〈論文〉

Emission Quota versus Emission Tax in a Mixed Duopoly with Foreign Ownership

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Abstract

The paper compares welfare under an emission tax with that under an emission quota in a mixed duopoly where a private firm is partly owned by foreign investors. It shows that an emission tax is more (less) welfare improving than an emission quota when the foreign investors' share is high (low). We note that the government chooses such a high tax level that it cannot earn the tax revenue from the private firm in the equilibrium.

Key words: environment; mixed duopoly; quota; tax

JEL classification: L33, Q58

1. Introduction

The last several years have seen research concerning environmental problems in the context of mixed oligopoly theory. Of particular interest is the effect of an emission tax, addressed in Bárcena-Ruiz and Garzón (2006), Beladi and Chao (2006), Chen and Wang (2010), Ohori (2006a, 2006b), Pal and Saha (2010), Wang and Wang (2009), and Wang et al. (2009). The effect of an emission tax has naturally been analyzed in the framework of pure oligopoly theory and further examined in the context of identifying a welfare-superior regime among market-based instruments, such as emission taxes and tradable emission permits or among those instruments and command-and-control regulations, such as emission standards and quotas. See, for example, Denicolò (1999), Kiyono and Okuno-Fujiwara (2003), Lahiri and Ono (2007), Requate (1993), and Spulber (1985).

Naito and Ogawa (2009) and Kato (2011) favor a welfare-superior regulation in the choice of an environmental policy in a mixed oligopoly. These studies consider only cases in which

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the competitor of the public firm is a private firm owned only by domestic investors. However, in the real globalized world, the competitor of the public firm is not always a private firm whose owners are only domestic investors.[1] This raises a question: Do the results of the previous studies also hold when foreign ownership of the private firm is allowed? This paper tries to answer this question by comparing welfare across emission tax and quota regimes in a mixed duopoly where the private firm is owned by both domestic and foreign investors.

The works most related to this paper are Ohori (2011) and Kato (2011). Ohori (2011) investigates the effects of foreign ownership of a domestic firm in a pure oligopoly on the design of environmental policy and, in particular, compares two environmental policy types: an emission tax and an emission quota.[2] He finds that the emission tax is welfare inferior to an emission quota in a pure duopoly where two private firms, one completely owned by foreign investors and the other by domestic investors, compete with each other. Kato (2011) shows that an emission tax is always welfare inferior to an emission quota in a mixed duopoly where the private firm is entirely owned by domestic investors and the public firm's objective is to maximize both the consumer's and the producer's surplus. These studies show that a command-and-control regulation is more welfare enhancing than a market-based instrument in a duopoly. In contrast, we show that the emission tax is more welfare improving than an emission quota in a mixed duopoly when the foreign investors' share in the private firm is high.

This paper is organized as follows. The next section describes our basic model. Sections 3 and 4 derive the equilibrium outcomes under an emission tax and an emission quota in a mixed duopoly, and Section 5 compares the equilibrium outcomes and welfare of the two regulations. Section 6 provides a brief remark of the model.

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1) The analyses associated with foreign ownerships in a mixed duopoly has been increasing in recent days (Cato and Matsumura, 2012; Han and Ogawa, 2009; Lin and Matsumura, 2012; Matsumura et al., 2009; Wang and Chen, 2011; Wang and Lee, 2013).

2) Ohori (2011) does not use the term "emission quota" but "emission standard". In his model, emission standard is such a regulation that the upper limit of net emission is imposed on firms. Kato (2011) calls this type of regulation as emission quota. In addition, in Naito and Ogawa (2009), emission standard is defined as imposing the minimum limit of abatement effort on firms. To uniform the terms in this paper, we describe emission standard in Ohori (2011) as emission quota.

2. Model

We basically follow the model used by Kato (2011). Consider an industry with two firms – one public (firm 0), whose objective is to maximize the sum of the consumer's and the producer's surplus, and the other private (firm 1), whose objective is to maximize its own profits. They produce a homogeneous good. The inverse demand function of the good is given by $p = \alpha - X$, where $X = x_0 + x_1$ denotes the total output, x_i $(i = 0, 1)$ denotes the output of firm i, p denotes the price of the good, and $\alpha > 0$. Both firms have symmetric production cost functions given by $c_i^p(x_i) = cx_i^2/2$.

Pollution e_i is generated by production. Producing one unit of output generates one unit of pollution. Firms can reduce their pollution by reducing their output or by investing abatement effort a_i. The emission of firm i can be represented as $e_i = max\{x_i - a_i, 0\}$. The abatement cost function of firm i is $c_i^a(a_i) = ka_i^2/2$. The profit of firm i is given by

$$\pi_i(x_0, x_1, a_i) = (\alpha - X)x_i - \frac{cx_i^2}{2} - \frac{ka_i^2}{2}. \quad (1)$$

Welfare is the sum of the consumer's surplus, producer's surplus, and environmental damage. It is given by

$$W(x_0, x_1, a_0, a_1) = \int_0^X (\alpha - s)ds - (\alpha - X)X + \pi_0 + (1 - \gamma)\pi_1 - \frac{(e_0 + e_1)^2}{2}, \quad (2)$$

where $\gamma \in [0, 1]$ represents the foreign private investors' share in the private firm and the last term of W represents the environmental damage.

