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An optimal and a heuristic approach to solve the route and spectrum allocation problem in OFDM networks

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by

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An optimal and a heuristic approach to solve the Route and Spectrum Allocation problem in OFDM networks

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ABSTRACT

To maximize the usage of optical resources, it is important to reduce the total bandwidth requirement for communication. Orthogonal Frequency Division Multiplexing (OFDM) has recently emerged as an encouraging competitor to Wavelength Division Multiplexing (WDM), which uses fixed capacity channels. A network using OFDM-based Spectrum-sliced Elastic Optical Path (SLICE) has a higher spectrum efficiency, due to the fine granularity of subcarrier frequencies used. To minimize the utilized spectrum in SLICE networks, the routing and spectrum allocation problem (RSA) has to be efficiently solved. We have solved the RSA problem using two Integer Linear Programming (ILP) formulations. Our first formulation provides an optimal solution, based on an exhaustive search and is useful as a benchmark. Our second approach reduces the time requirement by restricting the number of paths considered for each commodity, without significantly compromising on the solution quality. We have compared our approaches with another prominent formulation proposed recently.

DEDICATION

To my loving mumma, Sumita Paul

and my grandma, Gitarani Paul.

-Of all that walk the earth, you both are most precious to me.

ACKNOWLEDGEMENTS

I would like to take this opportunity to convey my sincere thanks to my supervisors, Dr. Subir Bandyopadhyay and Dr. Yash Aneja, for their guidance and support all throughout my graduate studies. This work could not have been achieved without their advice, suggestions and cooperation.

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Arijit Paul

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BILP - Binary Integer Linear Program

ILP - Integer Linear Program

LT - Logical Topology

MCDS - Minimum Connected Dominating Set

MCNF - Multi Commodity Network Flow

MILP - Mixed Integer Linear Program

OADM - Optical Add Drop Multiplexer

OEO - Optical Electronic Optical

OFDM - Orthogonal Frequency Division Multiplexing

OXC - Optical Cross Connect

QoT - Quality of Transmission

RSA - Route and Spectrum Allocation

RWA - Routing and Wavelength Assignment

WAN - Wide Area Network

WDM - Wavelength Division Multiplexing

Chapter 1

Introduction

1.1 Overview of Optical Networks

As we progress through the 21st century, we are witnessing dramatic changes in the telecommunications industry, driven by the relentless need for additional capacity in the network. This demand is fuelled by many factors. The tremendous growth of the internet in terms of the number of users, coupled with increasing demands for connectivity and the need to support enhanced services and applications, are some of the major factors. At the same time, businesses today rely heavily on high-speed networks to conduct their businesses. They use their digital presence to integrate and streamline business units, such as marketing, commercial transactions, inventory control, management and to facilitate end-user sales and support. All these factors have put tremendous pressures on the existing available capacity for data communication and it has made it very important to make an optimal utilization of the available finite communication resources.

Over the years, a wide range of traditional media have been explored, to facilitate data transmission. Some of the problems faced by these conventional media, (such as copper cables) include:

- Lack of bandwidth capacity,
- Sensitive to environmental noise,
- High latency,
- Low distance propagation.

Optical networks promises to resolve many of the key issues discussed above. As a result, optical communication has seen an unprecedented growth during the last decade, while the developments in relevant enabling technologies and the increasing research interest suggest an even more prosperous future. They have led to immense performance increases as well as cost reductions in the past decade. Recent innovations have also led to the surge in the rate of data transmission in optical networks, while maintaining an exceptionally low amount of error and impairments in the signal. Furthermore, these networks are increasingly able to deliver data communication rates in a flexible manner, i.e., as and when required. It is therefore anticipated that optical networks will establish themselves as the dominant telecommunication method in the foreseeable future.

Optical fibers provide much higher rate of data communication compared to conventional copper cables and are less susceptible to electromagnetic interferences and other undesirable effects. As a result, they are the favoured medium for transmission of data over any distance more than a kilometre at anything more than a few tens of megabits per second [3]. In fact, they offer significantly higher bandwidth capacities, of the order of Terabits per second (Tbps). A typical optical network may span several cities & countries and may act as a backbone to sustain other major forms of communication such as wireless. With recent developments in fiber-optic technology, newer concepts like Fiber to the Home (FTTH) technology have evolved, which are envisioned to support data communication rates of the order of several Tbps [4].

Google Fiber [5] is one such instance of the FTTH technology which is expected to flourish in the next few years.

In optical networks, the range of frequencies for which low attenuation data communication is feasible, is limited [2]. This range of frequencies is called the spectral bandwidth for optical networks and they limit the maximum data communication capability. Thus, maximal usage of the optical network resources can be made by reducing the total bandwidth requirement for communication.

Much of the success for exploitation of the aforementioned huge bandwidth capacity of the optical fibers may be attributed to the Wavelength Division Multiplexing (WDM) [2] technology. However, the rigid nature of wavelength-routed optical networks creates limitations on network utilization efficiency. These limitations originate from the fact that wavelength-routed networks require the allocation of a fixed bandwidth to a request for connection, even when the traffic between the corresponding end nodes is not sufficient to fill the entire data carrying capacity of that bandwidth. To address this problem, Orthogonal Frequency Division Multiplexing (OFDM) has been proposed as a modulation technique for optical networks, as it possesses better spectral efficiency and impairment tolerance [6]. Ideally, an adaptive Spectrum-sLICed Elastic optical path network (SLICE) network possesses greater flexibility, as it elastically delivers the requisite capacity of bandwidth according to the connection demands. Various bandwidth-variable transponders and other equipment have been designed for this purpose.

1.2 Principles of OFDM Network

OFDM is a special class of the Multi-Carrier Modulation (MCM) scheme, that communicates a data stream by dividing it into a number of channels, commonly referred to as *subcarriers*, each carrying a relatively-low data rate signal [7]. The recently proposed spectrum-sliced elastic optical path network (SLICE) is expected to mitigate the problem of network utilization inefficiency of WDM networks by adaptively allocating a portion of the available spectrum according to the traffic demands of each client. An adaptive network would elastically provide the required capacity to sub-wavelength or super-wavelength traffic demands. However, this new concept poses new challenges at the networking level, since the routing and wavelength assignment (RWA) algorithms of traditional WDM networks will no longer be directly applicable. A connection needing capacity greater than one OFDM subcarrier has to be allocated a number of contiguous subcarriers to achieve improved spectral efficacy [6]. The wavelength continuity constraint of traditional WDM networks corresponds to the spectrum continuity constraint in OFDM networks. To solve these issues, new route and spectrum allocation (RSA) algorithms, as well as appropriate extensions to the algorithms for network control are being researched.

1.3 Problems Addressed in This Research

The purpose of this work is to study the problem of RSA and propose an efficient scheme to minimize the total bandwidth requirement for a set of connection requests, by allocating a route and sufficient spectral resources to each connection in an optimal manner. We address the problem of RSA using Integer Linear Programming (ILP) formulations and have developed two approaches to solve the problem. Our first approach is an optimal ILP formulation(henceforth called ILP_1), which performs an exhaustive search to find an optimal path and an optimal bandwidth for each of the connection requests. To the best of our knowledge no researcher has developed an ILP formulation till date, to determine the optimal scheme for RSA in OFDM networks.

Due to the enormous computational resources needed to find an optimal scheme for RSA, ILP_1 cannot handle networks of practical sizes. To address this, our second ILP formulation (henceforth called ILP_2), further reduces the time required to solve the RSA problem by restricting the number of paths to be considered for each commodity. Though this restriction may not guarantee the optimality of the solution, it expedites the running time of the algorithm significantly.

The input to both our ILP formulations include the physical topology of the network and, for each request for communication, the corresponding source and the destination, as well as the number of subcarriers required for this request.

Finally, we have compared our formulations to another well-known algorithm, proposed recently [6], in terms of both the running time of the algorithms and the spectrum efficiency achieved. The number of integer variables in a mixed integer linear program (MILP) is critically important [8], since in general, the time needed to solve the formulation increases exponentially with the increase in the number of integer variables. Therefore, the underlying philosophy considered while designing the ILP formulations in our research was to reduce the number of integer variables in the formulations, to the extent possible.

1.4 Thesis organization

The rest of this thesis is organized as follows. In Chapter 2 we have reviewed basic concepts of OFDM optical networks, the notion of RSA, the k-shortest path algorithm which we have used in our investigation, the formulation proposed by Christodoulopoulos et al. [6] and a few other prominent investigations in this field. We have presented our work on RSA in Chapter 3. A detailed analysis, giving the number of integer variables generated by our formulations and the formulation proposed in [6] is also given in Chapter 3. Chapter 4 describes the implementation details, the testing workbench used to run the simulations, the simulation results and its associated analysis. Finally the conclusions and possible future work are presented in Chapter 5.

Chapter 2

Review on Related Topics

This chapter reviews the topics relevant to the research reported in this thesis, including

- Principles of OFDM Optical Networks.
- The concept of Route and Spectrum Allocation(RSA) in OFDM.
- k-shortest path algoritm.
- Christodoulopoulos formulation and a few other notable research works, related to our research.

2.1 Fundamental Principles of Fiber-Optic Communication

Fiber-optic communication is a system of transmitting information/data from one place to another by sending pulses of light via an optical fiber. The optical signal forms an electromagnetic carrier wave that is modulated to carry information over long distances [9]. This form of communication has largely replaced radio transmitter systems for long-haul data transmission.

In order to transmit the modulated optical signals, a special kind of cable is required. These cables are specifically known as optical cables. An optical cable is comprised of numerous long, thin strands of very pure glass about the diameter of a human hair; each being called an optical fiber. These fibers are bundled together to form a single cable. When an optical signal enters one end of the fiber, it travels (confined within the fiber) until it leaves the fiber at the other end. Due to this distinctive characteristic, the loss of signal during its journey along the fiber is very minimal.

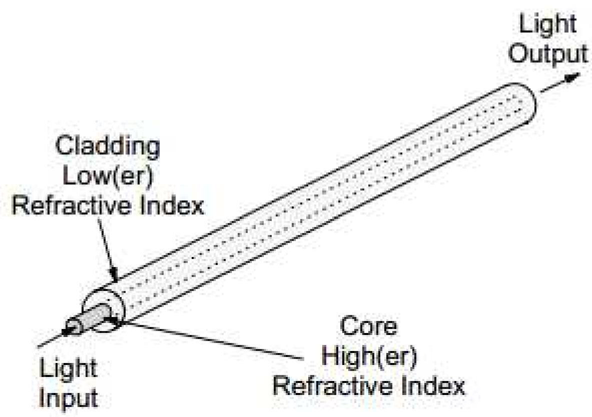

Fig. 2.1.1: Basic Principle of Light Transmission on Optical Fibre

Upon closely looking at a single optical fiber, we can see that it has the following parts:

- **Core:** Thin glass center of the fiber through which the light travels.
- **Cladding:** Outer optical material surrounding the core that reflects the light back into the core.
- **Buffer Coating:** Plastic coating that protects the fiber from damage and moisture.

The propagation of optical signals along the optical fiber is based on the laws of refraction and reflection. Refraction of light occurs when the light experiences a change in its speed while passing between mediums of different densities. Since optical cables are not always laid out perfectly straight, a ray of light entering the fibre is guided along the fibre by repeatedly bouncing off the interface between the (higher refractive index) core and the (lower refractive index) cladding. When light propagating through a medium having a refractive index of $n1$ encounters a second medium, having a refractive index of $n2$ ($n1 > n2$), at an incident angle greater than the critical angle $\sin^{-1}(\frac{n2}{n1})$, the light will follow the medium and will propagate without loss. This phenomena is called *total internal reflection*.

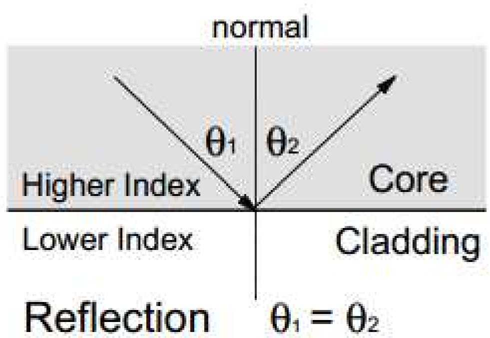

Fig. 2.1.2: Reflection of an optical signal

Thus, for an optical signal travelling from one optical medium to another, a change in refractive indices ensues, and if the refractive index of the former optical medium is greater than the latter, a total internal reflection may occur if the light passes in the medium at an angle exceeding the critical angle. The critical angle is determined based on the refractive index of the core and cladding by Snells Law.

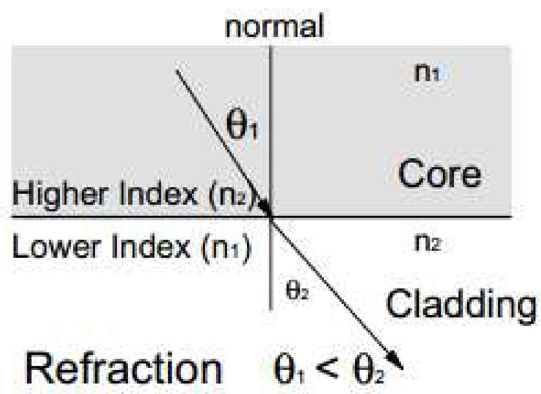

Fig. 2.1.3: Refraction of a light ray

Hence, the modulated optical signals must be guided at an angle above the critical angle, so that it is contained within the core until it reaches the destination.

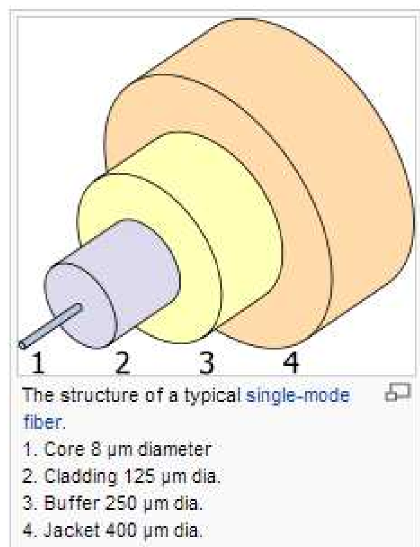

Fig. 2.1.4: (a) Single Mode optical fiber (b) Cross Section of a Single Mode optical fiber (referred from [1])

A typical *single mode* optical fiber has a core diameter between 8 and 10.5 μm and a cladding diameter of $125 \mu m$.

2.2 Optical Network Components

An optical network ordinarily consists of several components or devices, which help in the successful communication between a particular source destination pair via an optical medium. A few of the primary components: amplifiers, regenerators and switches are diagrammatically shown below.

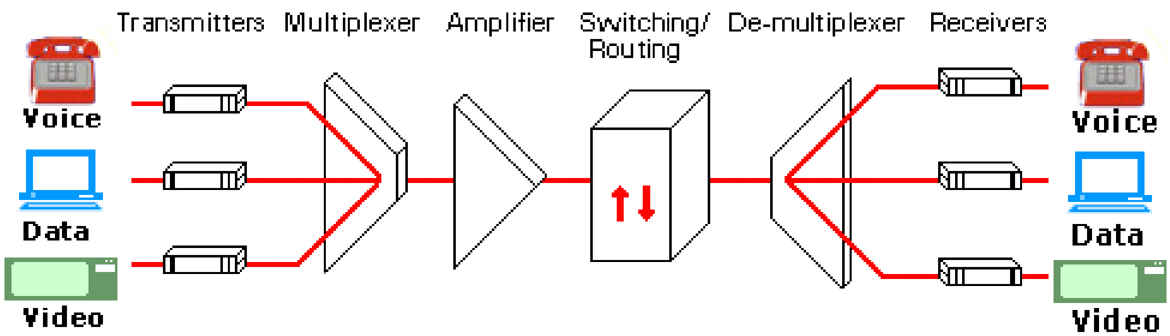

Fig. 2.2.1: Multi-mode Step Index Fiber

2.2.1 Transmitter and Receiver

As the name suggests, a transmitter is an electronic device which is used to generate light or optical signals of a specific carrier wavelength. With the assistance of multiple transmitters, numerous signals carrying different data can be transmitted by means of a single optical fiber, using a variable number of distinct carrier wavelengths. Several modulation schemes are used to convert data in electronic form to encoded optical signal. *On-off keying* (OOK) is a widely used modulation practise, which encodes a bit 0 (1) by turning light off (on)[2]. The receiver is used to extract the information from the encoded optical signal back into the electronic domain at the destination node.

