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〈論文〉

Price Competition in Mixed Duopoly with Increasing Cost Function

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Abstract

We analyze price competition under a mixed duopoly with homogeneous products and symmetric quadratic cost functions. We consider both the cases of domestic competitor (private firm) and foreign competitor, and all timings of pricing-sequential price setting with public leadership, sequential price setting with private leadership, and simultaneous price setting.

We arrive at the following main result: If the private firm is domestic, equilibrium price under sequential price setting with private leadership may exceed that under the other cases. If the private firm is foreign, such a price never appears at equilibrium.

JEL classification: D43, L32

Key words: mixed duopoly, price competition, Stackelberg.

Introduction

Mixed oligopolies, where welfare-maximizing public firms compete against profit-maximizing private firms, are common in some of the advanced, developing, and transitional countries. In many countries, the competition between public and private firms existed or still exists in many industries such as energy (electric power and natural gas), finance (banking, mortgage, and life insurance), transportation (railroad, airline, and overnight-delivery), telecommunications, education, and medical service[1]. From the seminal work of De Fraja and Delbono (1989), mixed oligopoly becomes one of the major topics in the theory of industrial organization. Recently, mixed oligopoly has been discussed in terms of both domestic competition and international competition. For example, Fjell and Pal (1996), Matsumura (2003), and Lu (2006) considers the case of foreign competitors.

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1) See the introduction section of Matsushima and Matsumura (2006) for the detailed example.

Many of the works in mixed oligopoly consider quantity competition and deals with the asymmetric linear cost function or the symmetric convex cost function[2)] . However, analyses with regard to price competition are relatively fewer than that on quantity competition[3)]. This paper tries to answer a part of left question.

In our analysis, we employ a model proposed by Dastidar (1995). He studies a price competition with homogeneous product markets under private oligopoly. He demonstrates that the equilibrium prices are continuous in a pure strategy, if the cost functions are symmetric between both firms. We switch the model to mixed duopoly. In other words, this paper analyzes the price competition in a homogeneous product market under a mixed duopoly. We consider the case where cost functions are symmetric between two firms and they are strictly convex[4)]. In our model, one private firm and one public (or privatized) firm exist. We consider both cases wherein the private firm is domestic and foreign because foreign firms might play a major role in some of the developing countries[5)].

The private firm maximizes its own profit. The public firm maximizes a weighted average of social welfare and its own profit[6)]. Since we are unsure about which firm is the first-mover in price competition[7)], we compare three timings of price setting: (1) (timing S) wherein both firms set those prices simultaneously. (2) (timing Pri) wherein the private firm sets its price first, followed by the public firm. We refer to this situation as "private price leadership." (3) (timing Pub) wherein the public firm sets its price first, followed by the private firm. We refer to this situation as "public price leadership."

We demonstrate that the equilibrium price under S has a range regardless of the private firm's origin and it equals that proposed by Dastidar (1995), despite the public firm existing in the market. We also find that if the private firm is domestic, the equilibrium price under

2) Pal (1998) and Lu (2006) provide examples of the former setting. De Fraja and Delbono (1989) and Delbono and Scarpa (1995) provide examples of the latter. Matsushima and Matsumura (2003) provides an example of the other type of cost function.

3) For example, Anderson, de Palma, and Thisse (1997), Matsumura and Matsushima (2004) and Ishibashi and Matsumura (2006) treat price competition. However, they consider differentiated products. Cremer, Marchand and Thisse (1991) deals with a price competition with homogenous products. However, in their model, each firm is differentiated by its location.

4) We employ a quadratic cost function.

5) We assume that the public firm is domestic.

6) For a rationalization of the objective, see Boš (1991) and Matsumura (1998). Using such an objective function, we can deal with several types of public firm.

7) In quantity competition, for example, Pal (1998) deals with the observable delay game in mixed oligopoly.

Pri is higher than that under *Pub* and may exceed the range of that under S under some condition. On the other hand, if the private firm is foreign, such pricing does not occur. In other words, if the competitor of the public firm is domestic, the existence of a public firm may hamper social welfare. This implies that the origin of the competitor is quite important. Even if the existence of public firms is justified by the reason of its objective of maximizing social welfare, the government of a country would benefit by limiting the primary role of the public firm in the market until domestic industry takes off.

This paper comprises five sections. Section 2 builds the model. Section 3 solves the equilibrium considering a domestic private firm. Section 4 analyzes the equilibrium with foreign private firm. Section 5 concludes the paper. We note that Section 3 is based on Ogawa and Kato (2006).

1. The Model

Suppose there is a homogeneous product market comprising one public firm (firm 0) and one private firm (firm 1). The demand function is given by $D(p) = a - p$, where a is positive and sufficiently large. The cost function is given by cq_i^2, where c is positive and q_i is the output of firm i, $i = 0, 1$. We do not assume any inefficiency of the public firm.

We introduce the following assumptions[8]:

Assumption

1. *Firms have to supply the demand that they face*[9].
2. *When both two firms choose the same price, they share the demand equally, that is, when they chooses the same price, p, each firm supplies* $\frac{1}{2}D(p)$ *respectively.*

The profits of firm i is given by

$$\pi_i = \begin{cases} p_i(a-p_i) - c(a-p_i)^2 & \text{if } p_i < p_j, \\ p_i\left\{\frac{1}{2}(a-p_i)\right\} - c\left\{\frac{1}{2}(a-p_i)\right\}^2 & \text{if } p_i = p_j, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

8) These assumptions are also considered in Dastidar (1997).

9) We note that our results are crucially depends on this assumption. Matsumura (2012) changes the assumption to “only the public firm has to supply the demand it faces.” They analyse a mixed duopoly with a domestic private firm (corresponding to section 3 of ours) and discuss for foreign private firm (section 4 of ours). They show that first best arises under the assumption, regardless of the origin of the private firm.

