〈研究ノート〉

Estimating Convex Adjustment Costs for Dynamic Labor Adjustments[1)]

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Abstract

This research note investigates the applicability of the Euler equation to estimate convex adjustment costs for dynamic labor adjustments. It is well established that there are convex adjustment costs for capital investment. However, observations of labor adjustments suggest that there are also convex adjustment costs for labor employments. The objective of this research is to estimate the convex adjustment costs. There are six parameters in a theoretical model, and this study estimates three of them that are associated with the Euler equation. The estimation is actually to solve a system of three nonlinear equations. This research note is primarily based on Dennis and Schnabel (1996) and Kelley (2003). Both books contain detailed discussions of programming as well as mathematics related to numerically solving systems of nonlinear equations.

Key words: Euler equation, Bellman equation, Newton's method

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1. Theoretical Model

A firm adjusts its labor employment every year. It hires two types of workers. One type is regular workers and the other is nonstandard workers. Adjusting the number of regular workers, l, incurs convex adjustment costs, while adjusting the number of nonstandard workers, n, is free of adjustment costs. The production function and the output demand are, respectively, Cobb-Douglas and iso-elastic. Then, the firm's problem becomes the following:

$$V[Z_0, l_0] = \max_{\{l_{s+1},\, n_s\}_{s=0}^{\infty}} E_0 \left\{ \sum_{s=0}^{\infty} \beta^s [R(Z_s, l_s, n_s) - W(l_s, n_s) - A(l_s, l_{s+1})] \right\} \tag{1}$$

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where V, R, W, A and $\beta \in (0, 1)$ are, respectively, the value function, the revenue function, the total wage, the convex adjustment costs, and the discount factor. The functions, R, W, and A are the following:

$$R\ (Z_s, l_s, n_s) = Z_s K_s^{\zeta} (l_s + \phi n_s)^{\gamma}, \qquad (\zeta + \gamma \leq 1) \tag{2}$$

$$W(l_s, n_s) = w_l l_s + w_n n_s, \qquad (w_n < w_l,\ \phi < w_n / w_l) \tag{3}$$

$$A(l_s, l_{s+1}) = C \left| \frac{l_{s+1} - l_s}{l_s} \right|^{\xi} l_s^{\omega}, \tag{4}$$

where Z, K, w_l, w_n, and ϕ are, respectively, a stochastic coefficient, the level of capital stock, the wage rates for regular and nonstandard workers, and the relative productivity of nonstandard workers to regular workers. There are six parameters : $\theta \equiv (\zeta, \gamma, \phi, C, \xi, \omega)$

Because nonstandard workers can be adjusted without adjustment costs, the optimal number of nonstandard workers can be written as follows:

$$n^* = \max \left[0, \frac{1}{\phi} \left\{ \left(\frac{\gamma \phi Z K^{\zeta}}{w_n} \right)^{\frac{1}{1-\gamma}} - l \right\} \right]. \tag{5}$$

Thus, the optimal employment of nonstandard workers is a function of variables Z and l. Entering the optimal employment of nonstandard workers into the functions R and W yields the following:

$$R(Z, l, n^*) - W(l, n^*) \equiv \pi(Z, l) = \begin{cases} \dfrac{(1-\gamma)\ w_n}{\gamma \phi} \left(\dfrac{\gamma \phi Z K^{\zeta}}{w_n} \right)^{\frac{1}{1-\gamma}} - \left(w_l - \dfrac{w_n}{\phi} \right) l & \text{if } n^* > 0 \\ Z K^{\zeta} l^{\gamma} - w_l l & \text{if } n^* = 0. \end{cases} \tag{6}$$

Then, equation (1) can be rewritten as follows:

$$V[Z, l] = \max_{\{l'\}} \{ r\ (Z, l, l') + \beta E_Z V\ [Z', l'] \}. \tag{7}$$

where r is the reward function, $r\ (Z, l, l') = \pi\ (Z, l) - A\ (l, l')$, and the prime (′) indicates the value of the next period. Equation (7) is the theoretical model or the Bellman equation for dynamic labor adjustments.

