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# Modeling reliability considerations in the design and analysis of cellular manufacturing systems.

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# MODELING RELIABILITY CONSIDERATIONS IN THE DESIGN AND ANALYSIS OF CELLULAR MANUFACTURING SYSTEMS

by

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**ABSTRACT**

Reliability plays a vital role in the overall performance of cellular manufacturing systems (CMSs). Machine failures significantly impact the fulfillment of due dates and other performance criteria, despite the option of part rerouting to alternative workstations. These facts suggest a need for the consideration of machine reliability during the operation allocation process.

Attempting to improve a system's reliability invariably results in higher costs. It follows that the ideal strategy for achieving optimum balance lies in an approach that integrates both cost and reliability information. A mixed integer multi-objective mathematical programming model that incorporates machine reliability and cost considerations is developed for the design of CMSs. The model selects processing route for each part type which maximizes the overall system reliability of machines along the route, while minimizing the overall costs. The proposed approach provides flexible routing, ensuring high CMS performance by minimizing the impact of machine failure through the provision of alternative process routes. To account for the constant and increasing failure pattern of manufacturing machines, the CMS design model considers both the exponential and Weibull distribution approaches. A performance evaluation criterion in terms of system availability for the part-process plan assignment based on the exponential distribution is also developed. Applicability of the model is demonstrated by solving example problems by following the $\epsilon$-constraint approach.

Optimization techniques for solving such models for large practical-size problems require a substantial amount of time and memory space; therefore, a heuristic, based on the basic steps to simulated annealing and solution generation procedure of genetic algorithm is developed. The heuristic is evaluated by comparing the solutions generated by the heuristic with the LP relaxation solution for the large problems and optimal solution for the smaller-sized problems. The results reveal that the heuristic performs well in various problem instances for reliability and cost combinations. The sensitivity of the model outputs to key factors has also been investigated

A reliability-based, preventive maintenance (PM) planning model is also incorporated, allowing CMS to restrict deterioration of machines due to usage and age and improve system reliability. A procedure for the integration of PM planning into the CMS design model is included for overall reliability and cost improvement of the CMS. Example problems are solved to illustrate the model's applicability.

## DEDICATION

To my son Amit Das and Sumit Das

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## NOMENCLATURE

**Indices and sets**

| | |
|---|---|
| $c \in \{1,2,..,C\}$ | cells |
| $i \in \{1,2,..,n\}$ | part types |
| $j \in \{1,2,..,m\}$ | machines |
| $J_{ipo} \subset \{j = 1,2,..,m\}$ | set of machines that can perform operation *o* of *(ip)* |
| $k \in \{1,2,..,N\}$ | index of the states of a cell in a Markovian transition |
| $o \in \{1,2,..,O(ip)\}$ | operations for part type *i* following process plan *p* |
| $p \in \{1,2,..,P(i)\}$ | process plan for part type *i* |
| *ip* | a part type-process plan combination |
| $s_j \in \{0,1\}$ | state of machine *j;* 1 = operating, 0 = not operating |
| $w_k = \{s_1.s_2,...s_m\}$ | cell states with *m* machines where each $s_j \in \{0,1\}$ |
| $W = \{w_1.w_2,...w_N\}$ | cell state space |
| $\Gamma$ | gamma function symbol |

**Parameters**

| | |
|---|---|
| $A_j\ (T)$ | availability of machine *j* in time period *T* |
| $b_j$ | amount of time available on machine *j* during the planned manufacturing period |
| $CO_{oj}(ip)$ | cost of performing operation *o* of (*ip*) on machine *j* |
| $cp_j$ | penalty cost for the non utilization proportion of machine *j* |
| $CR_{oj}(ip)$ | cost of refixturing a unit of (*ip*) for operation *o* on machine *j* |
| $d_i$ | demand for part type *i* distributed uniformly over the planning period |
| $H_{ijc\hat{j}\hat{c}}$ | cost of moving part type *i* from machine *j* in cell *c* to machine $\hat{j}$ in cell $\hat{c}$ for performing the next operation |
| $MS_{w_k j}$ | an indicator of the state of machine *j* in CMS state $w_k$; equals 1 if machine *j* is in operating condition in CMS state $w_k$; 0 otherwise |

| | |
|---|---|
| $MTBF_j$ | mean time between failures for machine $j$ |
| $MTTR_j$ | mean time to repair for machine $j$ |
| $r_j$ | repair rate for machine $j$ |
| $TO_{oj}(ip)$ | time for performing operation $o$ of $(ip)$ on machine $j$ |
| $TR_{oj}(ip)$ | time for refixturing $(ip)$ for operation $o$ on machine $j$ |
| $UM$ | maximum number of machines in a cell |
| $\lambda_j$ | failure rate of machine $j$ |
| $\pi_{w_k}$ | steady-state probability of CMS being in state $w_k$ |
| $\beta_j$ | shape factor for machine $j$ |

**Decision variables**

| | |
|---|---|
| $LIR_{ip}$ | system reliability measure corresponding to the machines performing the set of operations for $(ip)$ |
| $M_{jc}$ | 1 if machine $j$ is assigned to cell $c$;<br>0 otherwise |
| $SA(ip)$ | manufacturing system availability indicator corresponding to a given $(ip)$ in relation to the CMS state space $W$ |
| $SI_{w_k}^{ipoj}$ | 1 if, in CMS state $w_k$, machine $j$ is in operating condition to perform operation $o$ of $(ip)$;<br>0 otherwise |
| $TH_{w_k}(ip)$ | 1 if CMS state $w_k$ is selected in which machines needed to perform all the operations of $(ip)$ are in operating condition;<br>0 otherwise |
| $X_{ojc}(ip)$ | 1 if operation $o$ of $(ip)$ is performed on machine $j$ in cell $c$;<br>0 otherwise |
| $Y_{ojc\hat{j}\hat{c}}(ip)$ | 1 if part type $i$ moves to machine $\hat{j}$ in cell $\hat{c}$ to perform operation $(o + 1)$ after performing operation $o$ on machine $j$ in cell $c$, following process plan $p$;<br>0 otherwise |
| $Z(ip)$ | 1 if part type $i$ is processed following process plan $p$;<br>0 otherwise |

CHAPTER 1

# INTRODUCTION

## 1.1 Overview

The manufacturing environment and its production processes have changed substantially over the past two decades. Today's markets are characterized by the consumers' demand for an ever-increasing variety of products in smaller quantities. Rather than producing as much product as possible with limited options, market characteristics need just-in-time delivery, and customized, made-to-order products. To address this challenge effectively, manufacturers must be able to improve their efficiency, response time and quality quickly, and with a minimum investment of time and capital. Classical manufacturing systems—such as process layout and product layout—do not have the ability to respond quickly to these kinds of changes. As a result, manufacturing organizations with such systems are under considerable pressure to increase their responsiveness and flexibility, decrease set-up time and lower work-in-process inventory—all while maintaining an acceptable level of efficiency.

Group Technology (GT) has been recognized as an important, disciplined approach to low-volume/high-variety and mid-volume/mid-variety manufacturing. This includes 50 to 75 percent of manufactured parts—a number that is likely to increase (Zhao and Wu, 2000). GT is a manufacturing concept that seeks to identify and group similar parts, taking advantage of their similarities during the manufacturing and design process. GT implements the general philosophy that similar tasks should be performed in a similar manner (Askin and Estrada, 1999). Cellular Manufacturing is a practical application of Group Technology in which functionally dissimilar machines are grouped together to produce a family of parts.

Cellular manufacturing has recently been recognized as an important technological innovation for improving both productivity, and competitiveness. Dedicating a machine cell to the production of a part family allows much of the efficiency of mass production to be obtained in a less repetitive batch environment. Recent and past surveys (Askin and Estrada, 1999; Wemmerlov and Johnson, 1997; Wemmerlov and Hyer, 1989) indicate the following motivating factors for the implementation of a cellular manufacturing system (CMS):

1. Reduction of cycle time;
2. Reduction of work-in-process inventory (WIP);
3. Reduction of material handling costs;

4. Reduction of set-up time;
5. Reduction of indirect costs;
6. Reduction of scrap;
7. Improved product quality;
8. Improved shop floor control; and
9. Improved job satisfaction.

Although the benefits of CMSs are well documented, and many organizations have implemented them in order to benefit from these advantages, CMSs are not without drawbacks (Boughton and Arokiam, 2000; Agarwal and Sarkis, 1998; Suresh and Meredith, 1994; Morris and Tersine, 1990; Flynn and Jacobs, 1986). The following are the main problems drawn from these studies:

1. Cell formation reduces flexibility;
2. Cell formation reduces machine utilization by dedicating machines to cells;
3. Machine break-downs make it difficult for CMS users to meet the due-date;
4. Excessive inventories may result due to the dedication of machines to machine cells.

Some of the most influential factors in CMS performance include the structure of the machine-part matrix, the stability of the manufacturing system's product mix and the reliability of machines in manufacturing cells (Seifoddini and Djassemi, 2001). Reliability plays a vital role in the overall performance of CMSs. Traditionally, cell formation and work allocation are done with the consideration that all machines are 100% reliable. However, machines are definitively not 100% reliable. Machine failures cause the greatest impact on the fulfillment of due dates and other performance criteria, even when rerouting parts to alternative workstations is an option.

Machines are a major component of CMSs, and it is often not possible to handle a machine failure as quickly as the requirements of the order demand. Delays due to machine breakdowns not only affect the production rate, they also lead to scheduling problems—decreasing the overall productivity of the manufacturing operation. This issue creates a very important need for the reliability consideration of machines in cell formation decisions and during the operation allocation process. With increasing system complexity, this requirement has become a critical issue in the production planning of CMSs.

In order to minimize such disturbances, cell formation and work allocation can be achieved by considering machine reliability. However, considering reliability and ignoring costs can result in an increase in cost for an operations sequence. It follows that an optimization approach that integrates both cost and reliability considerations may be taken to achieve the best possible situation.

## 1.2 The Benefits of Cellular Manufacturing

Cellular Manufacturing is a discrete part-manufacturing system. Its main objective is to construct machine cells, identify part families and, ultimately, allocate part families to machine cells to overcome both the inherent inefficiencies of the job shop, and the inflexibilities of the flow shop.

**The Job Shop** is the oldest and most common type of manufacturing system. A typical job shop is characterized by its flexibility, variety of products, general-purpose machine tools, manual material handling system and functional layout. In general, job shops are designed to achieve maximum flexibility so that a wide variety of products can be manufactured in small lot sizes. Lots can be produced only once or at regular intervals, in order to satisfy the continuous demand for an item. The general practice in job shops is to produce in order to build the inventory for an item, then change to another item to fill other orders. Switching from one product to another involves changes in the set-ups of many machines. Machines are grouped functionally, according to the general manufacturing process types: lathes in one department, drill presses in another, and so forth. It is very common for most departments to have more than one copy of each machine. As such, part-processing operations in departments can be continued, despite breakdowns or other machine reliability-related issues. Products manufactured in job shops have different operations and operation sequences. When the processing of a part in a department has been completed, the part is usually moved a great distance for the next stage. Figure 1.1 illustrates a job shop. Sometimes the job may have to travel the entire distance of the facility to complete all the required processes. The following are the major advantages and limitations of job shops.

**Table 1.1: Advantages and limitations of the job shop**

| Advantages | Limitations |
|---|---|
| 1. Highly flexible in terms of quantity variation (it can handle any lot size) and design variation.<br>2. Due to functional layouts, nearby machines can easily handle the breakdown of one machine.<br>3. The potential to manufacture a wide variety of parts. | 1. Highly inefficient in controlling routing patterns.<br>2. High material handling time and cost.<br>3. Large in-process inventory.<br>4. Poor quality.<br>5. Cannot fulfill due date due to flow imbalance and other inefficiencies.<br>6. Large set-up times.<br>7. High product throughput time. |

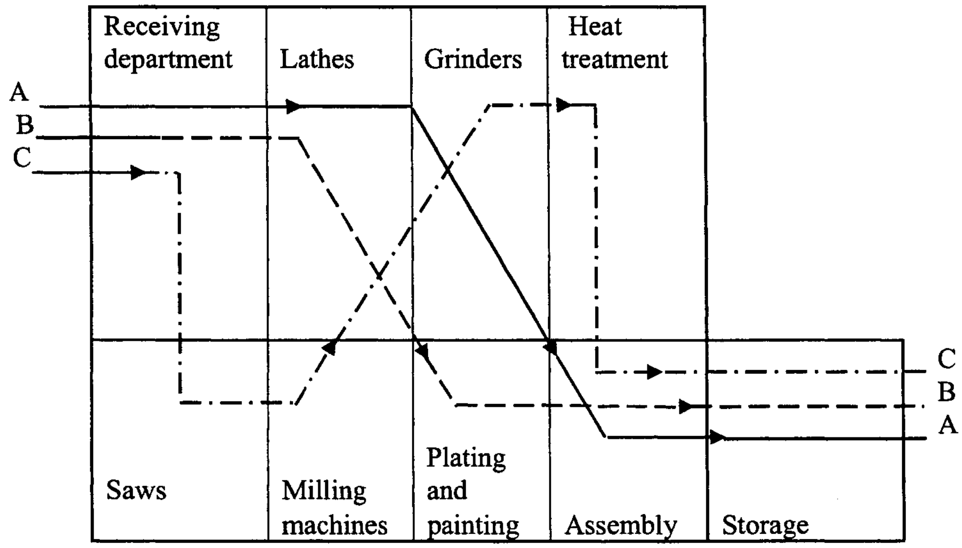


**Figure: 1.1: Job flow in a functional or process layout (job shop)**

**Flow Shops** are highly efficient in terms of throughput times, the balanced flow of jobs, quality level, material handling time considerations, the elimination of in-process inventory, high utilization rate of the machinery and low production costs. A flow shop is organized according to the sequence of operations required for a product. Specialized machines dedicated to the manufacture of the product are used to achieve high production rates. Flow line machines are expensive, so to justify the investment cost, a large volume must be produced. A major limitation of the flow line is its inflexibility, a characteristic that makes it impossible to produce products for which it is not designed. Figure 1.2 illustrates a flow shop.

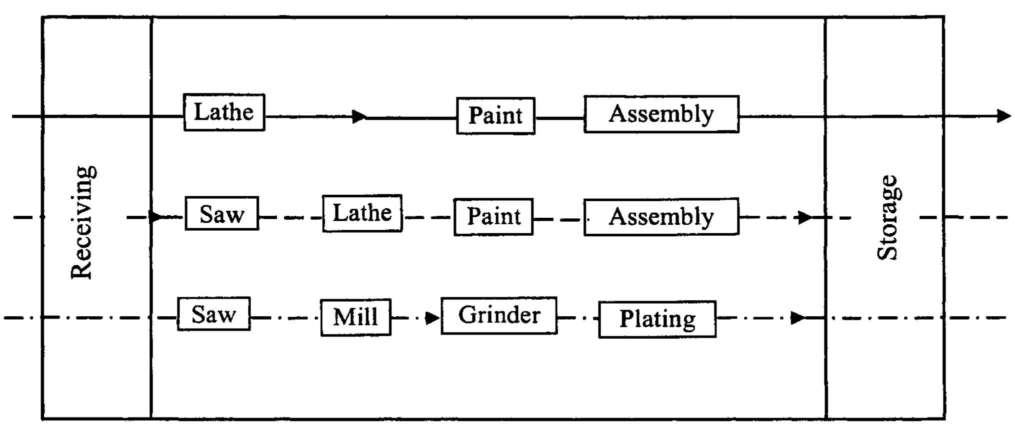


**Figure 1.2: Job flow in a line or product layout (Flow Shop)**

The inefficiencies existing in the job shop and the inflexibility of the flow shop—combined with external pressures such as technological advancements; economic uncertainties; and market pressures for quality, delivery, and customized products—are forcing discrete part manufacturers to reevaluate their manufacturing processes (Atmani et al., 1995). The present market calls for a manufacturing system that is both flexible and efficient enough to achieve the utilization level of mass production while retaining the flexibility of the job shop. This idea has given rise to the development of the flexible manufacturing system (FMS). The FMS is an automated system consisting of numerically controlled machines capable of performing multiple functions and linked together by an automated material handling system—all controlled by a computer system (Groover, 2001). In terms of flexibility and capability, a flexible manufacturing system is the most efficient for fulfilling all the requirements of the market. However, it is very costly and requires a significant financial investment. As a result, it is difficult for small and medium-sized organizations to justify an FMS to handle their production quantity within their income level. With FMSs lying beyond the reach of small manufacturing companies, these businesses must look for other options, and one possible solution is the CMS.

Figure 1.3 shows the logical conversion of the job shop and flow shop to a CMS. This reorganization leads to significant advantages, such as a reduction in material handling, refixturing and operation costs, as well as an improvement in space utilization and better control of quality and performances. Irani et al. (1999) presents the advantages of the CMS in Table 1.2.

### 1.3 Design of Cellular Manufacturing Systems

The design of cellular manufacturing systems (CMS) is a complex, multi-criterion, multi-step process. This problem has been identified as NP-complete in the CMS literature. The design of CMSs has been defined as cell formation (CF), part-family/machine cell formation, and manufacturing cell design. Given a set of part types, machines, demand for part types, and processing requirements, the design of CMSs consists of the following major steps:

- part family formation depending on processing requirements
- grouping of machines into cells
- assigning part families to cells

After completion of the design steps, a manufacturing cell configuration is created. At this stage, it is referred to as a cellular manufacturing system (CMS) which consists of a set of manufacturing cells with each cell made up of a group of machines and dedicated to producing a part family. The literature regarding CMSs follows different orders for implementing the above

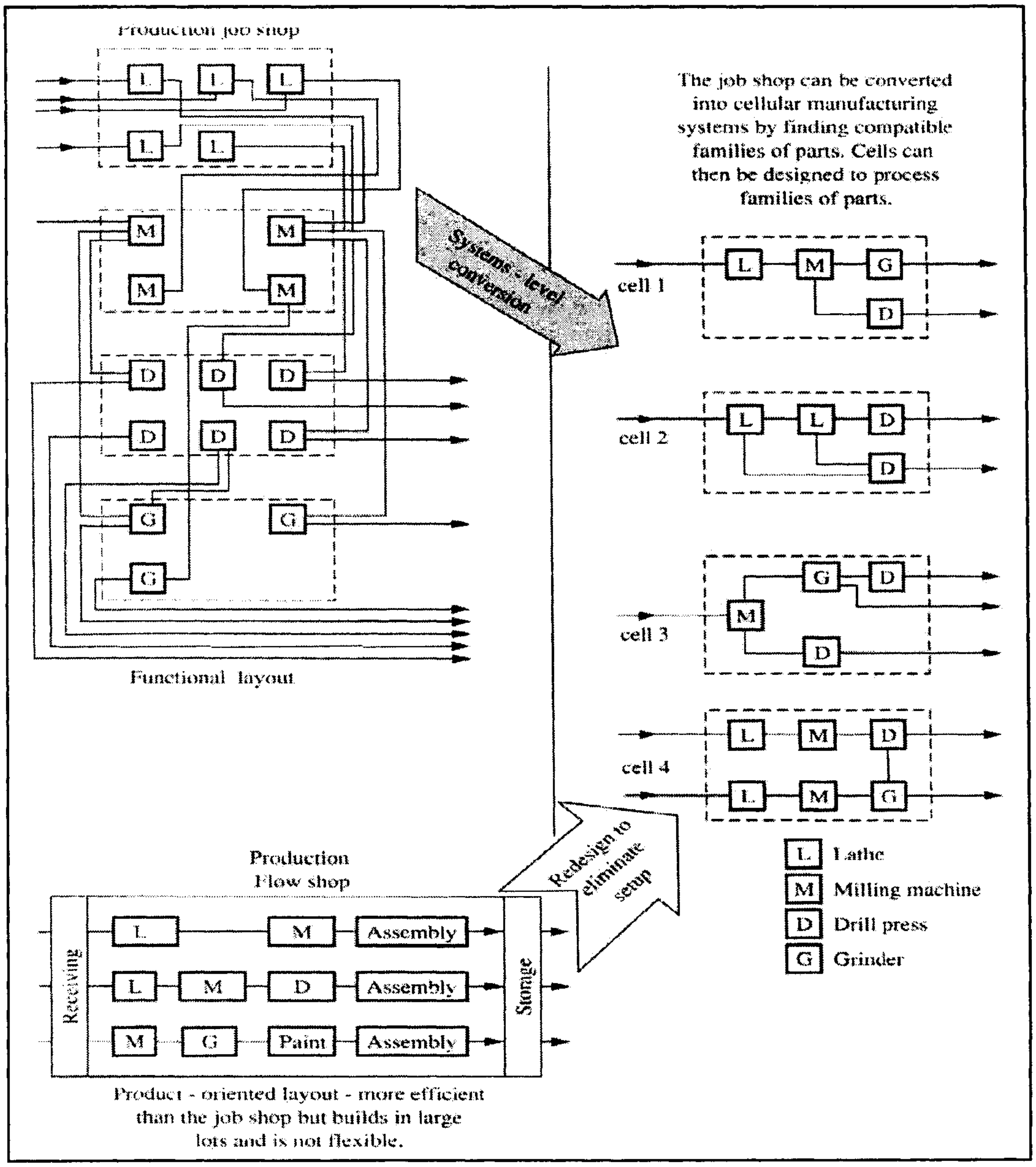


**Figure 1.3: Two classical manufacturing systems that require a system level conversion to be adapted into properly designed manufacturing cells (Black, 1991).**

**Table 1.2: Advantages of the cellular manufacturing systems (Irani et al. 1999)**

| |
|---|
| **Strategic Advantages:** |
| -On time delivery |
| -Improved response |
| -Reduced inventory |
| -Improved quality |
| -Improved workflow |
| -Increased operational flexibility |
| -Improvements in company culture |
| -Accountability |
| -Better equipment utilization |
| -Job Satisfaction |
| -Improved information flow |
| **Shop Floor Advantages:** |
| -Speed and throughputs in parts manufacturing and assembly |
| -Reduced work-in-process and finished goods inventory levels |
| -Elimination of non-value added operations (storage, inspection, and handling) |
| -Increased capacity by reducing set-up times and encouraging group scheduling of parts |
| -Accurate machine and manpower requirement analysis |
| -Improved quality by reducing scrap and process variations due to better monitoring of operations |
| -An autonomous team of workers who manage their own cells |
| -Sensible cost centers around each cell's activities and outputs |
| -Simplified production and assembly planning and control, scheduling, load balancing and capacity requirement analysis |
| -Introduction of just-in-time manufacturing and lean approach |
| -Provision of manufacturing capacity and schedule information promptly for sale and marketing personnel |

steps. Ballakur and Steudel (1987) suggest three strategies based on the approaches used to form the part families and manufacturing cells. The following strategies can be used as a basis to classify CMS design methods:

1. The part family-grouping solution strategy where part families are formed first, and then machines are grouped into cells as per part family.
2. The machine-grouping solution strategy where manufacturing cells are created first depending on the similarity of part routings, and then parts are allocated to the cells.
3. The simultaneous machine part-grouping strategy where part family and machine cells are formed simultaneously.

During the CMS design process, design objectives must be specified. The design objectives are typically to minimize the total sum of inter-cell material handling costs, equipment costs and operating costs. The following are the typical costs included in the CMS design objectives, as found in the literature:

1. Equipment costs;
2. Inter-cell material handling costs;
3. Machine relocation costs;
4. Inventory costs;
5. Machine non-utilization costs;
6. Operations costs;
7. Refixturing costs ; and
8. Set-up costs.

These objectives may conflict with each other and, as such, trade-offs may be necessary during the design process. For example, equipment costs can be avoided or reduced by including inter-cell material handling costs. On the other hand, cells can be created without inter-cell moves by increasing the number of machines. In addition to trade-offs, a practical design problem may exist in the conversion of the existing workshop into a CMS—considering the available resources. For this type of problem, the equipment cost consideration may be replaced by relocation costs, etc. Common design objectives in the existing literature include minimizing inter-cell moves, distances, processing costs and exceptional parts (parts that need more than one cell for processing).

Over and above these objectives, CMS design approaches also consider a number of strategic issues such as routing flexibility, machine flexibility, and cell layout in order to improve the performance of the CMS in the context of present market changes. Since machine reliability is critical to the performance of any manufacturing system (Seifoddini and Djassemi, 2001), another important design objective could be to maximize the system reliability of the machines. There is, however, very little research available in the CMS literature on the issue of reliability.

Furthermore, all CMS design problems need to satisfy a number of constraints:

1. Machine capacity;
2. Cell size;
3. Operational goals;
4. Machine utilization;
5. Number of cells; and
6. Production volume.

**Machine capacity** is the basic requirement of CMS design that necessitates the machines' adequate capacity to process all parts. Machine capacity is expressed in terms of the available time relevant to a machine during planning for a job order. Since machines are unreliable, effective machine capacity should be estimated by taking into account the total capacity and availability.

**Cell size** is defined as the number of machines (usually the range) allowed in a cell. Cell size needs to be controlled for several reasons. Foremost, the space limitation to accommodate the machines must be considered. Moreover, the visible control limit of a cell—if it is run by the operators—and the feeding capacity of raw materials or tools at the point of cell location must be taken into account.

**Operational goals** refer to the constraints developed in order to control the technological requirements. For example a machine can be in only one cell, a part can be processed by only one processing plan and a machine can perform more than one operation, etc.

Two types of **machine utilization** controls are usually used: the maximum utilization level—specified to ensure that the machine is not overloaded—and the minimum utilization level—specified to control the inclusion of new machines.

**The number of cells** is generally defined in terms of range. This control is used to force the model so that it does not form one cell, or too many cells, for fulfilling other requirements. This decision is influenced by the total number of machines, working groups, supports and space availability.

**Production volume** is the control used to ensure the production of required volume and volume mixes.

Extensive research has been conducted during the last three decades regarding the development of effective CMS design approaches. Reviews on these approaches can be found in Wommerlov and Hyer (1986); Singh, (1993); Offodile et al (1994); Joines et al. (1996); Selim et al. (1998); Agarwal and Sarkis (1998); and Mansuri et al. (2000). According to these reviews, CMS design approaches can be classified into the following major types:

- Part coding analysis;
- Cluster analysis techniques;
- Graph partitioning;
- Mathematical programming; and
- Artificial intelligence along with other advanced methods.

**Part coding analysis** employs a coding system to assign numerical weights to part characteristics using defined classification schemes to identify part families. This system is traditionally design-oriented or geometrical shape-based, which means that it is suitable to reduce component variety. The part coding system (Groover, 2001) incorporates production-based codes as supplemental codes, which can be used for production planning.

**Cluster analysis techniques:** array-based clustering is the most commonly used technique. In this system, the processing requirements of the parts on the machines are represented by a part machine incidence matrix. The incidence matrix has *zero* and *one* entries ($a_{ij}$). A *one* entry in the row $i$ and column $j$ ( $a_{ij} = 1$) signifies that part $j$ has one or more operations on machine $i$, whereas a *zero* entry indicates that it does not. As per this technique, block diagonal clusters of $a_{ij} = 1$ are created by appropriately rearranging rows and columns in the process of allocating machines to groups, and parts to families. Clustering techniques include the similarity coefficient algorithm by McAuley (1972), the rank order clustering algorithm by King (1980), and King and Nakoronchai (1982), the direct clustering algorithm by Chan and Milner (1982) and the cluster identification method by Kusiak and Chow (1987).

**Graph partitioning techniques**: graph partitioning considers machines and/or parts as the vertices, and the processing of parts as the arcs connecting these nodes. These models attempt to obtain disconnected sub-groups from machine-machine or machine-part graphs in order to identify manufacturing cells. Rajagopalan and Batra (1975) used Jaccard's similarity coefficient and graph theory to form machine groups. Faber and Carter (1986) developed a graph theoretic algorithm for grouping machines and parts into manufacturing cells by converting the machine similarity matrix into a cluster network. Wu and Salvendy (1999) developed a network model by converting a machine-machine graph into cells by considering its operation sequence.

**Mathematical programming techniques** : these methods deal with practical issues such as machine utilization, machine capacity, the consideration of various costs and safety factors, the upper and lower bounds of machine cells, the size of each cell, intercellular material movement, etc. Mathematical programming techniques can be further classified into linear programming (LP), linear and quadratic programming (LQP), dynamic programming (DP), and goal programming (GP). Typical examples of mathematical programming techniques can be found in Wicks and Reasor (1999), Sofianopoulou (1999), Heragu and Chen (1998), Chen (1998), Askin et al. (1997), Atmani et al. (1995), Lashkari and Kasilingam (1990).

Each of the above approaches has its advantages and limitations. Cluster analysis and graph partitioning techniques for cell formation are easy to understand and implement, but they do not consider practical design issues such as machine capacity, product demand, etc. (Chen and Heragu, 1999). Mathematical programming approaches take into account most of the reality of the design requirements, but they have the drawback of requiring high computation times for large-sized problems. Obtaining optimal solutions by solving mathematical programming problems sometimes becomes infeasible due to the combinatorial complexity of the CMS design problem (Selim et al., 1998). To overcome this limitation, heuristic procedures are developed to provide reasonably good solutions within an acceptable amount of time, and with the use of reasonable computational resources.

Two types of heuristics are available in the literature. One type is the problem-specific heuristic, which cannot be applied to solving other design models. The second type is meta-heuristics, which can generally be applied to solving most problems. These meta-heuristics include genetic algorithms, simulated annealing, tabu search, ant colony, etc. Examples of these heuristics being used advantageously can be found in studies by Venugopal and Narendran ( 1992), Vakharia and Chang (1997), Asokan et al.(2001), Uddin and Shankar (2002).

### 1.4 Limitations of Existing CMS Design Models

In this section, three major limitations in the existing CMS design and planning processes are discussed. The first is the lack of machine reliability consideration in the design to improve and maintain the effective performance of the system. The second is the lack of routing flexibility consideration when solving machine breakdown problems. The third is the lack of effective maintenance consideration regarding the machines in the CMS planning.

Most of the existing CMS design research considers cost factors, strategic factors and various technological requirements for developing a design. Implementations of CMSs by modern manufacturing industries center on the desire to achieve higher machine utilization, fulfillment of due-dates, cost effectiveness, flexibility and other performance goals. Machine breakdowns have the greatest impact on overall CMS performance by affecting due dates, product costs and other manufacturing aspects. To reduce the probability of machine breakdowns and improve the overall performance of the system, machine reliability should be integrated as a major performance factor in the design of CMSs.

Modern manufacturing machines are capable of performing more than one type of operation. As such, routing flexibility is almost an inherent quality that presents alternative process plans to the system. Existing CMS design research incorporates routing flexibility to achieve better utilization of machines, reduction of costs and improved cell configuration. However, current CMS design approaches often fail to address the fact that routing flexibility may also be utilized for solving machine non-availability situations.

Machines constitute the lion's share of a CMS's capital investment. Any dynamic manufacturing organization should develop preventive maintenance planning to restrict the deterioration of machines and improve their reliability. In a CMS the requirement is more prominent, because most of the machine cells are dedicated to a part family and a cell may have only one machine of each type. In addition, a CMS is a multi-machine situation imitating a line production system in relation to part-processing steps. Unplanned breakdowns in a CMS will halt the entire system and degrade its overall performance. The incorporation of appropriate preventive maintenance measures in the planning of a CMS may be considered one of the most important requirements in the modern manufacturing sector.

### 1.5 Research Objectives

The basic objective of this research is to incorporate machine reliability considerations into the planning framework of cellular manufacturing systems. The development of a CMS design that considers machine reliability in combination with cost and other factors will be able to fulfill industry expectations, as well as overcome limitations mentioned in recent research. As such, the CMS design model of this research considers system cost and system reliability simultaneously. Most of the reliability-related studies use exponential distribution for machine reliability analysis, because exponential distribution is mathematically tractable. Weibull distribution is a versatile approach that has also been used in the literature for analyzing the increasing, decreasing and constant failure rates of machines and systems. In this research, we include both exponential and Weibull distribution models to analyze machine reliability for cellular manufacturing systems. The research also focuses on the development of a preventive maintenance model for CMSs. To summarize, the objectives of the research are as follows:

1. To provide reliability analysis of the CMS machines by using the exponential distribution approach and the Weibull distribution approach.
2. To consider alternative part processing plans in CMS design models with the goal of rerouting the parts in the case of machine failure.
3. To develop a multi-objective, mathematical model for reorganizing job shops and flow shops into cellular manufacturing systems while minimizing system costs and maximizing system reliability.
4. To develop a performance evaluation model for the cellular manufacturing system and incorporate the performance model into the multi-objective CMS design model.
5. To develop a preventive maintenance model to improve system reliability and maintenance-related costs.
6. To develop an optimal solution procedure using an available software package.
7. To develop a heuristic approach to reduce the computational times for large problems.

## 1.6 Organization of the Dissertation

The remainder of the dissertation is organized as follows: Chapter 2 describes general reliability criteria in the design and analysis of CMSs, the development of a reliability objective considering machine reliability to follow the exponential distribution and the Weibull distribution and a performance evaluation model for CMSs based on the exponential distribution. Chapter 3 contains a review of the relevant literature on the design of cellular manufacturing systems—with and without consideration of reliability, the existing literature related to multi-equipment maintenance planning and the motivation and methodology of the proposed research. Chapter 4 is divided into four sections. The first section presents definitions, assumptions and a problem statement for the multi-objective CMS design model. The second section includes a multi-objective CMS design model and performance evaluation criteria using machine reliability considerations based on the exponential distribution. The third section offers a multi-objective CMS design model using machine reliability considerations based on the Weibull distribution with numerical examples given in sections two and three to analyze the applicability of the model. The fourth section presents a large-sized example problem to further illustrate the applicability of the models. Chapter 5 develops a heuristic to solve the multi-objective CMS design models for large practical-size problems and illustrates the heuristic by solving examples. Chapter 6 includes a sensitivity analysis of the model solutions for possible changes in the key factors. Chapter 7 develops a reliability-based preventive maintenance planning model for CMSs, integrates the preventive maintenance planning policies into the CMS design and illustrates the procedure with numerical examples. The final chapter discusses the results, conclusions and contributions—including suggestions for future research.

# CHAPTER 2
# RELIABILITY ANALYSIS IN THE DESIGN OF CMSs

## 2.1 CMS and Machine Reliability

While dedication of machines and part families to manufacturing cells does provide many advantages (see Table 1.2), it also decreases scheduling flexibility--especially in the case of machine breakdowns. This is because in functional layouts, the proximity of identical machines makes the rerouting of parts in the event of machine failure an easy exercise. The same task in cellular manufacturing leads to intercellular transportation, which goes against the very foundation of independent machine cells. Consequently, reliability changes have a more profound impact on cellular manufacturing systems than on job shop manufacturing systems (Seifoddini and Djassemi, 2001).

The reliability model for machines in a cellular manufacturing system is a series configuration. In the case of job shops, even though the system configuration is parallel, the machines are still visited in a serial fashion, and there are more process routes for processing of part types. Figures 2.1 and 2.2 show the general reliability structure for machines in CMSs and job shops, respectively:

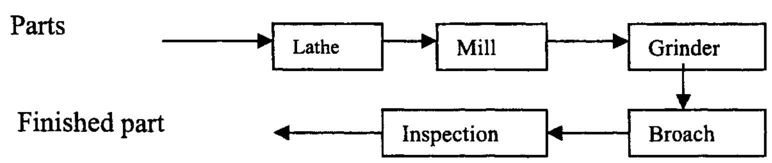


**Figure 2.1: Series reliability configuration in CMSs**

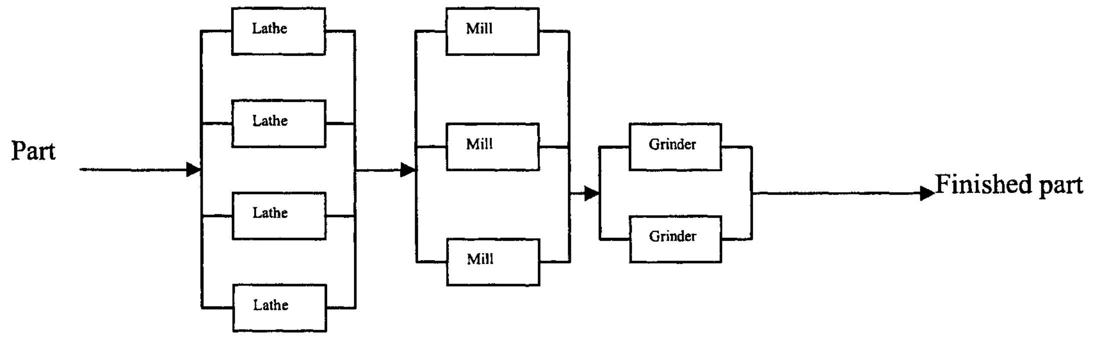


**Figure 2.2: Parallel reliability configuration in the job shop**

The two types of configurations for the same system reliability level illustrate that a CMS demands much higher reliability for individual machines (Ebeling, 1997).

Figure 2.1's simple reliability configuration explains the reliability approach taken for processing a job in a CMS, but it does not explain the complexity of the reliability in real cellular manufacturing. This is because the CMS

1) can produce part types in flexible routing and plan different machine routes for different parts;
2) simultaneously produces more than one part in different process plans; and
3) can have more than one throughput rate during any time period for part types. This happens due to the way process plans are changed in an effort to handle disturbances within the planning period.

When these points are considered in detail, it becomes very clear that to get optimum performance from the CMS, machine reliability should be integrated into CMS design. This study considers all machines to be unreliable. The CMS design approach that utilizes routing flexibility provides the option to change the process plan assignment in the event of machine breakdowns. In addition, one can try to select the processing route for the part types that have the highest system reliability for the machines along the route. The combination of these two aspects allow the CMS to react to internal disturbances in an efficient way by keeping the probability of failure at the lowest possible level, and by changing the process plan assignment when machine breakdowns occur. Since reliability considerations alone may not develop an optimum cell configuration in terms of cost, the cell formation decision should also include processing costs and resource utilization costs--combined with the other priorities of the user. Therefore, by integrating machine reliability with the existing design factors (cost, time, capacity, etc.), the designer can target an optimal process plan assignment with a more reliable set of machines and cost trade-offs to fulfill due date and other priorities of the business process. Implementing these approaches will improve most of the existing drawbacks of CMSs.

Achieving high reliability in CMSs is both complex, and difficult. Following the system reliability approach as described by Tillman et al. (1980), reliability can be improved by

1) reducing the complexity of the system;
2) increasing the reliability of the constituent components through product improvement programs;
3) using structural redundancy; and
4) planned maintenance and repair schedules.

These system reliability approaches are not easily applied in CMSs. The steps of reducing system complexity and increasing the reliability of constituent components are most applicable to the design phase of the machines, and as such, are not relevant during the cell formation phase. Consideration of redundancy allocation to machines is also not a viable approach as machines are the most costly component in a manufacturing business. Planned maintenance and improvements to machine reliability by modification are part of the continual improvement process necessary to an ongoing manufacturing business. As such, the basic approach of the CMS is to plan for achieving optimum reliability and cost considering the reliability status of the existing machines during the development of the system design.

## 2.2 Machine Reliability Related Functions

### 2.2.1 The Reliability Function

The reliability function $R(t)$ in the context of machines can be defined as the probability that the machine will perform its function over a given time period $t$. The reliability function is given by:

$$R(t) = Pr\{ t \leq T\} \quad \text{-----------(2.1)}$$

where $T$ is the continuous random variable to be the time to failure of the machine, $T \geq 0$, $R(t) \geq 0$, $R(0) = 1$, and $\lim_{t \to \infty} R(t) = 0$. For a given value of $t$, $R(t)$ is the probability that the time to failure is greater than or equal to $t$.

### 2.2.2 Failure Distribution Function

If we define:

$$F(t) = 1 - R(t) = \Pr\{T < t\} \quad \text{----------- (2.2)}$$

where $F(0)=0$ and $\lim_{t \to \infty} F(t) = 1$, then $F(t)$ is the probability that a machine failure occurs before time $t$, and $F(t)$ is defined as the cumulative distribution function ($CDF$) of the failure times for machines. The $pdf$ (probability density function) for the failure distribution is defined by :

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} \quad \text{----------- (2.3)}$$

where $f(t) \geq 0$ and $\int_0^\infty f(t)dt = 1$

### 2.2.3 Hazard Rate Function

The hazard rate or the instantaneous failure rate $\lambda(t)$ is defined as the failure per unit time (failure rate). The hazard rate function is represented by the following equation:

$$\lambda(t) = \frac{f(t)}{R(t)} \qquad \text{---------- (2.4)}$$

A hazard rate function uniquely determines a reliability function by the following functional relationship, which is derived by using (2.3) in (2.4):

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{-dR(t)}{dt}.\frac{1}{R(t)}$$

After integrating, the reliability function simplifies to the following equation:

$$R(t) = \exp[-\int_0^t \lambda(t)dt] \qquad \text{---------- (2.5)}$$

### 2.2.4 Reliability Function for Exponential Distribution

By assuming $\lambda(t) = \lambda$, $t \geq 0$, $\lambda \geq 0$, the reliability function for the exponential distribution or constant failure rate model can be obtained from equation (2.5) in the following form:

$$R(t) = \exp[-\lambda t], t \geq 0 \qquad \text{------------ (2.6)}$$

### 2.2.5 Reliability Function for Weibull Distribution

The Weibull distribution may be used to model both increasing and decreasing failure rates, as well as constant failure rates. Machine reliability function in the Weibull distribution is represented by the following equation:

$$R(t) = \exp[-(\frac{t}{\theta})^{\beta}] \qquad \text{------------ (2.7)}$$

where $t$ = time period under consideration,

$\theta$ is the characteristic life and

$\beta$ is the shape parameter.

$\beta >$ is considered for increasing failure rate

$\beta < 1$ is considered for decreasing failure rate and

when $\beta = 1$, exponential reliability function results with mean life $\theta = 1/\lambda$.

### 2.2.6 The Bathtub Curve

The bathtub curve is the most basic model used in reliability engineering to model various failure rates during the lifetime of a product or machine. Machines or systems having this hazard-rate function experience three distinct periods as shown in Figure 2.3. They experience decreasing failure rates early in their life cycle (burn-in period), followed by a nearly constant failure rate (useful life) period, followed by an increasing failure rate during the wear-out period.

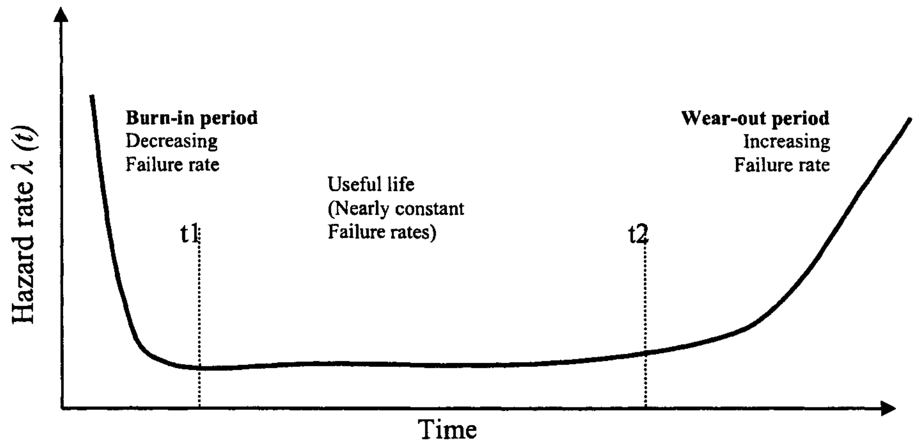


**Figure 2.3: The bathtub curve (hazard rate function over machine life)**

Machine reliability analysis for the burn-in and wear-out periods may be represented by using the Weibull distribution and that for the useful life period (t1 to t2) by the exponential distribution. During the useful life period, failures are random, and this is the only region where exponential distribution is valid. The burn-in period is quite short and is spent as a test-run period with the goal of removing various defects developed during the manufacturing of the machines (poor quality control for components, poor workmanship, defective parts, cracks during assembly, etc.). The wear-out period for machines arises due to aging, friction, cyclical loading, and fatigue. The wear-out period's effect on production machines can be reduced by preventive maintenance, modification, and parts replacement.

This dissertation addresses the reliability considerations of CMSs using two approaches:

1) exponential distribution; and
2) Weibull distribution.

### 2.3 Machine Reliability Analysis Based on Exponential Distribution:

#### 2.3.1 Machine Availability

Practically all machines can be considered unreliable or reliable with a reliability mark. When a machine is up, it produces; when a machine is down, it waits for repair. The machine reliability mark/definition in a manufacturing situation is generally represented by its availability. Availability is the probability that a system or component is performing its required function at a given point in time or over a stated period of time when operated and maintained in a prescribed manner (Ebeling, 1997). Availability may be interpreted as the probability that the machine is operational at a given point in time or during a percentage of time over some interval in which the machine is operational. Availability measures consist of the following types, irrespective of the distributions:

1) Point Availability: The point availability or instantaneous availability $A(t)$ at time $t \geq 0$ is the probability that the machine/system is functioning properly at time $t$.

2) Average availability $$A(T) = \frac{1}{T}\int_0^T A(t)dt \qquad \text{------------ (2.8)}$$

over time $[0, T]$. It can be generalized into interval availability between $t_1$ to $t_2$.

$$A_{t_2 - t_1} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} A(t)dt \qquad \text{----------- (2.9)}$$

3) Steady state availability $A = \lim_{T\to\infty} A(T)$. We can define it as Inherent Availability :

$$A_{inh} = \lim_{T\to\infty} A(T) = \frac{MTBF}{MTBF + MTTR} \qquad \text{----------(2.10)}$$

where $MTBF$ and $MTTR$ are the mean time between failures, and the mean time between repairs. Inherent availability is based solely on the failure distribution and repair time distribution of machines.

In a manufacturing environment, machine states are dynamic, and the probability of a machine being in an operative or inoperative state changes with respect to time, depending on repair, maintenance, and modifications. Therefore, in a CMS design, machine availability must be taken into account to determine the effective machine capacity during the cell formation and operation allocation processes. The most common approach for machine reliability representation is the Markovian model (Sulliman, 2000; Savsar, 2000; Diallo, 2001; Seifoddini and Djassemi, 2001). In this research, the appropriate model used to study the dynamic behavior of CMSs is a discrete state continuous time Markov process, assuming exponential distribution for machine failure and repair times and independence of the failure modes. The following is a brief description of the key concepts related to reliability/availability of machines in a CMS environment.

The simplest availability model with a repair rate $r$ and failure rate $\lambda$ for a single machine can be found by analyzing the transition diagram of the Markov process, as depicted in Figure 2.4:

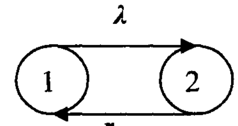


**Figure 2.4: Rate diagram for a single machine with repair (Ebeling, 1997)**

The following are the equations for the rate diagram:

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t) + rP_2(t) \quad \text{------------ (2.11)}$$

$$P_1(t) + P_2(t) = 1 \quad \text{------------ (2.12)}$$

Solving equations (2.11) and (2.12) yields

$$P_1(t) = \frac{r}{r+\lambda} + \frac{\lambda}{r+\lambda} e^{-(\lambda+r)t.} \quad \text{----------- (2.13)}$$

State (1) in Figure 2.4 is the available state for the machine, and $A\ (t)\ =P_1\ (t)$ is the point availability for this system. This provides the probability that the machine is in an operating state at time $t$. If machine $j$ is considered at time $t$, the point availability equation can be stated as per the following:

$$A_j(t) = \frac{r_j}{r_j + \lambda_j} + \frac{\lambda_j}{\lambda_j + r_j} e^{-(r_j+\lambda_j)t} \quad \text{---------- (2.14)}$$

The interval or mission availability for machine $j$ between times $t_1$ and $t_2$ may be stated as follows

$$A_j(t_2-t_1) = \frac{r_j}{r_j+\lambda_j} + \frac{\lambda_j}{(\lambda_j + r_j)^2(t_2 - t_1)}[e^{-(\lambda_j+r_j)t_1} - e^{-(\lambda_j+r_j)t_2}] \ , \quad t_2 \geq t_1 \text{------- (2.15)}$$

### 2.3.2 Machine Reliability Corresponding to a Process Plan

A number of research works on CMS design (Jeon et al., 1998; Seiffoddini and Djassemi, 2001; Diallo et al., 2001) have considered alternative process plans to handle machine breakdowns. To achieve this objective effectively, the reliability of the different machine routes for each part type needs to be determined. Of the available machine routes, the one with maximum system reliability may be assigned to the part to reduce the probability of breakdowns during the processing period.

To examine the concept of machine reliability corresponding to a part type-process plan assignment, we consider a manufacturing cell consisting of 5 machines processing 4 part types, as represented in Table 2.1. Each part type may be processed using either of the two process plans available, and each operation of a part type under a given process plan may be performed on one or more machines, giving rise to a number of machine routes (or processing routes). For example, part type 1 may be processed using any of the 8 processing routes as shown in Table 2.2, where each route is identified by a 4-digit number as *(part type, process plan,* and *processing route #)*. Thus, processing route 1203 represents *(part type 1, process plan 2,* and *processing route 03)*, and the system reliability corresponding to the machines along this route is:

$$R_S(1203) = R_4(T)R_3(T)R_1(T) \qquad \text{---------- (2.16)}$$

**Table 2.1: A typical routing table for a cell with 4 part types and 5 machines**

| Part types | Process plans | Operations | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| 1 | 1 | M3, M2 | M4, M5 | |
| | 2 | M2, M4 | M3 | M1, M4 |
| 2 | 1 | M2 | M4, M5 | M3 |
| | 2 | M1, M3 | M2 | M5 |
| 3 | 1 | M1, M4 | M3, M2 | M2 |
| | 2 | M4, M5 | M2, M4 | M1, M3 |
| 4 | 1 | M1, M3 | M2, M4 | M5 |
| | 2 | M4, M5 | M1 | M4 |

**Table 2.2: Processing routes for part type 1 in Table 2.1**

| Processing route number | Machine Sequence in Processing route |
|---|---|
| 1101 | M3-M4 |
| 1102 | M3-M5 |
| 1103 | M2-M4 |
| 1104 | M2-M5 |
| 1201 | M2-M3-M1 |
| 1202 | M2-M3-M4 |
| 1203* | M4-M3-M1 |
| 1204 | M4-M3-M4 |

where $R_j(T)$ is the reliability of machine $j$ at time $T$. Assuming that machine failures are exponentially distributed, the machine reliability is:

$$R_j(T) = EXP(-\lambda_j)T \qquad \text{---------- (2.17)}$$

where $\lambda_j$ is the failure rate of machine $j$, and the system reliability equation becomes:

$$R_S(1203) = EXP(-\sum_{j\in\{1,3,4\}} \lambda_j T) \qquad \text{---------- (2.18)}$$

which may be written as:

$$\frac{1}{\ln R_s(1203)} = \sum_{j\in 1,3,4} \lambda_j T$$

Since $T$, the planned time period, is common to all the machines under consideration, the above equation can be expressed as

$$\frac{1}{\ln R_S(1203)} * \frac{1}{T} = \sum_{j \in \{1,3,4\}} \lambda_j = LIR_{12} \qquad \text{---------} (2.19)$$

$LIR_{12}$ is defined as the system failure rate corresponding to machines M1, M3, and M4 along the processing route for the *part-process plan* combination (12). Obviously, the minimization of $LIR_{ip}$ values through the selection of machines and process plan assignments would lead to an optimum level of overall reliability for CMS.

## 2.4 Performance Evaluation of Cellular Manufacturing Systems

A major consideration in designing a manufacturing system is its performance (Zakarian and Kusiak, 1997). One factor that has a major influence on system performance is the unscheduled downtime of machines. Typically, failure of one machine in a CMS does not result in complete system failure, but it affects the system performance level significantly for the following reasons:

- All the parts that were planned to be processed on the failed machine need to be rerouted if alternative routes are available.
- If alternative routes do not exist, the parts have to wait for the machine until it is repaired and the subsequent operations will have to be halted while the repair process is underway. Often, this causes a chain reaction, reducing the utilization of subsequent machines and causing due date delays.

Because of the complexity of CMSs, the designer and user might be interested to know and verify the performance of the system to initiate preventive and/or improvement steps. The Markov approach is used extensively for performance evaluation modeling of manufacturing systems because of its simplicity and efficiency (Liu andYuan, 2001; Zakarian and Kusiak, 1997; Ram and Vishwanadham, 1994; Albino et al., 1990). This dissertation employs the Markov modeling approach for the evaluation of system availability as the performance index of the CMS.

### 2.4.1 The System Availability Model

The following assumptions will be included for the development of the system availability model:

1. Each machine has independent repair and failure modes.
2. Time to failure and time to repair follow exponential distributions.
3. For each machine, there is a maintenance history file in which *MTBF*, and *MTTR* and other failure and repair related information is available.

The system availability corresponding to a part type-process plan combination is the steady state probability of the machines required for that combination to be in operating condition. These steady state probabilities will be computed using a Markovian approach explained below.

In a cellular manufacturing system with $m$ machines, the state of a machine $j$ may be represented by the set $S_j = \{0,1\}$, $j = 1,2,...,m$, where $1$ indicates that the machine is *"up"*, and $0$ indicates that the machine is *"down."* Therefore, the CMS states may be defined by the set:

$$W = \{w_1, w_2, \ldots, w_N\}, \qquad \text{----------(2.20)}$$

where each element $w_k = \{S_1, S_2, \ldots S_m\}$, $k=1,2,...,N$, and $N=2^m$.