Domestic social welfare is given by

$$SW = \text{consumer's surplus} + \text{domestic producer's surplus},$$
$$= \begin{cases} \frac{1}{2}(q_0+q_1)^2 + \pi_0 + \pi_1, & \text{if firm 1 is domestic,} \\ \frac{1}{2}(q_0+q_1)^2 + \pi_0, & \text{otherwise.} \end{cases} \tag{2}$$

The objective function of the public firm U_0 and that of the private firm U_1 are given by

$$U_0 = \theta SW + (1-\theta)\pi_0, \tag{3}$$
$$U_1 = \pi_1. \tag{4}$$

where $\theta \in [0,1]$ indicates the share of ownership by states. In other words,

- If $\theta = 1$, the firm 0 is a purely public firm.
- If $\theta \in (0,1)$, the firm 0 is a partially privatized firm[10].
- If $\theta = 0$, the firm 0 is a perfectly privatized firm.

We consider the three types of the price competitions: simultaneous (S), sequential with private price leadership (Pri), and sequential with public price leadership (Pub). We employ the Nash equilibrium (at case "S") or the subgame perfect Nash equilibrium (SPNE; at cases "Pri" and "Pub") as the solution concept. In the following sections, we focus on the situation where each firm adopts pure strategies. We only consider the case $p_i \in [0,a]$ because the equilibrium must be in this region if it exists.

2. Equilibrium with a Domestic Private Firm

First we derive the best response correspondence of each firm.

Lemma 1

1. *The best response correspondence of firm 0 is given by*

$$BR_0^D(p_1) = \begin{cases} a\frac{2c+1-\theta}{2c+2-\theta}, & \text{if } p_1 > \sigma_1^D,\ \theta > \frac{1}{3}, \text{ and } c \geq \frac{2(1-\theta)^2}{3\theta-1}, \\ p_1 - \epsilon, & \text{if } p_1 \in \left(a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)}, a\frac{2c+1-\theta}{2c+2-\theta}\right], \text{ and } \begin{cases} \theta \leq \frac{1}{3}, \\ \theta > \frac{1}{3} \text{ and } c < \frac{2(1-\theta)^2}{3\theta-1}, \end{cases} \\ p_1 + \delta, & \text{if } p_1 < a\frac{(1-3\theta)c}{(1-3\theta)c+2(1-\theta)} \text{ and } \theta \leq \frac{1}{3}, \\ p_1, & \text{otherwise,} \end{cases}$$

10) To obtain the objective function for partial privatization, we follow Matsumura (1998).

$$\textit{where } \sigma_1^D = a\frac{c(1+\theta)+1-\theta+\sqrt{\frac{-\theta^3+2c\theta^2+3\theta+2c\theta-2}{2c+2-\theta}}}{c(1+\theta)+2}.$$

2. *The best response correspondence of firm* 1 *is given by*

$$BR_1^D(p_0) = \begin{cases} a\frac{2c+1}{2c+2}, & \textit{if } p_0 > a\frac{2c+1}{2c+2}, \\ p_0 - \epsilon, & \textit{if } p_0 \in (a\frac{3c}{3c+2}, a\frac{2c+1}{2c+2}], \\ p_0 + \delta, & \textit{if } p_0 < a\frac{c}{c+2}, \\ p_0, & \textit{otherwise.} \end{cases}$$

where δ (> 0) *is arbitrary, and* ϵ (> 0) *is sufficiently small.*

Proof. *See Appendix A.*

This lemma states the following:

- If the opponent firm sets a high price, the firm "undercuts," i.e., sets a lower price and take whole the demand.
- If the opponent firm sets a low price, the firm "pulls up," i.e., sets a higher price and supplies nothing.
- If the opponent firm sets a price not to be neither too low nor too high, the firm "follows," i.e., sets the same price so as to split the demand.

We note that the undercut incentive of the private firm is stronger than that of the public firm. In other words, the private firm is more aggressive than the public firm because the public firm considers the opponent's profits, whereas the private firm does not. On the other hand, the pull up incentive of the private firm is also stronger than the one of the public firm.

Using this lemma, we have the following proposition.

Proposition 1 *The equilibrium price is as follows:*

1. *If the price setting is simultaneous, the equilibrium price is continuous such that*

$$p_0^{S,D} = p_1^{S,D} \in \left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right].$$

2. *If the price setting is sequential with private leadership, the equilibrium price is unique such that*

$$p_0^{Pri,D} = p_1^{Pri,D} = \begin{cases} a\dfrac{c+1}{c+2}, & \textit{if} \begin{cases} \theta \le \theta_1, \ c \ge c_1, \\ \theta > \theta_1, \ c \ge c_2, \end{cases} \\ a\dfrac{c(1+\theta)+1-\theta+\sqrt{\frac{-\theta^3+2c\theta^2+3\theta+2c\theta-2}{2c+2-\theta}}}{c(1+\theta)+2}, & \textit{if } \theta > \theta_1, \ c \in [c_3, c_2), \\ a\dfrac{(3-\theta)c}{(3-\theta)c+2(1-\theta)}, & \textit{otherwise.} \end{cases}$$

$$\textit{where } \theta_1 = \frac{\sqrt{17}-3}{2},$$

$$c_1 = \frac{2(1-\theta)}{1+\theta},$$

$$c_2 = \frac{\theta^2-9\theta+2+\sqrt{\theta^4+14\theta^3+37\theta^2-4\theta+4}}{4\theta},$$

$$c_3 = \frac{2(\theta-1)^2}{3\theta-1}.$$

3. *If the price setting is sequential with public leadership, the equilibrium price is unique such that*

$$p_0^{Pub,D} = p_1^{Pub,D} = \begin{cases} a\dfrac{c(1+\theta)+1-\theta}{c(1+\theta)+2}, & \textit{if } c \ge \dfrac{2(1-\theta)}{1+\theta}, \\ a\dfrac{3c}{3c+2}, & \textit{otherwise.} \end{cases} \tag{5}$$

Proof. *See Appendix B.*

The focus of the proposition is as follows:

- At the equilibrium, both firms sets the same prices, i.e., $p_0 = p_1$ regardless of the timing.
- The equilibrium price at private price leadership may exceed not only that at public price leadership but also the range of the simultaneous price setting[11]: this results from the existence of a public or partially privatized firm[12].