In this paper, we assume that $c = k = 1$. Kato (2011) shows that welfare under the emission quota is always larger than that under emission tax when $c \geq 1$ and $k \geq 1$. As the main purpose of this paper is to examine whether there exists the case where the emission tax is a welfare-superior regulation to the emission quota in the framework of his setting except for the ownerships of the private firm, we assume $c = k = 1$.

The decision-making sequence of the government and firms is as follows. First, the government chooses the level of regulation given the kind of the environmental policy – emission tax or emission quota. Then, both firms simultaneously choose their outputs x_i and abatement efforts a_i. We analyze this game structure under an emission tax and an emission quota.

3. Emission tax

Consider a situation in which the government imposes an emission tax. The maximization problem of each firm is given by

$$\max_{x_0,a_0} \quad U^t(x_0,x_1,a_0,a_1,t), \text{ s.t. } e_0 \geq 0, \tag{3}$$
$$\max_{x_1,a_1} \quad \pi_1(x_0,x_1,a_1) - te_1, \text{ s.t. } e_1 \geq 0, \tag{4}$$

where $U^t(x_0,a_0,x_1,a_1,t) = \int_0^X (\alpha - s)ds - (\alpha - X)X + \pi_0 - te_0 + (1-\gamma)(\pi_1 - te_1)$. We note that the above maximization problems have the constraints; we use the Lagrange multiplier and denote the following Lagrangian function of each firm.

$$LU^t = U^t + \lambda_0(x_0 - a_0), \tag{5}$$
$$L\pi_1^t = \pi_1 - te_1 + \lambda_1(x_1 - a_1). \tag{6}$$

In the subsequent analyses, we calculate the first order condition of the above Lagrangian function of each firm and derive the equilibrium outcome in the second stage by considering whether the emission constraint of each firm is binding or not: We will separate four cases in this process: *Case (bn)* the constraint is binding only for the public firm, $\lambda_0 > 0$ and $\lambda_1 = 0$; *Case (nb)* the constraint is binding only for the private firm, $\lambda_0 = 0$ and $\lambda_1 > 0$; *Case (bb)* the constraints are binding for both firms, λ_0 and $\lambda_1 > 0$; *Case (nn)* the constraint is non-binding for neither firms, λ_0 and $\lambda_1 = 0$.

Calculating the equilibrium outcome in the second stage with paying attention to the above cases, we obtain the following Lemma 1.

Lemma 1 *In the second stage, the equilibrium outcome is as follows:*

$$(x_0^t, x_1^t, a_0^t, a_1^t) = \begin{cases} \left(\frac{(2+\gamma)(\alpha-t)}{5+\gamma}, \frac{\alpha-t}{5+\gamma}, t, t\right) & if \quad t \in \left[0, \frac{\alpha}{6+\gamma}\right), \\ \left(\frac{(3+\gamma)\alpha-4t}{7+\gamma}, \frac{\alpha+t}{7+\gamma}, t, \frac{\alpha+t}{7+\gamma}\right) & if \quad t \in \left[\frac{\alpha}{6+\gamma}, \frac{(3+\gamma)\alpha}{11+\gamma}\right), \\ \left(\frac{(3+\gamma)\alpha}{11+\gamma}, \frac{2\alpha}{11+\gamma}, \frac{(3+\gamma)\alpha}{11+\gamma}, \frac{2\alpha}{11+\gamma}\right) & if \quad t \in \left[\frac{(3+\gamma)\alpha}{11+\gamma}, \infty\right), \end{cases}$$

Proof See Appendix A. □

The superscript t (q) on x_i and a_i represents the equilibrium outcome in the second stage under the emission tax (quota). Lemma 1 implies that both firms discharge the emission and choose their abatement efforts so as to be equal to the emission tax level when the emission tax level is low. However, when the emission tax level increases, first, the private firm stops discharging its emission. This is caused by the difference of the objectives between the public and private firms: The public firm has an stronger incentive to produce since the objective of the public firm includes consumers' surplus and does not include the environmental damage. The output and the gross emission of the private firm are smaller than those of the public firm because the strategic substitution effect works. The abatement effort increases with the emission tax level, and thus, the private firm decides not to discharge its emission. Finally, both firm stops discharging when the tax level is sufficiently large.

The government chooses the second-best emission tax level to maximize welfare given Lemma 1. Note that welfare under emission tax is defined as $W^t(q_0, q_1, a_0, a_1, t) = \int_0^X (\alpha - s)ds - (\alpha - X)X + \pi_0 - te_0 + (1-\gamma)(\pi_1 - te_1) + t(e_0 + e_1) - (e_0 + e_1)^2/2$. By the simple calculation, we obtain the following proposition.