In our example of the cell with five machines there would be $N = 2^5 = 32$ states. As time goes on, the state of the cell changes, depending on whether a machine fails, or a repaired machine returns to work. We may represent such state changes using transition probabilities, $P_{w_k w_l}$, of going from state $w_k$ to state $w_l$. Assuming that the transition probabilities are stationary, and that the individual machine states are independent, we can compute all the $P_{w_k w_l}$ terms, then describe the transition probability matrix *TM*. Suppose states 1 and 5 are defined by:

$$w_1 = \{1,1,1,1,1\} \quad \text{and } w_5 = \{1,1,0,1,1\}$$

Thus, a transition from $w_1$ to $w_5$ implies that machine 3 has failed, and the corresponding transition probability is:

$$P_{w_1 w_5} = P^{u,u}_{j=1} \times P^{u,u}_{j=2} \times P^{u,d}_{j=3} \times P^{u,u}_{j=4} \times P^{u,u}_{j=5}$$

where $P^{u,d}_{j=3}$ is the probability that machine 3 makes a transition from "*up*" to "*down*" in a short time period, $\Delta t$. and in general $P_{w_k w_l}$ is defined as

$$P_{w_k w_l} = \prod_{j=1}^{m} P_j^{S_j(k)S_j(l)} \qquad \text{--------- (2.21)}$$

where $w_k$, $w_l$ are the states $k$ and $l$ of the cell. $S_j(k)$ and $S_j(l)$ are the states of machine $j$ at cell states $k$ and $l$. State of the machine can be *up* $(u=1)$ *or down* $(d=0)$ at cell states; therefore, as explained previously, $S_j(k)$ and $S_j(l) = \{0,1\}$. We may represent the transition probability matrix *TM* in terms of $P_{w_k w_l}$ in the following way. It is an $N \times N$ matrix, where $N$ indicates the number of cell states. As discussed for a 5-machine cell, the number of states is $(2^5) = 32$ and they are given as 11111, 11110, 11101, 11100 ...00001, 00000. Following the Markov chain model, the manufacturing system makes a transition from one state to another according to the transition probabilities during its manufacturing lifetime. These transition probabilities can be estimated from the maintenance file data of each individual machine.

$$TM = \begin{array}{c|cccc|} & w_1 & w_2 & .... & w_N \\ \hline w_1 & P_{w_1w_1} & P_{w_1w_2} & .. & P_{w_1w_N} \\ w_2 & P_{w_2w_1} & P_{w_2w_2} & .. & P_{w_2w_N} \\ . & .. & .. & .. & .. \\ . & .. & .. & .. & .. \\ w_N & P_{w_Nw_1} & P_{w_Nw_2} & .. & P_{w_Nw_N} \end{array}$$

Let the steady state probability vector

$$V = [\pi_{w_1}, \pi_{w_2}, \ldots, \pi_{w_N}] \qquad \text{--------- (2.22)}$$

where $\pi_{w_1}, \pi_{w_2}$ .... are steady state probabilities for the cell states *w1,w2*........From the approach of Markov chain analysis:

$$V = V \times TM \qquad \text{---------- (2.23)}$$

which results in the following equations:

$$\pi_{w_1} = P_{w_1w_1}\pi_{w_1} + P_{w_2w_1}\pi_{w_2} + \ldots\ldots.P_{w_Nw_1}\pi_{w_N}$$

$$\pi_{w_2} = P_{w_1w_2}\pi_{w_1} + P_{w_2w_2}\pi_{w_2} + \ldots\ldots.P_{w_Nw_2}\pi_{w_N}$$

$$\cdot$$

$$\cdot$$

$$\pi_{w_N} = P_{w_1w_N}\pi_{w_1} + P_{w_2w_N}\pi_{w_2} + \ldots\ldots.P_{w_Nw_N}\pi_{w_N} \qquad \text{-------- (2.24)}$$

while the normalizing equation is

$$\pi_{w_1} + \pi_{w_2} + \pi_{w_3} \ldots\ldots\pi_{w_N} = 1 \qquad \text{---------- (2.25)}$$

From these systems of equations $\pi_{w_k}$, k =1, 2. .. .*N* values can be calculated.

Now **system availability (SA)** for a part type *i* for the selected process plan *p* can be given:

$$SA_{ip} = \sum_{w_k} \pi_{w_k} \qquad \text{------------ (2.26)}$$

$SA_{ip}$ is the summation of steady state probabilities of the cell states where the relevant machines needed for performing the required operations on part type *i*, as per selected process plan *p*, are in the *up* condition.

In our example above, according to the selected processing route, part type 1 under process plan 2 needs machines 1, 3 and 4 to perform the required operations, and the relevant cell states are: 11111, 11110, 10111, and 10110, and the system availability corresponding to this part type-process plan combination, therefore, is:

$$SA(12) = \pi_{(11111)} + \pi_{(11110)} + \pi_{(10111)} + \pi_{(10110)}.$$

System availability evaluates the percentage of time the relevant machines required for part types under a selected process plan are in *up* condition.

### 2.5 Machine Reliability Analysis Based on Weibull Distribution

#### 2.5.1 Machine Availability

Definitions in relation to point availability, interval availability, average availability and inherent availability as described under section 2.3.1 are the same for all types of reliability distributions. The CMS design model based on Weibull distribution estimates effective machine capacity by considering inherent availability as the relevant availability model.

#### 2.5.2 Machine Reliability Corresponding to a Process Plan

Exponential distribution has been demonstrated to provide good approximations of machine failure distribution when the failure rate is constant, and, as such, is widely used in the literature (Yazhou et al. 1995; Hariga, 1996; Savsar, 2000; Diallo et al.2001; etc.). However, the Weibull distribution approach is considerably more versatile than the exponential distribution, and can be expected to fit many different failure patterns (Ireson et al., 1995). It has the advantage in reliability analysis of being able to adjust distribution parameters in order to address increasing, decreasing, and constant failure distributions. Abernathy (1996) mentions examples of machine tools and other engineering problems solved with Weibull analysis. Dai et al. (2003) conclude that the data on the distribution of time between failures for machine centers follow the Weibull distribution. Motivated by these discussions and conclusions, this research has developed a machine reliability analysis along the part processing route with the Weibull distribution approach.

The typical routing table (Table 2.1 in section 2.3.2) provides an example of number machine routes available for a part type. Taking a similar approach to section 2.3.2, system reliability for the machines along the part processing route 1203 (Table 2.2) is reproduced below:

$$R_S(1203) = R_4(T)R_3(T)R_1(T) \qquad \text{---------- (2.26)}$$

The Weibull reliability function for machine $j$ is defined:

$$R_j(T) = \exp[-(\frac{T}{\theta_j})^{\beta_j}] \qquad \text{----------- (2.27)}$$

where $T$= time period for the part time under consideration,

$\theta_j$ is the characteristic life for the machine $j$ and

$\beta_j$ is the shape factor for the machine $_j$

$\beta_j > 1$ is used to consider increasing failure rate and

$\beta_j < 1$ is used to consider decreasing failure rate analysis

when $\beta_j = 1$, exponential reliability function results with mean life $\theta_j = 1/\lambda$.

Shape factor value can be evaluated by studying and analyzing the failure data for the type of machine/components under consideration. In this research, the Weibull distribution is used to analyze an increasing machine failure rate.

The *MTBF* and *MTTR* data can be obtained from the maintenance files of manufacturing organizations. We assumed that *MTBF* for all the machines under consideration are known. As per the Weibull failure model:

$$MTTF = \theta\ \Gamma(1 + \frac{1}{\beta}) \qquad \text{----------} (2.28)$$

*MTTF* can be considered equal to *MTBF* for a repairable system when complete samples (failures) are analyzed for the estimation of *MTTF* (Abernathy, 1996). For this study, we also assume that *MTTF* equals to *MTBF*.

System reliability equation (2.26) for the part processing route 1203 can be represented by the following equation by considering the Weibull reliability function for machine $j$:

$$R_s(1203) = \exp[-(\frac{T}{\theta_1})^{\beta_1}] \times \exp[-(\frac{T}{\theta_3})^{\beta_3}] \times \exp[-(\frac{T}{\theta_4})^{\beta_4}] \qquad \text{---------} (2.29)$$

The equation can be simplified as:

$$R_s(1203) = \exp[-\{(\frac{T}{\theta_1})^{\beta_1} + (\frac{T}{\theta_3})^{\beta_3} + (\frac{T}{\theta_4})^{\beta_4}\}] \qquad \text{----------} (2.30)$$

The equation can be further simplified as per following:

$$\ln \frac{1}{R_s(1203)} = \sum_{j=1,3,4} \left( \frac{T}{\theta_j} \right)^{\beta_j} = LIR_{12} \qquad \text{-------------} (2.31)$$

Expressing $\theta$ in terms of *MTBF* and $\beta$:

$$\theta_j = \frac{MTBF_j}{\Gamma(1 + \frac{1}{\beta_j})} \qquad \text{------------} (2.32)$$

the equation takes the form:

$$\ln\frac{1}{R_s(1203)} = \sum_{j=1,3,4}\left(\frac{T}{\theta_j}\right)^{\beta_j} = \sum_{j=1,3,4}\left(\frac{T\,\Gamma(1+\frac{1}{\beta_j})}{MTBF_j}\right)^{\beta_j} = LIR_{12} \qquad \text{------------- (2.33)}$$

$LIR_{12}$ is the inverse of system reliability for machines M1, M3 and M4 along the part processing route for part type1 and process plan 2, in the natural logarithmic scale. It may also be explained as the cumulative number of failures for the machines along the part processing route during the time *t*. Minimization of $LIR_{ip}$ will increase the system reliability of the machines along the part processing routes for the combination (*ip*).

It may be noted that $LIR_{ip}$ is the measure of system reliability for the machines along the part processing route, both in the exponential distribution and in the Weibull distribution based approaches as presented respectively in section 2.3.2 and in this section. The measure (system reliability) is computed as the system failure rate in section 2.3.2 when machines are assumed to follow constant failure rates, and it is evaluated in terms of the inverse of system reliability in this section expecting machines to follow the increasing failure rates in the Weibull distribution based approach.

# CHAPTER 3
# LITERATURE SURVEY

## 3.1 Scope

In an effort to remain within the scope of this dissertation, the literature review in this chapter focuses on the effects of machine reliability on existing design approaches and recently developed methodologies of cellular manufacturing systems. Having discussed the impact of reliability and design methodologies on the performance of cellular manufacturing systems in many relevant ways, we must now provide a comprehensive review of these effects. The relevant aspects of the advanced techniques and solution procedures are highlighted in this chapter in order to focus on their usefulness in resolving the complexity of mathematical programming models. This study focuses solely on the review of mathematical programming techniques because a CMS design approach utilizing this technique provides a basis for considering most practical manufacturing issues. We do not cover layout, scheduling, flow balancing, product quality, maintenance management systems, or labor-related aspects. The purpose of this review is to provide the necessary background for the model developed in this dissertation.

The survey of literature is divided into two main sections, each of which has a major impact on defining and solving the problem. The first section deals with cell formation and work allocation, while the second section includes literature on reliability consideration. Significant research studies will be reviewed by highlighting the following salient features (see tables 3.1 and 3.2):

- Factors considered by the research
- Modeling descriptions
- Objectives of the study
- Major findings
- Relevance of the research to this study

In addition to the main sections listed above, a small sub-section covers significant literature that addresses multi-component/multi-unit maintenance planning. The review of these studies focuses on preventive maintenance and maintenance planning-related aspects of manufacturing environments similar to CMSs.

### 3.2 Cell Formation and Operation Allocation

Cell formation is one of the basic problems in the design of cellular manufacturing systems (Heragu and Chen, 1998). Diallo et al. (2001) consider cell formation to be both an important issue, and a complex optimization problem. It is also believed to be the central issue in group technology (GT) applications (Spiliopoulos and Sofianopoulou, 1998). This research aims to develop a mathematical programming technique for CMS design, as well as a procedure for solving large problems using a simulated annealing-based algorithm. Therefore, following the scope and area of interest for this study, this review will be limited to the recent literature on CMS design by mathematical programming techniques and advanced techniques such as Genetic Algorithm, simulated annealing, tabu search, neural networks, and fuzzy logic.

#### 3.2.1 Cell Formation by Mathematical Programming Techniques

These methods deal with practical issues such as machine utilization and capacity, cost considerations, safety factors, upper and lower bounds for the number of machine cells, cell size, intercellular material movement, simultaneous cell formation, and work allocation. Most of the works use zero-one or mixed integer programming to solve the cell formation problem.

A two phase approach to design manufacturing cells was developed by Albadawi et al. (2005). The first phase determined machine cells by applying the factor analysis technique to the similarity coefficient matrix. In the second phase, an integer programming model was used to assign the parts to cells with the objective of minimizing exceptional elements. The factor analysis technique is a multi-variate analysis tool used to translate the relationships between a large number of variables into a smaller set of independent variables. This study applied this tool to reduce the initial part-machine matrix to the machine cells matrix. The analysis was started with the development of a Jaccard similarity coefficient matrix, followed by an eigen value matrix and an eigen vector matrix of the similarity coefficients. The cell relationship of the machines was computed depending on the criteria (defined as Kaiser's criteria) of the highest eigen value and the corresponding value of the eigen vector. Finally, the approach used an algorithm to determine the optimal positioning of the machines in the cells. Factor analysis can be done using an SPSS statistical software package. Using this approach, the authors solved six test problems and claimed that the approach was capable of generating good solutions.

An iterative heuristic procedure was developed by Malakooti et al. (2004) to solve the cell formation problem by breaking down their proposed nonlinear integer programming model into three steps. Their study solved the process planning, production planning and cell formation problems simultaneously with the objective of minimizing inter-cell traffic. The first step found the best process and production plan considering a given cell configuration, while the second step found cell configuration with the process and production plans selected in the first step. In the final step, capacity constraint and inter-cell traffic were addressed using the results of the first two steps.

One of the major motivations for manufacturing cell design is the reduction of workload content. Workload content includes set-up times and processing costs for a part type. Kizil and Ozbayrak (2004) developed an algorithm to evaluate the trade-off between process plan selection and material handling cost for solving the cell formation problem. The study proposed an MIP model to select the process plan with the objective of minimizing workload content. The algorithm first solved the MIP model for part routing selection. This solution gave a lower bound value on the total machine requirements by type, as well as the part machine processing matrix used in cell formation. Cell formation was then planned using the ROC (rank order clustering) algorithm, and the inter-cell material movement cost for each randomly selected part, evaluated for all the possible alternative routes. The route with the minimum workload content subject to the capacity limitations and upper bound of set-up time for the machines was selected. If the route exceeded the machine capacity and set-up time limitations, the next route with higher workload content was checked. Following these steps, the inter-cell movement cost for the entire set of parts was evaluated by selecting the process plan with the minimum workload content. The approach ensured that the minimum workload content was achieved, without optimizing the inter-cell material movement simultaneously.

Won and Lee (2004) proposed modified *p*-median formulations for solving GT-cell formation problems efficiently. The GT cell is an independent cell dedicated to a part or part family. This approach was developed in an attempt to reduce the computational complexity of the classical *p*-median approach developed by Kusiak (1987). The *p*-median formulation set out to maximize the sum of similarity coefficients defined between the pairs of parts. The study included two modifications to reduce the limitations of the classical model. The first modification is the imposing of lower and upper limits on the number of machines in a cell, and the inclusion of a formula (defined as Kernn's formula) to evaluate which similarity coefficient to use in objective function. In addition to these two new steps, the first modification also introduced a candidate set of median machines and a procedure for generating a candidate set of median machines. The second modification was to remove all the machines which fall outside of the candidate set of machines in order to further improve the algorithm in terms of reducing computation time. By solving many problems with the modified formulation, the study concluded that the modified formulation can be easily applied to solve large size cell formation problems.

A step-by-step procedure defined as incremental cell formation was developed by Mahesh and Srinivasan (2002) to solve the 0-1 integer programming model proposed by the study. In their step-by-step approach, the first cell was formed from the high volume parts and the best machines capable of performing the tasks on the parts with the objective of achieving the minimum cycle time. The next cell was formed using the same approach, choosing the parts with the second highest volume, and so on, until all the part types were dealt with. Both a branch and bound approach, and a heuristic-based multi-stage programming approach were used to solve the problem.

Arzi et al. (2001) proposed a method to design CMS in "lumpy" or variable demand conditions. The study developed an MIP bi-objective model which took into account demand variability, as well as correlation among the parts. The first objective optimized grouping efficiency - defined as the ratio of the total number of unused part-machine combinations and the total number of these combinations over all the groups when inter-cell movement was not allowed. The second objective minimized the overall cost of the machine by considering the variable demand conditions of the environment. There are four traditional methods for solving the problems incurred by lumpiness: fixed additional capacity, extra inventories, order rejection, and temporary capacity usage. To determine the system capacity, the study considered the variability of the demands, and the correlation of the demand of the part types assigned to the cells and trade-off decisions in relation to keeping the extra inventory. The study also assumed that the required planned capacity of the machines was normally distributed.

A holonistic approach was proposed by Akturk and Turkan (2000) to solve the part family and machine cell formation problem. The paper proposed a MIP model to develop the CMS design with the overall objective of maximizing profit by taking into account raw material costs, inter-cell material handling costs, variable production costs, and machine investment costs. The research used the holonistic approach to develop independent cells that used machine duplication in the initial steps of their solution algorithm. A holon cell can be defined as a cell which can earn profit to become economically self-sufficient. The study, however, ultimately considered cell development to be using the inter-cell movement of parts to minimize the machine investment costs. The proposed model also considered the machine layout problem within the cell to reduce the cost of intra-cell material movement. It is apparent that simultaneous consideration of inter-cell movement from the first step could generate a mixed solution by reducing machine investment costs or developing a holon cell solution - depending on the profit objective. The step-by-step approach proposed by the study may be considered effective for small size problems where the inter-cell movement decisions were introduced depending on the profits and machine investment costs of the holon cell or independent cell.

Flexibility in relation to CMS can be defined as a system's ability to adjust its resources to account for any changes in relevant factors such as product, process, loads, and machine failures. Vakharia et al. (1999) studied flexibility considerations in cell design. This study defined various general flexibility types and developed formulations for flexibility measures relevant to CMS. The research proposed a flexible cell design model similar to that of Askin et al. (1997).

Wicks and Reasor (1999) proposed a multi-period, non-linear MIP model to design the CMS. The model minimized inter-cell material handling costs, machine duplication costs, and machine relocation costs. The authors considered the effect of the redesign of cells and the relocation of machines on the efficient handling of product demand. A genetic algorithm was used to solve the part family and machine cell formation (PF/MC) problems.

In an ideal CMS, the operations of parts are completed within a single cell. Parts that are processed by more than one cell are exceptional parts, and machines that are required by more than one part family are called bottleneck machines. Together, they are called exceptional elements (EEs). Berardi et al. (1999) have reviewed the literature on exceptional elements and investigated the mathematical programming model developed by Shafer et al. (1992). This model considered three cost categories when designing CMSs: sub-contracting costs, machine duplication costs, and intercellular part transfer costs. Berardi et al. (1999) proposed the inclusion of intra-cellular part movement costs and machine relocation costs in addition to the above three costs considered by Shafer et al. (1992).

Heragu and Chen (1998) developed a MIP model for cellular manufacturing system design. The objective function of the model minimized intercellular material movement and resource under-utilization costs to develop the cell formation. Benders' decomposition approach was used to solve the model.

To design a sustainable cellular manufacturing system by accommodating the dynamic manufacturing environment in terms of part mix and product mix change, Chen (1998) proposed an integer programming model with the objective of minimizing material handling costs, machine costs and related reconfiguration costs in a planning horizon of multiple periods. To solve the problem efficiently, the study broke the problem down into two parts. The first part of the problem was solved without reconfiguration costs and the constraints related to them. A dynamic programming solution approach was followed for generating cell configuration by considering machine costs, material handling costs, and relevant constraints for the planned time periods. Then the system reconfiguration costs were calculated depending on the solutions already obtained from the first part over the time periods. In the final stage, the best, most feasible cell configuration was selected - considering the overall costs and constraints involved. The procedure was illustrated by solving both a small and a large size problem.

Askin et al. (1997) developed an interactive flexible cell formation method to handle dynamic and random variations in part demand. The study had four phases. In the first phase, a linear 0-1 mathematical model solved the problem of operation assignment to the machines. The second phase then assigned each part–operation to specific machines of each type, so that similarity between the part operations was maximized. Graph partitioning and similarity coefficient measures were used to solve the problem at this stage. In the third phase, a machine clustering technique was used to identify candidate manufacturing cells. The fourth and final pahse improved routing flexibility and volume flexibility by rearranging and changing the assignments of the previous phases.

Heady (1997) proposed a 0-1 integer programming model to develop minimum cost machine cells by optimizing the cost savings obtained by inter-cell movement of parts, in place of outsourcing or sub-contracting. The study dealt with exceptional parts and compared the cost savings obtained by inter-cell movement to cost savings produced by outsourcing.

Lee and Chen (1997) developed a multi-criterion CMS design model. Their study employed a weighting approach that combined two criteria such as minimizing the intercellular movement of parts and maximizing workload balance among duplicated machines. This solution method followed a three-phase approach that determined machine cells and part families, as well as allowing for machine duplication where necessary. In the first phase, an estimation procedure balanced the work-load for the duplicated machines. In the second phase, machine cells and part families were constructed using a heuristic algorithm. During the third and final phase, an additional heuristic procedure was employed to improve the solution quality of the cell formation results.

The transformation of functional layouts to cellular layouts reduces routing flexibility by dedicating machines to cells. However, the advantages obtained by the reduction of set-up times, material handling times and other planning-related aspects have been found to outperform the functional layouts within the limited routing flexibility of CMS. Shafer and Charnes (1997) investigated the effects of transforming functional layouts to cellular layouts by using queuing network models where individual cells and functional layouts were represented by $M/M/1$ and $M/M/k$ ($k$ = the number of servers), respectively. Their analysis illustrated that although the dedication of machines to cells resulted in reduced routing flexibility in CMS, there were four characteristics of CMS which offset this loss, allowing CMS to outperform job shop. The characteristics studied were operation overlapping, set-up time reduction, simultaneous setting of equipment, and reduction in move times. By using computer simulation models in stochastic environments, the authors concluded that these benefits have the potential to offset the loss of routing flexibility reduction of CMSs due to a conversion from a functional layout.

Atmani et al. (1995) used a 0-1 integer programming model for the simultaneous solution of the cell formation and operation allocation problem in CMSs. The objective of the model was to simultaneously form machine groups and allocate part types to regrouped machines in such a way that the total sum of operation, refixturing and transportation costs were minimized.

To conclude on the subject of cell formation by mathematical programming techniques: Mathematical programming techniques consider real world factors related to cell formation. A large number of research papers have been published on the design of CMSs during the last three decades. Table 3.1 highlights and summarizes the most significant studies. Most of the research considers machines to be 100% reliable, which is unrealistic. Although some of the studies (e.g., Atmani et al, 1995, Askin et al, 1997, etc.) used multiple routes for the processing of parts, this routing flexibility was not considered to tackle machine reliability related issues.

### 3.2.2 Recent Approaches to CMS Design

Although small and medium-sized cell formation problems may be solved optimally, most real world cell formation problems are either not optimally solvable by the existing algorithms, or they involve very high computational times and memory requirements. To overcome this limitation, Yasuda et al. (2005) adopted Genetic Algorithm (GA) to solve their multi-objective cell formation problem. The authors followed Falkenauer's (Falkenauer, 1998) Grouping Genetic Algorithm (GGA) to encode their problem and apply GA. The GGA uses a special technique of adopting the structure of the grouping problem (combinations of part, machine and cell) as genes of the chromosomes. This structure is different from the classic GA based techniques where a chromosome has a separate part string and machine string. The GGA technique does not require input for pre-defined number of cells; it has the advantage of the solution algorithm's ability to generate the required number of cells. The study compared their solution quality with similar problem instances in the literature ( e.,g., Venugopal and Narendran, 1992) and concluded their algorithm to be effective in solving large cell formation problems.

Solaimanpur et al. (2004) developed a multi-objective model and proposed a genetic algorithm-based approach for solving the cell formation problem. The objectives of the model included minimizing processing costs, processing time, and machine investment costs, and maximizing part similarity for the formulation. To evaluate the fitness function of GA, the study converted the multi-objective model into a single objective model with weight factors. The solution approach explored different directions of the solution space by changing the weight factor values.

Peker and Kara (2004) used a neural network method to solve the cell formation problem. This method can be applied in the case of both binary (where the cell entry represents the operational relationship between part and machine in the matrix) and non-binary (where the cell entry represents machine capacity, processing times, etc.) incidence matrices in the cell formation process. Their paper attempted to validate this method by solving many cell formation examples from the literature.

A mixed integer, non-linear model with the objective of minimizing material handling costs, machine tool costs and cell set-up costs was proposed by Cao and Chen (2004) for designing CMSs. The model assumed a single processing plan for each part type, and did not consider part processing costs. To accommodate the production times within the limited capacity of the machine, multiple copies of each machine type were allowed in a cell. The study converted the model to a penalty function formulation and then solved it with a heuristic algorithm using a tabu search approach. By comparing the solution obtained from the heuristic (tabu search) and optimal solution procedure (LINDO), the authors claimed their heuristic algorithm to be efficient in generating near optimal solutions within an acceptable time limit.

Spiliopoulos and Sofianopoulou (2003) developed a three-stage procedure for CMS design. In the first stage, part families were formed according to the design similarity of the parts. In the second stage, a 0-1 integer programming model with the objective of minimizing inter-cell traffic was used for solving the cell formation problem based on the part family formed in the first stage. The outcome obtained from the second stage was investigated in the third stage to eliminate intercellular traffic and make decisions regarding machine duplication, part sub-contracting, and change of part routing. A tabu search algorithm was used to solve the model at the second stage with the goal of developing manufacturing cells. The research claimed their tabu search algorithm to be effective in generating quality solutions within the reasonable time requirements for large-sized problems.

A mathematical programming model with the multiple objectives of minimizing inter-cell and intra-cell part movement costs, total cell load variations and exceptional elements was proposed by Zhao and Wu (2000). Their study used the Genetic Algorithm approach to solve the model. In addition to the multiple objectives, the problem also considered multiple routing in their cell formation process. The authors recommended the solution approach as suitable for solving the machine component grouping problem in complicated working environments.

Plaquin and Pierreval (2000) developed a cell design methodology with an evolutionary algorithm. The algorithm addressed workshop specific constraints in addition to the general technological and capacity-related constraints. The workshop specific constraint considered by the study was to design cells around certain machine aggregates where certain specific machines were needed to stay together. The approach was tested on two problems, and the authors claimed to obtain the optimal solution. The population generation procedure, crossover, mutation and other steps of the proposed evolutionary algorithm were found to be similar to Genetic Algorithm.

Moon and Gen (1999) proposed a 0-1 integer programming model to develop independent cells. The objective function of the model minimized machine duplication costs and processing costs with the consideration of other manufacturing constraints and parameters such as machine capacity, alternative process plans, production volume, number of cells, and cell size. The study solved the cell formation problem using a Genetic Algorithm.

Tsai et al. (1997) proposed an MIP model to form manufacturing cells. Their study dealt with the exceptional elements of the cell formation problem while minimizing machine duplication costs, sub-contracting costs, and intercellular parts movement costs. Fuzzy membership functions and operators were used to solve the problem after converting the original model to a fuzzy MIP problem. The application of neural networks to the cell formation problem was also proposed by Rao and Gu (1993), and Kaparthi and Suresh (1991).

### 3.2.3 Cell Formation by Simulated Annealing Algorithm

A MIP model which included alternate routings, machine replication and operation sequence was proposed by Jayaswal and Adil (2004) to design CMSs. Their model used both machine replication and alternate routings to minimize inter-cell moves. An algorithm consisting of simulated annealing and a local search method was developed to solve the model for large-sized problems. The solution quality of the algorithm was found to be impressive when compared with the optimal solutions computed for small size problems. The algorithm was also found to be efficient in terms of its computational time for large-sized problems. The initial solution and neighborhood solution of the algorithm were generated by randomly selecting operation allocation variable values, and then changing the operation assignment of parts between machines and/or cells.

Asokan et al. (2001) used Simulated Annealing (SA) and Genetic Algorithms for solving cell formation problems. Their study was based on two mathematical models from the literature. The first model minimized inter-cell and intra-cell moves as proposed by Logendran (1990), while the second model minimized cell load variation (the difference between the workload variation among the machines and the average load on the cell) as proposed by Venugopal and Narendran (1992). Asokan et al. (2001) then combined the two models to solve the cell formation problem through SA and GA. The study used a random perturbation scheme for solving the problem with SA. The scheme consisted of developing a seed sequence of machines. If there were five machines, the sequence array would have five positions. Perturbation is achieved by changing the sequence of machines. Since each position of the sequence represents a cell number, when the sequence of machine is changed, the allocation of machines to cells also changes. The cost of solutions for these schemes was optimized using heuristics to achieve an acceptable CMS design. The study compared the performance of SA and GA for machine cell grouping problems. By numerical example, it was concluded that SA performed better when trying to achieve these objectives.

The combination of a heuristic method with a local search method (e.g., Jayaswal and Adil, 2001) or the pairing of other methods like the branch and bound methods, are applied by the solution procedures to improve the solution quality and reduce the computation time. Caux et al. (2000) used a combination of SA and branch and bound methods to solve their non-linear integer programming model for cell formation with alternative process plans and machine capacity constraints. The model considered the problem of minimizing inter-cell traffic by selecting alternative routing. A combined iterative approach was followed in the solution procedure by simultaneously solving two sub-problems. The SA algorithm generateed the machine cells respecting the maximum number of cells allowed in a cell, while the branch and bound method optimally assigned one routing to each part - respecting machine capacity and keeping inter-cell moves to a minimum considering the feasible partitions found in the SA. The study concluded that the solution methodology should be useful in the case of large-sized, un-constrained problems only.

Abdelmola and Taboun (1999) used an SA algorithm to solve the cell formation problem. The study proposed a non linear 0-1 integer programming model to optimize total productivity. Total productivity was defined as the ratio of intercellular material handling costs and the total sales amount (revenue) of the parts. The paper reported good performance of the heuristic by comparing the solution quality obtained by the heuristic for a 10-machine-10-part problem with the optimal solution generated by LINGO.

A Simulated Annealing algorithm was used by Zolfagari and Liang (1998) to solve their cell formation problem based on the grouping efficiency measure involving processing times, machine capacity limitation and machine duplication. The grouping measure proposed in the study was an extension of the grouping efficiency introduced by Chandrasekhran and

Rajagopalan (1986). The study developed an SA algorithm to solve the grouping problem of machines. The SA algorithm was applied to the seed solution generated by the neural network. The neighborhood solution for the algorithm was generated by randomly reassigning machines from one cell to another. The part assignment for each neighborhood solution was done to match the parts with the newly generated machine cells. The performance of the algorithm was compared with the results reported in the literature on the 16 machine type and 43 part type example studied by Burbidge (1975). The performance was found to be impressive in terms of computation time and solution quality.

Su and Hsu (1998) considered a three-objective cell formation model. The three objectives were: minimizing the total cost of inter-cell transportation, intra-cell transportation and machine investment; minimizing intra-cell machine load unbalance; and minimizing inter-cell machine load unbalance. The study unified these objectives by weighting them, and solved the model by means of simulated annealing algorithm. The study used a genetic algorithm as the generation mechanism of simulated annealing to reduce computational times.

Sofianopoulou (1997) used a simulated annealing approach for solving the proposed linear integer programming cell formation model. The objective of this mathematical model was to minimize the number of intercellular moves. The study considered the perturbation scheme for generating neighboring solutions by changing the cell membership of a randomly selected machine and, thus, reassigning the said machine to the same cell with another randomly selected machine. Performance of the heuristic has been compared with the optimal solution for the model. The results indicated that this algorithm was a good performer in terms of the solution quality and computational effort.

To generate solutions to CMS design problems with reduced computational complexity, and to investigate the performance of the heuristics, Vakharia and Chang (1997) solved their cell formation model by simulated annealing (SA) and tabu search. The research developed a MIP model with the objective of minimizing the procurement cost of machines (number of machines needed) and inter-cell material handling costs. The authors solved their model with eight randomly generated data sets and one published data set to evaluate the performance of these heuristics. Numerical results indicated that the SA algorithm performed better than tabu search, and provided close to optimal or optimal solutions for the tested problems.

Venugopal and Narendran (1992) developed a SA algorithm to solve the machine component grouping problem. Their study proposed a mathematical model with the objective of minimizing cell load variation among machines. When compared with that of K-means algorithm, the SA algorithm was found to yield better results.

To conclude on the subject of cell formation by recent approaches including simulated annealing: The primary reason for using a simulated annealing algorithm in CMS design is to obtain acceptable solutions within a reasonable amount of time. Although the goal of all studies is to achieve optimal solutions, sometimes this is computationally prohibitive. According to the reviewed research, the solutions obtained by simulated annealing algorithms are promising. Among the three meta-heuristics investigated, simulated annealing has the advantage of being easy to implement, while still providing good solutions. The comparative study between GA and SA by Asokan et al. (2001), and the comparison of solution quality between tabu search and SA by Vakharia andand Chang (1997) in the design of CMSs show that simulated annealing performs better than the other two searches. These findings have motivated us to select the SA algorithm solution approach for the design of our CMS.

### 3.3 Reliability Considerations in the CMS Design

This section reviews the literature on reliability considerations in the design and operation of cellular, flexible, and automated manufacturing systems (AMS).

#### 3.3.1 Performance Evaluation Studies in Cellular and Flexible Manufacturing

Seifoddini and Djassemi (2001) evaluated the effect of reliability considerations on the performance of job shops and cellular manufacturing systems. Their study considered mean flow time and work-in-process inventory as measures of performance. A simulation model was developed for estimating the system performance under varying reliability levels. Fixed reliability levels of 100%, 90%, 80% and 70% were assigned to both job shop and cellular manufacturing systems to compare their performances. The results indicated that, although both systems deteriorate as the machine reliability drops from the 100% to the 70% level, the CMS was more severely affected than the job shop. Considering the fact that CMSs follow a series reliability structure, and job shop follows a series-parallel (mixed), it was obvious that the authors of this study got only the expected outcomes.

Rupe and Kuo (2001) developed a Markov chain model for the flexible manufacturing system (FMS) states in terms of the number of unavailable machines, available inventory and the repair process. The objective of the study is to determine the performance of the FMS in terms of the availability of a number of machines. The analytical approach constructed useful models of the FMS failure and repair under general FMS architecture so that the model could be applied to a wide variety of FMS configurations. From their results the study concluded that the performability measure correctly reflected the system effectiveness at achieving a defined goal.

**Table 3.1 Summary of significant literatures on cell formation where machine reliability is not considered**

| Reference | Factors considered ** | Modeling descriptions | Solution Techniques | Demand | Process Plan type | Other significant factors |
|---|---|---|---|---|---|---|
| 1. Abdelmola and Taboun (1999) | M, n, NMC, ICM | A non-Linear IP model to maximize productivity. Productivity defined as the ratio of total sales turnover and inter-cell and intra-cell part movement cost. | A simulated annealing (SA) algorithm is used. | Fixed demand. | Multiple routes. | Sales price of product is used. |
| 2. Akturk and Turkan (2000) | M, n, PCM, U, SM, NMC, K, ICM, CM | MIP model to maximize profit. | A local search heuristic. | Fixed demand. | Multiple routes. | Holonic approach. Profit for each cell and within cell machine layout considered. |
| 3. Askin et al. (1997) | M, n, PCM, U, SM, NMC, K, ICM, CM | Integer programming (IP) model for operations assignment by minimizing operation cost and procurement cost. Graph partitioning and part similarity coefficient for assigning part operation to machines. - Clustering techniques to form cells. | An algorithm is proposed to systematically carry out the steps. | Dynamic random variation of demands. | Multiple routes. | Flexible cell in terms of routing flexibility Demand flexibility and availability of machines. |
| 4. Askin et al. (1998) | M, n, PCM, U, SM, NMC, K, ICM, CM | IP model for operations assignment by minimizing operations and procurement cost. Graph partitioning and part similarity coefficient for assigning operations to machines. Clustering techniques to form cells defined and formulated flexibility terms. | An algorithm is proposed to systematically carry out the steps. | Dynamic random variation of demands. | Multiple routes. | Flexible cell in terms of routing flexibility demand flexibility and availability of machines. |
| 5. Asokan et al. (2001) | M,n, K,NMC, CM | Three mathematical models : Model 1 (LP) minimizes inter-cell and intra-cell moves. Model 2 ( 0-1 IP) minimizes total within cell load variation. Model 3 combination of 1 and 2. | SA and Genetic Algorithm (GA) are used. | Fixed demand. | Multiple routings. | SA performed better than GA. |

**** Symbols used in Factors considered column**

| | | | | | |
|---|---|---|---|---|---|
| **M** | Number of machines | **n** | Number of part types | **K** | Number of cells desired |
| **ICM** | Inter-cell part movement | **NMC** | Number of machines in a cell | **PCM** | Procurement cost of machines |
| **CM** | Capacity of machines | **U** | Machine utilization | **SM** | Set-up time or cost |

**Table 3.1: cont'd**

| Reference | Factors considered** | Modeling descriptions | Solution Techniques | Demand | Process Plan type | Other significant factors |
|---|---|---|---|---|---|---|
| 6. Baykasoglu et al. (2001) | M, n, K, NMC, U | Non-linear IP and multiple objective model to minimize cell load imbalance and part dissimilarity based on the processing sequence and extra capacity requirements. | SA algorithm used. | Fixed demand. | *Multiple routes.* | Resource element approach considered in processing parts and capacity deviation of cells. |
| 7. Berardi et al. (1999) | M,n,CM, K, NMC, U, PCM, ICM | Reviewed models dealing with exceptional elements (EEs). Investigated Model of Shafer et al (1992). An IP model which minimizes machine duplication, inter-cell movement, and subcontracting cost. | Solved by LINDO and OSL without relaxation of integer restrictions. | Fixed demand. | Multiple routes. | Exceptional elements, subcontracting cost, machine duplication cost , bottleneck machine. |
| 8. Cao and Chen (2004) | M,n,CM, NMC, U, PCM. | Non-linear MIP model to minimize material handling, machine tool set-up and operational cost. Number of cells not fixed. Capacity for each operation is in terms of number of machines. | A heuristic algorithm and tabu search used to solve the problem. | Fixed demand. | Single process plan for each part. | Model converted to a penalty formulation and solved with tabu search procedure. |
| 9. Caux et al. (2000) | M,n,CM, K, NMC, ICM | 0-1 integer non-linear model. Objective function minimizes inter-cell traffic. | Combination of SA and branch and bound methods used. | Fixed demand. | Multiple process plan. | |
| 10. Dahel and Smith (1993) | M,n,CM, K, ICM, SM | 0-1 IP , multi-objective model. | Lexicographic approach and non-dominated solutions are generated by LINDO systems. | Fixed demand. | Multiple process plan. | Number of machine types in each cell is maximized to get operational flexibility. |
| 11. Heady (1997) | M, CM, K, NMC, ICM, SM | 0-1 IP model, minimizes opportunity cost for parts considering savings obtained by inter-cell part movement compared to outsourcing. | Lindo software used to solve the model. | Fixed demand. | Multiple routes. | Bottleneck machine and exceptional operations. |

****** Symbols used in Factors considered column****

| | | | | | |
|---|---|---|---|---|---|
| **M** | Number of machines | **n** | Number of part types | **K** | Number of cells desired |
| **ICM** | Inter-cell part movement | **NMC** | Number of machines in a cell | **PCM** | Procurement cost of machines |
| **CM** | Capacity of machines | **U** | Machine utilization | **SM** | Set-up time or cost |

**Table 3.1: cont'd**

| Reference | Factors considered** | Modeling descriptions | Solution Techniques | Demand | Process Plan type | Other significant factors |
|---|---|---|---|---|---|---|
| 12. Heragu and Chen (1998) | M,n,CM, K, NMC, U, ICM | MIP model to minimize inter-cell movement and resource under-utilization. | Benders decomposition. | Fixed demand. | Multiple routes. | |
| 13. Jayaswal and Adil (2004) | M, n, CM, K, ICM, PCM, NMC | A non-linear IP model to minimize operational cost, inter-cell moves cost, and machine investment. | SA algorithm and local search. | Fixed demand. | Multiple routes. | Machine replication and alternate routings used to minimize inter-cell moves. |
| 14. Logendran and Ramkrishnan (1997) | M,n,CM, SM Various cost factors | MIP model to maximize cost savings by machine duplication and part subcontracting. | A heuristic algorithm with tabu search is used. | Fixed demand. | Alternative process plan within the cell. | Duplication of bottleneck machine and subcontracting of exceptional parts. |
| 15. Mahesh and Srinivasan (2002) | M,n, CM,K No inter-cell movement | Non-linear 0-1 IP model to minimize cycle time of a part. | Branch and bound method and also a heuristic based multistage programming approach. | Fixed demand. | Multiple routes. | Precedence of operations, cycle time, equivalent part approach. |
| 16. Malakooti et al. (2004) | M, n, CM,K,NMC,ICM, U | IP model for selection part and operation plans and iterative approach for cell formation. | A problem specific heuristic used. | Fixed demand. | Multiple routes. | |
| 17. Moon and Gen (1999) | M, n,CM, K, NMC, U, PCM, | 0-1 IP model to minimize sum of machining and machine duplication costs. | GA used to solve the model. | Fixed demand. | Alternative process plan within the cell. | Both machine duplication and independent cell approach when only one machine of each type exists in a cell. |
| 18. Peker and Kara (2004) | M,n,K,CM, K, ICM,NMC, U, | Binary and non-binary part machine incidence matrix developed. | Fuzzy neural network solution approach used. | Fixed demand. | Single route. | Solve both binary ( 0-1) and non-binary part-machine incidence matrix. |

**<u>** Symbols used in Factors considered column</u>**

| | | | | | |
|---|---|---|---|---|---|
| **M** | Number of machines | **n** | Number of part types | **K** | Number of cells desired |
| **ICM** | Inter-cell part movement | **NMC** | Number of machines in a cell | **PCM** | Procurement cost of machines |
| **CM** | Capacity of machines | **U** | Machine utilization | **SM** | Set-up time or cost |

**Table 3.1: cont'd**

| Reference | Factors considered** | Modeling descriptions | Solution Techniques | Demand | Process Plan type | Other significant factors |
|---|---|---|---|---|---|---|
| 19. Shafer and Charnes (1997) | WIP, number of departments, M, lot sizes, K | Queuing network and simulation models used to analyze the models. | Analyzed with computer simulation. | Fixed demand. | Single process plan. | Operations overlapping, set up reduction, reduction in move times, simultaneous set up offsets loss in routing flexibility. |
| 20. Solaimanpur et al. (2004) | M,n, K,CM, U, PCM | Multi-objective IP model, minimize similarity between the parts, minimize processing cost, minimize processing time, minimize investment cost. | GA used. Weights to make single objective. | Fixed demand. | Multiple process plan. | Independent cells, efficient frontier. |
| 21.Sofianopoulou (1999) | M,n, K, NMC, U, ICM, PCM | Two non-linear IP model, 1) machine allocation to cells, and 2) part to machines. Model 1 minimizes inter-cell traffic and Model 2 maximizes part allocation to cells by the most advantageous process plan selected by the model 1 | SA algorithm | Fixed demand. | Multiple process plan. | Replicate machines are considered. |
| 22. Tsai et al. (1997) | M,n,CM, K, NMC, U, ICM, PCM | MIP model to minimize cost machine procurement, inter-cell material movement, and subcontracting. | Model converted to fuzzy MIP to achieve improved computational performance. | Fixed demand. | Multiple routes. | Fuzzy operators and membership function included. |
| 23. Wicks and Reasor (1999) | M,n, K, NMC, U, ICM, PCM, | MIP model. Considers multi-period forecast demands to minimize inter-cell movement, duplication of parts, and relocation cost of the machines. | GA used to solve the model. | Fixed demand. | Multiple process plan. | Considered trade-off between inter-cell movement and machine relocation cost. |
| 24. Vakharia and Chang (1997) | M, n, K, ICM, NMC,CM, U, PCM | 0-1 IP model, objective of minimizing inter-cell moves and procurement cost of machines. | Tabu search and SA algorithm | Fixed demand. | Multiple process plan. | SA performed better than tabu search. |

**** Symbols used in Factors considered column**

| | | | | | |
|---|---|---|---|---|---|
| **M** | Number of machines | **n** | Number of part types | **K** | Number of cells desired |
| **ICM** | Inter-cell part movement | **NMC** | Number of machines in a cell | **PCM** | Procurement cost of machines |
| **CM** | Capacity of machines | **U** | Machine utilization | **SM** | Set-up time or cost |

**Table 3.1: cont'd**

| Reference | Factors considered** | Modeling descriptions | Solution Techniques | Demand | Process Plan type | Other significant factors |
|---|---|---|---|---|---|---|
| 25. Venugopal and Narendran (1992) | M, n, K,NMC, CM | 0-1 IP model, objective of minimizing. cell load variation. | SA used. | Fixed demand. | Fixed process plan. | |
| 26. Xambre and Vilarinho (2003) | M,n, K, ICM, NMC, CM | MIP model to minimize inter-cellular flow. | SA algorithm used. | Fixed demand. | Multiple routes. | Functionally similar multiple machines used. |
| 27. Zhao and Wu (2000) | M, n, K, NMC, ICM, PCM, | Multi-objective model to minimize inter-cell and intra-cell movement cost, cell load variation and exceptional elements. | GA used. | Fixed demand. | Multiple process plan. | |
| 28. Zolfaghari and Liang (1998) | M, n, K, NMC, ICM, U | Generalized grouping efficiency measures for cell formation used as a model. | SA used. | Fixed demand. | Fixed process plan. | Machine duplication used. |

**<u>** Symbols used in Factors considered column</u>**

| | | | | | |
|---|---|---|---|---|---|
| **M** | Number of machines | **n** | Number of part types | **K** | Number of cells desired |
| **ICM** | Inter-cell part movement | **NMC** | Number of machines in a cell | **PCM** | Procurement cost of machines |
| **CM** | Capacity of machines | **U** | Machine utilization | **SM** | Set-up time or cost |

Through the proposed measure the analyst would be able to compare various FMS designs. The study is a typical reliability related study in FMS, and does not have much relevance to our objective.

There are a considerable number of performance evaluation studies which focus on the comparison of job shop and CMS in terms of reliability measures, or some output measures. Logendran and Talkington (1997) compared cellular and functional layouts (CL and FL) considering machine breakdowns and batch sizes as the two important factors which could influence the performance of the system. To analyze the impact of machine break-downs, repair policies based on breakdown and preventive maintenance were included in the study. Two performance measures - namely mean work-in-process-inventory and mean throughput times - were used in the comparison. A SLAM-II-based simulation was conducted to evaluate and compare the performances. The study results indicated that, in the absence of any preventive maintenance or of any machine reliability considerations, the functional layout out-performs the CMS.

Zakarian and Kusiak (1997) proposed an analytical approach for the availability evaluation of a cellular manufacturing system. The study broke down the manufacturing system into machining and material handling sub-systems, and a Markovian approach was applied to find the system availability of the machines and the material handling systems in the cells. The overall manufacturing system availability was evaluated by considering the probabilities of subsets of machines in working condition in each cell. A manufacturing system was considered acceptable when it fulfilled the production capacity requirement for the part to be manufactured. This is analogous to the requirement of a processing route for a part type to fulfill all the necessary operations to complete its processing.

The dependability of a manufacturing system is the measure of the system's availability. Simeu-Abazi et al. (1997) developed an analytical model to evaluate the dependability of a manufacturing system. The study took a decomposition approach for dividing manufacturing systems into elementary machine cells. Each machine represented a decomposed system. Each elementary cell is modeled as a combination of the Markov process and Stochastic Petri Nets (SPN). The whole system was then recomposed to determine the availability of individual machines as members of a manufacturing system set. The method was found suitable for very small production systems with a series-flow type of machine-product combination.

The Markovian approach of analyzing manufacturing system states is one of the established procedures for estimating performance of the automated manufacturing systems (AMSs).

Viswanadham and Ram (1994) developed a Markovian approach-based model for the transient analysis of manufacturing systems in the presence of failures and repairs. Their study decomposed the system into structure state processes, and a performance model to analyze both the processing and movement of materials. Structure state process was defined as the modes of operation during the time interval of interest, when a machine in operating condition failed, and again when a failed machine was repaired and returned to operation. The study used a time-based decomposition where structure state process occurred at a much slower scale than the part processing and movement. Considering two automatic machine centres and one AGV, the study evaluated the performance of a flexible manufacturing cell, in terms of availability, throughput, and lead times.

Ram and Viswanadham (1994) presented a framework for performance evaluations of automated manufacturing systems (AMSs) subject to failure and repair. The framework assessed the system productivity in the face of internal disturbances arising from equipment failure. The study developed a formula to measure the performance of AMS based on the assumption that the system was fault tolerant. A fault tolerant system can perform its function at a decreased performance level after facing a minor breakdown. That means it can tolerate fault up to a certain limit and perform at a lower production rate or performance level. A Markovian procedure was followed to formulate throughput and lead time related performance, and to assess these measures through a structure state process. The research developed a numerical approach to the computation of the distribution of mean lead time through the use of a generalized stochastic Petri net (GSPN) reward model.

Ram and Viswanadham (1992) proposed a performance evaluation technique for cellular flexible manufacturing systems (CFMS) using a decomposition approach. CFMSs were defined as the CMSs developed by configuring FMSs applying the GT concept. The study divided the parts to be manufactured into two types. One type, defined as local parts (the majority portions of the parts to be manufactured), needs machining only in a single cell. The remaining parts are defined as rare parts, and need inter-cell movement and machining in more than one cell. The Markov chain was used to evaluate the states and to obtain steady state average values of performance measures in terms of throughput, utilization, queue length, and waiting time.

In addition to the Markovian approach-based analytical model, there are performance evaluation studies which used simulation models. Nagarajah et al. (1992) investigated the effect of machine reliability and transporter speed on the performance of FMCs (Flexible manufacturing cells). The study used simulation to determine the performance of FMC in terms of the total number of parts produced and the average flow time by assigning various fixed reliability levels to machines. Two configurations, namely the cell layout and aggregated cell or process layout, were compared in terms of average throughput times when they were producing a mix of 5 part types. The study showed that using machine duplication to counter the effects of reduction in machine reliability could be counter productive when available equipment had been consistently displaying high reliability.

Albino et al. (1990) proposed an integrated performance reliability model to evaluate different measures for a flexible automated production system (FAPS). FAPSs are fault tolerant, and can react to a detected failure by reconfiguring to a state with a decreased level of performance, resulting in a gracefully degrading system. The model consisted of a Markovian model to determine the probability of each state in a specific FAPS state space, and a general analytical model that evaluated the performance relating to the state space. A homogeneous, continuous time discrete state Markovian model was used to evaluate the steady state probabilities of the major components of the FAPS.

### 3.3.2 Reliability Related Studies in FMSs and AMSs

The provision of buffer capacity is an established procedure in the discrete part manufacturing industry to solve the machine reliability problem. Kalir and Arzi (1998) developed an MIP formulation to determine the profit maximizing configuration of work stations along a flexible production line, with unreliable machines for finite and infinite buffers. They first developed a formulation for general problems that may occur in a finite buffer case. The study then included buffers with infinite capacities when buffer costs were considered negligible compared to the workstation's operational costs. An optimal solution algorithm was developed to solve cases of infinite buffer capacity. The study also developed a heuristic to solve large-scale problems arising in cases of infinite buffer capacity. The authors concluded that without this heuristic approach, the proposed optimal solution algorithm might not be suitable to handle large-scale problems.

Classical queuing theory was used by Lin et al. (1994) to study the characteristics of the fractional utilization of the FMS. A mathematical program was constructed to determine both the optimal number of floats for an important module of FMS (defined as key FMC) and the optimal capacity of the repair station. A float was defined as both the failed units undergoing repair, and the units that were on standby status. The paper used an $M/M/1/F$ queue for the repair system, and a closed queuing structure model where a certain number of independent and identical machines or processors were required to be in operation. $F$ was the total number of float modules, of which one was engaged in operation, and $(F-1)$ modules were standbys or in the repair shop. A maintenance float policy was used in both manufacturing and service sectors to keep the operating equipment at a high level of availability. With a maintenance float policy, several units of the key module were kept in the production line with one unit in operation while the remaining units were kept on standby.

Yuanidis et al. (1993) proposed a model for assessing the reliability of the FMS. Their model established a functional relationship between operational parameters and system responses. Operation times, *MTBF, MTTR,* etc., were considered as the input parameters. System responses included the number of parts produced in a given interval, and other output variables. An algorithm called GMDH (Group Method of Data Handling) was used to determine system responses against the input data. This model can be used to assess the reliability of FMS in the design and development process.

Wang and Wan (1993) developed a dynamic reliability model using a fuzzy logic approach for FMS. Their study considered the initial reliability of the machines, maintenance policies, and system failure mode analysis in the dynamic reliability model. Using the fuzzy logic approach, the model defined many types of failure modes, the initial system reliability, the production environment, the equipment's inherent quality, training of the manpower, and system degradation to establish reliability parameters.

Miriyala and Vishwanadam (1989) analyzed the reliability of FMS considering part reliability and machine reliability using process spanning graphs (PSG). Given an FMS configuration, the part reliability of a given part type is the steady state probability of producing that part by using at least one of the machine routes, and is equal to the probability that at least one PSG for the part type is in operation. A PSG represents a particular route taken by a work piece of some part type in the FMS. Markov chain modeling was used to evaluate the dynamic reliability and availability of the system.

Chaharbagi and Davies (1986) developed an approach for assessing the reliability of FMS. Their study proposed a simulation model to analyze alternative system design approaches. Manufacturing capability, production efficiency, and demanded production lead time success ratio were defined for the assessment of system performance measures. A simulation study-based technique was viewed to be appropriate for reliability analysis of FMSs because it can include machine flexibility, and job flexibility - which cannot be included in analytical techniques.

### 3.3.3 Reliability Studies Relevant to CMSs

The Markovian approach was used by Liu and Yuan (2001) to consider a finite inter-station buffer, uncertain service times, and random breakdowns of the assembly equipment to construct and derive formulas for throughput, loss of probability of types of work pieces, and mean flow times. Loss of probability of types of work pieces was the ratio of *difference between parts flow and throughput* to *parts flow*, where throughput was assumed to be less than parts flow. The study then developed an optimization model by including these parameters to maximize the system throughput of unreliable assembly lines, while maintaining the required customer service level. Two types of work pieces (Type 1 and Type 2) were considered in an assembly line consisting of two parallel work centers to conduct the numerical experiments and to analyze the system behavior. While Type 1 work pieces had infinite buffer capacity in front of work center 1, Type 2 work pieces had finite buffer capacity in front of work center 2. The study found that in the steady state, the loss of probability of Type 2 work pieces was independent of their relevant buffer sizes. Other conclusions of the study were dependent on the numerical values of flow rate, service level, etc.

Savsar (2000) developed a mathematical model to compare the performance of a reliable and an unreliable flexible manufacturing cell (FMC). The study assumed operation times, loading/ unloading times, and material handling times to be random in the stochastic analysis of system performance. Formulas for system performance (in terms of utilization rates of machines, robots, and pallet handling systems) were derived using the Markov process by considering the state probability definition and transition rate diagram of unreliable FMC operations. This study established that there was a significant difference in utilization between the reliable and unreliable cells in relation to machine, robot, and pallet handling systems. The utilization of the components is higher in reliable cells than in unreliable ones.

Gupta and Kavusturucu (1998) proposed a method to analyze finite buffer CMSs with unreliable machines. An open stochastic queuing network was used to model the system. Throughput calculation of the network was done using decomposition, isolation, and expansion methods. An expansion node was created as an extra node in front of each buffer of the machine to act as an apparent holding node for jobs which could not enter destination node because the buffer was full. The blocked job stayed in the expansion node until a space became available at the full buffer. The method broke the network down into a set of $M/M/1(BD)/K_i$ nodes. $K_i$ represented the job holding capacity of node *i*. $M/M/1(BD)/K_i$ represented an $M/M/1/K_i$ node when the machine was unreliable (i.e. subject to breakdown).

