

〈研究ノート〉

Point Estimates of Parameters for Convex Adjustment Costs of Labor Employment

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Abstract

This research note reports the point estimates of parameters for convex adjustment costs of labor employment. When an economic model is nonlinear, its corresponding econometrics model is usually a linear approximation, and is subject to linearization errors. In order to avoid such errors, the research note directly estimates a nonlinear econometrics model. The economic model is dynamic labor adjustments with convex adjustment costs, which is nonlinear, and the target industry is the automobile-parts manufacturing industry. By numerically solving the nonlinear economic model, the research note shows that the l_1 norm is superior to the l_2 norm, the l_∞ norm, and the GMM minimand, and reports best estimates by the grid search method.

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Key words: Bellman equation, Euler equation, Newton's method

1. Economic Model

A firm adjusts its labor employment every year in accordance with the fluctuating economic environment surrounding it. A firm hires two types of workers: regular workers and nonstandard workers. Adjusting the number of regular workers, l , incurs convex adjustment costs, while adjusting the number of nonstandard workers, n , is free of adjustment costs. The production function and the output demand are, respectively, Cobb-Douglas and iso-elastic. Then, the firm's problem becomes the follow-

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ing:

$$V[Z_0, l_0] = \max_{\{l_{s+1}, n_s\}_{s=0}^{\infty}} E_0 \left\{ \sum_{s=0}^{\infty} \beta^s [VA(Z_s, l_s, n_s) - TW(l_s, n_s) - AC(l_s, l_{s+1})] \right\} \quad (1)$$

where V , VA , TW , AC and $\beta \in (0, 1)$ are, respectively: the value function; the value-added function, i.e., $VA = \text{revenue } (R) - \text{material costs } (C)$; the total wage; the convex adjustment costs; and the discount factor. In addition, Z is a stochastic coefficient representing economic environment. The functional forms of VA , TW , and AC are the following:

$$VA(Z_s, l_s, n_s) = Z_s K_s^\zeta (l_s + \psi n_s)^\gamma, \quad (\zeta + \gamma \leq 1) \quad (2)$$

$$TW(l_s, n_s) = w_l l_s + w_n n_s, \quad (w_n < w_l, \psi < w_n/w_l) \quad (3)$$

$$AC(l_s, l_{s+1}) = C \left| \frac{l_{s+1} - l_s}{l_s} \right|^\xi l_s^\omega, \quad (\xi > 1) \quad (4)$$

where K , w_l , w_n , and ψ are, respectively: the level of capital stock; the wage rates for regular and non-standard workers; and the relative productivity of nonstandard workers to regular workers. In the economic model, there are seven parameters: $\theta \equiv (\beta, \zeta, \gamma, \psi, C, \xi, \omega)$. The wage rates are obtained from the “Monthly Labour Survey” conducted by the Ministry of Health, Labour and Welfare.

Because nonstandard workers can be adjusted without adjustment costs, the optimal number of nonstandard workers can be written as follows:

$$n^* = \max \left[0, \frac{1}{\psi} \left\{ \left(\frac{\gamma \psi Z K^\zeta}{w_n} \right)^{\frac{1}{1-\gamma}} - l \right\} \right]. \quad (5)$$

Thus, the optimal employment of nonstandard workers is a function of the variable l . Entering the optimal employment of nonstandard workers into the functions VA and TW yields the following:

$$VA(Z, l, n^*) - TW(l, n^*) = \begin{cases} \frac{(1-\gamma) w_n}{\gamma \psi} \left(\frac{\gamma \psi Z K^\zeta}{w_n} \right)^{\frac{1}{1-\gamma}} - \left(w_l - \frac{w_n}{\psi} \right) l & \text{if } n^* > 0 \\ ZK^\zeta l^\gamma - w_l l & \text{if } n^* = 0. \end{cases} \quad (6)$$

Then, equation (1) can be rewritten as follows:

$$V[Z, l] = \max_{l'} \{r(Z, l, l') + \beta E_Z V[Z', l']\}. \quad (7)$$

where r is the reward function, i.e., $r(Z, l, l') = VA(Z, l) - TW(l) - AC(l, l')$; the prime ($'$) indicates the value of the next period. Equation (7) is the theoretical model or the Bellman equation for dynamic labor adjustments.