2.2.2 Optical Amplifiers

While traversing via a transmission medium, an inevitable reduction in the intensity of the optical signal occurs with respect to distance travelled through the medium, known as *attenuation*. This reduction in the intensity of the signal may result in the erroneous interpretation of the signal at the destination. Therefore, to boost the strength of a propagating optical signal, optical amplifiers are placed at periodic intervals along the optical fiber. These amplifiers enhance the signal strength without reconverting the signal into electronic domain.

2.2.3 Optical Cross-Connects (OXC)

An *optical cross-connect* (OXC) is a device that is utilized to switch high-speed optical signals in a fiber-optic network. Optical cross-connects work entirely at the optical layer and are usually capable of operating without having to convert optical signals to electrical signals and back again. They are normally placed at any network junction points or router nodes. In an OXC, optical signals from an incoming fiber are first demultiplexed, before being eventually switched by optical switching modules. After switching operation, the optical signals are finally multiplexed onto an outgoing fiber by optical multiplexers [10].

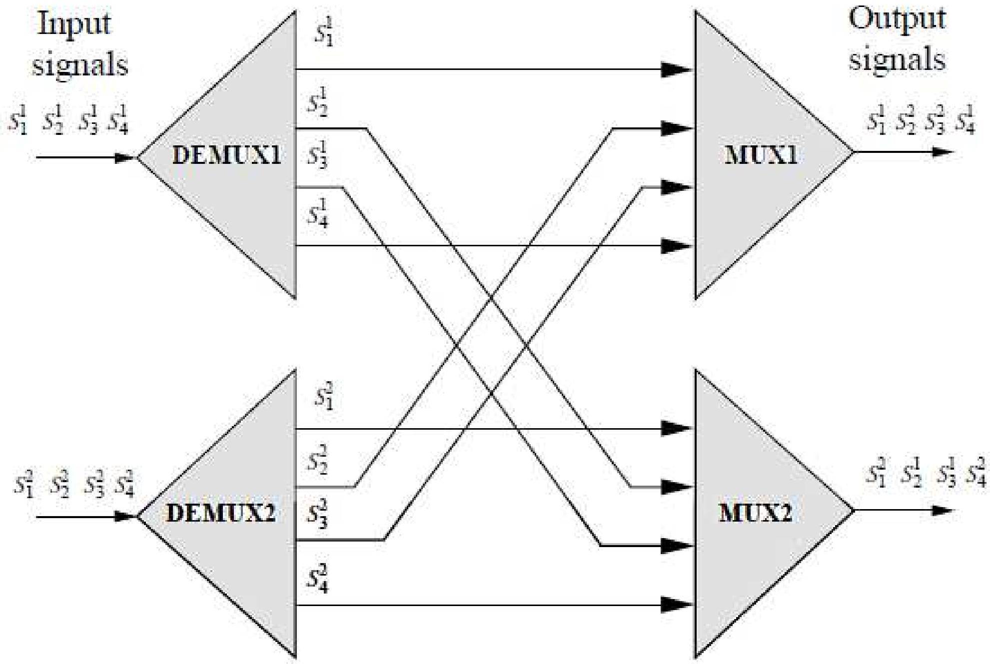

Fig. 2.2.2: An optical cross-connect switch(static) [2]

OXCs may be ideally categorized as *static* or *dynamic*[2]. The cross-connect switch presented in Fig.2.2.2 is a static switch, since the connections between the output terminals of demultiplexers and the input terminals of multiplexers are fixed.

2.2.4 Multiplexers & Demultiplexers

A *multiplexer* or MUX is used to combine optical signals on different individual channels, onto a single optical fiber. It selects one of several analog or digital input signals and forwards the selected input onto a single fibre. A multiplexer is also known as a *data selector*. Conversely, a *demultiplexer* (or demux) is a device that takes a single input signal and selects one of several data-output-lines, which is connected to the single input. A multiplexer is often accompanied with a complementary demultiplexer on the receiving end of the fiber.

2.3 Optical OFDM

The sustained growth of data traffic in recent years calls for the pressing need of an efficient and scalable transport platform for links of 100 Gb/s and beyond in optical networks. Consequently, in order to maximize the potential use of optical network resources, it is vital to reduce the total bandwidth requirement for communication. Orthogonal frequency division multiplexing (OFDM) has recently emerged as a promising alternative to Wavelength Division Multiplexing (WDM) due to its elastic band-width allocation property. A network using OFDM-based Spectrum-sliced Elastic Optical Path Network (SLICE) has a higher spectrum efficiency, compared to a WDM network, due to the fine granularity of sub-carrier frequencies used. For a connection needing a capacity larger than a single OFDM subcarrier, a number of contiguous subcarriers have to be allocated to achieve improved spectral efficacy [6]. The OFDM technology, enables both sub-wavelength and super-wavelength traffic accommodation by allotting appropriate number of sub-carriers according to the demand requirement.

In a typical OFDM network, a fiber usually carries a multitude of optical signals in the low attenuation bandwidth being used. These optical signals, therefore, must clearly be allotted different carrier wavelengths as the fiber carrying them is common for all the signals.

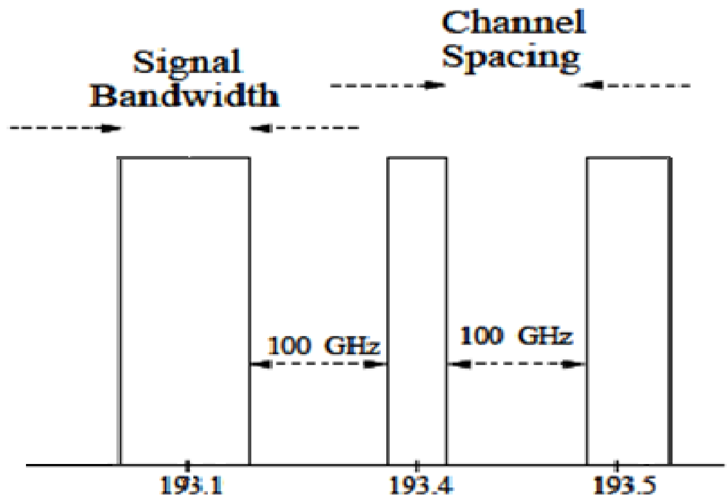

Fig. 2.3.1: Signal Bandwidth and Channel Spacing in OFDM Networks(modified from [2])

As apparent from Fig.2.3.1, each optical signal is assigned a distinct channel, such that each channel has an adequate flexible bandwidth, corresponding to its requirement, to accommodate the modulated signal. Furthermore, with a view to avoid the interference between different optical signals, each channel is separated from the other by a certain bandwidth termed as *channel spacing* or *guard band*. In the above figure, the value of channel spacing is taken a typical value of 100GHz.

Compared to WDM scheme, where a fixed channel spacing between the wavelengths is usually desirable to eradicate crosstalk, OFDM permits the spectrum of individual subcarriers to overlap because of its property of *orthogonality*, as depicted in Fig.2.3.3.

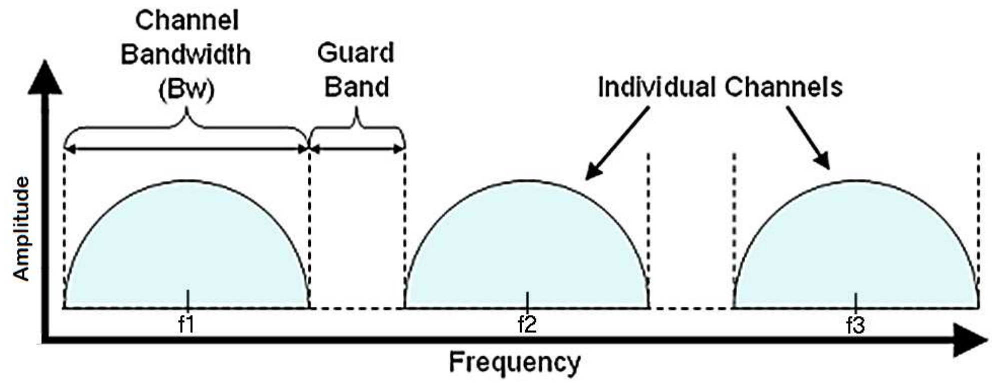

Fig. 2.3.2: Spectrum of WDM signals

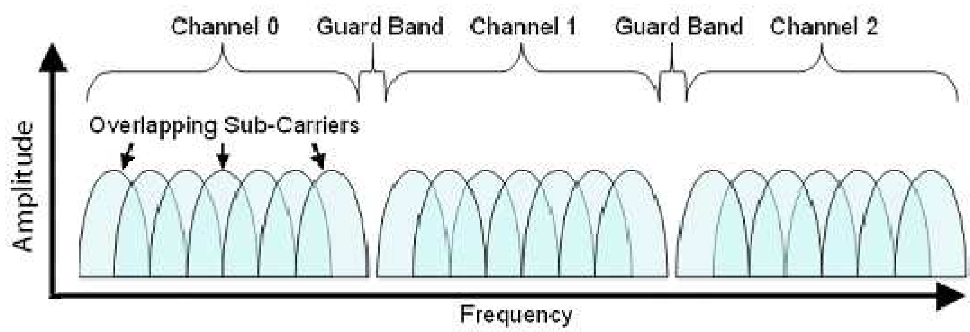

Fig. 2.3.3: Spectrum of OFDM signals

The orthogonality property between multiple subcarriers is fulfilled when the central frequencies of subcarriers are spaced $(\frac{n}{T_s})$ apart, where T_s is the symbol duration and n is a positive integer[7]. It can be noted from Fig.2.3.4 that the peak point of a subcarrier's spectrum coincides with the zero point of other subcarriers' spectra. This is because, when a subcarrier is sampled at its peak, all other subcarriers have zero-crossings at that point and hence do not interfere with the subcarrier being sampled.

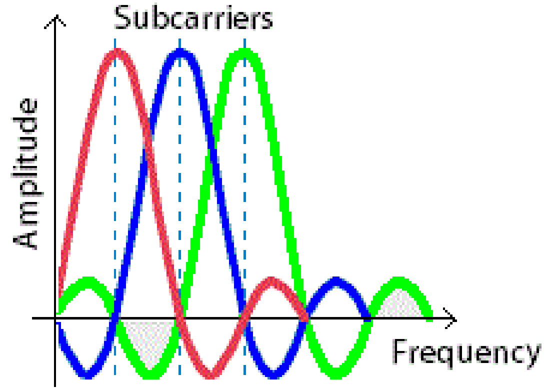

Fig. 2.3.4: Frequency domain expression of OFDM signal (with 3 subcarriers)

Thus orthogonality leads to a greater efficiency in the usage of spectral resources.

2.4 Route and Spectrum Allocation (RSA)

Given a network topology and a predefined set of demand-set requests, route and spectrum allocation (RSA) is the problem of determining the path for each request and assigning a bandwidth to it. The main objective of solving the RSA problem in OFDM is to establish the connections so as to achieve satisfactory spectrum allocation, with the constraint that the overlapping of spectrum is not permitted for the requests whose paths share some edges; and to minimize the total spectrum required to service all the requests. While designing the scheme for RSA, two important constraints need to be considered:

Spectrum Continuity Constraint: Due to limitations in optical technology, spectrum conversion at the optical layer is not economically feasible. Therefore, the spectrum assigned to a particular lightpath should remain the same all along its path from the source to the destination of the lightpath. This constraint is applicable for all the optical lightpaths to be established.

Spectrum Clash Constraint: This constraint states that any two lightpaths which share a common optical fiber, should be assigned non-overlapping bandwidths, separated by at least a guard band.

Two versions of the RSA problem have been considered by researchers for various kinds of traffic demands, namely static and dynamic. If the set of lightpaths to be set up is known a-priory to the network engineer, the problem is called as static or offline RSA problem. In static RSA, the lightpaths, once established, are not modified until there is a significant change in the traffic pattern, sufficient enough to warrant a different set of lightpaths. Thus, the lightpaths in this scheme exist for relatively long periods of time until the RSA algorithm is recomputed with a newer set of lightpaths to accommodate the changed traffic pattern. These newer set of lightpaths which represent the changed traffic pattern, will replace the existing lightpaths.

In contrast, the dynamic or online traffic demands are not known in advance and are established on demand. The requests for the data communication in this scheme are considered as and when they arrive in the system. In this scheme for dynamic RSA, while creating a new lightpath for a communication request, all the existing lightpaths have to be considered. When the communication is finished, all the resources dedicated for this communication is again reclaimed back for possible use in future communication[11]. In short, the dynamic lightpath allocation is done by setting up the lightpaths when needed and reclaiming them back when the communication is over.

The over-all objective of RSA, whether dynamic or static, is to maximize the number of established lightpath requests within a given finite spectrum, so that the optimal usage of the available spectrum is made. Static RSA is known to be an NP-complete problem [12] and it is more challenging than Routing and Wavelength Assignment (RWA) in fixed bandwidth wavelength-routed networks due to the existence of the spectrum contiguity constraint, which states that a connection needing capacity greater than one OFDM subcarrier has to be assigned a number of contiguous subcarriers to obtain increased spectral efficiency. The dynamic RSA is considered an even more difficult problem, since the dynamic connection requests arrive arbitrarily and persist in the network for a random extent of time.

Let us try to understand the notion of spectrum allocation in RSA with an example:

Let us assume that two lightpaths L1 and L2 have been assigned the paths in the network (arbitrarily taken) as shown in the figure.

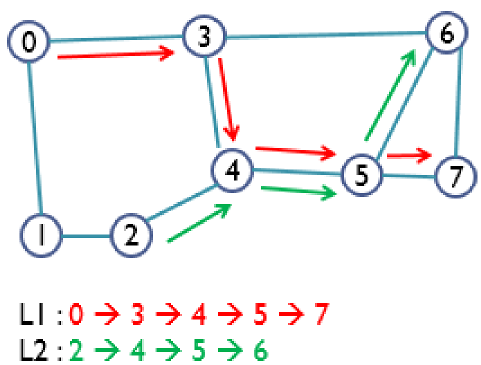

Fig. 2.4.1: Illustrative Example Network

As evident from Fig.2.4.1, the lightpaths L1 and L2 share a common edge/fiber from the node 4 to node 5. Thus, as per the spectrum clash constraint, the spectrums of lightpath L1 and L2 cannot overlap with each other. They must be assigned distinct spectrums that are separated at least by a guard band. Lightpaths L1 and L2 must adhere to the spectrum continuity constraint by selecting the same spectrum, throughout its path from source to destination.

Moreover, it is critically important to determine and allocate an efficient path for a commodity, while performing the RSA. The following scenario illustrates the importance of selection of an efficient path verses an inefficient paths for the commodities. Let us assume a sample network and a set of commodities, with their path allocation scheme as shown in the figure 2.4.2.