### 3.3.4 Reliability Studies in CMS

A cellular line production system design model considering machines to be unreliable was developed by Kuroda and Tomita (2005). A cellular line was defined as a line of flow shops when the flow shops were performing a sequence of intended operations and acting as a cell unit. Each cell unit of the line was composed of dissimilar facilities to perform similar operations of the process sequence. The proposed model followed a two step process. In the first step, dissimilar machines or facilities were grouped considering similarity of operations to be performed. To balance the part flow in the group/cell, the model considered the number of operation sequences assigned to each facility and the required number of machines in each stage of the operations sequence. In the balancing process, machine un-reliability - determined by considering the expected value of overloads at each stage - was incorporated. The expected value of overload was estimated with overloads required at a stage due to failure of a facility and probability of failure of the relevant facility. To estimate the probability of failure, the authors proposed a steady state probability based on machine availability. The exponential distribution was used for time to failure and time to repair of the machines. The objective of the study was to minimize the number of facilities needed for the stages. In the second step, each group of facilities were arranged to construct a balanced, one-way flow. A genetic algorithm-based procedure was used to solve an example problem. By including stochastic factors related to machine failure and repair, the authors claimed the approach to be robust.

Diallo et al. (2001) proposed an approach to design manufacturing cells which can change process plans to handle machine breakdowns. The study carried out reliability analysis of the individual machines and manufacturing system states in the presence of unreliable machines. Exponential distribution approach was used for the reliability analysis of the machines and manufacturing systems. The model allocates demand of part types to each of the available process plans. While the model selected the best process plans to satisfy the demand for parts, the cell configuration addressed the problem of manufacturing the parts in alternative process plans when the best plan was not available.

Jeon et al. (1998a) proposed a CMS design model to consider alternative routes during machine failure. The study considered a predefined number of breakdowns for each of the machines, and developed the model to reduce waiting costs, early/late finish penalty costs, and the sum of inventory holding costs by selecting alternative routes to handle the breakdowns. The study did not include a reliability analysis of the machines. Alternative routes have been considered by several studies to improve resource utilization, reduce costs by selecting the best process plan, and get minimum interaction between cells by choosing the best cell configuration. Among these we can mention Rajamani et al. (1996) and Chen (1998). These studies, however, did not consider alternative routes to solve the machine breakdown problems. All the traditional CMS design models, including these studies, considered machines to be reliable resources. For maintaining expected overall performance of the CMS, alternative routes should be considered in the design phase to plan to reroute the parts in case of machine failure (Jeon et al., 1998).

Li and Shaw (1998) developed a simulation model to study dynamic job shop rescheduling where work stations were not always available due to breakdown or preventive maintenance. By considering job priority, machine states, and consequent bottlenecks of part routes, this study attempted to determine the complete picture of the anticipated waiting time for rescheduling the works. This study is indirectly a reliability consideration in terms of the expected outcomes of waiting times.

To conclude: most of the reliability consideration-based studies on CMS deal primarily with performance evaluation issues. A considerable number of these studies (Seifoddini and Djassemi, 2001; Logendran and Talkington, 1997, etc.) emphasize the importance of machine reliability on the expected output of the CMS. This emphasis is one of the motivating factors for considering reliability in the design of a CMS in this work. However, Jeon et al. (1998) and Diallo et al. (2001) are the only two studies to consider machine reliability in their analysis and development of the CMS. Jeon et al. (1998) considered alternative routes to develop cell configurations to handle the problem of a predefined number of machine breakdowns. Their model aimed to minimize waiting costs, late and early finish costs and machine investment costs to solve the machine breakdown problem. Diallo et al. (2001) considered all their machines to be unreliable and consequently attempted to develop a cell configuration with alternative process plans to solve machine breakdown problems. These studies, however, do not include the proactive approach of considering both system costs and reliability to optimize costs and have the lowest failure probability during processing, while maintaining alternative routes for rerouting the parts in case of machine failure. Table 3.2 summarizes the major factors and objectives included in reliability related studies reviewed here.

### 3.4 Preventive Maintenance Studies in CMSs

This review focuses on the significant maintenance studies which are relevant to the maintenance planning of multi-machine systems similar to CMSs. Rao and Bhadhury (2000) presented a case study on the preventive maintenance of a multi-equipment, 210-MW thermal power station fueled by six independent and identical coal pulverizers. Failure characteristics of the pulverizer sub-systems were developed from the analysis of the data collected between two system overhauls. Based on this data, the decision concerning which components needed PM and which could be replaced after a predetermined age was made. For the increasing failure rate components the Weibull distribution was used. The small components of pulverizers were

**Table 3.2: Summary of significant reliability related studies on CMS and FMS**

| Study | Factors considered | Modeling descriptions | Objectives | Major findings | Relevance |
|---|---|---|---|---|---|
| **CMS RELIABILITY RELATED STUDY (Direct and Indirect)** | | | | | |
| 1. Diallo et al. (2001) | Steady state probability for system states, multiple process plans to handle breakdowns. | Markov optimization model for intercellular interaction and process plan. | Cell formation in presence of unreliable machines and optimization of inter-cell interactions of parts. | Design of manufacturing cells to improve throughput by using alternative process plan in case of machine failure. | The study is related to CMS reliability analysis. |
| 2. Li and Shaw (1998) | *MTBF, MTTR*, scheduling of jobs, part demand, WIP, job priority. | Visual interactive simulation. | Scheduling of the jobs considered priority and bottleneck. | Status of the workstation used for effective job rescheduling. | The study is related to the reliability considerations of the job shop. |
| 3. Gupta and Kavusturucu (1998) | Finite buffer, unreliable machines, system throughput. | Stochastic queuing network, simulation. | Performance evaluation of cellular manufacturing with unreliable machines. | Use of expansion node and decomposition to evaluate performance measures. | Performance evaluation of single flow line cells having unreliable machines. |
| 4. Jeon et al. (1998a) | Part similarity coefficient, fixed number of m/c breakdowns, alternative routes, and cost associated with inventory holding, early/late finish penalty, and operation. | Mixed integer programming model for grouping the machines into cells. | Minimization of m/c failure costs, inventory costs, early and late finish costs by considering scheduling and operational aspects. | Alternative process plans can improve resource utilization during machine failure. | It is a CMS study. It has considered m/c breakdowns; and the research is indirectly a reliability study. |
| 5. Jeon et al. (1998b) | Part family, alternative routes for parts. | Calculation of similarity ratio with algebraic relationship model. | Determination of part similarity coefficient considering number of alternatives routes during machine failure. | New similarity coefficient can be used for identifying part family. | The study considered machine breakdowns in CMS. |
| 6. Kuroda and Tomita (2005) | *MTBF*, *MTTR*, cycle time<br>Expected value of over load from machine failure<br>Assumed identical availability for all facilities | Cellular line design by MIP non-linear model | Objective to minimize number of facilities. | Using of *MTBF* and *MTTR* in designing a cellular line | Cellular line, each unit of line a cell. Exponential distribution for machine failure |

**Table 3.2 cont'd**

| Study | Factors considered | Modeling descriptions | Objectives | Major Findings | Relevance |
|---|---|---|---|---|---|
| **CMS RELIABILITY RELATED STUDY (Direct and Indirect)** | | | | | |
| 7.Seifoddini and Djassemi (2001 | Intercellular workload, mean flow time, total work load, M/C reliability, WIP. | Simulation modeling. | Performance evaluation and relative sensitivity of job shop and CMS with machine reliability change. | CMS is more sensitive to reliability changes than job shop. | Reliability has been considered for the performance evaluation of the CMS. |
| 8. Savsar (2000) | Pallet capacity, failure and repair rates and processing rates for machines, robot, and material handling device. Loading and unloading rate of jobs. | Markov model for a single machine FMC. | Comparison of performance of a reliable and unreliable FMC. | Development of formulas for the utilization of machine, robot, and material handling devices under stochastic failure conditions. | The study is related to the flexible manufacturing cell. |
| 9.Zakarian and Kusiak (1997) | Production capacity, availability, imperfect repair, imperfect coverage, repair rate, failure rates. | Analytical model , Markov chain, transient analysis. | Performance evaluation in terms of system availability. | Impact of imperfect repair and imperfect coverage, system availability by decomposition approach. | Performance evaluation considering machine failure for automatic manufacturing shops. |
| **FMS RELIABILITY STUDY** | | | | | |
| 10. Miriyala and Vishwanadam (1989) | Part routing and reliability of individual machines and subsystems. | Process spanning graph, general probability law and exponential distribution. | To study part reliability defines as the steady state probability of producing a part types by using at least one of the available routes and FMS reliability. | Routing and operational flexibility can contribute to increased reliability in FMS. | FMS reliability consideration study. |
| 11. Nagarajah et al. (1992) | M/C reliability, transporter speed, average flow times, total number of parts produced. Cell layout and process layout. | Simulation model with software MAST, data obtained from FMS users. | To study the effect of machine reliability and transporter speeds on FMS performance. | Cellular layout is more efficient than process layout when all the machines have high reliability. | FMS related study. The result or findings are similar to CMS. |

**Table 3.2 cont'd**

| Study | Factors considered | Modeling descriptions | Objectives | Major Findings | Relevance |
|---|---|---|---|---|---|
| **FMS RELIABILITY STUDY cont'd** | | | | | |
| 12. Lin et al. (1994) | Maintenance floats, failure rate, repair rate, production cost, availability. | Queuing models. | To implement maintenance float policy to keep operating equipment at a high level of availability. | Finding out the optimal maintenance float to achieve optimal repair capacity. | It is an FMS reliability study. |
| 13. Rupe and Kuo (2001) | Machine failures, spare inventory and repair process. | Markov model. | Performability measure for mission effectiveness. | Performability measure which reflects system effectiveness. | FMS performance study. Can be used for similar analysis of CMS. |
| 14. Wang and Wan (1993) | Initial reliability of the machines, failure rate, production performance requirement and cost. | Dynamic reliability model with fuzzy information. | Effect of failure modes, training of engineers and initial failure rates. | Identified 17 failure modes of FMS. If initial reliability is handled carefully and engineers are trained to handle failure system, performance will improve. | It is an FMS reliability study. |
| 15. Yuanidis et al. (1994) | System throughput, average production rate, *MTBF*, *MTTR*, operation times for jobs. | Statistical and analytical model, established functional relationship to map parameter space into the response space. | Development of a model or tool which closely resembles the FMS so that it can be used for system evaluation. | The algorithm GMDH which has been used to analyse the FMS, found to closely predict FMS output with the given input information. | It is an FMS reliability study. |

included in a single group, and a block replacement policy at fixed intervals was adopted. According to data analysis, the pulverizers were treated as a series system, and the failure process of the sub-systems was modeled as a renewal process. Due to the series structure, the opportunistic maintenance (OM) was considered to be appropriate. The study developed an $(n_{ij}, N_i)$ policy for OM maintenance where $n_{ij}$ represented OM age of component *i,* when the system was taken down for maintenance on component *j*, and $N_i$ represented the age at which preventive maintenance(PM) was carried out on component *i*. If $N_i$ was ∞, the policy became $(n_{ij}, \infty)$, and only OM was carried out when failure maintenance (FM) was conducted for *j*. OM ages for each component was specified corresponding to each opportunity class regardless of whether it was taken down for FM or PM. A simulation model was developed to validate the approach. This approach was suitable for small number of machines (systems as defined by the study); consequently, for a reasonable-sized CMS structure, it would be very complex to follow this procedure.

Sherwin (1997) constructed a block renewal model and illustrated it by assuming bad-as-old repairs between scheduled renewals. The study considered a total expected maintenance cost model which took into account the cumulative hazard rate function, planned renewal costs, and repair costs. The Weibull distribution was used for defining the cumulative hazard rate function. The best value for the number of intervals and the optimal interval were estimated from the model equation such that the minimum rate of change of expected cost per unit time was achieved. The paper emphasized the usefulness of the proposed component renewal model and illustrated the model with an example by considering *n* equal intervals and *n-1* renewals. Although the model was developed for single machine systems, a similar approach may also be implemented for multi-machine situations.

Talukder and Knapp (2002) developed a heuristic method for grouping equipment that would allow the application of PM in a series system with the goal of minimizing the total maintenance-related costs. The Weibull distribution was applied to represent increasing failure rates of the equipment. The paper derived a total cost model, and evaluated the PM intervals by minimizing the total cost for individual equipment groups. Decisions on the number of groups and the assignment of individual equipment to the groups were made using a similarity coefficient -based heuristic method. The authors used a group forming methodology similar to the group technology (GT) approach and claimed that the approach generated excellent results for their problems.

Kardon and Fredendall (2002) developed a preventive maintenance model to balance breakdown and preventive maintenance costs through the evaluation of preventive maintenance intervals that can limit the probability of system breakdown at the maximum tolerable limit set by the management. Using the Weibull distribution, their model was developed to decide the preventive maintenance interval so that the failure probability of a machine stayed below the specified limit set by the user organization. For multi-machine, multi-component systems the study considered four maintenance policies. The first policy was the replacement of components individually, which would incur very high costs and involve high downtime for conducting preventive maintenance in a serial manufacturing scenario similar to CMSs and, as such, was not a feasible proposition for multi-machine systems. The second policy was the block replacement or replacement of all components at the first failure. This is not a very efficient maintenance planning approach because it requires an organization to carry a high volume of spare parts to be ready to carry out the maintenance at the first failure. In addition, conducting preventive maintenance on all machines at every failure occurrence would be likely to incur very high costs. The third policy was a block replacement approach in which items were classified into a few categories or blocks depending on the similarity of maintenance intervals of the components as determined to ensure the minimum probability of failure. The fourth policy was the replacement of all components by determining the shortest maintenance interval to ensure a tolerable overall failure probability. After comparing the policies, the authors finally suggested to apply trial and error to adjust he interval to achieve the minimum possible total cost in a specific situation. The third and fourth policies may be applied to make a decision on preventive maintenance intervals.

To conclude on the studies related to preventive maintenance of CMSs or similar manufacturing environments: it is evident from these preventive maintenance studies for multi-unit situations similar to CMSs that almost all research considered a cost-based approach for deciding on preventive maintenance intervals. As Wang (2002) pointed out, all maintenance actions aim to improve the reliability performance of the system, but most of the maintenance models treated in the literature used cost optimization criteria, ignoring the reliability performance. Although Kardon and Fredendall (2002) did take machine reliability into consideration when developing the PM plans, their study did not consider the cost of performing unnecessary maintenance actions on some of the machines, as called for in block replacement policies. Considering the methodologies available in the literature, this dissertation proposes a new approach to PM planning in CMSs that addresses both machine reliability and cost. This dissertation also presents a CMS design model that incorporates machine reliability and PM planning concept to improve the performance of the system in terms of system reliability.

### 3.5 Motivation for the Proposed Research

This literature review establishes that reliability is one of the major factors affecting the performance of the CMS. Machine downtimes in CMSs can be accommodated when they are preplanned (e.g., planned maintenance), and cell configuration is capable of rerouting the parts. The literature review also indicates that research considering machine reliability and maintainability of cellular manufacturing systems is somewhat limited. Most of the existing works on CMS design consider machines to be 100% reliable, which is clearly an impractical assumption. The limited research which does consider the reliability of CMS machines has mainly emphasized performance evaluation and comparison between manufacturing systems. Only a limited number of studies have considered reliability in the planning and design stages. This situation calls for an integrated approach to consider machine reliability in the cell formation and operation allocation processes to improve the performance of CMSs in terms of efficiency and cost-effectiveness.

Manufacturing organizations pursue preventive maintenance steps on the machines and equipment to restrict deterioration and improve the reliability performance of machines. As such, maintenance considerations of the machines should also be integrated in the CMS design and planning process. This dissertation focuses on considering and analyzing machine reliability and maintainability in the design of the cellular manufacturing system to achieve the objectives listed in section 1.5 while attempting to address most of the drawbacks of CMS design models identified in the literature (Agarwal and Sarkis, 1998; Boughton and Arokiam ,2000; Savsar , 2000, and Diallo et al.,2001).

As discussed previously, modern manufacturing machines (CNCs and others) are capable of performing multiple operations, which facilitates each part type to have more than one processing route. Uncertainties in the manufacturing system may arise from both internal and external disturbances (Garret, 1986). Internal disturbances such as machine breakdowns, variable task times, and queuing delays can be solved with the help of routing flexibility. Routing flexibility improves the CMS performance in the following two ways:

- It facilitates the selection of alternate processing routes for a part type.
- It allows a design model to select a processing route with the highest system reliability.

Routing flexibility provisions can be used to overcome the challenges of machine breakdowns, machine non-availability, or changes in machine reliability resulting in under capacity use of the cells. The CMS is often considered flexible in its ability to respond to machine capability changes, volume, and mix changes. Manufacturing cells should be designed with these types of changes in mind, because failure to consider these changes results in poorer CMS performance

( Vakharia et al, 1999). Two limitations related to routing flexibility in the existing CMS design approaches are:

- There are several design methods in the literature which do not consider routing flexibility (Askin and Chieu, 1990; Ballkur and Steudel, 1987, etc). These CMS design approaches assume that each operation of a part type can be processed on one of the specific types of machines. Therefore, they cannot handle unexpected changes in part demand or changes to product mix; they also fail to address the possible under capacity of the shop floor due to uncertainties such as worker absenteeism, queuing delays, and machine breakdowns. As such, these approaches develop an inferior cell configuration and reduce machine utilization.
- There are studies which take into account alternative routings in their design, and aim to achieve objectives other than solving machine reliability related issues. By considering machines to be reliable, these approaches use routing flexibility to determine the best processing route assignment and the best cell configuration that will minimize interaction between cells and improve resource utilization (Gindy et al. 1996; Rajamani et al., 1990; Shankar and Agarwal,1997; Caux et al.,2000).

Possible rerouting of the parts should be considered while developing the CMS design to handle machine breakdown situations (Jeon et al., 1998).

### 3.6 Research Methodology

From the literature review it is concluded that reliability consideration of machines in the design of CMSs have not received much attention. In addition, recent research reveals that CMSs have not met the expected performance criteria due to machine failures and machine non-availability problems. Therefore, this research attempts to fill this gap. There are two common ways to ensure that the desired performance is achieved by the CMS. The first may be keeping more than one copy of each machine in the cells which will ensure very high reliability and availability levels. However, machines are the most costly resource in the CMS and, as such, this approach are not economically viable. The second approach focuses on integrating reliability considerations of the existing machines into the design phase so that the processing route selected for a part type has the highest system reliability for the machines along the route. Pursuing the second approach by giving due consideration to machine availability and alternate routing during operation allocation and machine loading will ensure a high level of system reliability. This is, however, an economic issue as we must balance the expected costs against the increased reliability to achieve an optimal solution.

The methodology of this research includes the following factors to reduce the machine reliability related impacts and accomplish the set objectives:

- consideration of machine availability to estimate effective machine capacity.
- selection of the most reliable machine routes for part types.
- provision of alternative process plans for rerouting parts in case of machine failure.
- the optimization of operation and machine utilization related costs.
- simultaneous considerations of system costs and system reliability along the part processing route to ensure the development of an effective and economically viable cell configuration.
- consideration of preventive maintenance in the CMS design process
- evaluation of the alternative options in the light of reliability/ cost trade-offs.
- evaluation of system availability as a measure of performance.
- generation of a solution procedure for large scale CMS design problems.

## CHAPTER 4
## DESIGN OF CMS WITH MACHINE RELIABILITY CONSIDERATIONS

This chapter is divided into 4 sections. Section 4.1 covers the assumption, definitions, problem statement and explanations of the multi-objective MIP model for simultaneous cell formation and operation allocation with consideration of machine reliability.

Section 4.2 presents the multi-objective mathematical model for designing cellular manufacturing systems (CMSs) with machine reliability considerations based on the exponential distribution. This model also evaluates CMS performance in terms of cell system availability for part type-process plan assignment by following the Markovian approach. For details on the analysis and development of the performance evaluation model, refer to section 2.4. System availability is defined as the total probability of cell states where relevant machines needed for processing a part type under a selected process plan are in the *up* (operating) condition. The steady state probability of the cell-machine states has been considered in the assessment of system availability. The applicability of the model is illustrated through an example solved by the developed procedure.

The development of the CMS design model with machine reliability considerations based on the Weibull distribution is presented in section 4.3. This section also addresses the optimal solution procedure for the model, and illustrates the comparison between outputs of the model based on Weibull distribution and the model based on exponential distribution by taking into consideration the same numerical example. The fact that the performance evaluation model is developed based on the Markovian approach makes it relevant to the exponential distribution model only. Therefore, section 4.3 does not include performance evaluation in terms of system availability.

In section 4.4, a large-sized problem is solved following the exponential distribution as well as the Weibull distribution based models in an effort to further illustrate the applicability of the models.

### 4.1 Model Development

The main goal of this research is to develop a multi-objective, mixed-integer programming model for simultaneous solutions of the cell formation and operation allocation problems in the design of CMSs with the considerations of machine reliability. The model, which follows the approach of Atmani et al. (1995), is based on the selection of a process plan for each part type which maximizes the overall system reliability, while minimizing the overall costs. The model has two objectives: The first objective is to optimize the variable costs of machining and intercell material movement as well as penalty costs for machine under-utilization in the joint cell formation and operation allocation problems. Detailed definitions of costs will follow.

The second objective is to optimize the system reliability of machines along the part-processing routes in the cell formation and operation allocation problem. The analysis and development of the system reliability of machines along part processing routes has been detailed in sections 2.3.2 and 2.5.2 for the two reliability distribution approaches.

### 4.1.1 Assumptions

The development of the mathematical model is based on the following assumptions:

1. The machines have been in the cell for some time, either in a functional layout (job shop-like) or a flow shop-like layout.
2. The machines are labeled by unit number (machine number), not by type.
3. The capabilities and capacities of the machines are known.
4. There is a set of part types to be processed, and the demand for each part type is fixed, and is chosen randomly from a uniform distribution.
5. Operation costs, refixturing costs, operation times and refixturing times for each part type to be processed on the machines are known.
6. Each operation of the part types may be performed on more than one machine.
7. Material handling costs from machine to machine within the cell (intracell) is negligible.
8. Material handling costs from cell to cell (intercell) is known. Material handling is done batch-wise. The total demand of each part type for a period is considered as a batch.
9. The number of cells to be used is specified in advance.
10. Bounds on the number of the machines in each cell are specified in advance.
11. Set up times are equivalent to refixturing times.
12. Since demand is uniform, the model is developed for a typical period of the demand.
13. There are certain machine reliability assumptions:
    - 13.1 The maintenance files of the machines contain updated information on *MTBF* (mean time to failure) and *MTTR* (mean time to repair).
    - 13.2 Machine availability for the model based on the exponential distribution is estimated by the interval availability approach ( equation 2.15), while that for the model based

on the Weibull distribution is estimated by the inherent availability approach (equation 2.10).

13.3 The multi-objective CMS design model and performance evaluation model presented in section 4.2 are based on the exponential distribution.

13.4 The multi-objective CMS design model presented in section 4.3 is based on the Weibull distribution. Machines in this model are assumed to be independent and follow increasing failure rates with different assumed shape factors. The characteristic life for each machine is estimated based on its *MTBF* and shape factor.

14. Cell layout and job scheduling issues of the CMS design are not addressed in this dissertation.

### 4.1.2 Definitions of Objective Function Parameters

**Processing/Operating cost** is the cost of performing specific operations on part types, by individually capable machines. This cost depends on the type of machine and the number of hours needed to perform said operations. The cost per piece for each of the specific part types and operations can be determined by considering the type of machine and the number of hours needed. This model uses cost per part type for each operation to be performed on each individual machine.

**Intercell material handling cost** is the cost of transferring parts between cells when all the operations of a part type cannot be completed within a single cell. This intercell transfer can occur because not all the machine types required to process the parts are available in the cell to which the parts are allocated, or because the cell does not have sufficient processing capacity.

**Refixturing cost** is the cost of loading and fixing each part on machines for each of the operations. This cost depends on the refixturing time, fixture type, type of operation and type of machine. Considering all these factors, the cost is converted to cover the cost per part type for each operation on individual machines.

**Machine under-utilization cost** is the penalty cost for the proportion of the effective available time a machine is idle in a planning period. It is determined by multiplying the penalty cost coefficient by the proportion of non-productive time associated with a machine during a planning period. The penalty cost coefficient for a machine type may be estimated from the fixed cost, indirect labor and other factory overhead chargeable to the machine for the planning period under consideration.

**Machine reliability** is the survival probability of individual machines for the planned operation period of the system. The objective function of the model incorporates the system failure rate and the inverse of system reliability for the machines along the part-processing routes as a measure of reliability in the case of the exponential distribution and the Weibull distribution, respectively. The system failure rate is estimated from the *MTBF* values of the machines, and the inverse of system reliability is estimated from the *MTBF* and shape factor values. Most researchers have developed cell formation and operation allocation models that consider machines to be 100% reliable. The proposed model considers machines to be unreliable, and includes the reliability and effective availability of the machines in the process of cell formation and operation allocation to optimize reliability and achieve an optimum/desired cost. The description in Chapter 2 forms the basis for considering machine reliability for the CMS design models in this chapter.

The three cost elements of the first objective function are interrelated, and could conflict. For example, operation costs can be minimized by selecting the machines which have the lowest processing cost, but penalty costs force the model to utilize all the machines. Similar arguments can be made for intercell material handling costs and processing costs. Processing costs may be minimized by selecting machines in two or more cells but intercell material handling costs force the model to limit intercell movement. Therefore, decisions for these costs need to be made simultaneously.

The first and second objective functions have contradictory requirements. While the first objective targets simultaneous cell formation and operation allocation by optimizing cost, the second objective targets the development of CMS design to optimize machine reliability only. Often, machines with higher reliabilities or lower failure rates have higher processing and refixturing costs. The penalty costs for machine under-utilization and intercell material movement are also in opposition in relation to machine reliability. Therefore, cost and reliability need to be integrated in the CMS design approach in order to successfully make trade off decisions and fulfill business goals.

### 4.1.3 Problem Definition

We assume that there is a set of $m$ machines with indices $j=1,2,..,m$ to process a set of $n$ part types with indices $i= 1,2,..,n$, where the part type $i$ has uniform demands $d_i$ during the planning period $T$. Each machine $j$ has a reliability level defined by its $MTBF_j$ and $MTTR_j$ (mean time between failure and mean time to repair), and a specified capacity $b_j$ for the entire planning period in terms of total available hours. A part type $i$ may be processed under any of the process plans $p=1,2,..,P(i)$. For a process plan $p$ of a part type $i$, the operations are represented by the indices $o=1,2,\dots, O(ip)$, and the machines that can perform operation $o$ of $(ip)$ are represented by the set $J_{ipo}$.

The objective is to group the machines into a number of cells, and to assign the part types to one or more cells for processing so as to minimize the total costs and maximize the over all system reliability.

### 4.1.4 Notations Used

**Indices and sets**

| | |
|---|---|
| $c \in \{1,2,..,C\}$ | cells |
| $i \in \{1,2,..,n\}$ | part types |
| $j \in \{1,2,..,m\}$ | machines |
| $J_{ipo} \subset \{j = 1,2,..,m\}$ | set of machines that can perform operation $o$ of $(ip)$ |
| $k \in \{1,2,..,N\}$ | index of the states of a cell in a Markovian transition |
| $o \in \{1,2,..,O(ip)\}$ | operations for part type $i$ following process plan $p$ |
| $p \in \{1,2,..,P(i)\}$ | process plan for part type $i$ |
| $ip$ | a part type-process plan combination |
| $s_j \in \{0,1\}$ | state of machine $j$; 1 = operating, 0 = not operating |
| $w_k = \{s_1.s_2,...s_m\}$ | cell states with $m$ machines where each $s_j \in \{0,1\}$ |
| $W = \{w_1.w_2,...w_N\}$ | cell state space |
| $\Gamma$ | gamma function symbol |

**Parameters**

| | |
|---|---|
| $A_j(T)$ | availability of machine $j$ in time period $T$ |
| $b_j$ | amount of time available on machine $j$ during the planned manufacturing period |
| $CO_{oj}(ip)$ | cost of performing operation $o$ of $(ip)$ on machine $j$ |
| $cp_j$ | penalty cost for the non utilization proportion of machine $j$ |
| $CR_{oj}(ip)$ | cost of refixturing a unit of $(ip)$for operation $o$ on machine $j$ |
| $d_i$ | demand for part type $i$ distributed uniformly over the planning period |
| $H_{ijc\hat{j}\hat{c}}$ | cost of moving part type $i$ from machine $j$ in cell $c$ to machine $\hat{j}$ in cell $\hat{c}$ for performing the next operation |

| | |
|---|---|
| $MS_{w_k j}$ | an indicator of the state of machine $j$ in CMS state $w_k$; equals 1 if machine $j$ is in operating condition in CMS state $w_k$, 0 otherwise |
| $MTBF_j$ | mean time between failures for machine $j$ |
| $MTTR_j$ | mean time to repair for machine $j$ |
| $r_j$ | repair rate for machine $j$ |
| $TO_{oj}(ip)$ | time for performing operation $o$ of $(ip)$ on machine $j$ |
| $TR_{oj}(ip)$ | time for refixturing $(ip)$ for operation $o$ on machine $j$ |
| $UM$ | maximum number of machines in a cell |
| $\lambda_j$ | failure rate of machine $j$ |
| $\pi_{w_k}$ | steady-state probability of CMS being in state $w_k$ |
| $\beta_j$ | shape factor for machine $j$ |

**Decision variables**

| | |
|---|---|
| $LIR_{ip}$ | system reliability measure corresponding to the machines performing the set of operations for $(ip)$ |
| $M_{jc}$ | 1 if machine $j$ is assigned to cell $c$; 0 otherwise |
| $SA(ip)$ | manufacturing system availability indicator corresponding to a given $(ip)$ in relation to the CMS state space $W$ |
| $SI_{w_k}^{ipoj}$ | 1 if, in CMS state $w_k$, machine $j$ is in operating condition to perform operation $o$ of $(ip)$; 0 otherwise |
| $TH_{w_k}(ip)$ | 1 if CMS state $w_k$ is selected in which machines needed to perform all the operations of $(ip)$ are in operating condition; 0 otherwise |
| $X_{ojc}(ip)$ | 1 if operation $o$ of $(ip)$ is performed on machine $j$ in cell $c$; 0 otherwise |
| $Y_{ojc\hat{j}\hat{c}}(ip)$ | 1 if part type $i$ moves to machine $\hat{j}$ in cell $\hat{c}$ to perform operation $(o + 1)$ after performing operation $o$ on machine $j$ in cell $c$, following process plan $p$; 0 otherwise |
| $Z(ip)$ | 1 if part type $i$ is processed following process plan $p$; 0 otherwise |

## 4.2 Mathematical Model Based on Exponential Distribution for Machine Reliability

### 4.2.1 The Mathematical Model

The following is a detailed description of the multi-objective mathematical model for the manufacturing cells.

**Objective function:** The first objective function (defined as objective function I) computes the total system costs consisting of the variable cost of machining (*VCM*), the inter-cell material handling cost (*MHC*) and the penalty cost of machine under utilization (*MNC*):

$$\textit{Minimize}\ \text{Objective function I} = VCM+MHC+MNC \qquad \text{----------- (4.1)}$$

The variable cost of machining *VCM* takes into account the operation cost $CO_{oj}(ip)$ and the refixturing cost $CR_{oj}$ $(ip)$. The 0-1 decision variable $X_{ojc}(ip)$ equals 1 if operation $o$ of $(ip)$ is performed on machine $j$ in cell $c$ , and is zero otherwise. Thus, *VCM* may be expressed as:

$$VCM = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \{CO_{oj}(ip) + CR_{oj}(ip)\} \sum_{c=1}^{C} X_{ojc}(ip) \qquad \text{-------- (4.1a)}$$

The inter-cell material handling cost *MHC* computes the cost of moving parts from cell $c$ to cell $\hat{c}$. $H_{ijc\hat{j}\hat{c}}$ is the cost of moving a unit of part type $i$ from machine $j$ in cell $c$, after performing operation $o$, to machine $\hat{j}$ in cell $\hat{c}$ for the next operation, $(o+1)$:

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j \in J_{ipo}} \sum_{\hat{j} \in J_{ip(o+1)}} \sum_{1 \le c, \hat{c} \le C} H_{ijc\hat{j}\hat{c}} X_{ojc}(ip) X_{(o+1)\hat{j}\hat{c}}(ip)$$

It is noted that *MHC* is a non-linear function, which may be linearized following the procedure described in Taha (1992), by replacing the product term $X_{ojc}(ip) X_{(o+1)\hat{j}\hat{c}}(ip)$ by a binary linearization variable, $Y_{ojc\hat{j}\hat{c}}(ip)$ which satisfies the following two constraints:

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - 2Y_{ojc\hat{j}\hat{c}}(ip) \ge 0, \qquad \text{--------- (4.2)}$$
$$\forall i, p, o \in \{1,2,\ldots, O(ip)-1\},\ j \in \{J_{ipo},\ \hat{j} \in J_{ip(o+1)}, c, \hat{c}$$

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - Y_{ojc\hat{j}\hat{c}}(ip) \le 1 \qquad \text{--------- (4.3)}$$
$$\forall i, p, o \in \{1,2,\ldots, O(ip)-1\},\ j \in J_{ipo},\ \hat{j} \in J_{ip(o+1)},\ c,\ \hat{c}$$

It is evident that $Y_{ojc\hat{j}\hat{c}}(ip)$ takes the value of 1 if and only if a unit of part type $i$ is moved from machine $j$ in cell $c$, after performing operation $o$, to machine $\hat{j}$ in cell $\hat{c}$ for operation $(o+1)$. Thus, the expression for *MHC* is:

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j \in J_{ipo}} \sum_{\hat{j} \in J_{ip(o+1)}} \sum_{1 \le c, \hat{c} \le C} H_{ijc\hat{j}\hat{c}} Y_{ojc\hat{j}\hat{c}}(ip) \qquad \text{--------- (4.1b)}$$

Finally, the term *MNC* computes the penalty cost for the proportion of the time machine $j$ is under utilized:

$$MNC = \sum_{j=1}^{m} cp_j \left( 1 - \left[ \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \frac{TO_{oj}(ip) + TR_{oj}(ip)}{A_j(T) b_j} \right] \right) \sum_{c=1}^{C} X_{ojc}(ip)$$

----------- (4.1c)

where $b_j$ is the total capacity and $A_j(T)$ is the availability of machine $j$ during the planning period $T$, and therefore, $A_j(T)b_j$ represents the effective capacity of machine $j$. In addition, $TO_{oj}(ip)$ and $TR_{oj}(ip)$ are the operation and refixturing times, respectively, corresponding to operation $o$ of $(ip)$ on machine $j$, and $cp_j$ is the penalty cost of the non-utilized fraction of the effective capacity of machine $j$.

The second objective function (defined as objective function II) computes the system reliability in terms of system failure rate over the set of all part-process plan combinations:

$$\textit{Minimize} \text{ Objective function II} = \sum_{i=1}^{n} \sum_{p=1}^{P(i)} LIR_{ip} \qquad \text{----------- (4.4)}$$

where,

$$LIR_{ip} = \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} \lambda_j X_{ojc}(ip) \qquad \forall i, p$$

Assuming the failure rate of machine $j$ follows an exponential distribution,

$$\lambda_j = \frac{1}{MTBF_j}$$

$LIR_{ip}$ calculates the system failure rate corresponding to the machines which perform the set of operations for $(ip)$. Objective function II seeks to select the set of process plans for all part types that results in the minimum system failure rate, and thus maximizes the system reliability. A detailed derivation for $LIR_{ip}$ is described in section 2.3.2

**Constraints:** The following constraints are defined:

Constraint set (4.5) assigns each part to a single process plan. The binary variable $Z(ip)$ equals one if and only if part type $i$ is processed under process plan $p$.

$$\sum_{p=1}^{P(i)} Z(ip) = 1 \qquad \forall i \qquad \text{----------- (4.5)}$$

Constraint set (4.6) ensures that when a process plan for a part type is selected, each operation of the process plan is assigned to one of the available machines in one of the cells.

$$\sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) = Z(ip) \qquad \forall i, p, o \qquad \text{------------ (4.6)}$$

The next set of constraints ensures that machine $j$ is assigned to at most one of the cells. Variable $M_{jc}$ equals one if machine $j$ is assigned to cell $c$, and is zero otherwise.

$$\sum_{c=1}^{C} M_{jc} \leq 1 \qquad \forall j \qquad \text{----------- (4.7)}$$

Constraint set (4.8) enforces an upper limit, $UM$, on the number of machines allowed in each cell. This upper limit is a design factor set by the CMS user.

$$\sum_{j=1}^{m} M_{jc} \leq UM \qquad \forall c \qquad \text{------------ (4.8)}$$

The next constraint set ensures that a machine $j$ has to be assigned to a cell $c$ before any operation could be allocated to that machine.

$$\sum_{i=1}^{n} \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} X_{ojc}(ip) \geq M_{jc} \qquad \forall j, c \qquad \text{------------ (4.9)}$$

Constraint set (4.10) ensures that the allocated operations do not overload a machine beyond its effective capacity.

$$\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} [\ TO_{oj}(ip) + TR_{oj}(ip)\ ] X_{ojc}(ip) \leq b_j M_{jc} A_j(T) \qquad \forall j, c \quad \text{----------- (4.10)}$$

As explained earlier, $TO_{oj}(ip)$ and $TR_{oj}(ip)$ are, respectively, the operation time and the refixturing time of operation $o$ of $(ip)$ on a machine $j$. The effective capacity of machine $j$ is estimated by multiplying its capacity $b_j$ by the availability factor $A_j(T)$ which is computed as explained in section 2.3.1.

Constraint sets (4.11)-(4.14) evaluate the performance of the CMS when it is processing the $(ip)$ combination. While the operation allocation variable $X_{ojc}(ip)$ selects machine $j$, in cell $c$, to perform operation $o$ of $(ip)$, constraint (4.11) selects the cell state $w_k$ in which machine $j$ is in operating condition. The zero-one variable $SI_{w_k}^{ipoj}$ equals 1 when machine $j$ (which is slated to perform operation $o$ of $(ip)$) is in operating condition in cell state $w_k$, and is zero otherwise. The parameter $MS_{w_k j}$ assumes a value of 1 or zero, depending on whether machine $j$ is in operating condition in cell state $w_k$, or not. The value of $MS_{w_k j}$ at each cell state $w_k$ may be determined by the probable machine state analysis as per the Markov chain approach.

$$\sum_{c=1}^{C} X_{ojc}(ip) MS_{w_k j} - SI_{w_k}^{ipoj} = 0 \qquad \forall w_k \ , i, p, o, j \quad \text{--------- (4.11)}$$

Constraint sets (4.12) and (4.13) work together to ensure the selection of only those cell states $w_k$ where all the machines needed to perform all the operations of *(ip)* are in operating condition. $TH_{w_k}(ip)$ equals 1 when in the cell state $w_k$, all the machines needed to complete the processing of *(ip)* are in operating condition. For example, if a part type *i* needs machines *j=2, 3, 5* to perform operations 1, 2, and 3, respectively, then $TH_{w_k}(ip)$ will be 1 if and only if in the cell state $w_k$ all three machines are in operating condition, depending on the decision made by operation allocation variables $X_{ojc}(ip)$. Here *MO* is a large number.

$$\sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) - \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} SI_{w_k}^{ipoj} \le MO(\ 1 - TH_{w_k}(ip)\ ) \quad \forall w_k, i, p \quad \text{--------- (4.12)}$$

$$\sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) - \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} SI_{w_k}^{ipoj} \ge (\ 1 - TH_{w_k}(ip)\ ) \quad \forall w_k\ , i, p \quad \text{----------- (4.13)}$$

Constraint set (4.14) calculates the system availability *SA(ip)* of the manufacturing cell for the selected *(ip)* combination depending on the steady state probability of the cell state space *W*. $TH_{w_k}(ip)$ selects the steady state probability $\pi_{w_k}$ if the cell state $w_k$ is available or if it can perform all the needed operations for the (*ip*) combination. Steady state probability for each of the cell states can be calculated as per the approach described in section 2.4.

$$SA(ip) = \sum_{w_k = w1}^{w_N} \pi_{w_k} TH_{w_k}\ (ip) \qquad \forall i, p \qquad \text{--------- (4.14)}$$

The last constraint set enforces the integrality of the variables.

$$X_{ojc}(ip), Y_{ojc\hat{j}\hat{c}}(ip), Z(ip), M_{jc}, SI_{w_k}^{ipoj}, TH_{w_k}\ (ip) \in \{0,1\} \ldots \forall i, p, o, j, c, \hat{j}, \hat{c}, w_k \quad \text{------- (4.15)}$$

### 4.2.2 Model Summary

Assembling the above, the model may now be presented as follows:

*Minimize* objective function I

$$\text{Objective function I} = VCM + MHC + MNC \qquad \text{-------- (4.16)}$$

where,

$$VCM = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \sum_{j \in Jipo} \{CO_{oj}(ip) + CR_{oj}(ip)\} \sum_{c=1}^{C} X_{ojc}(ip) \qquad \text{--------- (4.16a)}$$

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j \in J_{ipo}} \sum_{\hat{j} \in J_{ipo}} \sum_{1 \le c, \hat{c} \le C} H_{ijc\hat{j}\hat{c}} Y_{ojc\hat{j}\hat{c}}(ip) \qquad \text{----------- (4.16b)}$$

$$MNC = \sum_{j=1}^{m} cp_j \Big( 1 - \Big[\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \frac{TO_{oj}(ip) + TR_{oj}(ip)}{A_j(T) b_j}\Big] \Big) \sum_{c=1}^{C} X_{ojc}(ip) \qquad \text{------- (4.16c)}$$

*Minimize* objective function II:

$$\text{Objective function II} = \sum_{i=1}^{n} \sum_{p=1}^{P(i)} LIR_{ip} \qquad \text{------------ (4.17)}$$

where,

$$LIR_{ip} = \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} \lambda_j X_{ojc}(ip) \qquad \forall i, p \qquad \text{------------ (4.17a)}$$

subject to the following constraint sets:

$$\sum_{p=1}^{P(i)} Z(ip) = 1 \qquad \forall i \qquad \text{------------ (4.18)}$$

$$\sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) = Z(ip) \qquad \forall i, p, o \qquad \text{------------ (4.19)}$$

$$\sum_{c=1}^{C} M_{jc} \le 1 \qquad \forall j \qquad \text{------------ (4.20)}$$

$$\sum_{j=1}^{m} M_{jc} \le UM \qquad \forall c \qquad \text{------------ (4.21)}$$

$$\sum_{i=1}^{n} \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} X_{ojc}(ip) \ge M_{jc} \qquad \forall j, c \qquad \text{----------- (4.22)}$$

$$\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} [\ TO_{oj}(ip) + TR_{oj}(ip)\ ] X_{ojc}(ip) \le b_j M_{jc} A_j(T) \qquad \forall j, c \qquad \text{---------- (4.23)}$$

$$\sum_{c=1}^{C} X_{ojc}(ip) MS_{w_k j} - SI_{w_k}^{ipoj} = 0 \qquad \forall w_k, i, p, o, j \qquad \text{----------- (4.24)}$$

$$\sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) - \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} SI_{w_k}^{ipoj} \le MO\big( 1 - TH_{w_k}(ip) \big) \qquad \forall w_k, i, p \qquad \text{---------- (4.25)}$$

$$\sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) - \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} SI_{w_k}^{ipoj} \ge \big( 1 - TH_{w_k}(ip) \big) \qquad \forall w_k, i, p \qquad \text{---------- (4.26)}$$

$$SA(ip) = \sum_{w_k=w1}^{w_N} \pi_{w_k} TH_{w_k}(ip) \quad \forall i,p \qquad \text{----------- (4.27)}$$

$$X_{ojc}(ip)+X_{(o+1)\hat{j}\hat{c}}(ip)-2Y_{oj\hat{j}c\hat{c}}(ip)\geq 0, \ldots\ldots \forall i,p,o\in\{1,2,\ldots\ldots O(ip)-1\}, j\in\{J_{ipo}, \hat{j}\in J_{ip(o+1)}, c,\hat{c} \qquad \text{----------- (4.28)}$$

$$X_{ojc}(ip)+X_{(o+1)\hat{j}\hat{c}}(ip)-Y_{oj\hat{j}c\hat{c}}(ip)\leq 1 \quad \forall i,p,o\in\{1,2,..O(ip)-1\}, j\in J_{ipo}, \hat{j}\in J_{ip(o+1)}, c,\hat{c} \qquad \text{------------ (4.29)}$$

$$X_{ojc}(ip), Y_{oj\hat{j}c\hat{c}}(ip), Z(ip), M_{jc}, SI_{w_k}^{ipoj}, TH_{w_k}(ip)\in\{0,1\} \ldots \forall i,p,o,j,c,\hat{j},\hat{c},w_k \qquad \text{------------ (4.30)}$$

The above mathematical model incorporates a pre-determined number of cells. Venugapal and Narendran (1992) show that the number of ways in which $m$ machines may be assigned to $C$ cells is given by the Stirling number of the second kind.

$$S^{(C)} = \frac{\sum_{c=1}^{C}(-1)^{C-c}\binom{C}{c}c^m}{C!}$$

For example, there are only 34,105 distinct partitions of 10 machines into 4 cells, but this number increases to 11,259,666,000 approximately, if 19 machines are to be partitioned into 4 cells, resulting in a combinatorial explosion. However, if $C$ is not pre-specified, with $m$ machines, the total possible number of cells ranges from 1 (every machine is assigned to the same cell) to $m$ (each cell has only 1 machine). The total number of ways in which machine-cell assignment may be made explodes to $\sum_{C=1}^{m} S^{(C)} = \sum_{C=1}^{m}\left(\frac{\sum_{c=1}^{C}(-1)^{C-c}\binom{C}{c}c^m}{C!}\right)$

It has been shown that this class of problem is NP-complete (Gary and Johnson, 1979). Thus, in the problem formulation, the number of cells needs to be pre-specified in order to maintain tractability. This decision is generally based on several factors such as total number of machines to be assigned into cells, physical constraints on the workshop floor, labor relation and other management decision issues.

In a survey of 32 US manufacturing firms, Wemmerlov and Hyer (1989) reported that the average for manned cells was 6.2 machines. The second largest size was 15 machines (the largest is 40 machines). The smallest typical cell size for a manned cell was 2 machines.

### 4.2.3 Optimal Solution Procedure

The two objective functions and all the constraints are linear equations. The objectives have competing interests that conflict with each other; therefore, they cannot be converted into a common scale of cost or benefits. The model is solved utilizing a hierarchical approach/preemptive approach that selects each objective as the most important (first priority) to decide the desired level of performance. Finally, the problem is solved with the $\epsilon$-constraint approach in order to provide options for the users—allowing them to make tradeoff decisions. Efficient frontier diagram and the data for efficient frontier solutions show the influence of reliability on the cell configuration, work allocation and system costs.

### 4.2.4 Numerical Examples

To demonstrate the applicability of the model, we present two examples. Example 1 is a cell design problem involving 7 machines and 12 part types, and Example 2 is one involving 10 machines and 19 part types. The examples are solved using LINGO 07 on a PENTIUM 4, 2.26 GHZ, 712 MB RAM computer. Example 2 is presented at the end of the chapter in section 4.4.

**Values for the parameters:** For examples, part and machine related information is generated randomly by considering similar data in the example problems of the literature. For the first example, the planning period (*T*) is considered to be 1500 hours. It is also assumed that all the machines are not available for the total planning period. Accordingly, machine capacity has been randomly generated using a uniform distribution with parameters [950 - 1500] hours. For the second example, the planning period is 1875 hours and all the machines are assumed to have the same capacity. Machine reliability related information (*MTBF* and *MTTR)* is generated randomly using a uniform distribution with the following parameters in order to maintain machine availability up to a maximum of 90% for Example 1, and 95% for Example 2.

| | Example 1 | Example 2 |
|---|---|---|
| *MTBF* | (80-200) | (90-225) |
| *MTTR* | (8-30) | (4-75) |

We assumed the maximum number of cells for both the example problems to be 3.

#### 4.2.4.1 Example 1: a 7-machine-12-part cell design problem.

**Input data**: Demands, processing times and costs, operation sequences and process plan information for the part types are presented in Table 4.1. Machine related information is given in Table 4.2. According to the input information, there are two process plans for each part type. A part can be allocated to different machine routes following a process plan with different machine combinations. Each part needs two to three operations for its complete processing. Each operation can be performed on two machines. The inter-cell transfer cost for a batch of a part type is assumed to be \$50.

Based on the *MTBF* and *MTTR* values from Table 4.2 the availability and effective available capacity of each machine during the planning period are evaluated. For example, machine 1 (M1) has an availability of 0.896 (using equation 2.15, $t_1$=0, and $t_2$ =1500 hours) and an effective capacity of 1344 hours. From Table 4.1, we can see that whenever a machine is not available due to failure or the processing of other parts, there are other options for a part to be processed by other machines in alternative routes. The available alternative routes for each part type under each process plan are shown in Table 4.3. Table 4.4 illustrates a list of alternative routes for a typical part (part type 1) whenever machine *j (1, 2,..,7)* is not available.

**Calculation of steady state probability for cell states:** To evaluate the CMS performance, we need the steady state probabilities of the cell states. With 7 machines in the cell, we have $2^7$ = 128 probable cell states, designated as $W$ = {1111111, 1111110,..., 0000000}. To develop the transition probability matrix TM, we need to compute, for each machine, the probability of making a transition within a short time interval, $\Delta t$, as explained in Section 2.4.1. As an example, consider machine M2 for which we need to compute $P_2^{1,1}, P_2^{1,0}, P_2^{0,1}, P_2^{0,0}$, where, for instance, $P_2^{1,1}$ = probability that machine 2 is making a transition from "operating" to "operating" within $\Delta t$; the probability $P_2^{1,1}$ may be assumed to be the interval availability of machine 2 over the planning horizon under consideration, and therefore, $P_2^{1,0}$ = (*1- availability of machine 2*). In a similar fashion, $P_2^{1,0}, P_2^{0,1}, P_2^{0,0}$ may be defined.