Solving equation (7) yields the policy function or the predicted number of regular workers, $\hat{l}'(Z, l)$, as well as the value function V . With the policy function, the predicted values of the labor adjustment rate, $(l' - l)/l$, become computable. The conditional expectation of residuals, u , is zero, i.e.,

$$E[u(\hat{\theta}) | X] = 0 \quad \text{where } u(\hat{\theta}) \equiv \frac{l' - l}{l} - \frac{\hat{l}'(X | \hat{\theta}) - l}{l}, \quad (8)$$

and X is a vector of economic variables or $X \equiv (1, l, K, R, C)^T$. The variable vector X includes the unity for convenience and excludes the variable Z because the research note estimates Z by the variables in X . Because there are no analytical solutions of the Bellman equation, the research note resorts to numerical solutions, particularly the value function iterations.

The maximization of equation (7) also yields the Euler equation as the first order condition of maximization. The Euler equation can be written as follows:

$$\frac{\partial \pi(l, l', Z; \theta)}{\partial l'} + \beta E_Z \left[\frac{\partial \pi(l', l'', Z'; \theta)}{\partial l'} \right] = 0, \quad (9)$$

where

$$\frac{\partial \pi(l, l', Z; \theta)}{\partial l'} = \begin{cases} C \left| \frac{l'' - l'}{l'} \right|^{\xi-1} l'^{\omega-2} \left\{ \xi \cdot \text{sign} \left(\frac{l'' - l'}{l'} \right) l'' - \omega \left| \frac{l'' - l'}{l'} \right| l' \right\} - \left(w_1 \frac{w_n}{\phi} \right) & \text{if } n' > 0 \\ C \left| \frac{l'' - l'}{l'} \right|^{\xi-1} l'^{\omega-2} \left\{ \xi \cdot \text{sign} \left(\frac{l'' - l'}{l'} \right) l'' - \omega \left| \frac{l'' - l'}{l'} \right| l' \right\} + \gamma Z K^\gamma l'^{\gamma-1} - w_1 & \text{if } n' = 0 \end{cases}$$

and $\frac{\partial \pi(l, l', Z; \theta)}{\partial l'} = -\text{sign} \left(\frac{l' - l}{l} \right) \xi C \left| \frac{l' - l}{l} \right|^{\xi-1} l^{\omega-1}$.

Then, the following conditional expectation holds:

$$E[v(l, l', l'', Z, Z'; \theta) | X] = 0 \quad (10)$$

where $v(l, l', l'', Z, Z'; \theta) \equiv \frac{\partial \pi(l, l', Z; \theta)}{\partial l'} + \beta E_Z \left[\frac{\partial \pi(l', l'', Z'; \theta)}{\partial l'} \right]$. Equation (10) is the conditional expectation for the Euler equation. The variables l' and l'' are respectively replaced by the policy function $\hat{l}'(Z, l)$ and $\hat{l}''(Z', l) = \hat{l}'(Z', \hat{l}')$ in the numerical solutions.

2. Selection of Norm by Grid Search

Equations (8) and (10) yield the orthogonality conditions, i.e., $E[Xu] = 0$ and $E[Xv] = 0$. Because the values of X are greatly different, which makes it impractical to compare moments, the sample mo-

ments are augmented by the corresponding sample means of X . Thus, the augmented sample moments are the following zero expectations:

$$m(\theta) = \left(E_n[u], E_n\left[\frac{lu}{E_n[l]}\right], E_n\left[\frac{Ku}{E_n[K]}\right], E_n\left[\frac{Ru}{E_n[R]}\right], E_n\left[\frac{Cu}{E_n[C]}\right], \right. \\ \left. E_n[v], E_n\left[\frac{lv}{E_n[l]}\right], E_n\left[\frac{Kv}{E_n[K]}\right], E_n\left[\frac{Rv}{E_n[R]}\right], E_n\left[\frac{Cv}{E_n[C]}\right] \right) \theta = 0. \quad (11)$$