Proposition 1 *1. The equilibrium emission tax level is*

$$t^T = \frac{(34 + 14\gamma + \gamma^2)\alpha}{197 + 40\gamma + 2\gamma^2}.$$

2. The equilibrium outcome in the full game under the emission tax is as follows:

$$\begin{aligned}
x_0^T &= \frac{(65 + 28\gamma + 2\gamma^2)\alpha}{\Delta^T}, \quad x_1^T = \frac{3(11+\gamma)\alpha}{\Delta^T}, \\
a_0^T &= \frac{(34 + 14\gamma + \gamma^2)\alpha}{\Delta^T}, \quad a_1^T = \frac{3(11+\gamma)\alpha}{\Delta^T}, \\
e_0^T &= \frac{(31 + 14\gamma + \gamma^2)\alpha}{\Delta^T}, \quad e_1^T = 0, \quad t^T = \frac{(34 + 14\gamma + \gamma^2)\alpha}{\Delta^T}, \\
X^T &= \frac{(98 + 31\gamma + 2\gamma^2)\alpha}{\Delta^T}, \quad A^T = \frac{(67 + 17\gamma + \gamma^2)\alpha}{\Delta^T}, \\
E^T &= \frac{(31 + 14\gamma + \gamma^2)\alpha}{\Delta^T}, \quad W^T = \frac{(104 - 2\gamma - \gamma^2)\alpha^2}{2\Delta^T},
\end{aligned}$$

where $\Delta^T = 197 + 40\gamma + 2\gamma^2 > 0$.

Proof See Appendix B. □

The superscript T (Q) represents the equilibrium outcome in the full game under the emission tax (quota). The rational intuition behind Proposition 1 is as follows. There are two distortions in this economy: the environmental problem and mixed oligopoly. However, the government can use only one instrument: an emission tax. To control the production of the public firm with increasing the abatement effort, the government chooses the emission tax level so as to be moderately high. If its level is sufficiently high, the output of the public firm does not depend on the emission tax level, and thus the government cannot control it. We should note that this result can be obtained regardless of γ. The government cannot earn the tax revenue from the private firm even if the private firm is perfectly owned by foreign investors.

4. Emission quota

In this section, we derive the equilibrium outcome for an emission quota. The maximization problems of firm 0 and firm 1 are given by

$$\max_{x_0, a_0} \quad W(x_0, x_1, a_0, a_1) \;\; \text{s.t. } \bar{e} \geq e_0, \tag{7}$$

$$\max_{x_1, a_1} \quad \pi_1(x_0, x_1, a_1) \;\; \text{s.t. } \bar{e} \geq e_1. \tag{8}$$

As is the similar manner to the emission tax, we derive the equilibrium outcome in the second stage by separating whether the emission quota of each firm is binding or not: $\bar{e} \geq e_i$. We use the Lagrange multiplier and denote the following Lagrangian function of each firm.

$$LU^q = U^q + \mu_0(\bar{e} - x_0 + a_0), \tag{9}$$

$$L\pi_1^q = \pi_1 + \mu_1(\bar{e} - x_1 + a_1), \tag{10}$$

where $U^q = \int_0^X (\alpha - s)ds - (\alpha - X)X + \pi_0 + (1 - \gamma)\pi_1$.

In the subsequent analyses, we calculate the first order condition of the above Lagrangian function of each firm and derive the equilibrium outcome in the second stage by considering whether the emission quota of each firm is binding or not: We will separate four cases in this process: *Case (BN)* the quota is binding only for the public firm, $\mu_0 > 0$ and $\mu_1 = 0$; *Case (NB)* the quota is binding only for the private firm, $\mu_0 = 0$ and $\mu_1 > 0$; *Case (BB)* the quotas are binding for both firms, μ_0 and $\mu_1 > 0$; *Case (NN)* the quota is non-binding for neither firms, μ_0 and $\mu_1 = 0$.

Calculating the equilibrium outcome in the second stage with paying attention to the above cases, we obtain the following Lemma 2.

Lemma 2 *In the second stage, the equilibrium outcome is as follows:*

$$(x_0^q, x_1^q, a_0^q, a_1^q) = \begin{cases} \left(\dfrac{(\alpha+\bar{e})(3+\gamma)}{11+\gamma}, \dfrac{2(\alpha+\bar{e})}{11+\gamma}, \dfrac{(3+\gamma)\alpha-8\bar{e}}{11+\gamma}, \dfrac{2\alpha-(9+\gamma)\bar{e}}{11+\gamma}\right) \\ \qquad\qquad if \quad \bar{e} \in \left[0, \dfrac{2\alpha}{9+\gamma}\right), \\ \left(\dfrac{(2+\gamma)\alpha+3\bar{e}}{8+\gamma}, \dfrac{2\alpha-\bar{e}}{8+\gamma}, \dfrac{(2+\gamma)\alpha-(5+\gamma)\bar{e}}{8+\gamma}, 0\right) \\ \qquad\qquad if \quad \bar{e} \in \left[\dfrac{2\alpha}{9+\gamma}, \dfrac{(2+\gamma)\alpha}{5+\gamma}\right), \\ \left(\dfrac{(2+\gamma)\alpha}{5+\gamma}, \dfrac{\alpha}{5+\gamma}, 0, 0\right) \\ \qquad\qquad if \quad \bar{e} \in \left[\dfrac{(2+\gamma)\alpha}{5+\gamma}, \infty\right), \end{cases}$$

Proof See Appendix C. □

Lemma 2 implies that both firms discharge the emission by the upper limit of the emission quota when the quota level is low. However, when the quota level increases, the emission constraint becomes loosen. First, the private firm stops abating investment, and finally both firm stops abating when the quota level is sufficiently high.