The probability of making a transition from "down" to "up" within a short time interval $\Delta t$, $P_j^{0,1}$ depends on the reparability and maintainability of the machine. Considering an exponential distribution, the probability of completing the repair work within the time $t$ is (Ebeling, 1997):

$$H(t) = 1 - e^{-t/MTTR}$$

The total down time, $t$, is usually higher than *MTTR* due to supply delay time, waiting time, etc. Assuming $t \geq 1.5\ MTTR$, $P_j^{0,1}$ is computed to be approximately in the range of 0.77 to 0.80, and $P_j^{0,0}$ can be estimated from $P_j^{0,0} = 1 - P_j^{0,1}$. Table4.5 presents the probability for the four states of each machine including the basis for the probability values. Taking these individual machine state probabilities as the input data, the elements of the transition probability matrix have been

**Table 4.1: Demands (units), processing times (hours), and costs ($) for all part types (Example 1)**

| PART TYPE | DEMAND | DATA TYPE | PROCESS PLAN 1 OPERATIONS 1 | 2 | 3 | PROCESS PLAN 2 OPERATIONS 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | M/C | M1 M4 | M1 M5 | M4 M6 | M1 M5 | M4 M6 | |
| | | Time | 0.94 1.34 | 0.85 2.42 | 2.25 2.02 | 1.83 2.39 | 2.76 1.78 | |
| | | Cost | 0.32 0.86 | 0.77 0.61 | 0.55 0.43 | 1.0 0.86 | 1.00 0.72 | |
| 2 | 125 | M/C | M2 M5 | M3 M7 | M2 M7 | M3 M5 | M2 M7 | M3 M7 |
| | | Time | 1.98 0.99 | 2.44 2.44 | 1.54 0.94 | 1.70 1.75 | 1.91 1.70 | 1.75 1.65 |
| | | Cost | 0.51 0.29 | 0.66 0.29 | 0.4 0.36 | 0.38 0.81 | 0.25 0.79 | 0.22 0.69 |
| 3 | 110 | M/C | M3 M7 | M3 M6 | M4 M6 | M4 M7 | M3 M6 | M2 M4 |
| | | Time | 2.02 2.5 | 2.34 2.76 | 2.21 2.39 | 1.71 1.26 | 2.16 0.86 | 1.35 1.53 |
| | | Cost | 0.76 0.39 | 0.73 0.83 | 0.46 0.53 | 0.72 0.26 | 0.76 0.57 | 0.52 0.51 |
| 4 | 120 | M/C | M1 M4 | M3 M6 | M2 M5 | M1 M5 | M2 M6 | |
| | | Time | 1.86 1.86 | 0.95 1.76 | 1.47 2.09 | 1.83 2.76 | 2.76 1.16 | |
| | | Cost | 0.70 0.51 | 0.2 0.61 | 0.51 0.69 | 0.57 0.43 | 0.4 0.50 | |
| 5 | 200 | M/C | M2 M5 | M1 M5 | M4 M6 | M2 M7 | M1 M6 | M4 M7 |
| | | Time | 1.2 2.47 | 2.27 2.09 | 2.41 1.77 | 1.67 2.28 | 2.45 0.86 | 1.12 2.33 |
| | | Cost | 0.68 0.46 | 0.24 0.7 | 0.43 0.62 | 0.55 0.54 | 0.53 0.55 | 0.44 0.73 |
| 6 | 125 | M/C | M3 M7 | M1 M5 | M2 M7 | M3 M7 | M1 M5 | M3 M7 |
| | | Time | 0.87 1.64 | 1.24 1.42 | 1.14 2.36 | 2.01 1.81 | 1.08 2.24 | 0.86 0.84 |
| | | Cost | 0.61 0.27 | 0.35 0.32 | 0.66 0.6 | 0.43 .315 | 0.34 0.34 | 0.61 0.34 |
| 7 | 90 | M/C | M2 M5 | M1 M4 | M1 M4 | M2 M6 | M1 M4 | |
| | | Time | 1.13 1.0 | 1.15 0.85 | 1.47 2.09 | 1.84 1.80 | 1.41 2.61 | |
| | | Cost | 0.31 0.72 | 0.58 0.82 | 0.74 0.61 | 0.55 0.74 | 0.66 0.63 | |
| 8 | 50 | M/C | M2 M5 | M2 M7 | M4 M6 | M3 M5 | M1 M7 | M3 M6 |
| | | Time | 1.43 2.48 | 2.11 1.02 | 0.91 1.11 | 1.44 2.31 | 2.42 1.26 | 1.22 2.36 |
| | | Cost | 0.3 0.68 | 0.75 0.41 | 0.36 0.3 | 0.70 0.75 | 0.93 0.41 | 0.26 0.48 |
| 9 | 80 | M/C | M3 M7 | M1 M4 | M3 M6 | M1 M3 | M4 M7 | M1 M6 |
| | | Time | 2.32 1.73 | 0.85 2.11 | 1.73 1.35 | 2.39 2.48 | 2.23 2.29 | 1.46 1.35 |
| | | Cost | 0.31 0.59 | 0.81 0.28 | 0.52 0.78 | 0.58 0.72 | 0.51 0.67 | 0.46 0.78 |
| 10 | 50 | M/C | M2 M5 | M3 M6 | M1 M5 | M2 M6 | M3 M5 | M1 M5 |
| | | Time | 2.17 2.25 | 1.44 1.77 | 2.19 1.22 | 2.01 2.03 | 2.08 2.29 | 1.52 1.09 |
| | | Cost | 0.37 0.62 | 0.25 0.48 | 0.44 0.56 | 0.49 0.24 | 0.78 0.70 | 0.44 0.31 |
| 11 | 60 | MC | M1 M7 | M3 M7 | | M2 M5 | M3 M6 | |
| | | Time | 4.9 5.2 | 2.4 2.3 | | 3.5 4.0 | 3.2 3.1 | |
| | | Cost | 1.0 0.9 | 0.8 0.8 | | 0..9 0.75 | 1.2 1.0 | |
| 12 | 45 | M/C | M1 M4 | M3 M6 | M2 M5 | M3 M7 | M2 M6 | |
| | | Time | 0.8 0.9 | 1.2 1.5 | 2.0 1.9 | 1.7 1.6 | 1.8 1.9 | |
| | | Cost | 0.4 0.4 | 0.3 0.3 | 0.4 0.5 | 0.5 0.5 | 0.6 0.6 | |

**Table 4.2: Machine information (Example 1)**

| Data Type | MACHINES M1 | M2 | M3 | M4 | M5 | M6 | M7 |
|---|---|---|---|---|---|---|---|
| Capacity (Hrs) | 1500 | 1400 | 1200 | 1100 | 1300 | 1000 | 1400 |
| MTBF (Hrs) | 187 | 89 | 160 | 131 | 83 | 181 | 130 |
| *MTTR* (Hrs) | 22 | 10 | 18 | 14 | 10 | 28 | 12 |
| Penalty cost for % non utilization | 256 | 292 | 454 | 270 | 391 | 300 | 283 |

**Table 4.3: Available alternative routes for the part types (Example 1)**

| Part type | Process plan | Alternative machine routes | Total |
|---|---|---|---|
| 1 | 1 | M1-M1-M4, M1-M1-M6, M1-M5-M4, M1-M5-M6, M4-M1-M4, M4-M1-M6, M4-M5-M4, M4-M5-M6 | 8 |
| | 2 | M1-M4, M1-M6, M5-M4, M5-M6 | 4 |
| 2 | 1 | M2-M3-M2, M2-M3-M7, M2-M7-M2, M2-M7-M7, M5-M3-M2, M5-M3-M7, M5-M7-M2, M5-M7-M7 | 8 |
| | 2 | M3-M2-M3, M3-M2-M7, M3-M7-M3, M3-M7-M7, M5-M2-M3, M5-M2-M7, M5-M7-M3, M5-M7-M7 | 8 |
| 3 | 1 | M3-M3-M4, M3-M3-M6, M3-M6-M4, M3-M6-M6, M7-M3-M4, M7-M3-M6, M7-M6-M4, M7-M6-M6 | 8 |
| | 2 | M4-M3-M2, M4-M3-M4, M4-M6-M2, M4-M6-M4, M7-M3-M2, M7-M3-M4, M7-M6-M2, M7-M6-M4 | 8 |
| 4 | 1 | M1-M3-M2, M1-M3-M5, M1-M6-M2, M1-M6-M5, M4-M3-M2, M4-M3-M5, M4-M6-M2, M4-M6-M5 | 8 |
| | 2 | M1-M2, M1-M6, M5-M2, M5-M6 | 4 |
| 5 | 1 | M2-M1-M4, M2-M1-M6, M2-M5-M4, M2-M5-M6, M5-M1-M4, M5-M1-M6, M5-M5-M4, M5-M5-M6 | 8 |
| | 2 | M2-M1-M4, M2-M1-M7, M2-M6-M4, M2-M6-M7, M7-M1-M4, M7-M1-M7, M7-M6-M4, M7-M6-M7 | 8 |
| 6 | 1 | M3-M1-M2, M3-M1-M7, M3-M5-M2, M3-M5-M7, M7-M1-M2, M7-M1-M7, M7-M5-M2, M7-M5-M7 | 8 |
| | 2 | M3-M1-M3, M3-M1-M7, M3-M5-M3, M3-M5-M7, M7-M1-M3, M7-M1-M7, M7-M5-M3, M7-M5-M7 | 8 |
| 7 | 1 | M2-M1-M1, M2-M1-M4, M2-M4-M1, M2-M4-M4, M5-M1-M1, M5-M1-M4, M5-M4-M1, M5-M4-M4 | 8 |
| | 2 | M2-M1, M2-M4, M6-M1, M6-M4 | 4 |
| 8 | 1 | M2-M2-M4, M2-M2-M6, M2-M7-M4, M2-M7-M6, M5-M2-M4, M5-M2-M6, M5-M7-M4, M5-M7-M6 | 8 |
| | 2 | M3-M1-M3, M3-M1-M6, M3-M7-M3, M3-M7-M6, M5-M1-M3, M5-M1-M6, M5-M7-M3, M5-M7-M6 | 8 |
| 9 | 1 | M3-M1-M3, M3-M1-M6, M3-M4-M3, M3-M4-M6, M7-M1-M3, M7-M1-M6, M7-M4-M3, M7-M4-M6 | 8 |
| | **2** | M1-M3-M1, ***M1-M3-M6***, M1-M7-M1, M1-M7-M6, M3-M3-M1, M3-M3-M6, M3-M7-M1, M3-M7-M6 | 8 |
| 10 | 1 | M2-M3-M1, M2-M3-M5, M2-M6-M1, M2-M6-M5, M5-M3-M1, M5-M3-M5, M5-M6-M1, M5-M6-M5 | 8 |
| | 2 | M2-M3-M1, M2-M3-M5, M2-M5-M1, M2-M5-M5, M6-M3-M1, M6-M3-M5, M6-M5-M1, M6-M5-M5 | 8 |
| 11 | 1 | M1-M3, M1-M7, M7-M3, M7-M7 | 4 |
| | 2 | M2-M3, M2-M6, M5-M3, M5-M6 | 4 |
| 12 | 1 | M1-M3-M2, M1-M3-M5, M1-M6-M2, M1-M6-M5, M4-M3-M2, M4-M3-M6, M4-M6-M2, M4-M6-M5 | 8 |
| | 2 | M3-M2, M3-M6, M7-M2, M7-M6 | 4 |
| | | Total | 168 |

***M1-M3-M6** means: operations *1*, *2* and *3* of part type *9* are performed under process plan *2* by machine route M1-M3-M6,

**Table 4.4: Alternative routes for part 1 when machine *j* is not available (Example 1)**

| Part types | Unavailable machine *j* | Alternatives machine routes available after failure | Total |
|---|---|---|---|
| 1 | MI | M4-M5-M4, M4-M5-M6, M5-M4, M5-M6 | 4 |
| | M2 | All possible routes are available, no effect | 12 |
| | M3 | All possible routes are available, no effect | 12 |
| | M4 | M1-M1-M6, M1-M5-M6, M1-M6, M5-M6 | 4 |
| | M5 | M1-M1-M4, M1-M1-M6, M4-M1-M4, M4-M1-M6, M1-M4, M1-M6 | 6 |
| | M6 | M1-M1-M4, M1-M5-M4, M4-M1-M4, M4-M5-M4, M1-M4, M5-M4 | 6 |
| | M7 | All possible routes are available, no effect. | 12 |

computed by solving the equation (2.21) using LINGO 07. For a 7-machine cell we have a *128 X 128* matrix. For example, the element TM (*3, 5*) of the matrix (i.e., the probability of transition from cell state $w_3$={1111101} to cell state $w_5$={1111011}) is computed as follows:

$$TM(3,5) = P_{w_3, w_5} = P_1^{1,1} * P_2^{1,1} * P_3^{1,1} * P_4^{1,1} * P_5^{1,0} * P_6^{0,1} * P_7^{1,1}$$

= 0.896*0.899*0.900*0.905*(1-0.893)*0.78*0.894 = 0.0489. Similarly, the other elements of the transition probability matrix TM are constructed. The complete representation of the TM occupies a large amount of space, and therefore, only a part of the matrix is included in Table 4.6

From the relationship of *steady state probability vector*, TM and *normality equation*, steady state probabilities have been estimated following the Markovian analysis (refer to section 2.4.1 for a detailed description). The steady state probabilities have been evaluated using LINGO 07 and are shown in Table 4.7.

#### 4.2.4.2 Solution and Analysis

The total number of variables, integer variables and constraints in the model are 52228, 49088 and 18948, respectively.

The model solution determines the cell formation and the allocation of operations for each *(ip)* combination. It then calculates the system availability indicators *SA (ip)* as per equation (4.14). Table 4.8 shows the optimum cell formation, operation allocation and reliability related information when only the first objective function (total costs) is optimized. In this scenario, cell 1 consists of machines M1, M2, M3 and M4 while cell 2 consists of machines M5, M6 and M7. As an example, part type 1 is processed in cell 2 where operations 1 and 2 are performed on machines M5 and M6, respectively. The total costs (first objective function) equal $1,771.46, and the overall system failure rate (second objective function) evaluated at this solution point is 0.2621467. It is also noted that, for instance, SA (1, 2) = 0.682, implying that the machine route for part type 1 under process plan 2 has an expected availability of 68.2% over the planning period.

In a similar fashion, Table 4.9 shows the results when only the second objective function (overall system failure rate) is optimized. In this case, the overall system failure rate (second objective function) is 0.2019891, and the total costs (first objective function) evaluated at this solution point is $3,068.32. Between these two extremes there is the collection of efficient solutions to the problem (i.e., the efficient frontier) which are obtained by using the Є-constraint method as follows:

**Table 4.5: Individual machine state probabilities using exponential distribution, (Example 1)**

| Machines | Machine state matrix | | | Availability and reparability | |
|---|---|---|---|---|---|
| | states | states | | Parameters | Values |
| | | 1 | 0 | | |
| M1 | 1 | 0.896 | 0.104* | $A_1(T)$ | 0.896 |
| | 0 | 0.80 | 0.20 | $H_1(t)$ | 0.80 |
| M2 | 1 | 0.899 | 0.101 | $A_2(T)$ | 0.899 |
| | 0 | 0.79 | 0.21 | $H_2(t)$ | 0.79 |
| M3 | 1 | 0.90 | 0.10 | $A_3(T)$ | 0.90 |
| | 0 | 0.80 | 0.20 | $H_3(t)$ | 0.80 |
| M4 | 1 | 0.905 | 0.096 | $A_4(T)$ | 0.904 |
| | 0 | 0.80 | 0.20 | $H_4(t)$ | 0.80 |
| M5 | 1 | 0.893 | 0.107 | $A_5(T)$ | 0.893 |
| | 0 | 0.79 | 0.21 | $H_5(t)$ | 0.79 |
| M6 | 1 | 0.869 | 0.131 | $A_j(T)$ | 0.869 |
| | 0 | 0.78 | 0.22 | $H_6(t)$ | 0.78 |
| M7 | 1 | 0.894 | 0.107 | $A_7(T)$ | 0.893 |
| | 0 | 0.8 | 0.2 | $H_7(t)$ | 0.8 |

$A_j(T)$ = Availability of machine *j*, in planning period *T* which may be assume to be $p_j^{u,u}$, *u: up (1)*

Hj(t) = Reparability of machine j, by the time *t* which may be assumed to be $p_j^{d,u}$ *d :down (0)*

* $p_j^{1,0}$ = 0.104, when 1 = machine is *up* , 0 = machine is *down*

**Table 4.6: Transition probability matrix for the cell states (partial results, Example 1)**

| | | Machine states | Cell states | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | $w_8$ | | | $w_{124}$ | $w_{125}$ | $w_{126}$ | $w_{127}$ | $w_{128}$ |
| | | | 1111111 | 1111110 | 1111101 | 1111100 | 1111011 | 1111010 | 1111001 | 1111000 | | | 0000100 | 0000011 | 0000010 | 0000001 | 0000000 |
| | $w_1$ | 1111111 | 0.4555 | 0.0542 | 0.0685 | 0.0082 | 0.0545 | 0.0065 | 0.0082 | 0.0010 | | | 0.0000012 | 0.0000082 | 0.0000010 | 0.0000012 | 0.0000001 |
| | $w_2$ | 1111110 | 0.4078 | 0.1019 | 0.0613 | 0.0153 | 0.0488 | 0.0122 | 0.0073 | 0.0018 | | | 0.0000023 | 0.0000074 | 0.0000018 | 0.0000011 | 0.0000003 |
| | $w_3$ | 1111101 | 0.4087 | 0.0486 | 0.1153 | 0.0137 | 0.0489 | 0.0058 | 0.0138 | 0.0016 | | | 0.0000021 | 0.0000074 | 0.0000009 | 0.0000021 | 0.0000002 |
| | $w_4$ | 1111100 | 0.3659 | 0.0915 | 0.1032 | 0.0258 | 0.0437 | 0.0109 | 0.0123 | 0.0031 | | | 0.0000039 | 0.0000066 | 0.0000017 | 0.0000019 | 0.0000005 |
| | $w_5$ | 1111011 | 0.4029 | 0.0479 | 0.0606 | 0.0072 | 0.1071 | 0.0127 | 0.0161 | 0.0019 | | | 0.0000011 | 0.0000162 | 0.0000019 | 0.0000024 | 0.0000003 |
| | $w_6$ | 1111010 | 0.3607 | 0.0902 | 0.0542 | 0.0136 | 0.0959 | 0.0240 | 0.0144 | 0.0036 | | | 0.0000021 | 0.0000145 | 0.0000036 | 0.0000022 | 0.0000005 |
| | $w_7$ | 1111001 | 0.3615 | 0.0430 | 0.1020 | 0.0121 | 0.0961 | 0.0114 | 0.0271 | 0.0032 | | | 0.0000018 | 0.0000145 | 0.0000017 | 0.0000041 | 0.0000005 |
| C | $w_8$ | 1111000 | 0.3236 | 0.0809 | 0.0913 | 0.0228 | 0.0860 | 0.0215 | 0.0243 | 0.0061 | | | 0.0000035 | 0.0000130 | 0.0000033 | 0.0000037 | 0.0000009 |
| E | $w_9$ | 1110111 | 0.4029 | 0.0479 | 0.0606 | 0.0072 | 0.0482 | 0.0057 | 0.0072 | 0.0009 | | | 0.0000026 | 0.0000173 | 0.0000021 | 0.0000026 | 0.0000003 |
| L | $w_{10}$ | 1110110 | 0.3606 | 0.0902 | 0.0542 | 0.0136 | 0.0431 | 0.0108 | 0.0065 | 0.0016 | | | 0.0000049 | 0.0000154 | 0.0000039 | 0.0000023 | 0.0000006 |
| L | $w_{11}$ | 1110101 | 0.3615 | 0.0430 | 0.1020 | 0.0121 | 0.0432 | 0.0051 | 0.0122 | 0.0015 | | | 0.0000043 | 0.0000155 | 0.0000018 | 0.0000044 | 0.0000005 |
| | $w_{12}$ | 1110100 | 0.3236 | 0.0809 | 0.0913 | 0.0228 | 0.0387 | 0.0097 | 0.0109 | 0.0027 | | | 0.0000082 | 0.0000139 | 0.0000035 | 0.0000039 | 0.0000010 |
| S | $w_{13}$ | 1110011 | 0.3563 | 0.0424 | 0.0536 | 0.0064 | 0.0947 | 0.0113 | 0.0142 | 0.0017 | | | 0.0000023 | 0.0000339 | 0.0000040 | 0.0000051 | 0.0000006 |
| T | $w_{14}$ | 1110010 | 0.3190 | 0.0797 | 0.0480 | 0.0120 | 0.0848 | 0.0212 | 0.0128 | 0.0032 | | | 0.0000043 | 0.0000304 | 0.0000076 | 0.0000046 | 0.0000011 |
| A | $w_{15}$ | 1110001 | 0.3197 | 0.0380 | 0.0902 | 0.0107 | 0.0850 | 0.0101 | 0.0240 | 0.0029 | | | 0.0000038 | 0.0000305 | 0.0000036 | 0.0000086 | 0.0000010 |
| T | $w_{16}$ | 1110000 | 0.2862 | 0.0716 | 0.0807 | 0.0202 | 0.0761 | 0.0190 | 0.0215 | 0.0054 | | | 0.0000072 | 0.0000273 | 0.0000068 | 0.0000077 | 0.0000019 |
| E | $w_{17}$ | 1101111 | 0.4048 | 0.0482 | 0.0609 | 0.0072 | 0.0484 | 0.0058 | 0.0073 | 0.0009 | | | 0.0000025 | 0.0000165 | 0.0000020 | 0.0000025 | 0.0000003 |
| S | $w_{18}$ | 1101110 | 0.3624 | 0.0906 | 0.0545 | 0.0136 | 0.0433 | 0.0108 | 0.0065 | 0.0016 | | | 0.0000046 | 0.0000148 | 0.0000037 | 0.0000022 | 0.0000006 |
| | $w_{19}$ | 1101101 | 0.3632 | 0.0432 | 0.1025 | 0.0122 | 0.0434 | 0.0052 | 0.0122 | 0.0015 | | | 0.0000042 | 0.0000148 | 0.0000018 | 0.0000042 | 0.0000005 |
| | $w_{20}$ | 1101100 | 0.3252 | 0.0813 | 0.0917 | 0.0229 | 0.0389 | 0.0097 | 0.0110 | 0.0027 | | | 0.0000078 | 0.0000133 | 0.0000033 | 0.0000037 | 0.0000009 |
| | ….. | | | | | | | | | | | | | | | | |
| | …… | | | | | | | | | | | | | | | | |
| | $w_{121}$ | 0000111 | 0.2807 | 0.0334 | 0.0422 | 0.0050 | 0.0336 | 0.0040 | 0.0050 | 0.0006 | | | 0.0000209 | 0.0001394 | 0.0000166 | 0.0000210 | 0.0000025 |
| | $w_{122}$ | 0000110 | 0.2512 | 0.0628 | 0.0378 | 0.0094 | 0.0300 | 0.0075 | 0.0045 | 0.0011 | | | 0.0000392 | 0.0001248 | 0.0000312 | 0.0000188 | 0.0000047 |
| | $w_{123}$ | 0000101 | 0.2518 | 0.0300 | 0.0710 | 0.0085 | 0.0301 | 0.0036 | 0.0085 | 0.0010 | | | 0.0000351 | 0.0001251 | 0.0000149 | 0.0000353 | 0.0000042 |
| | $w_{124}$ | 0000100 | 0.2254 | 0.0564 | 0.0636 | 0.0159 | 0.0270 | 0.0067 | 0.0076 | 0.0019 | | | 0.0000660 | 0.0001119 | 0.0000280 | 0.0000316 | 0.0000079 |
| | $w_{125}$ | 0000011 | 0.2482 | 0.0295 | 0.0373 | 0.0044 | 0.0660 | 0.0079 | 0.0099 | 0.0012 | | | 0.0000184 | 0.0002741 | 0.0000326 | 0.0000412 | 0.0000049 |
| | $w_{126}$ | 0000010 | 0.2222 | 0.0556 | 0.0334 | 0.0084 | 0.0591 | 0.0148 | 0.0089 | 0.0022 | | | 0.0000347 | 0.0002453 | 0.0000613 | 0.0000369 | 0.0000092 |
| | $w_{127}$ | 0000001 | 0.2227 | 0.0265 | 0.0628 | 0.0075 | 0.0592 | 0.0070 | 0.0167 | 0.0020 | | | 0.0000310 | 0.0002459 | 0.0000293 | 0.0000694 | 0.0000083 |
| | $w_{128}$ | 0000000 | 0.1994 | 0.0498 | 0.0562 | 0.0141 | 0.0530 | 0.0133 | 0.0149 | 0.0037 | | | 0.0000584 | 0.0002201 | 0.0000550 | 0.0000621 | 0.0000155 |

Table 4.7: Steady state probability of cell states (Example 1)

| States, ($w_k$) | $\pi_{w_k}$ | States ($w_k$) | $\pi_{w_k}$ | States ($w_k$) | $\pi_{w_k}$ |
|---|---|---|---|---|---|
| 1111111 | 0.4319963 | 1010011 | 0.0002896 | 0101000 | 0.0000283 |
| 1111110 | 0.0563236 | 1010010 | 0.0000378 | 0100111 | 0.0003383 |
| 1111101 | 0.1085529 | 1010001 | 0.0000728 | 0100110 | 0.0000441 |
| 1111100 | 0.0141531 | 1010000 | 0.0000095 | 0100101 | 0.0000850 |
| 1111011 | 0.0743690 | 1001111 | 0.0036138 | 0100100 | 0.0000111 |
| 1111010 | 0.0096962 | 1001110 | 0.0004712 | 0100011 | 0.0000582 |
| 1111001 | 0.0186876 | 1001101 | 0.0009081 | 0100010 | 0.0000076 |
| 1111000 | 0.0024365 | 1001100 | 0.0001184 | 0100001 | 0.0000146 |
| 1110111 | 0.0291598 | 1001011 | 0.0006221 | 0100000 | 0.0000019 |
| 1110110 | 0.0038018 | 1001010 | 0.0000811 | 0011111 | 0.0019938 |
| 1110101 | 0.0073273 | 1001001 | 0.0001563 | 0011110 | 0.0002600 |
| 1110100 | 0.0009553 | 1001000 | 0.0000204 | 0011101 | 0.0005010 |
| 1110011 | 0.0050199 | 1000111 | 0.0002439 | 0011100 | 0.0000653 |
| 1110010 | 0.0006545 | 1000110 | 0.0000318 | 0011010 | 0.0003432 |
| 1110001 | 0.0012614 | 1000101 | 0.0000613 | 0011011 | 0.0000448 |
| 1110000 | 0.0001645 | 1000100 | 0.0000080 | 0011000 | 0.0000863 |
| 1101111 | 0.0626395 | 1000011 | 0.0000420 | 0011001 | 0.0000112 |
| 1101110 | 0.0081669 | 1000010 | 0.0000055 | 0010110 | 0.0001346 |
| 1101101 | 0.0157402 | 1000001 | 0.0000106 | 0010110 | 0.0000175 |
| 1101100 | 0.0020522 | 1000000 | 0.0000014 | 0010101 | 0.0000338 |
| 1101011 | 0.0107835 | 0111111 | 0.0345597 | 0010100 | 0.0000044 |
| 1101010 | 0.0014060 | 0111110 | 0.0045059 | 0010011 | 0.0000232 |
| 1101001 | 0.0027097 | 0111101 | 0.0086842 | 0010010 | 0.0000030 |
| 1101000 | 0.0003533 | 0111100 | 0.0011322 | 0010001 | 0.0000058 |
| 1100111 | 0.0042282 | 0111011 | 0.0059495 | 0010000 | 0.0000008 |
| 1100110 | 0.0005513 | 0111010 | 0.0007757 | 0001111 | 0.0002891 |
| 1100101 | 0.0010625 | 0111001 | 0.0014950 | 0001110 | 0.0000377 |
| 1100100 | 0.0001385 | 0111000 | 0.0001949 | 0001101 | 0.0000726 |
| 1100011 | 0.0007279 | 0110111 | 0.0023328 | 0001100 | 0.0000095 |
| 1100010 | 0.0000949 | 0110110 | 0.0003041 | 0001011 | 0.0000498 |
| 1100001 | 0.0001829 | 0110101 | 0.0005862 | 0001010 | 0.0000065 |
| 1100000 | 0.0000238 | 0110100 | 0.0000764 | 0001001 | 0.0000125 |
| 1011111 | 0.0249229 | 0110011 | 0.0004016 | 0001000 | 0.0000016 |
| 1011110 | 0.0032494 | 0110010 | 0.0000524 | 0000111 | 0.0000195 |
| 1011101 | 0.0062627 | 0110001 | 0.0001009 | 0000110 | 0.0000025 |
| 1011100 | 0.0008165 | 0110000 | 0.0000132 | 0000101 | 0.0000049 |
| 1011011 | 0.0042905 | 0101111 | 0.0050112 | 0000100 | 0.0000006 |
| 1011010 | 0.0005594 | 0101110 | 0.0006534 | 0000011 | 0.0000034 |
| 1011001 | 0.0010781 | 0101101 | 0.0012592 | 0000010 | 0.0000004 |
| 1011000 | 0.0001406 | 0101100 | 0.0001642 | 0000001 | 0.0000008 |
| 1010111 | 0.0016823 | 0101011 | 0.0008627 | 0000000 | 0.0000001 |
| 1010110 | 0.0002193 | 0101010 | 0.0001125 | | |
| 1010101 | 0.0004227 | 0101001 | 0.0002168 | | |
| 1010100 | 0.0000551 | | | | |

*Minimize:* Objective function I

*Subject to the original constraints, and*

Objective function II ≤ Є, and

0.2019891 ≤ Є ≤ 0.2621467

The summary of the results related to the efficient frontier is shown in Table 4.10, and the efficient frontier diagram is displayed in Figure 4.1. Following this Є-constraint approach, the efficient point # 2, Table 4.10 is the one that optimizes cost subject to achievement of the: Objective function II ≤0.2019891. Details of this solution are presented in Table 4.11. The result (point # 2, Table 4.10) illustrates the benefits of simultaneously considering cost and reliability. The solution achieves the desired reliability level while lowering the overall cost to $2,781.38 from $3,068.32, which has been the cost when considering only reliability. Quite convincingly, the cell configuration has also changed to a better machine combination to achieve the optimum cost (lower intercell material handling cost) and ensure the expected system reliability. Comparisons of cell configurations, system costs and system failure rates for the three solutions: 1) optimizing objective function I only; 2) optimizing objective function II only and 3) efficient point # 2, Table 4.10 are presented in Table 4.12 to highlight the advantages of simultaneously considering system reliability and system costs.

**Table 4.8: Model results when optimizing objective function I only (exponential model, Example 1)**

| Solution type | Part –process plan | Machine routes in cell 1 | Machine routes in cell 2 |
|---|---|---|---|
| *Min* = Objective Function I only | 1-2 | | M5-M6 |
| | *2-2 | M3-M2-M3 | |
| | 3-1 | | M7-M6-M6 |
| | 4-2 | M1-M2 | |
| | **5-1 | M1(2)-M4(3) | M5(1) |
| | 6-2 | | M7-M5-M7 |
| | 7-2 | M2-M4 | |
| | 8-2 | | M5-M7-M6 |
| | 9-1 | M3-M4-M3 | |
| | 10-1 | M2-M3-M1 | |
| | 11-1 | M1-M3 | |
| | 12-1 | M4-M3-M2 | |
| Objective function I (total costs) = $1,771.46 (VCM= $1410.34 MHC= $50.00 MNC = $311.12) | | | |
| Objective function II (system failure rate) = 0.2621467 | | | |
| **Performance** | | | |
| **Machine utilization** | ***MU(1)= 0.80, MU(2) = 0.74, MU (30) = 0.95, MU (4) = 0.93, MU(5) = 0.97, MU(6) = 0.99, MU(7) = 0.53 | | |
| **System availability** | SA (1,2) = 0.682, SA( 2,2) =0.826, SA(3,1) = 0.707, SA(4, 2) = 0.875, SA(5,1) = 0.74, SA( 6,2) = 0.755, SA(7,2) = 0.876, SA(8,2) = 0.603, SA (9,1) = 0.808, SA(10, 1) = 0.765, SA( 11,1) = 0.808, ****SA(12,1) = 0.764. | | |

*Part 2 uses process plan 2, operations sequence: M3-M2-M3 processed in cell 1.
** Part 5, uses process plan 1, operations sequence: M5-M1-M4, 1st operation is performed on M5 in cell 2, 2nd and 3rd operations performed on M1 and M4 respectively in cell 1.
***MU(1) : machine utilization for machine 1
**** SA (12,1) = System availability for part type 12, process plan 1

**Table 4.9: Model results when optimizing objective function II only (exponential model, Example 1)**

| Solution type | Part –process plan | Machine routes in cell 1 | Machine routes in cell 2 |
|---|---|---|---|
| *Min* = Objective Function II only | 1-2 | M1-M4 | |
| | *2-2 | M3-M7-M3 | |
| | 3-2 | M4(1)-M4(3) | M6(2) |
| | 4-2 | M1(1) | M6(2) |
| | **5-2 | M7(1) M 4(3) | M6 (2) |
| | 6-2 | M7- M1- M3 | |
| | 7-2 | M1(2) | M6(1) |
| | 8-2 | M3- M 1- M 3 | |
| | 9-1 | M 7(1)- M 1(2) | M6(3) |
| | 10-2 | M3(2)- M1(3) | M6(1) |
| | 11-1 | M1-M3 | |
| | 12-2 | M3(1) | M6(2) |
| Objective function II (system failure rate) = 0.2019891 | | | |
| Objective function I (total costs) = $3,068.32, (VCM = $1,790.38, MHC = $450.00, MNC = $827.94) | | | |
| **Performance** | | | |
| **Machine utilization** | ***MU(1)= 0.91, MU (3) = 0.93, MU (4) = 0.86, MU(6) = 0.99, MU(7) = 0.82 | | |
| **System availability** | SA(1,2) = 0.858, SA(2,2) = 0.773, SA(3,2) = 0.739, SA( 4,2) = 0.74, SA(5,2) =0.662, SA(6,2) = 0.715, SA( 7,2) = 0.74, SA(8,2) = 0.809, SA(9,1) = 0.646, SA(10,2) = 0.646, SA(11,1) = 0.809, ****SA(12, 2) =0.697 | | |

*Part 2 uses process plan 2, operations sequence: M3-M7-M3, processed in cell 1.
** Part 5 uses process plan 2, operations sequence: M7-M6-M4, M6 in cell 2 performs 2nd, M7 and M4 in cell 1 performs 1st and 3rd operations.
***MU(1) : machine utilization for machine 1
**** SA (12,2) = System availability for part type 12, process plan 2

**Table 4.10: Information for the efficient frontier diagram (exponential model, Example 1)**

| Points | Objective function I ($) | Objective function I components | | | Objective function II | epsilon € | Cells | |
|---|---|---|---|---|---|---|---|---|
| | | VCM ($) | MHC ($) | MNC ($) | | | 1 | 2 |
| 1 | 3068.32 | 1790.38 | 450.00 | 827.94 | 0.2019891 | N/A | M1,M3,M4,M7 | M6 |
| 2 | 2781.38 | 1768.28 | 200.00 | 813.10 | 0.2019891 | 0.2019891 | M1,M3,M6,M7 | M4 |
| 3 | 2594.14 | 1759.00 | 50.00 | 785.14 | 0.2044610 | 0.2044610 | M1,M3,M4,M6 | M2,M7 |
| 4 | 2389.10 | 1737.05 | 50.00 | 602.05 | 0.2096879 | 0.2096879 | M1,M3,M4,M6 | M2,M5,M7 |
| 5 | 2196.82 | 1573.91 | 100.00 | 522.91 | 0.2193262 | 0.2193262 | M1,M3,M4,M6 | M2,M5,M7 |
| 6 | 2034.84 | 1574.78 | 50.00 | 410.06 | 0.2289411 | 0.2289411 | M1,M3,M4,M6 | M2,M5,M7 |
| 7 | 1913.28 | 1490.68 | 100.00 | 322.60 | 0.2371155 | 0.2371155 | M1,M3,M4,M6 | M2,M5,M7 |
| 8 | 1851.77 | 1425.60 | 100.00 | 326.17 | 0.2486248 | 0.2486248 | M1,M2,M3,M4 | M5,M6,M7 |
| 9 | 1775.15 | 1410.40 | 50.00 | 314.75 | 0.254513 | 0.254513 | M1,M2,M3,M4 | M5,M6,M7 |
| 10 | 1771.46 | 1410.34 | 50.00 | 311.12 | 0.2621467 | 0.26215 | M1,M2,M3,M4 | M5,M6,M7 |
| 11 | 1771.46 | 1410.34 | 50.00 | 311.12 | 0.2621467 | N/A | M1,M2,M3,M4 | M5,M6,M7 |

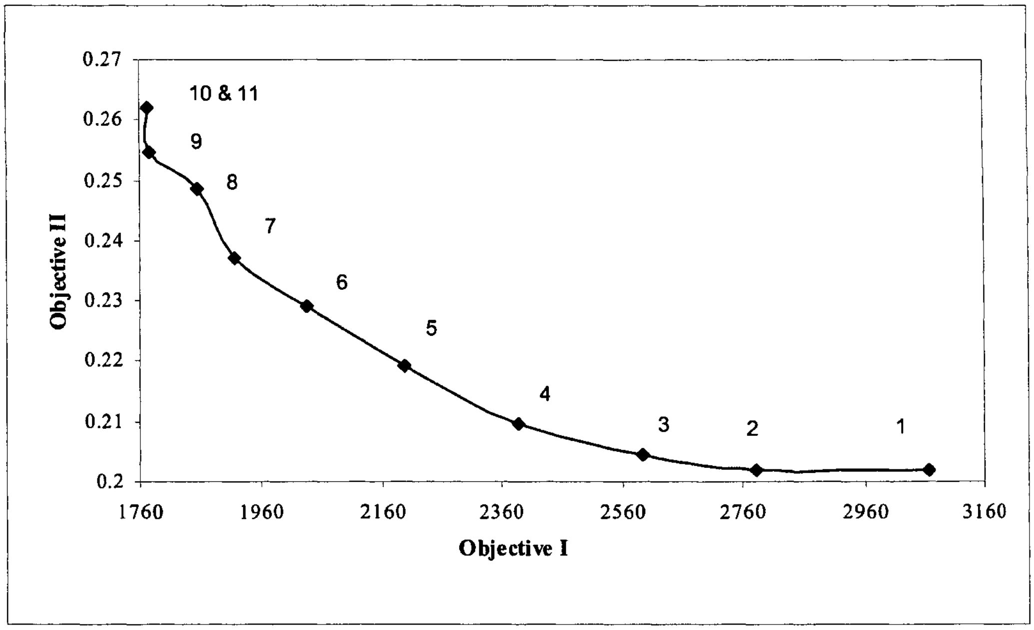


**Figure 4.1: Efficient frontier depicting cost and reliability optimization (exponential model, Example 1)**

**Table 4.11: Model results corresponding to point #2, in Table 4.10**

| Solution type | Part–process plan | Machine routes in cell 1 | Machine routes in cell 2 |
|---|---|---|---|
| *Min* = Objective function I<br>*s.t* objective function II ≤ ***0.2019891***<br>***and other constraints*** | 1-2 | M1(1) | M4(2) |
| | *2-2 | M3-M7-M3 | |
| | 3-2 | M6(2) | M4(1) M4(3) |
| | 4-2 | M1-M6 | |
| | **5-2 | M7(1)- M6 (2) | M4(3) |
| | 6-2 | M7-M1-M3 | |
| | 7-2 | M6-M1 | |
| | 8-2 | M3-M1-M3 | |
| | 9-1 | M3-M1-M6 | |
| | 10-2 | M6-M3-M1 | |
| | 11-1 | M1-M7 | |
| | 12-2 | M3-M6 | |
| Objective function II(system failure rate) = 0.2019891 | | | |
| Objective function I (total costs) = $2,781.38, (VCM = $1768.28, MHC = $200.00, MNC = $813.10) | | | |
| **Performance** | | | |
| **Machine utilization** | ***MU(1)= 0.91, MU (3) = 0.96, MU (4) = 0.86, MU(6) = 0.99, MU(7) = 0.82 | | |
| **System availability** | SA(1,2) = 0.858, SA(2,2) = 0.773, SA(3,2) = 0.739, SA(4,2) = 0.74 SA(5,2) =0.662, SA(6,2) = 0.715, SA( 7,2) = 0.74 SA(8,2) = 0.809, SA(9,1) = 0.646,SA(10,2) =0.646, SA(11,1) = 0.819, ****SA(12,2) = 0.697 | | |

*Part 2 uses process plan 2, operations sequence: M3-M7-M3, cell 1,

** Part 5 uses process plan 2, operations sequence: M7-M6-M4, M4 in cell 2 performs 3rd and M7 and M6 in cell 1 perform 1st and 2nd operations.

***MU(1) : machine utilization for machine 1

**** SA (12,2) = System availability for part type 12, process plan 2

**Table 4.12: Comparison of model results for cost-only, reliability-only optimization and considering cost and reliability simultaneously (Example 1)**

| Comparison focus | Optimizing: Only cost *(I)* | Only reliability *(II)* | Cost and reliability together *(III)* | Tangible benefits from *(III)* |
|---|---|---|---|---|
| Total costs | \$1,771.46 | \$3,068.00 | \$2,781.39 | (I) Very economic but probability of machine failure is very high<br>(II) Very expensive<br>(III) Effectively competitive |
| System failure rates | 0.2621467 | 0.2019891 | 0.2019891 | (III)Highest reliability with better cost than (II) |
| Cell configuration | Cell 1: M1, M2, M3, M4 | Cell 1: M1,M3, M4, M7 | Cell 1: M1,M3, M6, M7 | (III) Ensured high utilization of reliable machines |
| | Cell 2: M5, M6, M7 | Cell 2: M6 | Cell2: M4 | |

The efficient frontier analysis approach is developed by solving various problem instances for different combinations of system reliability and cost, so that the user/designer can study the pattern of solutions in terms of cell configuration, costs and desired machine system reliability in terms of system failure rates in order to make a suitable decision. In addition to this, the model also evaluates system availability as a performance indicator. This provides the user with the option of emphasizing the system availability of a part type-process plan combination depending on the delivery priority or customer importance.

The solution presented here shows the influence of machine reliability on the cell configuration and cost. Depending on the business perspective and priority, the model will help the user make an effective design decision.

### 4.3 Mathematical Model Based on Weibull Distribution for Machine Reliability

In this section, the multi-objective CMS design model considers machine reliability by following the Weibull distribution. In this model, the second objective function is different from the exponential distribution approach presented in Section 4.1. Other than the second objective function, the first objective function and the constraints—represented by model equation (4.1) to (4.3) and (4.5) to (4.10), including their relevant explanations—are similar to Section 4.1. The constraint sets—represented by equations (4.11) to (4.14) of Section 4.2—are performance-evaluation related and, as such, are not applicable to this section. The model summary based on the Weibull distribution is presented in this section and explanations of the second objective function based on Weibull distribution are described within the model summary.

### 4.3.1 The Mathematical Model

The Weibull distribution-based model is presented in an assembled condition by providing a relevant explanation for the second object function only. The first objective function and the constraints from the previous section are incorporated as discussed above.

*Minimize* objective function I:

$$\text{Objective function I} = VCM+MHC+MNC \quad \text{----------- (4.31)}$$

where,

$$VCM = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \sum_{j\in J_{ipo}} \{CO_{oj}(ip) + CR_{oj}(ip)\} \sum_{c=1}^{C} X_{ojc}(ip) \quad \text{----------- (4.31a)}$$

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j\in J_{ipo}} \sum_{\hat{j}\in J_{ipo}} \sum_{1\le c,\hat{c}\le C} H_{ijc\hat{j}\hat{c}} Y_{ojc\hat{j}\hat{c}}(ip) \quad \text{--------- (4.31b)}$$

$$MNC = \sum_{j=1}^{m} cp_j \left( 1 - \left[\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \frac{TO_{oj}(ip) + TR_{oj}(ip)}{A_j(T)b_j}\right] \right) \sum_{c=1}^{C} X_{ojc}(ip) \quad \text{--------- (4.31c)}$$

The second objective function computes the system reliability over the set of all the part type-process plan combinations:

*Minimize* objective function II:

$$\text{Objective function II} = \sum_{i=1}^{n} \sum_{p=1}^{P(i)} LIR_{ip} \quad \text{----------- (4.32)}$$

where

$$LIR_{ip} = \sum_{o=1}^{O(ip)} \sum_{j\in J_{ipo}} \sum_{c=1}^{C} \left[ \frac{TT(1+\frac{1}{\beta_j})}{MTBF_j} \right]^{\beta_j} X_{ojc}(ip) \qquad \forall i,p \quad \text{---------- (4.32a)}$$

Assuming the failure rate of machine $j$ follows the Weibull distribution, $LIR_{ip}$ calculates the inverse of system reliability in logarithmic scale corresponding to the machines which perform the set of operations for the $(ip)$. The second objective function seeks to select the set of process plans for all the part types that results in the minimum value for the inverse of system reliability, and thus maximizes system reliability. Detailed derivation of $LIR_{ip}$ for the Weibull distribution approach is described in Section 2.5.2

**Constraints**

The following constraints are defined:

$$\sum_{p=1}^{P(i)} Z(ip) = 1 \quad \forall i \qquad \text{------------ (4.33)}$$

$$\sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) = Z(ip) \quad \forall i,p,o \qquad \text{------------- (4.34)}$$

$$\sum_{c=1}^{C} M_{jc} \le 1 \quad \forall j \qquad \text{------------- (4.35)}$$

$$\sum_{j=1}^{m} M_{jc} \le UM \quad \forall c \qquad \text{------------- (4.36)}$$

$$\sum_{i=1}^{n} \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} X_{ojc}(ip) \ge M_{jc} \quad \forall j,c \qquad \text{------------ (4.37)}$$

$$\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} [\ TO_{oj}(ip) + TR_{oj}(ip)\ ] X_{ojc}(ip) \le b_j M_{jc} A_j(T) \quad \forall j,c \qquad \text{---------- (4.38)}$$

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - 2Y_{ojc\hat{j}\hat{c}}(ip) \ge 0, \ldots\ldots \forall i,p,o \in \{1,2,\ldots\ldots O(ip)-1\}, j \in \{J_{ipo}, \hat{j} \in J_{ip(o+1)}, c, \hat{c}$$

$$\text{---------- (4.39)}$$

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - Y_{ojc\hat{j}\hat{c}}(ip) \le 1 \quad \forall i,p,o \in \{1,2,\ldots O(ip)-1\}, j \in J_{ipo}, \hat{j} \in J_{ip(o+1)}, c, \hat{c}$$

$$\text{------------ (4.40)}$$

$$X_{ojc}(ip), Y_{ojc\hat{j}\hat{c}}(ip), Z(ip), M_{jc} \in \{0,1\} \ldots \forall i,p,o,j,c,\hat{j},\hat{c} \qquad \text{--------------- (4.41)}$$

### 4.3.2 Numerical Example

**Input data:** Example 1—with the same part and machine information as given in Tables 4.1 and 4.2—is illustrated in this section to show the applicability of this model. We also present Example 2 in section 4.4 to further investigate the applicability of the model. We assume that the machines follow increasing failure rates with different β (shape factor) and θ (characteristic life) values. For our Example 1, the model is studied for the following three ranges of β values, with the β values generated randomly following the uniform distribution with the parameters given: We first illustrate the model solution by the lowest range (β = 1.1 -1.4), and then extend the analysis for the other two ranges.

| Parameters/Ranges | Individual machine β Values | Range description |
|---|---|---|
| (1.1 -1.4) | 1.11, 1.25, 1.15, 1.34, 1.18, 1.2, 1.33 | low value |
| (1.35 -1.75) | 1.38, 1.5, 1.72, 1.63, 1.38, 1.7, 1.39 | high value |
| (1.1-1.8) | 1.21, 1.26, 1.78, 1.16, 1.56, 1.59, 1.33 | long range |

It may be mentioned here that, since objective function II, equation (4.32.a), is developed by replacing $\theta_j$ in terms of $MTBF_j$ and $\beta_j$ by following equation (2.32), we do not need to address $\theta_j$ values separately for the model solution.

The Є-constraint method is also utilized for the solution and analysis of this model. Following the same steps outlined in Section 4.2.6, the multi-objective model is first solved by the hierarchical approach considering objective functions I and II separately. The next steps are determining the efficient point solutions, and constructing an efficient frontier diagram.

### 4.3.3 Solution and Analysis

The model is solved using LINGO 07. The total number of variables (continuous plus integer), integer variables and constraints are 32260, 32193 and 2051, respectively.

Table 4.13 shows the optimum cell formation, operation allocation and reliability-related information when only the first objective function (total costs) is optimized. Under this cost optimization scenario, the cell configuration and operation allocation is identical to the exponential distribution-based model solution presented in Table 4.8. For example, cell 1 consists of machines M1, M2, M3 and M4 while cell 2 consists of machines M5, M6 and M7. Part type 1 is processed in cell 2 where operations 1 and 2 are performed on machines M5 and M6. There is, however, a $3.01 difference in the total cost as a result of the difference in the machine availability calculation basis. The total costs (objective function I) equal $1768.45and objective function II, which is the inverse value of system reliability given in natural logarithmic scale, evaluated at this solution point is 667.27.

Table 4.14 details the cell formation, operation allocation, reliability information and system cost when only reliability (objective function II) is optimized. As is evident in Tables 4.13 and 4.14, there are significant differences in the part routes and cell configuration of the two solutions. The model selected part processing routes (Table 4.14) for this solution point to get the optimum value of the inverse of system reliability, the objective function II value from this solution is 474.5, and the total costs (objective function I) value evaluated from this solution is $3,026.55. In the first step, cost is optimized. Consequently, the objective function II value (667.27) is very high, while in the next step—when reliability is optimized to obtain the lowest objective function

II value (474.5)—the system costs become very high in the process of achieving the optimum reliability, which is exhibited in the above cases.

However, the two extreme values of the second objective function obtained from the above solution instances are the *bounds* on objective function II for the following Є-constraint method. Between these bounds there is a collection of efficient solutions. The summary of the efficient solutions are given in Table 4.15, and the efficient frontier diagram is presented in Figure 4.2.

*Minimize:* Objective function I

*Subject to the original constraints, and*

Objective function II $\leq \epsilon$, and

$474.5 \leq \epsilon \leq 667.27$

Following the Є-constraint approach, the efficient point #2, Table 4.15, Figure 4.2, for example, is solved by optimizing system costs subject to the achievement of the : Objective function II $\leq$474.5. The detail of this solution is presented in Table 4.16. This solution illustrates the benefits of simultaneously considering cost and reliability by reducing cost from $3,026.55 to $2,776.55, while achieving desired reliability level (objective function II value 474.5). The efficient frontier analysis for different combinations of system reliabilities and costs offers the user/designer the opportunity to study different solutions in terms of cell configuration, costs and expected machine reliabilities for a system to make suitable trade off decisions. The solution results presented here show the influence of machine reliability on the cell configuration and processing routes of part types.

**Table 4.13: Model results when optimizing objective function I only (Weibull model, Example 1, $\beta$ = 1.1 to 1.35)**

| Solution type | Part –process plan | Machine routes in cell 1 | Machine routes in cell 2 |
|---|---|---|---|
| *Min* = Objective function I only | 1-2 | | M5-M6 |
| | *2-2 | M3-M2-M3 | |
| | 3-1 | | M7-M6-M6 |
| | 4-2 | M1-M2 | |
| | **5-1 | M1(2)-M4(3) | M5(1) |
| | 6-2 | | M7-M5-M7 |
| | 7-2 | M2-M4 | |
| | 8-2 | | M5-M7-M6 |
| | 9-1 | M3-M4-M3 | |
| | 10-1 | M2-M3-M1 | |
| | 11-1 | M1-M3 | |
| | 12-1 | M4-M3-M2 | |
| Objective function I( total costs) = $1,768.45, (VCM= $1,410.33 MHC= $50.00 MNC = $308.12) | | | |
| Objective function II = 667.27 | | | |
| **Performance** | | | |
| **Machine utilization** | ***MU(1) = 0.80, MU(2) = 0.74, MU (3) = 0.96, MU (4) = 0.93, MU(5) = 0.97, MU(6) = 0.99, MU(7) = 0.53 | | |

*Part 2 uses process plan 2, operations sequence: M3-M2-M3 , processed in cell 1

** Part 5 uses process plan 1, operations sequence: M5-M1-M4, M5 performs 1st operation in cell 2 and M1 and M4 perform 2nd and 3rd operations in cell 1.

*** MU (1) Machine utilization for machine 1

**Table 4.14: Model results when optimizing objective function II only (Weibull model, Example 1, β =1.1 to 1.35)**

| Solution type | Part –process plan | Machine routes in cell 1 | Machine routes in cell 2 |
|---|---|---|---|
| *Min*=Objective function II only | 1-2 | M1(1) | M4(2) |
| | 2-2 | M2(2)-M3(3) | M5(1) |
| | 3-2 | | M4-M6-M4 |
| | 4-2 | M1(1) | M6(2) |
| | **5-2 | M2(1) | M6 (2)-M4(3) |
| | 6-1 | M3-M1-M2 | |
| | 7-2 | M1(2) | M6(1) |
| | *8-2 | M3-M1-M3 | |
| | 9-1 | M3(1) -M1(2) | M6(3) |
| | 10-2 | M3(2) -M1(3) | M6(1) |
| | 11-1 | M1-M3 | |
| | 12-2 | M3(1) | M6(2) |
| Objective function II = 474.5 | | | |
| Objective function I ( total costs) = $3,026.55, (VCM = $1,800.00, MHC = $400.00, MNC = $826.55) | | | |
| **Performance** | | | |
| **Machine utilization** | ***MU(1) = 0.93, MU (2) = 0.57, MU (3) = 0.91, MU(4) = 0.86, MU(5) = 0.19 MU(6) = 0.99, MU(7) = 0.00 | | |

*Part 8 uses process plan 2, operations sequence: M3-M1-M3, processed in cell 1,

** Part 5 uses process plan 2, operations sequence: M2-M6-M4, M2 performs 1st operation in cell 1, M6 and M4 performs 2nd and 3rd operations in cell 2

*** MU (1) Machine utilization for machine 1

**Table 4.15: Information for efficient frontier diagram (Weibull model, β =1.1 to 1.35, Example 1)**

| Points | Objective function I ($) | Objective function I components | | | Objective function II | epsilon ϵ | Cells | |
|---|---|---|---|---|---|---|---|---|
| | | VCM ($) | MHC ($) | MNC ($) | | | 1 | 2 |
| 1 | 3026.55 | 1800.00 | 400.00 | 826.55 | 474.5 | N/A | M1,M2,M3 | M4,M5,M6 |
| 2 | 2776.55 | 1800.00 | 150.00 | 826.55 | 474.5 | 474.5 | M1,M3,M4,M6 | M2,M5 |
| 3 | 2334.65 | 1677.00 | 100.00 | 557.65 | 504.7 | 504.7 | M1,M3,M4,M6 | M2,M5,M7 |
| 4 | 1991.32 | 1559.78 | 50.00 | 381.54 | 559.45 | 559.45 | M1,M3,M4,M6 | M2,M5,M7 |
| 5 | 1891.28 | 1467.68 | 100.00 | 323.60 | 593.6 | 593.6 | M1,M3,M4,M6 | M2,M5,M7 |
| 6 | 1848.78 | 1425.59 | 100.00 | 323.19 | 622.3 | 622.3 | M1,M2,M3,M4 | M5,M6,M7 |
| 7 | 1772.12 | 1410.38 | 50.00 | 311.74 | 643.8 | 643.8 | M1,M2,M3,M4 | M5,M6,M7 |
| 8 | 1768.45 | 1410.33 | 50.00 | 308.12 | 667.3 | 667.3 | M1,M2,M3,M4 | M5,M6,M7 |
| 9 | 1768.45 | 1410.33 | 50.00 | 308.12 | 667.3 | N/A | M1,M2,M3,M4 | M5,M6,M7 |

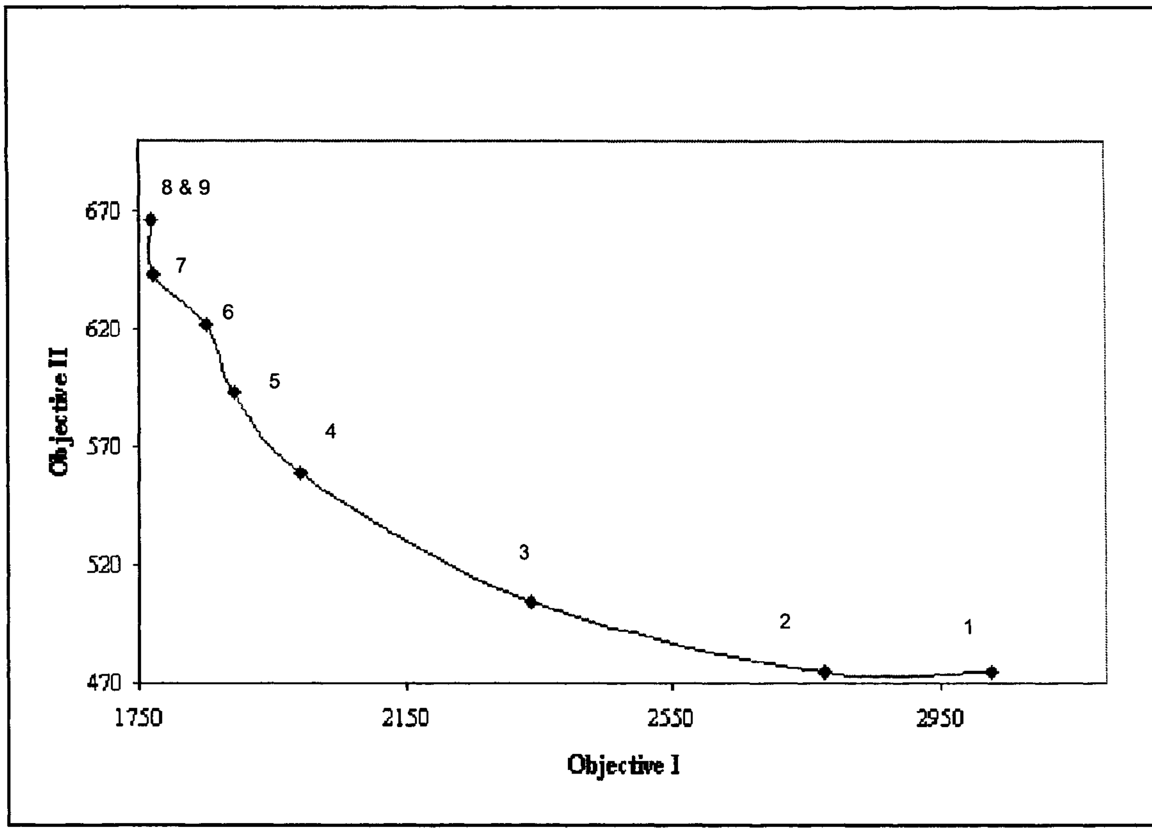


**Figure 4.2: Efficient frontier depicting cost and reliability optimization (Weibull model, β=1.10-1.35, Example 1)**

**Table 4.16: Model results corresponding to point # 2, in Table 4.15**

| Solution type | Part –process plan | Machine routes in cell 1 | Machine routes in cell 2 |
|---|---|---|---|
| *Min*=objective function I<br>*s.t Objective II ≤ 474.5*<br>*and other constraints* | *1-2 | M1-M4 | |
| | 2-2 | M3(3) | M5(1)-M2(2) |
| | 3-2 | M4-M6-M4 | |
| | 4-2 | M1-M6 | |
| | **5-2 | M6(2)-M4(3) | M2(1) |
| | 6-2 | M3(1)-M1(2) | M2(3) |
| | 7-2 | M6-M1 | |
| | 8-2 | M3-M1-M3 | |
| | 9-1 | M3-M1-M6 | |
| | 10-2 | M6-M3-M1 | |
| | 11-1 | M1-M3 | |
| | 12-2 | M3-M6 | |
| Objective function II = 474.5 | | | |
| Objective function I(total costs) = $2,776.55, (VCM = $1,800.00, MHC = $150.00, MNC = $826.55) | | | |
| **Performance** | | | |
| **Machine utilization** | ***MU(1) = 0.93, MU(2) =0.57, MU(3) = 0.91, MU(4) = 0.86, MU(5)= 0.19, MU(6) = 0.99 | | |

*Part 1 uses process plan 2, operations sequence: M1-M4, processed in cell 1

* Part 5 uses process plan 2, operations sequence: M2-M6-M4, M2 performs 1st operation in cell 2, M6, and M4 performs 2nd and 3rd operations in cell 1.