The equilibrium price is not extremely high or low: If the price is extremely high, at least one firm engages in undercutting. If the price is extremely low, at least one firm pulls up. Thus, the equilibrium price is in the range where the opponent firm “follows.” This is the driving force of the former point.

Since the public firm gives consideration to the private firm’s profits, it has weak incentive to “undercut.” Understanding this behavior, the private firm sets a high price at the sequential

11) If $c < 2$, $a\frac{c+1}{c+2} > a\frac{3c}{3c+2}$.

12) If the public firm is perfectly privatized, the equilibrium price must not exceed the price range appeared as the equilibrium under the simultaneous price setting.

price setting with private leadership. This is the mechanism behind the latter point.

3. Equilibrium with a Foreign Private Firm

First we derive the best response correspondence of each firm.

Lemma 2

1. *The best response correspondence of firm* 0 *is obtained by*

$$BR_0^F(p_1) = \begin{cases} a\frac{2c+1-\theta}{2c+2-\theta}, & \text{if } p_1 > a\frac{2c+1-\theta}{2c+2-\theta}, \\ p_1 - \epsilon, & \text{if } p_1 \in \left(a\frac{3c}{3c+2}, a\frac{2c+1-\theta}{2c+2-\theta}\right], \\ p_1 + \delta, & \text{if } p_1 < a\frac{c}{c+2}, \\ p_1, & \text{otherwise.} \end{cases}$$

2. *The best response correspondence of firm* 1 *is given by*

$$BR_1^F(p_0) = \begin{cases} a\frac{2c+1}{2c+2} & \text{if } p_0 > a\frac{2c+1}{2c+2}, \\ p_0 - \epsilon, & \text{if } p_0 \in (a\frac{3c}{3c+2}, a\frac{2c+1}{2c+2}], \\ p_0 + \delta, & \text{if } p_0 < a\frac{c}{c+2}, \\ p_0, & \text{otherwise.} \end{cases}$$

where δ (> 0) *is arbitrary, and* ϵ (> 0) *is sufficiently small.*

Proof. *See Appendix C.*

This lemma is similar to Lemma 1. However, the public firm facing foreign competitor has more incentive to undercut and pull up as compared to the case where it faces domestic competitor because the public firm does not take care of the foreign competitor's profits though they consider domestic competitor's profits. Since the difference of the incentives between the public and the private firm results from whether or not the firm considers the profits of its competitor, the incentives are symmetric between the firms unless the price is too high in the case of foreign competitor[13).

Using this lemma, we have the following proposition.

Proposition 2 *The equilibrium price is as follows:*

13) This result is crucially dependent on assumption 1. The assumption guarantees that (1) the consumer surplus does not change if a firm pulls up and (2) the consumer surplus is almost unchanged if a firm undercuts marginally.

1. *If the price setting is simultaneous, the equilibrium price is continuous such that*

$$p_0^{S,F} = p_1^{S,F} \in \left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right].$$

2. *If the price setting is sequential with private leadership, the equilibrium price is unique such that*

$$p_0^{Pri,F} = p_1^{Pri,F} = \begin{cases} a\dfrac{c+1}{c+2}, & \text{if } c \geq 2, \\ a\dfrac{3c}{3c+2}, & \text{otherwise.} \end{cases}$$

3. *If the price setting is sequential with public leadership, the equilibrium price is unique such that*

$$p_0^{Pub,F} = p_1^{Pub,F} = \begin{cases} a\dfrac{3c}{3c+2}, & \text{if } c \leq -4\theta + 2, \\ a\dfrac{c}{c+2}, & \text{if } c \leq 4\theta - 2, \\ a\dfrac{c-2\theta+1}{c-2\theta+2}, & \text{otherwise.} \end{cases}$$

Proof. *See Appendix D.*

Since the price range that satisfies $BR_i^F(p_j) = p_j$ $(i \neq j)$ is symmetric between the firms in foreign competitor's case, the equilibrium price at the sequential timing never exceeds the range of equilibrium price at simultaneous timing. This result contrasts with the result shown in Proposition 1.

Comparing both propositions, we have the following results:

- If the timing is simultaneous, the range of the equilibrium price shown at each proposition is identical because the incentives of the private firm to "undercut" and "pull up" are binding.

- If the timing is sequential, the equilibrium price with the foreign private firm is no more than the one with the domestic private firm.

As we mentioned after Lemma 2, the difference in the incentives between the firms is much smaller in the foreign competitor's case because the public firm does not give consideration to the profits of the private firm. Thus, the equilibrium prices with a foreign competitor at sequential timings tend to be lower than those with a domestic competitor.

4. Concluding Remarks

We analyzed three types of price competitions with homogeneous products and symmetric quadratic cost functions under mixed duopoly. We have arrived at the following results:

- Both firms set the same price regardless of the timing to select their price.
- Both firms set the same price regardless of whether or not the private firm is domestic.
- If the timing is simultaneous, the equilibrium price exists continuously: if not, the equilibrium price is unique.
- The equilibrium price under sequential price setting with private leadership may be higher than that under simultaneous price setting, provided the private firm is domestic.

