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Design of cellular manufacturing systems with refixturing considerations.

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University of Windsor

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DESIGN OF CELLULAR MANUFACTURING SYSTEMS
WITH
REFIXTURING CONSIDERATIONS

by

Vijayakanthan Damodaran

A Thesis
submitted to the
Faculty of Graduate Studies and Research
through the Department of
Industrial Engineering in Partial Fulfillment
of the requirements for the Degree
of Master of Applied Science at
the University of Windsor

Windsor, Ontario, Canada

1990

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ABSTRACT

Flexible manufacturing systems have received considerable attention in the modern manufacturing environment. The concept of cell formation emerged as a byproduct of flexible manufacturing systems to significantly reduce setup times and batch sizes. The objective of this research is to deal with some specific production planning and design problems during cell formation. Important manufacturing realities such as refixturing and material handling during operation allocation and cell formation were considered. Model 1 and model 2 consider the problem of assigning operation(s) of part types to one or more machines in a cellular manufacturing environment. Model 1 is developed for the case of a single cell and model 2 extends the operation allocation problem for multiple cells. Model 3 and model 4 are developed to determine machine groups and assignment of part operations to machines. Model 3 is used to simultaneously assign operations of parts to machines and form machine groups. Model 4 is developed for the situation where new machines are procured for the cellular manufacturing environment. Considerations of physical limitations such as an upper bound on cell size, machine capacity, etc., were also incorporated into cell formation and presented. These models bring forth the trade-offs between refixturing, inter cell movement and investment costs. A few illustrative examples were solved using the package LINDO (PC version) and the results were analyzed.

DEDICATION

To my parents

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NOMENCLATURE

INDEXING SETS:

i = (1,2,.......I) parts

j = (1,2,.......J) machines

k = (1,2,.......K_i) operations of part i

c = (1,2,.......C) cells

DECISION VARIABLES:

$$L_{ikj} = \begin{cases} 1 & \text{if operation k of part type i is performed on machine j} \\ 0 & \text{otherwise} \end{cases}$$

X_{i1j} = Number of units of part i which go to machine j for the first operation

X^{i1}_{jc} = Number of units of part i which go to machine j in cell c for the first operation.

$X_{ikj'j}$ = Number of units of part i which move from machine j', after performing operation (k-1), to machine j to perform operation k.

$X^{ik}_{j'c'jc}$ = Number of units of part i which move from machine 6j' in cell c', after performing operation (k-1), to machine j in cell c to perform operation k.

$$Y_{ikj'j} = \begin{cases} 1 & \text{if part i moves from machine j' to machine j to perform operation k} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ikj'jc} = \begin{cases} 1 & \text{if part i moves from machine j' after performing operation (k-1) to machine j to perform operation k in cell c} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{jc} = \begin{cases} 1 & \text{if machine j is allotted to cell c} \\ 0 & \text{otherwise} \end{cases}$$

W_{jc} = Number of machines of type j in cell c

COEFFIECIENTS

C_{ikj} = Cost to perform operation k of part type i on machine j

$C_{ikj'j}$ = Setup cost associated with refixturing if part i moves from machine j' after performing operation (k-1) to machine j to perform operation k

$\overline{C}_{ikj'j}$ = Variable cost associated with refixturing when part i moves from machine j' after performing operation (k-1) to machine j to perform operation k

$\overline{C}_{ij'j}$ = Cost associated with moving a part i from machine j' to machine j.

$\overline{C}_{i}$ = Average cost of inter-cell movement of part type i

C_j = Discounted cost of machine j

D_i = Demand for part i to be processed

G_c = Maximum number of machines in cell c

M = A very large number

t_{ikj} = Time for performing operation k of part type i on machine j

$t_{ikj'j}$ = Set-up time associated with refixturing if part i moves from machine j' after performing operation (k-1) to machine j to perform operation k

$\overline{t}_{ikj'j}$ = Variable time associated with refixturing if part i moves from machine j' after performing operation (k-1) to machine j to perform operation k

t_j = Time available on machine j

t_{jc} = Time available on machine j in cell c

CHAPTER 1

INTRODUCTION

The recent trend in the manufacturing environment is towards manufacturing a wide variety of parts in small batches. The aim of such a manufacturing environment is to combine the flexibility of a job shop and the productivity of a transfer line. The concept of flexible manufacturing systems (FMS) has emerged as an answer to the problems of low volume, high variety production. Cellular manufacturing in FMS is used to process clusters of similar parts (part families) on clusters of machines or manufacturing processes (machine groups). The focus of this thesis is on the design and production planning problems of cellular manufacturing systems .

1.1 Flexible Manufacturing Systems (FMS)

A FMS is an integrated computer controlled complex of automated material handling devices and numerically controlled (NC) machine tools that can simultaneously process medium sized volumes of a variety of part types with minimum manual intervention (Stecke 1983).

FMS evolved as the result of the integration of a number of independent NC machine tools. It consists of equipment work stations with automatic tool interchange capability linked by a computer controlled material handling system. NC programs and (often) computer aided process planning are used to generate a process plan for each part in a FMS.

The function of the computer in a FMS environment is to keep track of the availability and location of cutting tools, the processing times of various parts, the operation of the material handling system, and the sequencing and the scheduling of parts to be processed on the machines.

A typical FMS can process a variety of part types at the same time. Usually there is a certain demand to manufacture a certain number of parts of each part type. The manufacture of a part requires a certain number of processing steps to be performed on the part. These processing steps are grouped together into operations which require one or more cutting tools. The essential features that constitute a workable part family in a FMS are:

(1) Common shape - Prismatic and rotational surfaces cannot be produced by the same set of machines

(2) Size - The size of the parts cannot exceed a certain maximum size.

(3) Material - Certain types of materials like plastic and metal cannot be mixed.

(4) Tolerance - The level of tolerance necessary for a set of parts must be in a common range.

The main difference between FMS and a conventional manufacturing system is that in a job or a flow shop each operation of each part type is processed in a lot and this lot moves from machine to machine for processing. The potentially large number of parts in each lot causes large in-process-inventories. Since the FMS is considered to process the required number of part types to demand, work-in-process inventories are reduced greatly.

Some common benefits of FMS are: reduced throughput time, reduced set up time and costs, high equipment (machine tool) utilization, better quality, reduced scrap level and reduction in the economic order quantity.

1.2 Problems in Flexible Manufacturing Systems

It is well known that the economic benefits of any manufacturing system depends on the efficient design, planning, scheduling and control of the system. These issues have been the focus of attention of several researchers. The problems of FMS can be broadly classified into two categories: static and dynamic. FMS design, planning and control problems are classified as static as they deal with the one time allocation of FMS resources while the scheduling problem are classified as dynamic as they deal with the real time allocation of FMS resources (Stecke 1985).

1.2.1 FMS design problems

The design of a FMS has a great impact on the operational run of the system. Kusiak (1985) had discussed the following issues for the design of an efficient FMS:

(1) Selection of part families and determination of how these parts

families are to be manufactured.

(2) Type of flexibility desired in the FMS.

The types of flexibilities can be broadly classified into the following categories

Machine flexibility: Ease of change to process a given set of part types.

Process flexibility: Ability to produce a given set of part types.

Product flexibility: Ability to change to process new part types.

Routing Flexibility: Ability to process a given set of parts on alternate machines.

Volume Flexibility: Ability to operate profitably at varying overall levels of production.

Expansion flexibility: Ability to add capability and capacity.

Operational flexibility: Ability to interchange ordering of operations on a part.

Material handling system flexibility: Capability to handle a variety of part types.

(3) Based on the desired flexibility the type of FMS is chosen to handle the increase in the number of parts or the increase in production requirements.

(4) Selection of the material handling system and resources such as fixtures and pallets.

(5) Control of computers and their hierarchy.

(6) Layout and integration of the above system.

1.2.2 FMS Planning Problems

These problems involve the decisions to be taken before the parts are loaded on the FMS. They are as follows:

(1) Operation allocation problem

It is the problem of assigning each operation of each part type to one or more specific machines before the actual production can begin subject to the capacity and technological constraints of the system.

(2) Part-mix determination problem

This determines the relative proportion or number of parts that has to be maintained in the system at all times such that the total number of parts in the system remains fixed.

(3) Partitioning machines into machine groups or cells.

(4) Allocation of resources as fixtures and pallets.

1.2.3 FMS scheduling problem

The scheduling problem is a multi-faceted issue involving scheduling of parts, fixtures, pallets, tools and the material handling system. Some of the common problems faced are readjustment of the schedule subject to failure of machines, arrival of new parts and loading of high priority work parts. Some examples of scheduling rules are simple priority rules related to processing times, due dates, number of operations, cost rules, setup rules and setup times; combination of some simple priority rules; and development of heuristics to solve the scheduling problem.

1.2.4 FMS control problems

These problems are associated with the continuous monitoring of the system keeping track of the production to be certain that the production requirements and the due dates are met as scheduled. The most important of these are:

(1) Determination of periodic, preventive, predictive or breakdown maintenance policies.

(2) Determination of inspection policy of in-process and finished goods.

1.3 Cell formation problems in FMS

Cells are formed in FMS to group processes, peoples and machines to process clusters of similar parts (part families). This is an application of Group Technology (GT). Group Technology is a philosophy that exploits the proximity among the attributes of given objects. Cellular manufacturing is an application of group technology in manufacturing. The cell formation problem in FMS is the decomposition of the manufacturing system into cells. These cells are formed to capture the inherent advantages of group technology like reduced setup times, reduced in-process inventories, smaller lot sizes, reduction in production equipment, improved productivity and better overall control of operations. The common disadvantages are high investment and lower machine utilization.

The essential problem in cellular manufacturing is the formation of part families and machine groups. There are essentially two solution approaches to the above problem based on the way part families and machine groups are identified (Wemmerlov and Hyer 1986). They are:

(1) Sequential approach

This approach identifies machine groups and assigns part operations to machines or identifies part families and assigns machines to these part families.

(2) Simultaneous approach

Simultaneous approach forms machine groups and assigns part operations to these machine groups.

In cellular manufacturing, the following situations can be visualized for the formation of cells:

(i) Rearranging the existing machines to form cells.

(ii) New machines are procured to create cells.

Desirable design goals during cell formation include minimizing of investment, inter cell movement, refixturing and operating cost.

The common problems faced during cellular manufacture are:

(1) Production of parts with similar processing requirements in machine groups.

(2) Formation of part families under some criterion as shape, dimension, range, geometry and similar operational sequences.

(3) Formation of machine groups to efficiently produce a family of parts requiring almost similar machining sequences.

(4) Allocation of machines to cells and part families to these machine groups in order to meet the objectives of cell formation.

(5) Allocation of parts to machine groups based on the capability of the machine to process the part and handling the system changes as new parts are being introduced into the system and allowing for redesign and subcontracting of the parts.

1.4 EFFECT OF REFIXTURING AND MATERIAL HANDLING DURING CELL FORMATION

Refixturing and material handling play a crucial role during operation allocation and cell formation in a cellular manufacturing system. In any manufacturing activity, it is essential to fixture the parts. A part may require a number of operations to be performed. Having finished an operation(s) on a machine using a fixture, it may be necessary to re-orient the part using the same fixture or require a completely different type of fixture on the same or different machine to perform subsequent operation(s). This process is referred to as refixturing. The cost of refixturing depends on the sequence of machine visits selected for the manufacture of a part. We can have a number of process routes (sequence of machine visits) to manufacture parts, if operation(s) of parts are permitted to be performed on alternate machines. For each sequence, the costs of refixturing can be attributed to the level of fixture complexity. Few examples of fixtures include milling, plate, assembly, welding, multistation, duplex, indexing, vice jaw fixtures, etc. Refixturing is necessitated by the following reasons:

(1) The reference surface for the next operation may not be accessible due to the current pattern of holding the part.