Even though the population moments are zero, the sample moments should be nonzero. Therefore, as its measure of greatness for equation (11), the analysis employs three norms; the l_1 norm, the l_2 norm and the l_∞ norm. Econometric analyses usually choose the l_2 norm, or the square of it. In addition, the research note attempts the l_1 norm and the l_∞ norm.

In econometrics, estimations based on equation (11) usually rely upon the generalized method of moments (GMM) which is the following minimization:

$$\min_{\theta} m(\tilde{\theta})^T \cdot W \cdot m(\tilde{\theta}) \quad (12)$$

where W is a positive definite matrix. The optimal choice for the matrix W is the variance matrix of $m(\theta)$. When the number of the moments exceeds the number of parameters, —an over-identified case— the GMM minimand, $m^T \cdot W \cdot m (\equiv Qn)$, follows the chi-squared distribution under the null hypothesis that the population means are all zero.

As the first step of minimizing the norms or the GMM minimand, the research note resorts to the grid search for parameters θ . The grid search is a primitive and demanding way to find the solution of the minimization, but it provides the properties and characteristics of the minimization problem. In particular, when the minimization problem is ill-conditioned, the grid search method is very helpful. Table 1 shows the grid points of the parameters. The grid search requires 3,888 calculations. If the calculations are done by a personal computer, each calculation requires about thirty minutes and it

Table 1. Parameter Values for Grid Search

θ	
β	0.93, 0.94, 0.95, 0.96
ζ	0.30, 0.35, 0.40
γ	0.40, 0.50, 0.60
ψ	0.20, 0.225, 0.25
C	0.20, 0.40, 0.60, 0.80
ξ	1.50, 1.75, 2.00
ω	1.50, 1.75, 2.00

would take a couple of thousand hours. Because the research note relies upon a super computer, its calculations took about forty hours. Thus, this specific method of research is possible only by using a super computer.

3. Results

Figure 1 panel (a) shows the entire graph of the grid search. There are many points along the horizontal line around $E[u] = 0.04$, where the predicted adjustment rate is close to zero. Those points therefore result from too great adjustment costs. Figure 1 panel (b) is the scatter plot around the origin. Some points are very close to the origin of the $E[u]-E[v]$ plane. However, those points may not be close to zero for other moments.

