theta5), -cos(alpha5)*sin(theta5), sin(alpha5)*sin(theta5),
a5*cos(theta5);       sin(theta5),       cos(alpha5)*cos(theta5),       -
sin(alpha5)*cos(theta5), a5*sin(theta5); 0, sin(alpha5), cos(alpha5), d5;
0, 0, 0, 1 ];
A56 = [cos(theta6), -cos(alpha6)*sin(theta6), sin(alpha6)*sin(theta6),
a6*cos(theta6);       sin(theta6),       cos(alpha6)*cos(theta6),       -
sin(alpha6)*cos(theta6), a6*sin(theta6); 0, sin(alpha6), cos(alpha6), d6;
0, 0, 0, 1 ];

A06 = A01*A12*A23*A34*A45*A56

HOME = plot3(A06(1,4), A06(2,4), A06(3,4), 'r*','LineWidth',7);
```

E2

```
%Workspace of general CNC Machine
clear vars;
i = 0;
% D-H Parameters
a1 = 0;    d1 = 4715.22;    alpha1 = (90*pi)/180;    theta1 = (90*pi)/180;
a2 = 0;    d2 = 2714.59;    alpha2 = (90*pi)/180;    theta2 = (90*pi)/180;
a3 = 0;    d3 = 0;           alpha3 = (0*pi)/180;     theta3 = (0*pi)/180;
a4 = 0;   d4 = -2664.68;   alpha4 = (-90*pi)/180;   theta4 = (180*pi)/180;
a5 = 0;    d5 = 0;           alpha5 = (90*pi)/180;    theta5 = (0*pi)/180;
a6 = 0;    d6 = -281.25;    alpha6 = (0*pi)/180;     theta6 = (0*pi)/180;

% Axis Properties
X=[0 0 d2 d2 d2 d2 d2];
Z=[0 d1 d1 d1 d1 d1 d1];
Y=[0 0 0 0 d4 d4 d4+d6];
Tool = plot3(X,Y,Z,'b','LineWidth',5);

axis([-0.2*(d1) 1.2*(d1) -1.2*(d1) 0.2*(d1) -0.2*(d1) 1.3*(d1)]);

disp ('CNC Workspace-Monika')
grid on
hold('all')

% All the possible points in the workspace
for d1_VAR = -1000:200:1000
    for d2_VAR = -1000:200:1000
        for d3_VAR = -1000:200:1000
           i = i+1;
           x(i) = sin(theta4)*sin(theta5)*d6+(d2+d2_VAR);
           y(i) = cos(theta5)*d6+d4+(d3+d3_VAR);
           z(i) = cos(theta4)*sin(theta5)*d6+(d1+d1_VAR);
    hold on;
    plot3(x,y,z,'g.');
    refreshdata(Tool,'caller')
    drawnow
    pause(.001)
   fail = 1;
        end
    end
end
```


Appendix F

Matlab codes for FK (F1), Workspace (F2) and interior singularity locus (F3) of the Mitsubishi MELFA RV-3SDB

F1

```
%Mitsubishi RV-3SDB_FK

clc
clear all
% D-H Parameters
a1 = 95;   % length of first arm
a2 = 245;   % length of second arm
a3 = -135;   % length of third arm
a4 = 0;     % length of fourth arm
a5 = 0;     % length of fifth arm
a6 = 0;     % length of sixth arm

d1 = 350;   % offset of first arm
d2 = 0;     % offset of second arm
d3 = 0;     % offset of third arm
d4 = 270;  % offset of fourth arm
d5 = 0;     % offset of fifth arm
d6 = 85;   % offset of sixth arm


X=[0 0 a1 a1 a1 a1+d4 a1+d4+d6];
Z=[0 d1 d1 d1+a2 d1+a2-a3 d1+a2-a3 d1+a2-a3];
Y=[0 0 0 0 0 0 0];
Tool = plot3(X,Y,Z,'b','LineWidth',5);

axis([-1.5*(a1+a2) 2*(a1+a2) -1500 1500 -0.5*(a1+a2) 2.5*(a1+a2)]);

disp ('RV-3SDB Forward Kinematics Monika')
grid on
hold('all')

ALPHA_1 = (-90*pi)/180;
ALPHA_2 = (0*pi)/180;
ALPHA_3 = (90*pi)/180;
ALPHA_4 = (-90*pi)/180;
ALPHA_5 = (90*pi)/180;
ALPHA_6 = (0*pi)/180;

theta1_0 =  0;
theta2_0 = -90;
theta3_0 = 180;
theta4_0 =  0;
theta5_0 =  0;
theta6_0 =  0;

theta1_Pendent = 0;
theta2_Pendent = -83.206;
theta3_Pendent = 114.59;
```

```
theta4_Pendent = 0;
theta5_Pendent = 0;
theta6_Pendent = 0;

theta1 = (theta1_Pendent+theta1_0)*pi/180;
theta2 = (theta2_Pendent+theta2_0)*pi/180;
theta3 = (theta3_Pendent+theta3_0+theta2_Pendent)*pi/180;
theta4 = (theta4_Pendent+theta4_0)*pi/180;
theta5 = (theta5_Pendent+theta5_0)*pi/180;
theta6 = (theta6_Pendent+theta6_0)*pi/180;