*** MU (1) Machine utilization for machine 1

### 4.3.4 Comparisons of the Weibull and the Exponential Distribution Based Model

As discussed above, the cell configuration and part processing routes for optimizing "cost only" or objective function I is the same for both approaches—based on the solution of the model example. The objective function II values at this solution point vary due to the fact that they are in different scales, with one evaluating system failure rates and the other dealing with the inverse value of system reliability, along the part processing routes.

For ease of comparison regarding the model solutions for optimizing objective function II only and efficient point #2 solutions, the objective function values, machine utilization and cell configurations for these solution instances are reproduced in Table 4.17.

For optimizing "reliability only" case, (instance 1, Table 4.17) cell configuration generated by the two models is completely different. The model has selected part processing routes with the aim of achieving the highest reliability, based on the two different failure distributions—as can be seen in Tables 4.9 and 4.14. Although which solution is best cannot be decided, the model based on the Weibull distribution generated better cell configuration in terms of inter-cell material handling costs and total costs of the model, as shown in Table 4.17.

**Table 4.17: Comparisons of the Weibull and the exponential distribution based model solutions**

| | **The exponential distribution based solutions** | **The Weibull distribution based solutions** |
|---|---|---|
| | | |
| INSTANCE 1 | *Min* = Objective function II | |
| Objective function I (total costs) | **$3,068.00** | **$3,026.55** |
| System cost components | VCM = $1,790.00, MHC= $450.00, MNC = $828.00 | VCM =$1,800.00, MHC= $400.00, MNC = $826.55 |
| Objective function II (system reliability) | **0.2019891** | **474.5** |
| Cell configuration | **Cell1**: M1, M3, M4, M7, **Cell2**:M6 | **Cell1**: M1,M2, M3 **Cell2**: M4, M5, M6 |
| Machine utilization | MU1= 0.91, MU2 = 0.00, MU3= 0.93, MU4 = 0.86, MU5= 0.00, MU6 = 0.99, MU7= 0.82 | MU1= 0.93, MU2 = 0.57, MU3= 0.91, MU4 = 0.86, MU5= 0.19, MU6 = 0.99, MU7= 0.00 |
| | | |
| INSTANCE 2 | **Efficient point # 2 in Table 4.11 and Figure 4.1** | **Efficient point # 2 in Table 4.15 and Figure 4.2** |
| | Minimize Objective function I, s.t. Objective function II≤ *0.2019891* | Minimize Objective function I, s.t. Objective function II≤ *474.5* |
| Objective function I (total costs) | **$2,781.38** | **$2,776.55** |
| System cost components | VCM = $1,768.28, MHC= $200.00, MNC = $81310 | VCM = $1,800.00, MHC= $50.00, MNC = $826.55 |
| Objective function II (system reliability) | **0.2019891** | **474.5** |
| Cell configuration | **Cell1**: M1, M3, M6, M7, **Cell2**:M4 | **Cell1**: M1 M3, M4, M6, **Cell2**: M2,M5 |
| Machine utilization | MU1= 0.91, MU2 = 0.00, MU3= 0.93, MU4 = 0.86, MU5= 0.00, MU6 = 0.99, MU7= 0.82 | MU1= 0.93, MU2 = 0.57, MU3= 0.91, MU4 = 0.86, MU5= 0.19, MU6 = 0.99, MU7= 0.00 |

For the model solution corresponding to efficient point # 2, for optimizing cost subject to achievement of the highest reliability (instance 2, Table 4.17), the Weibull distribution based model generates better cell configurations in consideration of machine utilizations and inter-cell material movement with almost same cost.

For the intermediate efficient points (Tables 4.11 and 4.15), a comparison in terms of the reliability criteria and cost is difficult to make because of the different scales of $\epsilon$. What is clear from the cell configuration, however, is that the solutions are almost similar for the data ranges under consideration.

The Weibull-based model evaluates the inverse of system reliability, which gives a better insight than the system failure rates of the exponential-based model when making decisions regarding machine reliability. The Weibull distribution-based models develop better cell configurations when the expected reliability level is higher (points 1, 2 and 3 of Table 4.15 compared to points 1, 2 and 3 of Table 4.11). To further explore the conclusion, we took both a higher range of β (1.35-1.75), as well as a long-range β (1.16-1.80). The summary of efficient points and the efficient frontier diagram for these two ranges of β are presented in Table 4.18, Figure 4.3 and Table 4.19, Figure 4.4, respectively. Comparing the efficient points of Table 4.18 and Table 4.19 with those of Table 4.11 reveals that the solutions for efficient points 2, 3 and 4 in the Weibull distribution-based model for a higher range β (1.35-1.75) and a long range β (1.16-1.80) are completely different from the exponential-based model solutions. We may also observe from these solutions that with higher range β values the Weibull distribution-based model develops better cell configuration when the expected reliability level is higher.

The cost of solutions for both the Weibull and exponential distribution-based models are, however, considerably similar. As a result, it cannot be clearly concluded which of the two cell configurations is better. Practically failure rates of machines used in manufacturing increase with the increase of machine usage, and also from the illustration of this solution analysis, Weibull approach may be considered to generate better solution when reliability is more emphasized than the cost.

The solutions of the example problem obtained by following the model based on the exponential distribution, as well as the Weibull distribution clearly indicate that machine reliability influences cell configuration, part processing routes, and cost in both approaches. In general, the $\epsilon$-constraint method of solution procedure facilitates the selection of appropriate trade-off options between machine reliability and cost.

**Table 4.18: Information for efficient frontier diagram (Weibull model, β = 1.35 to 1.75, Example 1)**

| Points | Objective function I ($) | Objective function I components | | | Objective function II | epsilon ε | Cells | | |
|---|---|---|---|---|---|---|---|---|---|
| | | VCM ($) | MHC ($) | MNC($) | | | 1 | 2 | 3 |
| 1 | 3292.3 | 1754.05 | 700.00 | 838.31 | 965.9 | N/A | M1,M6 | M3,M7 | M4 |
| 2 | 2670.19 | 1683.99 | 150.00 | 836.20 | 965.9 | 965.9 | M1,M3,M6,M7 | M4,M5 | |
| 3 | 2237.00 | 1704.00 | 100.00 | 433.00 | 1000.00 | 1000.00 | M1,M3,M6,M7 | M4,M5 | |
| 4 | 2060.09 | 1520.18 | 50.00 | 489.91 | 1049.6 | 1049.6 | M1,M5,M6,M7 | M2,M3,M4 | |
| 5 | 1946.65 | 1465.38 | 100.00 | 381.27 | 1097.8 | 1097.8 | M1,M3,M4,M6 | M2,M5,M7 | |
| 6 | 1891.28 | 1467.68 | 100.00 | 323.60 | 1148.1 | 1148.1 | M1,M3,M4,M6 | M2,M5,M7 | |
| 7 | 1814.43 | 1413.08 | 50.00 | 351.35 | 1197.7 | 1197.7 | M1,M2,M3,M4 | M5,M6,M7 | |
| 8 | 1772.11 | 1410.37 | 50.00 | 311.74 | 1226.6 | 1226.6 | M1,M2,M3,M4 | M5,M6,M7 | |
| 9 | 1768.45 | 1410.34 | 50.00 | 308.11 | 1271.02 | 1410.34 | M1,M2,M3,M4 | M5,M6,M7 | |
| 10 | 1768.45 | 1410.34 | 50.00 | 308.11 | 1271.02 | N/A | M1,M2,M3,M4 | M5,M6,M7 | |

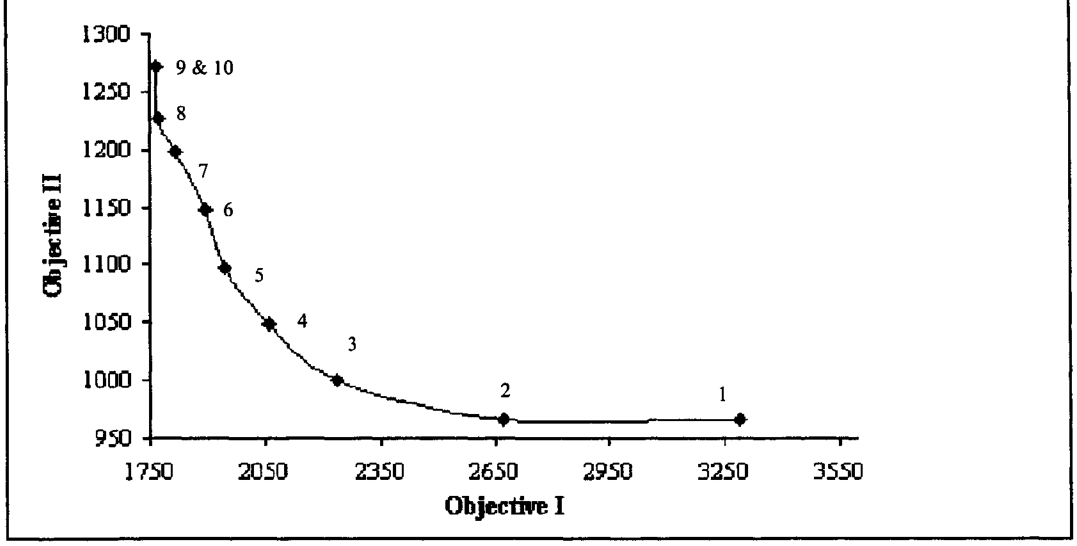


**Figure 4.3: Efficient frontier depicting cost and reliability optimization ( Weibull model, β=1.35-1.75, Example 1)**

**Table 4.19: Information for efficient frontier diagram (Weibull model, β = 1.1 to 1.8, Example 1)**

| Points | Objective function I ($) | Objective function I Components | | | Objective function II | epsilon € | Cells | | |
|---|---|---|---|---|---|---|---|---|---|
| | | VCM ($) | MHC ($) | MNC ($) | | | 1 | 2 | 3 |
| 1 | 3320.19 | 1712.43 | 650.00 | 957.76 | 715.4 | N/A | M1,M7 | M4,M6 | M2 |
| 2 | 2822.90 | 1632.92 | 200.00 | 989.98 | 715.4 | 715.42 | M1,M2,M4,M6 | M7 | |
| 3 | 2528.14 | 1495.68 | 150.00 | 882.46 | 746.9 | 747.9 | M1,M4,M6,M7 | M2,M3 | |
| 4 | 2310.87 | 1571.66 | 100.00 | 639.21 | 796.4 | 796.4 | M1,M2,M3,M4 | M7,M6 | |
| 5 | 2108.26 | 1411.98 | 150.00 | 546.28 | 872.9 | 872.9 | M1,M4,M6,M7 | M2,M3,M5 | |
| 6 | 1951.08 | 1419.10 | 100.00 | 431.98 | 946.5 | 946.5 | M1,M2,M3,M4 | M5,M6,M7 | |
| 7 | 1806.53 | 1399.08 | 50.00 | 357.45 | 1021.7 | 1021.7 | M1,M2,M3,M4 | M5,M6,M7 | |
| 8 | 1772.12 | 1410.38 | 50.00 | 31274 | 1094.8 | 1094.8 | M1,M2,M3,M4 | M5,M6,M7 | |
| 9 | 1768.45 | 1410.33 | 50.00 | 308.12 | 1110.7 | 1110.7 | M1,M2,M3,M4 | M5,M6,M7 | |
| 10 | 1768.45 | 1410.33 | 50.00 | 308.12 | 1110.7 | N/A | M1,M2,M3,M4 | M5,M6,M7 | |

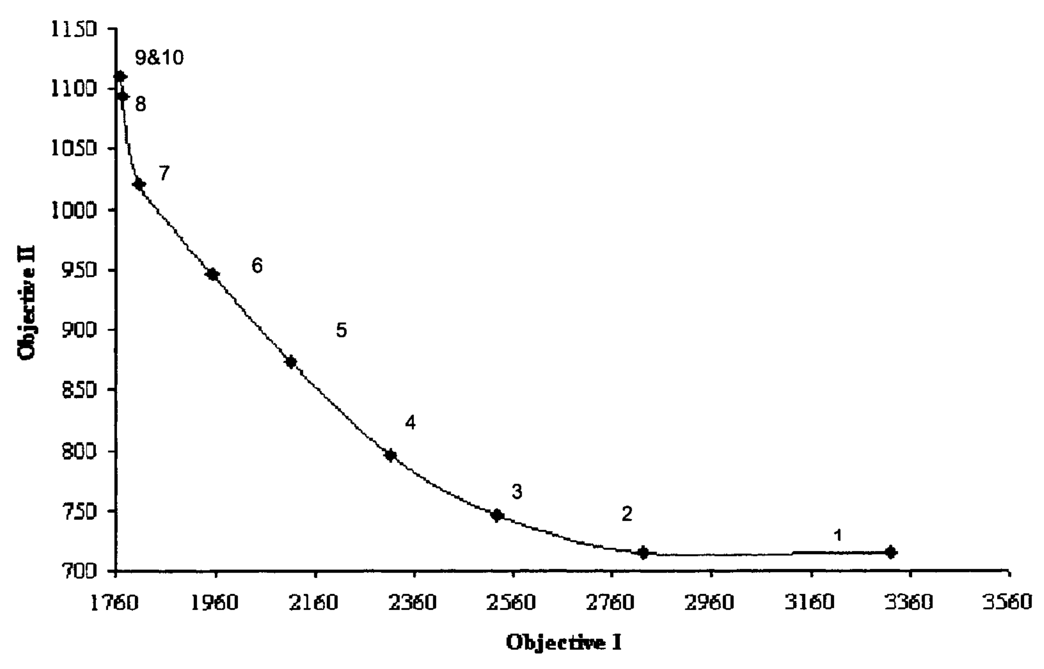


**Figure 4.4: Efficient frontier depicting cost and reliability optimization (Weibull model, β = 1.1 to 1.80, Example 1)**

### 4.4 Solution and Analysis of Example 2

**Input Data:** Example 2 concerns a CMS design problem involving 10 machines and 19 part types. Table 4.20 and Table 4.21 present the part types and machine related data for this problem. Each part type requires two to six operations. For example, part type 19 needs 2 operations, while part types 8 and 10 need as many as 6 operations. As is evident, most of the parts have two process plans, and each operation of the part type can be performed by more than one machine. As a result, there are options for the part types to be processed in alternative routes whenever a machine breaks down, or when a machine is busy with the operations of other parts.

Unlike Example 1, the part types in this example have limited options for selecting the rerouting provision. For example, according to the input information presented in Table 4.20, part type 1 can be processed in alternative routes if machine M4 fails, but the part does not have the option to reroute when any one of the other machines (M1, M5, M3 or M2) fail. Some of the part types have multiple options for selecting alternative routes in the case of machine failure. Part type 18 is an example of one that has several options. This scenario of alternative routes is near to a practical situation where one may not have the rerouting option for any machine failure while alternative options are provided for the important machines and parts.

Using the machine data in Table 4.21, machine availability and effective machine capacity are evaluated by following the interval availability, equation (2.15) for an exponential-based model, and the inherent availability, equation (2.10) for a Weibull-based model. The shape factor ($\beta$) values for the Weibull-based model solutions are generated randomly from the uniform distribution U (1.10, 1.8). For other input data, we have followed the basis assumed in Section 4.2.4.

#### 4.4.1 Exponential Distribution Based Model Solution–Example 2

The example problem is composed of 205743 total variables, 205670 integer variables and 1790 constraints.

Table 4.22 presents the optimum cell formation and operation allocation, as well as the reliability-related information when only the first objective function (total costs) is optimized. As displayed in Table 4.22, the model solution has three cells—machines M9 and M10 are in cell 1; machines M2, M3, M5, and M6 form cell 2; and machines M1, M4, M7, and M8 are in cell 3.

**Table 4.20: Demands, processing times (hours) and costs ($) for all part types (Example 2)**

| Part (i) (Demand, *di*) | Data type | Process plan 1 | | | | | | Process plan 2 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Operations | | | | | | Operations | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 |
| 1 | M/C | M1 | M5 | M3 | M5 M4 | M2 | | | | |
| (900) | Time | 2.78 | 2.76 | 3.48 | 3.88 2.18 | 3.92 | | | | |
| | Cost | 4.73 | 4.42 | 4.18 | 6.21 3.92 | 7.06 | | | | |
| 2 | M/C | M7 | M1 | M4 M5 | M5 | | | M7 M3 | M1 | M4 |
| (7700) | Time | 1.76 | 4.13 | 1.39 3.04 | 3.17 | | | 2.21 3.19 | 2.35 | 3.2 |
| | Cost | 2.46 | 7.02 | 2.5 4.86 | 5.07 | | | 3.09 3.83 | 4.0 | 5.76 |
| 3 | M/C | M8 M1 | M7 | M4 | M6 | | | M5 M1 | M7 | M6 |
| (2000) | Time | 2.29 2.88 | 2.77 | 2.68 | 2.97 | | | 2.87 2.77 | 2.47 | 2.86 |
| | Cost | 4.12 4.90 | 3.88 | 4.80 | 4.75 | | | 4.59 4.71 | 3.46 | 4.58 |
| 4 | M/C | M8 | M4 | M9 M7 | M4 M1 | M2 | | | | |
| (3000) | Time | 2.82 | 2.45 | 2.53 2.57 | 2.33 2.83 | 2.35 | | | | |
| | Cost | 5.08 | 4.41 | 4.81 3.60 | 4.19 4.81 | 4.23 | | | | |
| 5 | M/C | M8 M1 | M9 M7 | M1 M5 | M2 | | | M6 | M5 | M2 |
| (4200) | Time | 2.51 4.15 | 2.52 2.19 | 2.36 3.87 | 1.98 | | | 2.06 | 2.98 | 2.38 |
| | Cost | 4.52 7.06 | 4.79 3.07 | 4.01 6.19 | 3.56 | | | 3.30 | 4.77 | 4.28 |
| 6 | M/C | M9 | M5 | M5 M10 | M6 | | | M7 | M5 | M6 |
| (4500) | Time | 2.77 | 3.29 | 2.85 2.18 | 2.09 | | | 2.65 | 3.86 | 2.9 |
| | Cost | 5.26 | 5.26 | 4.56 3.49 | 3.34 | | | 3.71 | 6.18 | 4.64 |
| 7 | M/C | M1 | M9 | M5 M9 | M5 M10 | | | M4 | M5 M9 | M5 M10 |
| (3824) | Time | 2.9 | 3.09 | 3.03 2.46 | 3.27 2.14 | | | 3.03 | 2.82 2.45 | 3.98 2.59 |
| | Cost | 4.93 | 5.87 | 4.85 4.67 | 5.23 3.42 | | | 5.45 | 4.51 4.66 | 6.37 4.14 |
| 8 | M/C | M7 M9 | M1 | M4 | M3 | M5 | M1 | | | |
| (464) | Time | 2.48 2.79 | 2.84 | 2.48 | 3.37 | 3.01 | 4.18 | | | |
| | Cost | 3.47 5.30 | 4.83 | 4.64 | 4.04 | 4.82 | 7.11 | | | |
| 9 | M/C | M1 | M4 | M3 | M5 | M1 M4 | | | | |
| (3120) | Time | 2.47 | 1.88 | 2.99 | 3.23 | 2.75 3.18 | | | | |
| | Cost | 4.20 | 3.38 | 3.59 | 5.17 | 4.68 5.72 | | | | |

**Table 4.20 cont'd**

| Part (i) (Demand, $d_i$) | Data type | Process plan 1 | | | | | | Process plan 2 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Operations | | | | | | Operations | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 |
| 10 | M/C | M1 | M4 | M9 | M5 | M5 M10 | M6 | | | |
| (6496) | Time | 2.83 | 2.2 | 3.02 | 2.8 | 3.96 2.89 | 2.83 | | | |
| | Cost | 4.81 | 3.96 | 5.74 | 4.48 | 6.34 4.62 | 4.53 | | | |
| 11 | M/C | M1 | M5 M9 | M5 M10 | M6 | | | | | |
| (3690) | Time | 4.07 | 3.04 2.5 | 3.27 2.25 | 2.78 | | | | | |
| | Cost | 6.92 | 4.86 4.75 | 5.23 3.60 | 4.45 | | | | | |
| 12 | M/C | M8 M1 | M4 | M7 M9 | M2 | M6 | | | | |
| (4140) | Time | 2.16 2.40 | 1.88 | 1.86 2.67 | 3.85 | 2.18 | | | | |
| | Cost | 3.89 4.08 | 3.84 | 2.60 5.07 | 6.93 | 3.49 | | | | |
| 13 | M/C | M1 M10 | M2 M6 | M5 | | | | M5 | M2 M4 | M5 M9 |
| (1686) | Time | 2.76 2.13 | 2.33 3.00 | 3.9 | | | | 3.04 | 1.87 2.25 | 3.23 3.09 |
| | Cost | 4.42 3.41 | 4.19 4.80 | 6.24 | | | | 4.86 | 3.37 4.05 | 5.17 5.87 |
| 14 | M/C | M5 M10 | M2 | | | | | M4 | M6 | |
| (4135) | Time | 2.77 2.61 | 2.27 | | | | | 2.56 | 2.88 | |
| | Cost | 4.43 4.17 | 4.09 | | | | | 4.61 | 4.61 | |
| 15 | M/C | M5 M10 | M2 | | | | | M4 | M6 M10 | |
| (4805) | Time | 3.88 2.97 | 3.98 | | | | | 2.2 | 2.03 2.25 | |
| | Cost | 6.21 4.75 | 7.16 | | | | | 3.96 | 3.25 3.60 | |
| 16 | M/C | M8 | M4 | M2 | M6 M10 | | | | | |
| (3928) | Time | 2.17 | 1.89 | 2.15 | 2.11 2.10 | | | | | |
| | Cost | 3.91 | 3.40 | 3.87 | 3.77 3.36 | | | | | |
| 17 | M/C | M8 M4 | M7 M9 | M2 | M6 M10 | | | | | |
| (4475) | Time | 2.82 2.21 | 2.19 2.47 | 2.0 | 2.91 2.69 | | | | | |
| | Cost | 5.08 3.98 | 3.07 4.69 | 3.60 | 4.66 4.30 | | | | | |
| 18 | M/C | M5 M10 | M1 M5 | | | | | M8 M4 | M1 M7 | |
| (4582) | Time | 3.04 2.90 | 2.89 3.20 | | | | | 2.48 2.46 | 2.82 2.73 | |
| | Cost | 4.86 4.64 | 4.91 5.12 | | | | | 4.46 4.43 | 4.79 3.80 | |
| 19 | M/C | M5 M10 | M1 | | | | | M4 | M1 M7 | |
| (2508) | Time | 2.79 2.23 | 4.07 | | | | | 2.18 | 2.45 2.54 | |
| | Cost | 4.46 3.57 | 6.92 | | | | | 3.92 | 4.16 3.57 | |

**Table 4.21 Machine information (Example 2)**

| Machine number | Capacity ( Hours) | *MTBF* (Hours) | *MTTR* (Hours) | Shape factor β | Penalty cost for % non - utilization (CP) |
|---|---|---|---|---|---|
| 1 | 1875 | 97 | 17 | 1.80 | 230 |
| 2 | 1875 | 58 | 6 | 1.13 | 275 |
| 3 | 1875 | 211 | 70 | 1.11 | 255 |
| 4 | 1875 | 91 | 10 | 1.14 | 250 |
| 5 | 1875 | 208 | 37 | 1.64 | 220 |
| 6 | 1875 | 101 | 25 | 1.15 | 265 |
| 7 | 1875 | 134 | 57 | 1.32 | 270 |
| 8 | 1875 | 186 | 21 | 1.48 | 150 |
| 9 | 1875 | 184 | 10 | 1.21 | 175 |
| 10 | 1875 | 122 | 22 | 1.55 | 200 |

Part type 2, for instance, is processed in cell 3 where operations 1, 2 and 3 are performed sequentially on machines M7, M1, and M4. The total costs (objective function I) equal $19,448.72, and the overall system failure rate (objective function II) evaluated at this solution point is 0.63288.

Table 4.23 displays the solution when only the second objective (system failure rate) is optimized. As expected, the objective function II value at this solution instance is very low which is 0.60397 and objective function I value as obtained by this solution is $23,590.40, which is much higher than the solution for optimizing objective I only.

The two extreme outcomes of the objective function II values obtained from the above solutions are the *bounds on* objective function II. Between these bounds there is a collection of efficient solutions which may be solved using the following Є-constraint method model:

*Minimize:* Objective function I

*Subject to the original constraints, and*

Objective function II $\leq$ Є, and

0.60397 $\leq$ Є $\leq$ 0.63288

Table 4.24 summarizes the efficient solutions for various combinations of reliability and cost. The efficient frontier diagram is presented in Figure 4.5. For example, the solution of efficient point #2 in Table 4.24, Figure 4.5 optimizes cost (objective function I) subject to the achievement of: Objective function II $\leq$0.60397. This solution illustrates the benefits of simultaneously considering cost and reliability. Details of the solution are presented in Table 4.27. The solution achieves the expected highest reliability (with the lowest system failure rate 0.60397) while lowering the overall cost from $23,590.40 to $20,600.40.

**Table 4.22: Model results when optimizing objective function I only (exponential model, Example 2)**

| Solution type | Part -process plan | Part processing route |
|---|---|---|
| *Min*=Objective function I only | 1-1 | M1-M5-M3-M5-M2 |
| | *2-2 | M7-M1-M4 |
| | 3-2 | M1-M7-M6 |
| | 4-1 | M8-M4-M7-M4-M2 |
| | 5-2 | M6-M5-M2 |
| | 6-2 | M7-M5-M6 |
| | 7 -2 | M4-M9-M10 |
| | 8-1 | M7-M1-M4-M3-M5-M1 |
| | 9-1 | M1-M4-M3-M5-M1 |
| | 10-1 | M1-M4-M9-M5-M10-M6 |
| | 11-1 | M1-M9-M10-M6 |
| | 12-1 | M8-M4-M7-M2-M6 |
| | 13-2 | M5-M2-M5 |
| | 14-1 | M5-M2 |
| | 15-2 | M4-M10 |
| | 16-1 | M8-M4-M2-M6 |
| | 17-1 | M8-M7-M2-M6 |
| | 18-2 | M8-M7 |
| | 19-1 | M10-M1 |
| Machine cells | *CELL* 1= M9, M10<br>*CELL* 2 =M2, M3, M5, M6<br>*CELL* 3 =M1, M4, M7, M8 | |
| Objective function I (total costs) = $19,448.72, (VCM= $18,182.52, MHC= $600.00, MNC= $666.20) | | |
| Objective function II = 0.63288 | | |
| **Performance** | | |
| **Machine utilization** | **MU(1) =0.93, MU(2) =0.65, MU(3) =0.16, MU(4) =0.96, MU(5) =0.91, MU(6) = 0.96, MU(7) =0.91 MU(8) =0.49, MU(9) =0.36, MU(10) =0.56, | |
| *2-2: part type 2 uses process plan 2, operations 1, 2, and 3 are performed on machines M7, M1, and M4, respectively.<br>**MU(1) : utilization of machine M1 | | |

**Table: 4.23: Model results when optimizing objective function II only (exponential model, Example 2)**

| Solution type | Part –process plan | Part processing route |
|---|---|---|
| *Min*=Objective function II only | 1-1 | M1-M5-M3-M5-M2 |
| | *2-2 | M3-M1-M4 |
| | 3-2 | M5-M7-M6 |
| | 4-1 | M8-M4-M9-M1-M2 |
| | 5-2 | M6-M5-M2 |
| | 6-2 | M7-M5-M6 |
| | 7-2 | M4-M9-M10 |
| | 8-1 | M9-M1-M4-M3-M5-M1 |
| | 9-1 | M1-M4-M3-M5-M1 |
| | 10-1 | M1-M4-M9-M5-M10-M6 |
| | 11-1 | M1-M9-M10-M6 |
| | 12-1 | M8-M4-M9-M2-M6 |
| | 13-1 | M5-M6-M5 |
| | 14-2 | M4-M6 |
| | 15-2 | M4-M10 |
| | 16-1 | M8-M4-M2-M10 |
| | 17-1 | M8-M9-M2-M10 |
| | 18-2 | M8-M7 |
| | 19-1 | M5-M1 |
| Machines cells | *CELL* 1: M1, M4, M6, M10<br>*CELL* 2: M2, M3, M7, M9<br>*CELL* 3: M5, M8 | |
| Total Cost = $23,590.40, (VCM= $18,786.05, MHC= $41,20, MNC= $684.35) | | |
| Objective function II = 0.60397 | | |
| **Performance** | | |
| **Machine utilization** | * *MU(1) =0.96, MU(2) =0.53, MU(3) =0.45, MU(4) =0.99, MU(5) =0.93, MU(6) = 0.91, MU(7) =0.36 MU(8) =0.49, MU(9) =0.64, MU(10) =0.71, | |
| *2-2: part type 2 uses process plan 2, operations 1, 2, and 3 are performed on machines M3, M1, and M4, respectively<br>**MU(1) : utilization of machine M1 | | |

**Table: 4.24: Information for efficient frontier diagram (exponential model, Example 2)**

| Points | Objective function I ($) | Objective function I components | | | Objective function II | Epsilon € | Cells | | |
|---|---|---|---|---|---|---|---|---|---|
| | | VCM ($) | MHC ($) | MNC ($) | | | 1 | 2 | 3 |
| 1 | 23590.40 | 18786.05 | 4120.00 | 684.35 | 0.60397 | N/A | M1,M4, M6,M10 | M2,M3,M7, M9 | M5,M8 |
| 2 | 20600.40 | 18786.05 | 1130.00 | 684.35 | 0.60397 | 0.60397 | M1,M2, M3,M4 | M5,M6,M7 | M8,M9, M10 |
| 3 | 20079.65 | 18495.39 | 930.00 | 654.26 | 0.60973 | 0.60973 | M1,M2, M3,M4 | M5,M6,M7, M8 | M9,M10 |
| 4 | 19816.00 | 18454.10 | 660.00 | 701.90 | 0.61395 | 0.61395 | M1,M4, M9,M10 | M2,M3,M5, M6 | M7,M8 |
| 5 | 19751.09 | 18337.30 | 750.00 | 663.79 | 0.61529 | 0.61529 | M1,M3, M5,M6 | M2,M9,M10 | M4,M7, M8 |
| 6 | 19587.55 | 18224.34 | 690.00 | 673.21 | 0.61971 | 0.61971 | M1,M4, M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 7 | 19541.63 | 18210.19 | 660.00 | 671.44 | 0.62174 | 0.62174 | M1,M4, M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 8 | 19448.72 | 18182.52 | 600.00 | 666.20 | 0.63288 | 0.63288 | M1,M4, M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 9 | 19448.72 | 18182.52 | 600.00 | 666.20 | 0.63288 | N/A | M1,M4, M7,M8 | M2,M3,M5, M6 | M9,M10 |

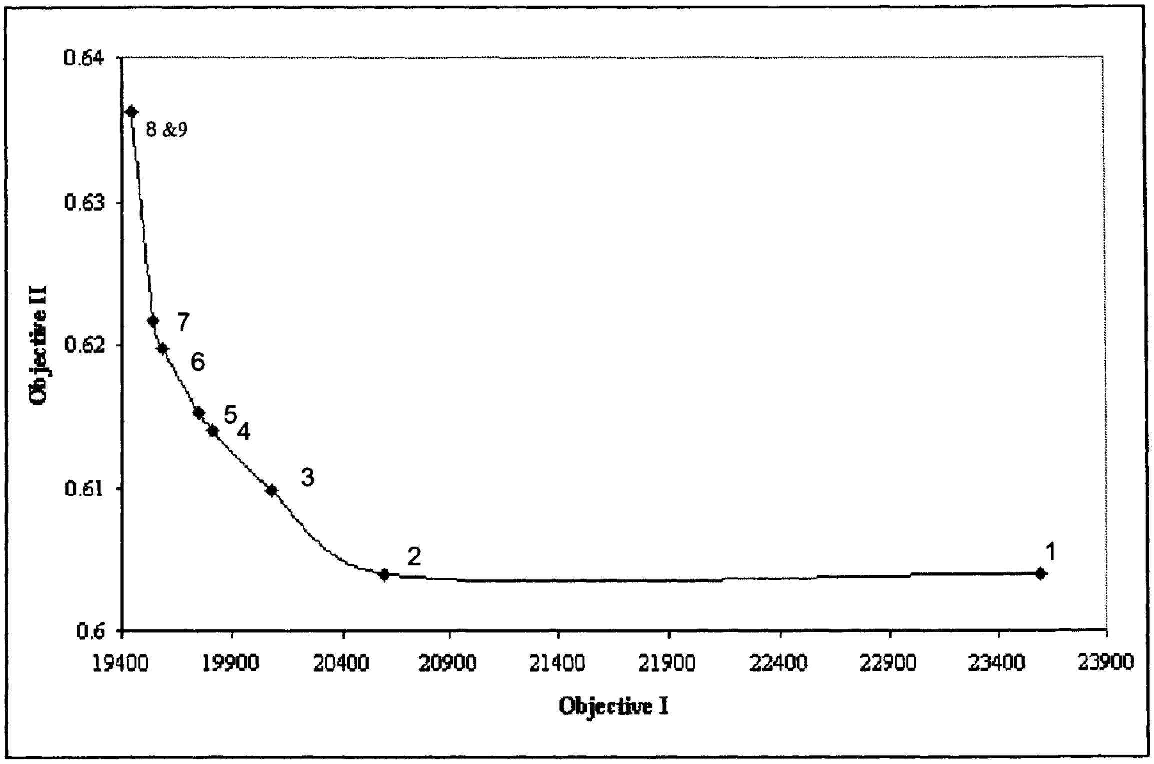


**Figure: 4.5: Efficient frontier depicting cost and reliability optimization (exponential model, Example 2)**

Table 4.25: Model results corresponding to point #2, in Table 4.24

| Solution type | Part -process plan | Part processing routes |
|---|---|---|
| *Min*=Objective function I s. t. Objective function II≤ 0.60397 | 1-1 | M1-M5-M3-M5-M2 |
| | *2-2 | M3-M1-M4 |
| | 3-3 | M5-M7-M6 |
| | 4-1 | M8-M4-M9-M1-M2 |
| | 5-2 | M6-M5-M2 |
| | 6-2 | M7-M5-M6 |
| | 7-2 | M4-M9-M10 |
| | 8-1 | M9-M1-M4-M3-M5-M1 |
| | 9-1 | M1-M4-M3-M5-M1 |
| | 10-1 | M1-M4-M9-M5-M10 |
| | 11-1 | M1-M9-M10-M6 |
| | 12-1 | M8-M4-M9-M2-M6 |
| | 13-1 | M5-M6-M5 |
| | 14-2 | M4-M6 |
| | 15-2 | M4-M10 |
| | 16-1 | M8-M4-M2-M10 |
| | 17-1 | M8-M9-M2-M10 |
| | 18-2 | M8-M7 |
| | 19-1 | M5-M1 |
| Machines cells | *CELL* 1: M5, M6 M7<br>*CELL* 2: M1, M2, M3, M4<br>*CELL* 3: M8, M9, M10 | |
| Objective function I (total costs) = $20,600.40, ( VCM= $18,786.05, MHC=$1,130, MNC= $684.35) | | |
| Objective function II (system failure rate) = 0.60397 | | |
| **Performance** | | |
| **Machine utilization** | **MU(1) =0.96, MU(2) =0.53, MU(3) =0.45, MU(4) =0.99, MU(5) =0.93, MU(6) = 0.91, MU(7) =0.36 MU(8) =0.49, MU(9) =0.64, MU(10) =0.71, | |
| *2-2: part type 2 uses process plan 2, operations 1, 2, and 3 are performed on machines M3, M1, and M4, respectively<br>**MU(1) : utilization of machine M1 | | |

The above results indicate that both the model and the Є-constraint method of solution procedure may be applied to develop an effective CMS design for large, realistic-sized problems by simultaneously considering the machine reliabilities, and the system costs.

### 4.4.2 Weibull Distribution Based Model Solution - Example 2

The solution of Example 2—following the Weibull distribution-based model—involved the same number of continuous variables, integer variables and constraints as the exponential-based model solution.

To generate the efficient solutions based on the Є-constraint method, the example is first solved for the two extreme cases: optimizing cost (objective function I) only; and optimizing reliability (objective function II) only—following the same procedure of exponential distribution -based solution presented in the previous section. As explained earlier, the second objective function in the Weibull distribution -based model evaluates the inverse of the system reliability in the natural logarithmic scale. This is unlike exponential-based model solutions, where the system failure rate is evaluated. Table 4.26 and 4.27 present the operation allocations, cell configurations, costs and reliability-related solutions, respectively, for optimizing the first objective and optimizing the second objective only. As expected, under the cost optimization scenario the cell configurations and operation allocation solutions are identical to the exponential-based solutions demonstrated in the previous section. For example, cell 1 consists of machines M9 and M10; cell 2 of machines M2, M3, M5 and M6; and cell 3 of machines M1, M4, M7 and M8. Part 2, for instance, is processed using process plan 2 in cell 3 on machines M7, M1 and M4 sequentially, as represented in Table 4.26. There is, however, a $4.66 difference in the total cost as a result of difference in the machine availability calculation basis. The total costs (objective function I) equal $19,444.06, objective function II evaluated at this solution point is 3619.68.

As expected, the reliability optimization solution summarized in Table 4.27 generated significantly different cell configurations and operation allocation solutions when compared to the cost optimization solutions in Table 4.26. For example in this solution instance, machines M2, M3, M9 and M10 formed cell 1, machines M1, M4 and M7 formed cell 2, and cell 3 consisted of machines M5, M6 and M8. Part 2 now is processed following process plan 1, which requires four operations. Operations 1, 2 and 3 are performed in cell 2 using the machine sequence M7-M1-M4, while operation 4 is processed in cell 3 on machine M5.

As expected, optimizing cost only generates the lowest objective function I value ($19,444.06) but a very high objective function II value (3619.68), while optimizing reliability produces a low objective function II value (3191.19) but a very high cost of $22,808.00. This outcome is similar to the exponential-based solution presented in the previous section, as well as the solutions obtained for Example I in the similar scenario. However, from these two extreme values of the second objective, the following the Є-constraint method is solved to generate a collection of efficient solution sets:

*Minimize:* Objective function *I*

*Subject to the original constraints, and*

Objective function II $\leq$ Є, and

$3191.19 \leq Є \leq 3619.68$

Table 4.28 presents the efficient set of solutions as generated from the above Є-constraint approach for various combinations of reliability and cost. Figure 4.6 presents the efficient frontier diagram. To illustrate, let us take the efficient point #2 in Table 4.28, Figure 4.6 where the system cost (objective function I) is optimized to achieve the highest reliability. Table 4.29 presents the details of the efficient point #2 solution. From this solution, we get the minimum objective function II value of 3191.19, and at the same time the total cost is reduced to $21,368.00 from

**Table 4.26: Model results when optimizing objective function I only(Weibull model, Example 2)**

| Solution type | Part -process plan | Part processing routes |
|---|---|---|
| *Min*=Objective function I only | 1-1 | M1-M5-M3-M5-M2 |
| | *2-2 | M7-M1-M4 |
| | 3-2 | M1-M7-M6 |
| | 4-1 | M8-M4-M7-M4-M2 |
| | 5-2 | M6-M5-M2 |
| | 6-2 | M7-M5-M6 |
| | 7-2 | M4-M9-M10 |
| | 8-1 | M7-M1-M4-M3-M5-M1 |
| | 9-1 | M1-M4-M3-M5-M1 |
| | 10-1 | M1-M4-M9-M5-M10-M6 |
| | 11-1 | M1-M9-M10-M6 |
| | 12-1 | M8-M4-M7-M2-M6 |
| | 13-2 | M5-M2-M5 |
| | 14-1 | M5-M2 |
| | 15-2 | M4-M10 |
| | 16-1 | M8-M4-M2-M6 |
| | 17-1 | M8-M7-M2-M6 |
| | 18-2 | M8-M7 |
| | 19-1 | M10-M1 |
| Machine cells | *CELL* 1= M10, M9<br>*CELL* 2 =M2, M3, M5, M6<br>*CELL*3 =M1, M4, M7, M8 | |
| Objective function I(System cost) = 19444.06 (VCM= 18182.52, MHC=600.00, MNC=661.54) | | |
| Objective function II = 3619.68. | | |
| **Performance** | | |
| **Machine utilization** | **MU(1) = 0.93, MU(2) = 0.65, MU(3) = 0.16, MU(4) = 0.96, MU(5) = 0.91, MU(6) = 0.96, MU(7) = 0.91 MU(8) = 0.49, MU(9) = 0.36, MU(10) = 0.56, | |

*2-2: part type 2 uses process plan 2, operations 1, 2, and 3 are performed on machines M7, M1, and M4, respectively.
**MU(1) : utilization of machine M1

**Table 4.27: Model results when optimizing objective function II only (Weibull model, Example 2)**

| Solution type | Part -process plan | Part processing routes |
|---|---|---|
| *Min*=Objective function II only | 1-1 | M1-M5-M3-M4-M2 |
| | *2-1 | M7-M1-M4-M5 |
| | 3-1 | M8-M7-M4-M6 |
| | 4-1 | M8-M4-M9-M4-M2 |
| | 5-2 | M6-M5-M2 |
| | 6-2 | M7-M5-M6 |
| | 7-2 | M4-M9-M10 |
| | 8-1 | M9-M1-M4-M3-M5-M1 |
| | 9-1 | M1-M4-M3-M5-M4 |
| | 10-1 | M1-M4-M9-M5-M10-M6 |
| | 11-1 | M1-M9-M10-M6 |
| | 12-1 | M8-M4-M9-M2-M6 |
| | 13-2 | M5-M4-M9 |
| | 14-1 | M10-M2 |
| | 15-1 | M10-M2 |
| | 16-1 | M8-M4-M2-M6 |
| | 17-1 | M8-M9-M2-M6 |
| | 18-2 | M8-M7 |
| | 19-2 | M4-M7 |
| Machines ce l ls | *CELL* 1: M2, M3, M9, M10<br>*CELL* 2: M1, M4, M7<br>*CELL* 3: M5, M6,M8 | |
| Objective function I (total Cost) = 22808.00, (VCM= 19515, MHC=2670.00, MNC=623.00) | | |
| Objective function II = 3191.19 | | |
| **Performance** | | |
| **Machine utilization** | **MU(1) = 0.82, MU(2) = 0.81, MU(3) =0.17, MU(4) = 0.98, MU(5) = 0.96, MU(6) = 0.96, MU(7) = 0.63, MU(8) = 0.54, MU(9) = 0.70, MU(10) = 0.65, | |

*2-1: part type 2 uses process plan 1, operations 1, 2, 3, and 4 are performed on machines M7, M1, M4, and M5, respectively.
**MU(1) : utilization of machine M1

**Table 4.28: Information for efficient frontier diagrams (Weibull model, Example 2)**

| Points | Objective function I($) | Components of objective function I | | | Objective function II | epsilon € | Cells | | |
|---|---|---|---|---|---|---|---|---|---|
| | | VCM ($) | MHC ($) | MNC ($) | | | 1 | 2 | 3 |
| 1 | 22808.00 | 19515.00 | 2670.00 | 623.00 | 3191.19 | N/A | M1,M4,M7 | M2,M3,M9, M10 | M5,M6, M8 |
| 2 | 21368.00 | 19515.00 | 1230.00 | 623.00 | 3191.19 | 3191.19 | M1,M2 | M3,M8,M9, M10 | M4,M5, M6,M7 |
| 3 | 19932.68 | 18406.48 | 890.00 | 636.20 | 3299.98 | 3299.98 | M1,M4,M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 4 | 19631.62 | 18401.77 | 600.00 | 629.85 | 3334.44 | 3334.44 | M1,M4,M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 5 | 19470.26 | 18178.62 | 630.00 | 661.64 | 3482.94 | 3482.94 | M1,M4,M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 6 | 19444.06 | 18182.52 | 600.00 | 661.54 | 3619.68 | 3619.68 | M1,M4,M7,M8 | M2,M3,M5, M6 | M9,M10 |
| 7 | 19444.06 | 18182.52 | 600.00 | 661.54 | 3619.68 | N/A | M1,M4,M7,M8 | M2,M3,M5, M6 | M9,M10 |

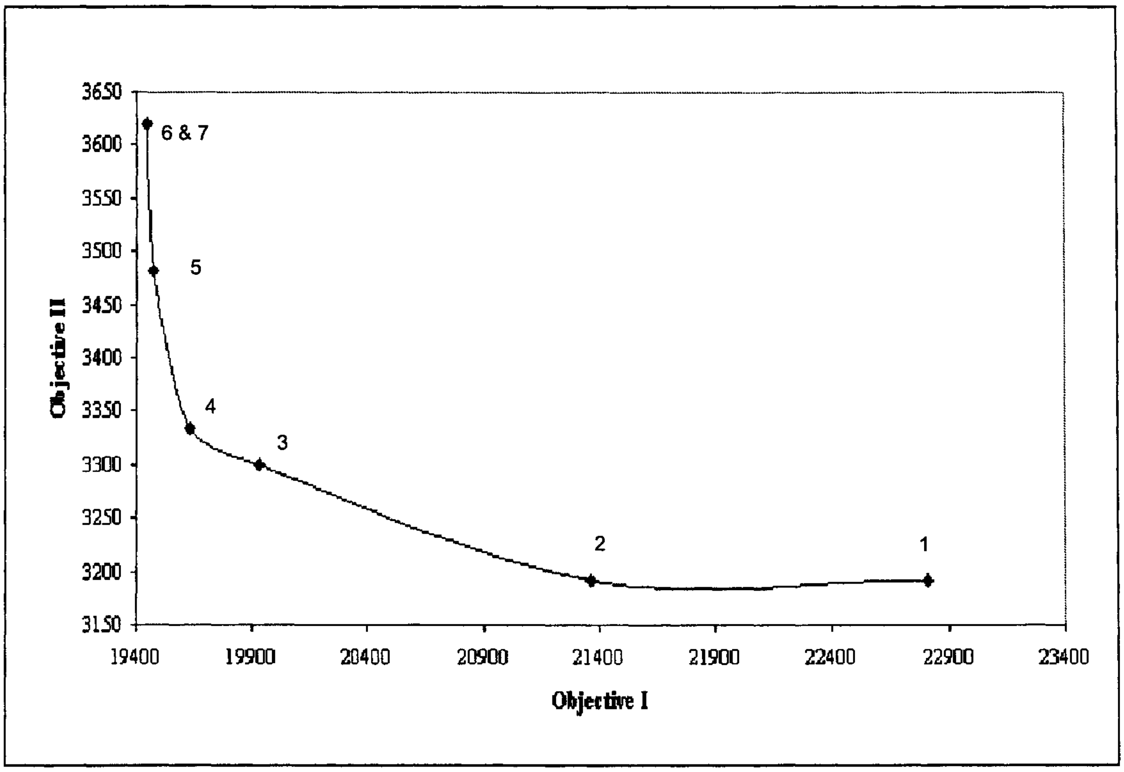


**Figure 4.6: Efficient frontier depicting cost and reliability optimization ( Weibull model, Example 2)**

**Table 4.29: Model results corresponding to point# 2 in Table 4.28**

| Solution type | Part -process plan | Part processing routes |
|---|---|---|
| *Min*=Objective function I s. t. Objective function II≤3191.19 . | 1-1 | M1-M5-M3-M4-M2 |
| | 2-1* | M7-M1-M4-M5 |
| | 3-1 | M8-M7-M4-M6 |
| | 4-1 | M8-M4-M9-M4-M2 |
| | 5-2 | M6-M5-M2 |
| | 6-2 | M7-M5-M6 |
| | 7-2 | M4-M9-M10 |
| | 8-1 | M9-M1-M4-M3-M5-M1 |
| | 9-1 | M1-M4-M3-M5-M4 |
| | 10-1 | M1-M4-M9-M5-M10-M6 |
| | 11-1 | M1-M9-M10-M6 |
| | 12-1 | M8-M4-M9-M2-M6 |
| | 13-2 | M5-M4-M9 |
| | 14-1 | M10-M2 |
| | 15-1 | M10-M2 |
| | 16-1 | M8-M4-M2-M6 |
| | 17-1 | M8-M9-M2-M6 |
| | 18-2 | M8-M7 |
| | 19-2 | M4-M7 |
| Machines cells | *CELL* 1: M3, M8, M9, M10<br>*CELL* 2: M4, M5, M6, M7<br>*CELL* 3: M1, M2 | |
| Objective function I (total costs) = 21368.00, (VCM= 19515.00, MHC=1230.00, MNC= 623.00) | | |
| Objective function II = 3191.19 | | |
| **Performance** | | |
| **Machine utilization** | **MU(1) = 0.82, MU(2) = 0.81, MU(3) = 0.17, MU(4) = 0.98, MU(5) = 0.96, MU(6) = 0.96, MU(7) = 0.63, MU(8) = 0.54, MU(9) = 0.70, MU(10) = 0.65 | |
| *2-1: part type 2 uses process plan 1, operations 1, 2, 3, and 4 are performed on machines M7, M1, M4, and M5, respectively.<br>**MU(1) : utilization of machine M1 | | |

$22,808.00— the cost obtained by optimizing objective function II only. The efficient point #2 solution establishes the advantage of considering cost and reliability simultaneously.

The solution and analysis of this large size problem further establishes that machine reliability has a major influence on the cell configuration as well as the overall performance of the CMS. The above results also indicate that the proposed mathematical models may be used to develop effective CMS designs by considering system cost and machine reliability to follow exponential ( constant failure rate) as well as Weibull distribution ( increasing failure rate) for realistic size problems.

# CHAPTER 5
# HEURISTIC SOLUTION APPROACH

## 5.1 Introduction

Mathematical models for solving realistic size cell formation problems with various conflicting and practical requirements are often computationally expensive, if not intractable. The partitioning of manufacturing systems into cells has been identified as an NP-complete problem in several studies ( Zolfagari and Liang, 1998; Sofianopoulou, 1997) and is considered unsolvable by the traditional optimization methods. Vakharia and Chang (1997) pointed out that most of the CMS design models in literature are computationally intractable for large size problems, creating the need for heuristic methods to obtain reasonably good solutions within an acceptable amount of time. In recent years, heuristic methods such as simulated annealing (SA), genetic algorithm (GA), and tabu search have been widely applied to CMS design problems. Among the heuristics, simulated annealing is the easiest to implement and has been recognized by a number of researchers to generate better quality solutions when compared to other meta-heuristics. Vakharia and Chang (1997) used SA and tabu search-based heuristics to solve their cell formation model. Their paper concluded that the SA-based heuristic performed better than the tabu search for their CMS design model. Asokan et al. (2001) solved three CMS design models selected from the literature using GA and SA algorithms. The study compared the solution quality for different sized problems. SA performed better for the type of models under consideration.

For a large, practical size problem, the multi-objective model presented in Chapter 4 involves too many 0-1 variables and a large number of constraints, causing the optimal solution—if attainable—to be computationally expensive. Therefore, we present a heuristic solution approach which incorporates the basic steps of SA to obtain near-optimal solutions. The solutions are further improved in terms of quality and computational time by applying the crossover and mutation operations—as in GAs—to generate better solutions, or neighboring solutions from a pair of good solutions. The proposed algorithm can generate near-optimal solutions for fairly large size problems within acceptable amounts of CPU time. It may also generate global optimum solutions for reasonable size problems, but at higher levels of computational effort.

## 5.2 Simulated Annealing (SA)

Simulated annealing is a random search method proposed by Kirkpatrick, et al. (1983) for solving combinatorial optimization problems. The method differs from the local search method in its ability to escape local optima by considering a probability according to the Metropolis criterion (Metropolis et al.1953) of accepting a neighboring solution worse than the current solution. The SA algorithm generally accepts all solutions that improve the objective function, while those which do not result in improvements may be accepted by the acceptance probability criterion. The acceptance probability is determined by a control parameter defined as the annealing temperature, which decreases with the progress of the SA steps. According to the Metropolis criterion, if the difference between the cost function values of the current and the newly produced solutions $\Delta E \leq 0$, a random number $\delta$ [0, 1] is generated from a uniform distribution, and if:

$$\delta \leq e^{(-\frac{\Delta E}{T_a})}$$

where $T_a$ is the annealing temperature, then the newly produced solution is accepted as the current solution. Otherwise, the current solution remains unchanged.

The performance of the SA based solution algorithm depends largely on the clarification and definition of the following basic elements:

- solution
- cost function
- generating initial solution
- initial temperature
- neighborhood of a solution
- annealing schedule and
- termination criteria

## 5.3 Definitions of the Basic Elements of the Algorithm

### 5.3.1 A Solution

A solution is represented by an assignment of machines to cells, and processing routes to parts so that it is feasible with respect to the cell size and machine capacity constraints. Considering the numerical example of a cell with 5 machines processing 4 part types, in Section 2.3.2, a typical solution $S$ is represented as an array with a total of 9 bits of information arranged as follows:

The first five bits describe the assignment of the five machines to cells (the cell assignment part). In this case, machines 1, 2, and 3 are assigned to cell 2, and machines 4 and 5 are assigned to cell 1. The remaining bits in the solution represent the processing routes assigned to the four part types (the processing route assignment part). For example, part type 1 is processed under process plan 2 through processing route # 04. The processing routes for part type 1 in this example are listed in Table 2.2.

$$S = (2, 2, 2, 1, 1)\ (1204, 2101, 3103, 4202)$$

**Cell assignment part**

- Position of digits indicates machines
- Digits indicate cell number

Position 1: Machine 1 in cell 2
Position 2: Machine 2 in cell 2
Position 3: Machine 3 in cell 2
Position 4: Machine 4 in cell 1
Position 5: Machine 5 in cell 1

**Processing route assignment**

1204: for part 1
2101: for part 2
3103: for part 3
4202: for part 4

It may be noted that the solution automatically satisfies constraint equations (4.5), (4.6), (4.7) and (4.9) of the CMS design model presented in section 4.2.1. Each solution is also checked for the cell size constraint, equation (4.8), and the machine capacity constraint (4.10).