The government chooses the second-best emission quota level to maximize welfare given Lemma 2. By the simple calculation, we obtain the following proposition.

Proposition 2 *1. The equilibrium emission quota level is*

$$\bar{e}^Q = \frac{(59-6\gamma-\gamma^2)\alpha}{667+17\gamma-15\gamma^2-\gamma^3}.$$

2. Under the emission quota, the equilibrium outcome is given by

$$\begin{aligned} x_0^Q &= \frac{(6-\gamma)(3+\gamma)(11+\gamma)\alpha}{\Delta^Q}, \quad x_1^Q = \frac{2(6-\gamma)(11+\gamma)\alpha}{\Delta^Q}, \\ a_0^Q &= \frac{(139+57\gamma-7\gamma^2-\gamma^3)\alpha}{\Delta^Q}, \quad a_1^Q = \frac{(73-4\gamma-\gamma^2)\alpha}{\Delta^Q}, \\ \bar{e}^Q = e_0^Q = e_1^Q &= \frac{(59-6\gamma-\gamma^2)\alpha}{\Delta^Q}, \quad X^Q = \frac{(6-\gamma)(5+\gamma)(11+\gamma)\alpha}{\Delta^Q}, \\ A^Q &= \frac{(212+53\gamma-8\gamma^2-\gamma^3)\alpha}{\Delta^Q}, \quad E^Q = \frac{2(59-6\gamma-\gamma^2)\alpha}{\Delta^Q}, \\ W^Q &= \frac{(6-\gamma)(59-6\gamma-\gamma^2)\alpha^2}{2\Delta^Q}, \end{aligned}$$

where $\Delta^Q = 667+17\gamma-15\gamma^2-\gamma^3 > 0$.

Proof See Appendix D. □

We note that in the equilibrium, the emission quota is binding for both firms, that is,

$\bar{e}_i = e_i$. The rational intuition behind Proposition 2 is as follows. Unlike the emission tax, the emission quota can give firms room to choose the combination of output and abatement effort: the emission tax basically requires firms to choose their abatement effort so as to be equal to the emission tax level. When the regulation level is severe, the government can not control both firms' behaviors under the emission tax but can still control them to some extent under the emission quota. Thus, the government can impose the severe emission constraint on firms.

5. Comparison of the equilibrium outcome under an emission tax and that under an emission quota

Using the results of the previous section, we compare the equilibrium outcome and welfare under the emission tax and those under the emission quota. First, we obtain the following relationships of these equilibrium outcomes.

Proposition 3

$$x_1^T < x_1^Q < x_0^Q < x_0^T, \; \textit{for all } \gamma \in [0,1],$$
$$a_1^Q < a_1^T < a_0^T < a_0^Q, \; \textit{for all } \gamma \in [0,1],$$
$$e_1^T < e_1^Q = e_0^Q < e_0^T, \; \textit{for all } \gamma \in [0,1],$$
$$X^Q < X^T, \; \textit{for all } \gamma \in [0,1],$$
$$A^Q < A^T, \; \textit{if and only if } \gamma \in [0,\gamma^A),$$
$$E^T < E^Q, \; \textit{if and only if } \gamma \in [0,\gamma^E),$$

where A and E denote the total abatement effort and total emission, respectively and $\gamma^A \simeq 0.427$ and $\gamma^E \simeq 0.327$.

Proof A simple comparison of the equilibrium outcomes yields the results in Proposition 3. □

The intuition behind Proposition 3 is as follows. As the public firm's objective includes consumer's surplus, the public firm produces more than the private firm. Under the emission tax, both firms basically choose the abatement effort level which is independent of the output level, whereas the firm can adjust the abatement effort level by changing its output level as long as the emission constraint is binding under the emission quota. This leads to the output of the public firm is larger and that of the private firm is smaller under the emission tax than those under the emission quota. Further it also causes the total output under the emission tax to be larger than that under the emission quota.

With respect to the abatement effort and the emission of each firm, the emission tax essentially causes the abatement effort of each firm to be equated and the emission quota does the emission of each firm to be equated in this model.[3] Thus, the abatement efforts of both firms under the emission tax are in the range of (a_1^Q, a_0^Q) and the emissions of the both firm under the emission quota are in the range of (e_1^T, e_0^T).

With respect to the total abatement effort and the total emission, the magnitude relationships varies with the value of γ. When γ increases, the gross emission increases ($dX^T/d\gamma > 0$ and $dX^Q/d\gamma > 0$), and, therefore, each environmental regulation becomes severe ($dt^T/d\gamma > 0$ and $d\bar{e}^Q/d\gamma < 0$); the public firm produces more to decrease the revenue of the private firm. However, it does not reduce the emission under emission tax, though it does under emission quota ($dE^T/d\gamma > 0$ and $dE^Q/d\gamma < 0$); the output-expansion effect dominates an increase of abatement effort under emission tax. As a result, the magnitude relationships of the total emission between emission tax and emission quota changes when γ exceeds some threshold γ^E.