Solving equation (7) yields the policy equation $\hat{l}'(Z, l)$, as well as the value function V. With the policy function, the estimated values of the labor adjustment rate, $(l' - l)/l$, become computable. The conditionally expected value of residuals, u, is zero, i.e.,

$$E[u(\hat{\theta}) \mid l, Z] = 0 \quad \text{where } u(\hat{\theta}) \equiv \frac{l' - l}{l} - \frac{\hat{l}'(Z, l \mid \hat{\theta}) - l}{l}. \tag{8}$$

When there are no analytical solutions of the Bellman equation, researchers would resort to numerical solutions of it, either value function iterations or policy function iterations.

The maximization of equation (7) also yields the Euler equation as the first order condition of maximization. The Euler equation has an advantage over the Bellman equation because it is analytically computable. The Euler equation can be written as follows:

$$\frac{\partial r\ (l, l', Z; \theta)}{\partial l'} + \beta E_Z \left[\frac{\partial r\ (l', l'', Z'; \theta)}{\partial l'} \right] = 0, \tag{9}$$

where

$$\frac{\partial r\ (l', l'', Z'; \theta)}{\partial l'} = \begin{cases} C \left| \frac{l''-l'}{l'} \right|^{\xi-1} l'^{\omega-2} \left\{ \xi \cdot sign\left(\frac{l''-l'}{l'} \right) l'' - \omega \left| \frac{l''-l'}{l'} \right| l' \right\} - \left(w_1 \frac{w_n}{\phi} \right) & if\ n' > 0 \\ C \left| \frac{l''-l'}{l'} \right|^{\xi-1} l'^{\omega-2} \left\{ \xi \cdot sign\left(\frac{l''-l'}{l'} \right) l'' - \omega \left| \frac{l''-l'}{l'} \right| l' \right\} + \gamma Z K^{\xi} l'^{\gamma-1} - w_1 & if\ n' = 0 \end{cases}$$

and $\frac{\partial r\ (l, l', Z; \theta)}{\partial l'} = -\text{sign}\left(\frac{l'-l}{l} \right) \xi C \left| \frac{l'-l}{l} \right|^{\xi-1} l^{\omega-1}.$

Then, the following conditional expectation holds:

$$E\left[v(l, l', l'', Z, Z'; \theta) \mid l, Z\right] = 0 \tag{10}$$

where $v(l, l', l'', Z, Z'; \theta) \equiv \frac{\partial r\ (l, l', Z; \theta)}{\partial l'} + \beta E_Z \left[\frac{\partial r\ (l', l'', Z'; \theta)}{\partial l'} \right]$. The variables l' and l'' are respectively replaced by the policy function $\hat{l}'(Z, l)$ and $\hat{l}''(Z', l') = \hat{l}'(Z', \hat{l}')$.

2. System of Nonlinear Equations

Equations (8) and (10) yield orthogonality conditions and the following zero expectations:

$$\begin{cases} E[u] = 0 \\ E[lu] = 0 \\ E[Zu] = 0 \\ E[v] = 0 \\ E[lv] = 0 \\ E[Zv] = 0. \end{cases} \tag{11}$$

Or, $m\ (\theta) \equiv\ (E[u], E[lu], E[Zu], E[v], E[lv], E[Zv])^T = \mathbf{0}$. Equation (11)´s system of six nonlinear equations just identifies six parameters. Empirical researchers replace the expectations with the sample equivalents, and employ the Generalized Method of Moments (GMM) to estimate the parameters, θ. The GMM estimation is to minimize the quadratic form of the moments, $\boldsymbol{m}$, with a symmetric, positive definite weighing matrix W, i.e., minimizing $J \equiv \boldsymbol{m}^T W \boldsymbol{m}$. However, the local minimum of J may not be equal to zero, i.e., $\boldsymbol{m} \neq \boldsymbol{0}$. This research note pursues $\boldsymbol{m} = \boldsymbol{0}$, or the global minimum of J with the identity matrix replacing W. If its Jacobian matrix, $\partial \boldsymbol{m}(\theta)/\partial \theta$, is block diagonal, the numerical solution of equation (11) may be easier.