P1 = [cos(theta1), -cos(ALPHA_1)*sin(theta1), sin(ALPHA_1)*sin(theta1),
a1*cos(theta1);      sin(theta1),      cos(ALPHA_1)*cos(theta1),      -
sin(ALPHA_1)*cos(theta1), a1*sin(theta1); 0, sin(ALPHA_1), cos(ALPHA_1),
d1; 0, 0, 0, 1 ];
P2 = [cos(theta2), -cos(ALPHA_2)*sin(theta2), sin(ALPHA_2)*sin(theta2),
a2*cos(theta2);      sin(theta2),      cos(ALPHA_2)*cos(theta2),      -
sin(ALPHA_2)*cos(theta2), a2*sin(theta2); 0, sin(ALPHA_2), cos(ALPHA_2),
d2; 0, 0, 0, 1 ];
P3 = [cos(theta3), -cos(ALPHA_3)*sin(theta3), sin(ALPHA_3)*sin(theta3),
a3*cos(theta3);      sin(theta3),      cos(ALPHA_3)*cos(theta3),      -
sin(ALPHA_3)*cos(theta3), a3*sin(theta3); 0, sin(ALPHA_3), cos(ALPHA_3),
d3; 0, 0, 0, 1 ];
P4 = [cos(theta4), -cos(ALPHA_4)*sin(theta4), sin(ALPHA_4)*sin(theta4),
a4*cos(theta4);      sin(theta4),      cos(ALPHA_4)*cos(theta4),      -
sin(ALPHA_4)*cos(theta4), a4*sin(theta4); 0, sin(ALPHA_4), cos(ALPHA_4),
d4; 0, 0, 0, 1 ];
P5 = [cos(theta5), -cos(ALPHA_5)*sin(theta5), sin(ALPHA_5)*sin(theta5),
a5*cos(theta5);      sin(theta5),      cos(ALPHA_5)*cos(theta5),      -
sin(ALPHA_5)*cos(theta5), a5*sin(theta5); 0, sin(ALPHA_5), cos(ALPHA_5),
d5; 0, 0, 0, 1 ];
P6 = [cos(theta6), -cos(ALPHA_6)*sin(theta6), sin(ALPHA_6)*sin(theta6),
a6*cos(theta6);      sin(theta6),      cos(ALPHA_6)*cos(theta6),      -
sin(ALPHA_6)*cos(theta6), a6*sin(theta6); 0, sin(ALPHA_6), cos(ALPHA_6),
d6; 0, 0, 0, 1 ];

    A1_6 = P1*P2*P3*P4*P5*P6

    HOME = plot3(A1_6(1,4), A1_6(2,4), A1_6(3,4), 'r*');
```

F2

```
%Mitsubishi RV-3SDB Workspace%
clc
clear all
% D-H Parameters
a1 = 95;    d1 = 350;  alpha1 = (-90*pi)/180;  theta1 = (0*pi)/180;
a2 = 245;   d2 = 0;    alpha2 = (0*pi)/180;    theta2 = (-pi/2*pi)/180;
a3 = -135;  d3 = 0;    alpha3 = (90*pi)/180;   theta3 = (180*pi)/180;
a4 = 0;     d4 = 270;  alpha4 = (-90*pi)/180;  theta4 = (0*pi)/180;
a5 = 0;     d5 = 0;    alpha5 = (90*pi)/180;   theta5 = (0*pi)/180;
a6 = 0;     d6 = 85;   alpha6 = (0*pi)/180;    theta6 = (0*pi)/180;

% Axis Properties
X=[0 0 a1 a1 a1 a1+d4 a1+d4+d6];
Z=[0 d1 d1 d1+a2 d1+a2-a3 d1+a2-a3 d1+a2-a3];
```



```
Y=[0 0 0 0 0 0 0];
Tool = plot3(X,Y,Z,'b','LineWidth',5);
axis([-1.5*(a1+a2-a3) 2*(a1+a2-a3) -1000 1000 -0.5*(a1+a2) 2.5*(a1+a2)]);

disp ('RV-3SDB Workspace Monika')
grid on
hold('all')