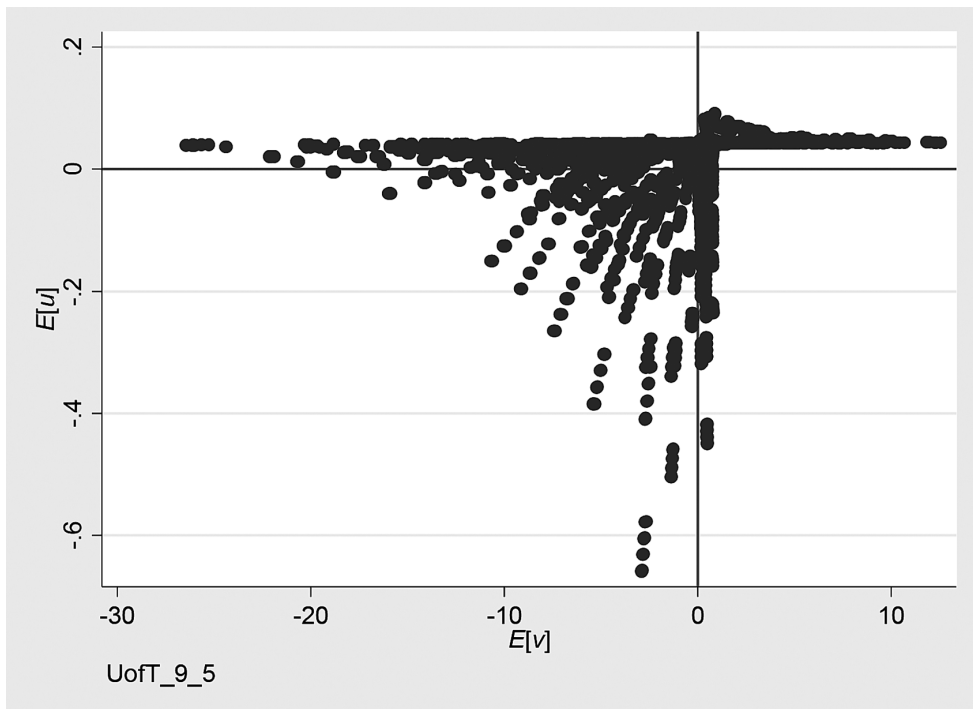
Panels of figure 2 show the scatter plots for the three examined norms and the GMM minimand, Q_n . Figure 2 panel (a) demonstrates that the l_1 norm is minimized on the x -axis. Thus, the predicted adjustment rates are close to the actual rates on average. The other three panels illustrate that their parameter values, at minimum, are corresponding to zero adjustments, so that predicted adjustment costs are too great. Table 2 shows the values of the parameters and the moments. Table 2 also shows that, although all estimated moments are far less than the corresponding standard errors, the p -values of the chi-squared test are too little to reject the null hypothesis so that some of the population moments are nonzero.

4. Further Research

The research note finds that the l_1 norm is superior to the two other examined norms and the GMM minimand because the latter three tend to over-estimate the parameter values. Because the grid search is an inefficient method for solving minimization problems, the next step would be Newton's method or the quasi-Newton method. However, although it is not stated in the research note, both the Hessian matrix for minimization problems and the Jacobian matrix for solving nonlinear equations seem close to singular; that is, the estimation is ill-conditioned.