Next, the following proposition shows the results of welfare comparison between the emission tax and the emission quota.

Proposition 4

$$W^Q > W^T \quad if\ \gamma \in [0, \bar{\gamma}),$$
$$W^T \geq W^Q \qquad otherwise,$$

where $\bar{\gamma}$ ($\simeq 0.073$) *is the sollution of* $W^T - W^Q = 0$ *and the strict inequality holds when* $\gamma \neq \bar{\gamma}$.

Proof See Appendix E. □

The intuition behind proposition 4 is as follows. When γ is low, welfare is smaller under the emission tax than under the emission quota, though consumer's surplus is larger and the environmental damage is smaller under the emission tax than emission quota. This is because the proportion of producer's surplus ($\pi_0 + (1-\gamma)\pi_1$) in welfare is larger under low γ and the public firm produces its output such that the price ($P(X)$) is slightly smaller than the

3) In fact, as the emission constraint of the private firm is binding under the emission tax, the abatement efforts of both firms are not the same but still similar by compared with those under the emission quota.

marginal cost, $c'(q_0)$. In this case, the degradation of producer's surplus by the inefficiency of the production allocation is quite large under the emission tax and this effect affects on welfare in a large scale. However, when γ is high, this result is opposite. As a result, the degradation of producer's surplus does not much matter by compared to the case where γ is low, the emission tax is superior to the emission quota.

6. Concluding remarks

The paper compares an emission tax with an emission quota in a mixed duopoly where a private firm is owned by not only domestic but foreign investors. We show that emission tax is more welfare improving (worsening) than an emission tax when the share of the foreign investors for the private firm is high (low). From our results and Kato (2011), we should pay attention to the share of foreign investors in the private firm when we determine the environmental policies.

We'll mention one remark. Under the emission tax, the government cannot earn the tax revenue from the private firm. Here, we relax the assumption of the emission; firms can abate not only its own emission but also the rival's emission, that is, e_i is allowed to be negative under the emission tax. In this case, when γ is high, the emission of the private firm is negative: The government pays the reward for the abatement to the private firm. Even in this setting, the similar results to the main text are still obtained. From these results, we should mind that the result that the emission tax is superior to the emission quota when the private firm is owned by a large proportion of the foreign investors is not obtain from the reason that the government can earn the positive tax revenue from the private firm.

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Appendix A

Proof of Lemma 1 Case (bb): First, emission constraint of each firm is binding, therefore, $\lambda_0 > 0$ and $\lambda_1 > 0$. The first-order condition of the maximization problem of each firm is as follows.

$$\frac{\partial LU^t}{\partial x_0} = \alpha + \lambda_0 - 2x_0 - x_1(1-\gamma) - t = 0, \tag{11}$$

$$\frac{\partial LU^t}{\partial a_0} = -a_0 - \lambda_0 + t = 0, \tag{12}$$

$$\frac{\partial LU^t}{\partial \lambda_0} = x_0 - a_0 = 0, \tag{13}$$

$$\frac{\partial L\pi_1^t}{\partial x_1} = \alpha + \lambda_1 - x_0 - 3x_1 - t = 0, \tag{14}$$

$$\frac{\partial L\pi_1^t}{\partial a_1} = -a_1 - \lambda_1 + t = 0, \tag{15}$$

$$\frac{\partial L\pi_1^t}{\partial \lambda_1} = x_1 - a_1 = 0. \tag{16}$$

To solve the above first-order conditions, we obtain

$$x_0^{bb} = \frac{(3+\gamma)\alpha}{11+\gamma}, \quad x_1^{bb} = \frac{2\alpha}{11+\gamma},$$
$$a_0^{bb} = \frac{(3+\gamma)\alpha}{11+\gamma}, \quad a_1^{bb} = \frac{2\alpha}{11+\gamma},$$
$$e_0^{bb} = 0, \quad e_1^{bb} = 0,$$
$$\lambda_0^{bb} = -\frac{(3+\gamma)\alpha - (11+\gamma)t}{11+\gamma}, \quad \lambda_1^{bb} = -\frac{2\alpha - (11+\gamma)t}{11+\gamma},$$

where the superscript *bb* denotes the equilibrium outcome in Case (bb). In what follows, this superscript is also used to represent the above meaning.

As both λ_0^{bb} and λ_1^{bb} have to be positive, and therefore, Case (bb) exists in $t > (3+\gamma)\alpha/(11+\gamma)$. From the comparison of λ_0^{bb} and λ_1^{bb}, the emission constraint is more severe for firm 1: There exist such tax levels that $\lambda_1^{bb} < 0$ and $\lambda_0^{bb} > 0$. And therefore, there does not exist Case (bn): Emission constraint of firm 0 is binding and that of firm 1, not binding, that is, $\lambda_0 > 0$ and $\lambda_1 = 0$.