(2) The machine spindle may not be able to reach and/or move on the

work surface as required because of the current position of the part.

(3) A specific type of fixture may be needed to perform the next operation due to the nature of the machining process involved.

Similarly, the load on the material handling system is directly related to the layout of the manufacturing system, assignment of part operations to machines and the proportion of each part type to be manufactured. For, example assignment of operations of parts to machines in alternate cells would result in more inter-cell movement and thereby increase the load of the material handling device.

The practical considerations of refixturing and material handling as well as the physical limitation on cell configuration may lead to a trade-off between refixturing and material handling movement, and thereby influence operation allocation and cell design. For example, if the refixturing costs are higher to manufacture a part within a cell, then the part may have to move to another cell where the total costs of refixturing costs and material handling are lesser for its subsequent operation(s). Also, the cell could be designed such that the machines necessary to manufacture the part are located within the same cell. Therefore, there is a need to consider refixturing and material handling together during cell formation.

1.5 Organization of the research

The research is organized as follows. A review of modeling approaches to operation allocation and cell formation problems are given in chapter 2. Motivation and objectives of the proposed research is also included in chapter 2. Operation allocation problems in cellular manufacturing systems are discussed in chapter 3. Design of cellular manufacturing systems are presented in chapter 4. Computational results are presented in chapter 5. The conclusions are presented in the chapter 6.

CHAPTER 2

LITERATURE SURVEY

Modelling of cell formation problems and operation allocation problems have been discussed quite extensively in literature. A comprehensive review of FMS modelling techniques was presented by Wilhem and Sarin (1983) and Buzzacot and Shanthikumar (1980). With regard to cell formation a review of available literature was discussed by Wemmerlov and Hyer (1986) while Chu and Pan (1988) provided a review on the use of clustering techniques in cell formation.

2.1 Review of mathematical programming approaches

Mathematical programming is applied to some static planning models to provide optimal resource input to the simulation models in manufacturing systems. As this research deals with the mathematical programming formulations a review of the literature in this area is presented. On this basis the available literature is classified into one of the following.

1. Mathematical models for operation allocation.
2. Mathematical models for cell formation problems.

2.1.1. Mathematical models for operation allocation

The problem of machine loading and the allocation of operations of each part type to one or more machines have been solved by various authors.

Stecke (1981) discussed several integer programming formulations of the loading problem. The principle to balance the work load was also discussed. Though it is quite adequate, it had been shown that in certain configurations of FMS this does not achieve better system performance.

Stecke (1983) has discussed the computational behavior of the integer programming formulation of the FMS loading problem. Kusiak (1983) has reported some simple and computationally attractive integer programming formulations in FMS.

Kimemia and Gershwin (1985) proposed a network flow optimization approach to determine the optimal part routing policy for FMS after part mix and operation allocation decisions have been taken.

Shanker and Tzen (1985) reported a analysis on the loading and dispatching problem and attempted to verify the quality of the solution by employing a simulation procedure using the solution from the mathematical models as inputs.

Padhye (1986) used the outputs of the operation allocation problem and developed a part mix determination problem as a general integer program considering important real life planning aspects as refixturing and limited tool availability.

Lashkari et al.(1987) extended the formulation of the operation allocation problem to include the important aspects of refixturing and limited tool availability.

Leung and Tanchoco (1987) also studied the part assignment model with the objective of maximizing profit in a multi-machine, multi-product environment and reported the economics of equipment replacement in such an environment.

Singh et al. (1990) discussed a min-max approach with often conflicting multi objectives for the allocation of parts processed on NC machines with multi-tools and fixtures.

2.1.2 Mathematical models for cell formation problems

A review of the literature indicates that a sequential or simultaneous approach is usually adopted for the formation of manufacturing cells. A sequential approach first forms part families or machine groups followed by the machine assignment or part allocation respectively. The simultaneous approach determines the part families and machine groups simultaneously.

Dutta et al. (1986) suggested a heuristic procedure for determining manufacturing families from design based grouping for FMS. Algorithms are developed for clustering parts into families and relocates to reflect identical processing and tools required.

Askin and Chiu (1988) proposed a mathematical model and solution procedure for the group technology configuration problem. This is a simultaneous approach for the grouping of individual machines into cells and routing of components to machines within cells. Various costs as inventories, machine depreciation, machine setup and material handling are first incorporated into a mathematical programming formulation and a heuristic graph partitioning procedure is fabricated and solved for each sub-problem.

Choobineh (1988) proposed a two stage sequential approach to cell design. Clustering techniques are used for forming part families. An integer programming model is then used to specify the type and number of machines in each cell and the assignment of part families to cells.

Co and Arrar (1988) suggested a three stage sequential approach partitioning machines into manufacturing cells and assigning cells to process a specific set of parts. First there is a operation allocation of assigning parts to machines. The resulting machine part matrix is manipulated using some existing algorithms to form part-machine groups. Then a search algorithm is used to determine the number of cells and composition of each cell.

Kasilingam (1989) developed number of mathematical models and used both sequential and simultaneous approach for the cell formation problem. The sequential approach first identifies machine groups and then allots parts to these machine groups while the simultaneous approach uses the multiple objectives of compatibility maximization between the part and the machine and the cost trade-offs between the inter cell movements and the duplication of machines. The machine-part grouping problem was extended to account for the presence of alternate process plans for various part types.

Shtub (1989) considered several process plans for each part type in the cell formation problem. Seifoddini (1989) considered the economic trade off between machine duplication in cells and inter cell movement based on decomposing the part machine matrix.

Srinivasan et al. (1990) presented an assignment model to solve the grouping problem using a similarity coefficient matrix as an input to the assignment model. The basis of grouping was the identification of closed loops as sub tours after solving the assignment problem.

Rajamani et al. (1990) developed mathematical models in the presence of alternate process plans and analyzed how alternate process plans influence the resource utilization when part families and machine groups are formed simultaneously. An efficient solution scheme using the column generation procedure was also reported to solve large scale instances of the problem.

Formulations of cell formation often result in product terms of 0-1 integer variables. Commercially available integer programming packages often require the model to be in a completely linear form. Most extensively used linearization strategies in literature are based on reports by Watters (1967) and the modifications of these linearization strategies by Glover and Woolsey (1974). Glover (1975) also discussed computational efficiency of existing integer programming algorithms with an increase in the number of integer variables.

2.2 Motivation for proposed research

A review of the programming approaches to the operation allocation and the design of cells was discussed in the previous section. Most of the earlier approaches do not take into account important production planning aspects such as refixturing and part travel together during operation allocation and cell formation. Also each operation of a part is restricted to one machine which is not realistic since in practice the operation of a particular part may be performed on alternate machines. Moreover even the simple models developed for the simultaneous grouping of part families and machine groups were non-linear and needed efficient linearization strategies to effectively solve them.

This indicates the importance of developing procedures to generate optimal solutions in the planning stage of the manufacturing system to overcome the operational difficulties during the production.

2.3 Objectives of the proposed research

The major objectives of the research are:

1. Develop mathematical models for production planning and cell design aspects of cellular manufacturing systems considering important manufacturing realities as refixturing and inter-cell movement.
2. Develop mathematical models to simultaneously form machine groups and assign part operations to these machine groups selected considering the real life planning aspects such as processing capacities and production volumes.
3. Identification of cost trade-offs between inter-cell movements,

refixturings, and the duplication of machines while taking into consideration that operations on a particular part can be performed on alternate machines.

4 Development of mixed integer linear models for these problems.

CHAPTER 3

PRODUCTION PLANNING MODELS WITH REFIXTURING CONSIDERATIONS

In this chapter we consider the problem of assigning operations of part types to one or more machines in a cellular manufacturing system. We develop mixed integer linear models considering the trade-off between refixturing and material handling movement. Accordingly, the chapter is organized as follows. In section 2 we discuss the possible trade-off that may exist between refixturing and material handling and in section 3 we develop two mathematical programming models to explicitly consider refixturing, material handling and processing costs during operation allocation.

3.1 THE PROBLEM SITUATION:

The practical considerations of refixturing and material handling as well as the physical limitation on the cell configuration may lead to a trade-off between refixturing and material handling and, therefore, influence operation allocation. For example, consider a part having two operations to be manufactured in a cellular manufacturing system having two cells. Cell 1 contains machine M1 and machine M2 while cell 2 contains machine M3. Suppose the first operation is to be performed on machine M1 and the second operation can be performed on either machine M2 or machine M3. If refixturing costs on machine M2 and machine M3 are RFC_2 and RFC_3, respectively, material handling cost between machine M1 and machine M2 is MHC_{1-2} , and the material handling cost between machine M1 and machine M3 is MHC_{1-3} then, the second operation will be allocated to machine M2 in cell 1 if

$$RFC_2 + MHC_{1-2} \leq RFC_3 + MHC_{1-3}$$

Otherwise, it will be allocated to machine M3 in cell 2. In the next section we develop mathematical models to explicitly consider this trade-off for operation allocation.

3.2 MATHEMATICAL MODELS:

In this section the mathematical models are presented. Model 1 is developed for the case of a single cell and Model 2 extends the operation allocation problem for multiple cells.

The models presented are formulated under the following assumptions:

1. Detailed process sheets are available for each part type giving information about the sequence of operations to be performed on each part.
2. Part types have been selected for production and demand for each part type is known.
3. For each part type, the total number of units produced may be divided into few batches, each one following a different process plan. However within each process plan, the batch would be treated as a single lot size.
4. The number of cells and the number of machine types are given.

3.2.1 MODEL 1

We assume that each part type i (i = 1,2,.....I) requires k (k = 1,2,.....K_i) operations to be performed on one or more of machines j (j

= 1,2,.....J) in a single cell. The objective is to assign operations of parts to machines to minimize the cost of performing the operations, the material handling cost and the cost associated with having a part refixtured depending on which machine the next operation is performed on.

$$\text{Minimize Z1} = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq 1}} \left[\sum_{1 \leq j,\, j' \leq J} C_{ikj'j} Y_{ikj'j} + \sum_{1 \leq j,\, j' \leq J} \overline{C}_{ikj'j} X_{ikj'j} \right.$$

$$\left. + \sum_{1 \leq j,\, j' \leq J} \overline{C}_{ij'j} X_{ikj'j} + C_{i1j} X_{i1j} + \sum_{1 \leq j,\, j' \leq J} C_{ikj} X_{ikj'j} \right] \quad (1)$$

The constraints of the system are as follows:

i) $$\sum_{j \in J} X_{i1j} = D_i \qquad \forall\ i \qquad (2)$$

These constraints ensure that the demands for the parts are met.

i i) $$\sum_{j \in J} X_{i2j'j} = X_{i1j'} \qquad \forall\ i, j'$$

$$\sum_{j \in J} X_{ikj'j} = \sum_{j'' \in J} X_{i(k-1)j''j'} \qquad \forall\ i, j', k \neq 1,2 \qquad (3)$$

This set of constraints ensures that the number of units of part i moving from machine j' to machine j to perform a given operation k is equal to the number of units of part i that had moved from all machines j" to machine j' to perform operation k-1.

iii) $$\sum_{i \in I} t_{i1j} X_{i1j} + \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq 1}} \sum_{j' \in J} t_{ikj} X_{ikj'j} + \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq 1}} \sum_{j' \in J} t_{ikj'j} Y_{ikj'j}$$

$$+ \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} \overline{t}_{ikj'j} \; X_{ikj'j} \leq t_j \qquad \forall \; j \qquad (4)$$

These constraints guarantee that the capacity of each machine is not violated

iv) $$X_{ikj'j} \leq M \, Y_{ikj'j} \qquad \forall \; i,k,j',j \qquad (5)$$

This constraint forces the setup charge to take a value of one if there is any movement of part i from machine j' to machine j for performing operation k and zero otherwise.

v) X_{i1j} , $X_{ikj'j} \geq 0$, $Y_{ikj'j} = 0$ or 1 $\qquad \forall \; i,k,j',j \qquad (6)$

Constraints (6) indicate the 0-1 integer and continuous variables.