Figure 1. Grid Search

(a) Entire graph



(b) Partial graph around the origin

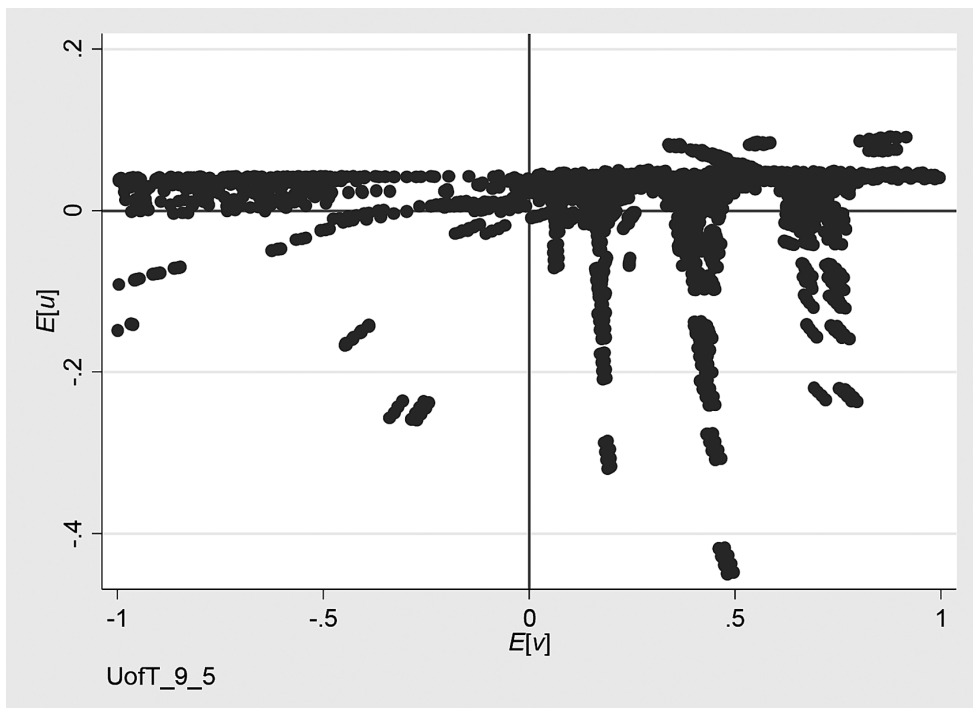