The strategy that the research note pursues is, therefore, to divide equation (11) into two subsets. In addition, the parameters are also split into two subsets, i.e., $\theta_1 \equiv (\zeta, \gamma, \psi)$ and $\theta_2 \equiv (C, \xi, \omega)$. The first subset of the moments is that of orthogonality conditions for the residuals, u, and the second subset of the moments is that for the Euler equation, v, i.e., $\boldsymbol{m}_1(\theta_1, \theta_2) \equiv (E[u], E[lu]$ and $E[Zu])$ and $\boldsymbol{m}_2(\theta_1, \theta_2) \equiv (E[v], E[lv]$ and $E[Zv])$. The first subset consists of revenue function parameters, and the second contains parameters of convex adjustment costs. Then, the research note applies the Gauss-Seidel procedure, which is one iterative method for solving the system of equations. The Gauss-Seidel procedure is usually applied to a system of equations on the single-equation base. The research note, however, applies the procedure on the block-of-equations base. The first step of each iteration simultaneously solves $\boldsymbol{m}_2 = 0$ for θ_2 with θ_1 given. Then, the second step simultaneously solves $\boldsymbol{m}_1 = 0$ for θ_1 with the solution of θ_2 derived from the first step. The procedure repeats these steps until the solutions of $\boldsymbol{m}_1 = \boldsymbol{0}$ and $\boldsymbol{m}_2 = \boldsymbol{0}$ converge. However, the research note deals only with the first step in evaluating the Gauss-Seidel procedure on the block-of-equations base.

The first-order Taylor approximation of the function $\boldsymbol{m}_2$ around its solution, θ_2^*, is the following:

$$\theta_2^* \approx \theta_2 - J(\theta_2)^{-1} \boldsymbol{m}_2(\theta_2) \tag{12}$$

where the function J is the Jacobian matrix. Then, modifying equation (12) yields Newton's method for numerical solutions:

$$\theta_2^{(i+1)} \approx \theta_2^{(i)} - J(\theta_2^{(i)})^{-1} \boldsymbol{m}_2(\theta_2^{(i)}), \tag{13}$$

where (i) indicates the i-th order of the iterations. The research note sets the Newton direction, $s(\theta_2^{(i)})$, identically equal to the second term normalized by the l-2 norm, i.e., $s(\theta_2^{(i)}) \equiv -J(\theta_2^{(i)})^{-1} \boldsymbol{m}_2(\theta_2^{(i)}) / \| J^{-1}.\boldsymbol{m}_2 \|_2$. The research note then multiplies the Newton direction by a step size, λ, so that equation (13) becomes the following:

$$\theta_2^{(i+1)} = \theta_2^{(i)} + \lambda s(\theta_2^{(i)}) \tag{14}$$

The step size varies from about one thousandth to about one tenth, and the best one is chosen such

Table 1. Newton's method

(1) Case 1: $\zeta = 0.525$. $\gamma = 0.375$, $\psi = 0.2475$

C	ξ	ω	$E[v]$	$E[lv]/E[l]$	$E[Zv]/E[Z]$	*Sum of Sq.*
6	1.15	0.112	−0.19586	0.00721	−0.14970	0.06082
6.77513	1.00803	−0.02593	−0.14920	−0.05669	−0.11361	0.03838
10.17235	1.08030	−0.14265	−0.04384	−0.00702	0.03963	0.00334
10.18735	1.08002	−0.14238	−0.04426	−0.00707	0.03642	0.00334
10.40696	1.07300	−0.15338	−0.04224	−0.00871	0.03829	0.00333
10.62649	1.06061	−0.16070	−0.04468	−0.01648	0.03420	0.00330
10.62869	1.06070	−0.16071	−0.04465	−0.01162	0.03425	0.00330

Remark: The iteration was terminated because the improvement in the sum of squares is below 10^{-6}.