3.2.2 MODEL 2

This model considers the situation with multiple cells. The problem is to assign operations of parts to machines in various cells considering the practical aspects of refixturing and inter-cell movement.

The objective function minimizes the setup and variable costs associated with refixturing, cost of inter-cell movement, and the operating cost.

The setup cost associated with refixturing is:

$$(B1) = \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{1\leq j,\, j'\leq J} \sum_{c\in C} C_{ikj'j} \; Y_{ikj'jc}$$

The variable cost associated with refixturing is:

$$(B2) \; = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq 1}} \sum_{1 \leq j, j' \leq J} \sum_{1 \leq j, j' \leq J} \overline{C}_{ikj'j} \, X^{ik}_{j'c'jc}$$

The cost of inter-cell movement is:

$$(B3) \; = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq 1}} \sum_{1 \leq j, j' \leq J} \sum_{\substack{1 \leq c, c' \leq C \\ c \neq c'}} \overline{C}_i \, . \, X^{ik}_{j'c'jc}$$

The operating cost is:

$$(B4) = \sum_{i \in I} \sum_{j \in J} \sum_{c \in C} C_{ij1} X^{i1}_{jc} + \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq 1}} \sum_{1 \leq j, j' \leq J} \sum_{1 \leq c, c' \leq C} C_{ijk} \, X^{ik}_{j'c'jc}$$

The objective function would be:

$$\text{Minimize } Z2 = (B1) + (B2) + (B3) + (B4) \qquad (7)$$

The constraints of the system are as follows:

$$\text{i)} \quad \sum_{j \in J} \sum_{c \in C} X^{i1}_{jc} = D_i \qquad \forall \; i \qquad (8)$$

These constraints ensure the fact that a given part can be manufactured by different machines and that the demandsfor the parts are met.

$$\text{ii)} \quad \sum_{j \in J} \sum_{c \in C} X^{i2}_{j'c'jc} = X^{i1}_{j'c'} \qquad \forall \; i, j', c' \qquad (9)$$

$$\sum_{j \in J} \sum_{c \in C} X^{ik}_{j'c'jc} = \sum_{j'' \in J} \sum_{c'' \in C} X^{i(k-1)}_{j''c''j'c'} \qquad \forall \; i, j, c, k \neq 1, 2 \qquad (10)$$

This set of continuity equations ensures that the number of units of part i moving from machine j' in cell c' to all machines j in cell c to perform operation k is equal to the number of units of part i moving from all machines j" in cells c" to machine j' in cell c' to perform operation k-1.

iii) $$\sum_{i\in I} t_{ij1} X^{i1}_{jc} + \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} \sum_{c'\in C} t_{ijk} X^{ik}_{j'c'jc}$$

$$+ \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} t_{ikj'j} \cdot Y_{ikj'jc}$$

$$+ \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} \sum_{c'\in C} \overline{t}_{ikj'j} X^{ik}_{j'c'jc} \leq t_{jc} \qquad \forall\ j,c \qquad (11)$$

In equation (11) the first and second term represent the processing time, whereas the third and fourth terms represent the refixturing time. These constraints ensure that the capacity of each machine is not violated.

iv) $$\sum_{c'} X^{ik}_{j'c'jc} \leq M\, Y_{ikj'jc} \qquad \forall\ i,k,j',j \qquad (12)$$

This constraint forces the setup charge associated with refixturing to take a value whenever part i moves from machine j' in any cell c' to machine j in cell c to perform operation k and no value otherwise.

v) $X^{ik}_{j'c'jc} \geq 0$, $Y_{ikj'jc} = 0$ or 1 $\qquad \forall\ j,c,j',c',i,k$ (13)

These constraints indicate the 0-1 and continuous variables.

3.2.2.1 MODEL 2.1

In model 2 the cost of material handling movement within a cell was assumed to be negligible compared to the cost of inter cell movement. If, however, the factory layout warrants significant costs associated with intra-cell movements then we can consider intra-cell material handling costs by adding the following terms to the objective function.

The cost of intra-cell movement:

$$(B5) = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq i}} \sum_{\substack{1 \leq j, j' \leq J \\ j \neq j'}} \sum_{\substack{1 \leq c, c' \leq C \\ c = c'}} \overline{C}_{ij'j} \; X^{ik}_{j'c'jc}$$

The model could then be briefly stated as follows

Minimize Z3 = (B1) + (B2) + (B3) + (B4) + (B5)

subject to the constraints (8), through (13).

A few illustrative examples are solved and the results are reported in chapter 5.

CHAPTER 4

MATHEMATICAL FORMULATIONS FOR CELL DESIGN

In this chapter, refixturing and material handling aspects of cellular manufacturing systems are modeled. For this purpose two mathematical models are developed to determine machine groups and assignment of part operations to machines. In model 3 the part families and machine groups are formed simultaneously when parts and machines for cellularization have been selected. Model 4 is developed for the situation where new machines are procured for a cellular manufacturing environment. Accordingly, the chapter is organized as follows. In section 4.1 we discuss the problem situation and in section 4.2 we develop two mixed integer linear models that explicitly consider refixturing, material handling, processing capacities and production volumes in the design of cells.

4.1. THE PROBLEM SITUATION

We consider a cell design problem where a part may have more than one process plan and where each operation of the part may be performed on more than one machine. The costs of refixturing and inter-cell movement during the manufacture of the part depend on the sequence of machine visits which, in turn, is related to the way the cells are formed.

The situation may be illustrated by considering the following simple example. Suppose a part with two operations is to be manufactured in a cellular manufacturing system consisting of two cells. The first operation has to be done on machine M1 while the second operation could be done on either machine M2 or machine M3. Let the refixturing cost on machine M2 and machine M3 be RFC_2 and RFC_3, respectively, material handling cost between machine M1 and machine M2 be MHC_{1-2} , and the material handling costs between machine-M1 and machine-M3 be MHC_{1-3}. Assuming only one unit of each machine is available, the three possible designs are given below:

	Cell-1	Cell-2
Design 1	machine M1 machine M2	machine M3
Design 2	machine M1 machine M3	machine M2
Design 3	machine M2 machine M3	machine M1

Design-1 is an optimal design if

$$RFC_2 + MHC_{1-2} \le RFC_3 + MHC_{1-3}$$

and if the second operation is assigned to machine M2 in cell 1. However, machine M2 may not have enough capacity. In that case, the second operation of some of the parts may have to be performed on machine M3 in cell 2.

Design 2 may be an optimal design if

$$RFC_2 + MHC_{1-2} > RFC_3 + MHC_{1-3}$$

and if the second operation is assigned to machine M3 in cell 1. The capacity limitation on machine M3 may force the second operation for some of the parts to be performed in cell 2.

It may, however, be pointed out that the choice of cell design also depends on other cost elements such as the operating cost, discounted investment cost as well as the constraints imposed on the system operation. This might lead to a different cell design such that machines M2 and M3 are in cell 1 and machine M1 in cell 2 as in design 3. The models developed in the next section consider all such possibilities to optimally allocate the machines to cells and operations of parts to these machine groups.

4.2 MATHEMATICAL MODELS

The models presented in this paper are formulated under the following assumptions:

1. Parts and machines for cellularization are selected.
2. For each part type, detailed process sheets are available providing information about the sequence of operations to be performed on each part, as well as refixturing requirements between operations.
3. For each part type, the total number of units produced may be divided into few batches, each one following a different process plan. However within each process plan, the batch would be treated as a single lot size.
4. The number of cells, the number of machine types and the maximum number of machines to be allotted to each cell are given.

4.2.1 MODEL 3

In this section we developed a model for the situation where the parts and machines for cellularization have been selected. The objective is to simultaneously allocate machines to cells and operations of parts to machines such that the costs of refixturing, inter cell movement and the operating costs are minimized, considering demand and capacity constraints.

The setup cost associated with refixturing is:

$$(B1) = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq i}} \sum_{1 \leq j,\, j' \leq J} \sum_{c \in C} C_{ikj'j} \, Y_{ikj'jc}$$

The variable cost associated with refixturing is:

$$(B2) = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq i}} \sum_{1 \leq j,\, j' \leq J} \sum_{1 \leq j,\, j' \leq J} \overline{C}_{ikj'j} \, X^{ik}_{j'c'jc}$$

The cost of inter-cell movement is:

$$(B3) = \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq i}} \sum_{1 \leq j,\, j' \leq J} \sum_{\substack{1 \leq c,\, c' \leq C \\ c \neq c'}} \overline{C}_i . \, X^{ik}_{j'c'jc}$$

The operating cost is:

$$(B4) = \sum_{i \in I} \sum_{j \in J} \sum_{c \in C} C_{ij1} X^{i1}_{jc} + \sum_{i \in I} \sum_{\substack{k \in K_i \\ k \neq i}} \sum_{1 \leq j,\, j' \leq J} \sum_{1 \leq c,\, c' \leq C} C_{ijk} \, X^{ik}_{j'c'jc}$$

The objective function would be:

Minimize Z1 = (B1) + (B2) + (B3) + (B4) (15)

The constraints of the system are as follows:

i) $$\sum_{j\in J}\sum_{c\in C} X^{i1}_{jc} = D_i \qquad \forall\ i \qquad (16)$$

These constraints ensure the fact that a given part can be manufactured by different machines and that the demandsfor the parts are met.

ii) $$\sum_{j\in J}\sum_{c\in C} X^{i2}_{j'c'jc} = X^{i1}_{j'c'} \qquad \forall\ i, j', c' \qquad (17)$$

$$\sum_{j\in J}\sum_{c\in C} X^{ik}_{j'c'jc} = \sum_{j''\in J}\sum_{c''\in C} X^{i(k-1)}_{j''c''j'c'} \qquad \forall\ i, j, c, k\neq 1,2 \qquad (18)$$

This set of continuity equations ensures that the number of units of part i moving from machine j' in cell c' to all machines j in cell c to perform operation k is equal to the number of units of part i moving from all machines j" in cells c" to machine j' in cell c' to perform operation k-1.

iii) $$\sum_{i\in I} t_{ij1} X^{i1}_{jc} + \sum_{i\in I}\sum_{\substack{k\in K_i\\ k\neq 1}}\sum_{j'\in J}\sum_{c'\in C} t_{ijk} X^{ik}_{j'c'jc}$$

$$+ \sum_{i\in I}\sum_{\substack{k\in K_i\\ k\neq 1}}\sum_{j'\in J} t_{ikj'j} Y_{ikj'jc}$$

$$+ \sum_{i\in I}\sum_{\substack{k\in K_i\\ k\neq 1}}\sum_{j'\in J}\sum_{c'\in C} \overline{t}_{ikj'j} X^{ik}_{j'c'jc} \leq t_j Z_{jc} \qquad \forall\ j, c \qquad (19)$$

In equation (19) the first and second terms represent the processing time, whereas the third and fourth terms represent the refixturing time.