Source → Destination	Path	Traffic
3 → 5	3--1--2--5	10
1 → 5	1--2--5	12
4 → 5	4--2--5	15
4 → 6	4--6	6

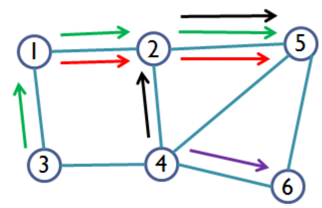

Fig. 2.4.2: Sample network & set of commodities with an inefficient path allocation scheme

As all the commodities in figure 2.4.2 have to adhere to the spectrum continuity and spectrum clash constraints, the total spectrum requirement in this case would be 37.

However, for the very same network and commodity set, if the paths are allocated by the scheme as shown in the figure 2.4.3, the total spectrum requirement would drastically reduce to 21.

Source → Destination	Path	Traffic
3 → 5	3--4--5	10
1 → 5	1--2--5	12
4 → 5	4--6--5	15
4 → 6	4--6	6

Fig. 2.4.3: Same set of commodities with an efficient path allocation scheme

Thus, it is apparent from the above illustration that choosing an efficient path selection criteria while performing RSA, will be beneficial in reducing the total spectrum requirement for satisfying the commodities.

2.5 Some Useful techniques/algorithms used in the research

The optimization of optical networks problems, in general, are viewed as the Multi-Commodity Network Flow (MCNF) problems [13]. To solve a MCNF problem, one approach is to define an appropriate formulation using an Integer Linear Program (ILP) or Mixed Integer Linear Program (MILP) and solve the formulation using a solver, such as the CPLEX Optimizer [14]. Solving ILPs in general, is known to be NP-Complete[15][13]. A majority of the MILPs for designing optical networks can find acceptable solutions within a reasonable amount of time only for comparatively smaller networks. Heuristics are mostly used to attain faster results for larger networks.

2.5.1 *k*-Shortest Path Algorithm

One of the key components of this research work is the implementation of the *k-shortest path* algorithm. For a given graph G(V,E), with |V| vertices and |E| edges, a *k*-shortest path algorithm can find the first *k loopless* shortest paths between any two vertices. A path is termed as a loopless path when none of the nodes appearing in the path are traversed more than once. If only one path is considered while computing the path in RSA, then it is very likely that the total spectrum requirement may not be optimal. In other words, when k-shortest path algorithm is used, where *k* paths are considered for each request for communication, the algorithm has additional options of trying alternative paths if the current path being considered leads to inefficient usage of available bandwidth.

Yen's algorithm [16] is a general algorithm for finding k-shortest loopless paths from a given source to a given destination in a graph with non-negative edge costs. It employs any shortest path algorithm to find the best path, then proceeds to find k - 1 deviations of the best path. Each path is computed in a manner such that, it is the next available shortest path to the previous computed path, and it does not feature in the finalized list of the shortest paths previously computed. The actual algorithm can be broken down into two stages. In the first stage, the algorithm finds the shortest path for the (s, d) pair in the given network. The second stage involves of a number of iterations to determine successive shortest paths. In each iteration, the next shortest path is found. A detailed explanation of the algorithm and its working is provided in [16].

2.5.2 Christodoulopoulos Algorithm for RSA

Christodoulopoulos et al [6] is among the first of the papers to address the Routing and Spectrum Allocation problem, and as such, does not mention any shortcomings of the previous papers.

In this paper, the authors introduced the Routing and Spectrum Allocation problem and addressed it by presenting various algorithms for solving the RSA. They presented an ILP RSA algorithm that tries to minimize the spectrum used to serve the set of requests for communication, and also proposed a decomposition method that splits RSA into two sub-problems, namely, (i) routing and (ii) spectrum allocation (R+SA) and solved them sequentially. The authors also proposed a heuristic algorithm that served connections one-by-one and used it to resolve the planning problem by sequentially serving all the requests for communication. Two ordering policies were planned to feed the sequential algorithm; a simulated annealing meta-heuristic was also used to find superior orderings.

The authors used simulation experiments to evaluate the performances of their proposed algorithms. They used Matlab to implement the algorithms, LINDO API for ILP solving, and Matlab built-in simulated annealing meta-heuristic. The authors analyzed their results for the low and high load cases.

According to the authors, for low load condition, the MSF ordering of demands in the sequential heuristic algorithm performed the best among all the proposed algorithms in terms of the time required for execution. Moreover, Simulated Annealing enhanced the performance of the sequential heuristic algorithm. For high load cases, the decomposed R+SA ILP algorithm found the best solutions.

Notations used in Christodoulopoulos Algorithm

$\boldsymbol{P_{sd}}$ **:** the set of all the paths from source s to destination d.

$\boldsymbol{T_{sd}}$ **:** the number of subcarriers required for the communication between source s and destination d.

$\boldsymbol{x_p}$ **:** Boolean variable denoting the utilization of path $p \in P$.
(x_p equals to 0 if path p is not utilized, and 1 if p is utilized).

$\boldsymbol{f_{sd}}$ **:** Integer variable denoting the starting frequency for connection (s,d).

$\boldsymbol{T_{total}}$ $= \sum_{(s,d)} T_{sd}$.

$\boldsymbol{\delta_{sd,s'd'}}$ **:** Boolean variable that equals 0 if the starting frequency of connection (s', d') is smaller than the starting frequency of connection (s,d) (i.e., $f_{s'd'} < f_{sd}$), and 1 otherwise (i.e., $f_{sd} < f_{s'd'}$).

$\boldsymbol{G}$ **:** Guard Band.

$\boldsymbol{c}$ **:** maximum utilized spectrum slot.

The formulation for Christodoulopoulos Algorithm

Objective Function

Minimize c

Subject to the following Constraints

1. Calculate the cost function

$$c \geq f_{sd} + T_{sd} \quad \text{for all (s,d) pairs} \tag{2.1}$$

2. Satisfy the single path routing constraints

$$\sum_{p \in P_{sd}} x_p = 1 \quad \text{for all (s,d) pairs} \tag{2.2}$$

3. Impose the starting frequencies ordering constraints

For all commodities (s,d) and (s',d') that have $p_i \in P_{sd}$ and $p_j \in P_{s'd'}$, with p_i and p_j sharing at least one common link l,

$(\forall(s,d),(s',d') : \exists p_i \in P_{sd} \cap \exists p_j \in P_{s'd'} \cap (l \in p_i \cap l \in p_j))$

$$\delta_{sd,s'd'} + \delta_{s'd',sd} = 1, \tag{2.3}$$

$$f_{s'd'} - f_{sd} < T_{total}.\delta_{sd,s'd'}, \tag{2.4}$$

$$f_{sd} - f_{s'd'} < T_{total}.\delta_{s'd',sd}, \tag{2.5}$$

4. Satisfy the spectrum continuity and non-overlapping spectrum allocation constraints

For all commodities (s,d) and (s',d') that have $p_i \in P_{sd}$ and $p_j \in P_{s'd'}$, with p_i and p_j sharing at least one common link l,

$$f_{sd} + T_{sd} + G - f_{s'd'} \leq (T_{total} + G).(1 - \delta_{sd,s'd'} + 2 - x_{pi} - x_{pj}) \tag{2.6}$$

$$f_{s'd'} + T_{s'd'} + G - f_{sd} \leq (T_{total} + G).(1 - \delta_{s'd',sd} + 2 - x_{pi} - x_{pj}) \tag{2.7}$$

Justification of Christodoulopoulos Algorithm

The objective of Christodoulopoulos algorithm was to minimize c, the maximum utilized spectrum slot, required in fulfilling all the demand requests. Constraint (2) is the single-path routing constraint, which ensures that only a single path is selected for routing a particular commodity, out of all the precomputed paths for that commodity. Constraints (3)-(5) guarantee that either $\delta_{sd,s'd'}$=1, implying that the starting frequency f_{sd} of connection (s,d) is smaller than the starting frequency $f_{s'd'}$ of (s',d')(i.e. $f_{sd} < f_{s'd'}$), or $\delta_{s'd',sd}$=1, implying that $(f_{sd} > f_{s'd'})$.

When one (or both) of the paths p_i and p_j is not utilized. (i.e. $x_{pi} \neq 1$ or $x_{pj} \neq 1$), constraints (6) and (7) are redundant (hold always, irrespective of f_{sd} and $f_{s'd'}$), since the right hand side of the constraints take a value greater than T_{total}, which is always higher than the left hand side.

Now, considering the case when both the paths p_i and p_j are utilized ($x_{pi} = 1$ and $x_{pj} = 1$), either of the constraints (6) or (7) are activated according to the values of $\delta_{sd,s'd'}$ and $\delta_{s'd',sd}$. When $\delta_{sd,s'd'} = 1$, constraint (6) is activated and it becomes:

$$f_{sd} + T_{sd} + \mathrm{G} \leq f_{s'd'}$$

guaranteeing that the spectrum utilized by the two connections (s, d) and (s', d') do not overlap. Constraint (7), in this case, is trivially satisfied, since (7) becomes:

$$f_{s'd'} + T_{s'd'} \text{ - } f_{sd} \leq T_{total}$$

which holds always irrespectively of $f_{s'd'}$ and f_{sd}. Similarly, when $\delta_{s'd',sd} = 1$, constraint (7) is activated and constraint (6) is trivially satisfied. Thus, constraints (6) and (7) together ensure that the spectrums assigned to connections that utilize paths that share a common link, do not overlap.

2.5.3 Other Related Works on RSA

Varvarigos et al. [17] have extensively studied the routing, modulation level and spectrum allocation (RMLSA) problem in the SLICE network, proved that RMLSA is NP-complete and presented various algorithms to resolve this problem. They presented an ILP RSA algorithm to minimize the spectrum used to handle all the requests for data communication, and also proposed a decomposition method that splits RMLSA into its two sub-problems, namely, (i) routing and modulation level (ii) spectrum allocation (RML+SA) and solved them in sequence. The authors also proposed a heuristic algorithm that serves connections one-by-one and used it to resolve the planning problem by sequentially handling all requests for data communication.

The authors used simulation experiments to evaluate the performance of their proposed algorithms. They used Matlab to implement the algorithms, LINDO API for ILP solving and Matlab built-in simulated annealing meta-heuristic. The authors observed the performance of the proposed algorithms through simulation experiments and assessed the spectrum utilization benefits that can be attained by utilizing OFDM elastic bandwidth allocation.

The authors stated that their results indicated that the proposed sequential heuristic combined with a suitable ordering discipline could deliver close to optimum solutions in low running times. They demonstrated the OFDM-based networks to have substantial spectrum benefits over classic fixed-grid WDM networks, specifying that the OFDM architecture offers a promising solution for future high capacity transport networks.

Sen et al. [12] introduced the Routing and Spectrum Allocation problem (RSA problem) and proved that it is NP-complete even when the optical network topology is as simple as a chain. They proposed approximation algorithms for the RSA problem when the network topology is a binary tree or a ring. They introduced the Spectrum Constrained RSA (SCRSA) problem where the goal was to satisfy as many requests as possible, subject to the constraint that only a finite size spectrum is available for satisfying connection requests. Also, they proposed a heuristic algorithm that with arbitrary topology and measured the effectiveness of the heuristic with extensive simulation.

All the three heuristics SPSR, BLSA and DPH, proposed by them, operate in two phases. In the first phase they computed the routes (paths) and in the second phase they allocated spectrum to these paths. In the spectrum allocation phase of the SPSR and BLSA, the computed paths were partitioned into sets of disjoint paths (starting from the path with the largest demand).

The authors stated that, in all their performed tests, DPH is more efficient than all the other heuristics, even though SPSR and BLSA use the same spectrum allocation technique as DPH. They verified that the routing scheme used in DPH plays a significant role in improving its performance over SPSR and BLSA.

Klinkowski et al. [18] noted the inefficiency of First-Fit frequency assignment (FA-FF) algorithm discussed in Jinno et al.[19].

In addition to proposing an ILP algorithm, the authors also proposed a novel heuristic algorithm called AFA-CA (Adaptive Frequency Assignment - Collision Avoidance), which adaptively selects the sequence of processed demands in order to minimize the spectrum used in the network. The authors compared the RSA performance results obtained with ILP, AFA-CA, and two reference algorithms, namely, FAFF and MSF.

The researchers indicated that AFA-CA offers improved performance (approx. 7.5%) compared to MSF. The authors noted that in all investigated cases, their method AFA-CA delivers superior results than the reference algorithms. They mentioned that although algorithm AFA-CA needs more time to find the solution compared to FA-FF and MSF, the execution time of AFA-CA is less than 1 second even for most demanding case.

Wang et al. [20] formulated an optimal ILP RSA algorithm that tries to optimally minimize the maximum number of sub-carriers necessary on any fiber of a SLICE network. They then analyzed the lower/upper bounds for the sub-carrier number in a network with general or specific topology. They proposed two efficient algorithms, namely, balanced load spectrum allocation (BLSA) algorithm and shortest path with maximum spectrum reuse (SPSR) algorithm to decrease the requisite sub-carrier number in a SLICE network.

The authors used the ILOG CPLEX for implementing the ILP model. They conducted simulation tests for the proposed ILP model, heuristic algorithms and the lower bound analysis and proved the NP hardness of the optimal RSA problem.

The authors stated that the simulations which they conducted, have established that for ring networks with various uniform traffic demand and guard-carrier size, the ILP model can achieve the lower bound produced by the cut-set (CS) method. Their simulation results further confirmed that both BLSA and SPSR algorithms produce results close to the optimal ILP solution for uniform traffic demands.

Wang et al. proposed in [21], two efficient heuristic algorithms to minimize the required sub-carrier number in a large SLICE network when the ILP model becomes intractable.

The authors studied the routing and spectrum allocation (RSA) problem in the SLICE network by using a set of proposed Integer Linear Programming (ILP) formulations to achieve different optimization objectives. New approaches to find the lower/upper bounds for the sub-carrier number in a SLICE network were examined. Two heuristic algorithms, namely Shortest Path with maximum Reuse (SPSR) and Balanced Load Spectrum Allocation (BLSA) were also studied in their simulation under different optimization goals.

The authors noted that BLSA needs more sub-carriers than SPSR, which may entail that the shortest path routing facilitates the objective of minimizing the total sub-carrier number. They showed that in general, their results indicate that SPSR outperforms BLSA when minimizing the total sub-carrier number due to its shortest path routing, while BLSA outperforms SPSR when minimizing the maximum sub-carrier index.

Klinkowski [22] introduced the problem of static Routing and Spectrum Assignment (RSA) in a flexible grid optical network with dedicated path protection (DPP) consideration. The author developed a Genetic Algorithm-based algorithm that provides, a near-optimal solution to the offline RSA with DPP problem in a flexgrid-based optical network (FG-ON).

The author proved via his experiments that his algorithm significantly outperforms the heuristic algorithms referenced in his literature and it provides results close to the optimal ones for both smaller and larger networks.

Chapter 3

Optimal and Heuristic Approaches to Solve the Route and Spectrum Allocation Problem in OFDM Networks

It is convenient to view the problem of static RSA as a multi-commodity network flow (MCNF) [13] problem, where a connection request for source-destination pair ($O(k)$,$D(k)$) corresponds to a distinct commodity k, to be shipped from the source $O(k)$ to the destination $D(k)$. To derive an optimal solution, this problem may be specified as a formulation using a Mixed Integer Linear Program (MILP).