theta1_0 = 0;
% Conditions for joint's limits
for theta1_0 = -170:40:170;
  for theta2_0 = -90:25:135;
   for theta3_0 = -20:25:171;
       if  (((theta2_0+theta3_0)  >=  -110)  &&  ((theta2_0+theta3_0)  <=
306))
        theta1 = (theta1_0)*pi/180;
        theta2 = (-90+theta2_0)*pi/180;
        theta3 = (180+theta3_0)*pi/180;
% Homogeneous transformation matrices
A01  =  [cos(theta1),  -cos(alpha1)*sin(theta1),  sin(alpha1)*sin(theta1),
a1*cos(theta1);      sin(theta1),      cos(alpha1)*cos(theta1),      -
sin(alpha1)*cos(theta1), a1*sin(theta1); 0, sin(alpha1), cos(alpha1), d1;
0, 0, 0, 1 ];
A12  =  [cos(theta2),  -cos(alpha2)*sin(theta2),  sin(alpha2)*sin(theta2),
a2*cos(theta2);      sin(theta2),      cos(alpha2)*cos(theta2),      -
sin(alpha2)*cos(theta2), a2*sin(theta2); 0, sin(alpha2), cos(alpha2), d2;
0, 0, 0, 1 ];
A23  =  [cos(theta3),  -cos(alpha3)*sin(theta3),  sin(alpha3)*sin(theta3),
a3*cos(theta3);      sin(theta3),      cos(alpha3)*cos(theta3),      -
sin(alpha3)*cos(theta3), a3*sin(theta3); 0, sin(alpha3), cos(alpha3), d3;
0, 0, 0, 1 ];
A34  =  [cos(theta4),  -cos(alpha4)*sin(theta4),  sin(alpha4)*sin(theta4),
a4*cos(theta4);      sin(theta4),      cos(alpha4)*cos(theta4),      -
sin(alpha4)*cos(theta4), a4*sin(theta4); 0, sin(alpha4), cos(alpha4), d4;
0, 0, 0, 1 ];
A45  =  [cos(theta5),  -cos(alpha5)*sin(theta5),  sin(alpha5)*sin(theta5),
a5*cos(theta5);      sin(theta5),      cos(alpha5)*cos(theta5),      -
sin(alpha5)*cos(theta5), a5*sin(theta5); 0, sin(alpha5), cos(alpha5), d5;
0, 0, 0, 1 ];
A56  =  [cos(theta6),  -cos(alpha6)*sin(theta6),  sin(alpha6)*sin(theta6),
a6*cos(theta6);      sin(theta6),      cos(alpha6)*cos(theta6),      -
sin(alpha6)*cos(theta6), a6*sin(theta6); 0, sin(alpha6), cos(alpha6), d6;
0, 0, 0, 1 ];

A06 = A01*A12*A23*A34*A45*A56;
Envelope_1 = plot3(A06(1,4), A06(2,4), A06(3,4),'g.');
refreshdata(Tool,'caller')
axis equal
drawnow
pause(.001)
else
   fail = 1;
        end
    end
  end
end
```

F3

```
%Singularity _Mitsubishi RV-3SDB
clc
clear all
% D-H Parameters
a1 = 95;    d1 = 350;  alpha1 = (-90*pi)/180;  %theta1 = (0*pi)/180;
a2 = 245;   d2 = 0;    alpha2 = (0*pi)/180;    %theta2 = (-90*pi)/180;
a3 = -135;  d3 = 0;    alpha3 = (90*pi)/180;   %theta3 = (180*pi)/180;
a4 = 0;     d4 = 270;  alpha4 = (-90*pi)/180;  %theta4 = (0*pi)/180;
a5 = 0;     d5 = 0;    alpha5 = (90*pi)/180;   %theta5 = (0*pi)/180;
a6 = 0;     d6 = 85;   alpha6 = (0*pi)/180;    %theta6 = (0*pi)/180;

% Axis Properties
X=[0 0 a1 a1 a1 a1+d4 a1+d4+d6];
Z=[0 d1 d1 d1+a2 d1+a2-a3 d1+a2-a3 d1+a2-a3];
Y=[0 0 0 0 0 0 0];
Tool = plot3(X,Y,Z,'b','LineWidth',5);
%axis([-1.5*(a1+a2-a3)    2*(a1+a2-a3)    -1500    1500    -0.5*(a1+a2)
2.5*(a1+a2)]);
disp ('RV-3SDB Singularities Monika')
grid on
hold('all')

X_m=[]; Y_m=[]; Z_m=[];

XmM=[]; YmM=[]; ZmM=[];

%Conditions for Jacobian, IK and singularities

for theta1_0 = -170:10:170;
    for theta4_0 = -160:20:160;
        for theta5_0 = -120:20:120;
            for theta6_0 = -360:20:360;

            theta1 = (0+theta1_0)*pi/180;
            theta4 = (0+theta4_0)*pi/180;
            theta3_0 = atan2(d4,a3);
            theta2_0          =          atan2(a1-a2-a3*cos(theta3_0)-
d4*sin(theta3_0),a3*sin(theta3_0)-d4*cos(theta3_0));
            theta2 = (-pi/2+theta2_0);
            theta3 = (pi+theta3_0+theta2_0);
            theta5 = (0+theta5_0)*pi/180;
            theta6 = (0+theta6_0)*pi/180;

   % Homogeneous transformation matrices

    A01        =        [cos(theta1),        -cos(alpha1)*sin(theta1),
sin(alpha1)*sin(theta1),          a1*cos(theta1);          sin(theta1),
cos(alpha1)*cos(theta1),  -sin(alpha1)*cos(theta1),  a1*sin(theta1);  0,
sin(alpha1), cos(alpha1), d1; 0, 0, 0, 1 ];
    A12        =        [cos(theta2),        -cos(alpha2)*sin(theta2),
sin(alpha2)*sin(theta2),          a2*cos(theta2);          sin(theta2),
cos(alpha2)*cos(theta2),  -sin(alpha2)*cos(theta2),  a2*sin(theta2);  0,
sin(alpha2), cos(alpha2), d2; 0, 0, 0, 1 ];
    A23        =        [cos(theta3),        -cos(alpha3)*sin(theta3),
sin(alpha3)*sin(theta3),          a3*cos(theta3);          sin(theta3),
cos(alpha3)*cos(theta3),  -sin(alpha3)*cos(theta3),  a3*sin(theta3);  0,
sin(alpha3), cos(alpha3), d3; 0, 0, 0, 1 ];
    A34        =        [cos(theta4),        -cos(alpha4)*sin(theta4),
sin(alpha4)*sin(theta4),          a4*cos(theta4);          sin(theta4),
cos(alpha4)*cos(theta4),  -sin(alpha4)*cos(theta4),  a4*sin(theta4);  0,
sin(alpha4), cos(alpha4), d4; 0, 0, 0, 1 ];
    A45        =        [cos(theta5),        -cos(alpha5)*sin(theta5),
sin(alpha5)*sin(theta5),          a5*cos(theta5);          sin(theta5),
cos(alpha5)*cos(theta5),  -sin(alpha5)*cos(theta5),  a5*sin(theta5);  0,
sin(alpha5), cos(alpha5), d5; 0, 0, 0, 1 ];
    A56        =        [cos(theta6),        -cos(alpha6)*sin(theta6),
sin(alpha6)*sin(theta6),          a6*cos(theta6);          sin(theta6),
cos(alpha6)*cos(theta6),  -sin(alpha6)*cos(theta6),  a6*sin(theta6);  0,
sin(alpha6), cos(alpha6), d6; 0, 0, 0, 1 ];