These constraints ensure that the capacity of each machine is not violated.

iv) $$\sum_{j \in J} Z_{jc} \le G_c \qquad \forall\ c \qquad (20)$$

This constraint restricts the number of machines allotted to each cell.

v) $$\sum_{c \in C} Z_{jc} \le 1 \qquad \forall\ j \qquad (21)$$

This constraint ensures that a machine is not alloted to more than one cell.

vi) $$\sum_{c' \in C} X^{ik}_{j'c'jc} \le M\ Y_{ikj'jc} \qquad \forall\ i,k,j',j \qquad (8)$$

This constraint forces the setup charge associated with refixturing to take on a value whenever part i moves from machine j' in any cell c' to machine j in cell c to perform operation k and no value otherwise.

v) $$X^{ik}_{j'c'jc} \ge 0\ ,\ Y_{ikj'jc}\ ,\ Z_{jc} = 0 \text{ or } 1 \qquad \forall\ j,c,j',c',i,k \qquad (22)$$

These constraints indicate the 0-1 and continuous variables.

4.2.2 MODEL 4

This model considers the situation where new machines are procured. The problem is to simultaneously determine the number of units of each machine types to be procured, allocate these machines to cells, and assign operations of the parts to these machine groups.

The objective function minimizes the setup and variable costs associated with refixturing, the cost of inter-cell movement, operating cost and the investment cost.

The cost of investment is:

$$(B5) = \sum_{j\in J} \sum_{c\in C} C_j W_{jc}$$

The model could be briefly stated as follows

$$\text{Minimize } (Z2) = (B1) + (B2) + (B3) + (B4) + (B5) \tag{23}$$

subject to the constraints (2), (3), (4), (8) and

i)

$$\sum_{i\in I} t_{ij1} X^{i1}_{jc} + \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} \sum_{c'\in C} t_{ijk} X^{ik}_{j'c'jc}$$

$$+ \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} t_{ikj'j} Y_{ikj'jc}$$

$$+ \sum_{i\in I} \sum_{\substack{k\in K_i \\ k\neq 1}} \sum_{j'\in J} \sum_{c'\in C} \overline{t}_{ikj'j} X^{ik}_{j'c'jc} \leq t_j W_{jc} \quad \forall \; j,c \tag{24}$$

In equation (24) the first and the second terms represent the processing time, whereas the third and fourth terms represent the refixturing time. These constraints ensure that the capacity of each machine is not violated.

ii)

$$\sum_{j\in J} W_{jc} \leq G_c \qquad \forall \; c \tag{25}$$

This constraint restricts the number of machines allotted to each cell.

iii) $X^{ik}_{j'c'jc} \geq 0$, $Y_{ikj'jc} = 0$ or 1 , $W_{jc} =$ integer $\forall \; j,c,j',c',i,k$

(26)

These constraints indicate the 0-1, integer and continuous variables.

Few illustrative examples are solved for the models developed and the results are reported in Chapter 5.

CHAPTER 5

COMPUTATIONAL RESULTS

The formulations presented in the previous chapters are applied to a few examples. These results are presented and then analyzed in the following section.

5.1 DISCUSSION OF RESULTS

To illustrate the influence of refixturing and material handling costs on operation allocation and cell design, we consider a few numerical examples consisting of two part types with known demands. Refixturing information for the two part types are given in Tables 1 and 2, respectively. The time and costs required for processing the parts are given in Table 3 along with the material handling costs. Table 4 gives the other necessary data required for solving model 1 and model 2 along with the cases analyzed for solving model 2. The data required for solving model 3 and model 4 are given in Table 5. The various cases for the analysis of model 3 and model 4 are shown in Table 6.

The results of model 1 are given in Table 7. It is observed that there is one process plan for the production of part type 1 and four process plans for the production of part type 2. All units of part type 1 are manufactured by Machine M1. This could be due to the following reasons: Machine M1 has a large capacity, manufacture of part type 1

Table 1: Refixturing information for part type 1

Setup cost for refixturing $\left(C_{ikj'j} \right)$, From-To Matrix

(a)

		Machine (j)		
		1	2	3
Machine (j')	1	6	11	9
	2	10	12	8
	3	∞	∞	∞

Variable cost for refixturing $\left(\overline{C}_{ikj'j} \right)$, From-To Matrix

(b)

		Machine (j)		
		1	2	3
Machine (j')	1	.05	.09	.1
	2	.07	.12	.08
	3	∞	∞	∞

Setup time for refixturing $\left(t_{ikj'j} \right)$, From-To Matrix

(c)

		Machine (j)		
		1	2	3
Machine (j')	1	2	5	4
	2	5	4	3
	3	∞	∞	∞

Variable time for refixturing $\left(\overline{t}_{ikj'j} \right)$, From-To Matrix

(d)

		Machine (j)		
		1	2	3
Machine (j')	1	.02	.1	.1
	2	.07	.09	.05
	3	∞	∞	∞

∞ indicates that an operation cannot be performed on machine j or machine j' or both.

Table 2: Refixturing information for part type 2

Setup cost for refixturing $\left(C_{ikj'j} \right)$, From-To Matrix

Machine (j)

(a) Machine (j')

	1	2	3
1	0	0	10
2	0	0	9
3	11	7	12

Variable cost for refixturing $\left(\overline{C}_{ikj'j} \right)$, From-To Matrix

Machine (j)

(b) Machine (j')

	1	2	3
1	0	0	.1
2	0	0	.09
3	.1	.05	.1

Setup time for refixturing $\left(t_{ikj'j} \right)$, From-To Matrix

Machine (j)

(c) Machine (j')

	1	2	3
1	0	0	3
2	0	0	3
3	4	2	4

Variable time for refixturing $\left(\overline{t}_{ikj'j} \right)$, From-To Matrix

Machine (j)

(d) Machine (j')

	1	2	3
1	0	0	.09
2	0	0	.08
3	.08	.04	.01

∞ indicates that an operation cannot be performed on machine j or machine j' or both.

Table 3: Operation time $\left(t_{ikj}\right)$ and costs $\left(C_{ikj}\right)$ for various machines

PARTS	1 OPERATION TIME, min COST ,$		2 OPERATION TIME, min COST ,$	
OPERATIONS	1	2	1	2
MACHINE 1	6 (.7)	8 (.9)	9 (1.1)	10 (1.2)
MACHINE 2	4 (.9)	6 (1.0)	6 (1.0)	7 (1.0)
MACHINE 3	∞ (∞)	9 (.9)	8 (.9)	11 (.8)

∞ indicates that an operation cannot be performed on machine j.

Data on the costs of material handling for Model-1 $\left(\overline{C}_{ij'j}\right)$

From-to Matrices

Machine (j)

		1	2	3
Machine (j')	1	0	.02	.05
	2	.02	0	.03
	3	.05	.03	0

PART-1

Machine (j)

		1	2	3
Machine (j')	1	0	.04	.1
	2	.04	0	.06
	3	.1	.06	0

PART-2

Table 4: Other data required for solution of Models 1 and 2

a) **Model 1**

$D_1 = 175$ $t_1 = 2500$
$D_2 = 200$ $t_2 = 1500$
$t_3 = 1500$

b) **Model 2**

$D_1 = 350$ $G_c = 3$ $t_{11} = 2000$ $t_{12} = 4000$
$D_2 = 450$ $t_{21} = 2000$ $t_{22} = 2000$
$t_{31} = 2000$

Case 1: (Base Case) $\overline{C}_1 = .01$ $\overline{C}_2 = .05$

Case 2: Material handling costs are increased by a factor of ten.

Case 3: Material handling costs are decreased by a factor of ten.

Case 4: Refixturing costs are increased by a factor of ten.

Case 5: Refixturing costs are decreased by a factor of ten.

Table 5: Other data required for solving Models 3 and 4

a) **Model 3**

$D_1 = 350$ $t_1 = 2000$ $G_1 = 2$
$D_2 = 450$ $t_2 = 2000$ $G_2 = 2$
$t_3 = 2000$

$\overline{C}_1 = .01$ $\overline{C}_2 = .05$

b) **Model 4**

$D_1 = 500$ $t_1 = 2500$ $C_1 = 5500$
$D_2 = 525$ $t_2 = 2000$ $C_2 = 6500$
$t_3 = 3000$ $C_3 = 6000$

$\overline{C}_1 =.01$ $\overline{C}_2 =.05$
$G_1 = 3$ $G_2 = 3$

Table 6:

a) Cases analyzed for Model 3

Case 1: Base Case
Case 2: Refixturing costs were decreased by a factor of ten
Case 3: Refixturing costs were increased by a factor of ten

b) Cases analyzed for Model 4

Case 1: Base Case
Case 2: Refixturing costs were decreased by a factor of ten
Case 3: Refixturing costs were increased by a factor of ten

c) Cases analyzed for restrictions on machine availabilty in Model 4

Case 4: Machine M1 is restricted to a maximum of one unit
Case 5: Machine M1 is not available
Case 6: Machine M2 is not available
Case 7: Machine M3 is restricted to a maximum of two units
Case 9: Machine M3 is restricted to a maximum of one unit
Case 10: Machine M3 is not available

Table 7: Solution for Model 1

a) Objective function value = $ 6644

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1		PART #2							
	PLAN #1		PLAN #1		PLAN #2		PLAN #3		PLAN #4	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
MACHINE 1	175	175	5							
MACHINE 2				5	58	58		101		36
MACHINE 3							101		36	

by Machine M1 has the minimum cost of refixturing, and there is no material handling movement associated with the manufacture of part type 1 by Machine M1. Plans 1,3 and 4 of part type 2 indicate that there is considerable material handling movement during the manufacture of that part. These material handling movements could be due to the trade-off between refixturing and material handling.

Five different cost combinations of refixturing and material handling are considered for model 2 to analyze the impact on operation allocation and to study the possible trade-off between refixturing and material handling. Different types of material handling devices include AGV's (automated guided vehicles), conveyors, fork lifts, etc. Similarly, there could be various refixturing costs depending on the type of machines, and the type of operations performed on those machines. In case 1 (Base Case) the refixturing costs are given in Tables 1 and 2 and the material handling cost is fixed as given in Table 5. The optimal allocation of parts to machines are given in Table 8. Here, part 1 is produced by two process plans while part type 2 is produced by four process plans. A fair amount of inter-cell movement is permitted to manufacture part type one.

In case 2 only the cost associated with material handling is increased by a factor of ten above the base level. As expected there is no inter-cell movement because the increase in material handling cost forces a part to undergo refixturing rather than inter-cell movement. In case 3 the material handling cost is decreased by a factor of ten

Table 8: Solution for Model 2

Case 1:

a) Objective function value =$ 1486

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2							
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3		PLAN #4	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1												
MACHINE 1	217				35							
MACHINE 2						35						250
MACHINE 3											250	
CELL 2												
MACHINE 1		217	133	133			23					
MACHINE 2								23	142	142		

Case 2:

a) Objective function value = $ 1490

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2							
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3		PLAN #4	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1												
MACHINE 1	136	136										
MACHINE 2							19	19				
MACHINE 3											250	250
CELL 2												
MACHINE 1			214	214	59							
MACHINE 2						59			122	122		

Case 3:

a) Objective function value = $ 1485

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2							
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3		PLAN #4	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1												
MACHINE 1	152				23		35					
MACHINE 2								35				250
MACHINE 3											250	
CELL 2												
MACHINE 1		152	198	198		23			142	142		
MACHINE 2												

Case 4:

a) Objective function value = $ 1861

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2							
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3		PLAN #4	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1												
MACHINE 1	152					109		6				
MACHINE 2					109		6					181
MACHINE 3											181	
CELL 2												
MACHINE 1		152	198	198								
MACHINE 2									154	154		

Case 5:

a) Objective function value = $ 1444

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2									
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3		PLAN #4		PLAN #5	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1														
MACHINE 1	143	143												
MACHINE 2							71	71					180	
MACHINE 3														180
CELL 2														
MACHINE 1			207	207	30	30				29				
MACHINE 2									29		140	140		

below the base level while the other costs remain the same. The optimal solution allows for two process plans for part type one and four process plans for part type two. Plan 1 of part type 1 and plan 1 of part type 2 show inter-cell movement. Although, the number of units of part type 1 which move between cells movement has decreased compared to the base case, the overall cost of optimally allocating the operations has decreased.