In this chapter we have presented our proposed MILP approach for optimally solving the Route and Spectrum Allocation (RSA) problem in OFDM networks. The objective of our algorithm is to determine an optimal path and an optimal bandwidth allocation scheme for each of the request, such that the total spectrum requirement to satisfy the set of demand requests is as small as possible. The existing approaches for RSA consider a limited set of potential routes for each request, while selecting an appropriate route and allocating spectrum for the request. This leads to an incomplete exploration of the solution space, which in turn, does not guarantee the optimality of the derived solution. On the other hand, our optimal ILP formulation carries out an exhaustive search and leaves no route unexplored in order to establish the connection request. The solution obtained using this ILP formulation, if CPLEX solver terminates within a specified CPU time limit, is guaranteed to be optimal. We will use the term ILP_1 to denote this optimal ILP formulation.

Solving a MILP with a large number of binary variables is generally time consuming, as the time required to solve such problems increases exponentially with the number of binary variables [13].

Due to this reason, our ILP_1 approach is not able to handle larger networks, since it requires unrealistic amount of time for optimally solving the problem for larger networks. Therefore, we have proposed a modified ILP formulation where we will restrict the search space by limiting the number of paths to be considered for each commodity. The number of actual binary variables used in our second formulation is significantly less than that for the first formulation. We will use the term ILP_2 to denote the second ILP formulation.

Finally, we have compared the results obtained using our approaches to another popular algorithm recently proposed by Christodoulopoulos et al. [6]. A review and a summary of this approach is in Chapter 2.

3.1 Notations used in the ILP Algorithms

3.1.1 Parameters

***K* :** a set of commodities, where commodity $k \in K$ is specified by

- *O(k)*, the source of the commodity.
- *D(k)*, the destination of the commodity.

- T_k, the bandwidth needed, specified by the number of subcarriers required for the commodity k.

$\boldsymbol{A}$ **:** the set of all edges in the graph.

$\boldsymbol{N}$ **:** the set of all nodes in the graph.

$\boldsymbol{Total_k} = \sum_{k \in K} T_k$ is the sum of all bandwidths.

$\boldsymbol{G}$ **:** guard band.

$\boldsymbol{R^k}$ **:** the set of precomputed routes for commodity k.

3.1.2 Decision Variables

$\boldsymbol{x_{ij}^k}$ **:** a binary variable denoting whether the path chosen for k^{th} commodity uses edge $(i, j) \in A$, where

$$x_{ij}^k = \begin{cases} 1 & \text{if the path chosen for } k^{th} \text{ commodity uses edge } (i, j) \in A, \\ 0 & \text{otherwise.} \end{cases}$$

$\boldsymbol{\delta_{kl}^{ij}}$ **:** a (non-negative) continuous variable denoting whether edge (i, j) is used by both the paths for commodities k and l, so that

$$\delta_{kl}^{ij} = \begin{cases} 1 & \text{if the edge } (i, j) \in A \text{ is used by both the paths for commodities } k \text{ and } l, \\ 0 & \text{otherwise.} \end{cases}$$

$\boldsymbol{\theta_{kl}}$ a (non-negative) continuous variable denoting whether at least one edge is shared by the paths for commodities k and l, so that

$$\theta_{kl} = \begin{cases} 1 & \text{if the commodities } k \text{ and } l \text{ share at least one edge,} \\ 0 & \text{otherwise.} \end{cases}$$

$\boldsymbol{f_k}$ **:** a (non-negative) continuous variable representing the starting frequency of the commodity k, $(k \in K)$.

$\boldsymbol{\partial_{kl}}$ **:** a binary variable denoting the ordering of the starting frequencies for commodities k and l, so that

$$\partial_{kl} = \begin{cases} 1 & \text{if } f_k < f_l \text{ and the commodities } k \text{ and } l \text{ share at least one edge,} \\ 0 & \text{otherwise.} \end{cases}$$

$\boldsymbol{\lambda}$ **:** the maximum utilized spectrum.

$\boldsymbol{P_r^k}$ **:** a binary variable $P_r^k(r \in R^k, k \in K)$ such that

$$P_r^k = \begin{cases} 1 & \text{if the } k^{th} \text{ commodity uses route } r \in R^k, \\ 0 & \text{otherwise.} \end{cases}$$

3.2 An approach to solve the Route and Spectrum Allocation problem in OFDM networks optimally

3.2.1 The formulation for ILP_1

Objective Function

$$\text{Minimize } \lambda$$

Subject to the following constraints

1. Compute the value of cost function λ

$$\lambda \geq f_k + T_k \quad \text{,for all k} \in \text{K} \tag{3.1}$$

2. Satisfy the flow-balance equations [23]

$$\sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(j,i)\in A} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k), \\ -1 & \text{if } i = D(k), i \in N, k \in K, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

3. Define continuous variable δ_{kl}^{ij} whose value becomes equal to 1, if and only if the paths of commodities k and l share edge (i,j), for all $k, l \in K$, for all $(i,j) \in A$.

$$\delta_{kl}^{ij} \leq x_{ij}^k \tag{3.3}$$

$$\delta_{kl}^{ij} \leq x_{ij}^l \tag{3.4}$$

$$\delta_{kl}^{ij} \geq x_{ij}^k + x_{ij}^l - 1 \tag{3.5}$$

4. Define continuous variables θ_{kl}, whose value becomes 1, if and only if, the paths of commodities k and l share at least one edge, for all $k, l \in K$. It is important to note that the value of the variable θ_{kl} is independent of the total number of shared edges between commodities in the network.

$$\theta_{kl} \geq \delta_{kl}^{ij}, \forall (i,j) \in A \tag{3.6}$$

$$\theta_{kl} \leq \sum_{j:(i,j)\in A} \delta_{kl}^{ij} \tag{3.7}$$

$$\theta_{kl} \leq 1 \tag{3.8}$$

5. Ensure the starting frequency ordering constraint

Define binary variable ∂_{kl},$(k,l) \in K$, such that ∂_{kl} is 1 iff $f_k < f_l$ and commodities k and l share edges.

$$\partial_{kl} + \partial_{lk} = \theta_{kl}, \quad \forall (k, l \in K) \tag{3.9}$$

6. Specify spectrum non-overlapping constraints for commodities k and l

$$f_l - f_k \geq T_k + G + Total_k(\partial_{kl} - 1) \quad (3.10)$$

$$f_k - f_l \geq T_l + G + Total_k(\partial_{lk} - 1) \quad (3.11)$$

3.2.2 Justification of ILP_1

The objective of the formulation ILP_1 is to minimize λ, the maximum utilized spectrum slot, required to fulfil all the requests for communication. Equation 3.1 specifies that λ must be greater than or equal to the maximum value of the subcarrier wavelengths required by the commodities. Since the objective is to minimize λ, the net effect is that λ is set to the value of the largest subcarrier wavelength used. In other words, λ is set to the spectrum required to handle all commodities in K. In Equation 3.2, $\sum_{j:(j,i)\in A} x_{ji}^k$ is the total incoming flows for commodity k, into node i. Similarly, $\sum_{j:(i,j)\in A} x_{ij}^k$, is the total outgoing flows for commodity k, using edges from node i. The intent of equation 3.2 is to specify that the difference between the sum of outgoing flows and incoming flows is :

- 1, if node i is the source, $O(k)$.
- -1, if node i is the destination, $D(k)$.
- 0, if node i is any other intermediate node in the path from the source to the destination for commodity k.

Equations 3.3, 3.4 and 3.5 restrict the value of continuous variable δ_{kl}^{ij} to 1, if and only if both x_{ij}^k & x_{ij}^l is 1. The value of δ_{kl}^{ij} is 0 for all other combinations of x_{ij}^k & x_{ij}^l. This can easily be verified from the following truth table obtained by putting all the possible combinations of x_{ij}^k & x_{ij}^l in the equations 3.3, 3.4 and 3.5. If both x_{ij}^k & x_{ij}^l is 1, equations 3.3, 3.4 and 3.5 become $\delta_{kl}^{ij} \leq 1$, $\delta_{kl}^{ij} \leq 1$ and $\delta_{kl}^{ij} \geq 1$ respectively.

Likewise, when either of x_{ij}^k or x_{ij}^l is 1 and the other 0, equations 3.3, 3.4 and 3.5 become $\delta_{kl}^{ij} \leq 0$, $\delta_{kl}^{ij} \leq 1$ and $\delta_{kl}^{ij} \geq 0$, thereby limiting the value of δ_{kl}^{ij} to 0. Similarly, for the case when both x_{ij}^k & x_{ij}^l is 0, equations 3.3, 3.4 and 3.5 become $\delta_{kl}^{ij} \leq 0$, $\delta_{kl}^{ij} \leq 0$ and $\delta_{kl}^{ij} \geq$ -1, so that δ_{kl}^{ij} is constrained to be 0.

x_{ij}^k	x_{ij}^l	δ_{kl}^{ij}
0	0	0
0	1	0
1	0	0
1	1	1

To understand the significance of the equations 3.6, 3.7 and 3.8, let us consider two scenarios. In scenario 1, let us assume the paths of commodities k & l have n edges ($n \geq 1$) in common. In that case, the equations 3.6, 3.7 and 3.8 become $\theta_{kl} \geq 1$, $\theta_{kl} \leq n$ and $\theta_{kl} \leq 1$. Thus, the equations 3.6, 3.7 and 3.8 restrict the value of continuous variables θ_{kl} for the shared edges to 1.

In scenario 2, let us assume that the paths of commodities k & l have no edges in common. In that case, equations 3.6, 3.7 and 3.8 become $\theta_{kl} \geq 0$, $\theta_{kl} \leq 0$ and $\theta_{kl} \leq 1$. Thus, equations 3.6, 3.7 and 3.8 effectively restrict the value of continuous variables θ_{kl} to 0.

In summary, equations 3.6, 3.7 and 3.8 restrict the value of continuous variables θ_{kl} to 1, if and only if the paths of commodities k and l share at least one edge, for all $k, l \in K$. For the scenario where the paths of commodities k & l have no edges in common, the value of θ_{kl} is restricted to 0. The point to be noted here is that δ_{kl}^{ij} and θ_{kl} are continuous variables whose values are restricted to 0 or 1, using the constraints mentioned above. Such use of continuous variables to replace binary variables drastically improves the performance of the formulation.

Constraints 3.9, 3.10 & 3.11 ensure the allocated spectrum to be non-overlapping, for the commodities that share one or more edge(s) in their path. They ensure that the value of ∂_{kl} and ∂_{lk} both cannot be simultaneously 1. ie., either $f_k < f_l$ or $f_l < f_k$ always holds true for the commodities that share edges on their paths.

When θ_{kl} is 0, i.e. if k and l do not share an edge, then it implies $\partial_{kl} = \partial_{lk} = 0$, equations 3.10 & 3.11 become $f_l + Total_k \geq f_k + T_k + G$ and $f_k + Total_k \geq f_l + T_l + G$ respectively. Thus, both the constraints 3.10 & 3.11 are trivially satisfied.

When $\theta_{kl} = 1$, then only one of the constraints is relevant and the other becomes trivially satisfied or redundant. For instance, when ($\partial_{kl} = 0$ and $\partial_{lk} = 1$), the equations 3.10 & 3.11 become

$$f_l + Total_k \geq f_k + T_k + G$$

$$f_k \geq f_l + T_l + G$$

respectively. The first is trivially satisfied and the second ensures that

1) the bandwidth for k and l are non-overlapping and

2) the bandwidth for f_k follows bandwidth for f_l.

The case when $\partial_{kl} = 1$ and $\partial_{lk} = 0$ is similar.

3.3 A fast approach to approximately solve the Route and Spectrum Allocation problem in OFDM networks

3.3.1 The formulation for ILP$_2$

For each commodity $k \in K$, we precompute $|R^k|$ routes, all from source *O(k)* to destination *D(k)*, to be used in the formulation.

Objective Function

$$\text{Minimize } \lambda$$

Subject to the following constraints

1. Satisfy the single path routing constraint by ensuring that only a single path is chosen among the $|R^k|$ precomputed paths for commodity k.

$$\sum_{r \in R^k} P_r^k = 1, \quad \forall (k \in K) \tag{3.12}$$

2. Compute continuous variable δ_{kl}^{ij}, such that δ_{kl}^{ij} is 1 *iff* the selected route for commodity l and k both use edge (i, j), for all commodities $k, l \in K$.

$$\delta_{kl}^{ij} \leq \sum_{(r \in R^k : (i,j) \in r)} P_r^k \tag{3.13}$$

$$\delta_{kl}^{ij} \leq \sum_{(r \in R^l : (i,j) \in r)} P_r^l \tag{3.14}$$

$$\delta_{kl}^{ij} \geq \sum_{(r \in R^k : (i,j) \in r)} P_r^k + \sum_{(r \in R^l : (i,j) \in r)} P_r^l \quad - \quad 1 \tag{3.15}$$

The other constraints of this formulation are identical to the constraints 3.1, 3.6, 3.7, 3.8, 3.9, 3.10 and 3.11 of the ILP_1 formulation.

3.3.2 Justification of ILP_2

Equation 3.12, $\sum_{r \in R^k} P_r^k = 1$, ensures that exactly one route must be selected from the possible R^k routes for the k^{th} commodity from source $O(k)$ to destination $D(k)$.

Equations 3.13, 3.14 and 3.15 ensure that the value of continuous variable δ_{kl}^{ij} is 1 *iff* the selected route for both commodities l and k use the edge (i,j). Otherwise, the value of δ_{kl}^{ij} is constrained to be 0. The explanations for other equations are identical to 3.6, 3.7, 3.8, 3.9, 3.10 and 3.11.

3.4 Analysis of ILP Formulations

3.4.1 Analysis of ILP_1 Formulation

There are two sets of binary (0/1) variables - x_{ij}^k and ∂_{kl}. There is one variable x_{ij}^k for each edge $(i, j) \in A$, and for each value of k, $1 \leq k \leq |K|$. There is one variable ∂_{kl} for each combination of k and l. Therefore, the formulation has $(\frac{|K|.(|K|-1)}{2} + |K|.|A|)$ binary variables.

There are three sets of continuous variables - δ_{kl}^{ij}, θ_{kl} and f_k. There is one variable δ_{kl}^{ij} for each edge $(i, j) \in A$, and for each combination of k and l. There is one variable θ_{kl}, for every combination of k and l. Further, there is one variable f_k, for each value of k, $k \in K$. Thus, the formulation has $\frac{|K|(|K|-1)}{2}(|A|+1+\frac{2}{|k|-1})$ continuous variables. The number of constraints in the formulation is $|K||N+1|+\frac{5}{2}(|A|+1)(|K|.(|K|-1))$.

3.4.2 Analysis of ILP_2 Formulation

There are two sets of binary (0/1) variables - P_r^k and ∂_{kl}. There is one variable P_r^k for each value of k, $1 \leq k \leq |K|$ and for each value of r, $r \in R^k$. There is one variable ∂_{kl} for each combination of k and l. Therefore, the formulation has $(\frac{|K|(|K|-1)}{2} + |K|.|R^k|)$ binary variables, which are integers.

The number of continuous variables generated, i.e. δ_{kl}^{ij}, θ_{kl} and f_k, are of the same order as in the ILP_1 formulation. Thus, the formulation has $\frac{|K|(|K|-1)}{2}(|A| + 1 + \frac{2}{|k|-1})$ continuous variables. The number of constraints in the formulation is $2|K| + \frac{5}{2}(|A| + 1)(|K|.(|K| - 1))$.

3.4.3 Analysis of Christodoulopoulos Formulation

Let the total number of (s, d) pairs be denoted by K. Also, let the total number of paths generated for each commodity k, (denoted as P_{sd} in the original formulation) be represented by R^k. There are two classes of binary (0/1) variables - x_p and $\delta_{sd,s'd'}$.