    A06 = A01*A12*A23*A34*A45*A56;

    X_m=[X_m A06(1,4)]; Y_m=[Y_m A06(2,4)]; Z_m=[Z_m A06(3,4)];

    plot3(A06(1,4), A06(2,4), A06(3,4),'r.','LineWidth',5);

            end
        end
    end
    XmM=[XmM; X_m];
    X_m=[];
    YmM=[YmM; Y_m];
    Y_m=[];
    ZmM=[ZmM; Z_m];
    Z_m=[];
end
    axis equal
    refreshdata(Tool,'caller')
    drawnow
```

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Appendix G

Matlab codes for FK (G1), Workspace (G2) and interior singularity locus for the left arm(G3) and the right arm(G4) of the Yaskawa Motoman DA-20

G1

```
%Motoman DA20 forward kinematics
clc
clear all
% D-H Parameters
a0 = 295;
a1 = 135;   % length of first joint
a2 = 250;   % length of second joint
a3 = 390;   % length of third joint
a4 = 0;     % length of fourth joint
a5 = 0;     % length of fifth joint
a6 = 0;     % length of sixth joint

d0 = 965;
d1 = 136.5;   % offset of first joint
d2 = 0;     % offset of second joint
d3 = 0;     % offset of third joint
d4 = -230;  % offset of fourth joint
d5 = 0;     % offset of fifth joint
d6 = -65;   % offset of sixth joint

% Axis Properties Left Arm

XL=[0 0 a0 a0 a0+a1 a0+a1 a0+a1+a3 a0+a1+a3 a0+a1+a3 a0+a1+a3];
ZL=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YL=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];

plot3(XL,YL,ZL,'b','LineWidth',5);
grid on
hold('all')
disp ('DA20 Left Arm FK-Monika')

% Axis Properties Right Arm

XR=[0 0 -a0 -a0 -a0-a1 -a0-a1 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3];
ZR=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YR=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];

plot3(XR,YR,ZR,'b','LineWidth',5);
grid on
hold('all')
disp ('DA20 Right Arm FK-Monika')
%axis([-2.5*(a1+a2+a3) 2.5*(a1+a2+a3) -1500 1500 0 2*(a1+a2+a3)]);
alpha0L = (90*pi)/180;
alpha0R = (-90*pi)/180;
alpha1 = (90*pi)/180;
alpha2 = (180*pi)/180;
alpha3 = (-90*pi)/180;
alpha4 = (90*pi)/180;
```



```
alpha5 = (-90*pi)/180;
alpha6 = (180*pi)/180;

theta0_0L = 0;
theta0_0R = 180;
theta1_0 =  0;
theta2_0 =  90;
theta3_0 =  90;
theta4_0 =  0;
theta5_0 =  0;
theta6_0 =  0;

theta0_Pendent = 0;
theta1_Pendent = 0;
theta2_Pendent = 0;
theta3_Pendent = 0;
theta4_Pendent = 0;
theta5_Pendent = 0;
theta6_Pendent = 0;

theta0L = (theta0_Pendent+theta0_0L)*pi/180;
theta0R = (theta0_Pendent+theta0_0R)*pi/180;
theta1 = (theta1_Pendent+theta1_0)*pi/180;
theta2 = (theta2_Pendent+theta2_0)*pi/180;
theta3 = (theta3_Pendent+theta3_0)*pi/180;
theta4 = (theta4_Pendent+theta4_0)*pi/180;
theta5 = (theta5_Pendent+theta5_0)*pi/180;
theta6 = (theta6_Pendent+theta6_0)*pi/180;

% Homogeneous transformation matrices Left and Right arm

A00L        =        [cos(theta0L),        -cos(alpha0L)*sin(theta0L),
sin(alpha0L)*sin(theta0L),        a0*cos(theta0L);        sin(theta0L),
cos(alpha0L)*cos(theta0L),  -sin(alpha0L)*cos(theta0L),  a0*sin(theta0L);
0, sin(alpha0L), cos(alpha0L), d0; 0, 0, 0, 1 ];
A00R        =        [cos(theta0R),        -cos(alpha0R)*sin(theta0R),
sin(alpha0R)*sin(theta0R),        a0*cos(theta0R);        sin(theta0R),
cos(alpha0R)*cos(theta0R),  -sin(alpha0R)*cos(theta0R),  a0*sin(theta0R);
0, sin(alpha0R), cos(alpha0R), d0; 0, 0, 0, 1 ];