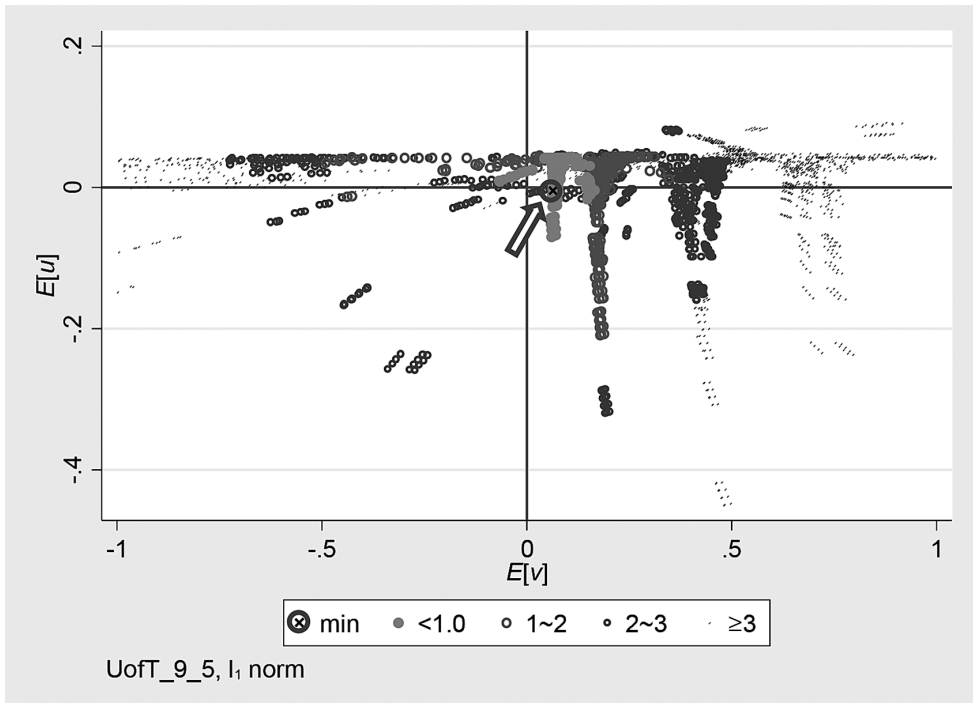
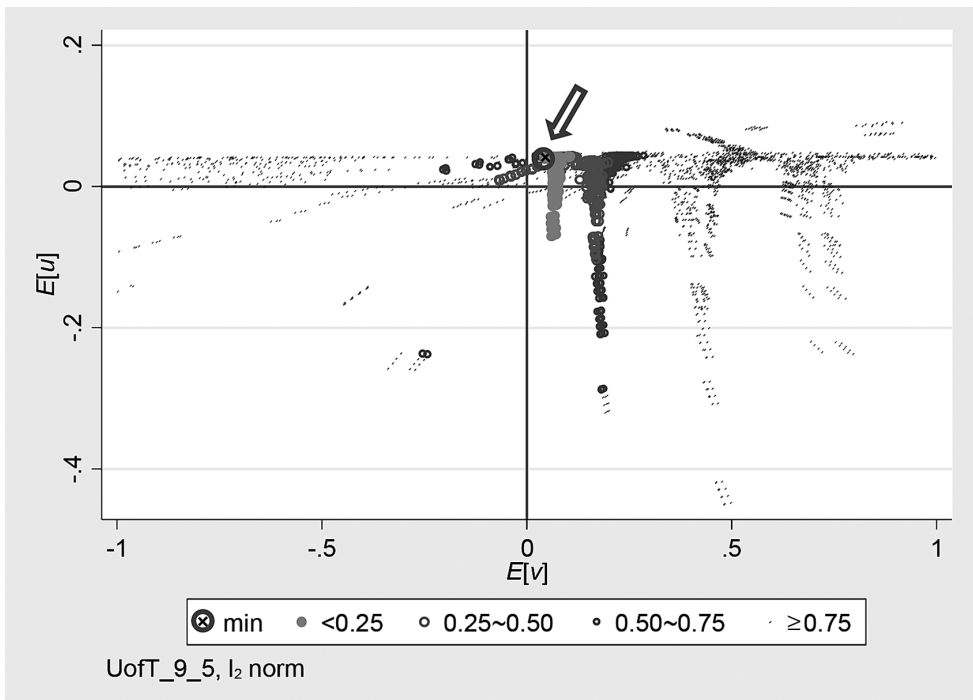