There is one variable x_p for each value of k, $1 \leq k \leq |K|$ and for each value of p, $p \in R^k$. There is one variable $\delta_{sd,s'd'}$ for every combination of sd and $s'd'$. There is one integer variable f_{sd} for each value of k, $1 \leq k \leq |K|$. Thus, the formulation has $(|K| + |K|^2 + |K|.|R^k|)$ integer variables.

The number of constraints in the formulation is $(2|K| + 5|K|^2)$.

Table 3.4.1: Analysis of the ILP approaches

Formulation	*Number of integer variables*	*Number of constraints*
ILP_1	$(\frac{\|K\|.(\|K\|-1)}{2} + \|K\|.\|A\|)$	$\|K\|\|N+1\| + \frac{5}{2}(\|A\|+1)(\|K\|.(\|K\|-1))$
ILP_2	$(\frac{\|K\|(\|K\|-1)}{2} + \|K\|.\|R^k\|)$	$2\|K\| + \frac{5}{2}(\|A\|+1)(\|K\|.(\|K\|-1))$
Christodoulopoulos	$\|K\| + \|K\|^2 + \|K\|.\|R^k\|$	$2\|K\| + 5\|K\|^2$

To compare the number of integer variables in the formulations, let a network have 10 nodes, (i.e., $N = 10$), 25 edges (i.e., $|A| = 25$), and let the number of commodities, be 20 (i.e., $|K| = 20$), where we supply 4 paths for each commodity (i.e., $|R^k| = 4$). The number of integer variables generated by each formulation is shown in table 3.4.2.

Table 3.4.2: Comparative analysis of the ILP approaches with a sample network

Formulation	*Number of integer variables*
ILP_1	690
ILP_2	270
Christodoulopoulos	500

Thus, the above scenario demonstrates a significant reduction in the number of integer variables by our ILP$_2$ formulation, in comparison with the Christodoulopoulos formulation, with both of them being provided the same number of paths, the same set of commodities and the ame network topology.

Chapter 4

Experimental Results

Simulation is a widely used technique in computer networks to study the performance of the system, without having to set up the network physically. In order to effectively evaluate the performance of our ILP formulations for RSA, a suite of simulation tools with an interface has been developed. Testing these tools with identical configuration across all the formulations will allow precise and trustworthy performance comparison.

Our ILP_1 formulation always generates the optimum solution and was developed with the intention of acting as a benchmark for comparison with other formulations. As per our knowledge, none of the researchers have solved the problem of static RSA optimally. Our ILP_2 formulation and the formulation proposed by Christodoulopoulos et al. in [6][1], accept a set of paths for each commodity as an input. Each path in the set is from the source to the destination of the commodity. It is logical to include the first k shortest paths between the source and the destination of the commodity as the set of paths for some suitable value of k. In our experiments, we have used the k-shortest path algorithm [16] by Yen, to compute the first k shortest paths for a given commodity. Since we supply both ILP_2 and CHR formulations with the same set of paths, the objective values produced by both of them should be the same.

The primary objective of the simulation study reported below is to evaluate and

[1] henceforth referred as CHR in the thesis

compare the performances of our proposed formulations ILP_1 and ILP_2, with those of CHR. We also studied the performance of our ILP_2 formulation on Deutsche Telekom (DT) network, previously studied in [6]. We conducted four sets of experiments to study the efficacy of our formulations. We started our study by solving the RSA optimization problem under different sets of randomly generated network topologies. For our first set of experiments to compare the performances, we have generated 8, 12 and 15 node networks, where the edges of the networks were randomly chosen node-pairs. An edge between two nodes in the network consists of 2 separate unidirectional optical fibers in our experiments. For a given size of the network, we have generated 5 random physical topologies, and have run all the three formulations on them[2]. For each set, we have randomly chosen the degree of each node to lie between 2 and 3. We have also generated 5 instances of commodity sets, consisting of 8, 12 and 15 connection requests. Each of the connection requests in these commodity sets consists of the source node, the destination node and the number of subcarriers required by the connection request.

For a given size of the network and a set of commodities, we have solved each of the formulations and noted the execution time and the objective values obtained. For these randomly generated networks, each of our results reported below represent the average of 25 simulation runs using five topologies and five sets of commodities. The detailed results of the simulation runs can be found in the appendix section of the thesis.

We specified an upper limit of 3600 seconds as the maximum allowed computation time for solving each formulation. When solving a given topology and a given set of commodities, if the solution for a formulation required more than this upper limit, the process was automatically killed by the CPU. When computing the averages, we have excluded the cases where the solver could not find a solution within 3600 seconds.

[2]We were limited to networks with 15 or fewer nodes, since all the formulations take an unacceptable amount of time to solve, if the network has more than 15 nodes.

In our second series of experiments, we have studied the times required to solve ILP_2 and the corresponding objective values, by varying the number of precomputed paths for each commodity. Our objective was to find the tradeoff between the quality of the solutions and the execution times, when we varied the number of supplied paths for each commodity. Table 4.2.1 shows our experimental results of varying the number of paths for each commodity in the case of a 12 node network.

For our third set of experiments to evaluate and compare the performance of the formulations, we have used a realistic network topology, namely the Deutsche Telekom (DT) topology consisting of 14 nodes and 46 directed links. We have created 10 instances of commodity sets, consisting of 12, 15, 20, 25, 35 and 40 commodities. We have extensively tested all the three formulations with these commodity sets. For each value of the number of requests in the set of commodities, each reported result represents the average values for 10 sets of commodities.

Our fourth set of experiments tests our ILP_2 formulation using various network sizes to evaluate the maximum number of commodities which the formulation can handle in a reasonable time. A comprehensive description of the studies and their results have been presented in the subsequent sections.

4.1 Performance study of ILP_1, ILP_2 and CHR formulations

Table 4.1.1 compares the average execution times (given in seconds) needed to solve the RSA problem and standard deviations of the formulations ILP_1, ILP_2 and CHR considering networks with 8 nodes. We have considered 8, 12, 15, 18 and 20 commodities. In the cases of ILP_2 and CHR, we used 3 precomputed paths for each commodity.

Table 4.1.1: Comparison of the average execution times(Avg.) and standard deviations(SD) of ILP_1, ILP_2 and CHR for 8-node networks.

Ratios		Commodities		
		8	12	15
$\frac{ILP_1(time)}{CHR(time)}$	Avg.	0.98	5.65	2.28 [a]
	SD	1.11	3.23	6.19 [b]
$\frac{ILP_1(time)}{ILP_2(time)}$	Avg.	0.88	124.40	164.36
	SD	0.69	226.47	123.90
$\frac{CHR(time)}{ILP_2(time)}$	Avg.	0.90	22.00	72.15
	SD	0.68	26.92	227.28
$\frac{ILP_1(obj.value)}{ILP_2(obj.value)}$	Avg.	1.00	0.994	0.986

[a] The instances for which CHR was unable to solve in a reasonable time have been excluded while computing the averages.

[b] The instances for which CHR was unable to solve in a reasonable time have been excluded while computing the averages.

The results show that the formulation ILP_2 needs considerably less time than both ILP_1 and CHR to solve the RSA problem for any 8-node networks when the number of commodities was 12 or more. For instance, ILP_2 is approximately 22 (125) times faster than CHR (ILP_1) formulation for 12 commodities. The relative execution time of ILP_2 compared to ILP_1 and CHR increases even more, when the size of the set of commodities increases. ILP_2 is approximately 72(164) times better than CHR (ILP_1)

formulations for 15 commodity sets.

The objective values obtained using ILP_2 and CHR were remarkably close to those obtained using ILP_1, since it was approximately 98-99% of the optimal ILP_1 objective value. ILP_2 was able to handle 18(20) commodities and the average time was 1.80 (4.28) seconds.

Table 4.1.2: Comparison of the average execution times(Avg.) and standard deviations(SD) of ILP_1, ILP_2 and CHR for 12-node networks.

Ratios		Commodities		
		8	12	15
$\frac{ILP_1(time)}{CHR(time)}$	Avg.	1.94	27.35	13.33 [a]
	SD	1.25	51.40	106.57 [b]
$\frac{ILP_1(time)}{ILP_2(time)}$	Avg.	3.08	535.73	387.20 [c]
	SD	23.86	773.55	1179.19
$\frac{CHR(time)}{ILP_2(time)}$	Avg.	1.58	19.59	29.03[d]
	SD	7.94	82.49	50.97
$\frac{ILP_1(obj.value)}{ILP_2(obj.value)}$	Avg.	0.99	0.98	0.96

[a]The instances in the above table for which CHR & ILP_1 were unable to solve in reasonable time, have been excluded while averaging and taking ratio.
[b]The instances for which CHR was unable to solve in a reasonable time have been excluded while computing the averages.
[c]There were 2 instances of CHR which exceeded 3600 seconds.
[d]There were 5 instances of ILP_1 which exceeded 3600 seconds.

Similarly, Table 4.1.2 presents the comparison of the average execution times (in seconds) required to solve the RSA problem, standard deviations and the objective values for 12 node networks. The columns represent the commodity sets that were used for testing the formulations. In the cases of ILP_2 and CHR, we used 3 pre-computed paths for each commodity.

As evident from Table 4.1.2, ILP_2 significantly outperforms ILP_1 and CHR in terms of execution time. The ratios in the table confirm the superior and outstanding performance achieved by ILP_2. It is noteworthy that the execution time of ILP_2 shows substantial improvement by being approximately 20 (29) times better than CHR for 12 (15) commodity sets. The performance obtained in terms of objective values is also exceptionally good - in the range 96-98% of the optimal objective values obtained by ILP_1.

Table 4.1.3: Comparison of the average execution times(Avg.) and standard deviations(SD) of ILP_1, ILP_2 and CHR for 15-node networks.

Ratios		Commodities		
		8	12	15
$\frac{ILP_1(time)}{CHR(time)}$	Avg.	2.39	2.49 [a]	13.9 [b]
	SD	4.12	269.43	489.77
$\frac{ILP_1(time)}{ILP_2(time)}$	Avg.	2.65	86.71 [c]	1157.46 [d]
	SD	2.74	1056.00	2223.40
$\frac{CHR(time)}{ILP_2(time)}$	Avg.	1.11	34.76	83.28
	SD	0.66	82.13	155.41
$\frac{ILP_1(obj.value)}{ILP_2(obj.value)}$	Avg.	0.93	0.96	0.92

[a]The instances in the above table for which CHR & ILP_1 were unable to solve in reasonable time, have been excluded while averaging and taking ratio.
[b]The instances in the above table for which CHR & ILP_1 were unable to solve in reasonable time, have been excluded while averaging and taking ratio.
[c]There was 1 instance of ILP_1 which exceeded 3600 seconds.
[d]There was 1 instance of ILP_1 which exceeded 3600 seconds.

In the same way, Table 4.1.3 gives the comparison of the objective values, standard deviations and the average execution times (in seconds) required to solve the RSA problem for 15 node networks. The columns in the table represent the commodity sets that were used for testing the formulations. In the cases of ILP_2 and CHR, we used 3 pre-computed paths for each commodity.

As obvious from Table 4.1.3, ILP_2 again demonstrates that it is significantly better than ILP_1 and CHR in terms of computation time. The computation time of ILP_2 is approximately 35 (83) times better than CHR for 12 (15) commodities.

A major improvement in the computation time of ILP_2 is noticed as compared with ILP_1. ILP_2 is faster by approximately 86 (1157) times better than ILP_1 for 12 (15) commodity sets. The performance obtained in terms of objective values is also exceptionally good- in the range 92-96% of the optimal objective values obtained by ILP_1.

4.2 Effect of varying the number of considered paths in ILP_2

Table 4.2.1: Effect of changing the search space for ILP_2 by varying the number of paths (12-node networks).

Ratios of the number of paths	Ratios of time and obj val	Commodities 12	15	20
1 paths/4 path	Time(sec)	0.57	0.12	0.02
	Obj Val	1.48	1.45	1.58
2 paths/4 paths	Time(sec)	0.98	0.17	0.11
	Obj Val	1.09	1.09	1.15
3 paths/4 paths	Time(sec)	0.95	0.56	0.27
	Obj Val	1.02	1.02	1.02

In both ILP_2 and CHR, we restricted the number of paths considered by the formulation for each commodity to k paths, for some predetermined k. If we increase the value of k, better solutions are expected, at the cost of increased solution times since the search spaces are increased.

Table 4.2.1 illustrates a comparison of the achieved objective values and the average execution times (given in seconds) needed to solve the RSA problem using the formulation ILP_2, for the same topologies and commodity sets, when the number of paths for each commodity was varied. The results were initially garnered by providing

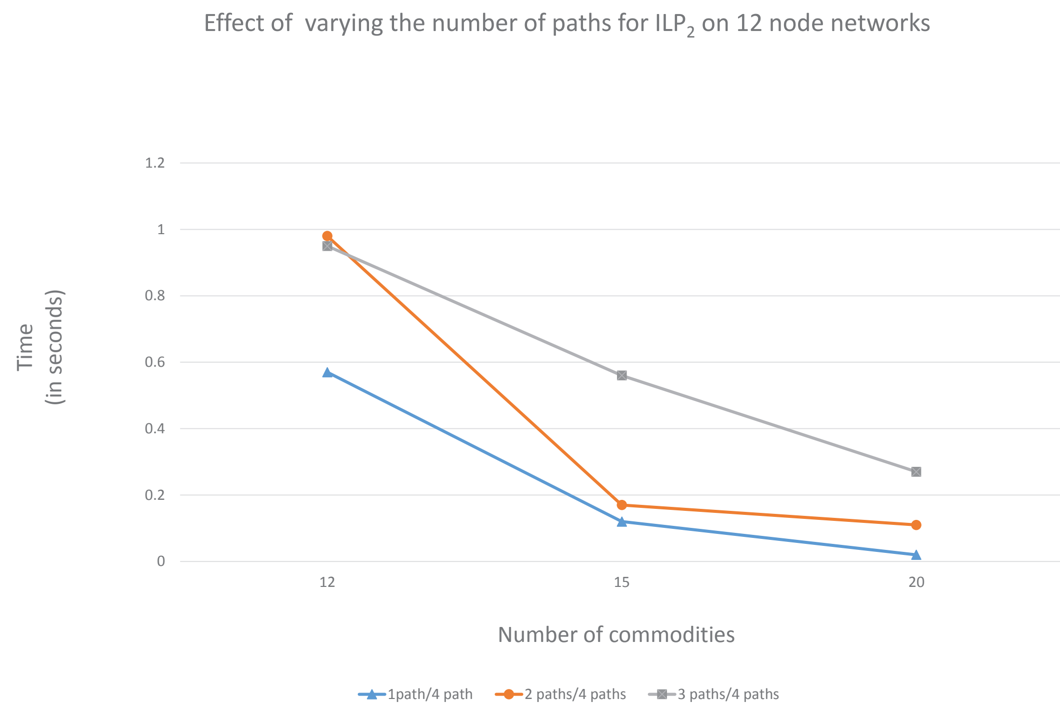

Fig. 4.2.1: Effect of changing the search space for ILP_2 by varying the number of paths (12-node network)

the formulation with 1, 2, 3 and 4 paths for each commodity. We computed these paths using Yen's [16] k-shortest path algorithm and supplied the paths as input. The spectrum values obtained using 4 paths are, as expected, better than those obtained using 1, 2 or 3 paths. To show this, the results shown in Table 4.2.1 give the ratio of the times (objective values) for 1, 2 and 3 paths, to the corresponding times (objective values) using 4 paths.