A01  = [cos(theta1),  -cos(alpha1)*sin(theta1),  sin(alpha1)*sin(theta1),
a1*cos(theta1);       sin(theta1),       cos(alpha1)*cos(theta1),       -
sin(alpha1)*cos(theta1), a1*sin(theta1); 0, sin(alpha1), cos(alpha1), d1;
0, 0, 0, 1 ];
A12  = [cos(theta2),  -cos(alpha2)*sin(theta2),  sin(alpha2)*sin(theta2),
a2*cos(theta2);       sin(theta2),       cos(alpha2)*cos(theta2),       -
sin(alpha2)*cos(theta2), a2*sin(theta2); 0, sin(alpha2), cos(alpha2), d2;
0, 0, 0, 1 ];
A23  = [cos(theta3),  -cos(alpha3)*sin(theta3),  sin(alpha3)*sin(theta3),
a3*cos(theta3);       sin(theta3),       cos(alpha3)*cos(theta3),       -
sin(alpha3)*cos(theta3), a3*sin(theta3); 0, sin(alpha3), cos(alpha3), d3;
0, 0, 0, 1 ];
A34  = [cos(theta4),  -cos(alpha4)*sin(theta4),  sin(alpha4)*sin(theta4),
a4*cos(theta4);       sin(theta4),       cos(alpha4)*cos(theta4),       -
sin(alpha4)*cos(theta4), a4*sin(theta4); 0, sin(alpha4), cos(alpha4), d4;
0, 0, 0, 1 ];
```

```
A45 = [cos(theta5), -cos(alpha5)*sin(theta5), sin(alpha5)*sin(theta5),
a5*cos(theta5);      sin(theta5),      cos(alpha5)*cos(theta5),      -
sin(alpha5)*cos(theta5), a5*sin(theta5); 0, sin(alpha5), cos(alpha5), d5;
0, 0, 0, 1 ];
A56 = [cos(theta6), -cos(alpha6)*sin(theta6), sin(alpha6)*sin(theta6),
a6*cos(theta6);      sin(theta6),      cos(alpha6)*cos(theta6),      -
sin(alpha6)*cos(theta6), a6*sin(theta6); 0, sin(alpha6), cos(alpha6), d6;
0, 0, 0, 1 ];

%Forward Kinematics
A06L = A00L*A01*A12*A23*A34*A45*A56
A06R = A00R*A01*A12*A23*A34*A45*A56

    HOME1 = plot3(A06L(1,4), A06L(2,4), A06L(3,4), 'r*','LineWidth',10);
    HOME2 = plot3(A06R(1,4), A06R(2,4), A06R(3,4), 'm*','LineWidth',10);
```

G2

```
% Motoman DA20 Workspace
clc
clear all
a0 = 295;  d0 = 965;    alpha0L =(90*pi)/180;    %theta0L = (0*pi)/180;
                         alpha0R  =(-90*pi)/180;          %theta0R  =
(180*pi)/180;
a1 = 135;  d1 = 136.5;  alpha1 = (90*pi)/180;    %theta1L = (0*pi)/180;
                                                   %theta1R          =
(0*pi)/180;
a2 = 250;  d2 = 0;      alpha2 = (180*pi)/180;   %theta2 = (90*pi)/180;
a3 = 390;  d3 = 0;      alpha3 = (-90*pi)/180;   %theta3 = (90*pi)/180;
a4 = 0;     d4 = -230;   alpha4 = (90*pi)/180;     theta4 = (0*pi)/180;
a5 = 0;     d5 = 0;      alpha5 = (-90*pi)/180;    theta5 = (0*pi)/180;
a6 = 0;     d6 = -65;    alpha6 = (180*pi)/180;    theta6 = (0*pi)/180;

% Axis Properties Left Arm

XL=[0 0 a0 a0 a0+a1 a0+a1 a0+a1+a3 a0+a1+a3 a0+a1+a3 a0+a1+a3];
ZL=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YL=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];

Tool1 = plot3(XL,YL,ZL,'b','LineWidth',5);

grid on
hold('all')

% Axis Properties Right Arm

XR=[0 0 -a0 -a0 -a0-a1 -a0-a1 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3];
ZR=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YR=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];

Tool2 = plot3(XR,YR,ZR,'b','LineWidth',5);

grid on
hold('all')
disp ('DA20 Workspace-Monika')
```



```
% Mechanical Limits for joints
for theta0L_0 = -180:360:180;
    for theta0R_0 = -180:360:180;
        for theta1L_0 = -190:40:80;
            for theta1R_0 = -260:40:170;
                for theta2_0 = -40:30:220;
                    for theta3_0 = -35:30:215;

        if (((theta2_0+theta3_0) >= -75) && ((theta2_0+theta3_0) <= 435))

        theta0L = (theta0L_0)*pi/180;
        theta0R = (180+theta0R_0)*pi/180;
        theta1L = (theta1L_0)*pi/180;
        theta1R = (theta1R_0)*pi/180;
        theta2 = (90+theta2_0)*pi/180;
        theta3 = (90+theta3_0)*pi/180;