In case 4 the refixturing cost is increased by a factor of ten while the other costs remain the same as in the base case. The results indicate that there are two process plans for part type 1 and five process plans for part type 2. The results show that plan 1 for part type one allows for a considerable amount of inter-cell movement. Since the refixturing cost is higher the part is manufactured in different cells having less refixturing costs. In case 5 the refixturing cost is decreased by a factor of ten. The optimal allocation does not allow for any inter-cell movement since the parts could be manufactured within the cells optimally because the associated cost of refixturing is less. These results highlight the impact of refixturing and material handling during operation allocation. The process plan selection in all the above cases is also dependant on operating costs and the capacity available on the machines to process the part types.

The results of model 3 are given in Table 9. In case 1 (Base Case) it is observed that there are two process plans for the manufacture of

Table 9: Solution for Model 3
Base Case:

a) Objective function value = $ 845

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2			
	PLAN #1		PLAN #2		PLAN #1		PLAN #2	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1								
MACHINE 2			22		25	25		225
MACHINE 3				22			225	
CELL 2								
MACHINE 1	178	178						

Case 2:

a) Objective function value =$ 804

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2	
	PLAN #1		PLAN #2		PLAN #1	
	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1						
MACHINE 2				38		250
MACHINE 3					250	
CELL 2						
MACHINE 1	162	162	38			

Case 3:

a) Objective function value = $ 984

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2			
	PLAN #1		PLAN #2		PLAN #1		PLAN #2	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1								
MACHINE 3				22			223	
CELL 2								
MACHINE 1	178	178						
MACHINE 2			22		27	27		223

part type 1 and two process plans for part type 2. Machines M2 and M3 are allocated to cell 1 and Machine M1 is allocated to cell 2. The results indicate that there is no inter-cell movement for the manufacture of either part type 1 or part type 2; however, there is considerable refixturing involved in the manufacture of both parts as indicated in the process plans.

In case 2 the refixturing costs are decreased by a factor of ten. The cell design remains the same but the operation allocation differs from the base case. The results show that there are two process plans for the manufacture of part type 1 and one process plan for the manufacture of part type 2. The results also show some inter-cell movement in the manufacture of part type 1. As the refixturing cost is now less compared to the base case, the amount of refixturing involved is greater as compared to base case allocation.

In case 3 the costs of refixturing are increased by a factor of ten. The cell design has now changed and Machine 3 is allocated to cell 1 while Machine M1 and Machine M2 are allocated to cell 2. There are two process plans each for the manufacture of part type 1 and part type 2. The results indicate that there is considerable inter-cell movement for the manufacture of both these parts. Since the refixturing cost is higher the part is rerouted to different cells in order to avoid refixturing as much as possible.

The results of model 4 are given in Table 10. For the base case, the

Table 10: Solution for Model 4

Base Case:

a) Objective function value = $ 37453

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2					
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1										
MACHINE 3 (3 units)					432	432	50		43	
CELL 2										
MACHINE 1 (2 units)	76	76		424				50		
MACHINE 2			424							43

(Case 2)

a) Objective function value = $ 37320

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2					
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1										
MACHINE 3 (3 units)					432	432	50		43	
CELL 2										
MACHINE 1 (2 units)	76	76		424				50		
MACHINE 2			424							43

(Case 3)

a) Objective function value = $ 38046

b) Indicates the plan selected for each part, machine selected for each operation and allocation of machines to cells

Operation allocation	PART #1				PART #2							
	PLAN #1		PLAN #2		PLAN #1		PLAN #2		PLAN #3		PLAN #4	
	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2	K=1	K=2
CELL 1												
MACHINE 2 (2 units)			144		64	64		371				
MACHINE 3							371		4			
CELL 2												
MACHINE 1 (2 units)	356	356		144						4		
MACHINE 3											86	86

* The quantities of parts processed in various machines were rounded off.

optimal solution indicates that there are three units of machine M3 in cell 1, two units of machine M1 in cell 2 and one unit of machine M2 in cell 2. In case 2 the refixturing costs are decreased by a factor of ten; however, the optimal design remains unchanged, although the objective function value decreases due to the decrease in refixturing costs. In case 3, where the cost of refixturing is increased by a factor of ten, the optimal design changes such that two units of machine M2 and one unit of machine M3 are now allocated to cell 1 while two units of machine M1 and one unit of machine M3 are allocated to cell 2. Since the costs associated with refixturing are higher, more parts are allowed to be manufactured in different cells in order to avoid refixturing as much as possible. It is also noted that, in this case, the number of machines for which refixturing costs are high, are reduced.

5.2 Sensitivity analysis of machine cost.

In model 2 the purchase price of a machine may be an important deciding factor. To investigate the effect of machine procurement cost on cell design, sensitivity analyses were performed on the base-case model, considering variations in the price of machine M1. The results are reported in Table 11. For a unit price of up to $5427 for machine M1, the results indicate that two units of machine M1 are procured for cell 2. For a unit price range of $5428 to $6030, the machine allocation in cell 2 changes: only one unit of machine M1 is now procured and machine M3 replaces the previous allocation of machine M1. For a unit price greater than $6030 the machine allocation in cell 2 again changes:

TABLE 11: Sensitivity Analysis of machine cost for Model 4

COST RANGES OF MACHINE M1	MACHINE ALLOCATION CELL 1	CELL 2
$5427 AND LESS	M3, M3, M3	M1, M1, M2
$5427 TO $6030	M3, M3, M3	M1, M2, M3
$6030 AND MORE	M3, M3, M3	M2, M2, M3

Machine M1 is completely eliminated from cell design as the requirements are met by additional units of machines M2 and M3. Similar analyses could be performed on machines M2 and M3 to determine their respective price ranges.

5.3 Sensitivity analysis of machine availability.

To investigate the effect of machine availability on the cell design, limits on the number of machines in the system were introduced in Model 2. The various cases analyzed are given in Table 6(c) and the results are given in Table 12. For the base case, 3 units of machine M3 were allocated to cell 1, and 2 units of machine M1 and 1 unit of machine M2 were allocated to cell 2 (See Table 10(a)). The objective function value is $37453. From Table 12 it is observed that if Machine M1 is restricted to one unit (case 4), then the overall cost increases by $427, whereas if Machine M1 is not available at all (case 5), then the cost increases by $958. However, if machine M2 is not available (case 6), then the solution becomes infeasible. If machine M3 is restricted to two, one and zero units (cases 7,8,and 9,respectively), then the cost increases by $462, $1002 and $2591, respectively. These results indicate that Machine M2 is critical because its non-availability renders the solution infeasible. However, either one of machines M1 and M3 are not critical; if machine M1 is not available, it could be replaced by machine M2 or machine M3, and if machine M3 is not available, it could be replaced by machines M1 and M2.

TABLE 12: **Sensitivity Analysis of Machine availability for Model 4**

CASES	MACHINE ALLOCATION CELL 1	 CELL 2	OBJECTIVE FUNCTION VALUE
CASE 1	M3, M3, M3	M1, M1, M2	$37453
CASE 4	M3, M3, M3	M1, M2, M3	$37880
CASE 5	M2, M3, M3	M3, M3, M3	$38411
CASE 6			infeasible
CASE 7	M2, M2, M3	M1, M1, M3	$37905
CASE 8	M2, M2, M3	M1, M1, M2	$38455
CASE 9	M2, M2, M2	M1, M2, M2	$40045

CHAPTER 6

CONCLUSIONS

In this research mathematical models were developed to address the production planning and cell formation problems in cellular manufacturing systems. The proposed formulations were applied to a planning situation and the optimal solutions were obtained using the LINDO (PC version) package. The contributions of the research are given in the next section followed by a brief discussion on the directions of future research.

6.1 Contributions of the research

In this work, mathematical models were developed for the following problems in cellular manufacturing systems:

(a) Production planning problems

(b) Cell design problems

Most of the models currently available in literature do not consider important manufacturing realities such as refixturing and material handling together during operation allocation and cell formation. Also, the flexibility of a part to have alternate process routes (sequence of machine visits) and the ability of a part to be manufactured in alternate machines have not been considered. Fixing a machine for an operation does not select the machines optimally, thereby increasing capital cost. Authors who have addressed the issue of cell formation often decompose a part machine matrix into clusters of cells. Here costs are not explicitly considered. Processing times and production volumes are often ignored. Moreover no mathematical formulations have been reported in cell formation for identifying the potential trade-offs between the important aspects of refixturing and inter-cell movement. Similarly the trade-offs between the duplication of machines with these operational issues have also not been considered.

In this research, an attempt was made to consider these realities. All the models developed allow the operations of parts to be manufactured in alternate machines and allow the flexibility of a part to be manufactured through alternate process routes. Model 1 is for the case of a single cell and model 2 extends the production planning problem for the case of multiple cells. Both these models explicitly consider refixturing and material handling during operation allocation. Model 3 simultaneously formed machine groups and assigned part operations to these machine groups. Model 4 extended this approach for the situation where new machines are procured for the cellular manufacturing environment. A few illustrative examples were solved and the trade-offs between refixturing and material handling were identified and the results were discussed in chapter 5.

6.2 Directions for future research

1. There is a need to develop heuristic or approximate procedures for solving large scale instances of the present problem. This may be achieved by relaxing the assumption of having a setup charge associated with refixturing. This assumption would greatly

decrease the number of integer variables, thereby, decreasing the model complexity.

2 Inclusion of performance measures such as the utilization of machines has to be looked into. Such modeling would most probably, require the incorporation of the analytical results of queuing networks. For the present model machine utilization may be maximized by minimizing the slack variable corresponding to the machine capacity constraint.

3. It may be an interesting area to investigate solution methodologies required to solve planning problems where the demand is not deterministic, where the parts do not arrive at the system in well defined batches.

4. There is a need to consider other factors such as tool magazine capacity of machines, tool life, number of tools available, etc., during cell formation.