It can be inferred from the above table that there exists only a 2% improvement in the objective values achieved by using 4 paths, as compared to 3 paths. However, the computation times when we used 3 paths is significantly lower, compared to the corresponding times when we used 4 paths. This gain increases substantially with the increase in the number of commodities. For example, the ratios of the times when 3 paths were used, to that when 4 paths were used, decrease drastically from 0.95% for 12 commodities to 0.27% for 20 commodities, implying a huge performance gain in terms of computation time.

4.3 Performance studies of formulations on the Deutsche Telekom(DT) network

In [6], the authors categorically mention that CHR was "unable to produce results for this network in reasonable time".

Table 4.3.1: Simulation results of ILP_1, ILP_2 and CHR on DT network with 3 paths.

Ratios		Commodities		
		12	15	20
$\frac{ILP_1(time)}{CHR(time)}$	Avg.	52.12	8.49	1.88
	SD	174.48	18.98	72.30
$\frac{ILP_1(time)}{ILP_2(time)}$	Avg.	40.54	139.85	1029.86
	SD	45.60	916.17	1296.11
$\frac{CHR(time)}{ILP_2(time)}$	Avg.	0.78	16.43	548.30
	SD	1.00	63.46	435.24
$\frac{ILP_1(obj.value)}{ILP_2(obj.value)}$	Avg.	0.92	0.93	0.87

The above table shows that, our ILP_2 formulation performs extremely well as compared to both ILP_1 and our implementation of CHR formulation on the DT network. Since both CHR and ILP_2 are expected to give the same results in terms of objective values, we conclude that ILP_2 significantly outperforms CHR in terms of computation times. For instance, the average ratio of computation times required by CHR and ILP_2 for a 20 commodity set was approximately 548. Moreover, it is worth mentioning that ILP_2 was able to handle upto 40 commodities on the DT network in a reasonable time, while our implementation of CHR and ILP_1 could only handle upto 20 commodities. We note that for 25, 35 and 40 commodities, ILP_2 gave average times of 2.16, 96.2 and 213.6 seconds respectively. ILP_2 failed on 3 instances for 40 commodities to give a solution within a reasonable time. We have excluded the results that took more than 3600 seconds while averaging and calculating the ratios.

4.4 Study of the number of commodities that the ILP_2 formulation can handle.

As previously observed from our studies on the DT network topology, it is obvious that the ILP_2 formulation can handle more commodities within an acceptable amount of time. However, it is interesting to find the largest problem that ILP_2 can handle. To illustrate this, we have taken an example of 12 node networks and made exhaustive simulations on it to gather the data for the analysis.

Table 4.4.1 shows the running time of the ILP_2 formulation for different sets of commodities on 12 node networks, using 2 and 3 paths for each commodity. It is observed that there is a significant increase in the average time required by ILP_2 to solve a 30 commodity problem using 2 paths/commodity as compared to a 20 commodity problem, also with 2 paths/commodity. The formulation, however took an unreasonably long time to solve the instances of commodity sets beyond 30 commodities.

For 3 paths, the ILP_2 formulation could handle up to 20 commodities in a reasonable time. However, as we moved to 30 commodities and beyond, the average computation time of the formulation exceeded the time limit of 3600 seconds for most of the instances. The results for those cases have not been reported in the table. It is therefore observed that ILP_2 can handle upto 30 commodities with 2 paths and up to 20 commodities with 3 paths for 12 node networks.

Table 4.4.1: Analysis results to check the number of commodities that can be handled by ILP_2 on 12-node networks.

Paths	Number of commodities	Time (in sec)
2	8	0.37
	15	0.43
	20	1.74
	30	101.67
3	8	0.37
	12	0.47
	15	1.43
	18	1.79
	20	4.27

For 15 node networks, Table 4.4.2 shows the computation time of the ILP_2 formulation for various sets of commodities, using 3 paths for each commodity. It is observed that there is a significant increase in the average time required by ILP_2 to solve a 25 commodity problem using 3 paths/commodity as compared to a 20 commodity problem, also with 3 paths/commodity. The average computation time for the ILP_2 formulation exceeded the time limit of 3600 seconds for most of the instances of commodity sets beyond 25 commodities. Hence, the results for these sizes of networks have not been included in the Table 4.4.2.

Table 4.4.3 demonstrates the computation time of the ILP_2 formulation for various sets of commodities on 20 node networks, using 3 paths/commodity. It is observed that there is a notable increase in the average computation time required by ILP_2 to solve a 30 commodity problem using 3 paths/commodity as compared to a 25 commodity problem, also with 3 paths/commodity. The average computation time for the ILP_2 formulation exceeded the time limit of 3600 seconds for most cases with

30 commodities and were automatically terminated by the server. Hence, the results for the sizes of networks beyond 30 commodities have not been included in the Table 4.4.2.

Table 4.4.2: Analysis results to check the number of commodities that can be handled by ILP_2 on 15-node networks.

Paths	Number of commodities	Time (in sec)
3	8	0.47
	12	0.47
	15	0.72
	20	6.74
	25	35.52

Table 4.4.3: Analysis results to check the number of commodities that can be handled by ILP_2 on 20-node networks.

Paths	Number of commodities	Time (in sec)
3	20	3.03
	25	11.35
	30	286.93

Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this Masters thesis, we have presented two novel formulations to find the solutions to the static RSA problem in OFDM networks. We have presented a formulation (which we called ILP_1), that always finds the optimal solution for the RSA problem. To our knowledge, this is a novel formulation and none of the previous researchers have solved the RSA problem with an optimal ILP formulation. We have investigated and proposed a modification to the ILP_1 formulation by restricting the search space used by the formulation. This second formulation, (which we called ILP_2), takes as input a set of pre-computed paths for each commodity and selects exactly one path for each commodity. An implementation of the CHR formulation proposed in [6] was done by us for comparison purposes. We have used Yen's k-shortest path algorithm to pre-compute the k-shortest paths, between the source and the destination, corresponding to each commodity. We have supplied these pre-computed paths as an input to both CHR and ILP_2.

In Chapter 3, we have examined and analysed our formulations ILP_1, ILP_2 and CHR with respect to the basis size and the number of integer variables. In Chapter

4, we have performed an exhaustive study of the performances of all the formulations ILP_1, ILP_2 and CHR. We have reported our studies, in four separate sections as follows:

- A comparative performance study of ILP_1, ILP_2 and CHR formulations for 8 and 12 node networks.
- The effect of changing the search space for ILP_2 by varying the supplied number of paths for each commodity.
- A comparative study of all formulations on the Deutsche Telekom(DT) network.
- An analysis to determine the largest problem (in terms of the size of the network and the number of commodities) that the ILP_2 formulation can handle.

With this extensive simulation experiments, we have demonstrated the effectiveness and efficiencies of our proposed formulations. ILP_2 was found to be much faster compared to CHR formulation under almost all cases. For the Deutsche Telekom network reported in [6], our ILP_2 formulation also reported excellent results compared to CHR, both in terms of computation time and the number of commodities that it could handle. We have also given an instance of 12-node networks to test the largest problem that our ILP_2 formulation can handle.

5.2 Future Work

OFDM networks offer a huge promise in terms of the efficiency of network utilization by adaptively allocating a portion of the available spectrum according to the traffic demands. If successfully implemented, it can offer huge spectrum gains as compared to WDM networks. Our ILP_2 formulation and CHR were unable to handle large number of commodities in networks of practical size. Therefore, there is an urgent need for other approaches or heuristics that can handle these problems. We expect our ILP_1 formulation to act as a benchmark for these heuristics.

Another possible future work will be investigating techniques to improve the resiliency of the OFDM networks. To our best knowledge, none of the researchers have studied the area of path-protection in OFDM networks. It would be interesting to study dedicated or shared path protection schemes for OFDM networks.

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APPENDIX: RESULT DATA

This appendix section contains the data from our simulation experiments in a tabulated format. The tables have been arranged in the order of increasing number of nodes. The formulations may be assumed to use 3 paths, unless otherwise specified.

8 nodes-8 commodities								
CHR-3paths		ILP1		ILP2-3paths		ILP1 time/ CHR time	ILP1 obj / CHR obj	CHR time/ ILP2 time
Time	Obj Val	Time	Obj Val	Time	Obj Val			
0.12	85	0.15	85	0.10	85	1.28	1	1.16
0.28	135	0.13	135	0.40	135	0.48	1	0.69
0.33	144	0.39	144	0.69	144	1.17	1	0.48
0.42	138	0.79	138	0.65	138	1.90	1	0.64
0.32	38	0.66	38	0.31	38	2.05	1	1.04
0.11	48	0.30	48	0.78	48	2.79	1	0.14
0.32	48	0.15	48	0.38	48	0.48	1	0.84
0.96	51	0.27	51	0.43	51	0.28	1	2.23
0.14	51	0.57	51	0.48	51	4.10	1	0.29
0.34	38	0.88	38	0.30	38	2.56	1	1.13
0.12	47	0.27	47	0.72	47	2.20	1	0.17
0.40	48	0.19	48	0.26	48	0.47	1	1.57
0.28	49	0.58	49	0.24	49	2.10	1	1.17
0.10	80	0.39	80	0.49	80	3.70	1	0.21
0.53	49	0.60	49	0.49	49	1.12	1	1.08
0.36	47	0.65	47	0.39	47	1.82	1	0.91
0.23	48	0.21	48	0.24	48	0.94	1	0.94
0.36	49	0.23	49	0.32	49	0.63	1	1.13
0.97	51	0.52	51	0.32	51	0.53	1	3.01
0.57	38	0.22	38	0.48	38	0.39	1	1.20
0.20	63	0.48	63	0.85	63	2.40	1	0.23
0.62	48	0.26	48	0.33	48	0.42	1	1.91
0.54	72	0.19	72	0.43	72	0.35	1	1.26
0.58	51	0.24	51	0.34	51	0.42	1	1.70
0.46	38	0.15	38	0.30	38	0.32	1	1.53

Topologies	8nodes-12commodities													
	CHR-3paths		ILP2-1path		ILP2-2paths		ILP2-3paths		ILP2-4paths		1/CHR obj	2/CHR obj	3/CHR obj	4/CHR obj
	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val				
T0	1.24	77	0.11	77	0.17	77	0.52	77	0.32	77	1.00	1.00	1	1.00
	16.32	124	1.00	124	0.70	124	0.66	124	0.14	124	1.00	1.00	1	1.00
	0.36	154	0.12	154	0.13	154	0.10	154	0.39	154	1.00	1.00	1	1.00
	0.54	117	0.96	117	0.99	117	0.14	117	0.17	117	1.00	1.00	1	1.00
	11.67	157	0.86	157	0.92	157	0.66	157	0.24	157	1.00	1.00	1	1.00
T1	0.36	49	0.65	112	0.76	49	0.18	49	0.19	49	2.29	1.00	1	1.00
	39.81	79	0.34	105	0.36	83	0.33	79	5.71	78	1.33	1.05	1	0.99
	7.29	76	0.87	120	0.22	83	0.71	76	6.67	72	1.58	1.09	1	0.95
	10.70	84	0.74	128	0.32	91	0.43	84	10.13	84	1.52	1.08	1	1.00
	4.26	75	0.29	89	0.15	75	0.14	75	0.84	75	1.19	1.00	1	1.00
T2	0.99	49	0.61	49	0.12	49	0.54	49	0.13	49	1.00	1.00	1	1.00
	0.27	76	0.74	80	0.45	76	0.48	76	0.21	76	1.05	1.00	1	1.00
	2.38	74	0.11	74	0.13	74	0.13	74	2.26	74	1.00	1.00	1	1.00
	0.44	72	0.96	96	0.14	74	0.15	72	0.67	72	1.33	1.03	1	1.00
	0.53	74	0.77	94	0.13	74	0.17	74	1.23	74	1.27	1.00	1	1.00
T3	0.65	49	0.57	88	0.75	49	0.84	49	0.16	49	1.80	1.00	1	1.00
	3.88	57	0.99	94	0.99	57	0.19	57	0.20	54	1.65	1.00	1	0.95
	2.83	60	0.97	82	0.13	60	0.10	60	0.29	60	1.37	1.00	1	1.00
	3.55	58	0.74	122	0.91	72	0.15	58	0.64	58	2.10	1.24	1	1.00
	4.99	66	0.68	82	0.12	66	0.19	66	0.77	66	1.24	1.00	1	1.00
T4	20.60	77	0.74	88	0.38	77	1.15	77	2.59	77	1.14	1.00	1	1.00
	1.56	76	0.11	113	0.11	84	0.21	76	0.43	76	1.49	1.11	1	1.00
	2.54	68	0.72	97	0.10	68	0.40	68	0.72	68	1.43	1.00	1	1.00
	75.59	93	0.38	128	0.81	102	0.95	93	11.82	93	1.38	1.10	1	1.00
	0.84	64	0.59	76	0.93	67	0.23	64	0.37	64	1.19	1.05	1	1.00

Topologies	8 Node - 12 Commodity								
	ILP1		CHR-3paths		ILP2-3paths		ILP1 time / CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	0.61	77	1.14	77	0.52	77	0.54	1.00	2.21
	1.28	124	16.31	124	0.66	124	0.08	1.00	24.63
	2.63	154	0.33	154	0.10	154	8.07	1.00	3.16
	0.39	117	0.48	117	0.14	117	0.81	1.00	3.52
	2.48	157	11.57	157	0.66	157	0.21	1.00	17.50
T1	1.24	49	0.36	49	0.18	49	3.46	1.00	2.04
	13.73	78	39.81	79	0.33	79	0.35	0.99	120.29
	23.12	72	7.29	76	0.71	76	3.17	0.95	10.34
	12.89	83	10.70	84	0.43	84	1.20	0.99	25.07
	6.30	75	4.26	75	0.14	75	1.48	1.00	29.61
T2	0.28	49	0.99	49	0.54	49	0.29	1.00	1.85
	0.30	76	0.27	76	0.48	76	1.12	1.00	0.57
	13.66	74	2.38	74	0.13	74	5.74	1.00	18.07
	1.51	71	0.44	72	0.15	72	3.47	0.99	2.96
	1.94	74	0.53	74	0.17	74	3.67	1.00	3.08
T3	3.48	49	0.65	49	0.84	49	5.37	1.00	0.77
	1.39	54	3.88	57	0.19	57	0.36	0.95	20.76
	9.37	60	2.83	60	0.10	60	3.31	1.00	27.12
	1.31	58	3.55	58	0.15	58	0.37	1.00	23.28
	1.74	64	4.99	66	0.19	66	0.35	0.97	26.87
T4	15.73	77	20.60	77	1.15	77	0.76	1.00	17.92
	0.81	76	1.56	76	0.21	76	0.52	1.00	7.44
	4.80	68	2.54	68	0.40	68	1.89	1.00	6.41
	1088.79	93	75.59	93	0.95	93	14.40	1.00	79.43
	0.87	64	0.84	64	0.23	64	1.03	1.00	3.59