% Homogeneous transformation matrices Left and Right arm

A00L        =        [cos(theta0L),        -cos(alpha0L)*sin(theta0L),
sin(alpha0L)*sin(theta0L),        a0*cos(theta0L);        sin(theta0L),
cos(alpha0L)*cos(theta0L), -sin(alpha0L)*cos(theta0L), a0*sin(theta0L);
0, sin(alpha0L), cos(alpha0L), d0; 0, 0, 0, 1 ];
A00R        =        [cos(theta0R),        -cos(alpha0R)*sin(theta0R),
sin(alpha0R)*sin(theta0R),        a0*cos(theta0R);        sin(theta0R),
cos(alpha0R)*cos(theta0R), -sin(alpha0R)*cos(theta0R), a0*sin(theta0R);
0, sin(alpha0R), cos(alpha0R), d0; 0, 0, 0, 1 ];

A01L        =        [cos(theta1L),        -cos(alpha1)*sin(theta1L),
sin(alpha1)*sin(theta1L),        a1*cos(theta1L);        sin(theta1L),
cos(alpha1)*cos(theta1L), -sin(alpha1)*cos(theta1L), a1*sin(theta1L); 0,
sin(alpha1), cos(alpha1), d1; 0, 0, 0, 1 ];
A01R        =        [cos(theta1R),        -cos(alpha1)*sin(theta1R),
sin(alpha1)*sin(theta1R),        a1*cos(theta1R);        sin(theta1R),
cos(alpha1)*cos(theta1R), -sin(alpha1)*cos(theta1R), a1*sin(theta1R); 0,
sin(alpha1), cos(alpha1), d1; 0, 0, 0, 1 ];
A12 = [cos(theta2), -cos(alpha2)*sin(theta2), sin(alpha2)*sin(theta2),
a2*cos(theta2);      sin(theta2),      cos(alpha2)*cos(theta2),      -
sin(alpha2)*cos(theta2), a2*sin(theta2); 0, sin(alpha2), cos(alpha2), d2;
0, 0, 0, 1 ];
A23 = [cos(theta3), -cos(alpha3)*sin(theta3), sin(alpha3)*sin(theta3),
a3*cos(theta3);      sin(theta3),      cos(alpha3)*cos(theta3),      -
sin(alpha3)*cos(theta3), a3*sin(theta3); 0, sin(alpha3), cos(alpha3), d3;
0, 0, 0, 1 ];
A34 = [cos(theta4), -cos(alpha4)*sin(theta4), sin(alpha4)*sin(theta4),
a4*cos(theta4);      sin(theta4),      cos(alpha4)*cos(theta4),      -
sin(alpha4)*cos(theta4), a4*sin(theta4); 0, sin(alpha4), cos(alpha4), d4;
0, 0, 0, 1 ];
A45 = [cos(theta5), -cos(alpha5)*sin(theta5), sin(alpha5)*sin(theta5),
a5*cos(theta5);      sin(theta5),      cos(alpha5)*cos(theta5),      -
sin(alpha5)*cos(theta5), a5*sin(theta5); 0, sin(alpha5), cos(alpha5), d5;
0, 0, 0, 1 ];
A56 = [cos(theta6), -cos(alpha6)*sin(theta6), sin(alpha6)*sin(theta6),
a6*cos(theta6);      sin(theta6),      cos(alpha6)*cos(theta6),      -
```

```
sin(alpha6)*cos(theta6), a6*sin(theta6); 0, sin(alpha6), cos(alpha6), d6;
0, 0, 0, 1 ];

%Forward Kinematics
A06L = A00L*A01L*A12*A23*A34*A45*A56;
A06R = A00R*A01R*A12*A23*A34*A45*A56;

Envelope_1 = plot3(A06L(1,4), A06L(2,4), A06L(3,4),'g.');
Envelope_2 = plot3(A06R(1,4), A06R(2,4), A06R(3,4),'c.');
refreshdata(Tool1,'caller')
refreshdata(Tool2,'caller')
axis equal
drawnow
pause(.001)
        else
        end
                    end
                end
            end
        end
    end
end
```

G3

```
%Singularity_DA20Left Arm
clc
clear all
% D-H Parameters
a0 = 295;
a1 = 135;
a2 = 250;
a3 = 390;
a4 = 0;
a5 = 0;
a6 = 0;

d0 = 965;
d1 = 136.5;
d2 = 0;
d3 = 0;
d4 = -230;
d5 = 0;
d6 = -65;

% Axis Properties Left Arm
XL=[0 0 a0 a0 a0+a1 a0+a1 a0+a1+a3 a0+a1+a3 a0+a1+a3 a0+a1+a3];
ZL=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YL=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];
Tool1 = plot3(XL,YL,ZL,'b','LineWidth',5);
grid on
hold('all')

% Axis Properties Right Arm
XR=[0 0 -a0 -a0 -a0-a1 -a0-a1 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3];
```



```
ZR=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YR=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];
Tool2 = plot3(XR,YR,ZR,'b','LineWidth',5);
grid on
hold('all')

disp ('DA20 SingularitiesL-Monika')

ALPHA_0 =(90*pi)/180;
theta0 = (0*pi)/180;
ALPHA_1 = (90*pi)/180;
ALPHA_2 = (180*pi)/180;
ALPHA_3 = (-90*pi)/180;
ALPHA_4 = (90*pi)/180;
ALPHA_5 = (-90*pi)/180;
ALPHA_6 = (180*pi)/180;

X_mo=[]; Y_mo=[]; Z_mo=[];