We have the following intuition from the results. Public enterprises are often justified by the fact that they are conscious of social welfare and enhance it. However, even if they act for the improvement of social welfare, their existence may lead to a worse outcome because domestic private firms would exploit such a situation: Even if the domestic private firm sets a high price, the public firm may hesitate to undercut because such an action might damage the private firm and result in an increase total cost. Since the private firms understand this situation, they sets a high price[14)].

Therefore, monitoring the market price and market share is quite important. If the market share of domestic competitors is high, the competition may be eased because of the public firm's behavior, and therefore, a highly marked-up price may be sustained. The government should privatize the public firm when they encounter such a situation. Even in the developing countries, the government should limit the activity of public firms until the domestic industry takes off.

14) If the private firm is a foreign firm, the public firm never gives consideration to its profits, and thus, this situation should not occur.

Appendix

A．Proof of Lemma 1

A．1　Best response correspondence of the public firm

First we consider the best response of firm 0. Substituting equation (1) into (3), we obtain

$$U_0 = \begin{cases} V^D_{0h}, & \text{if } p_0 > p_1, \\ V^D_{0e}, & \text{if } p_0 = p_1, \\ V^D_{0l}, & \text{otherwise.} \end{cases} \tag{6}$$

where

$$\begin{aligned} V^D_{0h} &= \theta\left[\frac{(a-p_1)^2}{2} + p_1(a-p_1) - c(a-p_1)^2\right] + (1-\theta)\cdot 0, \\ &= \frac{1}{2}\theta(a-p_1)\big[2p_1 + (1-2c)(a-p_1)\big], \\ V^D_{0e} &= \theta\left[\frac{(a-p_1)^2}{2} + 2p_1\cdot\frac{a-p_1}{2} - 2c\Big(\frac{a-p_1}{2}\Big)^2\right] + (1-\theta)\left[p_1\cdot\frac{a-p_1}{2} - c\Big(\frac{a-p_1}{2}\Big)^2\right], \\ &= \frac{1}{4}\theta(a-p_1)\big[2p_1 + (2-c)(a-p_1)\big] + \frac{1}{4}(a-p_1)\big[2p_1 - c(a-p_1)\big], \\ V^D_{0l} &= \theta\left[\frac{(a-p_0)^2}{2} + p_0(a-p_0) - c(a-p_0)^2\right] + (1-\theta)[p_0(a-p_0) - c(a-p_0)^2], \\ &= \frac{1}{2}\theta(a-p_0)^2 + (a-p_0)\big[p_0 - c(a-p_0)\big]. \end{aligned} \tag{7}$$

Now, we intercompare V^D_{0h}, V^D_{0e}, and V^D_{0l}.

$$\begin{aligned} V^D_{0h} > V^D_{0e} \iff\ & 2\theta(a-p_1)\big[2p_1 + (1-2c)(a-p_1)\big] \\ & > \theta(a-p_1)\big[2p_1 + (2-c)(a-p_1)\big] + (a-p_1)\big[2p_1 - c(a-p_1)\big], \\ \iff\ & p_1 < a\frac{(1-3\theta)c}{(1-3\theta)c + 2(1-\theta)}, \quad \left(\text{if } \theta < \frac{1}{3}\right). \end{aligned} \tag{8}$$

Thus, if firm 1 sets the price so as to satisfy the condition mentioned at (8), firm 0 sets the higher price and supply nothing[15].

Comparison between V^D_{0e} and V^D_{0l} is a little more complicated than the one between V^D_{0h} and V^D_{0e} because V^D_{0l} is not a given value but a function of p_0. Thus, we must maximize V^D_{0l} subject to $p_0 < p_1$ in order to compare them. Since

$$\frac{\partial V^D_{0l}}{\partial p_0} = 0 \iff p_0 = a\frac{2c+1-\theta}{2c+2-\theta}, \tag{9}$$

15) If $\theta \geq \frac{1}{3}$, $V^D_{0h} \leq V^D_{0e}\ \forall p_1 \in [0, a]$.

we have

$$\max_{p_0<p_1} V_{0l}^D = \begin{cases} V_{0l}^D\left(a\frac{2c+1-\theta}{2c+2-\theta}\right), & \text{if } p_1 > a\frac{2c+1-\theta}{2c+2-\theta}, \\ V_{0l}^D(p_1-\epsilon), & \text{otherwise.} \end{cases} \tag{10}$$

and

$$\begin{aligned} V_{0l}^D\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) &= \frac{a^2}{2(2c+2-\theta)}, \\ \lim_{\epsilon\to 0} V_{0l}^D(p_1-\epsilon) &= \frac{(a-p_1)(2p_1-2ac+a\theta+2cp_1-p_1\theta)}{2}. \end{aligned} \tag{11}$$

Comparing them, we obtain[16)]

$$V_{0l}^D\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) > V_{0e}^D \iff \begin{cases} p_1 > \sigma_1^D, & \text{if } c \geq \frac{(\theta+2)(\theta-1)^2}{2\theta(\theta+1)}, \\ \forall p_1 \in [0,a], & \text{otherwise,} \end{cases}$$

$$\text{where} \quad \sigma_1^D = a\frac{c(1+\theta)+1-\theta+\sqrt{\frac{-\theta^3+2c\theta^2+3\theta+2c\theta-2}{2c+2-\theta}}}{c(1+\theta)+2}, \tag{12}$$

$$\lim_{\epsilon\to 0} V_{0l}^D(p_1-\epsilon) > V_{0e}^D \iff p_1 > a\frac{a(3-\theta)c}{(3-\theta)c+2(1-\theta)}. \tag{13}$$

We note that if $\theta > \frac{1}{3}$,

$$\begin{aligned} \sigma_1^D &\gtreqless a\frac{2c+1-\theta}{2c+2-\theta} && \iff c \gtreqless \frac{2(\theta-1)^2}{3\theta-1}, \\ a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)} &\gtreqless a\frac{2c+1-\theta}{2c+2-\theta} && \iff c \gtreqless \frac{2(\theta-1)^2}{3\theta-1}, \end{aligned} \tag{14}$$

and

$$\frac{(\theta+2)(\theta-1)^2}{2\theta(\theta+1)} < \frac{2(\theta-1)^2}{3\theta-1}. \tag{15}$$

If $\theta \leq \frac{1}{3}$,

$$a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)} < \frac{a(2c+1-\theta)}{2c+2-\theta} \tag{16}$$

always holds regardless of the c.