### 5.3.2 Cost Function

The **cost function *(Z)*** corresponding to a solution is defined as:

$$\textit{Minimize } \mathbf{Z} = w_1 \bullet \textit{Objective function I} + w_2 \bullet \textit{Objective function II}$$
$$= w_1 \bullet F_1 + w2 \bullet F_2$$

where objective function I and objective function II are described by equations (4.1) and (4.4) respectively, in Section 4.2.1, and $w_1$ and $w_2$ are the user-specified weights assigned to the two objectives. We may use an appropriate method of scaling the objective functions in order to assign the weights in a systematic and logical way.

To determine the range of variations for the two objective functions, we solve the model for the two extreme cases of ($w_1 = 1$, $w_2 = 0$) and ($w_1 = 0$, $w_2 = 1$), where, respectively, $F_1$ and $F_2$ are individually optimized. In this case, $F_1$ represents the total costs (typically, in the range of thousands of dollars), and $F_2$ evaluates the overall system failure rate (typically very small, less than one in our case). Following the method of range equalization (Steuer, 1986), we can estimate appropriate values for $w_1$ and $w_2$. This will be discussed further when the numerical example is represented.

### 5.3.3 Initial Solution and the Initial Temperature

The following steps are taken to generate the initial solution and the initial temperature:

1. Ten random, feasible solutions ($S_1$, $S_2$, ....., $S_{10}$) are generated;
2. Among the solutions, $Z_{best}$ and $Z_{worst}$ are determined, and the solution corresponding to $Z_{best}$ is set as the initial solution;
3. $diff = Z_{worst} - Z_{best}$ is evaluated;
4. The initial annealing temperature is determined by the following equation derived from the Metropolis criterion described above: $T_a = (\frac{-diff}{\log p})$, where $p$ is the probability of accepting bad solutions.

### 5.3.4 Neighborhood Solution

A neighborhood solution in SA is generated by a perturbation scheme that makes a slight change in the current solution to obtain a new solution that can be traced from the previous one. According to the reviewed research, the neighborhood solution generation procedure is problem-specific, and differs from one study to the next. The algorithm proposed in this research does not follow all the steps of the SA and has a major difference with respect to the neighborhood solution generation procedure. This procedural difference has been incorporated to improve the solution quality and performance in terms of time for generating a quality solution for a multi-objective, multiple-process plan CMS design model.

The algorithm randomly selects cell configuration and part routes for generating neighborhood solutions. As discussed, if a newly generated neighborhood solution is feasible and has a better objective function value (cost) than the initial solution, it is replaced with the new, better solution. Otherwise, acceptance decisions about the new solution are made following the Metropolis criteria. Whenever the initial solution is replaced—with a neighboring solution with a better cost, or with a solution based on the Metropolis criterion of acceptance—the newly accepted solution and the initial solution are treated with two types of ***crossover*** and a predefined number of ***mutation operations***. The aim of these operations, as shown in the following illustration, is to generate an even better solution by applying these population generation procedures of GA. If an even better solution is generated out of these operations (*crossover* and *mutations*), the illustrated procedure is repeated; otherwise, the algorithm returns to the usual procedure of generating neighborhood solutions by randomly selecting part routes and cell configuration.

#### 5.3.4.1 The Crossover and Mutation Operations

These operations are applied to a pair of solutions (initial solution and newly obtained better solution) in the following manner:

1) first, a crossover operation is performed in which the processing route assignment parts of the two solutions are interchanged, keeping the cell assignment parts unchanged,
2) next, a single point crossover is performed at a random point within the processing route assignment part of the solutions, and
3) finally, a mutation operation is carried out on the processing route assignment part of the best solution.

To demonstrate, we consider, once again, the numerical example of Section 2.3.2 with the following two solutions (initial solution and a better solution):

**(1, 1, 2, 2, 1)** ⋮ **[1104, 2202, 3207, 4104]***initial solution*

*(1, 2, 2, 2, 1)* ⋮ *[1103, 2202, 3103, 4201]better or accepted solution*

[1]

The vertical line [1] indicates the first crossover operation which results in the interchange of the processing route assignment parts of the two solutions, as shown below:

**(1, 1, 2, 2, 1)** *[1103, 2202,* ⋮ *3103, 4201]new solution 1*

*(1, 2, 2, 2, 1)* **[1104, 2202,** ⋮ **3207, 4104]***new solution 2*

[2]

The vertical line [2] indicates the second crossover operation. A random number in the range of 1 to 3 (i.e., the number of part types-1) is generated. Suppose the random number is 2. Thus, the crossover operation takes place *after* the processing route assignment of part type 2, as shown below:

**(1, 1, 2, 2, 1) [1104, 2202,** *3103, 4201]* *new solution 3*

*(1, 2, 2, 2, 1) [1103, 2202,* **3207, 4104]** *new solution 4*

Finally, the ***mutation*** operation is performed on the best solution generated from the crossover operations by generating a random number in the range of 1 to 3 (i.e., the number of part types -1). Suppose the random number is 3, then the processing route of part type 3 is changed to the next one on the list. Thus, in this case, the processing route of part type 3 in the *new solution 3* is changed from #03 to #04:

**(1, 1, 2, 2, 1) [1104, 2202,** *3103,* 4201**]** »»» **(1, 1, 2, 2, 1) [1104, 2202,** *3104,* 4201**]**,

resulting in a new neighboring solution.

### 5.3.5 Annealing/Cooling Schedule and Limiting the Computational Time

The annealing/cooling schedule is a plan to reduce the annealing temperature after a specified number of iterations. The cooling schedule recommended in the literature is a geometric reduction function, $\alpha\ (t) = \boldsymbol{\alpha}^*T_a$ , where $\boldsymbol{\alpha}$ is the reduction factor. Generally, high values of $\boldsymbol{\alpha}$ perform the best, and most of the successful studies in the literature reported $\boldsymbol{\alpha}$ values from 0.8 to 0.99 (Dowsland, 1993).

To limit the computational time, we may specify limits on one or more of the following:

- the number of local searches at a temperature,
- the temperature reduction factor,
- the final temperature, and
- the total number of iterations.

### 5.4 The simulated Annealing Based Algorithm.

Most SA-related algorithms progress with an initial solution (considered to be the *best*) until it is replaced by a solution with a better cost, or a solution with a worse cost selected according to the Metropolis probability criterion. It is also possible (Xambre and Vilarinho, 2003; Zolfaghari and Liang, 1998) to isolate the *best* solution and move with two solutions, the *best* and the *current best*. The algorithm developed here incorporates the latter approach and progresses with two solutions: *a best* and a *current best*. Whenever a neighborhood solution has a better cost than the *current best* and the *best*, it replaces both the *current best* and the *best*. Otherwise, it replaces the *current best* in case it is better than the *current best* only. Alternatively, a worse solution is accepted in place of the *current best* only as per the Metropolis criterion. In this way, the algorithm ensures the selection and isolation of the *best* solution available for a search.

The following simulated annealing parameters have been used in the implementation of the CMS design algorithm:

- In the calculation of the initial temperature, the probability of accepting bad solutions, $p$, is suggested to be in the range of 0.5 to 0.99 (Zolfagaghari and Liang, 1998). In our study, we found 0.5 to be the most suitable value for the problems solved.
- As discussed in section 5.3.5, the temperature reduction factor, $\alpha$, usually ranges from 0.85 to 0.98. The best value in our study turned out to be 0.98.

- The number of iterations at a local temperature is a user-specified, problem-specific value. Our study chose a value of 750. The limit on the number of total iterations is set at 100,000.
- The final temperature is, again, a user-specified, problem-specific parameter which, in our study, ranges from 0.05 to 0.0005 with 0.005 being the best performer. The final temperature value is set so as to make the probability, *p*, very close to zero, thus avoiding the acceptance of any bad solutions at that stage.

The following algorithm (The Algorithm) and the flow charts (Figure 5.1 and 5.2) demonstrate detailed steps of the heuristic solution generations procedure:

**The Algorithm**

**Step 1.0** Parameter initialization

Step 1.1 Set simulated annealing parameters, *p, a, tf (final temperature)*
Total accepted iterations $T_{count}$, maximum iterations at local temperature *tL*, accepted iteration counter at local temperature *tl*

Step 1.2 Generate 10 feasible random solutions *S1* to *S10*

Step 1.3 Evaluate objective function value for $f(S1) = Z1$ to $f(S10) = Z10$

Step 1.4 Evaluate $diff = Z_{worst} - Z_{best}$, set initial temperature $Ta = (-\ diff / \log p)$

Step 1.5 Initialize solution for $Z_{best}$ as $S_i$, and *bS* (best solution),
$f(bS) = Z_{best}$ and $f(Si) = Z_{best}$

Step 1.6 Set iteration counter *count* for main loop, initialize *count* = 0

**Step 2.0** Execute main loop from step 2.1 to 2.8 until criteria at 2.9 is not met

*2.1* Initialize inner loop $i = 0$ counter $t_l = 0$, solution *S*, objective value *f*(*S*)

2.1.1 *Update i = i +1*
Randomly generate a solution *S*
*if* ( the solution is not feasible), *go to* step 2.1.1
*else* evaluate *f*(*S*), *diff =f(S)-f(Si),* ***go to*** step ***2.1.2****, tl = tl+1*

2.1.2 *if (diff<0); Si = S, f(Si)=f(S)* ***go to*** step ***2.2***
*else go to step 2.6*

2.2 *evaluate diff1=f(S)-f(bS)*
*if (diff1<0);* ***go to*** step ***2.3.1*** before updating and
update *bS* = S, *f*(*bS*) = *f*(*S*)
*else* ***go to*** step 2.3

2.3 *aS = S*, *f*(*aS*)=*f*(*S*), *gS = Si*, *f*(gS) = *f*(*Si*) ***go to*** step 2.3.2

2.3.1 *aS = S*, *f*(*aS*)=*f*(*S*), *gS = bS*, *f*(gS) = *f*(*bS*) ***go to*** step 2.3.2

2.3.2 generate neighborhood solution *gSc1* and *gSc2* by single point ***crossover*** between *aS*, and *gS* keeping cell configurations unchanged
*if(* solutions are feasible*),* ***go to*** step ***2.3.3*** *tl = tl+1*
*else* ***go to*** step 2.3.4

2.3.3 *if f(gSc1)<f(gSc2), evaluate gdiff =f(gSc1)-f(bS),* ***go to*** step 2.4
*else, if f(gSc2)<f(gSc1),*
*evaluate gdiff =f(gSc2)-f(bS),* ***go to*** step 2.4

2.3.4 generate neighborhood solution *gSc1* and *gSc2* by single point ***crossover*** between *aS*, and *gS* starting from a random part route
*If(* solutions are feasible*), go to* step *2.3.3* *tl = tl+1*
*else* ***go to*** step 2.5

2.4 *If(gdiff<0), go to step 2.4.1 before updating and update bS=gScj, f(bS)=f(gScj), j= 1 or 2*
*Else* ***go to*** step 2.5

2.4.1 *aS=gScj, f(aS)=f(gScj), gS = bS, f*(gS) = *f*(*bS*) **go to** step 2.3.1

2.5 *do* the random ***mutations*** on part routes of *bS* and generate neighborhood solution *gSm*
*If (the solution is feasible);* *tl = tl+1*
evaluate *diffm* = *f*(*gSm*)-*f*(*bS*) and ***go to*** step 2.5.1
*else* ***go to*** step *2.1.1*

2.5.1 *If (diffm <0); go to step 2.5.2 before updating and update bS=gSm, f(bS)=f(gSm)*
*else go to step 2.7*

2.5.2 *aS = gSm, f(aS) = f(gSm), gS = bS, f*(gS) = *f*(*bS*) ***go to*** step 2.3.1

2.6 $if(\delta \le e^{(-\frac{diff}{Ta})})$; where δ is a random number [0,1], ***go to*** step 2.3 before updating and update *Si* = *S*, *f*(*Si*) = *f*(*S*),
*else go to step 2.7*

2.7 *If(tl =tL)*; **terminate the inner loop,** ***go to*** step 2.8
*Else* ***go to*** step 2.1.1

2.8 *Update count =count +1,*
*Record Si, f(Si), bS, f(S)*
*Reduce cooling temperature Ta=a*Ta* and ***go to*** step 2.9

2.9 *If(count≥Tcount, or Ta≤ tf); terminate the main loop go to step 3*
*else continue with main loop go to step 2.1*

**Step 3.0** *Print the best solution bS and best objective function f(bS)*

**5.5 Illustrative Example**

The algorithm has been coded in C++ (see Appendix A.4, CD FORMAT) and run on a PC (Pentium 4, 2.26GHz, 760 MB RAM) to solve a numerical example involving 14 machines and 24 part types. Table 5.1 shows the machine-related information for the problem. The machine and part-related information is generated randomly by considering similar data from examples in the literature. Part type demands, processing times and costs, operation sequences and process plan information are presented in Table 5.2. The machine reliability-related information—*MTBF* and *MTTR*—are generated using uniform distributions of [160-360 hours] and [8-48 hours], respectively, to maintain machine availabilities of about 80% to 95% considering similar availability data in the literature (Askin et al.,1997). According to the input information, each part can be processed by either one of two process plans. Each operation of a part type may be performed on more than one machine in most cases, and each machine can perform more than one operation. As a result, each part type is associated with several processing routes. To illustrate, Table 5.3 shows the available processing routes for part type 1. The sequence of operations, operation times and the total cost for each route are also given in Table 5.3. The number of cells and the maximum number of machines allowed in a cell are assumed to be 3 and 5, respectively.

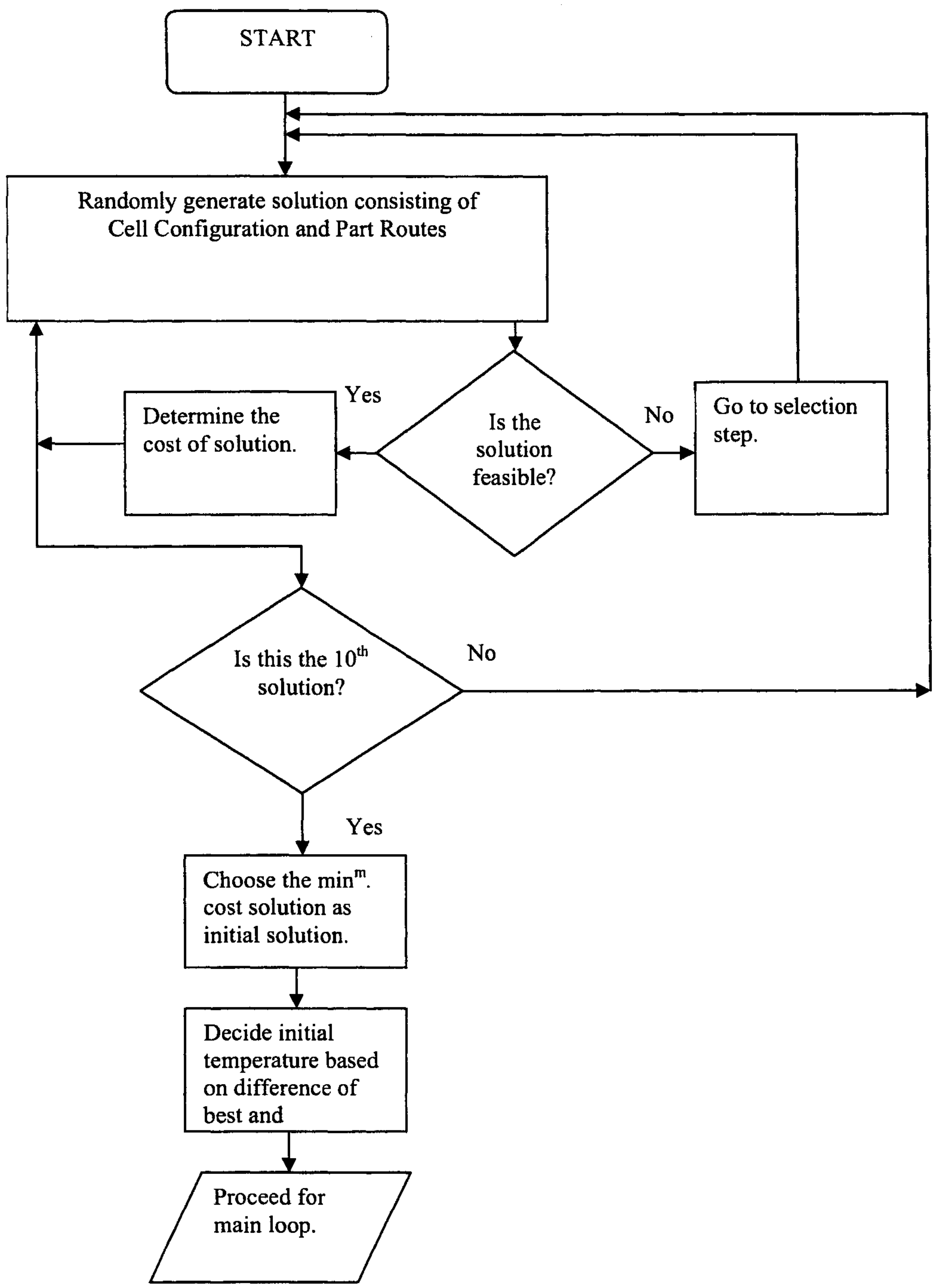


**Figure 5.1: Flow chart for generating initial solutions and determining of initial temperature**

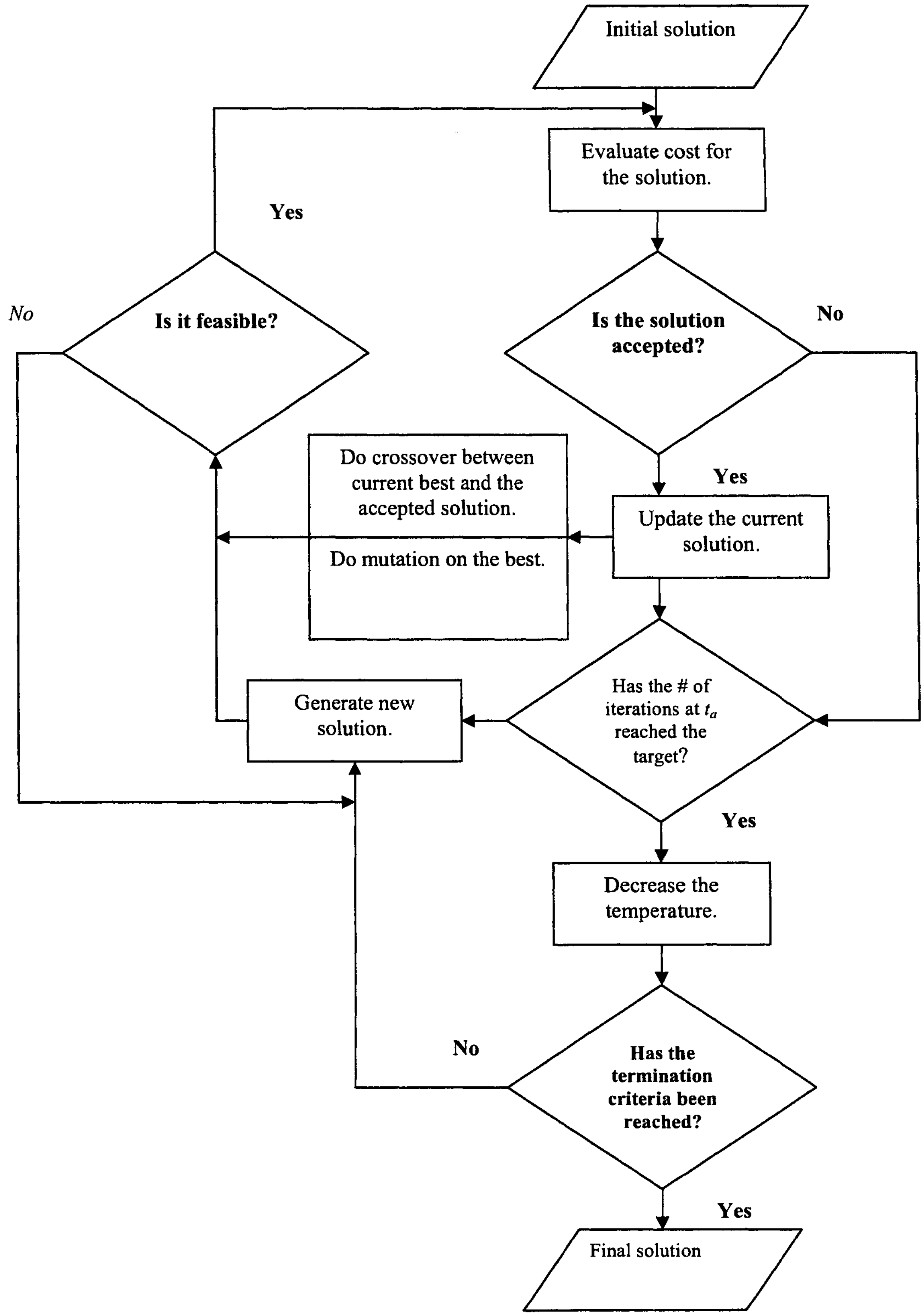


**Figure 5.2: Flow chart for the main loop of the algorithm**

**Table 5.1: Machine data for the numerical example**

| Machine | Capacity $b_j$ (Hour) | *MTBF* (Hour) | *MTTR* (Hour) | Penalty cost for % non utilization ($CP_j$) |
|---|---|---|---|---|
| 1 | 1000 | 282 | 35 | 425 |
| 2 | 1000 | 288 | 24 | 470 |
| 3 | 700 | 190 | 37 | 408 |
| 4 | 1000 | 198 | 24 | 319 |
| 5 | 700 | 241 | 18 | 375 |
| 6 | 2000 | 207 | 10 | 490 |
| 7 | 700 | 312 | 30 | 485 |
| 8 | 1800 | 311 | 35 | 430 |
| 9 | 1000 | 175 | 15 | 472 |
| 10 | 1000 | 200 | 27 | 336 |
| 11 | 1000 | 191 | 20 | 419 |
| 12 | 1000 | 168 | 30 | 470 |
| 13 | 2000 | 346 | 40 | 452 |
| 14 | 1200 | 217 | 40 | 444 |

**Table 5.2: Demand, processing times (hour) and processing costs ($) for all part types**

| Part type (Demand) | Data type | Process plan 1 | | | | Process plan 2 | | |
|---|---|---|---|---|---|---|---|---|
| | | Operations | | | | Operations | | |
| | | 1 | 2 | 3 | 4 | 1 | 2 | 3 |
| 1 | M/C | M4 | M1 M5 | M7 | | M1 | M7 M13 | |
| (20) | Time | 5 | 6 8 | 4 | | 3 | 6 5 | |
| | Cost | 9 | 7 7 | 8 | | 8 | 4 6 | |
| 2 | M/C | M4 M5 | M6 M7 | | | M1 M4 | M5 | M12 M13 |
| (10) | Time | 7 8 | 6 7 | | | 9 7 | 4 | 8 6 |
| | Cost | 8 7 | 8 9 | | | 8 9 | 8 | 5 9 |
| 3 | M/C | M2 | M3 | M10 | M9 M11 | M2 M3 M3 | M11 M13 | |
| (30) | Time | 8 | 3 | 6 | 6 5 | 6 7 10 | 9 7 | |
| | Cost | 6 | 4 | 8 | 5 7 | 2 3 9 | 8 7 | |
| 4 | M/C | M2 | M3 M5 | M11 M13 | | M2 | M10 M12 | M5 M11 |
| (40) | Time | 9 | 5 6 | 11 9 | | 6 | 7 6 | 7 8 |
| | Cost | 5 | 4 7 | 7 4 | | 8 | 4 4 | 9 4 |
| 5 | M/C | M8 | M9 | M11 | | M6 M12 | M9 M11 | M14 |
| (10) | Time | 4 | 7 | 5 | | 8 9 | 5 4 | 7 |
| | Cost | 9 | 7 | 7 | | 6 12 | 9 9 | 6 |
| 6 | M/C | M1 | M13 | | | M4 M7 | M5 M9 | M12 |
| (50) | Time | 6 | 6 | | | 7 8 | 5 6 | 7 |
| | Cost | 5 | 5 | | | 5 4 | 8 7 | 4 |
| 7 | M/C | M3 | M7 | M10 M12 | M13 | M3 M4 | M5 | M11 M12 |
| (20) | Time | 3 | 6 | 7 5 | 5 | 7 6 | 8 | 9 8 |
| | Cost | 6 | 6 | 6 4 | 5 | 9 6 | 7 | 9 6 |
| 8 | M/C | M12 | M13 | | | M4 M5 | M7 M8 | |
| (30) | Time | 5 | 7 | | | 9 10 | 4 4 | |
| | Cost | 6 | 8 | | | 7 8 | 6 4 | |
| 9 | M/C | M6 M8 | M8 | M9 M11 | M13 M14 | M2 | M8 M11 | M14 |
| (40) | Time | 5 7 | 4 | 8 6 | 6 5 | 5 | 5 4 | 7 |
| | Cost | 5 5 | 6 | 6 7 | 8 8 | 5 | 9 6 | 6 |
| 10 | M/C | M6 M10 | M8 M9 | | | M9 | M12 | M14 |
| (10) | Time | 4 5 | 7 6 | | | 9 | 7 | 6 |
| | Cost | 5 7 | 6 7 | | | 7 | 6 | 6 |

**Table 5.2 : cont'd**

| Part type (Demand) | Data type | Process plan 1 Operations 1 | 2 | 3 | 4 | Process plan 2 Operations 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| 11 | M/C | M6 M9 | M12 | M12 M14 | | M7 M9 | M10 M14 | |
| (20) | Time | 5 6 | 6 | 6 4 | | 5 6 | 7 6 | |
| | Cost | 6 7 | 8 | 7 6 | | 7 5 | 6 7 | |
| 12 | M/C | M6 | M8 M12 | M9 M14 | | M8 M12 | M10 M14 | |
| (10) | Time | 6 | 7 5 | 5 6 | | 8 4 | 7 6 | |
| | Cost | 6 | 9 9 | 5 4 | | 6 8 | 4 6 | |
| 13 | M/C | M9 M12 | M13 M14 | | | M6 M10 | M8 M13 | |
| (10) | Time | 9 7 | 7 8 | | | 7 8 | 6 5 | |
| | Cost | 5 5 | 9 7 | | | 9 4 | 9 8 | |
| 14 | M/C | M6 M8 | M9 M13 | | | M9 M12 | M13 M14 | |
| (50) | Time | 6 5 | 8 9 | | | 6 7 | 6 5 | |
| | Cost | 8 5 | 5 4 | | | 6 8 | 7 5 | |
| 15 | M/C | M6 M10 | M8 | M9 M13 | | M1 M3 M14 | M8 M10 | M14 |
| (30) | Time | 5 6 | 9 | 4 3 | | 9 7 4 | 8 7 | 6 |
| | Cost | 6 4 | 7 | 8 7 | | 7 4 8 | 6 8 | 5 |
| 16 | M/C | M6 M9 | M8 M13 | | | M9 M12 | M5 M14 | |
| (50) | Time | 8 6 | 7 8 | | | 6 5 | 6 7 | |
| | Cost | 4 9 | 7 4 | | | 9 8 | 4 8 | |
| 17 | M/C | M1 M4 | M5 M8 | M7 M13 | | M1 | M10 M12 | M13 M14 |
| (20) | Time | 6 4 | 5 4 | 5 6 | | 3 | 6 7 | 8 9 |
| | Cost | 9 4 | 4 5 | 3 8 | | 8 | 7 9 | 7 9 |
| 18 | M/C | M4 M13 | M12 M14 | | | M1 M13 | M6 M10 | |
| (30) | Time | 4 8 | 6 5 | | | 5 7 | 4 6 | |
| | Cost | 9 7 | 6 6 | | | 5 8 | 6 7 | |
| 19 | M/C | M4 M7 | | | | M1 M13 | M5 M9 | |
| (40) | Time | 7 6 | | | | 9 8 | 6 7 | |
| | Cost | 4 7 | | | | 7 5 | 7 7 | |
| 20 | M/C | M4 M7 | M5 M9 | M7 | | M1 M4 | M12 M14 | M9 M13 |
| (10) | Time | 6 5 | 3 4 | 3 | | 5 6 | 5 6 | 3 4 |
| | Cost | 3 5 | 4 5 | 3 | | 6 8 | 7 6 | 3 5 |
| 21 | M/C | M3 M7 | M11 M14 | | | M2 M6 | M10 M12 | |
| (20) | Time | 7 6 | 8 7 | | | 7 6 | 7 5 | |
| | Cost | 7 5 | 5 7 | | | 7 7 | 8 6 | |
| 22 | M/C | M6 M10 | M8 M13 | | | M9 M13 | M5 M14 | |
| (30) | Time | 6 5 | 6 7 | | | 6 7 | 6 8 | |
| | Cost | 3 7 | 9 4 | | | 7 7 | 7 8 | |
| 23 | M/C | M4 M13 | M5 | M11 M13 | | M1 | M6 M9 | M11 M13 |
| (50) | Time | 7 7 | 5 | 8 9 | | 8 | 4 5 | 6 7 |
| | Cost | 9 9 | 5 | 5 6 | | 8 | 5 7 | 5 6 |
| 24 | M/C | M10 M13 | M11 M12 | | | M2 M10 | M3 M13 | |
| (10) | Time | 5 6 | 7 8 | | | 7 8 | 5 6 | |
| | Cost | 8 5 | 5 7 | | | 5 7 | 4 6 | |

**Table 5.3: Processing routes for part type 1 of the numerical example**

| Routes | Machine sequence 1 | 2 | 3 | 4 | Operation times (hours) 1 | 2 | 3 | 4 | Total operation cost for the route |
|---|---|---|---|---|---|---|---|---|---|
| 1. 1101 | M4 | M1 | M7 | - | 5 | 6 | 4 | - | 24 |
| 2. 1102 | M4 | M5 | M7 | - | | 5 | 8 | 4 | 24 |
| 3. 1201 | M1 | M7 | - | - | 3 | 6 | - | - | 12 |
| 4. 1202 | M1 | M13 | - | - | 3 | 5 | | | 14 |

### 5.6 Performance Evaluation

The model has been solved for various values of the weight factors $w_1$ and $w_2$, which have been estimated using the method of range equalization (Steuer, 1986) as mentioned in Section 5.3.2, and the results are displayed in Table 5.4. The performance of the algorithm has been evaluated by comparing the objective function value of the example problem generated by the algorithm with that of the LP relaxation solution found by using LINGO 7. The objective function value of the LP relaxation solution is considered as a basis for evaluating the performance of the algorithm, since it provides a *Lower Bound* on the objective function value of the problem at hand. Following the approach of Vakharia and Chang (1997), the GAP (%) is introduced as a performance index for the algorithm, which is evaluated by the following formula:

$$\text{GAP (\%)} = 100\,[\{Z^* - LB\}/\,Z^*],$$

where,

$Z^*$ = the best objective function value found at the termination of the algorithm

$LB$ = the objective function value of the LP relaxation solution to the problem.

Table 5.4 lists the GAP (%) for each solution. This index may be used for assessing the solution quality of the algorithm. The assessment criterion is: the smaller the gap, the better the solution. However, it is noted that the actual gap between the objective function value of the integer program solved to optimality and that obtained by the algorithm is less than the GAP(%) values listed in Table 5.4.

**Table 5.4: Performance summary of the algorithm**

| Problem instance | Performance items | *Lower Bound* | Solution from algorithm | Gap | CPU time (minutes) |
|---|---|---|---|---|---|
| **Case 1** Z= *w1*F1+w2*F2* *w1=1* *w2=0* | Cost function *Z* | **9187** | **10173** | 9.69 % | 15 |
| | *F1* | | 10173 | | |
| | *w1*F1* | | 10173 | | |
| | *F2* | | 0.243538 | | |
| | *w2*F2* | | 0 | | |
| **Case 2** Z= *w1*F1+w2*F2* *w1= 0* *w2= 1* | Cost function *Z* | **0.204851** | **0.219749** | 6.78% | 16 |
| | *F1* | | 13348 | | |
| | *w1*F1* | | 0 | | |
| | *F2* | | 0.219749 | | |
| | *w2*F2* | | 0.219749 | | |
| **Case 3** Z= *w1*F1+w2*F2* *w1= 1* *w2= 25 000* | Cost function *Z* | **14929** | **16498** | 9.51% | 15 |
| | *F1* | | 10541 | | |
| | *w1*F1* | | 10541 | | |
| | *F2* | | 0.238274 | | |
| | *w2*F2* | | 5957 | | |
| **Case 4** Z= *w1*F1+w2*F2* *w1= 0.75* *w2= 25 000* | Cost Function *Z* | **12547** | **13807** | 9.1% | 15 |
| | *F1* | | 10795 | | |
| | *w1*F1* | | 8096 | | |
| | *F2* | | 0.228459 | | |
| | *w2*F2* | | 5711 | | |

**Table 5.4: Cont'd**

| Problem instance | Performance items | *Lower Bound* | Solution from algorithm | Gap | CPU time (minutes) |
|---|---|---|---|---|---|
| **Case 5** Z= *w1*F1+w2*F2* *w1= 0.5* *w2= 25 000* | Cost Function ***Z*** | **10159** | **11148** | 8.87% | 18 |
| | *F1* | | 10930 | | |
| | *w1*F1* | | 5465 | | |
| | *F2* | | 0.227306 | | |
| | *w2*F2* | | 5683 | | |
| **Case 6** Z= *w1*F1+w2*F2* *w1= 0.25* *w2= 25 000* | Cost Function ***Z*** | **7719** | **8377** | 7.85% | 18 |
| | *F1* | | 11088 | | |
| | *w1*F1* | | 2772 | | |
| | *F2* | | 0.224183 | | |
| | *w2*F2* | | 5605 | | |
| **Case 7** Z= *w1*F1+w2*F2* *w1= 1* *w2= 50 000* | Cost Function ***Z*** | **20318** | **22220** | 8.56% | 16 |
| | *F1* | | 10803 | | |
| | *w1*F1* | | 10803 | | |
| | *F2* | | 0.228348 | | |
| | *w2*F2* | | 11417 | | |
| **Case 8** Z= *w1*F1+w2*F2* *w1= 0.75* *w2= 50 000* | Cost Function ***Z*** | **17901** | **19522** | 8.3% | 16 |
| | *F1* | | 11126 | | |
| | *w1*F1* | | 8344 | | |
| | *F2* | | 0.223538 | | |
| | *w2*F2* | | 11177 | | |
| **Case 9** Z= *w1*F1+w2*F2* *w1= 0.50* *w2= 50 000* | Cost Function ***Z*** | **15439** | **16776** | 8.5% | 18 |
| | *F1* | | 11244 | | |
| | *w1*F1* | | 5622 | | |
| | *F2* | | 0.223084 | | |
| | *w2*F2* | | 11154 | | |
| **Case 10** Z= *w1*F1+w2*F2* *w1= 0.25* *w2= 50 000* | Cost Function ***Z*** | **12922** | **13963** | 7.45% | 16 |
| | *F1* | | 11652 | | |
| | *w1*F1* | | 2913 | | |
| | *F2* | | 0.221 | | |
| | *w2*F2* | | 11050 | | |
| **Case 11** Z= *w1*F1+w2*F2* *w1=1* *w2=100 000* | Cost function ***Z*** | **30877** | **33665** | 8.28% | 18 |
| | *F1* | | 11019 | | |
| | *w1*F1* | | 11019 | | |
| | *F2* | | 0.226465 | | |
| | *w2*F2* | | 22645 | | |
| **Case 12** Z= *w1*F1+w2*F2* *w1= 0.75* *w2=100 000* | Cost function ***Z*** | **28388** | **30743** | 7.66% | 18 |
| | *F1* | | 11222 | | |
| | *w1*F1* | | 8415 | | |
| | *F2* | | 0.223275 | | |
| | *w2*F2* | | 22328 | | |
| **Case 13** Z= *w1*F1+w2*F2* *w1= 0.5* *w2= 100 000* | Cost function ***Z*** | **25844** | **27830** | 7.13% | 18 |
| | *F1* | | 11322 | | |
| | *w1*F1* | | 5661 | | |
| | *F2* | | 0.22169 | | |
| | *w2*F2* | | 22169 | | |
| **Case 14** Z= *w1*F1+w2*F2* *w1=0.25* *w2= 100 000* | Cost function ***Z*** | **23215** | **24977** | 7.54% | 16 |
| | *F1* | | 11699 | | |
| | *w1*F1* | | 2925 | | |
| | *F2* | | 0.220525 | | |
| | *w2*F2* | | 22052 | | |

The computational times in each case are also listed in Table 5.4, and may be considered as a secondary measure of the performance of the algorithm—keeping in mind that the CMS design is a planning problem which is solved only once during the planning period and therefore the reduction of CPU time may not carry much importance.

### 5.7 Illustration of Detailed Solution Steps

To illustrate, we consider the model solution corresponding to $w_1 = 1$ and, $w_2 = 0$ (Case 1 in Table 5.4), which means optimizing *objective function I* only. To start, the algorithm randomly generates 10 feasible solutions with the following cost function values:

12788.6, 12903.4, 12692.8, 12886.9, 12875.6, ***12126.9***, ***14622.4***, 13130.6, 13202.6 and 12306.2. Thus, $Z_{best}$ = 12126.9 and $Z_{worst}$ = 14622.4, and $diff = Z_{worst} - Z_{best}$ = 2495.54. Now, considering $p$ = 0.5 in equation $T_a = \frac{-diff}{\log p}$, the initial temperature is computed as $T_a$ = 8290. The initial solution, $S_i$, is the one corresponding to $Z_{best}$:

$S_i$ = (1 1 2 2 3 3 1 2 1 2 2 3 1 3) (1201 2103 3102 4103 5201 6101 7202 8202 9202 10104 11104 12101 13103 14101 15203 16101 17101 18103 19102 20205 21104 22101 23202 24102)

The *objective function II* corresponding to this solution = 0.265115, and the corresponding cost function $Z$ is:

$$\boldsymbol{Z} = w_1 \bullet \textit{Objective function I} + w_2 \bullet \textit{Objective function II}$$

$$= 1 \bullet (12126.9) + 0 \bullet (0.265115) = 12126.9$$

The cell configuration generated by this solution is:

| | |
|---|---|
| Cell 1: | M1, M2, M7, M9, M13 |
| Cell 2: | M3, M4, M8, M10, M11 |
| Cell 3: | M5, M6, M12, M14 |

Depending on the temperature reduction factor ($\alpha$ = 0.98), the total number of iterations (100,000), the final temperature ($t_f$ = 0.005), and the number of iterations at each temperature (750), the best solution obtained at the termination of the algorithm has:

*Objective function I = 10173*

*Objective function II = 0.243538*

The solution details are presented below:

(1 3 3 1 1 2 1 3 2 3 1 2 3 2) (1201 2102 3202 4102 5201 6101 7204 8202 9202 10103 11204 12202 13204 14104 15203 16204 17105 18202 19102 20103 21204 22101 23101 24201), and the cell configuration is:

| | |
|---|---|
| Cell 1: | M1, M4, M5, M7, M11 |
| Cell 2: | M6, M9, M12, M14 |
| Cell 3: | M2, M3, M8, M10, M13 |

### 5.8 Discussion of the Results

The performance of the algorithm is discussed here in light of the fourteen solution cases listed in Table 5.4. The first two cases, 1 and 2, are designed to establish bounds on *F1* and *F2*. In case 1, we minimize $F_1$ only, resulting in the lowest possible total cost of $10173, as generated by the algorithm. Similarly, in case 2, we minimize $F_2$ only, resulting in the lowest possible total system failure rate of 0.219749, as generated by the algorithm.

In the next four cases—3 to 6—we test the algorithm by assigning $w_2$ = 25,000 and varying $w_1$ from 1 to 0.25, gradually decreasing the importance of $F_1$. As a consequence, the $F_1$ value increases (from 10,541 in case 3 to 11,088 in case 6), while the $F_2$ values decrease, (from 0.238274 in case 3 to 0.224183 in case 6), indicating the decreased importance of $F_1$ and the increased importance of $F_2$ in the optimization process. In the next four cases—7 to 10—the algorithm is tested by assigning $w_2$ = 50, 000 and varying $w_1$ from 1 to 0.25, gradually decreasing the importance of $F_1$ as before. Now, the $F_1$ value increases (from 10,803 in case 7 to 11,652 in case 10), while the $F_2$ value decreases, (from 0.228348 in case 7 to 0.221 in case 10), indicating once again the consequences of changing the levels of importance placed on each objective function.

The last four cases—11 to 14—show the solution results when $w_2$ = 100 000 and $w_1$ is varied from 1 to 0.25, as before. As a consequence, the $F_1$ value increases (from 11,019 in case 11 to 11,699 in case 14), while the $F_2$ values decrease, (from 0.226465 in case 11 to 0.220525 in case 14), indicating the decreased importance of $F_1$ and the increased importance of $F_2$ in the optimization process. These results are depicted in Figure 5.3, where, in as much as the solutions are obtained by a heuristic procedure, the results constitute only a set of 'pseudo-efficient' solution points for this example.

The performance index of the algorithm, i.e., the GAP (%) as presented in Table 5. 4 shows a range of 6.78 to 9.7%. As pointed out earlier, the actual gap between the objective function values obtained from the optimal integer programming solution (if attainable) and from the algorithm is in fact smaller, due to the fact that the LP relaxation solution to the model generates an objective function value that is, in all likelihood, smaller that that of the optimal integer programming solution.

**Table 5.5: Processing routes and machine sequences as generated by the algorithm for selected cases in Table 5.4**

| Part type | Case 1 | | Case 2 | | Case 3 | | Case 6 | | Case 7 | | Case 10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Processing route | Machine sequence | Processing route | Machine sequence | Processing route | Machine sequence | Processing route | Machine sequence | Processing route | Machine sequence | Processing route | Machine sequence |
| 1 | 1201 | M1-M7 | 1202 | M1-M13 | 1201 | M1-M7 | 1201 | M1-M7 | 1201 | M1-M7 | 1201 | M1-M7 |
| 2 | 2102 | M4-M7 | 2104 | M5-M7 | 2103 | M5-M6 | 2104 | M5-M7 | 2102 | M4-M7 | 2102 | M4-M7 |
| 3 | 3202 | M2-M13 | 3202 | M2-M13 | 3201 | M2-M11 | 3202 | M2-M13 | 3202 | M2-M13 | 3201 | M2-M11 |
| 4 | 4102 | M2-M3-M13 | 4202 | M2-M10-M11 | 4102 | M2-M3-M13 | 4101 | M2-M3-M11 | 4101 | M2-M3-M11 | 4102 | M2-M3-M13 |
| 5 | 5201 | M6-M9-M14 | 5101 | M8-M9-M11 | 5101 | M8-M9-M11 | 5101 | M8-M9-M11 | 5202 | M6-M11-M14 | 5101 | M8-M9-M11 |
| 6 | 6101 | M1-M13 | 6101 | M1-M13 | 6204 | M8-M9-M12 | 6101 | M1-M13 | 6101 | M1-M13 | 6101 | M1-M13 |
| 7 | 7204 | M4-M5-M12 | 7201 | M3-M5-M11 | 7204 | M4-M5-M12 | 7204 | M4-M5-M12 | 7102 | M1-M7-M12-M13 | 7203 | M4-M5-M11 |
| 8 | 8202 | M4-M8 | 8202 | M4-M8 | 8201 | M4-M7 | 8201 | M4-M7 | 8201 | M4-M7 | 8203 | M5-M7 |
| 9 | 9202 | M2-M12-M14 | 9103 | M6-M8-M11-M13 | 9202 | M2-M12-M14 | 9102 | M6-M8-M9-M14 | 9202 | M2-M12-M14 | 9108 | M8-M8-M11-M14 |
| 10 | 10103 | M10-M8 | 10103 | M10-M8 | 10101 | M6-M8 | 10103 | M10-M8 | 10101 | M6-M8 | 10101 | M6-M8 |
| 11 | 11204 | M9-M14 | 11202 | M7-M14 | 11203 | M9-M10 | 11204 | M9-M14 | 11202 | M7-M14 | 11202 | M7-M14 |
| 12 | 12202 | M12-M14 | 12102 | M6-M8-M14 | 12102 | M6-M8-M14 | 12102 | M6-M8-M14 | 12102 | M6-M8-M14 | 12102 | M6-M8-M14 |
| 13 | 13204 | M10-M13 | 13202 | M6-M13 | 13204 | M10-M13 | 13101 | M9-M13 | 13103 | M12-M13 | 13204 | M10-M13 |
| 14 | 14104 | M8-M13 | 14103 | M8-M8 | 14104 | M8-M13 | 14103 | M8-M8 | 14103 | M8-M8 | 14101 | M6-M8 |
| 15 | 15203 | M3-M8-M14 | 15102 | M6-M8-M13 | 15102 | M6-M8-M13 | 15102 | M6-M8-M13 | 15102 | M6-M8-M13 | 15205 | M14-M8-M14 |
| 16 | 16204 | M12-M14 | 16203 | M12-M5 | 16102 | M6-M13 | 16104 | M9-M13 | 16102 | M6-M13 | 16102 | M6-M13 |
| 17 | 17105 | M4-M5-M7 | 17108 | M4-M8-M13 | 17105 | M4-M5-M7 | 17101 | M1-M5-M7 | 17105 | M4-M5-M7 | 17102 | M1-M5-M13 |
| 18 | 18202 | M1-M9 | 18104 | M13-M14 | 18201 | M1-M5 | 18104 | M13-M14 | 18201 | M1-M5 | 18204 | M13-M9 |
| 19 | 19102 | M7 | 19102 | M7 | 19102 | M7 | 19101 | M4 | 19101 | M4 | 19101 | M4 |
| 20 | 20103 | M7-M5-M7 | 20103 | M7-M5-M7 | 20101 | M4-M5-M7 | 20103 | M7-M5-M7 | 20204 | M1-M14-M13 | 20103 | M7-M5-M7 |
| 21 | 21204 | M6-M12 | 21104 | M7-M14 | 21101 | M3-M11 | 21104 | M7-M14 | 21202 | M2-M12 | 21201 | M2-M10 |
| 22 | 22101 | M6-M9 | 22201 | M9-M5 | 22202 | M9-M14 | 22204 | M13-M14 | 22102 | M6-M13 | 22102 | M6-M13 |
| 23 | 23101 | M4-M5-M11 | 23202 | M1-M6-M13 | 2320 | M1-M6-M13 | 23202 | M1-M6-M13 | 23103 | M13-M5-M11 | 23204 | M1-M9-M13 |
| 24 | 24201 | M2-M3 | 24202 | M2-M13 | 24202 | M2-M13 | 24103 | M13-M11 | 24104 | M13-M12 | 24201 | M2-M3 |

**Table 5.6: Cell configurations generated by the algorithm for cases in Table 5.4**

| Cases | Cell 1 | Cell2 | Cell 3 |
|---|---|---|---|
| Case 1 | M1, M4, M5, M7, M11 | M6, M9, M12, M14 | M2, M3, M8, M10, M13 |
| Case2 | M1, M4, M5, M9, M11 | M2, M3, M12, M13, M14 | M6, M7, M8, M10 |
| Case 3 | M6, M8, M9, M10, M13 | M1, M4, M5, M7, M12 | M2, M3, M11, M14 |
| Case 4 | M2, M3, M10, M11, M12 | M1, M6, M8, M9, M13 | M4, M5, M7, M14 |
| Case 5 | M6, M8, M9, M11, M14 | M2, M3, M4, M12 | M1, M5, M7, M10, M13 |
| Case 6 | M6, M8, M9, M13, M14 | M2, M3, M10, M11 | M1, M4, M5, M7, M12 |
| Case 7 | M1, M4, M5, M7 | M3, M11, M14 | M2, M6, M8, M12, M13 |
| Case 8 | M1, M7, M10, M13, M14 | M4, M6, M8, M9 | M2, M3, M5, M11, M12 |
| Case 9 | M2, M3, M10, M12, M13 | M1, M4, M5, M7 | M6, M8, M9, M11, M14 |
| Case 10 | M1, M4, M5, M7 | M6, M8, M11, M14 | M2, M3, M9, M10, M13 |
| Case 11 | M1, M5, M7, M10, M13 | M4, M6, M8, M9, M14 | M2, M3, M11 |
| Case 12 | M4, M6, M9, M11 | M1, M5, M7, M8, M14 | M2, M3, M10, M12, M13 |
| Case13 | M2, M8, M9, M14 | M1, M6, M10, M11, M13 | M4, M7, M12 |
| Case14 | M2, M10, | M1, M6, M8, M9, M13 | M4, M5, M7, M11, M14 |

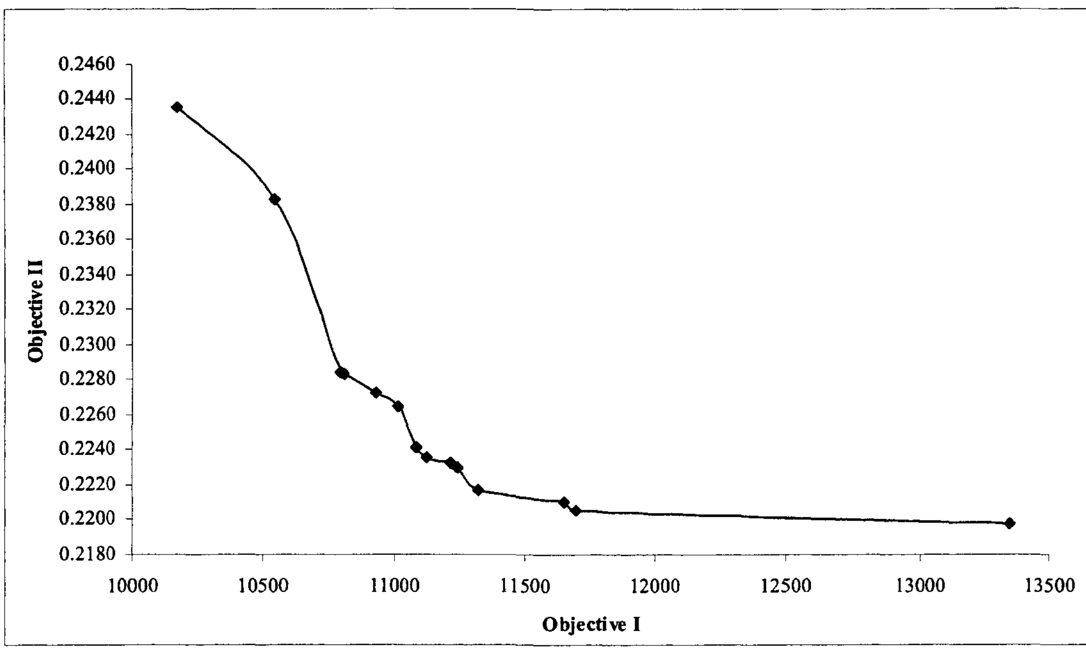


**Figure 5.3: The 'pseudo-efficient' set of solutions for the numerical example**

Table 5.5 displays, for each part type, the processing route and the machine sequence for selected cases in Table 5.4, and Table 5.6 presents the cell configurations corresponding to the cases in Table 5.4. As an example, in case 1, *part type1* is processed under *process plan 2* through *processing route #01*, and the machine sequence for this combination is M1-M7; according to Table 5.6, both the machines are in Cell #1. Similarly, in case 1, *part type 7* is processed using *process plan 2,* through *processing route #04,* and the machine sequence is M4-M5-M12, which involves processing in two cells; M4 and M5 are in *cell 1* and M12 is in *cell 2,* as shown in Table 5.6 .

To further explore the performance, Table 5.7 is presented to compare the solution from the algorithm with the optimal solutions of the 7 machine 12 parts problem (Example 1 in section 4.2.4) as obtained using LINGO 07. Machine data and part processing information for this problem are recorded in Tables 4.1, and 4.2. Table 5.7 also shows the comparison of cell formation by the optimal solution and the Algorithm. It is evident from the solution instances that the algorithm can generate near optimal solutions with the expense of a very small amount of CPU time. It may be mentioned here that the solution quality in terms of GAP% can be further improved by running the algorithm for longer times, or for a higher number of iterations.

**Table 5.7: Performance summary of the heuristic (7 machine -12 part problem)**

| Problem instance | Performance items | Optimal solutions* | Solutions by the algorithm | Gap | CPU time ( minutes) |
|---|---|---|---|---|---|
| **Case 1** Z= *w1***F1*+*w2***F2* *w1=1* *w2=0* | Cost function *Z* | 1771 | 1826 | 3.0% | 3.0 |
| | *F1* | 1771 | 1826 | | |
| | *w1***F1* | 1771 | 1826 | | |
| | *F2* | 0.2621467 | 0.273611 | | |
| | *w2***f2* | 0 | 0 | | - |
| | Cell formation | Cell1: 1,2,3,4<br>Cell 2: 5,6,7 | Cell1: 1,2,3,4<br>Cell 2: 5,6,7 | | |
| **Case 2** Z = *w1***F1*+*w2***F2* *w1= 0* *w2= 1* | Cost function *Z* | 0.2019891 | 0.2077 | 2.74% | 4.0 |
| | *F1* | 3068 | 2902 | | - |
| | *w1***F1* | 0 | 0 | | |
| | *F2* | 0.2019891 | 0.2077 | | - |
| | *w2***F2* | 0.2019891 | 0.2077 | | - |
| | Cell formation | Cell 1: 1,3,4,7<br>Cell 2: 6 | Cell1: 1,2,3,7<br>Cell 2: 4,6 | | |
| **Case 3** Z = *w1***F1*+*w2***F2* *w1= 1* *w2= 25000* | Cost function Z | 7630 | 7789 | 2.0% | 4.0 |
| | *F1* | 2497 | 2377 | | - |
| | *w1***F1* | 2497 | 2377 | | |
| | *F2* | 0.2053338 | 0.216466 | | - |
| | *w2***F2* | 5133 | 5412 | | |
| | Cell formation | Cell 1: 1,3,4,6<br>Cell 2: 2,7 | Cell1:1,2,3,4,6<br>Cell 2: 7 | | |
| **Case 4** Z = *w1***F1*+*w2***F2* *w1= 1* *w2= 15000* | Cost function *Z* | 5428 | 5641 | 3.78% | 4.5 |
| | *F1* | 1959.6 | 2077 | | - |
| | *w1***F1* | 1959.6 | 2077 | | - |
| | *F2* | 0.2312271 | 0.237581 | | - |
| | *w2***F2* | 3468 | 3563 | | |
| | Cell formation | Cell 1: 1,3,4,6<br>Cell 2: 2,7,5 | Cell: 1,3,4,6,7<br>Cell2:2,5 | | |
| **Case 4** Z = *w1***F1*+*w2***F2* *w1=1* *w2=5000* | Cost function *Z* | 3048 | 3177 | 4.0% | 4.5 |
| | *F1* | 1775 | 1884 | | - |
| | *w1***F1* | 1775 | 1884 | | - |
| | *F2* | 0.2545131 | 0.258501 | | - |
| | *w2***F2* | 1273 | 1293 | | |
| | Cell formation | Cell1: 1,2,3,4<br>Cell 2: 5,6,7 | Cell1: 1,2,3,4<br>Cell 2: 5,6,7 | | |

In this chapter we have presented an SA-based solution algorithm for the design of cellular manufacturing systems by considering machine reliability in the multi-objective model based on exponential distribution—which takes into account multiple part types, multiple machines and alternative process plans for each part type. The algorithm solves the CMS design model efficiently within reasonable limits of CPU time to provide a near-optimal solution. The algorithm incorporates an efficient neighboring solution generation procedure, using genetic algorithm-based operators (crossover and mutation), which improves the solution quality and reduces the computational time. The proposed algorithm is easy to implement and as such it can be applied to solve practical size CMS design problems to obtain reasonably good solutions.