XmoM=[]; YmoM=[]; ZmoM=[];

for theta1_0 = -190:10:80;
    for theta4_0 = -180:40:180;
        for theta5_0 = -120:40:120;
            for theta6_0 = -180:40:180;

            theta1 = (0+theta1_0)*pi/180;
            theta4 = (0+theta4_0)*pi/180;
            theta3_0 = -atan2(d4,a3);
            theta2_0                      =                      atan2((-
a1+a2+a3*cos(theta3_0)+d4*sin(theta3_0)),(a3*sin(theta3_0)+d4*cos(theta
3_0)));
            theta2 = (pi/2+theta2_0);
            theta3 = (pi/2+theta3_0+theta2_0);
            theta5 = (0+theta5_0)*pi/180;
            theta6 = (0+theta6_0)*pi/180;

    P0        =        [cos(theta0),        -cos(ALPHA_0)*sin(theta0),
sin(ALPHA_0)*sin(theta0),          a0*cos(theta0);          sin(theta0),
cos(ALPHA_0)*cos(theta0), -sin(ALPHA_0)*cos(theta0), a0*sin(theta0); 0,
sin(ALPHA_0), cos(ALPHA_0), d0; 0, 0, 0, 1 ];
    P1        =        [cos(theta1),        -cos(ALPHA_1)*sin(theta1),
sin(ALPHA_1)*sin(theta1),          a1*cos(theta1);          sin(theta1),
cos(ALPHA_1)*cos(theta1), -sin(ALPHA_1)*cos(theta1), a1*sin(theta1); 0,
sin(ALPHA_1), cos(ALPHA_1), d1; 0, 0, 0, 1 ];
    P2        =        [cos(theta2),        -cos(ALPHA_2)*sin(theta2),
sin(ALPHA_2)*sin(theta2),          a2*cos(theta2);          sin(theta2),
cos(ALPHA_2)*cos(theta2), -sin(ALPHA_2)*cos(theta2), a2*sin(theta2); 0,
sin(ALPHA_2), cos(ALPHA_2), d2; 0, 0, 0, 1 ];
    P3        =        [cos(theta3),        -cos(ALPHA_3)*sin(theta3),
sin(ALPHA_3)*sin(theta3),          a3*cos(theta3);          sin(theta3),
cos(ALPHA_3)*cos(theta3), -sin(ALPHA_3)*cos(theta3), a3*sin(theta3); 0,
sin(ALPHA_3), cos(ALPHA_3), d3; 0, 0, 0, 1 ];
    P4        =        [cos(theta4),        -cos(ALPHA_4)*sin(theta4),
sin(ALPHA_4)*sin(theta4),          a4*cos(theta4);          sin(theta4),
```

```
cos(ALPHA_4)*cos(theta4), -sin(ALPHA_4)*cos(theta4), a4*sin(theta4); 0,
sin(ALPHA_4), cos(ALPHA_4), d4; 0, 0, 0, 1 ];
    P5        =        [cos(theta5),        -cos(ALPHA_5)*sin(theta5),
sin(ALPHA_5)*sin(theta5),          a5*cos(theta5);          sin(theta5),
cos(ALPHA_5)*cos(theta5), -sin(ALPHA_5)*cos(theta5), a5*sin(theta5); 0,
sin(ALPHA_5), cos(ALPHA_5), d5; 0, 0, 0, 1 ];
    P6        =        [cos(theta6),        -cos(ALPHA_6)*sin(theta6),
sin(ALPHA_6)*sin(theta6),          a6*cos(theta6);          sin(theta6),
cos(ALPHA_6)*cos(theta6), -sin(ALPHA_6)*cos(theta6), a6*sin(theta6); 0,
sin(ALPHA_6), cos(ALPHA_6), d6; 0, 0, 0, 1 ];

    P0_6 = P0*P1*P2*P3*P4*P5*P6;

    X_mo=[X_mo P0_6(1,4)]; Y_mo=[Y_mo P0_6(2,4)]; Z_mo=[Z_mo P0_6(3,4)];

    plot3(P1_6(1,4), P1_6(2,4), P1_6(3,4),'g.','LineWidth',2)
            end
        end
    end
    XmoM=[XmoM; X_mo];
    X_mo=[];
    YmoM=[YmoM; Y_mo];
    Y_mo=[];
    ZmoM=[ZmoM; Z_mo];
    Z_mo=[];
end
```

G4

```
%Singularity_DA20Right Arm
clear all
% D-H Parameters
a0 = 295;
a1 = 135;
a2 = 250;
a3 = 390;
a4 = 0;
a5 = 0;
a6 = 0;

d0 = 965;
d1 = 136.5;
d2 = 0;
d3 = 0;
d4 = -230;
d5 = 0;
d6 = -65;

% Axis Properties Left Arm

XL=[0 0 a0 a0 a0+a1 a0+a1 a0+a1+a3 a0+a1+a3 a0+a1+a3 a0+a1+a3];
ZL=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YL=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];
Tool1 = plot3(XL,YL,ZL,'b','LineWidth',5);
grid on
hold('all')
```



```
% Axis Properties Right Arm
XR=[0 0 -a0 -a0 -a0-a1 -a0-a1 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3 -a0-a1-a3];
ZR=[0 d0 d0 d0 d0 d0 d0 d0 d0 d0];
YR=[0 0 0 -d1 -d1 -d1-a2 -d1-a2 -d1-a2+d4 -d1-a2+d4 -d1-a2+d4+d6];
Tool2 = plot3(XR,YR,ZR,'b','LineWidth',5);
grid on
hold('all')

disp ('DA20 SingularitiesR-Monika')