	8nodes-15commodities													
Topologies	CHR-3paths		ILP2-1path		ILP2-2paths		ILP2-3paths		ILP2-4paths		1/CHR obj	2/CHR obj	3/CHR obj	4/CHR obj
	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val				
T0	0.78	111	0.57	117	0.11	111	0.46	111	0.49	111	1.05	1.00	1	1.00
	15.26	189	0.10	189	0.41	189	0.70	189	1.34	189	1.00	1.00	1	1.00
	3600.67	174	0.46	175	0.84	175	1.81	174	1.90	174	1.01	1.01	1	1.00
	804.84	243	0.50	243	0.82	243	1.73	243	2.11	243	1.00	1.00	1	1.00
	86.65	195	0.14	195	0.20	195	0.83	195	0.73	195	1.00	1.00	1	1.00
T1	135.96	75	0.33	110	0.75	91	0.46	75	3.37	65	1.47	1.21	1	0.87
	244.67	88	0.18	126	0.44	93	3.50	88	13.71	87	1.43	1.06	1	0.99
	14.10	54	0.21	90	0.80	58	0.28	54	0.79	54	1.67	1.07	1	1.00
	61.10	85	0.28	109	0.31	86	3.64	85	6.18	85	1.28	1.01	1	1.00
	92.21	87	0.35	163	0.48	87	2.40	87	7.32	87	1.87	1.00	1	1.00
T2	3.61	80	0.31	92	0.76	80	0.57	80	0.53	75	1.15	1.00	1	0.94
	56.82	84	0.21	121	0.94	84	2.15	84	2.76	84	1.44	1.00	1	1.00
	120.62	67	0.23	88	0.53	71	0.21	67	0.88	67	1.31	1.06	1	1.00
	344.30	83	0.15	99	0.37	85	0.29	83	0.93	81	1.19	1.02	1	0.98
	18.35	83	0.24	94	0.10	83	0.56	83	0.65	78	1.13	1.00	1	0.94
T3	9.80	76	0.43	117	0.11	91	0.47	76	1.74	65	1.54	1.20	1	0.86
	24.80	80	0.36	134	0.78	80	0.75	80	3.25	80	1.68	1.00	1	1.00
	9.65	58	0.18	67	0.95	62	0.38	58	1.23	58	1.16	1.07	1	1.00
	14.30	83	0.24	93	0.11	85	0.54	83	0.78	83	1.12	1.02	1	1.00
	11.22	78	0.22	120	0.11	78	0.80	78	1.90	78	1.54	1.00	1	1.00
T4	3.59	91	0.33	111	0.76	91	0.23	91	0.92	91	1.22	1.00	1	1.00
	101.97	112	0.30	119	0.60	119	4.71	112	27.45	112	1.06	1.06	1	1.00
	20.45	64	0.22	86	0.12	64	0.94	64	3.96	64	1.34	1.00	1	1.00
	20.92	85	0.26	165	0.16	85	3.35	85	5.30	85	1.94	1.00	1	1.00
	37.53	81	0.24	118	0.20	85	0.79	81	1.97	81	1.46	1.05	1	1.00

Topologies	8 Node - 15 Commodity								
	ILP1		CHR-3paths		ILP2-3paths		ILP1 time / CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	11.83	111	0.78	111	0.46	111	15.12	1.00	1.71
	6.85	189	15.26	189	0.70	189	0.45	1.00	21.86
	13.96	174	3600.67	174	1.81	174	0.00	1.00	1994.56
	12.89	243	804.84	243	1.73	243	0.02	1.00	465.55
	6.33	195	86.65	195	0.83	195	0.07	1.00	103.90
T1	32.96	65	135.96	75	0.46	75	0.24	0.87	295.72
	23.27	86	244.67	88	3.50	88	0.10	0.98	69.92
	13.11	54	14.10	54	0.28	54	0.93	1.00	49.83
	92.14	85	61.10	85	3.64	85	1.51	1.00	16.81
	42.98	87	92.21	87	2.40	87	0.47	1.00	38.45
T2	3.24	75	3.61	80	0.57	80	0.90	0.94	6.34
	1108.65	84	56.82	84	2.15	84	19.51	1.00	26.39
	5.56	67	120.62	67	0.21	67	0.05	1.00	586.80
	5.61	81	344.30	83	0.29	83	0.02	0.98	1199.93
	8.18	78	18.35	83	0.56	83	0.45	0.94	33.05
T3	138.44	65	9.80	76	0.47	76	14.12	0.86	20.76
	35.53	80	24.80	80	0.75	80	1.43	1.00	33.02
	7.51	58	9.65	58	0.38	58	0.78	1.00	25.59
	13.33	83	14.30	83	0.54	83	0.93	1.00	26.40
	10.15	78	11.22	78	0.80	78	0.90	1.00	14.02
T4	61.28	91	3.59	91	0.23	91	17.07	1.00	15.95
	3600.10	112	101.97	112	4.71	112	35.31	1.00	21.63
	24.85	64	20.45	64	0.94	64	1.21	1.00	21.69
	43.16	85	20.92	85	3.35	85	2.06	1.00	6.25
	25.20	81	37.53	81	0.79	81	0.67	1.00	47.26

Topologies	12 Node - 8 Commodity								
	ILP1		CHR-3 paths		ILP2-3 paths		ILP1 time/ CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	0.13	48	0.53	48	0.54	48	0.25	1.00	0.98
	0.56	44	0.14	44	0.26	44	3.97	1.00	0.53
	0.12	45	0.23	45	0.21	45	0.50	1.00	1.08
	0.16	48	0.31	48	0.26	48	0.53	1.00	1.20
	0.16	35	0.18	35	0.40	35	0.88	1.00	0.46
T1	0.19	48	0.39	48	0.40	48	0.49	1.00	0.99
	0.62	44	0.39	44	0.39	44	1.60	1.00	0.99
	0.97	45	0.24	45	0.25	45	3.98	1.00	0.99
	0.11	48	0.24	48	0.20	48	0.43	1.00	1.21
	0.30	35	0.48	35	0.73	35	0.62	1.00	0.67
T2	0.37	48	0.37	48	0.41	48	1.02	1.00	0.90
	0.18	44	0.28	44	0.28	44	0.64	1.00	1.00
	0.13	45	0.29	45	0.23	45	0.46	1.00	1.24
	0.50	48	0.87	48	0.73	48	0.57	1.00	1.19
	0.77	35	0.28	41	0.54	41	2.72	0.85	0.53
T3	0.64	48	0.39	48	0.37	48	1.65	1.00	1.06
	0.13	44	0.39	44	0.30	44	0.33	1.00	1.31
	0.59	45	0.32	45	0.27	45	1.82	1.00	1.20
	0.20	48	0.35	48	0.34	48	0.58	1.00	1.04
	1.17	35	0.32	35	0.29	35	3.71	1.00	1.10
T4	1.62	61	0.46	61	0.64	61	3.53	1.00	0.72
	0.24	44	0.22	44	0.24	44	1.09	1.00	0.90
	0.23	45	0.25	45	0.33	45	0.95	1.00	0.74
	17.35	81	5.86	81	0.14	81	2.96	1.00	40.68
	1.11	41	0.92	42	0.54	42	1.21	0.98	1.70

Topologies	12 node-12 commodities															
	CHR		ILP1		ILP2-1paths		ILP2-2paths		ILP2-3paths		ILP2-4paths		1/ILP1 obj	2/ILP1 obj	3/ilp1 obj	4/ilp1 obj
	Time	Obj	Time	Obj	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val				
T0	7.00	52	13.63	49	0.55	97	0.12	52	0.13	52	0.14	49	1.98	1.06	1.06	1.00
	43.53	84	61.42	84	0.20	87	0.43	84	0.11	84	0.16	84	1.04	1.00	1.00	1.00
	3.29	63	115.82	55	0.21	120	0.41	63	0.78	63	0.85	55	2.18	1.15	1.15	1.00
	9.65	58	69.37	58	0.15	106	0.38	70	0.65	58	0.35	58	1.83	1.21	1.00	1.00
	0.32	50	6.49	50	0.21	95	0.26	50	0.54	50	0.14	50	1.90	1.00	1.00	1.00
T1	4.60	60	71.82	60	0.55	75	0.94	66	0.15	60	0.18	60	1.25	1.10	1.00	1.00
	13.59	74	44.72	74	0.19	117	0.63	84	0.16	74	0.14	74	1.58	1.14	1.00	1.00
	14.11	52	1.75	50	0.17	96	0.50	59	0.67	52	0.86	50	1.92	1.18	1.04	1.00
	9.88	55	347.44	55	0.36	69	0.70	56	0.16	55	0.33	55	1.25	1.02	1.00	1.00
	0.12	50	0.91	50	0.23	84	0.45	50	0.60	50	0.14	50	1.68	1.00	1.00	1.00
T2	0.32	49	12.21	49	0.29	49	0.35	49	0.63	49	0.46	49	1.00	1.00	1.00	1.00
	1.73	84	12.63	84	0.23	97	0.29	84	0.67	84	0.15	84	1.15	1.00	1.00	1.00
	6.20	53	68.34	50	0.25	134	0.53	70	0.72	53	0.28	52	2.68	1.40	1.06	1.04
	8.45	57	284.52	55	0.34	70	0.93	57	0.78	57	0.14	55	1.27	1.04	1.04	1.00
	0.14	50	11.45	50	0.27	50	0.37	50	0.77	50	0.45	50	1.00	1.00	1.00	1.00
T3	2.36	64	174.23	58	0.36	97	0.49	70	0.17	64	0.35	58	1.67	1.21	1.10	1.00
	7.31	54	222.66	54	0.20	86	0.58	54	0.23	54	0.85	54	1.59	1.00	1.00	1.00
	0.17	50	8.816	50	0.26	119	0.65	65	0.92	50	0.16	50	2.38	1.30	1.00	1.00
	11.94	55	574.8	51	0.29	83	0.55	57	0.18	55	0.65	55	1.63	1.12	1.08	1.08
	0.15	50	3.48	50	0.21	57	0.20	50	0.62	50	0.94	50	1.14	1.00	1.00	1.00
T4	3.78	64	152.14	64	0.41	100	0.13	87	0.14	64	0.53	64	1.56	1.36	1.00	1.00
	6.89	74	64.95	74	0.26	99	0.30	75	0.13	74	0.16	74	1.34	1.01	1.00	1.00
	63.97	113	3600	112	0.18	116	0.61	116	0.33	113	3.26	112	1.04	1.04	1.01	1.00
	0.72	70	15.12	70	0.20	70	0.41	70	0.39	70	0.11	70	1.00	1.00	1.00	1.00
	0.37	55	92.76	55	0.18	68	0.49	66	0.61	55	0.11	55	1.24	1.20	1.00	1.00

Topologies	12 Node - 15 Commodity								
	ILP1		CHR		ILP2		ILP1 time/ CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	58.24	48	0.13	48	0.14	48	448.00	1.00	0.93
	9.78	52	4.68	52	0.96	52	2.09	1.00	4.88
	1271.197	73	10.4	73	0.39	73	122.23	1.00	26.67
	215.98	67	26.21	67	0.76	67	8.24	1.00	34.49
	3420.76	69	25.74	72	0.79	72	132.90	0.96	32.58
T1	8.69	48	0.62	48	0.15	48	14.02	1.00	4.13
	31.18	61	36.73	63	0.18	63	0.85	0.97	204.06
	54.93	73	4.98	76	0.42	76	11.03	0.96	11.86
	173.32	60	5.73	60	0.32	60	30.25	1.00	17.91
	34.67	56	2.64	63	0.46	63	13.13	0.89	5.74
T2	30.83	48	0.4	48	0.4	48	77.08	1.00	1.00
	33.6	45	5.55	52	0.39	52	6.05	0.87	14.23
	3600.25	80	15.46	80	1.73	80	232.88	1.00	8.94
	3600.12	67	158.91	67	2.95	67	22.66	1.00	53.87
	1347.11	64	134.8	71	2.79	71	9.99	0.90	48.32
T3	58.15	48	54.53	50	0.6	50	1.07	0.96	90.88
	8.24	52	10.67	52	0.48	52	0.77	1.00	22.23
	460.86	73	14.49	77	0.85	77	31.81	0.95	17.05
	1041.3	58	183.71	63	1.76	63	5.67	0.92	104.38
	3600.11	67	78.8	70	1.19	70	45.69	0.96	66.22
T4	3600.42	81	5.38	99	0.57	99	669.22	0.82	9.44
	178.8	52	5.32	52	0.55	52	33.61	1.00	9.67
	3600.27	92	171.1	94	1.94	94	21.04	0.98	88.20
	88.43	104	3600.41	104	2.91	104	0.02	1.00	1237.25
	2572.63	105	3600.62	105	12.15	105	0.71	1.00	296.35

Topologies	12 nodes -15 commodities													
	ILP1		ILP2-1path		ILP2-2paths		ILP2-3paths		ILP2-4paths		1/ILP1 obj	2/ILP1 obj	3/ilp1 obj	4/ilp1 obj
	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val				
T0	58.24	48	0.41	58	0.86	48	0.14	48	0.22	48	1.21	1.00	1.00	1.00
	9.78	52	0.20	64	0.86	60	0.96	52	0.15	52	1.23	1.15	1.00	1.00
	1271.20	73	0.18	80	0.70	73	0.39	73	1.67	73	1.10	1.00	1.00	1.00
	215.98	67	0.83	103	0.24	67	0.76	67	1.82	67	1.54	1.00	1.00	1.00
	3420.76	69	0.24	155	0.18	86	0.79	72	3.68	72	2.25	1.25	1.04	1.04
T1	8.69	48	0.30	73	0.75	48	0.15	48	0.28	48	1.52	1.00	1.00	1.00
	31.18	61	0.28	100	0.12	63	0.18	63	0.77	63	1.64	1.03	1.03	1.03
	54.93	73	0.16	112	0.20	81	0.42	76	0.47	74	1.53	1.11	1.04	1.01
	173.32	60	0.20	123	0.16	70	0.32	60	0.54	60	2.05	1.17	1.00	1.00
	34.67	56	0.24	86	0.64	64	0.46	63	0.15	60	1.54	1.14	1.13	1.07
T2	30.83	48	0.38	60	0.56	48	0.4	48	0.15	48	1.25	1.00	1.00	1.00
	33.60	45	0.15	67	0.11	56	0.39	52	0.21	48	1.49	1.24	1.16	1.07
	4341.37	80	0.19	93	0.20	81	1.73	80	0.88	80	1.16	1.01	1.00	1.00
	1376.59	63	0.24	89	0.15	75	2.95	67	1.52	67	1.41	1.19	1.06	1.06
	1347.11	64	0.34	126	0.60	78	2.79	71	1.75	70	1.97	1.22	1.11	1.09
T3	58.15	48	0.30	57	0.20	57	0.6	50	0.28	48	1.19	1.19	1.04	1.00
	8.24	52	0.24	96	0.20	63	0.48	52	0.66	56	1.85	1.21	1.00	1.08
	460.86	73	0.32	119	0.28	83	0.85	77	1.13	76	1.63	1.14	1.05	1.04
	1041.30	58	0.15	82	0.90	71	1.76	63	1.39	58	1.41	1.22	1.09	1.00
	3600.00	67	0.24	118	0.16	83	1.19	70	1.41	67	1.76	1.24	1.04	1.00
T4	3600.00	81	0.36	112	0.29	105	0.57	99	2.39	83	1.38	1.30	1.22	1.02
	178.80	52	0.19	75	0.14	52	0.55	52	0.31	52	1.44	1.00	1.00	1.00
	3600.00	92	0.24	122	0.38	94	1.94	94	4.91	94	1.33	1.02	1.02	1.02
	88.43	104	0.11	116	0.32	106	2.91	104	3.77	104	1.12	1.02	1.00	1.00
	2572.63	105	0.93	142	1.47	105	12.15	105	33.62	105	1.35	1.00	1.00	1.00