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APPENDICES

APPENDIX A.1

SPECIAL CASE:

Model A1

The model developed here is for the special case when it is assumed that each part cannot have more than one process route (sequence of machine visits) towards its manufacture. For this purpose, a nonlinear integer programming model is developed to form machine groups and assign part operations to machines such that the total costs of refixturing, inter-cell movement and operating costs are minimized considering demand and capacity constraints. The model is as follows.

$$\text{Minimize } m1 = \sum_{\substack{i j j' k \\ k = 2}} (L_{ik-1j'} \, L_{ikj} \, C_{ikj'j}) + \sum_{\substack{i j j' k \\ k = 2}} d_i (L_{ik-1j'} \, L_{ikj} \, \overline{C}_{ikj'j})$$

$$+ \sum_{\substack{i j j' k c c' \\ j \neq j' \\ c \neq c' \\ k = 2}} (L_{ikj} \, Z_{jc})(L_{ik-1j'} \, Z_{j'c'}) d_i \overline{C}_i + \sum_{i j k} (C_{ikj} \, L_{ikj}) d_i \qquad (1)$$

The constraints of the system are as follows:

$$\sum_{j} L_{ikj} = 1 \qquad \forall \quad i,k \qquad (2)$$

$$\sum_{c} Z_{jc} = 1 \qquad \forall \quad j \qquad (3)$$

$$\sum_{ik} L_{ikj} \leq My_j \qquad \forall \quad j \qquad (4)$$

$$\sum_{j} y_j Z_{jc} \le G_c \qquad \forall \quad c \qquad (5)$$

$$\sum_{ik} d_i \left(t_{ikj} L_{ikj}\right) + \sum_{ikj'} \left(t_{ikj'j} L_{ik-1j'} L_{ikj}\right)$$

$$+ \sum_{ikj'} d_i \left(\overline{t}_{ikj'j} L_{ik-1j'} L_{ikj}\right) \le t_j \qquad \forall \quad j \qquad (6)$$

$$L_{ikj} , Z_{jc} , Y_j = 0 \text{ or } 1 \qquad \forall \ i, j, k, c \qquad (7)$$

The objective function minimizes the total cost of refixturing, intercell movement and operating costs. The constraints of the optimization model are given by (2) to (6). Constraint(2) ensures that each operation of each part type is assigned to one machine. Constraint (3) guarantees that each machine is allotted to a cell. Constraints (4) and (5) restricts the number of machines in each cell where Y_j is the binary indicator of potencial machine "j". Constraint (6) ensures that the capacity of each machine is not violated. The demand for the parts is also accounted for in these constraints. Constraint (6) indicates the 0-1 variables.

It can be seen that there are product terms of two or more 0-1 decision variables and this makes the model nonlinear. The method suggested by Glover and Woolsey (1974) can be used to linearize the constraints and is explained below.

For each relevant i, j, j', k define the variable $Y_{ikj'j}$ replacing the product terms L_{ikj} and $L_{ik-1j'}$ such that

$Y_{ikj'j} = 1$ if $L_{ikj} = 1$ and $L_{ik-1j'} = 1$

$= 0$ otherwise

Similarly define the variable $Z^{ik}_{j'c'jc}$ replacing the product terms L_{ikj} , $L_{ik-1j'}$, Z_{jc} , $Z_{j'c'}$ such that

$Z^{ik}_{j'c'jc} = 1$ if $L_{ikj} = 1$, $L_{ik-1j'} = 1$, $Z_{jc} = 1$, $Z_{j'c'} = 1$

$= 0$ otherwise

and the variable Y_{jc} replacing the product terms Y_j and Z_{jc} such that

$Y_{jc} = 1$ if $Y_j = 1$, $Z_{jc} = 1$

$= 0$ otherwise

Now the model A1 can be rewritten as:

$$\text{Minimize } m1 = \sum_{\substack{ijj'k \\ k = 2}} (Y_{ikjj'} \ C_{ikj'j}) + \sum_{\substack{ijj'k \\ k = 2}} (Y_{ikjj'} \ \overline{C}_{ikj'j}) d_i$$

$$+ \sum_{\substack{ijj'k \ c' \\ j \neq j' \\ c \neq c' \\ k = 2}} (Z^{ik}_{jcj'c'} \cdot \ d_i) \ \overline{C_i} \quad + \sum_{ijk} (C_{ikj} \ L_{ikj}) \ d_i \qquad (8)$$

subject to the following constraints:

$$\sum_{j} L_{ikj} = 1 \qquad \forall \quad i,k \qquad (9)$$

$$\sum_{c} Z_{jc} = 1 \qquad \forall \quad j \qquad (10)$$

$$\sum_{ik} L_{ikj} \leq My_j \qquad \forall \quad j \qquad (11)$$

$$\sum_{j} Y_{jc} \le G_c \qquad \forall \quad c \qquad (12)$$

$$\sum_{ik} d_i (t_{ikj}\, L_{ikj}) \;+ \sum_{ikj'} (t_{ikj'j}\, Y_{ikj'j}\,)$$

$$+ \sum_{ikj'} d_i (\overline{t}_{ikj'j}\, Y_{ikj'j}\,) \;\le t_j \qquad \forall \quad j \qquad (13)$$

$$L_{ik-1j'} + L_{ikj} - 1 \le Y_{ikj'j} \qquad \forall \; i,j,j',k \qquad (14)$$

$$L_{ikj} + Z_{jc} + L_{ik-1j'} + Z_{j'c'} - 3 \le Z^{ik}_{jcj'c'} \qquad \forall \; i,j,j',c,c',k \qquad (15)$$

$$Y_j + Z_{jc} - 1 \le Y_{jc} \qquad \forall \; j,c \qquad (16)$$

$$Y_j,\; L_{ikj},\; Z_{jc} = 0 \text{ or } 1 \qquad \forall \; i,j,k,c \qquad (17)$$

$$Y_{ikj'j}\;,\; Y_{jc}\;,\; Z^{ik}_{jcj'c'} \;\ge 0 \qquad \forall \; i,j,j'.c,c' \qquad (18)$$

The objective function (8) minimizes the operating cost, cost of refixturing and the cost of inter-cell movement. The constraints are given (9) to (16). Constraints (9) to (13) correspond to (2) to (6) of model A1. Constraints (17) and (18) indicate 0-1 and continuous variables. As the variables $Y_{ikj'j}$, Y_{jc}, $Z^{ik}_{jcj'c'}$ are associated with minimizing the cost there is no need for the inclusion of the corresponding family of inequalities of the form

$$X_{ik-1j'} \ge Y_{ikj'j}\;,\; X_{ikj} \ge Y_{ikj'j}$$

$$Y_j \ge Y_{jc}\;,\; Z_{jc} \ge Y_{jc}$$

$$X_{ikj} \ge Z^{ik}_{jcj'c'}\;,\; Z_{jc} \ge Z^{ik}_{jcj'c'}\;,\; X_{ik-1j'} \ge Z^{ik}_{jcj'c'}\;,\; Z_{j'c'} \ge Z^{ik}_{jcj'c'}$$

in the constraint set.

APPENDIX A.2

LINEARIZATION STRATEGY

The objective function and constraints of model A1 containes product terms of 0-1 variables. Commercially available integer programming packages require the model to be in a completely linear form. Therefore a linearization strategy based on Watters (1967) and modified by Glover and Woolsey (1974) was proposed. The procedure is as follows.

1. Replace each product term $(X_j)^k$. $(k \geq 0)$ by X_j .
2. Replace each product term "g" by a continuous linearization variable Z_g with the addition of the following set of constraints for each product term "g".

$$\sum_{j \in S'_g} X_j - Z_g \leq (n_g - 1) \qquad \text{for all } g \in P_I$$

$$X_j \geq Z_g \begin{cases} \text{for all } j \in S'_g \\ \text{for all } g \in p_I \end{cases}$$

$Z_g \geq 0$: $X_j = 0,1$ for all for all $j \in S'_g$, for all $g \in P_I$.

where P_I = Index set of all linearization variables in the formulation and S'_g = Index set of 0-1 integer variables occurring in a given product term g, $(g \in P_I)$.

Example:

To linearize the term X1.X2.X3, replace it by a continuous linearization variable Z1 and add the following constraints,

$$X1 + X2 + X3 - Z1 \leq (3 - 1)$$

$$X1 \geq Z1 \ ; \quad X2 \geq Z1$$

$$X3 \geq Z1 \ ; \quad Z1 \geq 0; \quad X1, X2, X3 = 0,1$$

APPENDIX A.3

EQUATIONS OF FORMULATIONS

Model 1 (Base Case):

```
MIN     6 Y1211 + 11 Y1212 + 9 Y1213 + 10 Y1221 + 12 Y1222 + 8 Y1223
     + 10 Y2213 + 9 Y2223 + 11 Y2231 + 7 Y2232 + 12 Y2233 + 9.05 X1211
     + 10.11 X1212 + 9.15 X1213 + 9.12 X1221 + 10.12 X1222 + 9.11 X1223
     + 8.2 X2213 + 8.96 X2223 + 12.2 X2231 + 10.11 X2232 + 8.1 X2233
     + 10.04 X2212 + 12.04 X2221 + 7 X111 + 9 X121 + 11 X211 + 10 X221
     + 9 X231 + 12 X2211 + 10 X2222
SUBJECT TO
       2)   X111 + X121 =    175
       3)   X211 + X221 + X231 =    200
       4)   X1211 + X1212 + X1213 - X111 =    0
       5)   X1221 + X1222 + X1223 - X121 =    0
       6)   X2213 + X2212 - X211 + X2211 =    0
       7)   X2223 + X2221 - X221 + X2222 =    0
       8)   X2231 + X2232 + X2233 - X231 =    0
       9)   2 Y1211 + 5 Y1221 + 4 Y2231 + 8.02 X1211 + 8.07 X1221
     + 10.08 X2231 + 10 X2221 + 6 X111 + 9 X211 + 10 X2211 <=   2500
      10)   5 Y1212 + 4 Y1222 + 2 Y2232 + 6.1 X1212 + 6.09 X1222
     + 7.04 X2232 + 7 X2212 + 4 X121 + 6 X221 + 7 X2222 <=   1500
      11)   4 Y1213 + 3 Y1223 + 3 Y2213 + 3 Y2223 + 4 Y2233 + 9.1 X1213
     + 9.05 X1223 + 11.09 X2213 + 11.08 X2223 + 11.01 X2233 + 8 X231
     <=   1500
      12) - 100000 Y1211 + X1211 <=   0
      13) - 100000 Y1212 + X1212 <=   0
      14) - 100000 Y1213 + X1213 <=   0
      15) - 100000 Y1221 + X1221 <=   0
      16) - 100000 Y1222 + X1222 <=   0
```

17) - 100000 Y1223 + X1223 <= 0
18) - 100000 Y2211 + X2211 <= 0
19) - 100000 Y2212 + X2212 <= 0
20) - 100000 Y2213 + X2213 <= 0
21) - 100000 Y2221 + X2221 <= 0
22) - 100000 Y2222 + X2222 <= 0
23) - 100000 Y2223 + X2223 <= 0
24) - 100000 Y2231 + X2231 <= 0
25) - 100000 Y2232 + X2232 <= 0
26) - 100000 Y2233 + X2233 <= 0