ALPHA_0 =(-90*pi)/180;
theta0 = (180*pi)/180;
ALPHA_1 = (90*pi)/180;
ALPHA_2 = (180*pi)/180;
ALPHA_3 = (-90*pi)/180;
ALPHA_4 = (90*pi)/180;
ALPHA_5 = (-90*pi)/180;
ALPHA_6 = (180*pi)/180;

X_mo=[]; Y_mo=[]; Z_mo=[];

XmoM=[]; YmoM=[]; ZmoM=[];

for theta1_0 = -190:10:80;
    for theta4_0 = -180:40:180;
        for theta5_0 = -120:40:120;
            for theta6_0 = -180:40:180;

            theta1 = (0+theta1_0)*pi/180;
            theta4 = (0+theta4_0)*pi/180;
            theta3_0 = -atan2(d4,a3);
            theta2_0                    =                    atan2((-
a1+a2+a3*cos(theta3_0)+d4*sin(theta3_0)),(a3*sin(theta3_0)+d4*cos(theta
3_0)));
            theta2 = (pi/2+theta2_0);
            theta3 = (pi/2+theta3_0+theta2_0);
            theta5 = (0+theta5_0)*pi/180;
            theta6 = (0+theta6_0)*pi/180;

    P0        =        [cos(theta0),        -cos(ALPHA_0)*sin(theta0),
sin(ALPHA_0)*sin(theta0),          a0*cos(theta0);          sin(theta0),
cos(ALPHA_0)*cos(theta0), -sin(ALPHA_0)*cos(theta0), a0*sin(theta0); 0,
sin(ALPHA_0), cos(ALPHA_0), d0; 0, 0, 0, 1 ];
    P1        =        [cos(theta1),        -cos(ALPHA_1)*sin(theta1),
sin(ALPHA_1)*sin(theta1),          a1*cos(theta1);          sin(theta1),
cos(ALPHA_1)*cos(theta1), -sin(ALPHA_1)*cos(theta1), a1*sin(theta1); 0,
sin(ALPHA_1), cos(ALPHA_1), d1; 0, 0, 0, 1 ];
    P2        =        [cos(theta2),        -cos(ALPHA_2)*sin(theta2),
sin(ALPHA_2)*sin(theta2),          a2*cos(theta2);          sin(theta2),
cos(ALPHA_2)*cos(theta2), -sin(ALPHA_2)*cos(theta2), a2*sin(theta2); 0,
sin(ALPHA_2), cos(ALPHA_2), d2; 0, 0, 0, 1 ];
    P3        =        [cos(theta3),        -cos(ALPHA_3)*sin(theta3),
sin(ALPHA_3)*sin(theta3),          a3*cos(theta3);          sin(theta3),
cos(ALPHA_3)*cos(theta3), -sin(ALPHA_3)*cos(theta3), a3*sin(theta3); 0,
sin(ALPHA_3), cos(ALPHA_3), d3; 0, 0, 0, 1 ];
```

```
    P4         =         [cos(theta4),         -cos(ALPHA_4)*sin(theta4),
sin(ALPHA_4)*sin(theta4),          a4*cos(theta4);          sin(theta4),
cos(ALPHA_4)*cos(theta4), -sin(ALPHA_4)*cos(theta4), a4*sin(theta4); 0,
sin(ALPHA_4), cos(ALPHA_4), d4; 0, 0, 0, 1 ];
    P5         =         [cos(theta5),         -cos(ALPHA_5)*sin(theta5),
sin(ALPHA_5)*sin(theta5),          a5*cos(theta5);          sin(theta5),
cos(ALPHA_5)*cos(theta5), -sin(ALPHA_5)*cos(theta5), a5*sin(theta5); 0,
sin(ALPHA_5), cos(ALPHA_5), d5; 0, 0, 0, 1 ];
    P6         =         [cos(theta6),         -cos(ALPHA_6)*sin(theta6),
sin(ALPHA_6)*sin(theta6),          a6*cos(theta6);          sin(theta6),
cos(ALPHA_6)*cos(theta6), -sin(ALPHA_6)*cos(theta6), a6*sin(theta6); 0,
sin(ALPHA_6), cos(ALPHA_6), d6; 0, 0, 0, 1 ];

    P0_6 = P0*P1*P2*P3*P4*P5*P6;

    X_mo=[X_mo P0_6(1,4)]; Y_mo=[Y_mo P0_6(2,4)]; Z_mo=[Z_mo P0_6(3,4)];

    plot3(P1_6(1,4), P1_6(2,4), P1_6(3,4),'g.','LineWidth',2)
            end
        end
    end
    XmoM=[XmoM; X_mo];
    X_mo=[];
    YmoM=[YmoM; Y_mo];
    Y_mo=[];
    ZmoM=[ZmoM; Z_mo];
    Z_mo=[];
end
```

VITA AUCTORIS

NAME:	Monika Filiposka
PLACE OF BIRTH:	Macedonia
YEAR OF BIRTH:	1985
EDUCATION:	BSc, Universitet Sveti Kliment Ohridski, Bitola, Macedonia, 2012
	MASc, University of Windsor, Windsor, ON, Canada 2014