Case (nb): Next, we consider Case (nb) where emission constraint of firm 0 is not binding and that of firm 1, binding, that is, $\lambda_0 = 0$ and $\lambda_1 > 0$. The first-order conditions are as follows:

$$\frac{\partial LU^t}{\partial x_0} = \alpha - 2x_0 - x_1(1-\gamma) - t = 0, \tag{17}$$

$$\frac{\partial LU^t}{\partial a_0} = -a_0 + t = 0, \tag{18}$$

$$\frac{\partial L\pi_1^t}{\partial x_1} = \alpha + \lambda_1 - x_0 - 3x_1 - t = 0, \tag{19}$$

$$\frac{\partial L\pi_1^t}{\partial a_1} = -a_1 - \lambda_1 + t = 0, \tag{20}$$

$$\frac{\partial L\pi_1^t}{\partial \lambda_1} = x_1 - a_1 = 0. \tag{21}$$

To solve the above first-order conditions, we obtain

$$x_0^{nb} = \frac{(3+\gamma)\alpha - 4t}{7+\gamma}, \quad x_1^{nb} = \frac{\alpha + t}{7+\gamma},$$
$$a_0^{nb} = t, \quad a_1^{nb} = \frac{\alpha + t}{7+\gamma},$$
$$e_0^{nb} = \frac{(3+\gamma)\alpha - (11+\gamma)t}{7+\gamma}, \quad e_1^{nb} = 0,$$
$$\lambda_0^{nb} = 0, \quad \lambda_1^{nb} = -\frac{\alpha - (6+\gamma)t}{7+\gamma}.$$

As both λ_1^{nb} and e_0^{nb} have to be positive, and therefore, Case (nb) exists when $\alpha/(6+\gamma) < t < (3+\gamma)\alpha/(11+\gamma)$.

Case (nn): Finally, we consider Case (nn): emission constraints are not binding for both firms, λ_0 and $\lambda_1 = 0$. The first-order conditions are as follows:

$$\frac{\partial LU^t}{\partial x_0} = \alpha - 2x_0 - x_1(1-\gamma) - t = 0, \tag{22}$$
$$\frac{\partial LU^t}{\partial a_0} = -a_0 + t = 0, \tag{23}$$
$$\frac{\partial L\pi_1^t}{\partial x_1} = \alpha - x_0 - 3x_1 - t = 0, \tag{24}$$
$$\frac{\partial L\pi_1^t}{\partial a_1} = -a_1 + t = 0. \tag{25}$$

To solve the above first-order conditions, we obtain

$$x_0^{nn} = \frac{(2+\gamma)(\alpha - t)}{5+\gamma}, \quad x_1^{nn} = \frac{\alpha - \gamma}{5+\gamma},$$
$$a_0^{nn} = t, \quad a_1^{nn} = t,$$
$$e_0^{nn} = \frac{(2+\gamma)\alpha - (7+2\gamma)t}{5+\gamma}, \quad e_1^{nn} = \frac{\alpha - (6+\gamma)t}{5+\gamma}.$$

In this case, e_0^{nn} and e_1^{nn} have to be positive, therefore, Case (nn) exists in the range where $t < \alpha/(6+\gamma)$.

Summing up these results, we obtain Lemma 1.

□

Appendix B

Proof of Proposition 1 Substituting x_0^t, x_1^t, a_0^t, and a_1^t into $W^t(x_0, x_1, a_0, a_1, t)$, we derive welfare level in each case and define $\tilde{W}(t) = W(x_0^t, x_1^t, a_0^t, a_1^t, t)$.

First, we consider the case where $t < \alpha/(6+\gamma)$. In this case, we calculate the first-order condition of the maximization problem of the government is

$$\tilde{W}'(t) = \frac{(38 + 27\gamma + 4\gamma^2)\alpha - (233 + 136\gamma + 23\gamma^2 + \gamma^3)t}{(5+\gamma^2)} = 0.$$

To solve the above equation, we obtain

$$t_{nn} = \frac{(38 + 27\gamma + 4\gamma^2)\alpha}{233 + 136\gamma + 23\gamma^2 + \gamma^3} \tag{26}$$

Here, we have to check $t_{nn} < \alpha/(6+\gamma)$. From the comparison, we obtain

$$t_{nn} < \frac{\alpha}{(6+\gamma)} \quad if \quad \gamma \in \left[0, \frac{-13+\sqrt{181}}{6}\right),$$
$$t_{nn} \geq \frac{\alpha}{(6+\gamma)} \quad if \quad \gamma \in \left[\frac{-13+\sqrt{181}}{6}, 1\right].$$

Note that $(-13+\sqrt{181})/6 \simeq 0.0756$. In the former case, t_{nn} satisfies the condition of the inner solution, therefore, welfare is given by

$$W_{nn}^{In} = \frac{(123+43\gamma-\gamma^3)\alpha^2}{2(233+136\gamma+23\gamma^2+\gamma^3)}.$$

In the latter case, t_{nn} does not satisfy the condition of the inner solution, we have to find the corner solution. However, correctly speaking, the maximization tax level is empty because the range in this case is open set. If we are allowed to use ε that is positive and sufficiently smal number, we find that the solution is $\alpha/(6+\gamma)-\varepsilon$. In this case, welfare is given by

$$W_{nn}^{Co} \simeq \frac{(19+2\gamma-\gamma^2)\alpha^2}{2(6+\gamma)^2}. \tag{27}$$