12 node-20 commodity								
Topologies	ILP2-1path		ILP2-2paths		ILP2-3paths		ILP2-4path	
	Time	Obj Val	Time	Obj Val	Time	Obj Val	Time	Obj Val
T0	0.55	117	0.55	68	1.80	64	5.41	64
	0.35	98	0.27	83	0.50	83	4.43	83
	0.18	87	0.31	81	1.46	57	1.33	53
	0.56	151	2.24	88	37.37	81	79.67	75
	0.26	132	1.48	108	2.28	85	10.93	84
T1	0.27	82	1.84	76	3.94	66	10.82	66
	0.32	104	0.75	83	1.58	77	4.70	77
	0.26	105	2.35	83	0.95	70	2.84	61
	0.43	162	8.50	84	6.34	77	19.30	77
	0.16	129	0.67	83	0.33	83	0.58	83
T2	0.44	114	1.44	82	2.92	68	5.50	68
	0.30	120	4.67	90	3.98	77	6.37	70
	0.31	137	0.51	81	0.74	67	2.78	56
	0.26	101	1.99	80	8.75	73	74.79	72
	0.27	132	0.39	88	1.50	83	3.84	83
T3	0.31	87	0.96	69	0.76	64	2.82	64
	0.26	101	0.62	83	4.68	83	8.93	77
	0.43	109	0.61	81	1.79	64	2.45	64
	0.67	136	6.73	83	9.54	82	56.50	82
	0.32	113	0.87	87	1.69	83	4.28	83
T4	0.37	106	2.15	95	1.60	92	2.93	92
	0.25	106	0.35	93	1.23	83	1.70	83
	0.28	84	0.76	75	0.57	69	4.00	69
	0.88	181	1.78	115	10.13	95	84.99	95
	0.31	139	0.60	93	0.60	77	1.81	77

The maximum number of commodities to be handled by ILP2 - 12 node 2 paths

12 node-20 commodity-2path	
Time	Obj Val
0.55	68
0.27	83
0.31	81
2.24	88
1.48	108
1.84	76
0.75	83
2.35	83
8.50	84
0.67	83
1.44	82
4.67	90
0.51	81
1.99	80
0.39	88
0.96	69
0.62	83
0.61	81
6.73	83
0.87	87
2.15	95
0.35	93
0.76	75
1.78	115
0.60	93

12 node-30 commodity-2path	
Time	Obj Val
427.20	92
8.70	85
27.52	99
76.54	97
149.21	107
38.88	92
35.24	108
85.74	108
169.17	108
192.92	107
21.95	93
18.32	89
3600.14	94
480.52	121
61.13	91
20.76	91
5.48	105
3600.31	104
64.33	109
4.38	89
720.25	123
87.43	152
3600.13	133
62.10	158
444.14	139

12 node-15 commodity-2path	
Time	Obj Val
0.86	48
0.86	60
0.70	73
0.24	67
0.18	86
0.75	48
0.12	63
0.20	81
0.16	70
0.64	64
0.56	48
0.11	56
0.20	81
0.15	75
0.60	78
0.20	57
0.20	63
0.28	83
0.90	71
0.16	83
0.29	105
0.14	52
0.38	94
0.32	106
1.47	105

The maximum number of commodities to be handled by ILP2 - 12 node 3 paths

12 node-20 commodity-3path	
Time	Obj Val
1.77	64
0.49	83
1.49	57
37.37	81
2.27	85
3.94	66
1.57	77
0.951	70
6.34	77
0.325	83
2.91	68
3.98	77
0.74	67
8.74	73
1.49	83
0.76	64
4.67	83
1.789	64
9.54	82
1.69	83
1.6	92
1.23	83
0.56	69
10.12	95
0.6	77

12 node-18 commodity-3path	
Time	Obj Val
6.25	87
3.56	66
0.54	48
0.74	56
4.41	56
4.29	82
1.36	64
1.13	56
1.11	63
4.25	63
1.85	84
1.68	78
0.17	47
0.58	50
2.36	60
3.86	90
0.53	77
0.62	49
0.54	57
0.81	63
0.22	82
2.206	86
0.86	66
0.86	55
0.15	66

12 node-15 commodity-3path	
Time	Obj Val
0.14	48
0.96	52
0.39	73
0.76	67
0.79	72
0.15	48
0.18	63
0.42	76
0.32	60
0.46	63
0.4	48
0.39	52
1.73	80
2.95	67
2.79	71
0.6	50
0.48	52
0.85	77
1.76	63
1.19	70
0.57	99
0.55	52
1.94	94
2.91	104
12.15	105

14 Node - Deutsche Telekom(DT) network

14 nodes 12 commodities					
ILP1		CHR		ILP2	
Time	Obj Val	Time	Obj Val	Time	Obj Val
1.17	48	0.89	70	0.72	70
6.48	50	0.47	54	0.77	54
7.19	66	1.23	68	0.35	68
8.67	84	0.16	84	0.68	84
57.99	58	0.15	58	0.56	58
23.80	50	0.29	63	0.67	63
0.57	50	0.46	50	0.37	50
11.76	80	0.24	89	0.34	89
23.33	66	0.14	66	0.39	66
77.98	59	0.16	59	0.58	59

14 nodes 15 commodities					
ILP1		CHR		ILP2	
Time	Obj Val	Time	Obj Val	Time	Obj Val
26.12	51	0.62	55	0.79	55
32.37	44	0.62	44	0.66	44
16.22	94	1.39	112	0.85	112
427.55	67	29.59	67	0.14	67
9.37	50	6.63	56	0.28	56
15.68	84	2.82	100	0.63	100
17.82	67	0.45	67	0.10	67
11.36	45	17.29	57	0.25	57
22.54	56	1.74	63	0.13	63
78.22	86	16.21	86	0.87	86

14 nodes 20 commodities					
ILP1		CHR		ILP2	
Time	Obj Val	Time	Obj Val	Time	Obj Val
82.41	45	366.2808	67	0.33	67
449.11	67	29.96407	75	0.70	75
227.98	70	6.532284	75	0.45	75
3600.93	100	48.97398	102	0.79	102
2254.39	75	3600.363	75	0.56	75
1573.12	71	8.32443	92	0.42	92
3600.80	87	46.44765	89	2.73	89
690.62	76	3600.332	107	0.96	107
915.77	83	81.39403	87	0.97	87
974.45	64	104.9146	78	0.79	78

14 nodes 25 commodities					
ILP1		CHR		ILP2	
Time	Obj Val	Time	Obj Val	Time	Obj Val
				0.21	75
				1.51	116
				1.31	73
				0.38	71
				6.47	116
				0.69	86
				1.46	122
				7.30	75
				0.77	83
				1.54	78

14 nodes 35 commodities					
ILP1		CHR		ILP2	
Time	Obj Val	Time	Obj Val	Time	Obj Val
				52.42	81
				12.33	85
				387.77	95
				19.27	103
				47.58	113
				17.62	128
				3.25	119
				3.84	123
				3600.48	92
				321.75	127

14 nodes 40 commodities					
ILP1		CHR		ILP2	
Time	Obj Val	Time	Obj Val	Time	Obj Val
				16.11	149
				37.13	114
				395.86	113
				50.86	150
				526.33	98
				3600.28	128
				264.41	106
				3600.39	122
				204.48	114
				3600.68	134

Topologies	15 Node - 8Commodity								
	ILP1		CHR - 3 paths		ILP2 -3 paths		ILP1 time/ CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	0.83	54	0.89	54	0.80	54	0.93	1.00	1.11
	0.15	49	0.50	77	0.38	77	0.30	0.64	1.31
	0.16	48	0.19	48	0.57	48	0.82	1.00	0.34
	0.38	50	0.25	50	0.32	50	1.52	1.00	0.78
	2.22	73	0.21	73	0.66	73	10.47	1.00	0.32
T1	3.18	51	0.95	52	0.65	52	3.34	0.98	1.46
	2.39	75	0.20	81	0.53	81	12.22	0.93	0.37
	0.78	91	0.75	91	0.67	91	1.03	1.00	1.13
	1.63	79	0.12	79	0.52	79	13.74	1.00	0.23
	1.56	47	0.14	54	0.48	54	11.24	0.87	0.29
T2	0.26	43	0.97	47	0.67	47	0.27	0.91	1.44
	0.31	49	0.37	49	0.32	49	0.83	1.00	1.17
	0.91	48	0.23	48	0.32	48	3.88	1.00	0.73
	2.56	57	0.90	71	0.74	71	2.83	0.80	1.22
	0.21	38	0.48	38	0.28	38	0.44	1.00	1.73
T3	0.35	43	0.31	43	0.43	43	1.13	1.00	0.72
	1.33	49	1.56	81	0.55	81	0.85	0.60	2.85
	0.69	48	0.76	48	0.32	48	0.91	1.00	2.39
	3.75	57	0.76	57	0.75	57	4.96	1.00	1.01
	0.12	38	0.26	38	0.29	38	0.48	1.00	0.90
T4	0.49	43	0.65	43	0.45	43	0.75	1.00	1.44
	0.61	49	0.22	49	0.22	49	2.75	1.00	0.98
	0.12	48	0.25	48	0.28	48	0.50	1.00	0.89
	5.80	57	0.92	71	0.43	71	6.30	0.80	2.16
	0.61	38	0.26	38	0.21	38	2.41	1.00	1.22

Topologies	15 Node - 12 Commodity								
	ILP1		CHR - 3 paths		ILP2 -3 paths		ILP1 time/ CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	6.37	68	0.24	68	0.61	68	26.44	1.00	0.40
	30.65	97	368.34	97	0.93	97	0.08	1.00	396.60
	8.60	84	0.42	86	0.60	86	20.37	0.98	0.70
	3.64	58	0.12	58	0.60	58	31.37	1.00	0.19
	14.72	47	0.31	47	0.52	47	47.34	1.00	0.60
T1	6.18	78	0.72	78	0.17	78	8.62	1.00	4.24
	7.47	94	1.39	94	0.11	94	5.38	1.00	12.88
	10.83	56	0.13	56	0.14	56	85.77	1.00	0.91
	32.23	66	1.47	67	0.11	67	21.90	0.99	13.12
	1.16	45	0.35	45	0.65	45	3.27	1.00	0.55
T2	31.27	70	2.18	70	0.11	70	14.34	1.00	20.19
	3600.65	76	8.91	78	0.22	78	404.17	0.97	40.45
	15.57	54	0.89	84	0.68	84	17.55	0.64	1.31
	8.82	48	0.18	48	0.18	48	49.60	1.00	1.00
	4.68	45	0.45	45	0.52	45	10.47	1.00	0.86
T3	706.14	62	0.54	62	0.14	62	1308.77	1.00	3.90
	3600.63	65	13.54	66	0.94	66	265.88	0.98	14.37
	11.96	43	0.19	43	0.67	43	61.53	1.00	0.29
	1.67	48	0.11	48	0.96	48	15.10	1.00	0.12
	0.55	45	0.43	45	0.37	45	1.28	1.00	1.15
T4	6.55	72	0.77	72	0.12	72	8.54	1.00	6.23
	5.66	49	0.70	75	0.71	75	8.13	0.65	0.98
	21.68	83	8.99	83	0.24	83	2.41	1.00	37.23
	5.37	58	0.63	67	0.73	67	8.58	0.87	0.86
	5.28	45	0.62	45	0.85	45	8.48	1.00	0.74

Topologies	15 Node - 15Commodity								
	ILP1		CHR - 3 paths		ILP2 -3 paths		ILP1 time/ CHR time	ILP1 obj / CHR obj	CHR time / ILP2 time
	Time	Obj	Time	Obj	Time	Obj			
T0	27.64	92	0.31	92	0.10	92	89.16	1.00	3.10
	512.47	71	0.90	73	0.11	73	569.41	0.97	8.18
	26.94	62	4.17	70	0.19	70	6.46	0.89	21.95
	0.90	59	0.18	59	0.95	59	5.00	1.00	0.19
	499.14	82	3.39	97	0.32	97	147.24	0.85	10.59
T1	63.91	47	2.40	47	0.42	47	26.59	1.00	5.66
	676.70	67	26.83	73	1.40	73	25.22	0.92	19.13
	1301.76	65	27.79	65	0.67	65	46.85	1.00	41.48
	332.27	58	3.91	59	0.41	59	84.90	0.98	9.50
	1496.92	66	443.96	74	4.83	74	3.37	0.89	91.89
T2	363.64	47	575.11	71	0.80	71	0.63	0.66	722.95
	3600.16	64	98.61	73	0.79	73	36.51	0.88	124.77
	2062.40	60	34.29	66	0.45	66	60.15	0.91	76.77
	2119.33	47	17.14	59	0.62	59	123.68	0.80	27.61
	2025.35	50	188.53	64	0.61	64	10.74	0.78	307.47
T3	58.24	47	0.21	47	0.14	47	275.89	1.00	1.56
	1858.14	67	14.40	75	0.82	75	129.02	0.89	17.49
	1966.55	63	8.33	66	0.46	66	236.22	0.95	18.21
	870.86	51	1.39	51	0.29	51	627.56	1.00	4.78
	76.80	47	1.29	47	0.12	47	59.39	1.00	11.04
T4	271.20	47	0.85	47	0.44	47	317.25	1.00	1.96
	87.90	85	21.28	85	1.41	85	4.13	1.00	15.10
	2132.52	66	0.90	83	0.23	83	2376.42	0.80	3.88
	936.00	59	45.41	61	0.31	61	20.61	0.97	147.62
	15.99	47	0.45	51	1.00	51	35.44	0.92	0.45

15 Node -20 Commodity	
ILP2	
Time	Obj
4.74	99
3.58	103
0.48	142
1.68	79
3.44	123
25.33	119
28.22	104
5.55	91
4.23	78
11.89	87
5.46	82
9.51	77
1.42	86
1.37	81
2.89	68
3.87	82
2.21	74
2.89	86
2.33	65
2.32	70
3.71	95
11.62	106
13.82	102
12.27	86
3.61	78

15 Node -25 Commodity	
ILP2	
Time	Obj
27.46	138
14.54	119
16.47	100
12.90	100
52.82	111
228.94	143
74.58	92
16.82	101
1.93	90
10.10	89
18.81	95
87.51	91
7.75	91
2.49	90
30.77	90
8.50	88
79.84	90
3.99	94
1.45	73
94.14	101
15.52	103
44.56	89
12.85	86
0.73	78
22.39	130

The maximum number of commodities to be handled by ILP2 - 12 node 3 paths

12 node-20 commodity-3path	
Time	Obj Val
1.77	64
0.49	83
1.49	57
37.37	81
2.27	85
3.94	66
1.57	77
0.951	70
6.34	77
0.325	83
2.91	68
3.98	77
0.74	67
8.74	73
1.49	83
0.76	64
4.67	83
1.789	64
9.54	82
1.69	83
1.6	92
1.23	83
0.56	69
10.12	95
0.6	77

12 node-18 commodity-3path	
Time	Obj Val
6.25	87
3.56	66
0.54	48
0.74	56
4.41	56
4.29	82
1.36	64
1.13	56
1.11	63
4.25	63
1.85	84
1.68	78
0.17	47
0.58	50
2.36	60
3.86	90
0.53	77
0.62	49
0.54	57
0.81	63
0.22	82
2.206	86
0.86	66
0.86	55
0.15	66

12 node-15 commodity-3path	
Time	Obj Val
0.14	48
0.96	52
0.39	73
0.76	67
0.79	72
0.15	48
0.18	63
0.42	76
0.32	60
0.46	63
0.4	48
0.39	52
1.73	80
2.95	67
2.79	71
0.6	50
0.48	52
0.85	77
1.76	63
1.19	70
0.57	99
0.55	52
1.94	94
2.91	104
12.15	105

VITA AUCTORIS

Arijit Paul was born at Kolkata, India in the year 1989. He passed his Secondary School Certificate examination in 2005 from GES HAL High School, Nasik, India. In 2007, he passed the Higher Secondary Certificate examination from GES HAL Junior College, Nasik, India. Later, he attended the University of Mumbai, where he was awarded the Bachelor of Engineering degree in Information Technology in the year 2011. He is currently a candidate for the Masters degree in Computer Science at the University of Windsor, Ontario and hopes to graduate in Spring 2014.