## CHAPTER 6
## SENSITIVITY ANALYSIS

### 6.1 Introduction

This chapter consists of three main sections. The key factors/parameters that have a major impact on the expected output of the model are identified in Section 6.2. Also included in this section are the necessary assumptions and preliminary considerations that form the basis of the analysis. Sections 6.3 and 6.4 examine the output of the model for possible changes in the key factors—based on the exponential and Weibull distributions, respectively.

### 6.2 Key Factors

Machine reliabilities undergo changes as a result of aging, routine maintenance, failure repairs, and modifications through time. When formulating CMS design models using machine reliability considerations based on the exponential distribution, the mean time between failure (*MTBF*) and mean time to repair (*MTTR*) are the parameters that impact the performance indices of the cell. Changes in *MTBF* impact both the system costs and the system reliability. Similarly, changes in *MTTR* impact machine availability, and thus the effective machine capacity, system utilization and system costs.

Characteristic life $\theta$ and shape factor $\beta$ are the two key factors that impact the model outputs (system reliability, system cost, and operation allocations) for the CMS design model using machine reliability considerations based on the Weibull distribution. Changes in shape factor $\beta$ may be considered to be very rare during the life time of a machine. We assume that the shape factor $\beta$ for the machine remains unchanged for the planning period under consideration. Since $\theta$ is a function of both *MTBF* and $\beta$, the model outputs are ultimately influenced by *MTBF*.

The processing cost is a function of processing time, operator salary, machine type, set-up time and set-up cost. A decrease in processing cost is very rare in a manufacturing environment and is usually a result of major changes in the system. An increase in processing cost usually occurs by a certain percentage identically applicable to all the machines. It is evident that such increase in processing cost will not affect cell formation or operation allocation, but rather it will increase the system cost by a certain percentage only. Cell formation and operation allocation will be affected if there is a major change in machine type, change of set up arrangement, or major modification. We have not included the impact of processing cost in our sensitivity analysis because we do not assume any such changes for this study.

Considering the above information, sensitivity analysis is performed to examine the impact of changes in *MTBF* and *MTTR* on the cell configuration, operation allocation, system reliability, and total costs in the case of exponential-based and the Weibull-based model.

### 6.2.1 Assumptions and Preliminary Considerations

1. Since failure rate $\lambda$ is the inverse of *MTBF* in the exponential distribution, to simplify the analysis we investigate the effect of changes in the failure rate $\lambda$—instead of *MTBF*—on the model outputs (total cost, system reliability, cell configuration, and operation allocations).
2. Only increases in failure rate (or decreases in *MTBF*) are included in the sensitivity analysis, because a decrease in failure rate (or an increase in *MTBF*) is very rare, and it can happen only when there is a major modification to the machine or the system. No such modification is assumed for the machines or the system under consideration in this study.
    a) To investigate the impact of an increase in failure rate $\lambda$ on the model solutions based on exponential distributions, we increment the failure rates of all the machines by 10% to a maximum of 50%. For brevity, a 10% increase in failure rates would be represented as $1.10\lambda$.
    b) To investigate the impact of a decrease in *MTBF* on model solutions based on the Weibull distributions, we decrement *MTBF* of all the machines by 10% to a maximum of 50%. For example, a 10% decrease in *MTBF* would be represented as 0.90*MTBF*.
3. While an increase in *MTTR* is a usual phenomenon, organizations often pursue efficient maintenance policies (employing efficient crews, going for maintenance contracts, using efficient maintenance aids and equipments, etc.) with the aim of completing repair work as quickly as possible. As such, the impact of both possible increases and decreases in *MTTR* on the model result are investigated. Thus, the *MTTR* values of all the machines are changed by ±10% up to a maximum of ±50%. A 10% increase in the *MTTR* values would be represented as 1.10*MTTR* and a 10% decrease will be represented by 0.90*MTTR*.
4. Example 1 in Chapter 4 has been used for this study. For the exponential distribution-based study, we follow the solution steps identical to section 4.2 ignoring the steps described for performance evaluation. For the Weibull distribution based study, we use

the solution procedure of section 4.3 and the model is solved for long range β (1.16-1.80) values, as defined in section 4.3.2.

5. The existing or initial values of the parameters (*MTBF, MTTR*), given in the input data tables (Table 4.1 and 4.2), are the basis of sensitivity analysis for the models.
6. An increase in failure rate or a decrease in *MTBF* does not impair the capability of the machines. The machines remain capable of performing the operations at the same rate as long as they are in operating condition either at the initial, or at the changed levels of their parameters.
7. Considering that the model in question is a multi-objective one, conducting sensitivity analyses for all the points on the efficient frontier of the model solution is prohibitively time consuming; thus, the analysis carried out in this chapter pertains to the efficient point #2 in Figures 4.1 (efficient frontier for the exponential model solutions) and 4.4 (efficient frontier for the Weibull model solutions), as an example.

### 6.3 Sensitivity of the Exponential Distribution-Based Model Outputs

#### 6.3.1 Effect of Increased Failure Rates

The effects of the increased machine failure rates on machine parameters (availability, utilization etc.), on the objective function values, and on the part processing routes are summarized, respectively, in Tables 6.1, 6.2, and 6.3, and displayed in Figure 6.1. In so far as these three tables are inter-related in terms of the information presented, the following analysis makes simultaneous references to them while discussing the results.

1) Table 6.1 summarizes the effects of increased failure rates on machine availability, available time and machine utilization. As machine failure rates increase, we expect a decrease in both availability and available time, and an increase in machine utilization which, in turn, results in a decreased non-utilized machine time (MNC) cost, as illustrated in Table 6.2. We have not included the cost of a decrease in machine availability combined with an increase in maintenance cost due to increased failure rate. However, a decrease in MNC creates insight into why the effective available times on machines are decreasing—adding to machine down time with the increase in machine failure rate. For example, Table 6.2 reveals that with a 50% increase in failure rates from the existing level, the decrease in MNC is $813-$764 = $ 49. Simultaneously, Table 6.1 shows that machine M1 lost (1344 -1277) = 67 hours of its effective capacity with a

**Table 6.1: Effect of increased failure rates (λ) on machine times (exponential model)**

| Parameter | Performance parameters | Machines | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| | | Total machine capacities (hours) | | | | | | |
| | | **1500** | **1400** | **1200** | **1100** | **1300** | **1000** | **1400** |
| 1.0λ | availability | 0.896 | 0.900 | 0.900 | 0.905 | 0.893 | 0.869 | 0.894 |
| | available time (hrs) | 1344.18 | 1259.49 | 1080.29 | 995.01 | 1161.17 | 869.28 | 1251.15 |
| | %utilization | 0.910 | 0.000 | 0.961 | 0.861 | 0.000 | 0.993 | 0.825 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1036.85 | 856.40 | 0.00 | 862.80 | 1032.75 |
| | **non-utilized time** (hrs) | **120.68** | **1259.49** | **42.44** | **138.61** | **1161.17** | **6.48** | **218.40** |
| 1.1λ | availability | 0.887 | 0.891 | 0.891 | 0.896 | 0.884 | 0.858 | 0.884 |
| | available time (hrs) | 1330.36 | 1246.98 | 1069.62 | 985.61 | 1148.91 | 858.06 | 1236.99 |
| | %utilization | 0.920 | 0.000 | 0.999 | 0.869 | 0.000 | 0.880 | 0.917 |
| | utilized time(hrs) | 1223.50 | 0.00 | 1068.75 | 856.40 | 0.00 | 754.80 | 1135.65 |
| | **non-utilized time** (hrs) | **106.86** | **1246.98** | **0.87** | **129.21** | **1148.91** | **103.26** | **102.34** |
| 1.2λ | availability | 0.878 | 0.882 | 0.883 | 0.888 | 0.875 | 0.847 | 0.875 |
| | available time (hrs) | 1316.82 | 1234.71 | 1059.16 | 976.38 | 1136.89 | 846.11 | 1225.10 |
| | %utilization | 0.966 | 0.000 | 0.992 | 0.867 | 0.000 | 0.891 | 0.956 |
| | utilized time (hrs) | 1272.30 | 0.00 | 1050.65 | 846.70 | 0.00 | 754.80 | 1171.35 |
| | **non-utilized time** (hrs) | **44.52** | **1234.71** | **8.51** | **129.68** | **1136.89** | **92.31** | **53.75** |
| 1.3λ | availability | 0.869 | 0.873 | 0.874 | 0.879 | 0.865 | 0.836 | 0.866 |
| | available time (hrs) | 1303.55 | 1222.68 | 1048.90 | 966.32 | 1125.13 | 836.47 | 1212.47 |
| | %utilization | 0.939 | 0.000 | 0.985 | 0.885 | 0.000 | 0.902 | 0.937 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1033.15 | 856.40 | 0.00 | 754.80 | 1135.65 |
| | **non-utilized time** (hrs) | **80.05** | **1222.68** | **15.75** | **110.92** | **1125.13** | **81.67** | **76.82** |
| 1.4λ | availability | 0.860 | 0.865 | 0.866 | 0.871 | 0.857 | 0.826 | 0.857 |
| | available time (hrs) | 1290.55 | 1210.88 | 1038.83 | 958.42 | 1113.61 | 826.08 | 1200.11 |
| | %utilization | 0.948 | 0.000 | 0.995 | 0.894 | 0.000 | 0.914 | 0.946 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1033.15 | 856.40 | 0.00 | 754.80 | 1135.65 |
| | **non-utilized time** (hrs) | **66.05** | **1210.88** | **5.68** | **102.02** | **1113.61** | **71.28** | **64.46** |
| 1.5λ | availability | 0.852 | 0.857 | 0.857 | 0.863 | 0.848 | 0.816 | 0.849 |
| | available time (hrs) | 1276.81 | 1199.31 | 1028.96 | 949.69 | 1102.32 | 815.94 | 1186.99 |
| | %utilization | 0.958 | 0.000 | 0.963 | 0.902 | 0.000 | 0.925 | 0.986 |
| | utilized time (hrs) | 1223.50 | 0.00 | 990.65 | 856.40 | 0.00 | 754.80 | 1171.35 |
| | **non-utilized time** (hrs) | **54.31** | **1199.31** | **38.31** | **93.29** | **1102.32** | **61.14** | **16.64** |

**Table 6.2: Effect of increased failure rates (λ) on the model solution (exponential model)**

| Parameter | Objective I | Components of objective I | | | Objective II |
|---|---|---|---|---|---|
| | | VCM | MHC | MNC | |
| 1.0λ | 2781 | 1768 | 200 | 813 | 0.2019891 |
| 1.1λ | 2712 | 1713 | 200 | 799 | 0.2271107 |
| 1.2λ | 2727 | 1701 | 250 | 776 | 0.2477571 |
| 1.3λ | 2719 | 1735 | 200 | 784 | 0.2684036 |
| 1.4λ | 2705 | 1735 | 200 | 770 | 0.2890500 |
| 1.5λ | 2733 | 1769 | 200 | 764 | 0.3096964 |

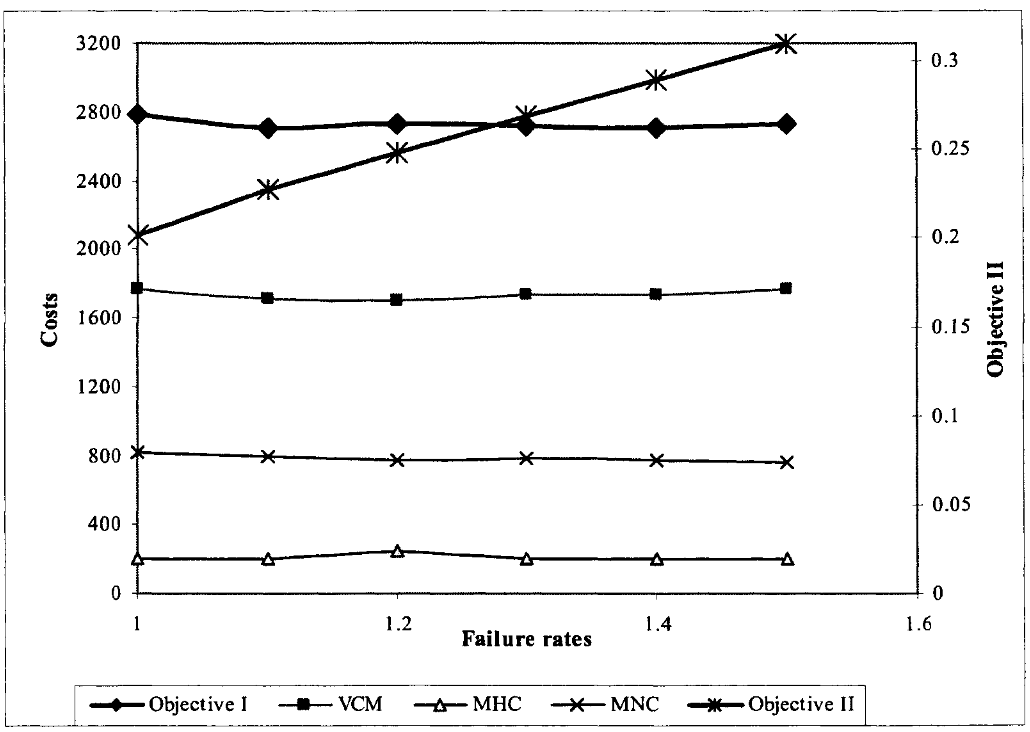


**Figure 6.1: Effect of increased failure rate (λ) on the model solution (exponential model)**

**Table 6.3: Effect of increased failure rate (λ) on process plan and part routes assignment (exponential model)**

| Part type | Plan | 1.0λ | 1.1λ | | 1.2λ | | 1.3λ | | 1.4λ | | 1.5λ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Processing route | Plan | Process route | Plan | Process route | Plan | Process route | Plan | Process route | Plan | Process route |
| 1 | 2 | M1-M4 | | | | | | | | | | |
| 2 | 2 | M3-M7-M3 | | | | | | | | | | |
| 3 | 2 | M4-M6-M4 | | | 2 | M7-M6-M4 | 2 | M4-M6-M4 | 2 | M4-M6-M4 | 2 | M4-M6-M4 |
| 4 | 2 | M1-M6 | | | | | | | | | | |
| 5 | 2 | M7-M6-M4 | | | | | | | | | | |
| 6 | 2 | M7-M1-M3 | **2 | M7-M1-M7 | 2 | M7-M1-M3 | 2 | M7-M1-M7 | 2 | M7-M1-M7 | 2 | M7-M1-M3 |
| 7 | 2 | M6-M1 | | | | | | | | | | |
| 8 | 2 | M3-M1-M3 | | | | | | | | | | |
| 9 | 1 | M3-M1-M6 | 1 | M3-M1-M3 | 2 | M3-M4-M1 | 1 | M7-M1-M3 | 1 | M7-M1-M3 | 1 | M7-M1-M3 |
| 10 | 2 | M6-M3-M1 | | | | | | | | | | |
| 11 | 1 | M1-M7 | | | | | 1 | M1-M3 | 1 | M1-M3 | 1 | M1-M7 |
| 12 | 2 | M3-M6 | | | | | | | | | | |
| | | | | | | | | | | | | |
| Cell 1 | M1,M3, M6, M7 | | | | | | | | | | | |
| Cell 2 | M4 | | | | | | | | | | | |

Notes:

1. (shaded) Indicates no change in part processing routes/cell configuration relative to the current state 1.0 λ

2. ** 2 M7-M1-M7 indicates a change in processing route for part type 6 from M7 – M1-M3 to M7-M1-M7, but no change in process plan

50% increase in failure rate. A similar amount of time is lost by other machines also, and these values are added to the down time. These insights into decreases in MNC help focus attention on the areas that contribute to the abnormal reliability performance of the CMS.

All other changes in the costs—recorded in Table 6.2—are due to adjustments in processing routes. When the available time on a machine decreases to a point where the current operation assignments are no longer feasible, the model attempts to adjust processing routes for the part types in order to optimize the costs (objective function I) and achieve the desired value for objective function II, as shown in Table 6.3.

2) Considering machine M1 as an example, as the failure rates increase by 10%, Table 6.1 indicates that the machine availability declines from the current level of 0.896 to 0.887. The available time decreases from 1344.18 units to 1330.36 units, and machine utilization increases from 0.910 to 0.920. A similar trend can be observed for machine M4. At the current level,

machine M6 is almost fully loaded. Accordingly, a 10% increase in machine failure rates prompts a shift in the model solution (documented in Table 6.3). The processing route of part type 9 changes from M3-M1-M6, to M3-M1-M3 while, that of part type 6 converts from M7-M1-M3, to M7-M1-M7. While the model decremented the load on M6 by shifting its work to machine M3 through the change of processing route for part type 9, it also readjusted the load of M3 by changing the processing route of part type 6. As a result of these changes, objective function I decreases by \$(2781-2712) = \$ 69 (or, -2.48%), and objective function II is increased by (0.2271107 - 0.2019891) = 0.0251216, a 12.4% increase.

Now, with a 10% increase in machine failure rates, machine M3 is almost fully loaded. Consequently, we observe the next shift in the model solution at 1.2λ, as seen in Table 6.3, when the processing routes of part types 3, 6, and 9 are changed. With an increase in failure rate from 10% to 20%, objective function I changed from \$2712 to \$2727, and objective function II increased from 0.2271107 to 0.2477571, a 9% increase, as can be seen in Table 6.2 and Figure 6.1. This trend continues for nearly every increase in the failure rates.

As discussed, during all the changes the model attempted to adjust machine times and change processing routes in order to optimize cost and achieve the desired value for objective function II as shown in Table 6.2. For example, the model solution has not selected machines M2 and M5 (see Table 6.3) during all the changes, and therefore their times have not been utilized. The reason machines M2 and M5 have not been selected lies in their high failure rates (revealed by the *MTBF* values in Table 4.2). Instead, the model attempted to complete the part assignments with a combination of more reliable machines to achieve comparatively higher system reliability.

As expected, Figure 6.1 illustrates how the value of objective function II steadily increases as the machine failure rates increase. As failure rates increase, machine reliabilities decrease, and it becomes more and more difficult to maintain a desired level of system reliability. From the above discussion, it may be concluded that the increased machine failure rates have a fairly insignificant effect on the total cost (objective function I), and a significant impact on system reliability. Within the range of the failure rate values under consideration, objective function I has decreased by \$(2781-2733) =\$ 48 (or 1.7%), which is mainly due the decrease in machine non-utilization cost (MNC). As indicated before, a decrease in MNC is an indirect indication of an increase in down time. However, the model could keep the variable cost of machining almost the same by changing the processing routes. For a 50% increase in failure rates, the increase in the objective function II value is about 53.3%—indicating that although machine performances are degrading as time passes, the model is fairly robust with respect to the operation assignments at a reasonable cost, as well as the cell configuration.

### 6.3.2 Effect of Changes in *MTTR*

The effect of an increase and/or decrease in the *MTTR* values of the machines on machine parameters, part processing routes and the objective function values are summarized, respectively, in Tables 6.4, 6.5, 6.6a and 6.6b (and displayed in Figure 6.2). Again, these four tables are inter-related in terms of the information presented, and require simultaneous references when discussing the results.

1) Table 6.4 summarizes the effects of changes in *MTTR* on machine availability, available time and machine utilization. As *MTTR* decreases, we expect an increase in machine availability and available time and, generally, a decrease in machine utilization that leads to an increase in MNC—the cost of non-utilized machine time—as shown in Table 6.5. When the available time on a machine increases, the model attempts to make changes to processing routes for the part types in order to generate a solution with a lower cost (objective function I) and achieve the desired value for objective function II (illustrated in Table 6.6b).

2) Table 6.4 reveals that machine availability for all the machines increased due to a 10% decrease in *MTTR* from the existing level. For example, the availability of machine M7 increases from the current levels (1.0*MTTR*) of 0.894 to 0.903, and available time increases from 1251 to 1264 units. With a 10% decrease in *MTTR* we observe a shift in the model solutions indicated in Table 6.6b; the processing route for part type 6 changes from M7-M1-M3 to M7-M1-M7, and that of part type 11 changes from M1-M7 to M1-M3. With these changes, the utilization of M7 increases from 0.825 to 0.924—suggesting that the model could generate lower cost solutions and reduce the objective function I value by ($ 2781 - $ 2739) = $ 42 (≈ 1.5% improvement) while maintaining the system reliability (objective function II value) at the existing level (recorded in Table 6.5 and displayed in Figure 6.2). This improvement was not previously possible due to the limitation of available time on M7, as shown in Table 6.4.

3) A further decrease in *MTTR* (from 0.9*MTTR* to 0.5*MTTR*) boosts the effective machine capacities (as is displayed in Table 6.4); however, these additional capacities (extra time) on a machine creates a slight deterioration in the cost picture because it is added to the machine non-utilization time and—in turn—cost, (shown in Table 6.5). For instance, the additional capacity added as a result of a 50% decrease in *MTTR* increases the MNC or machine non-utilization cost by ($856-$776) = $80 (shown in Table 6.5).

**Table 6.4: Effect of increased and decreased *MTTR* on machine times (exponential model)**

| Parameter | Performance parameters | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
|---|---|---|---|---|---|---|---|---|
| | | Total machine capacities (hours) | | | | | | |
| | | 1500 | 1400 | 1200 | 1100 | 1300 | 1000 | 1400 |
| 0.5*MTTR* | availability | 0.945 | 0.947 | 0.947 | 0.950 | 0.943 | 0.929 | 0.944 |
| | available time (hrs) | 1416.24 | 1325.78 | 1136.55 | 1044.54 | 1226.40 | 929.14 | 1321.08 |
| | %utilization | 0.950 | 0.000 | 0.910 | 0.991 | 0.000 | 0.929 | 0.662 |
| | utilized time (hrs) | 1346.70 | 0.00 | 1034.75 | 1034.80 | 0.00 | 862.80 | 874.75 |
| | **non-utilized time** (hrs) | **70.54** | **1325.78** | **101.80** | **9.74** | **1226.40** | **66.34** | **446.33** |
| 0.6*MTTR* | availability | 0.935 | 0.937 | 0.937 | 0.940 | 0.933 | 0.916 | 0.933 |
| | available time (hrs) | 1401.91 | 1311.93 | 1124.76 | 1034.19 | 1212.74 | 916.37 | 1306.42 |
| | %utilization | 0.971 | 0.000 | 0.900 | 0.836 | 0.000 | 0.954 | 0.834 |
| | utilized time (hrs) | 1361.23 | 0.00 | 1012.61 | 864.86 | 0.00 | 874.59 | 1089.89 |
| | **non-utilized time** (hrs) | **40.69** | **1311.93** | **112.15** | **169.33** | **1212.74** | **41.78** | **216.53** |
| 0.7*MTTR* | availability | 0.925 | 0.927 | 0.928 | 0.931 | 0.923 | 0.904 | 0.923 |
| | available time (hrs) | 1386.95 | 1298.39 | 1113.25 | 1024.07 | 1199.39 | 904.02 | 1292.11 |
| | %utilization | 0.897 | 0.000 | 0.970 | 0.836 | 0.000 | 0.954 | 0.904 |
| | utilized time (hrs) | 1243.50 | 0.00 | 1080.35 | 856.40 | 0.00 | 862.80 | 1168.50 |
| | **non-utilized time** (hrs) | **143.45** | **1298.39** | **32.90** | **166.67** | **1199.39** | **41.22** | **123.61** |
| 0.8*MTTR* | availability | 0.915 | 0.918 | 0.918 | 0.922 | 0.913 | 0.892 | 0.913 |
| | available time (hrs) | 1372.35 | 1285.14 | 1102.01 | 1014.17 | 1186.36 | 892.07 | 1278.14 |
| | %utilization | 0.906 | 0.000 | 0.980 | 0.844 | 0.000 | 0.967 | 0.914 |
| | utilized time (hrs) | 1243.50 | 0.00 | 1080.35 | 856.40 | 0.00 | 862.80 | 1168.50 |
| | **non-utilized time** (hrs) | **128.85** | **1285.14** | **21.66** | **156.77** | **1186.36** | **29.27** | **109.64** |
| 0.9*MTTR* | availability | 0.905 | 0.909 | 0.909 | 0.913 | 0.903 | 0.880 | 0.903 |
| | available time (hrs) | 1358.10 | 1272.18 | 1091.02 | 1004.49 | 1173.62 | 880.49 | 1264.49 |
| | %utilization | 0.916 | 0.000 | 0.990 | 0.853 | 0.000 | 0.980 | 0.924 |
| | utilized time (hrs) | 1243.50 | 0.00 | 1080.35 | 856.40 | 0.00 | 862.80 | 1168.50 |
| | **non-utilized time** (hrs) | **114.60** | **1272.18** | **10.67** | **148.09** | **1173.62** | **16.69** | **95.99** |
| 1.0*MTTR* | availability | 0.896 | 0.900 | 0.900 | 0.905 | 0.893 | 0.869 | 0.894 |
| | available time (hrs) | 1344.18 | 1259.49 | 1080.29 | 995.01 | 1161.17 | 869.28 | 1251.15 |
| | %utilization | 0.910 | 0.000 | 0.961 | 0.861 | 0.000 | 0.993 | 0.825 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1036.85 | 856.40 | 0.00 | 862.80 | 1032.75 |
| | **non-utilized time** (hrs) | **120.68** | **1259.49** | **42.44** | **138.61** | **1161.17** | **6.48** | **218.40** |

**Table 6.4 cont'd**

| Parameter | Performance parameters | Machines | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| | | Total machine capacities | | | | | | |
| | | 1500 | 1400 | 1200 | 1100 | 1300 | 1000 | 1400 |
| 1.0*MTTR* | availability | 0.896 | 0.900 | 0.900 | 0.905 | 0.893 | 0.869 | 0.894 |
| | available time (hrs) | 1344.18 | 1259.49 | 1080.29 | 995.01 | 1161.17 | 869.28 | 1251.15 |
| | %utilization | 0.910 | 0.000 | 0.961 | 0.861 | 0.000 | 0.993 | 0.825 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1036.85 | 856.40 | 0.00 | 862.80 | 1032.75 |
| | **non-utilized time** (hrs) | **120.68** | **1259.49** | **42.44** | **138.61** | **1161.17** | **6.48** | **218.40** |
| 1.1*MTTR* | availability | 0.887 | 0.891 | 0.891 | 0.896 | 0.884 | 0.858 | 0.884 |
| | available time (hrs) | 1330.58 | 1246.08 | 1069.79 | 985.74 | 1149.01 | 858.41 | 1238.11 |
| | %utilization | 0.920 | 0.000 | 0.999 | 0.869 | 0.000 | 0.879 | 0.917 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1068.75 | 856.40 | 0.00 | 754.80 | 1135.65 |
| | **non-utilized time** (hrs) | **106.08** | **1246.08** | **1.04** | **129.34** | **1149.01** | **103.61** | **102.46** |
| 1.2*MTTR* | availability | 0.878 | 0.882 | 0.883 | 0.888 | 0.875 | 0.848 | 0.875 |
| | available time (hrs) | 1316.29 | 1234.92 | 1059.53 | 976.66 | 1136.11 | 846.87 | 1225.36 |
| | %utilization | 0.966 | 0.000 | 0.992 | 0.867 | 0.000 | 0.890 | 0.956 |
| | utilized time (hrs) | 1272.30 | 0.00 | 1050.65 | 846.70 | 0.00 | 754.80 | 1171.35 |
| | **non-utilized time** (hrs) | **44.99** | **1234.92** | **8.88** | **129.96** | **1136.11** | **93.07** | **54.01** |
| 1.3*MTTR* | availability | 0.870 | 0.874 | 0.875 | 0.880 | 0.866 | 0.838 | 0.866 |
| | available time (hrs) | 1304.31 | 1223.01 | 1049.50 | 966.77 | 1125.48 | 836.64 | 1212.89 |
| | %utilization | 0.938 | 0.000 | 0.984 | 0.885 | 0.000 | 0.901 | 0.936 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1033.15 | 856.40 | 0.00 | 754.80 | 1135.65 |
| | **non-utilized time** (hrs) | **80.81** | **1223.01** | **16.35** | **111.37** | **1125.48** | **82.84** | **76.24** |
| 1.4*MTTR* | availability | 0.861 | 0.865 | 0.866 | 0.872 | 0.857 | 0.828 | 0.858 |
| | available time (hrs) | 1291.62 | 1211.35 | 1039.68 | 959.06 | 1114.10 | 826.72 | 1200.70 |
| | %utilization | 0.947 | 0.000 | 0.994 | 0.893 | 0.000 | 0.912 | 0.946 |
| | utilized time (hrs) | 1223.50 | 0.00 | 1033.15 | 856.40 | 0.00 | 754.80 | 1135.65 |
| | **non-utilized time** (hrs) | **68.12** | **1211.35** | **6.53** | **102.66** | **1114.10** | **72.92** | **65.05** |
| 1.5*MTTR* | availability | 0.853 | 0.857 | 0.858 | 0.864 | 0.848 | 0.818 | 0.849 |
| | available time (hrs) | 1279.21 | 1199.93 | 1030.07 | 950.53 | 1102.96 | 818.08 | 1188.77 |
| | %utilization | 0.956 | 0.000 | 0.962 | 0.901 | 0.000 | 0.923 | 0.985 |
| | utilized time (hrs) | 1223.50 | 0.00 | 990.65 | 856.40 | 0.00 | 754.80 | 1171.35 |
| | **non-utilized time** (hrs) | **55.71** | **1199.93** | **39.42** | **94.13** | **1102.96** | **63.28** | **16.42** |

**Table 6.5: Effect of increased and decreased *MTTR* on the model solution (exponential model)**

| Parameter | Objective I | Components of objective I | | | Objective II |
|---|---|---|---|---|---|
| | | VCM | MHC | MNC | |
| 0.5*MTTR* | 2795 | 1839 | 100 | 856 | 0.199227 |
| 0.6*MTTR* | 2787 | 1763 | 200 | 823 | 0.201989 |
| 0.7*MTTR* | 2771 | 1763 | 200 | 808 | 0.201989 |
| 0.8*MTTR* | 2755 | 1763 | 200 | 792 | 0.201989 |
| 0.9*MTTR* | 2739 | 1763 | 200 | 776 | 0.201989 |
| 1.0*MTTR* | 2781 | 1768 | 200 | 813 | 0.201989 |
| 1.1*MTTR* | 2713 | 1713 | 200 | 799 | 0.206464 |
| 1.2*MTTR* | 2728 | 1701 | 250 | 777 | 0.206464 |
| 1.3*MTTR* | 2721 | 1736 | 200 | 785 | 0.206464 |
| 1.4*MTTR* | 2706 | 1736 | 200 | 770 | 0.206464 |
| 1.5*MTTR* | 2736 | 1770 | 200 | 766 | 0.206464 |

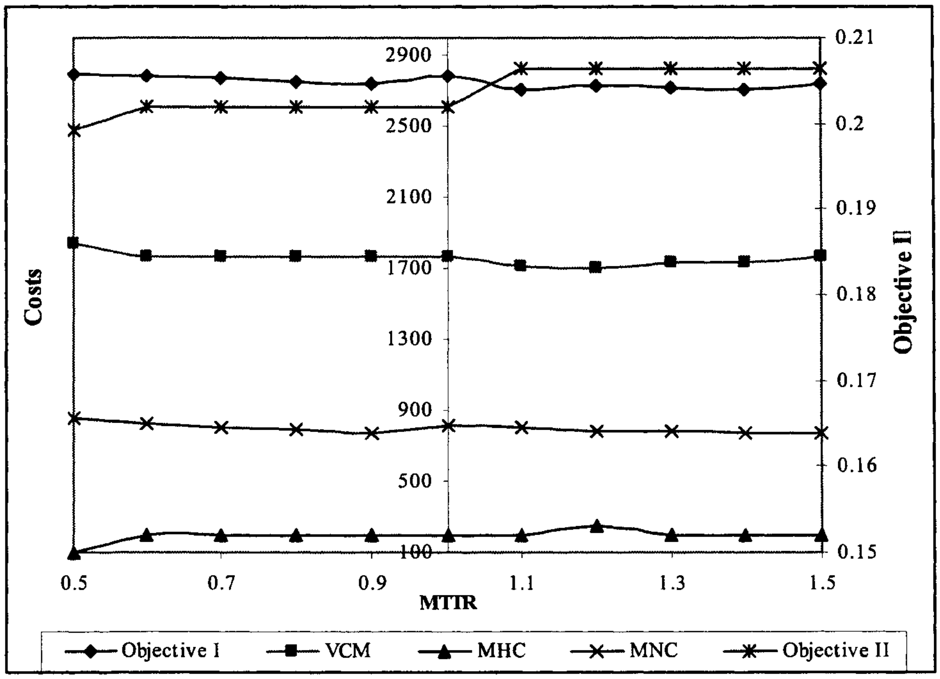


**Figure 6.2: Effect of increased and decreased *MTTR* on the model solutions (exponential model)**

**Table 6.6a: Effect of increased *MTTR* on process plan and part routes assignment (exponential model)**

| Part type | 1.0*MTTR* | | 1.1*MTTR* | | 1.2*MTTR* | | 1.3*MTTR* | | 1.4*MTTR* | | 1.5*MTTR* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | plan | process route | plan | process route | plan | process route | plan | process route | plan | process route | plan | process route |
| 1 | 2 | M1-M4 | | | | | | | | | | |
| 2 | 2 | M3-M7-M3 | | | | | | | | | | |
| 3 | 2 | M4-M6-M4 | | | 2 | M7-M6-M4 | 2 | M4-M6-M4 | 2 | M4-M6-M4 | 2 | M4-M6-M4 |
| 4 | 2 | M1-M6 | | | | | | | | | | |
| 5 | 2 | M7-M6-M4 | | | | | | | | | | |
| 6 | 2 | M7-M1-M3 | **2 | M7-M1-M7 | 2 | M7-M1-M3 | 2 | M7-M1-M7 | 2 | M7-M1-M7 | 2 | M7-M1-M3 |
| 7 | 2 | M6-M1 | | | | | | | | | | |
| 8 | 2 | M3-M1-M3 | | | | | | | | | | |
| 9 | 1 | M3-M1-M6 | 1 | M3-M1-M3 | 2 | M3-M4-M1 | 1 | M7-M1-M3 | 1 | M7-M1-M3 | 1 | M7-M1-M3 |
| 10 | 2 | M6-M3-M1 | | | | | | | | | | |
| 11 | 1 | M1-M7 | | | | | 1 | M1-M3 | 1 | M1-M3 | 1 | M1-M7 |
| 12 | 2 | M3-M6 | | | | | | | | | | |
| | | | | | | | | | | | | |
| Cell 1 | M1,M3,M6,M7 | | | | | | | | | | | |
| Cell 2 | M4 | | | | | | | | | | | |

Notes:

1. Indicates no change in part processing routes/cell configuration relative to the current state 1.0*MTTR*

2. ** 2 M7-M1-M7 indicates a change in processing route for part type 6 from M7 – M1-M3 to M7-M1-M7, but no change in process plan

**Table 6.6b: Effect of decreased *MTTR* on process plan and part routes assignment (exponential model)**

| Part type | 1.0*MTTR* | | 0.9*MTTR* | | 0.8*MTTR* | | 0.7*MTTR* | | 0.6*MTTR* | | 0.5*MTTR* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | plan | process route | plan | process route | plan | process route | plan | process route | plan | process route | plan | process route |
| 1 | 2 | M1-M4 | | | | | | | | | | |
| 2 | 2 | M3-M7-M3 | | | | | | | | | 2 | M3-M7-M7 |
| 3 | 2 | M4-M6-M4 | | | | | | | | | | |
| 4 | 2 | M1-M6 | | | | | | | | | | |
| 5 | 2 | M7-M6-M4 | | | | | | | | | | |
| 6 | 2 | M7-M1-M3 | ** 1 | M7-M1-M7 | 1 | M7-M1-M7 | 1 | M7-M1-M7 | 1 | M7-M1-M7 | 2 | M3-M1-M3 |
| 7 | 2 | M6-M1 | | | | | | | | | | |
| 8 | 2 | M3-M1-M3 | | | | | | | | | | |
| 9 | 1 | M3-M1-M6 | | | | | | | | | 2 | M1-M4-M6 |
| 10 | 2 | M6-M3-M1 | | | | | | | | | | |
| 11 | 1 | M1-M7 | 1 | M1-M3 | 1 | M1-M3 | 1 | M1-M3 | 1 | M1-M3 | 1 | M1-M3 |
| 12 | 2 | M3-M6 | | | | | | | | | | |
| | | | | | | | | | | | | |
| Cell 1 | M1,M3,M6,M7 | | | | | | | | | | M1,M4,M3,M6 | |
| Cell 2 | M4 | | | | | | | | | | M7 | |

Notes:

1. Indicates no change in part processing routes/cell configuration relative to the current state 1.0*MTTR*

2. ** 1 M7-M1-M7 indicates a change in processing route for part type 6 from M7 –M1- M3 to M7-M1-M7, and change in process plan from 2 to 1

Table 6.6b, however, observes that the model solutions shift with a 50% decrease in *MTTR* as the processing route of part types 2, 6, 9, and 11 are changed. As a result of this shift, Table 6.5 and Figure 6.2 reveal that the model achieved a higher reliability when the value of objective function II decreased from 0.201989 to 0.199227 (approximately 1.4% improvement). This improvement in system reliability was not possible previously due to the limitation of available time on machines M1 and M4, as can be seen in Table 6.4. Table 6.6b also shows that with a 50% decrease in *MTTR,* the model solution displays a change in cell configuration from {cell 1: M1, M3, M6, M7 and cell 2: M4} to {cell 1: M1, M3, M6, M4 and cell 2: M7}.

4) Conversely, Table 6.4 shows that as the *MTTR* increases by 10%, the availability for machine M7 decreases from the current level of 0.894 to 0.884, while the available time decreases from 1251 units to 1238 units and machine utilization increases from 0.825 to 0.917. This is the general trend for each increase in *MTTR*. At the current level (1.0*MTTR*), Table 6.4

displays machine M6 as almost 100% loaded. To accommodate the process assignment of M6 at a 10% increase in *MTTR*, we observe a shift in the model solutions (illustrated in Table 6.6a). The processing route of part type 6 changes from M7-M1-M3 to M7-M1-M7, while the processing route of part type 9 changes from M3-M1-M6 to M3-M1-M3. Objective function II increased from 0.201989 to 0.206464, a 2.2% deterioration in system reliability—recorded in Table 6.5 and displayed in Figure 6.2—as a result of this shift. The model solution could, however, reduce cost (objective function I) by $(2781-2713) = $68.

5) Table 6.4 indicates that with a 10% increase in *MTTR* (1.10*MTTR* levels), machine M3 is practically 100% loaded. This prompts the second shift in model solutions at 1.20*MTTR* (observed in Table 6.6a) where the processing route of part type 3 changes from M4-M6-M4 to M7-M6-M4; part type 6 reverts back to M7-M1-M3; and part type 9 changes process plans from #1 to #2 while the processing route switches from M3-M1-M3 to M3-M4-M1. At this point, the value of objective function I increased from $2713 to $2728 due to a change in machine processing assignments, and the objective function II value (0.206464) remained unchanged as shown in Table 6.5.

6) A study of Table 6.6a reveals a shift in model solutions and changes in selected processing routes at every increase in *MTTR*. It is also evident from Table 6.4 that as *MTTR* increases, machine availability decreases and—as discussed—the model solutions resort to changing the processing routes of part types to accommodate the processing times and achieve the desired reliability level. Table 6.5 and Figure 6.2 show that although *MTTR* increased from 10% to 50% and machine availability decreased at every increase in *MTTR*, the model solutions could maintain the objective function II value at the same level through the change of processing routes.

7) Table 6.5 and Figure 6.2 indicate that as the *MTTR* increases, the objective function I value tends to decrease. The total decrease in the objective function I value for an increase from current level to 50% is $(2781-2736) = $ 45. The variation between the highest (at 1.0MTTR) and the lowest (at 1.4*MTTR*) is $75 (approximately -2.7%). As previously explained, the decrease in the objective function I value is mainly due to a decrease in MNC, which contributed $(813-766) = $ 46 for the 50% increase in *MTTR* from the current level. As discussed, a decrease in MNC is the consequence of a decrease in availability. This decrease in machine availability may be considered equivalent to an increase in machine down-time and repair and maintenance costs—usually a significant amount although they are not accounted for in this study.

From the above discussion, it may be concluded that the model solutions in terms of processing route assignments are significantly sensitive to changes in *MTTR* values. We may also conclude that although there are changes in part processing routes for every increase or decrease in *MTTR,* the model solution (in terms of system costs and system reliability) is not overly sensitive to changes in the *MTTR* values, as displayed in Tables 6.5, 6.6a, 6.6b and Figure 6.2. When the *MTTR* values for machines change in the range of -50% to 50%, we can see the corresponding changes in the objective function I value from –0.5% to +2.7 %.(-$14 to +$75) and changes in the objective function II value from +1.4% to 2.2%.

### 6.4 Sensitivity of the Weibull Distribution-Based Model Outputs

#### 6.4.1 Effect of Decreases in *MTBF*

The effects of a decrease in *MTBF* on machine parameters, objective function values and part processing routes are summarized in Tables 6.7, 6.8 and 6.9 and displayed in Figure 6.3. The fact that these results are interrelated allows us to take a similar approach to Section 6.3, and we will be making simultaneous references to them during our discussion on sensitivity analysis.

1) Table 6.7 summarizes the effect of a decrease in *MTBF* on machine availability and machine utilization. As *MTBF* decreases, machine availability and the available time on machines decrease. Due to this decrease in availability, un-utilized time on machines decreases—implying a reduction in MNC (observed in Table 6.8). This effect of a decrease in *MTBF* is similar to that of an increase in failure rates, as explained in Section 6.3.1 for exponential distribution-based model solutions. With the decrease in *MTBF*, the available time on a machine reaches a point where it can no longer accommodate the process assignment it has been performing. The model attempts to select a different part processing route to accommodate the processing times and achieve the desired reliability (objective function II value) as shown in Tables 6.8 and 6.9.

2) Investigating the effect of a decrease in *MTBF* on machine times and considering machine M7 as an example, we can observe that with a 10% decrease in *MTBF*, availability decreases from the current 0.893 to 0.882 while available time decreases from 1250 to 1235 hours (shown in Table 6.7). Although M7 utilizes the same amount of time (836 hours) to complete its current assignment, as well as at the 10% decrease in *MTBF*, utilization increases from 0.670, to 0.678 due to the decrease in available time for the machine. This trend is generally followed for all the machines at all decrements of *MTBF* until the model attempts to shift the solutions and change the processing routes for part types to accommodate processing times, as discussed above. Due to the shift in solutions, there are changes in the machine utilization and utilized time. For example, M4 is practically 100% loaded at the current level. Thus, with a 10% decrease in *MTBF* we

**Table 6.7: Effect of decreased *MTBF* on machine times (Weibull model)**

| Parameter | Performance indicators | Machines | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| | | Total machine capacities ( hours) | | | | | | |
| | | 1500 | 1400 | 1200 | 1100 | 1300 | 1000 | 1400 |
| *MTBF* | availability | 0.895 | 0.899 | 0.899 | 0.903 | 0.892 | 0.866 | 0.893 |
| | available time ( hrs) | 1342.11 | 1258.59 | 1078.65 | 993.79 | 1160.22 | 866.03 | 1250.00 |
| | %utilization | 0.946 | 0.934 | 0 | 0.999 | 0 | 0.939 | 0.670 |
| | utilized time (hrs) | 1269.1 | 1175.1 | 0 | 993.7 | 0 | 813.3 | 836.5 |
| | **non-utilized time (hrs)** | **73.01** | **83.49** | **1078.65** | **0.09** | **1160.22** | **52.73** | **412.50** |
| 0.9*MTBF* | availability | 0.884 | 0.889 | 0.889 | 0.894 | 0.882 | 0.853 | 0.882 |
| | available time ( hrs) | 1326.59 | 1244.62 | 1066.67 | 983.24 | 1146.52 | 853.33 | 1235.29 |
| | %utilization | 0.964 | 0.930 | 0.000 | 0.928 | 0.000 | 0.996 | 0.678 |
| | utilized time ( hrs) | 1279.2 | 1158 | 0 | 912.00 | 0 | 849.79 | 836.5 |
| | **non-utilized time** ( hrs) | **46.39** | **86.62** | **1066.67** | **71.24** | **1146.52** | **3.53** | **396.79** |
| 0.8*MTBF* | availability | 0.872 | 0.877 | 0.877 | 0.882 | 0.869 | 0.838 | 0.870 |
| | available time ( hrs) | 1306.69 | 1226.59 | 1052.05 | 970.37 | 1129.84 | 836.96 | 1216.39 |
| | %utilization | 0.993 | 0.957 | 0.000 | 0.967 | 0.000 | 0.933 | 0.688 |
| | utilized time ( hrs) | 1298 | 1175.1 | 0 | 938.3 | 0 | 781.8 | 836.4999 |
| | **non-utilized time** | **9.69** | **52.49** | **1052.05** | **32.07** | **1129.84** | **56.16** | **379.89** |
| 0.7*MTBF* | availability | 0.856 | 0.862 | 0.862 | 0.868 | 0.853 | 0.819 | 0.854 |
| | available time ( hrs) | 1284.17 | 1206.36 | 1033.85 | 954.30 | 1109.10 | 819.00 | 1195.12 |
| | %utilization | 0.995 | 0.993 | 0.000 | 0.983 | 0.000 | 0.982 | 0.804 |
| | utilized time ( hrs) | 1278.00 | 1198.20 | 0.00 | 938.30 | 0.00 | 804.60 | 961.25 |
| | non-utilized time ( hrs) | **6.17** | **8.16** | **1033.85** | **16.00** | **1109.10** | **14.40** | **233.87** |
| 0.6*MTBF* | availability | 0.836 | 0.842 | 0.842 | 0.849 | 0.833 | 0.795 | 0.833 |
| | available time ( hrs) | 1254.10 | 1179.18 | 1010.53 | 933.69 | 1082.61 | 795.02 | 1166.67 |
| | %utilization | 0.879 | 0.997 | 0.000 | 0.861 | 0.000 | 0.953 | 0.985 |
| | utilized time ( hrs) | 1102 | 1175.1 | 0 | 804.3 | 0 | 756.8 | 1149.5 |
| | **non-utilized time**( hrs) | **152.10** | **4.08** | **1010.53** | **129.39** | **1082.61** | **36.22** | **16.17** |
| 0.5*MTBF* | availability | 0.810 | 0.817 | 0.816 | 0.824 | 0.806 | 0.764 | 0.806 |
| | available time ( hrs) | 1214.29 | 1143.12 | 979.59 | 906.29 | 1046.57 | 763.71 | 1129.03 |
| | %utilization | 0.936 | 0.979 | 0.000 | 0.995 | 0.000 | 0.901 | 0.915 |
| | utilized time ( hrs) | 1136.00 | 1119.60 | 0.00 | 901.90 | 0.00 | 686.80 | 1033.15 |
| | **non-utilized time** ( hrs) | **78.29** | **23.52** | **979.59** | **4.39** | **1046.57** | **75.91** | **95.88** |

**Table 6.8 : Effect of decreased *MTBF* on model solutions (Weibull model)**

| Parameter | Objective I | Objective I components | | | Objective II |
|---|---|---|---|---|---|
| | | VCM | MHC | MNC | |
| 1.0*MTBF* | 2822 | 1632 | 200 | 990 | 715.42 |
| 0.9*MTBF* | 2894 | 1708 | 200 | 986 | 823.79 |
| 0.8*MTBF* | 2848 | 1671 | 200 | 977 | 965.78 |
| 0.7*MTBF* | 2803 | 1640 | 250 | 913 | 1155.08 |
| 0.6*MTBF* | 2713 | 1630 | 150 | 932 | 1429.9 |
| 0.5*MTBF* | 2839 | 1667 | 250 | 922 | 1830.6 |

**Table 6.9: Effect of decreased *MTBF* on part processing routes (Weibull model)**

| Part type | 1.0*MTBF* | | 0.9*MTBF* | | 0.8*MTBF* | | 0.7*MTBF* | | 0.6*MTBF* | | 0.5*MTBF* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | plan | process route | plan | process route | plan | process route | plan | process route | plan | process route | plan | process route |
| 1 | 2 | M1-M6 | 2 | M1-M4 | 1 | M4-M1-M6 | 1 | M4-M1-M6 | 2 | M1-M6 | 2 | M1-M4 |
| 2 | 1 | M2-M7-M7 | | | | | | | | | 1 | M2-M7-M2 |
| 3 | 2 | M4-M6-M4 | **2 | M4-M6-M2 | | | | | | | | |
| 4 | 2 | M1-M6 | | | | | 2 | M1-M2 | 2 | M1-M6 | 2 | M1-M6 |
| 5 | 2 | M2-M6-M4 | | | | | | | | | | |
| 6 | 1 | M7-M1-M2 | | | | | 1 | M7-M1-M7 | 1 | M7-M1-M2 | 1 | M7-M1-M7 |
| 7 | 2 | M2-M4 | 2 | M6-M1 | 2 | M2-M1 | 2 | M6-M1 | 2 | M2-M1 | 2 | M2-M1 |
| 8 | 1 | M2-M2-M6 | 1 | M2-M2-M4 | 1 | M2-M2-M6 | 1 | M2-M2-M4 | 1 | M2-M2-M4 | 1 | M2-M7-M4 |
| 9 | 2 | M1-M4-M1 | 2 | M1-M4-M6 | 2 | M1-M4-M1 | 2 | M1-M4-M1 | 2 | M1-M4-M1 | 1 | M7-M1-M6 |
| 10 | 1 | M2-M6-M1 | | | | | | | | | | |
| 11 | 1 | M1-M7 | | | | | | | 1 | M7-M7 | | M7-M9 |
| 12 | 2 | M7-M6 | | | | | | | | | | |
| Cell 1 | M1, M2, M4, M6 | | | | | | | | | | | |
| Cell 2 | M7 | | | | | | | | | | | |

Note:

1. [shaded] Indicates no change in part processing routes/cell configuration relative to the current state 1.0*MTBF*

2. ** 2 M4-M6-M2 indicates a change in processing route for part type 3 from M4–M6-M4 to M4-M6-M2, but no change in process plan

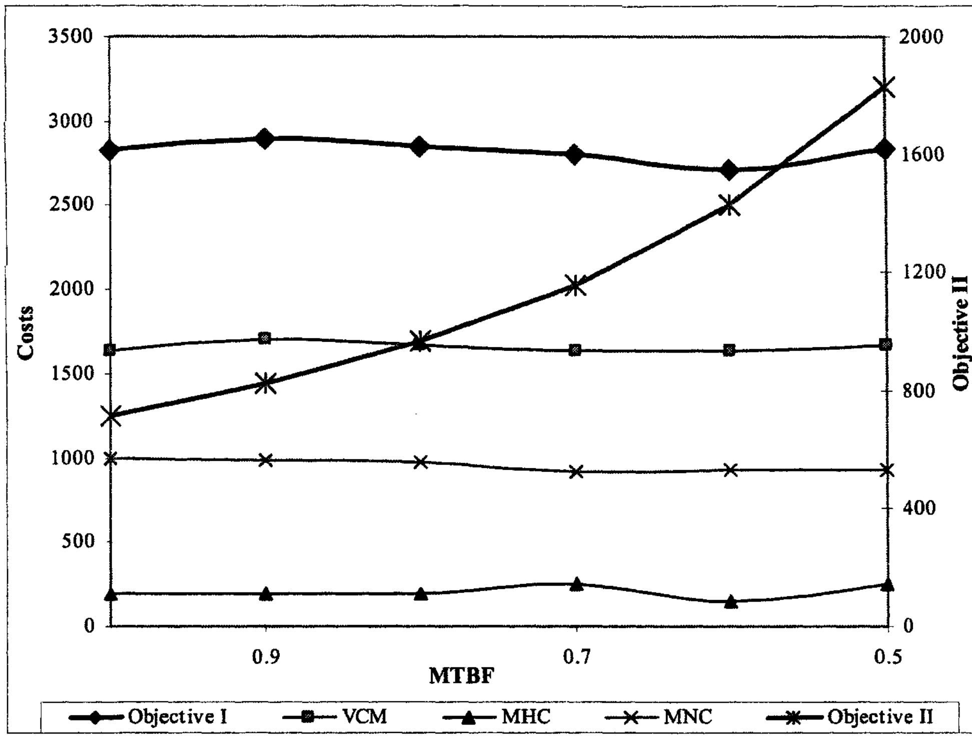


**Figure 6.3: Effect of decreased *MTBF* on model solutions (Weibull model)**

observe a shift in the model solution (recorded in Table 6.9) when the processing routes of the following part types are changed:

| Part type | Change of processing routes | Part type | Change of processing routes |
|---|---|---|---|
| 1 | from M1-M6 to M1-M4 | 7 | from M2-M4 to M6-M1 |
| 3 | from M4-M6-M4 to M4-M6-M2 | 8 | from M2-M2-M6 to M2-M2-M4 |
| 9 | from M1-M4-M1 to M1-M4-M6 | | |

While these processing route changes could accommodate the part processing times within the available times of the machines while achieving the desired objective function II value (823.79), the objective function I value increased by $(2,894-2,822) = $72 (illustrated in Table 6.8 and Figure 7.3). Again, at 0.9*MTBF* machine M6 is almost 100% loaded, which prompts the next shift in the model solution at a 20% decrease in *MTBF*, where the processing route of part type 1 changes from M1-M4 to M4-M1-M6 while that of part type 7 changes from M6-M1 to M2-M1. The processing routes of part types 8 and 9 revert back to the current state routes (M2-M2-M4 to M2-M2-M6 and M1-M4-M6 to M1-M4-M1, respectively) as shown in Table 6.8. With a 20% decrease in *MTBF*, the model solution improves the objective function I value by $(2894-2848) = $46 and achieves the desired reliability (objective function II value target 965.78) as recorded in

Table 6.8 and Figure 7.3. At each decrement of *MTBF*, similar changes in part processing routes and objective function values may be observed from Tables 6.8 and 6.9, and Figure 6.3.