Next, we consider the case where $\alpha/(6+\gamma) < t \leq (3+\gamma)\alpha/(11+\gamma)$. In this case, we calculate the first-order condition of the maximization problem of the government is

$$\tilde{W}'(t) = \frac{(34+14\gamma+\gamma^2)\alpha-(197+40\gamma+2\gamma^2)t}{(7+\gamma)^2} = 0. \tag{28}$$

To solve the above equation, we obtain

$$t_{nb} = \frac{(34+14\gamma+\gamma^2)\alpha}{197+40\gamma+2\gamma^2}. \tag{29}$$

From the simple calculation, we easily find that $\alpha/(6+\gamma) < t_{nb} < (3+\gamma)\alpha/(11+\gamma)$. Therefore, welfare is

$$W_{nb} = \frac{(104-2\gamma-\gamma^2)\alpha^2}{2(197+40\gamma+2\gamma^2)}. \tag{30}$$

Finally, we consider the case where $(3+\gamma)\alpha/(11+\gamma) < t$. In this case, the equilibrium output in the second stage does not depend on the tax level, so we can easily calculate welfare. Welfare is as follows.

$$W_{bb} = \frac{(59-6\gamma-\gamma^2)\alpha^2}{2(11+\gamma)^2}.$$

From the above results, we find that the government chooses the emission tax level in the following two cases: $\gamma \in [0, (-13+\sqrt{181})/6)$ and $\gamma \in [(-13+\sqrt{181})/6, 1]$. Fortunately, we find the following relationships with respect to welfare:

$$W_{nb} = \max\{W_{nn}^{In}, W_{nn}^{Co}, W_{nb}, W_{bb}\}. \tag{31}$$

Summing up these results, we obtain Proposition 1.

□

Appendix C

Proof of Lemma 2 Case (BB): First, we consider Case (BB) where the emission constraints are binding for both firms, μ_0 and $\mu_1 > 0$. The first-order conditions are as follows:

$$\frac{\partial LU^q}{\partial x_0} = \alpha - \mu_0 - 2x_0 - x_1(1-\gamma) = 0, \tag{32}$$

$$\frac{\partial LU^q}{\partial a_0} = -a_0 + \mu_0 = 0, \tag{33}$$

$$\frac{\partial LU^q}{\partial \mu_0} = \bar{e} - x_0 + a_0 = 0, \tag{34}$$

$$\frac{\partial L\pi_1^q}{\partial x_1} = \alpha - \mu_1 - x_0 - 3x_1 = 0, \tag{35}$$

$$\frac{\partial L\pi_1^q}{\partial a_1} = -a_1 + \mu_1 = 0, \tag{36}$$

$$\frac{\partial L\pi_1^q}{\partial a_1} = \bar{e} - x_1 + a_1 = 0. \tag{37}$$

To solve the above first-order conditions, we obtain

$$x_0^{BB} = \frac{(3+\gamma)(\alpha+\bar{e})}{11+\gamma}, \quad x_1^{BB} = \frac{2(\alpha+\bar{e})}{11+\gamma},$$
$$a_0^{BB} = \mu_0^{BB} = \frac{(3+\gamma)\alpha - 8\bar{e}}{11+\gamma}, \quad a_1^{BB} = \mu_1^{BB} = \frac{2\alpha-(9+\gamma)\bar{e}}{11+\gamma},$$
$$e_0^{BB} = e_1^{BB} = \bar{e}.$$

In this case, μ_0^{BB} and μ_1^{BB} have to be positive, therefore, Case (BB) exists in the range where $\bar{e} < 2\alpha/(9+\gamma)$. From the comparison of μ_0^{BB} and μ_1^{BB}, we find that Case (NB) where $\mu_0 = 0$ and $\mu_1 > 0$ does not exist.

Case (BN): Next, we consider Case (BN): $\mu_0 > 0$ and $\mu_1 = 0$. Increasing a_i monotonically decreases the value of the objective function of firm i and therefore, firm 1 does not invest the abatement effort at all, that is, $a_1 = 0$. With respect to other variables, the first-order conditions are as follows:

$$\frac{\partial LU^q}{\partial x_0} = \alpha - \mu_0 - 2x_0 - x_1(1-\gamma) = 0, \tag{38}$$

$$\frac{\partial LU^q}{\partial a_0} = -a_0 + \mu_0 = 0, \tag{39}$$

$$\frac{\partial LU^q}{\partial \mu_0} = \bar{e} - x_0 + a_0 = 0, \tag{40}$$

$$\frac{\partial L\pi_1^q}{\partial x_1} = \alpha - x_0 - 3x_1 = 0. \tag{41}$$

To solve them, we obtain

$$x_0^{BN} = \frac{(2+\gamma)\alpha + 3\bar{e}}{8+\gamma}, \quad x_1^{BN} = \frac{2\alpha - \bar{e}}{8+\gamma},$$
$$a_0^{BN} = \mu_0^{BN} = \frac{(2+\gamma)\alpha - (5+\gamma)\bar{e}}{8+\gamma}, \quad a_1^{BN} = \mu_1^{BN} = 0,$$
$$e_0^{BN} = \bar{e}, \quad e_1^{BN} = \frac{2\alpha - \bar{e}}{8+\gamma}.$$

As both μ_0^{BN} and e_1^{BN} have to be positive, and therefore, Case (BN) exists when $2\alpha/(9+\gamma) < \bar{e} < (2+\gamma)\alpha/(5+\gamma)$.