Hence, the best response correspondence is given by

16) We note that if $c < \frac{(\theta+2)(\theta-1)^2}{2\theta(\theta+1)}$, $V_{0l}^D\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) - V_{0e}^D = 0$ has no real solution and therefore $V_{0l}^D\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) > V_{0e}^D$.

$$BR_0^D(p_1) = \begin{cases} a\frac{2c+1-\theta}{2c+2-\theta}, \text{if } p_1 > \sigma_1^D,\ \theta > \frac{1}{3}, \text{and } c \geq \frac{2(1-\theta)^2}{3\theta-1}, \\ p_1 - \epsilon, \quad \text{if } p_1 \in \left(a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)}, a\frac{2c+1-\theta}{2c+2-\theta}\right],\ \text{and } \begin{cases} \theta \leq \frac{1}{3}, \\ \theta > \frac{1}{3} \text{ and } c < \frac{2(1-\theta)^2}{3\theta-1}, \end{cases} \\ p_1 + \delta, \quad \text{if } p_1 < a\frac{(1-3\theta)c}{(1-3\theta)c+2(1-\theta)} \text{ and } \theta \leq \frac{1}{3}, \\ p_1, \quad \text{otherwise.} \end{cases} \tag{17}$$

A.2　Best response correspondence of the private firm

Second we consider the best response of firm 1. Substituting equation (1) into (4), we obtain

$$U_1 = \begin{cases} V_{1h}^D, & \text{if } p_1 > p_0, \\ V_{1e}^D, & \text{if } p_1 = p_0, \\ V_{1l}^D, & \text{otherwise.} \end{cases} \tag{18}$$

where

$$\begin{aligned} V_{1h}^D &= 0, \\ V_{1e}^D &= \left[p_0 \cdot \frac{a-p_0}{2} - c\left(\frac{a-p_0}{2}\right)^2\right], \\ &= \frac{1}{4}(a-p_0)\big[2p_0 - c(a-p_0)\big], \\ V_{1l}^D &= [p_1(a-p_1) - c(a-p_1)^2], \\ &= (a-p_1)\big[p_1 - c(a-p_1)\big]. \end{aligned} \tag{19}$$

Now, we intercompare V_{1h}^D, V_{1e}^D, and V_{1l}^D.

$$\begin{aligned} V_{1h}^D > V_{1e}^D &\iff \frac{1}{4}(a-p_0)\big[2p_0 - c(a-p_0)\big] < 0, \\ &\iff p_0 < a\frac{c}{2+c}. \end{aligned} \tag{20}$$

Thus, if firm 0 sets the price so as to satisfy the condition mentioned at (20), firm 1 sets the higher price and supply nothing.

Comparison between V_{1e}^D and V_{1l}^D is a little more complicated than the one between V_{1h}^D and V_{1e}^D because V_{1l}^D is not a given value but a function of p_1. Thus, we must maximize V_{1l}^D subject to $p_1 < p_0$ in order to compare them. Since

$$\frac{\partial V_{1l}^D}{\partial p_1} = 0 \iff p_1 = a\frac{2c+1}{2c+2}, \tag{21}$$

we have

$$\max_{p_1 < p_0} V_{1l}^D = \begin{cases} V_{1l}^D\left(a\frac{2c+1}{2c+2}\right), & \text{if } p_0 > a\frac{2c+1}{2c+2}, \\ V_{1l}^D(p_0 - \epsilon), & \text{otherwise,} \end{cases} \tag{22}$$

and

$$V_{1l}^D\left(a\frac{2c+1}{2c+2}\right) = \frac{a^2}{4(c+1)},$$
$$\lim_{\epsilon\to 0} V_{1l}^D(p_0-\epsilon) = (a-p_0)(p_0-ac+cp_0). \tag{23}$$

Comparing them, we obtain

$$V_{1l}\left(a\frac{2c+1}{2c+2}\right) > V_{1e}^D \iff \forall p_0 \in [0,a],$$
$$\lim_{\epsilon\to 0} V_{1l}^D(p_0-\epsilon) > V_{1e}^D \iff p_0 > \frac{a\cdot 3c}{3c+2}. \tag{24}$$

We note that $a\frac{3c}{3c+2} < a\frac{2c+1}{2c+2}$. Hence, the best response correspondence is

$$BR_1^D(p_0) = \begin{cases} a\frac{2c+1}{2c+2}, & \text{if } p_0 > a\frac{2c+1}{2c+2}, \\ p_0-\epsilon, & \text{if } p_0 \in (a\frac{3c}{3c+2}, a\frac{2c+1}{2c+2}], \\ p_0+\delta, & \text{if } p_0 < a\frac{c}{c+2}, \\ p_0, & \text{otherwise.} \end{cases} \tag{25}$$

B. Proof of Proposition 1

B.1 Simultaneous price setting

Comparing BR_0^D and BR_1^D, we find

$$a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)} \geq a\frac{3c}{3c+2}, (\forall\theta \in [0,1],\ \forall c > 0),$$
$$a\frac{(1-3\theta)c}{(1-3\theta)c+2(1-\theta)} \leq a\frac{c}{c+2}, \left(\theta \leq \frac{1}{3},\ \forall c > 0\right). \tag{26}$$

Hence, we directly obtain

$$p_0^{S,D} = p_1^{S,D} \in \left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right]. \tag{27}$$

B.2 Sequential price setting with private leadership

The private firm maximizes V_{1e}^D subjected to the $BR_0^D(p_1) = p_1$ because of the following reason.