Figure 2. Values of the Norm and the GMM Minimand

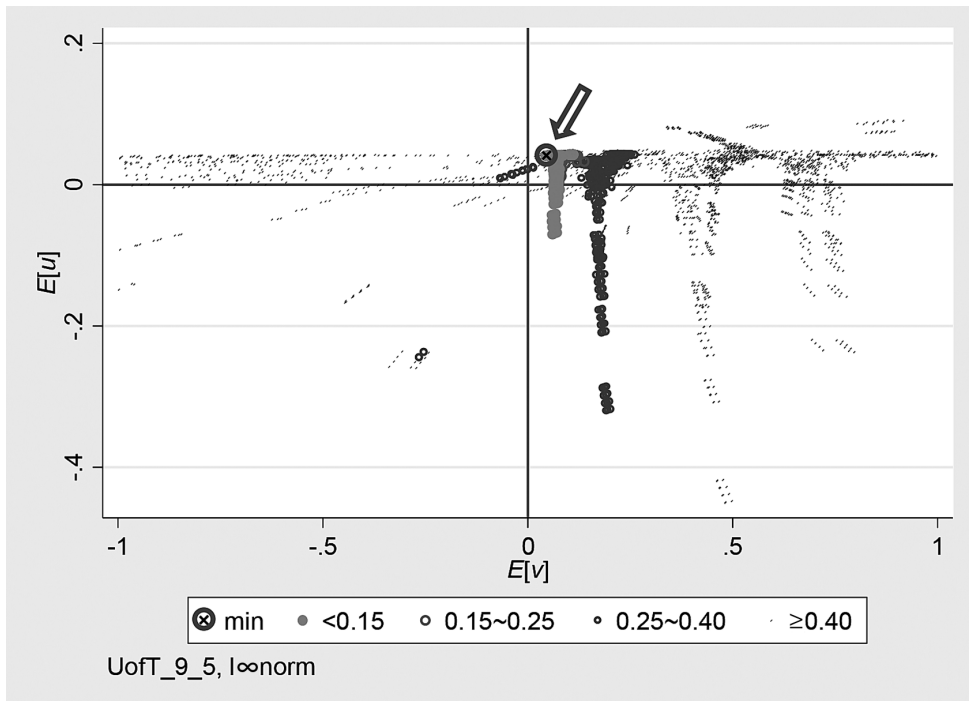
(a) l_1 norm



(b) l_2 norm



(c) $l\omega$ norm



(d) GMM minimand, Q_n

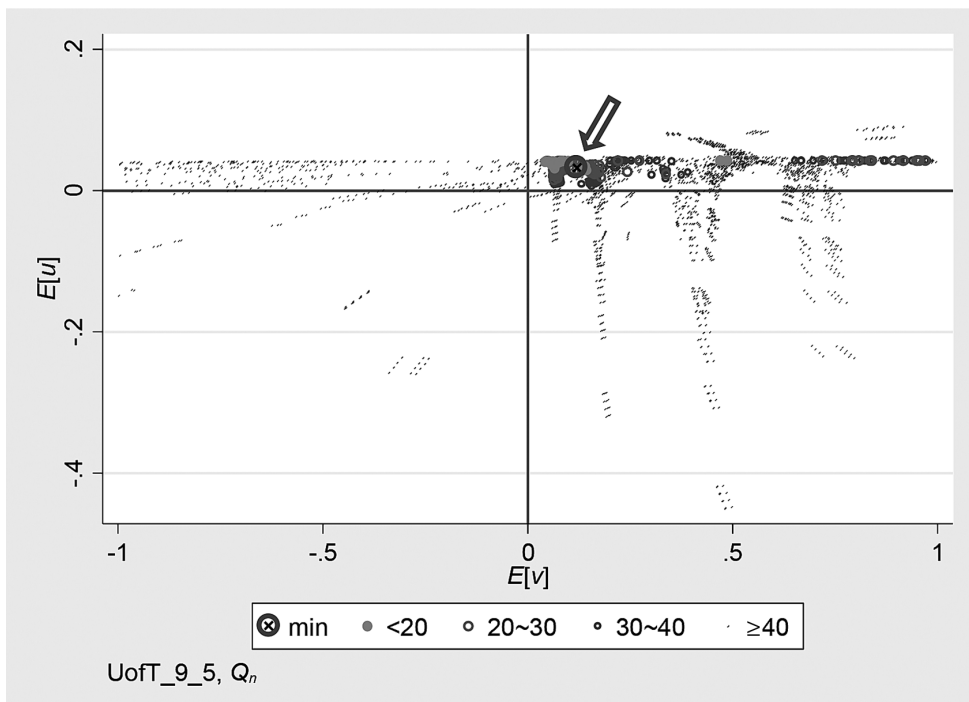


Table 2. Parameters and augmented moments at the least norms or GMM minimand

	l_1		l_2 & l_∞		Q_n	
value	0.4275		0.1511 & 0.0783		13.73	
	Parameters					
β	0.930		0.950		0.960	
ζ	0.300		0.300		0.350	
γ	0.600		0.600		0.600	
ψ	0.250		0.250		0.225	
C	0.400		1.000		1.000	
ξ	2.000		2.000		2.000	
ω	1.500		2.000		2.000	
	Augmented moments					
	std. error		std. error		std. error	
$E_n[u]$	-0.0047	0.2745	0.0416	0.2751	0.0325	0.2741
$E_n[lv/E_n[l]]$	-0.0094	0.1880	0.0263	0.1888	0.0191	0.1885
$E_n[Ku/E_n[K]]$	0.0006	0.3122	0.0468	0.3146	0.0385	0.3133
$E_n[Ru/E_n[R]]$	0.0004	0.2132	0.0323	0.2165	0.0257	0.2155
$E_n[Cu/E_n[C]]$	0.0204	0.2859	0.0421	0.2967	0.0364	0.2947
$E_n[v]$	0.0641	0.1446	0.0459	0.1926	0.1196	0.4838
$E_n[lv/E_n[l]]$	0.0729	0.1689	0.0505	0.2282	0.1372	0.5013
$E_n[Kv/E_n[K]]$	0.0512	0.1090	0.0346	0.1538	0.0949	0.3823
$E_n[Rv/E_n[R]]$	0.0868	0.2233	0.0586	0.3168	0.1718	0.6766
$E_n[Cv/E_n[C]]$	0.1171	0.3247	0.0783	0.4385	0.2358	0.9757
Q_n	62.39	(1.81E-13)	27.83	(3.94E-6)	13.73	(3.30E-3)

Remark: The p -values of the χ^2 tests in the parentheses (χ^2 distribution with 3 degrees of freedom)

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