Case (NN): Finally, we consider Case (NN) where the emission constraints are not binding for both firms, μ_0 and $\mu_1 = 0$. As mentioned above, we can easily find that both firms choose their abatement effort level to be equal to 0. Therefore, we have to consider the choice of the output of each firm. The first-order conditions are as follows:

$$\frac{\partial LU^q}{\partial x_0} = \alpha - 2x_0 - x_1(1-\gamma) = 0, \tag{42}$$

$$\frac{\partial L\pi_1^q}{\partial x_1} = \alpha - x_0 - 3x_1 = 0. \tag{43}$$

To solve the above first-order conditions, we obtain

$$x_0^{NN} = e_0^{NN} = \frac{(2+\gamma)\alpha}{5+\gamma}, \quad x_1^{NN} = e_1^{NN} = \frac{\alpha}{5+\gamma},$$
$$a_0^{NN} = a_1^{NN} = 0.$$

This corresponds to the case where $\bar{e} \geq (2+\gamma)/(5+\gamma)$.

Summing up these results, we obtain Lemma 2. □

Appendix D

Proof of Proposition 2 Substituting x_0^q, x_1^q, a_0^q, and a_1^q into $W(x_0, x_1, a_0, a_1)$ in each case, we derive welfare level in each case and we define $\hat{W}(\bar{e}) = W(x_0^q, x_1^q, a_0^q, a_1^q)$.

First, we consider the case where $\bar{e} < 2\alpha/(9+\gamma)$. In this case, we calculate the first-order condition of the maximization problem of the government is

$$\hat{W}'(\bar{e}) = \frac{(59-6\gamma-\gamma^2)\alpha - (667+17\gamma-15\gamma^2-\gamma^3)\bar{e}}{(11+\gamma)^2} = 0.$$

To solve the above equation, we obtain

$$\bar{e}_{BB} = \frac{(59-6\gamma-\gamma^2)\alpha}{\Delta^Q} \tag{44}$$

From the simple calculation, we find that $\bar{e}_{BB} < 2\alpha/(9+\gamma)$.

Therefore, welfare is

$$W_{BB} = \frac{(6-\gamma)(59-6\gamma-\gamma^2)\alpha^2}{2\Delta^Q}. \tag{45}$$

Next, we consider the case where $2\alpha/(9+\gamma) \leq \bar{e} < (2+\gamma)\alpha/(5+\gamma)$. In this case, we calculate the first-order condition of the maximization problem of the government is

$$\hat{W}'(\bar{e}) = \frac{\gamma\alpha - (11+2\gamma)\bar{e}}{8+\gamma} = 0. \tag{46}$$

To solve the above equation, we obtain

$$\bar{e}_{BN} = \frac{\gamma\alpha}{11+2\gamma}. \tag{47}$$

Here, we have to check whether $\bar{e}_{BN} \in [2\alpha/(9+\gamma), (2+\gamma)\alpha/(5+\gamma))$ or not. From the simple calculation, we easily find that $\bar{e}_{BN} < 2\alpha/(9+\gamma)$. Since $\hat{W}''(\bar{e}) < 0$, the solution is the corner solution, that is, $2\alpha/(9+\gamma)$. Therefore, welfare is

$$W_{BN} = \frac{(5-\gamma)(7+\gamma)\alpha^2}{2(9+\gamma)^2}. \tag{48}$$

Finally, we consider the case where $(2+\gamma)\alpha/(5+\gamma) \leq \bar{e}$. In this case, the equilibrium output in the second stage does not depend on the quota level, so we can easily calculate welfare. Welfare is as follows.

$$W_{NN} = \frac{(7-3\gamma-\gamma^2)\alpha^2}{2(5+\gamma)^2}.$$

From the above results, the government chooses the emission quota level that maximizes social welfare. From the simple calculation, we obtain the following results:

$$W_{BB} = \max\{W_{BB}, W_{BN}, W_{NN}\}. \tag{49}$$

Summing up there results, we obtain Proposition 2.

□

Appendix E

Proof of Proposition 4 We compare welfare under an emission tax and an emission quota. We obtain

$$W^T - W^Q = \frac{(-370+4989\gamma+831\gamma^2-98\gamma^3-23\gamma^4-\gamma^5)\alpha^2}{2\Delta^T\Delta^Q}. \tag{50}$$

Here, we define $f(\gamma) = W^T - W^Q$. First, we find the following facts: $f(0) < 0$ and $f(1) > 0$. Second, we easily find $f'(\gamma) > 0$ for all $\gamma \in [0,1]$. To sum up the above results, we obtain $\bar{\gamma}$ such that $f(\bar{\gamma}) = 0$: $\bar{\gamma}$ is the only solution of $W^T - W^Q = 0$. Thus, we obtain Proposition 4. □

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