(2) Case 2: $\zeta = 0.475$. $\gamma = 0.425$, $\psi = 0.25$

C	ξ	ω	$E[v]$	$E[lv]/E[l]$	$E[Zv]/E[Z]$	*Sum of Sq.*
5	1.1	0.0	−0.00608	0.05355	0.05430	0.00585
7.76841	0.78556	−0.27754	−0.02137	0.06277	0.02174	0.00487

Remark: The iteration was terminated because there was no improvement in the sum of squares.

(3) Case 3: $\zeta = 0.525$. $\gamma = 0.425$, $\psi = 0.2525$

C	ξ	ω	$E[v]$	$E[lv]/E[l]$	$E[Zv]/E[Z]$	*Sum of Sq.*
7	1.1	−0.1	−0.01090	0.02057	0.05652	0.00374
8.39440	1.00109	−0.17666	−0.01664	0.00322	0.04203	0.00205
8.39255	1.00001	−0.17686	−0.01683	0.00304	0.04167	0.00203

Remark: The iteration was terminated because the Jacobian matrix is not invertible.

that it yields m_2 closest to zero.

When an increment of θ_2, $\Delta\theta_2 \equiv \lambda s\ (\theta_2)$, improves the numerical solution, the following condition must hold:

$$f(\theta_2^{(i)} + \Delta\theta_2) \approx f(\theta_2^{(i)}) + \frac{\partial f}{\partial \theta_2} \Delta\theta_2 < f(\theta_2^{(i)}) \tag{15}$$

for some measure of performance f. When the measure is the square of the l-2 norm, i.e., the sum of squares, the equation (15) becomes the following:

$$m_2(\theta_2^{(i)})^T J(\theta_2^{(i)}) \Delta\theta < 0. \tag{16}$$

Equation (16) may not hold, however, because of truncation errors or other errors in the numerical solutions.

Table 1 shows three examples of Newton's method. For all cases, the values for the parameters, ζ, γ, and ψ, are chosen such that they show $E[u]\approx 0$. Every case is terminated with the sum of squares or the measure of performance at the 10^{-3} order. The termination criterion is usually the order of 10^{-6}. In other words, the iterations of Newton's method are prematurely terminated.

Table 1 also shows that when the sum of squares is small at the initial step, the required steps to reach the termination are also small. This is usually the case. In case (3), the Jacobian matrix becomes singular so that it is not invertible. Here, $\partial E[v]/\partial C=\partial E[v]/\partial \xi=\partial E[v]/\partial \omega=0$.

3. Further Research

This research note shows that the iterative method estimating θ_2 by Newton's method and the Gauss-Seidel procedure prematurely terminated. This section briefly discusses possible further research strategies.

When the Jacobian matrix is nearly singular, Newton's method is not applicable. In such a case, researchers can employ the following correction:

$$\Delta\theta=a\{J(\theta_2^{(i)})^T J(\theta_2^{(i)})+\mu I\}^{-1} J(\theta_2^{(i)})^T m_2(\theta_2^{(i)}) \qquad (16)$$

where a and μ are appropriate constants and I is the identity matrix.

Even though Newton's method for $m_2=\mathbf{0}$ is prematurely terminated, the Gauss-Seidel procedure may still be applicable when the iteration is far from the solution.

Although the Gauss-Seidel procedure for the block diagonal Jacobian matrix is unproductive, a numerical solution of some kind may be applicable to the entire system of six nonlinear equations.

Note

1) Please do not quote.

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