- If the private firm picks the price so that the public firm undercuts, $\pi_1(p_1) = V_{1h}^D = 0$.
- If the private firm chooses the price so that the public firm pulls up, $\pi_1(p_1) = V_{1l}^D < 0$ because $a\frac{(1-3\theta)c}{(1-3\theta)c+2(1-\theta)} \leq a\frac{c}{c+2}$ and thus $V_{1l}^D\left(a\frac{(1-3\theta)c}{(1-3\theta)c+2(1-\theta)}\right) \leq V_{1l}^D\left(a\frac{c}{c+2}\right) = 0$.

Since

$$\frac{\partial V_{1e}^D}{\partial p_1} = 0 \iff p_1 = a\frac{c+1}{c+2}, \tag{28}$$

the firm 1 sets price as follows:

$$p_1 = \begin{cases} a\frac{c+1}{c+2}, & \text{if } BR_0^D\left(a\frac{c+1}{c+2}\right) = a\frac{c+1}{c+2}, \\ \sigma_1^D & \text{if } BR_0^D\left(a\frac{c+1}{c+2}\right) \neq a\frac{c+1}{c+2},\ \theta > \frac{1}{3},\ \text{and } c \geq \frac{2(1-\theta)^2}{3\theta-1}, \\ a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)} & \text{otherwise.} \end{cases} \tag{29}$$

Now, we check whether the condition $BR_0^D(p_1) = p_1$ deters from setting $p_1 = a\frac{c+1}{c+2}$. Comparing $a\frac{c+1}{c+2}$ with the upper bound of p which satisfies $BR_0^D(p) = p$, we have

$$\begin{aligned} \sigma_1^D \leq a\frac{c+1}{c+2} &\iff c \leq \frac{\theta^2 - 9\theta + 2 + \sqrt{\theta^4 + 14\theta^3 + 37\theta^2 - 4\theta + 4}}{4\theta}, \\ a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)} \leq a\frac{c+1}{c+2} &\iff c \leq \frac{2(1-\theta)}{1+\theta}. \end{aligned} \tag{30}$$

We note that if $\theta = \frac{\sqrt{17}-3}{2}$, both conditions mentioned at (30) are the same: $c \leq \frac{\sqrt{17}-3}{2}$. If $\theta = c = \frac{\sqrt{17}-3}{2}$,

$$\sigma_1^D = a\frac{(3-\theta)c}{(3-\theta)c+2(1-\theta)} = a\frac{2c+1-\theta}{2c+2-\theta}, \tag{31}$$

and we note that $a\frac{(1-3\theta)c}{(1-3\theta)c+2(1-\theta)} < a\frac{c+1}{c+2}$.

Hence, we have the equilibrium price as follows:

$$p_0^{Pri,D} = p_1^{Pri,D} = \begin{cases} a\dfrac{c+1}{c+2}, & \text{if } \begin{cases} \theta \leq \theta_1,\ c \geq c_1, \\ \theta > \theta_1,\ c \geq c_2, \end{cases} \\ a\dfrac{c(1+\theta)+1-\theta+\sqrt{\frac{-\theta^3+2c\theta^2+3\theta+2c\theta-2}{2c+2-\theta}}}{c(1+\theta)+2}, & \text{if } \theta > \theta_1,\ c \in [c_3, c_2), \\ a\dfrac{(3-\theta)c}{(3-\theta)c+2(1-\theta)}, & \text{otherwise,} \end{cases} \tag{32}$$

where

$$\begin{aligned} \theta_1 &= \frac{\sqrt{17}-3}{2}, \\ c_1 &= \frac{2(1-\theta)}{1+\theta}, \\ c_2 &= \frac{\theta^2 - 9\theta + 2 + \sqrt{\theta^4 + 14\theta^3 + 37\theta^2 - 4\theta + 4}}{4\theta}, \\ c_3 &= \frac{2(\theta-1)^2}{3\theta-1}. \end{aligned} \tag{33}$$

B. 3 Sequential price setting with public leadership

The public firm maximizes V_{0e}^D subjected to the $BR_1^D(p_0) = p_0$ because both SW and π_0 are deteriorated if the private firm undercuts or pulls up.

- Social welfare SW is harmed if only one firm produced because of the cost function. Total cost is minimized if both firms produce $\frac{q}{2}$ in order to supply q.
- $\pi_0 = 0$ if the private firm undercuts, and $\pi_0 < 0$ if the private firm pulls up.

Since

$$\frac{\partial V_{0e}^D}{\partial p_0} = 0 \iff p_0 = a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}, \tag{34}$$

the firm 0 sets price as follows:

$$p_0 = \begin{cases} a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}, & \text{if } BR_1^D\big(a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}\big) = a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}, \\ a\frac{3c}{3c+2}, & \text{otherwise.} \end{cases} \tag{35}$$

Now, we check whether the condition $BR_1^D(p_0) = p_0$ prevents from setting $p_0 = a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}$. Comparing $a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}$ with the upper bound of p which satisfies $BR_1^D(p) = p$, we have

$$a\frac{3c}{3c+2} \leq a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2} \iff c \leq \frac{2(1-\theta)}{1+\theta}, \tag{36}$$

and we note that $a\frac{c}{c+2} < a\frac{c(1+\theta)+1-\theta}{c(1+\theta)+2}$.