where $Xi1j$, $Xikj'j \geq 0$, $Yikj'j = 0$ or 1 $\forall\ i,k,j',j$

Model 2 (Base Case)

```
MIN     6 Y12111 + 11 Y12121 + 9 Y12131 + 10 Y12211 + 12 Y12221
     + 8 Y12231 + 10 Y22131 + 9 Y22231 + 11 Y22311 + 7 Y22321 + 12 Y22331
     + 6 Y12112 + 11 Y12122 + 9 Y12132 + 10 Y12212 + 12 Y12222 + 8 Y12232
     + 10 Y22132 + 9 Y22232 + 11 Y22312 + 7 Y22322 + 12 Y22332
     + 1.1 X121122 + 1.1 X121221 + 1.01 X121132 + 1.01 X121231
     + 1.01 X122112 + 1.01 X122211 + 0.99 X122132 + 0.99 X122231
     + 0.96 X121211 + 0.96 X121112 + 1.13 X122221 + 1.13 X122122
     + 1.25 X221211 + 1.25 X221112 + 1.05 X222221 + 1.05 X222122
     + 0.95 X223231 + 0.95 X223132 + 1.05 X221122 + 1.05 X221221
     + 0.95 X221132 + 0.95 X221231 + 1.25 X222112 + 1.25 X222211
     + 0.94 X222132 + 0.94 X222231 + 1.35 X223112 + 1.35 X223211
     + 1.1 X223122 + 1.1 X223221 + 0.7 X1111 + 0.7 X1112 + 0.9 X1121
     + 0.9 X1122 + 1.1 X2111 + 1.1 X2112 + X2121 + X2122 + 0.9 X2131
     + 0.9 X2132 + 0.95 X121111 + 0.95 X121212 + 1.09 X121121
     + 1.09 X121222 + X121131 + X121232 + X122111 + X122212 + 1.12 X122121
     + 1.12 X122222 + 0.98 X122131 + 0.98 X122232 + 1.2 X221111
     + 1.2 X221212 + X221121 + X221222 + 0.9 X221131 + 0.9 X221232
     + 1.2 X222111 + 1.2 X222212 + X222121 + X222222 + 0.89 X222131
     + 0.89 X222232 + 1.3 X223111 + 1.3 X223212 + 1.05 X223121
     + 1.05 X223222 + 0.9 X223131 + 0.9 X223232
SUBJECT TO
       2)   X1111 + X1112 + X1121 + X1122 =    350
       3)   X2111 + X2112 + X2121 + X2122 + X2131 + X2132 =    450
       4)   X121122 + X121132 + X121112 - X1111 + X121111 + X121121
     + X121131 =    0
       5)   X121221 + X121231 + X121211 - X1112 + X121212 + X121222
     + X121232 =    0
       6)   X122112 + X122132 + X122122 - X1121 + X122111 + X122121
     + X122131 =    0
       7)   X122211 + X122231 + X122221 - X1122 + X122212 + X122222
     + X122232 =    0
       8)   X221112 + X221122 + X221132 - X2111 + X221111 + X221121
     + X221131 =    0
       9)   X221211 + X221221 + X221231 - X2112 + X221212 + X221222
```

```
+ X221232 =     0
 10)    X222122 + X222112 + X222132 - X2121 + X222111 + X222121
+ X222131 =     0
 11)    X222221 + X222211 + X222231 - X2122 + X222212 + X222222
+ X222232 =     0
 12)    X223132 + X223112 + X223122 - X2131 + X223111 + X223121
+ X223131 =     0
 13)    X223231 + X223211 + X223221 - X2132 + X223212 + X223222
+ X223232 =     0
 14)    2 Y12111 + 5 Y12211 + 4 Y22311 + 8.07 X122211 + 8.02 X121211
+ 10 X221211 + 10 X222211 + 10.08 X223211 + 6 X1111 + 17 X2111
+ 8.02 X121111 + 8.07 X122111 + 10 X221111 + 10 X222111
+ 10.08 X223111 <=    2000
 15)    2 Y12112 + 5 Y12212 + 4 Y22312 + 8.07 X122112 + 8.02 X121112
+ 10 X221112 + 10 X222112 + 10.08 X223112 + 6 X1112 + 17 X2112
+ 8.02 X121212 + 8.07 X122212 + 10 X221212 + 10 X222212
+ 10.08 X223212 <=    4000
 16)    5 Y12121 + 4 Y12221 + 2 Y22321 + 6.1 X121221 + 6.09 X122221
+ 7 X222221 + 7 X221221 + 7.04 X223221 + 4 X1121 + 6 X2121
+ 6.14 X121121 + 6.09 X122121 + 7 X221121 + 7 X222121 + 7 X223121
<=    2000
 17)    5 Y12122 + 4 Y12222 + 2 Y22322 + 6.14 X121122 + 6.09 X122122
+ 7 X222122 + 7 X221122 + 7 X223122 + 4 X1122 + 6 X2122 + 6.1 X121222
+ 6.09 X122222 + 7 X221222 + 7 X222222 + 7.04 X223222 <=    2000
 18)    4 Y12131 + 3 Y12231 + 3 Y22131 + 3 Y22231 + 4 Y22331
+ 9.1 X121231 + 9.05 X122231 + 11.1 X223231 + 11 X221231
+ 11.08 X222231 + 8 X2131 + 9.1 X121131 + 9.05 X122131 + 11.18 X221131
+ 11.08 X222131 + 11.1 X223131 <=    2000
 19)    4 Y12132 + 3 Y12232 + 3 Y22132 + 3 Y22232 + 4 Y22332
+ 9.1 X121132 + 9.05 X122132 + 11.1 X223132 + 11.18 X221132
+ 11.08 X222132 + 8 X2132 + 9.1 X121232 + 9.05 X122232 + 11 X221232
+ 11.08 X222232 + 11.1 X223232 <=    0
 20) - 1000000 Y12111 + X121211 + X121111 <=    0
 21) - 1000000 Y12121 + X121221 + X121121 <=    0
 22) - 1000000 Y12131 + X121231 + X121131 <=    0
 23) - 1000000 Y12231 + X122231 + X122131 <=    0
```

```
24) - 1000000 Y12221 + X122221 + X122121 <=   0
25) - 1000000 Y12211 + X122211 + X122111 <=   0
26) - 1000000 Y22111 + X221211 + X221111 <=   0
27) - 1000000 Y22121 + X221221 + X221121 <=   0
28) - 1000000 Y22131 + X221231 + X221131 <=   0
29) - 1000000 Y22211 + X222211 + X222111 <=   0
30) - 1000000 Y22221 + X222221 + X222121 <=   0
31) - 1000000 Y22231 + X222231 + X222131 <=   0
32) - 1000000 Y22311 + X223211 + X223111 <=   0
33) - 1000000 Y22321 + X223221 + X223121 <=   0
34) - 1000000 Y22331 + X223231 + X223131 <=   0
35) - 1000000 Y12112 + X121112 + X121212 <=   0
36) - 1000000 Y12122 + X121122 + X121222 <=   0
37) - 1000000 Y12132 + X121132 + X121232 <=   0
38) - 1000000 Y12232 + X122132 + X122232 <=   0
39) - 1000000 Y12222 + X122122 + X122222 <=   0
40) - 1000000 Y12212 + X122112 + X122212 <=   0
41) - 1000000 Y22112 + X221112 + X221212 <=   0
42) - 1000000 Y22122 + X221122 + X221222 <=   0
43) - 1000000 Y22132 + X221132 + X221232 <=   0
44) - 1000000 Y22212 + X222112 + X222212 <=   0
45) - 1000000 Y22222 + X222122 + X222222 <=   0
46) - 1000000 Y22232 + X222132 + X222232 <=   0
47) - 1000000 Y22312 + X223112 + X223212 <=   0
48) - 1000000 Y22322 + X223122 + X223222 <=   0
49) - 1000000 Y22332 + X223132 + X223232 <=   0
```

where Xikj'c'jc ≥ 0, Yikj'jc = o or 1 ∀ j,c,j',c',i,k

Model 3 (Base Case)

```
MIN     6 Y12111 + 11 Y12121 + 9 Y12131 + 10 Y12211 + 12 Y12221
     + 8 Y12231 + 10 Y22131 + 9 Y22231 + 11 Y22311 + 7 Y22321 + 12 Y22331
     + 6 Y12112 + 11 Y12122 + 9 Y12132 + .0 Y12212 + 12 Y12222 + 8 Y12232
     + 10 Y22132 + 9 Y22232 + 11 Y22312 + 7 Y22322 + 12 Y22332
     + 1.1 X121122 + 1.1 X121221 + 1.01 X121132 + 1.01 X121231
     + 1.01 X122112 + 1.01 X122211 + 0.99 X122132 + 0.99 X122231
     + 0.96 X121211 + 0.96 X121112 + 1.13 X122221 + 1.13 X122122
     + 1.25 X221211 + 1.25 X221112 + 1.05 X222221 + 1.05 X222122
     + 0.95 X223231 + 0.95 X223132 + 1.05 X221122 + 1.05 X221221
     + 0.95 X221132 + 0.95 X221231 + 1.25 X222112 + 1.25 X222211
     + 0.94 X222132 + 0.94 X222231 + 1.35 X223112 + 1.35 X223211
     + 1.1 X223122 + 1.1 X223221 + 0.7 X1111 + 0.7 X1112 + 0.9 X1121
     + 0.9 X1122 + 1.1 X2111 + 1.1 X2112 + X2121 + X2122 + 0.9 X2131
     + 0.9 X2132 + 0.95 X121111 + 0.95 X121212 + 1.09 X121121
     + 1.09 X121222 + X121131 + X121232 + X122111 + X122212 + 1.12 X122121
     + 1.12 X122222 + 0.98 X122131 + 0.98 X122232 + 1.2 X221111
     + 1.2 X221212 + X221121 + X221222 + 0.9 X221131 + 0.9 X221232
     + 1.2 X222111 + 1 2 X222212 + X222121 + X222222 + 0.89 X222131
     + 0.89 X222232 + 1.3 X223111 + 1.3 X223212 + 1.05 X223121
     + 1.05 X223222 + 0.9 X223131 + 0.9 X223232

SUBJECT TO
       2)   X1111 + X1112 + X1121 + X1122 =    200
       3)   X2111 + X2112 + X2121 + X2122 + X2131 + X2132 =    250
       4)   X121122 + X121132 + X121112 - X1111 + X121111 + X121121
     + X121131 =    0
       5)   X121221 + X121231 + X121211 - X1112 + X121212 + X121222
     + X121232 =    0
       6)   X122112 + X122132 + X122122 - X1121 + X122111 + X122121
     + X122131 =    0
       7)   X122211 + X122231 + X122221 - X1122 + X122212 + X122222
     + X122232 =    0
       8)   X221112 + X221122 + X221132 - X2111 + X221111 + X221121
     + X221131 =    0
```

```
 9)   X221211 + X221221 + X221231 - X2112 + X221212 + X221222
+ X221232 =    0
 10)   X222122 + X222112 + X222132 - X2121 + X222111 + X222121
+ X222131 =    0
 11)   X222221 + X222211 + X222231 - X2122 + X222212 + X222222
+ X222232 =    0
 12)   X223132 + X223112 + X223122   X2131 + X223111 + X223121
+ X223131 =    0
 13)   X223231 + X223211 + X223221 - X2132 + X223212 + X223222
+ X223232 =    0
 14) - 2500 Z11 + 2 Y12111 + 5 Y12211 + 4 Y22311 + 8.07 X122211
+ 8.02 X121211 + 10 X221211 + 10 X222211 + 10.08 X223211 + 6 X1111
+ 17 X2111 + 8.02 X121111 + 8.07 X122111 + 10 X221111 + 10 X222111
+ 10.08 X223111 <=   0
 15) - 2500 Z12 + 2 Y12112 + 5 Y12212 + 4 Y22312 + 8.07 X122112
+ 8.02 X121112 + 10 X221112 + 10 X222112 + 10.08 X223112 + 6 X1112
+ 17 X2112 + 8.02 X121212 + 8.07 X122212 + 10 X221212 + 10 X222212
+ 10.08 X223212 <=   0
 16) - 2000 Z21 + 5 Y12121 + 4 Y12221 + 2 Y22321 + 6.1 X121221
+ 6.09 X122221 + 7 X222221 + 7 X221221 + 7.04 X223221 + 4 X1121
+ 6 X2121 + 6.14 X121121 + 6.09 X122121 + 7 X221121 + 7 X222121
+ 7 X223121 <=   0
 17) - 2000 Z22 + 5 Y12122 + 4 Y12222 + 2 Y22322 + 6.14 X121122
+ 6.09 X122122 + 7 X222122 + 7 X221122 + 7 X223122 + 4 X1122 + 6 X2122
+ 6.1 X121222 + 6.09 X122222 + 7 X221222 + 7 X222222 + 7.04 X223222
<=   0
 18) - 2000 Z31 + 4 Y12131 + 3 Y12231 + 3 Y22131 + 3 Y22231 + 4 Y22331
+ 9.1 X121231 + 9.05 X122231 + 11.1 X223231 + 11 X221231
+ 11.08 X222231 + 8 X2131 + 9.1 X121131 + 9.05 X122131 + 11.18 X221131
+ 11.08 X222131 + 11.1 X223131 <=   0
 19) - 2000 Z32 + 4 Y12132 + 3 Y12232 + 3 Y22132 + 3 Y22232 + 4 Y22332
+ 9.1 X121132 + 9.05 X122132 + 11.1 X223132 + 11.18 X221132
+ 11.08 X222132 + 8 X2132 + 9.1 X121232 + 9.05 X122232 + 11 X221232
+ 11.08 X222232 + 11.1 X223232 <=   0
 20)   Z11 + Z21 + Z31 <=   2
 21)   Z12 + Z22 + Z32 <=   2
```

```
22)   Z11 + Z12 <=   1
23)   Z21 + Z22 <=   1
24)   Z31 + Z32 <=   1
25) - 1000000 Y12111 + X121211 + X121111 <=   0
26) - 1000000 Y12121 + X121221 + X121121 <=   0
27) - 1000000 Y12131 + X121231 + X121131 <=   0
28) - 1000000 Y12231 + X122231 + X122131 <=   0
29) - 1000000 Y12221 + X122221 + X122121 <=   0
30) - 1000000 Y12211 + X122211 + X122111 <=   0
31) - 1000000 Y22111 + X221211 + X221111 <=   0
32) - 1000000 Y22121 + X221221 + X221121 <=   0
33) - 1000000 Y22131 + X221231 + X221131 <=   0
34) - 1000000 Y22211 + X222211 + X222111 <=   0
35) - 1000000 Y22221 + X222221 + X222121 <=   0
36) - 1000000 Y22231 + X222231 + X222131 <=   0
37) - 1000000 Y22311 + X223211 + X223111 <=   0
38) - 1000000 Y22321 + X223221 + X223121 <=   0
39) - 1000000 Y22331 + X223231 + X223131 <=   0
40) - 1000000 Y12112 + X121112 + X121212 <=   0
41) - 1000000 Y12122 + X121122 + X121222 <=   0
42) - 1000000 Y12132 + X121132 + X121232 <=   0
43) - 1000000 Y12232 + X122132 + X122232 <=   0
44) - 1000000 Y12222 + X122122 + X122222 <=   0
45) - 1000000 Y12212 + X122112 + X122212 <=   0
46) - 1000000 Y22112 + X221112 + X221212 <=   0
47) - 1000000 Y22122 + X221122 + X221222 <=   0
48) - 1000000 Y22132 + X221132 + X221232 <=   0
49) - 1000000 Y22212 + X222112 + X222212 <=   0
50) - 1000000 Y22222 + X222122 + X222222 <=   0
51) - 1000000 Y22232 + X222132 + X222232 <=   0
52) - 1000000 Y22312 + X223112 + X223212 <=   0
53) - 1000000 Y22322 + X223122 + X223222 <=   0
54) - 1000000 Y22332 + X223132 + X223232 <=   0
```