Hence, we have the equilibrium price as follows:

$$p_0^{Pub,D} = p_1^{Pub,D} = \begin{cases} a\dfrac{c(1+\theta)+1-\theta}{c(1+\theta)+2}, & \text{if } c \geq \dfrac{2(1-\theta)}{1+\theta}, \\ a\dfrac{3c}{3c+2}, & \text{otherwise.} \end{cases} \tag{37}$$

C. Proof of Lemma 2

C. 1 Best response correspondence of the public firm

First we consider the best response of firm 0. Substituting equation (1) into (3), we obtain

$$U_0 = \begin{cases} V_{0h}^F, & \text{if } p_0 > p_1, \\ V_{0e}^F, & \text{if } p_0 = p_1, \\ V_{0l}^F, & \text{otherwise.} \end{cases} \tag{38}$$

where

$$
\begin{aligned}
V_{0h}^F &= \theta\frac{(a-p_1)^2}{2} + (1-\theta)\cdot 0,\\
V_{0e}^F &= \theta\left[\frac{(a-p_1)^2}{2} + p_1\cdot\frac{a-p_1}{2} - c\Big(\frac{a-p_1}{2}\Big)^2\right] + (1-\theta)\left[p_1\cdot\frac{a-p_1}{2} - c\Big(\frac{a-p_1}{2}\Big)^2\right],\\
&= \frac{1}{2}\theta(a-p_1)^2 + \frac{1}{4}(a-p_1)\big[2p_1 - c(a-p_1)\big],\\
V_{0l}^F &= \theta\left[\frac{(a-p_0)^2}{2} + p_0(a-p_0) - c(a-p_0)^2\right] + (1-\theta)[p_0(a-p_0) - c(a-p_0)^2],\\
&= \frac{1}{2}\theta(a-p_0)^2 + (a-p_0)\big[p_0 - c(a-p_0)\big]. \qquad (39)
\end{aligned}
$$

Now, we intercompare V_{0h}^F, V_{0e}^F, and V_{0l}^F.

$$
\begin{aligned}
V_{0h}^F > V_{0e}^F \iff& 0 < \frac{1}{4}(a-p_1)\big[2p_1 - c(a-p_1)\big],\\
\iff& p_1 < a\frac{c}{c+2}. \qquad (40)
\end{aligned}
$$

Thus, if firm 1 sets the price so as to satisfy the condition mentioned at (40), firm 0 sets the higher price and supply nothing.

Comparison between V_{0e}^F and V_{0l}^F is a little more complicated than the one between V_{0h}^F and V_{0e}^F because V_{0l}^F is not a given value but a function of p_0. Thus, we must maximize V_{0l}^F subject to $p_0 < p_1$ in order to compare them. Since

$$
\frac{\partial V_{0l}^F}{\partial p_0} = 0 \iff p_0 = a\frac{2c+1-\theta}{2c+2-\theta}, \qquad (41)
$$

we have[17]

$$
\max_{p_0<p_1} V_{0l}^F = \begin{cases} V_{0l}^F\left(a\frac{2c+1-\theta}{2c+2-\theta}\right), & \text{if } p_1 > a\frac{2c+1-\theta}{2c+2-\theta},\\ V_{0l}^F(p_1-\epsilon), & \text{otherwise.}\end{cases} \qquad (42)
$$

and

$$
\begin{aligned}
V_{0l}^F\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) &= \frac{a^2}{2(2c+2-\theta)},\\
\lim_{\epsilon\to 0} V_{0l}^F(p_1-\epsilon) &= \frac{(a-p_1)(2p_1 - 2ac + a\theta + 2cp_1 - p_1\theta)}{2}. \qquad (43)
\end{aligned}
$$

Comparing them, we obtain[18]

$$
V_{0l}^F\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) > V_{0e}^F \iff \begin{cases} p_1 > \sigma_1^F, & \text{if } \theta \ge \frac{1}{3} \text{ and } c \ge \frac{2(\theta^2-\theta+2)}{3\theta-2},\\ \forall p_1 \in [0,a], & \text{otherwise,}\end{cases}
$$

17) We note that $V_{0l}^F = V_{0l}^D$ because $\pi_1 = 0$.

18) We note that if $\theta < \frac{2}{3}$, $V_{0l}^F\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) - V_{0e}^F = 0$ has no real solution and therefore $V_{0l}^F\left(a\frac{2c+1-\theta}{2c+2-\theta}\right) > V_{0e}^F$.

$$\text{where} \qquad \sigma_1^F = a\frac{c-2\theta+1+\sqrt{\frac{3\theta-2}{2c+2-\theta}}}{c-2\theta+2}, \tag{44}$$

$$\lim_{\epsilon\to 0} V_{0l}^D(p_1-\epsilon) > V_{0e}^D \iff p_1 > a\frac{3c}{3c+2}. \tag{45}$$

We note that

$$\sigma_1^F - a\frac{2c+1-\theta}{2c+2-\theta} = a\frac{\sqrt{(3\theta-2)(2c+2-\theta)}-(c+\theta)}{(2c+2-\theta)(c+2-2\theta)} < 0,$$
$$a\frac{3c}{3c+2} - a\frac{2c+1-\theta}{2c+2-\theta} = -a\frac{c+2(1-\theta)}{(3c+2)(2c+2-\theta)} < 0. \tag{46}$$