where Xikj'c'jc >=0 , Yikj'jc , Zjc = 0 or 1   ∀ j,c,j',c',i,k

Model 4 (Base Case)

```
MIN      5500 Z11 + 5500 Z12 + 6500 Z21 + 6500 Z22 + 6000 Z31 + 6000 Z32
     + 6 Y12111 + 11 Y12121 + 9 Y12131 + 10 Y12211 + 12 Y12221 + 8 Y12231
     + 10 Y22131 + 9 Y22231 + 11 Y22311 + 7 Y22321 + 12 Y22331 + 6 Y12112
     + 11 Y12122 + 9 Y12132 + 10 Y12212 + 12 Y12222 + 8 Y12232 + 10 Y22132
     + 9 Y22232 + 11 Y22312 + 7 Y22322 + 12 Y22332 + 1.1 X121122
     + 1.1 X121221 + 1.01 X121132 + 1.01 X121231 + 1.01 X122112
     + 1.01 X122211 + 0.99 X122132 + 0.99 X122231 + 0.96 X121211
     + 0.96 X121112 + 1.13 X122221 + 1.13 X122122 + 1.25 X221211
     + 1.25 X221112 + 1.05 X222221 + 1.05 X222122 + 0.95 X223231
     + 0.95 X223132 + 1.05 X221122 + 1.05 X221221 + 0.95 X221132
     + 0.95 X221231 + 1.25 X222112 + 1.25 X222211 + 0.94 X222132
     + 0.94 X222231 + 1.35 X223112 + 1.35 X223211 + 1.1 X223122
     + 1.1 X223221 + 0.7 X1111 + 0.7 X1112 + 0.9 X1121 + 0.9 X1122
     + 1.1 X2111 + 1.1 X2112 + X2121 + X2122 + 0.9 X2131 + 0.9 X2132
     + 0.95 X121111 + 0.95 X121212 + 1.09 X121121 + 1.09 X121222 + X121131
     + X121232 + X122111 + X122212 + 1.12 X122121 + 1.12 X122222
     + 0.98 X122131 + 0.98 X122232 + 1.2 X221111 + 1.2 X221212 + X221121
     + X221222 + 0.9 X221131 + 0.9 X221232 + 1.2 X222111 + 1.2 X222212
     + X222121 + X222222 + 0.89 X222131 + 0.89 X222232 + 1.3 X223111
     + 1.3 X223212 + 1.05 X223121 + 1.05 X223222 + 0.9 X223131
     + 0.9 X223232


SUBJECT TO
          2)   X1111 + X1112 + X1121 + X1122 =     500
          3)   X2111 + X2112 + X2121 + X2122 + X2131 + X2132 =     525
          4)   X121122 + X121132 + X121112 - X1111 + X121111 + X121121
     + X121131 =     0
          5)   X121221 + X121231 + X121211 - X1112 + X121212 + X121222
     + X121232 =     0
          6)   X122112 + X122132 + X122122 - X1121 + X122111 + X122121
     + X122131 =     0
          7)   X122211 + X122231 + X122221 - X1122 + X122212 + X122222
     + X122232 =     0
          8)   X221112 + X221122 + X221132 - X2111 + X221111 + X221121
```

```
+ X221131 =    0
  9)   X221211 + X221221 + X221231 - X2112 + X221212 + X221222
+ X221232 =    0
 10)   X222122 + X222112 + X222132 - X2121 + X222111 + X222121
+ X222131 =    0
 11)   X222221 + X222211 + X222231 - X2122 + X222212 + X222222
+ X222232 =    0
 12)   X223132 + X223112 + X223122 - X2131 + X223111 + X223121
+ X223131 =    0
 13)   X223231 + X223211 + X223221 - X2132 + X223212 + X223222
+ X223232 =    0
 14) - 2500 W11 + 2 Y12111 + 5 Y12211 + 4 Y22311 + 8.07 X122211
+ 8.02 X121211 + 10 X221211 + 10 X222211 + 10.08 X223211 + 6 X1111
+ 17 X2111 + 8.02 X121111 + 8.07 X122111 + 10 X221111 + 10 X222111
+ 10.08 X223111 <=    0
 15) - 2500 W12 + 2 Y12112 + 5 Y12212 + 4 Y22312 + 8.07 X122112
+ 8.02 X121112 + 10 X221112 + 10 X222112 + 10.08 X223112 + 6 X1112
+ 17 X2112 + 8.02 X121212 + 8.07 X122212 + 10 X221212 + 10 X222212
+ 10.08 X223212 <=    0
 16) - 2000 W21 + 5 Y12121 + 4 Y12221 + 2 Y22321 + 6.1 X121221
+ 6.09 X122221 + 7 X222221 + 7 X221221 + 7.04 X223221 + 4 X1121
+ 6 X2121 + 6.14 X121121 + 6.09 X122121 + 7 X221121 + 7 X222121
+ 7 X223121 <=    0
 17) - 2000 W22 + 5 Y12122 + 4 Y12222 + 2 Y22322 + 6.14 X121122
+ 6.09 X122122 + 7 X222122 + 7 X221122 + 7 X223122 + 4 X1122 + 6 X2122
+ 6.1 X121222 + 6.09 X122222 + 7 X221222 + 7 X222222 + 7.04 X223222
<=    0
 18) - 2000 W31 + 4 Y12131 + 3 Y12231 + 3 Y22131 + 3 Y22231 + 4 Y22331
+ 9.1 X121231 + 9.05 X122231 + 11.1 X223231 + 11 X221231
+ 11.08 X222231 + 8 X2131 + 9.1 X121131 + 9.05 X122131 + 11.18 X221131
+ 11.08 X222131 + 11.1 X223131 <=    0
 19) - 2000 W32 + 4 Y12132 + 3 Y12232 + 3 Y22132 + 3 Y22232 + 4 Y22332
+ 9.1 X121132 + 9.05 X122132 + 11.1 X223132 + 11.18 X221132
+ 11.08 X222132 + 8 X2132 + 9.1 X121232 + 9.05 X122232 + 11 X221232
+ 11.08 X222232 + 11.1 X223232 <=    0
 20)   W11 + W21 + W31 <=    3
```

```
21)   W12 + W22 + W32 <=   3
22) - 1000000 Y12111 + X121211 + X121111 <=   0
23) - 1000000 Y12121 + X121221 + X121121 <=   0
24) - 1000000 Y12131 + X121231 + X121131 <=   0
25) - 1000000 Y12231 + X122231 + X122131 <=   0
26) - 1000000 Y12221 + X122221 + X122121 <=   0
27) - 1000000 Y12211 + X122211 + X122111 <=   0
28) - 1000000 Y22111 + X221211 + X221111 <=   0
29) - 1000000 Y22121 + X221221 + X221121 <=   0
30) - 1000000 Y22131 + X221231 + X221131 <=   0
31) - 1000000 Y22211 + X222211 + X222111 <=   0
32) - 1000000 Y22221 + X222221 + X222121 <=   0
33) - 1000000 Y22231 + X222231 + X222131 <=   0
34) - 1000000 Y22311 + X223211 + X223111 <=   0
35) - 1000000 Y22321 + X223221 + X223121 <=   0
36) - 1000000 Y22331 + X223231 + X223131 <=   0
37) - 1000000 Y12112 + X121112 + X121212 <=   0
38) - 1000000 Y12122 + X121122 + X121222 <=   0
39) - 1000000 Y12132 + X121132 + X121232 <=   0
40) - 1000000 Y12232 + X122132 + X122232 <=   0
41) - 1000000 Y12222 + X122122 + X122222 <=   0
42) - 1000000 Y12212 + X122112 + X122212 <=   0
43) - 1000000 Y22112 + X221112 + X221212 <=   0
44) - 1000000 Y22122 + X221122 + X221222 <=   0
45) - 1000000 Y22132 + X221132 + X221232 <=   0
46) - 1000000 Y22212 + X222112 + X222212 <=   0
47) - 1000000 Y22222 + X222122 + X222222 <=   0
48) - 1000000 Y22232 + X222132 + X222232 <=   0
49) - 1000000 Y22312 + X223112 + X223212 <=   0
50) - 1000000 Y22322 + X223122 + X223222 <=   0
51) - 1000000 Y22332 + X223132 + X223232 <=   0
```

Xikj'c'jc ≥ 0 , Yikj'jc = 0 or 1 , Wjc = integer   ∀ j,c,j',c',i,k

VITA AUCTORIS

1968	Born in Kumbakonum, India on 30th of August
1985	Completed High school education from R. S. Krishnan Hr. Sec. School, Trichy, India
1989	Graduated from Birla Institute of Technology and Science, Pilani, India with a Bachelor's degree (Honours) in Mechanical Engineering
1990	Currently a candidate for the degree of Master of Applied Science in Industrial Engineering at the University of Windsor, Windsor, Canada