Hence, the best response correspondence is given by

$$BR_0^F(p_1) = \begin{cases} \frac{a(2c+1-\theta)}{2c+2-\theta}, & \text{if } p_1 > a\frac{2c+1-\theta}{2c+2-\theta}, \\ p_1-\epsilon, & \text{if } p_1 \in \left(a\frac{3c}{3c+2}, a\frac{2c+1-\theta}{2c+2-\theta}\right], \\ p_1+\delta, & \text{if } p_1 < a\frac{c}{c+2}, \\ p_1, & \text{otherwise.} \end{cases} \tag{47}$$

C. 2 Best response correspondence of the private firm

The objective function of the private firm is the same at the both – domestic and foreign – cases, although those of the public firm are not. Hence, the best response correspondence is also the same as the one derived at appendix A. 2 , i.e.,

$$BR_1^F(p_0) = \begin{cases} a\frac{2c+1}{2c+2}, & \text{if } p_0 > a\frac{2c+1}{2c+2}, \\ p_0-\epsilon, & \text{if } p_0 \in (a\frac{3c}{3c+2}, a\frac{2c+1}{2c+2}], \\ p_0+\delta, & \text{if } p_0 < a\frac{c}{c+2}, \\ p_0, & \text{otherwise.} \end{cases} \tag{48}$$

D. Proof of Proposition 2

D. 1 Simultaneous price setting

Comparing BR_0^F and BR_1^F, we find that the price range that a firm follows its opponent price is the same between both firms. Therefore, obviously we obtain

$$p_0^{S,F} = p_1^{S,F} \in \left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right]. \tag{49}$$

D. 2 Sequential price setting with private leadership

The private firm maximizes V_{1e}^F subjected to the $BR_0^F(p_1) = p_1$ because of the following reason.

- If the private firm picks the price so that the public firm undercuts, $\pi_1(p_1) = V_{1h}^F = 0$.
- If the private firm chooses the price so that the public firm pulls up, $\pi_1(p_1) = V_{1l}^F < 0$ because $V_{1l}^F\left(a\frac{c}{c+2}\right) = 0$.

Since

$$\frac{\partial V_{1e}^F}{\partial p_1} = 0 \iff p_1 = a\frac{c+1}{c+2}, \tag{50}$$

the firm 1 sets price as follows:

$$p_1 = \begin{cases} a\frac{c+1}{c+2}, & \text{if } BR_0^F\left(a\frac{c+1}{c+2}\right) = a\frac{c+1}{c+2}, \\ a\frac{3c}{3c+2} & \text{otherwise.} \end{cases} \tag{51}$$

Now, we check whether the condition $BF_0^D(p_1) = p_1$ deters from setting $p_1 = a\frac{c+1}{c+2}$. Comparing $a\frac{c+1}{c+2}$ with the upper bound of p which satisfies $BR_0^D(p) = p$, we have

$$a\frac{3c}{3c+2} \leq a\frac{c+1}{c+2} \iff c \leq 2. \tag{52}$$

We note that $a\frac{c}{c+2} < a\frac{c+1}{c+2}$.

Hence, we have the equilibrium price as follows:

$$p_0^{Pri,F} = p_1^{Pri,F} = \begin{cases} a\dfrac{c+1}{c+2}, & \text{if } c \geq 2, \\ a\dfrac{3c}{3c+2}, & \text{otherwise.} \end{cases} \tag{53}$$

D. 3　Sequential price setting with public leadership

The public firm maximizes V_{0e}^F subjected to the $BR_1^F(p_0) = p_0$ because both SW and π_0 are deteriorated if the private firm undercuts or pulls up.

- Social welfare SW is harmed if only one firm produced because
  - if the private firm undercuts, π_0 damaged though consumer surplus is almost unchanged[19].
  - if the private firm pulls up, π_0 damaged though consumer surplus is exactly the same[20].

19) As we mentioned Appendix C. 2, the optimal undercut for the private firm is $p_0 - \epsilon$ and therefore the increase of consumer surplus is negligible.

20) If the private firm pulls up, i.e., $p_0 < a\frac{c}{c+2}$, $V_{0l}^F < 0$ and consumer surplus is not changed at all.

- $\pi_0 = 0$ if the private firm undercuts, and $\pi_0 < 0$ if the private firm pulls up.

Since

$$\frac{\partial V_{0e}^F}{\partial p_0} = 0 \iff p_0 = a\frac{c - 2\theta + 1}{c - 2\theta + 2}, \tag{54}$$

the firm 0 sets price as follows:

$$p_0 = \begin{cases} a\frac{c-2\theta+1}{c-2\theta+2}, & \text{if } BR_1^F\left(a\frac{c-2\theta+1}{c-2\theta+2}\right) = a\frac{c-2\theta+1}{c-2\theta+2}, \\ a\frac{3c}{3c+2}, & \text{otherwise.} \end{cases} \tag{55}$$

Now, we check whether the condition $BR_1^F(p_0) = p_0$ prevents from setting $p_0 = a\frac{c-2\theta+1}{c-2\theta+2}$. Comparing $a\frac{c-2\theta+1}{c-2\theta+2}$ with the upper bound of p which satisfies $BR_1^D(p) = p$, we have

$$\begin{aligned} a\frac{3c}{3c+2} \leq a\frac{c-2\theta+1}{c-2\theta+2} &\iff c \leq -4\theta + 2, \\ a\frac{c}{c+2} \geq a\frac{c-2\theta+1}{c-2\theta+2} &\iff c \leq 4\theta - 2. \end{aligned} \tag{56}$$

Hence, we have the equilibrium price as follows:

$$p_0^{Pub,F} = p_1^{Pub,F} = \begin{cases} a\dfrac{3c}{3c+2}, & \text{if } c \leq -4\theta + 2, \\ a\dfrac{c}{c+2}, & \text{if } c \leq 4\theta - 2, \\ a\dfrac{c-2\theta+1}{c-2\theta+2}, & \text{otherwise.} \end{cases} \tag{57}$$

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