3) As expected, Table 6.8 and Figure 6.3 reveal that the objective function II value steadily increases (implying deterioration of system reliability) with each decrement in *MTBF*. With each decrease in *MTBF*, achieving the desired reliability becomes more and more difficult. Consequently, the model solution attempts to make many changes in the part processing routes to achieve the desired reliability target and accommodate part processing times within the available times of the machines—details of which are made evident in Table 6.9.

From the above discussion it may be concluded that—in terms of system reliability and part processing routes—the Weibull distribution-based model solution is considerably sensitive to decreases in *MTBF*. As *MTBF* decreases the system reliability decreases, the objective function II value increases, and the model resorts to changes in part processing route in order to accommodate the operation assignments and maintain reasonable system reliability. In terms of system cost (objective function I) the model solution is less sensitive to a decrease in *MTBF*. For example, within the range of *MTBF* values under consideration, the objective function II value increases by (1830.6-715.42) = 1115.18, whereas, the highest change in objective function I value is $(2894-2713) = $181(which is 6.4% of the current solution cost). This finding is similar to the sensitivity exhibited by exponential distribution-based model solutions when subjected to fluctuations in machine failure rate. The difference lies in the fact that the sensitivity of the Weibull distribution-based model solution to decreases in *MTBF*—in terms of system reliability and part processing routes—is more prominent than that of the exponential distribution-based model's solution to increases in failure rate, which is exhibited by a larger number of changes in the processing route assignments.

### 6.4.2 Effect of Changes in *MTTR*

Table 6.10, 6.11 and 6.12a and 6.12b summarize the effects of the increase and decrease in *MTTR* on the model solutions in terms of machine parameters, objective function values and part processing routes. Figure 6.4 displays the effect of changes in *MTTR* on the objective function values. Similar to the effect of key factors discussed previously, Tables 6.10, 6.11, 6.12a and 6.12b are interrelated in terms of the information presented. Therefore, simultaneous references will be made to them during the following discussion.

The effect of an increase and a decrease in *MTTR* on machine availability, available time and machine utilization is presented in Table 6.10. As expected, with the increase in *MTTR* we observe a decrease in availability and available time, as well as an increase in machine

**Table 6.10: Effect of increased and decreased *MTTR* on machine times (Weibull model)**

| Parameter | Performance indicators | Machines | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| | | Total machine capacities (Hours) | | | | | | |
| | | 1500 | 1400 | 1200 | 1100 | 1300 | 1000 | 1400 |
| 0.5*MTTR* | availability | 0.944 | 0.947 | 0.947 | 0.949 | 0.943 | 0.928 | 0.943 |
| | available time (hrs) | 1416.66 | 1325.53 | 1136.09 | 1044.20 | 1226.14 | 928.21 | 1320.75 |
| | %utilization | 0.985 | 0.907 | 0.000 | 0.770 | 0.000 | 0.991 | 0.545 |
| | utilized time (hrs) | 1395.99 | 1202 | 0 | 804.3 | 0 | 919.8 | 720 |
| | **non-utilized time** (hrs) | **0.89** | **123.53** | **1136.09** | **239.90** | **1226.14** | **8.41** | **600.75** |
| 0.6*MTTR* | availability | 0.934 | 0.937 | 0.937 | 0.940 | 0.933 | 0.915 | 0.933 |
| | available time (hrs) | 1401.10 | 1311.58 | 1124.12 | 1033.72 | 1212.36 | 915.07 | 1305.97 |
| | %utilization | 0.994 | 0.916 | 0.00 | 0.996 | 0.00 | 0.811 | 0.551 |
| | utilized time (hrs) | 1392.00 | 1202.00 | 0.00 | 1029.30 | 0.00 | 741.80 | 720.00 |
| | **non-utilized time** (hrs) | **9.10** | **109.58** | **1124.12** | **4.42** | **1212.36** | **173.27** | **585.97** |
| 0.7*MTTR* | availability | 0.924 | 0.927 | 0.927 | 0.930 | 0.922 | 0.902 | 0.923 |
| | available time (hrs) | 1385.87 | 1296.92 | 1112.40 | 1023.44 | 1198.89 | 902.29 | 1291.51 |
| | %utilization | 0.993 | 0.944 | 0.000 | 0.786 | 0.000 | 0.840 | 0.653 |
| | utilized time (hrs) | 1376 | 1225.1 | 0 | 804.3 | 0 | 756.8 | 843.75 |
| | **non-utilized time** (hrs) | **9.87** | **72.82** | **1112.40** | **219.14** | **1198.89** | **144.49** | **446.76** |
| 0.8*MTTR* | availability | 0.914 | 0.918 | 0.917 | 0.921 | 0.912 | 0.890 | 0.912 |
| | available time (hrs) | 1370.97 | 1284.54 | 1100.92 | 1013.36 | 1185.71 | 889.87 | 1276.37 |
| | %utilization | 0.999 | 0.816 | 0.065 | 0.981 | 0.000 | 0.984 | 0.661 |
| | utilized time (hrs) | 1370.10 | 1048.10 | 72.00 | 993.70 | 0.00 | 875.80 | 843.75 |
| | **non-utilized time** (hrs) | **0.87** | **236.44** | **1028.92** | **19.66** | **1185.71** | **14.07** | **433.62** |
| 0.9*MTTR* | availability | 0.904 | 0.908 | 0.908 | 0.912 | 0.902 | 0.878 | 0.903 |
| | available time (hrs) | 1356.38 | 1271.43 | 1089.67 | 1003.48 | 1172.83 | 876.79 | 1263.54 |
| | %utilization | 0.990 | 0.812 | 0.095 | 0.802 | 0.000 | 0.965 | 0.668 |
| | utilized time (hrs) | 1342.5 | 1032 | 104 | 804.3 | 0 | 846.3 | 843.7501 |
| | **non-utilized time** | **13.88** | **239.43** | **985.67** | **199.18** | **1172.83** | **30.49** | **419.79** |
| 1.0*MTTR* | availability | 0.895 | 0.899 | 0.899 | 0.903 | 0.892 | 0.866 | 0.893 |
| | available time (hrs) | 1342.11 | 1258.59 | 1078.65 | 993.79 | 1160.22 | 866.03 | 1250.00 |
| | %utilization | 0.946 | 0.934 | 0.000 | 1.000 | 0 | 0.939 | 0.670 |
| | utilized time (hrs) | 1269.10 | 1175.10 | 0.00 | 993.70 | 0.00 | 813.30 | 836.50 |
| | **non-utilized time** (hrs) | **73.01** | **83.49** | **1078.65** | **0.09** | **1160.22** | **52.73** | **412.50** |

**Table 6.10 cont'd**

| Parameter | Performance indicators | Machines | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| | | Total machine capacities | | | | | | |
| | | 1500 | 1400 | 1200 | 1100 | 1300 | 1000 | 1400 |
| 1.0*MTTR* | availability | 0.895 | 0.899 | 0.899 | 0.903 | 0.892 | 0.866 | 0.893 |
| | available time (hrs) | 1342.11 | 1258.59 | 1078.65 | 993.79 | 1160.22 | 866.03 | 1250.00 |
| | %utilization | 0.946 | 0.934 | 0.000 | 1.000 | 0 | 0.939 | 0.670 |
| | utilized time (hrs) | 1269.10 | 1175.10 | 0.00 | 993.70 | 0.00 | 813.30 | 836.50 |
| | **non-utilized time** (hrs) | **73.01** | **83.49** | **1078.65** | **0.09** | **1160.22** | **52.73** | **412.50** |
| 1.1*MTTR* | availability | 0.885 | 0.890 | 0.890 | 0.895 | 0.883 | 0.855 | 0.883 |
| | available time (hrs) | 1328.13 | 1246.00 | 1066.85 | 984.29 | 1146.87 | 854.58 | 1236.75 |
| | %utilization | 0.963 | 0.929 | 0.000 | 0.927 | 0.000 | 0.994 | 0.677 |
| | utilized time (hrs) | 1279.20 | 1158.00 | 0.00 | 912.00 | 0.00 | 849.80 | 836.50 |
| | **non-utilized time** (hrs) | **48.92** | **88.00** | **1066.85** | **72.29** | **1146.87** | **4.78** | **399.25** |
| 1.2*MTTR* | availability | 0.876 | 0.881 | 0.881 | 0.886 | 0.874 | 0.843 | 0.874 |
| | available time (hrs) | 1314.43 | 1233.66 | 1056.27 | 974.97 | 1135.79 | 843.50 | 1223.78 |
| | %utilization | 0.987 | 0.953 | 0.000 | 0.962 | 0.000 | 0.927 | 0.684 |
| | utilized time (hrs) | 1298.00 | 1175.10 | 0.00 | 938.30 | 0.00 | 781.86 | 836.50 |
| | **non-utilized time** (hrs) | **16.43** | **58.56** | **1056.27** | **36.67** | **1135.79** | **61.63** | **386.28** |
| 1.3*MTTR* | availability | 0.867 | 0.873 | 0.872 | 0.878 | 0.865 | 0.833 | 0.865 |
| | available time (hrs) | 1301.02 | 1221.57 | 1046.89 | 965.82 | 1123.96 | 832.57 | 1211.07 |
| | %utilization | 0.998 | 0.962 | 0.000 | 0.972 | 0.000 | 0.939 | 0.692 |
| | utilized time (hrs) | 1298.00 | 1175.10 | 0.00 | 938.30 | 0.00 | 781.80 | 836.50 |
| | **non-utilized time** (hrs) | **3.02** | **46.47** | **1046.89** | **26.52** | **1123.96** | **50.77** | **373.57** |
| 1.4*MTTR* | availability | 0.859 | 0.864 | 0.864 | 0.870 | 0.856 | 0.822 | 0.856 |
| | available time (hrs) | 1286.88 | 1209.71 | 1036.72 | 956.84 | 1112.37 | 821.98 | 1198.63 |
| | %utilization | 0.992 | 0.926 | 0.000 | 0.981 | 0.000 | 0.951 | 0.746 |
| | utilized time (hrs) | 1278.00 | 1119.60 | 0.00 | 938.30 | 0.00 | 781.80 | 894.75 |
| | **non-utilized time**(hrs) | **9.88** | **90.11** | **1036.72** | **18.54** | **1112.37** | **40.18** | **303.88** |
| 1.5*MTTR* | availability | 0.850 | 0.856 | 0.856 | 0.862 | 0.847 | 0.812 | 0.847 |
| | available time (hrs) | 1275.00 | 1198.08 | 1026.74 | 948.03 | 1101.02 | 811.66 | 1186.44 |
| | %utilization | 0.986 | 0.875 | 0.071 | 0.903 | 0.000 | 0.993 | 0.828 |
| | utilized time (hrs) | 1256.00 | 1048.10 | 73.16 | 856.40 | 0.00 | 805.80 | 982.15 |
| | **non-utilized time** (hrs) | **18.00** | **149.98** | **953.58** | **91.63** | **1101.02** | **5.86** | **204.29** |

**Table 6.11: Effect of decreased and increased *MTTR* on model solutions (Weibull model)**

| Parameter | Objective I | Objective I components | | | Objective II |
|---|---|---|---|---|---|
| | | VCM | MHC | MNC | |
| 0.5*MTTR* | 2978 | 1658 | 250 | 1069 | 694.6 |
| 0.6*MTTR* | 2957 | 1651 | 250 | 1056 | 696.71 |
| 0.7*MTTR* | 2973 | 1605 | 300 | 1067 | 702.61 |
| 0.8*MTTR* | 2913 | 1638 | 300 | 975 | 706.51 |
| 0.9*MTTR* | 2899 | 1632 | 250 | 1017 | 714.23 |
| 1.0*MTTR* | 2822 | 1632 | 200 | 990 | 715.42 |
| 1.1*MTTR* | 2896 | 1708 | 200 | 988 | 715.42 |
| 1.2*MTTR* | 2854 | 1671 | 200 | 983 | 718.52 |
| 1.3*MTTR* | 2841 | 1671 | 200 | 970 | 718.52 |
| 1.4*MTTR* | 2807 | 1640 | 250 | 917 | 718.81 |
| 1.5*MTTR* | 2899 | 1719 | 250 | 930 | 722.9 |

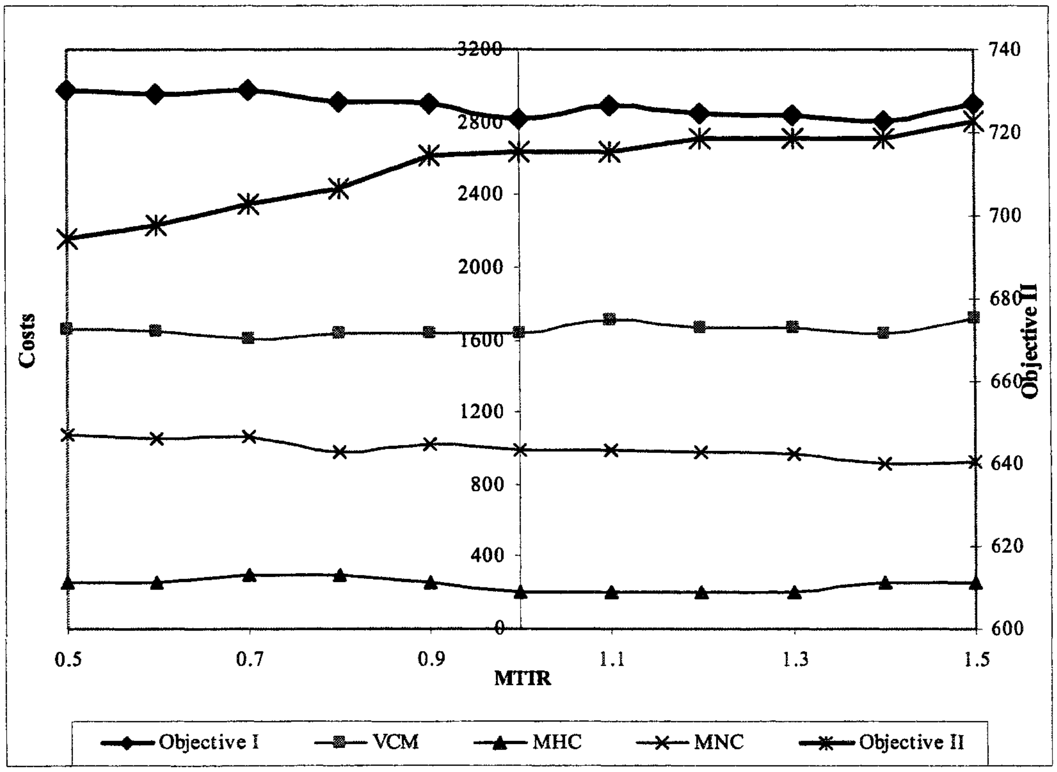


**Figure 6.4: Effect of increased and decreased *MTTR* on model solutions (Weibull model)**

**Table 6.12a: Effect of increased *MTTR* on part processing routes (Weibull model)**

| Part type | 1.0*MTTR* | | 1.1*MTTR* | | 1.2*MTTR* | | 1.3*MTTR* | | 1.4*MTTR* | | 1.5*MTTR* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | plan | process routes | plan | process routes | plan | process routes | plan | process routes | plan | process routes | plan | process routes |
| 1 | 2 | M1-M6 | | | 1 | M4-M1-M6 | 1 | M4-M1-M6 | 1 | M4-M1-M6 | 2 | M1-M4 |
| 2 | 1 | M2-M7-M7 | | | | | | | | | 1 | M2-M7-M2 |
| 3 | 2 | M4-M6-M4 | | | | | | | | | | |
| 4 | 2 | M1-M6 | | | | | | | 2 | M1-M2 | | |
| 5 | 2 | M2-M6-M4 | | | | | | | | | | |
| 6 | 1 | M7-M1-M2 | | | | | | | 2 | M7-M1-M7 | 2 | M7-M1-M7 |
| 7 | 2 | M2-M4 | **2 | M6-M1 | 2 | M2-M1 | 2 | M2-M1 | 2 | M6-M1 | 2 | M2-M1 |
| 8 | 1 | M2-M2-M6 | 1 | M2-M2-M4 | 1 | M2-M2-M4 | 1 | M2-M2-M4 | 1 | M2-M7-M4 | 2 | M3-M1-M6 |
| 9 | 2 | M1-M4-M1 | 2 | M1-M4-M6 | | | | | | | 1 | M7-M1-M6 |
| 10 | 1 | M2-M6-M1 | | | | | | | | | 1 | M2-M6-M1 |
| 11 | 1 | M1-M7 | | | | | | | | | | |
| 12 | 2 | M7-M6 | | | | | | | | | | |
| | | | | | | | | | | | | |
| Cell 1 | M1, M2, M4,M6 | | | | | | | | M1,M4,M6,M7 | | M1,M2,M6,M7 | |
| Cell 2 | M7 | | | | | | | | M2 | | M3,M4 | |

**Note:**

1. [shaded box] Indicates no change in part processing routes/cell configuration relative to the current state 1.0*MTTR*

2. ** 2 M6-M1 indicates a change in processing route for part type 7 from M2-M4 to M6-M1, but no change in process plan

**Table 6.12b: Effect of decreased *MTTR* on part processing routes (Weibull model)**

| Part type | 1.0*MTTR* | | 0.9*MTTR* | | 0.8*MTTR* | | 0.7*MTTR* | | 0.6*MTTR* | | 0.5*MTTR* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **plan** | **process route** | **plan** | **process route** | **plan** | **process route** | **plan** | **process route** | **plan** | **process route** | **plan** | **process route** |
| 1 | 2 | M1-M6 | | | | | | | 1 | M1-M1-M4 | 2 | M1-M6 |
| 2 | 1 | M2-M7-M7 | 1 | M2-M7-M2 | 1 | M2-M7-M2 | 1 | M2-M7-M2 | 1 | M2-M7-M2 | 1 | M2-M7-M2 |
| 3 | 2 | M4-M6-M4 | | | | | | | | | | |
| 4 | 2 | M1-M6 | | | | | | | | | | |
| 5 | 2 | M2-M6-M4 | | | | | | | | | | |
| 6 | 1 | M7-M1-M2 | **2 | M7-M1-M7 | 2 | M7-M1-M7 | 2 | M7-M1-M7 | 1 | M7-M1-M2 | 1 | M7-M1-M2 |
| 7 | 2 | M2-M4 | 2 | M6-M1 | 2 | M2-M4 | | | 2 | M6-M1 | 2 | M6-M1 |
| 8 | 1 | M2-M2-M6 | 1 | M2-M2-M4 | 2 | M3-M1-M6 | 1 | M2-M2-M4 | 1 | M2-M2-M4 | 1 | M2-M2-M4 |
| 9 | 2 | M1-M4-M1 | | | | | | | | | | |
| 10 | 1 | M2-M6-M1 | 2 | M6-M3-M1 | 1 | M2-M6-M1 | 1 | M2-M6-M1 | | | | |
| 11 | 1 | M1-M7 | | | | | | | | | | |
| 12 | 2 | M7-M6 | 2 | M7-M2 | 2 | M7-M6 | 2 | M7-M6 | | | | |
| | | | | | | | | | | | | |
| Cell 1 | M1, M2, M4, M6 | | M1,M3,M4,M6 | | M1,M4,M6,M7 | | | | | | | |
| Cell 2 | M7 | | M2, M7 | | M2,M3 | | | | | | | |

**Note:**

1. [shaded] Indicates no change in part processing routes/cell configuration relative to the current state 1.0*MTTR*

2. ** 2 M7-M1-M7 indicates a change in processing route for part type 6 from M7 –M1-M2 to M7-M1-M7, but no change in process plan

utilization—resulting in an overall decrease in the cost of machine non-utilized time (MNC). These results (recorded in Table 6.11.) show that a 50% increase in *MTTR* (i.e., at 1.5*MTTR* level) decreases MNC by $(990-930) = $60. Considering machine M1 as an example, Table 6.10 reveals that a 50% increase in *MTTR* decreases availability from 0.895 to 0.850, and available time is decreased by (1342-1275) = 65 hours. Similar losses in availability can be observed for other machines also. These decreases in available times are added to the machine down time, impacting machine maintenance and repair costs. As previously discussed, a decrease in MNC provides insights into areas of abnormal reliability performances.

Tables 6.10, 6.12a and 6.12b also depict the effect of each change in *MTTR* on the model solutions. For example, Table 6.10 shows that at the current level (1.0*MTTR*), machine M4 is practically 100% loaded. With a 10% increase in *MTTR* we observe a shift in the model solutions (recorded in Table 6.12a). The processing route of part type 7 is changed from M2-M4 to M6-M1, that of part type 8 is changed from M2-M2-M6 to M2-M2-M4 and that of part type 9 is changed from M1-M4-M1 to M1-M4-M6. This shift allows the model solutions to accommodate the processing times of part types within the available capacity of machines. The objective function I value increased by ($2896-2822) = $74 and the objective function II value remained the same as the current level solution. Tables 6.10 and 6.12a reveal that similar shifts in model solutions are observed for almost all increases in *MTTR*. The implications for these shifts are also displayed in Table 6.11 and Figure 6.4.

2) Conversely, Table 6.10 illustrates that as *MTTR* decreases, machine availability and available time increases and—consequently—machine utilization decreases, leading to an overall increase in the cost of non-utilized machine time (MNC) as shown in Table 6.11. A 50% decrease in *MTTR*, for example, increases MNC by ($1069-990) = $79. Considering the example of M1, when *MTTR* decreases by 50%, available time increases by (1416-1342) = 74 hours. The increase in machine availability is an advantage as long as the available time is utilized. Otherwise, it will only add to the non-utilized time and the cost.

To further study the effect of *MTTR* on model solutions, investigation of Table 6.12b shows as *MTTR* decreases, available time increases and, thus the model solution attempts to utilize the increase in available time and improve system reliability. For example, at a 10% decrease in *MTTR* we observe a shift in model solutions. The processing route of six part types (types 2, 6, 7, 8, 10 and 12) and their cell configurations changed. Table 6.11 explains how this shift allows the model solutions to improve system reliability. One example is the objective function II value decreasing from the current 715.42 to 714.23. Tables 6.10, 6.11, 6.12b and Figure 6.4 also show that the model solutions shift with almost every decrease in *MTTR* in their attempts to improve the system reliability

3) From the above analysis, we can observe that the system reliability deteriorates (with the objective function II value increasing) as *MTTR* increases. In contrast, system reliability improves (with the objective II value decreasing) as *MTTR* decreases. The effect of decreases and increases in *MTTR* on the objective II value are recorded in Table 6.11and displayed in Figure 6.4.

From the above analysis we may conclude that the model solution is somewhat sensitive to the changes in *MTTR*, which is displayed by many shifts in the processing route assignments and changes in cell configuration. However, if we examine the model solutions in terms of changes in the objective function I and objective function II values, the impact of a change in *MTTR* is very low. A -50% to +50% change in the *MTTR* prompts a change of -2.6% to +0.5% in the objective function I value (system cost) while the objective function II value changed from +2.9% to -1.04%.

## CHAPTER 7
## CONSIDERATION OF
## PREVENTIVE MAINTENANCE IN CMS

### 7.1 Introduction

Machines are the major components of a cellular manufacturing system (CMS), and they represent a significant share of the capital investment in such systems. Machines are subject to deterioration relative to both usage and age, which leads to reduced product quality and increased production costs (Valdez-Flores and Feldman, 1989). Manufacturing industries carry out preventive maintenance (PM) on machinery and equipment in an effort to prevent or slow down deterioration. Preventive maintenance is a scheduled downtime—usually periodical—during which a well-defined set of tasks (e.g., inspection, repair, replacement, cleaning, lubrication, adjustment and alignment) is performed (Ebeling, 1997). It is important to note that PM is justified only when it is cost effective, reduces unscheduled breakdowns, and extends the useful life of the equipment. Further, for the PM to be effective, the failure rate of the equipment must increase with time (Jardine, 1973; Ebeling, 1997), which is usually the case in the manufacturing industry (e.g., CNC machines). Figure 7.1 shows how preventive maintenance can result in improved reliability over time.

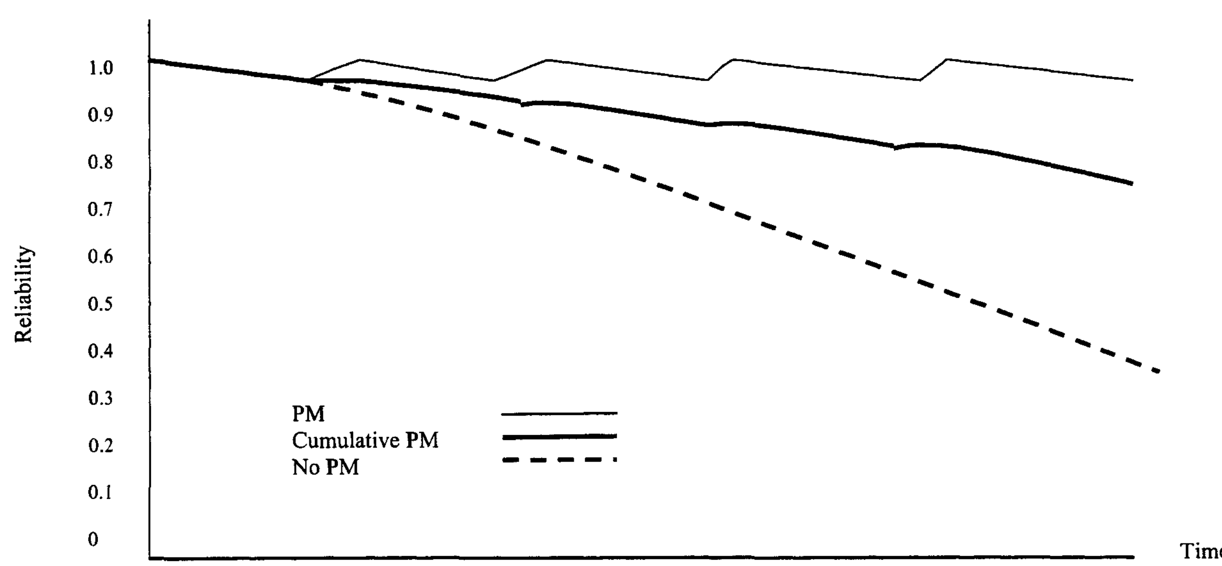


**Figure 7.1: A periodic maintenance reliability curve for an increasing failure rate (Ebeling, 1997)**

Chapter 2 analyzed the way a part type in a CMS is typically processed on several machines in a serial fashion, causing the system reliability to follow a series reliability structure. Therefore, a single-machine PM plan is not feasible in these cases, due to the interruptions suffered in the upstream and downstream operations when a machine undergoes the maintenance action. Instead, a multi-machine PM plan is required to address this kind of interdependent structure. A number of studies have reviewed the various PM policies in manufacturing systems (Wang, 2002; Dekker, *et al.*, 1997; Cho and Parlar, 1991). Among the policies that may be applicable to CMSs are the fixed group planned maintenance policy outlined by Dekker, *et al.* (1997), and the group maintenance policy mentioned by Wang (2002). Both are based on the concept of replacing a selected group of components after a fixed interval of time, while addressing the unplanned failures of the components during the interval through repairs or minimal repairs. Another group maintenance policy studied by Wilderman, *et al.* (1997) concerned the maintenance activities carried out on a group of equipment, and involved a system-dependent set up cost that was the same for all the activities. The grouping of machines saved costs, since the execution of a group of activities required only one set up.

Chapter 4 showed how machine reliability can be incorporated in the CMS design model with the aim of optimizing system reliability and system cost, simultaneously. To address the increasing failure rate of machines we have developed a CMS design model based on the Weibull distribution. The model attempts to select machines along the processing route for each part type in order to get the lowest overall probability of failure, or the highest system reliability. It is evident that despite these efforts, there is still the probability of failure for the machines. In addition to this, the CMS design model develops cell designs based on the existing reliability status of the machines. This existing status for the machines is subject to deterioration due to usage and age.

As discussed above, the objective of a PM plan is to prevent the deterioration and improve the reliability of the equipment—enhancing equipment's performance. Consequently, a PM policy becomes an important requirement for improving the overall performance of CMSs. In addition, because PM is a planning process, it may be possible to integrate it into the development of the CMS design along with the system costs and other relevant factors. In a complex system such as the CMS, the impact of unplanned shut-downs is significant. If the machine reliability and maintainability can be effectively managed during the planning phase, losses due to unplanned failures can be kept at a minimum.

The objective of this chapter is to develop a reliability-based PM planning in a cellular manufacturing environment. Analyses of the cost-based, reliability-based and combined approaches to PM planning are presented in Section 7.2. In Section 7.3, an algorithm that modifies the reliability-based model to develop a group PM plan based on an effective maintenance interval is presented. In Section 7.4, the proposed procedure for the integration of PM policies in the CMS design model is outlined. Numerical examples are provided throughout the chapter to illustrate the procedures discussed.

## 7.2. Preventive Maintenance Models

### 7.2.1 Cost-based Approach

The main objective of the cost-based approach to PM planning is to determine the optimum maintenance interval that will balance the system failure repair costs and the PM costs so that the system performance in terms of machine reliability, cost, and machine utilization is improved by reducing system down time due to unplanned failures.

The basic cost-based approach to maintenance planning was developed by Jardine (1973) and extended and refined over time in multiple studies (Talukder and Knapp, 2002; Sherwin, 1997). It estimates the optimal interval between preventive replacements of equipment/components subject to failures, and may be applied to preventive maintenance and overhaul—assuming that the overhaul returns the equipment to the as-good-as-new condition and that the failure repair between preventive maintenance actions makes it possible to run the machine up to the next interval (i.e., it results in a bad-as-old condition).

Using the approach suggested by Jardine (1973), and defining $tpc$ as the PM interval, the total maintenance cost per unit time for a group of $m$ machines in a cell may be represented by:

$$\frac{TC(tpc)}{tpc} = \frac{Co + \sum_{j=1}^{m} CPMR_j + \sum_{j=1}^{m} cf_j H_j(tpc)}{tpc} \qquad \text{---------- (7.1)}$$

The first expression in the numerator,

$$\left( Co + \sum_{j=1}^{m} CPMR_j \right)$$

computes the PM cost during the interval $tpc$, where $Co$ is the fixed cost incurred every time PM is carried out, and $CPMR_j$ is the estimated average maintenance cost to return machine $j$ to the as-good-as-new condition.

The second expression,

$$\sum_{j=1}^{m} cf_j H_j(tpc)$$

is the failure repair cost during the interval *tpc,* where $cf_j$ is the average cost of a failure repair on machine *j*, and $H_j(tpc)$ is the estimated cumulative number of failures of machine *j* during the interval *tpc*. Assuming the machine failure times are Weibull distributed, $H_j(tpc)$ is computed as (Talukder and Knapp, 2002; Sherwin, 1997):

$$H_j(tpc) = (\frac{tpc}{\theta_j})^{\beta_j}$$

where $\theta_j$ and $\beta_j$ are, respectively, the scale parameter and the shape parameter of the Weibull distribution for machine *j*.

Replacing $H_j(tpc)$ in equation (7.1), the total maintenance cost per unit time is:

$$Z = \frac{TC(tpc)}{tpc} = \frac{Co + \sum_{j=1}^{m} CPMR_j + \sum_{j=1}^{m} cf_j (\frac{tpc}{\theta_j})^{\beta_j}}{tpc} \qquad \text{---------- (7.2)}$$

The optimal preventive maintenance interval can be found by taking the first derivative of equation (7.2) and equating it to zero:

$$-( Co + \sum_{j=1}^{m} CPMR_j ) + \sum_{j=1}^{m} cf_j (\beta_j - 1)(\frac{tpc}{\theta_j})^{\beta_j} = 0 \qquad \text{---------- (7.3)}$$

Using a numerical search procedure (e.g., the golden section search), equation (7.3) can be solved to estimate *tpc*.

Let *n* be the total number of preventive maintenance actions during the planning period *T*, then $n = T/tpc$, and the corresponding total maintenance cost, *TC (T),* is:

$$TC(T) = n\left( Co + \sum_{j=1}^{m} CPMR_j \right) + n\sum_{j=1}^{m} cf_j (\frac{tpc}{\theta_j})^{\beta_j} \qquad \text{----------- (7.4)}$$

The following numerical example illustrates the above approach.

#### 7.2.1.1 Numerical Example 1

We consider a cell consisting of seven machines when demonstrating the cost-based maintenance model. The cost and maintenance related input data for the cell is given in Table 7.1. The preventive maintenance costs $CPMR_j$ and the failure repair costs $cf_j$ are randomly generated from the uniform distributions [30, 70] and [200, 500], respectively. The parameters $\beta_j$ and

$MTBF_j$ are also randomly generated from the uniform distributions [1.1, 1.80] and [160, 360], respectively. The parameter $\theta_j$ is computed from the relationship: $\theta_j = MTBF_j / \Gamma(1+1/\beta_j)$.

The optimal interval *tpc* has been determined by solving equation (7.3) using the golden section search in MATLAB 6.5. The results are shown in Table 7.2, where the preventive maintenance costs, failure repair costs, and total maintenance costs are also evaluated and displayed.

The optimal *tpc* value is computed to be 70 hours; thus, the set of machines in the cell will undergo a total of 1500/ 70 ≈ 22 PM actions during the planning period. The total PM cost for the 22 planned maintenance actions is $8,844 while the failure repair cost is $23,303, and the total maintenance cost equals $32,147.

**Table 7.1: Input data for numerical example 1**

| Data Type | MACHINES | | | | | | |
|---|---|---|---|---|---|---|---|
| | **M1** | **M2** | **M3** | **M4** | **M5** | **M6** | **M7** |
| *MTBF* (Hrs) | 187 | 89 | 160 | 131 | 83 | 181 | 130 |
| $\beta_j$ | 1.21 | 1.26 | 1.78 | 1.16 | 1.56 | 1.59 | 1.33 |
| $\theta_j$ | 199.61 | 97.83 | 179.85 | 138.09 | 92.36 | 201.73 | 110.00 |
| *Co* | $50 | | | | | | |
| $CPMR_j$ | $31 | $53 | $62 | $44 | $50 | $51 | $61 |
| $cf_j$ | $270 | $455 | $220 | $230 | $490 | $185 | $320 |
| Planning period, *T* | 1500 hours | | | | | | |

**Table 7.2: Solution to numerical Example 1 (cost based model solution)**

| | |
|---|---|
| *tpc* (hours) | 70.00 |
| Total preventive maintenance cost | $8,844 |
| Total failure repair cost | $23,303 |
| Total maintenance cost | $32,147 |

### 7.2.2 Machine Reliability-based Approach

The reliability-based group PM planning for CMSs aims to compute the largest possible interval to minimize the total maintenance cost by reducing the number of maintenance actions while keeping the individual machine failure probabilities below a predefined *Upper Bound.* For deciding the limit on the failure probability of machines we have followed the approaches of Johnson (1959) and Kardon and Fredendall (2002).

As usual, the total maintenance costs over the planning period consist of the cost of PM actions and the cost of unplanned failure repairs that occur between the PM actions on each machine.

#### 7.2.2.1 Development of the Preventive Maintenance Interval Model

We define *tpr* to be the PM interval when using a reliability-based approach. Assuming a Weibull distribution, the probability of failure for machine *j* at time *tpr* is:

$$F_j(tpr) = 1 - \exp(-(tpr/\theta_j)^{\beta_j}),\ (tpr,\ \theta_j,\ \beta_j) > 0 \qquad \text{------- (7.5)}$$

where $\theta_j$ and $\beta_j$ are as defined before.

Following the approach of Kardon and Fredendall (2002), the PM interval, *tpr,* may be derived from equation (7.5) when an organization sets an *Upper Bound* on the failure probability of machine *j*, i.e., $F_j(t)$:

$$tpr \le \theta_j \{\ln\left[\frac{1}{1-F_j\ (tpr)}\right]\}^{1/\beta_j},\quad j=1,2,\dots,m \qquad \text{----------(7.6)}$$

and

$$F_j\ (tpr) \le Upper\ Bound,\ j=1,2,\dots,m \qquad \text{--------(7.7)}$$

Now, the following optimization model (hereafter designated as *OptimInterval*) is proposed to estimate the optimal PM interval:

*OptimInterval*: *Maximize tpr*

*s.t.*

$$tpr \le \theta_j \{\ln\left[\frac{1}{1-F_j\ (tpr)}\right]\}^{1/\beta_j}\ \ j=1,2,\dots,m \qquad \text{- --------(7.8)}$$

$$F_j\ (tpr) \le Upper\ Bound,\quad j=1,2,\dots,m \qquad \text{----------(7.9)}$$

The model is illustrated by the following numerical example.

#### 7.2.2.2 Numerical Example 2

We illustrate the above approach using Numerical Example 1 with the same data given in Table 7.1, and the *Upper Bound* parameter set at 0.30. The model is solved using LINGO 9, and the results are shown in Table 7.3. The optimal PM interval *tpr* is computed as 42.28 hours, which implies that the set of machines in the cell will undergo a total of 1500/42.25 ≈ 36 PM actions during the planning period. Using equation (7.4), the corresponding total PM cost, failure repair cost and the total maintenance cost are computed as $19,026, $14,472 and $33,498, respectively as presented in Table 7.3.

A closer examination of Table 7.3 indicates that at *tpr* = 42.28 hours, only the failure probability of Machine 2 (M2) is at the *Upper Bound* of 0.30, whereas for other machines the failure probability is lower than the *Upper Bound.* This implies that, by implementing PM actions after every 42.25 hours, the failure probabilities of the machines are maintained at or below the reliability threshold set by the *Upper Bound* parameter.

**Table 7.3: Solution to numerical example 2 (reliability based model solution)**

| Data Type | MACHINES | | | | | | |
|---|---|---|---|---|---|---|---|
| | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| *tpr* (hours) | 42.28 | | | | | | |
| Failure probability for machine *j* at *tpr=42.28* | 14.18% | *30%* | 7.32% | 22.38% | 27.59% | 8% | 24.44 |
| Total PM cost | $19,026 | | | | | | |
| Total failure repair cost | $14,472 | | | | | | |
| Total cost (TCR) | $33,498 | | | | | | |

### 7.2.3 A Combined Approach

A more desirable approach is to combine the cost-based and the reliability-based models to determine the optimum PM interval. The cost-based approach focuses on the total maintenance costs at the expense of machine failure probabilities, thus it may generate maintenance plans that leave some machines with unacceptably high failure probabilities. For instance, in Example1, the failure probabilities of each machine, computed at the interval *tpc* = 70 hours, are given in Table 7.4. As can be observed, machine M2 has a 49% failure probability during the interval of 70 hours.

**Table 7.4: Failure probability of machines in Example 1 computed at *tpc* =70 hours**

| Data Type | MACHINES | | | | | | |
|---|---|---|---|---|---|---|---|
| | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| *tpc* (hours) | 70 | | | | | | |
| Failure probability for machine *j* at *tpc* =70 hours | 0.25 | 0.49 | 0.17 | 0.37 | 0.48 | 0.17 | 0.42 |

The cost based model computes the interval *tpc* based on the ratio of the PM cost, $CPMR_j$, to the failure repair cost, $cf_j$ (refer to section 7.2.1). In the example presented above, this ratio is approximately 1/7. To explore further, we consider a second example in which $CPMR_j$ and $cf_j$ are generated randomly from the uniform distributions U [20-35] and U[200-600], respectively, as given below:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $CPMR_j$ | \$35 | \$20 | \$33 | \$30 | \$32 | \$25 | \$23 |
| $cf_i$ | \$589 | \$550 | \$438 | \$480 | \$599 | \$399 | \$426 |

Now, the ratio is 1/15, and we obtain a PM interval of 40.71 hours. From these two examples, it is clear that in the cost–based approach the ratio of PM cost to failure repair costs has a significant effect on the length of the PM interval. When PM cost is << failure repair cost (a higher ratio, e.g., in the second example, 1/15), a shorter interval is generated which implies too many maintenance actions, but when the PM cost is < failure repair cost (a lower ratio, e.g., in the Example1, 1/7), a longer interval is generated, implying fewer maintenance actions, but more failure repair with the interval.

On the other hand, the reliability-based approach (refer to 7.2.2.1) maintains an *Upper Bound* on machine failure probabilities regardless of cost, thus it may generate very expensive maintenance plans.

The following multi-objective mathematical model is proposed for combining the cost and reliability-based maintenance planning approaches to determine a balanced maintenance plan.

**7.2.3.1 The Combined Model**

$$\textit{Maximize } tpm \qquad \text{------ (7.10)}$$

$$\textit{Minimize } \; TC = n\left(Co + \sum_{j=1}^{m} CPMR_j\right) + n\sum_{j=1}^{m} cf_j\left(\frac{tpm}{\theta_j}\right)^{\beta_j} \qquad \text{---------- (7.11)}$$

s.t.

$$tpm \le \theta_j \{\ln\left[\frac{1}{1-F_j\,(tpm)}\right]\}^{1/\beta_j} \quad j = 1,2,\ldots,m \qquad \text{---------- (7.12)}$$

$$F_j\,(tpm) \le \textit{Upper Bound}, \quad j = 1,2,\ldots,m \qquad \text{--------- (7.13)}$$

$$n = \frac{T}{tpm}, \textit{ an integer} \qquad \text{---------- (7.14)}$$

where *tpm* is the PM interval, *T* is the planning period, and *n* is the number of PM actions carried out during *T*.

#### 7.2.3.2 Numerical Example 3

The combined approach is applied to the same numerical example used previously. The solution results are obtained using a pre-emptive optimization process (Rardin, 1998), and are displayed in Table 7.5. The process computes an optimum cost and interval combination depending on the user-specified priority placed on the first objective function (i.e., the PM interval or *tpm*), or on the second (i.e., the total cost or *TC*).

**Table 7.5: Maintenance solution from the combined approach model**

| ***Upper Bound*** **on machine failure probabilities** | ***Priority on tpm*** | | **Priority on TC** | |
|---|---|---|---|---|
| | ***tpm*** **(hours)** | ***TC*** **($)** | ***tpm*** **(hours)** | ***TC*** **($)** |
| 55% | 79.95 | 31,340 | 70.17 | 31,236 |
| 50% | 71.63 | 31,238 | 70.17 | 31,236 |
| 45% | 63.7 | 31,295 | 63.7 | 31,295 |
| 40% | 56.22 | 31,555 | 56.22 | 31,555 |
| 35% | 49.11 | 32,088 | 49.11 | 32,088 |
| 30% | 42.28 | 33,011 | 42.28 | 33,011 |

For example, at an *Upper Bound* value of 0.55, the model is solved in the following manner: if priority is placed on *tpm,* we first maximize the first objective function (*tpm*), subject to constraints (7.12) – (7.14), and ignore the second objective function. The solution yields $tpm$ = 79.95, as shown in Column 2 of Table 7.5. In the next step, we minimize the second objective function (*TC*), subject to the same constraints (7.12 - 7.14), and the additional constraint $tpm \geq$ 79.97. The solution now is $tpm$ = 79.95, and $TC$ = $31,340—as shown in Column 3.

If priority is placed on *TC,* we first minimize the second objective function (*TC*), subject to constraints (7.12) – (7.14), and ignore the first objective function. The solution yields $TC$ = $31,236 (Column 5). Next, we minimize the first objective function (*tpm*), subject to the same constraints (7.12 - 7.14) and the additional constraint $TC \leq$ $31,236. The solution now is $tpm$ = 70.17 (Column 4), and $TC$ = $31,236.

The solutions corresponding to the other values of the *Upper Bound* parameter are obtained in a similar fashion, as listed in Table 7.5. It is evident that for *Upper Bound* values of 0.55 to 0.50, the model achieves a minimum total cost value of $ 31,236 when *TC* is optimized as a first priority and the corresponding *tpm* interval of 70.17 hours ensures that all the machine failure probabilities stay below the corresponding *Upper Bound.* However, when maximizing the *tpm,* interval is the first priority, we get a range of solutions with wider intervals and higher costs when compared to the previous cost-prioritized solution. For example, at *Upper Bound* = 0.55, we get *tpm* = 79.95 hours (compared to 70.17 hours) and *TC* = $31,340 (compared to $31,236). A wider interval implies fewer maintenance actions (e.g., 1500/79.95 ≈19 against 1500/70.17 ≈ 22), but it also means a higher number of unplanned failures during the interval, as well as higher total costs.

In this example, when the *Upper Bound* value falls below 45%, both the cost-prioritized and the *tpm*-prioritized solutions become identical. For example, at *Upper Bound* = 0.45, we have TC = $31,295, and *tpm* = 63.7 hours in both the solutions. The analysis indicates that as the *Upper Bound*—the failure probability tolerance for the set of machines—falls below a certain level, the emphasis shifts to machine reliability and the cost-prioritized approach can no longer generate a solution which reduces the total costs while satisfying the failure probability constraint at the same time.

### 7.3 The Group PM Planning Based on Effective Interval

In an effort to overcome the limitations of the cost-based and the reliability-based maintenance plans, we resorted to a multi-objective model that combines the two approaches with the expectation of determining a PM plan which will optimize the costs while at the same time ensure the desired machine reliability threshold. As noted above, the combined model fulfills this expectation up to a limit on the *Upper Bound* parameter, after which the model generates reliability-based solutions to satisfy the constraints on machine failure probabilities. Thus, although reliability-based maintenance planning is more costly, it turns out to be the only option for developing maintenance plans when the reliability expectations of the system are higher than threshold. Based on this observation, we modify the reliability-based model to develop a group PM plan centered on an 'effective' maintenance interval. An algorithm would outline the steps in the development of the methodology, and a numerical example would demonstrate its application.

To motivate the development of the algorithm, we note that the reliability approach determines *tpr* according to the *Upper Bound* parameter corresponding to a machine *j* for which the $\theta_j$ and $\beta_j$ values generate the optimum *tpr*. This is illustrated by examining Table 7.3, where it is observed that the failure rate of machine M2 at *tpr* = 42.25 hours is at the limit of 0.30, and that the failure rates of the other machines—evaluated at *tpr* = 42.25 hours—are all less than 0.30. Equivalently, we can evaluate 'maximum' PM intervals for the other machines—denoted as $Tmax_j$ and evaluated at the failure probability *Upper Bound* of 0.30—which are all higher than 42.25 hours. For instance, for machine M1 the failure probability at *tpr* = 42.28 hours is 0.142; however, at the failure probability *Upper Bound* of 0.30, the PM interval is 87.14 hours. This implies that if maintenance planning is carried out at intervals of *tpr* = 42.28 hours, machines such as M1 would undergo too many maintenance actions unnecessarily. By defining an 'effective' maintenance interval for a machine, we can avoid the unnecessary PM actions and still maintain a threshold on the machine failure probabilities. This idea underlies the development of the algorithm, which addresses the above limitation.

### 7.3.1 Algorithm for Effective Maintenance Planning

*Step1-* Input the values of *Co*, $CPMR_j$, $cf_j$, *Upper Bound*, $\theta_j$, $\beta_j$

*Step2-* Compute the optimum *tpr* using ***OptimInterval*** model, reproduced below:

***OptimInterval* Model**

Maximize *tpr* -------- (7.15)

Subject to:

$$tpr \le \theta_j \{\ln\left[\frac{1}{1-F_j(tpr)}\right]\}^{1/\beta_j} \quad j=1,2,\ldots,m \qquad \text{-------- (7.16)}$$

$$F_j(tpr) \le Upper\ Bound, \quad j=1,2,\ldots,m \qquad \text{-------- (7.17)}$$

*Step3-* Compute the maximum possible interval, $Tmax_j$, for each machine by setting $F_j(tpr)$ = *Upper Bound* in equation (7.16):

$$T\max_j = \theta_j \{\ln\left[\frac{1}{1-(Upper\ Bound)}\right]\}^{1/\beta_j} \qquad \text{--------- (7.18)}$$

*Step 4-* Compute the total cost *TCr (T)* for the planning period T using the above inputs in the following sequence.

$$T\max_j \le tprY_j, \quad \forall j \qquad \text{------- (7.19)}$$

$$efftp_j = tprY_j, \quad \forall j \qquad \text{------- (7.20)}$$

$$T \ge efftp_j N_j, \quad \forall j \qquad \text{--------- (7.21)}$$

$$N_{\max} = \max\{N_j, j=1,2,..m\} \qquad \text{--------- (7.22)}$$

$$CPcell = N_{\max} Co + \sum_{j=1}^{m} N_j CPMR_j \qquad \text{--------- (7.23)}$$

$$CFcell = \sum_{j=1}^{m} N_j cf_j \left(\frac{efftp_j}{\theta_j}\right)^{\beta_j} \qquad \text{---------- (7.24)}$$

$$TCr(T) = CPcell + CFcell \qquad \text{-------- (7.25)}$$

$Y_j$, $N_j$ *integer*

In this model, $Y_j$ computes the equivalent number of optimum intervals for the maximum interval value of machine *j*, $efftp_j$ represents the effective preventive maintenance interval applicable to machine *j*, and $N_j$ is the number of preventive maintenance actions to be scheduled for machine *j*.

*CPcell* = preventive maintenance cost, and *CFcell* = failure repair cost

*Step 5:* record *CPcell, CFcell, TCr(T),* $N_j$, *Nmax*

*Step 6:* develop preventive maintenance schedule for the group of machines according to $N_j$

### 7.3.2 Numerical Example 4

The algorithm is illustrated by Numerical Example 1, which is used to demonstrate the cost-based approach. The input data required in *Step 1* of the algorithm is given in Table 7.1.

In *Step 2*, *tpr* is computed assuming a maximum acceptable failure probability (*Upper Bound)* of 0.30. The solution of the *OptimInterval* model is presented in Table 7.6. The optimum interval *tpr* turns out to be 42.28 hours. Using this *tpr*, the model next computes the probability of failure for each machine, as shown in the second row of the Table. It is noted at this stage that the solution of the *OptimInterval* model is the same as presented in Table 7.3. For ease of reference when considering the algorithm steps, the results are reproduced in Table 7.6. Table 7.6 reveals that the optimum *tpr* corresponds to machine M2 whose failure probability is at the *Upper Bound* level of 0.30. It is evident from the model solution that M2 has the minimum *tpr* for the combination $MTBF_2$, $\theta_2$, and $\beta_2$ among the seven machines. For the other machines, the failure probabilities computed at *tpr = 42.28* are less than 0.30. Equivalently, the corresponding *tpr* for these machines would be higher than *42.28* hours if their failure probabilities are set at the *Upper Bound* value. This is done by implementing *Step 3* of the algorithm, which computes the maximum possible intervals ($Tmax_j$) for machines other than M2, as given in the third row of Table 7.6.

Detailed output from *Step 4* of the algorithm is presented in Table 7.7. For practical considerations, we set *tpr* = 42. Using this *tpr*, equation (7.19) evaluates the equivalent number of common preventive maintenance intervals according to the maximum possible interval $Tmax_j$ for machine *j*. For example, the maximum preventive maintenance interval of 100.78 hours—computed for M3—can be written as: $100.78 \leq 42.00*2$, that is, $Y_3 = 2$.

Equation (7.20) computes the effective preventive maintenance interval for machine *j* as a multiple of the optimum common preventive maintenance interval *tpr*. It may be noted that this interval is within the allowable limit of failure probability. For example, the effective interval for machine M3 is 84 hours, which is two times the common preventive maintenance interval (i.e., $Y_3$ =2). Thus, $Y_j$ determines the preventive maintenance schedule. In our example, there are 36 PM actions, so that when $Y_j$ = 1, machine *j* is scheduled for preventive maintenance in every period and the schedule is 1, 2,…, 36. When $Y_j$ = 2, machine *j* is scheduled for preventive maintenance every other period and the schedule is 1, 3, 5, .., 35, and so on.

Equation (7.21) computes the number of times preventive maintenance action is carried out on each machine *j* during the planning period *T*. For example, M3 undergoes preventive maintenance 18 times during the planning period of 1500 hours. Equation (7.22) computes $N_{max}$ —the maximum number of times preventive maintenance is carried out in the cell—a parameter which is used in the calculation of the total PM costs in equation (7.23).

Equation (7.25) computes the total maintenance cost or *TCr(T)* for the cell over the planning period *T*. The components of *TCr(T)* are the fixed costs incurred whenever a preventive maintenance action takes place, plus the total preventive maintenance costs (Equation 7.23), as well as the total failure repair costs (Equation 7.24). It is noted that $N_{max}$ in this case equals 36, as seen in Table 7.7.

**Table 7.6: Solution of the *OptimInterval* model**

| Data Type | MACHINES | | | | | | |
|---|---|---|---|---|---|---|---|
| | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| *tpr* (hours) | 42.28 | | | | | | |
| Failure probability for machine *j* at *tpr=42.28* | 14.18% | *30%* | 7.32% | 22.38% | 27.59% | 8% | 24.44 |
| *Tmaxj* at 30% failure rate | 87.14 | 42.28 | 100.78 | 56.77 | 47.69 | 107.48 | 50.67 |

**Table 7.7: Solution of the *TotalCost* Model**

| Data Type | MACHINES | | | | | | |
|---|---|---|---|---|---|---|---|
| | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
| Effective interval for machine *j*, (*efftp<sub></sub>*$_j$) | 84 | 42 | 84 | 42 | 42 | 84 | 42 |
| Number of preventive maintenance actions ($N_j$) | 18 | 36 | 18 | 36 | 36 | 18 | 36 |
| Maintenance schedule for machine *j* | 1, 3,..,35 | 1,2, .., 36 | 1,3,.., 35 | 1,2,.., 36 | 1,2,.., 36 | 1,3, ., 35 | 1,2, .., 36 |
| *CPcell* (Total preventive maintenance cost) | $11,880 | | | | | | |
| *CFcell* (Total failure repair cost) | $19,789 | | | | | | |
| Total maintenance cost | $31,669 | | | | | | |

## 7. 4. Integrating PM Interval in the CMS Design Model

### 7.4.1 Machine Reliability Analysis in a Process Plan Route

To incorporate PM planning in machine reliability analysis along a process plan route for a CMS, we shall follow the approach developed in Chapter 2. Considering the numerical example of a 5 machine cell processing 4 part types, in section 2.5.2, system reliability equation (2.29) for the part processing route (1203), considering machine reliability to follow the Weibull distribution, is :

$$R_s(1203) = \exp[-(\frac{t}{\theta_1})^{\beta_1}] \times \exp[-(\frac{t}{\theta_3})^{\beta_3}] \times \exp[-(\frac{t}{\theta_4})^{\beta_4}] \qquad \text{--------------- (7.26)}$$

### 7.4.2 Preventive Maintenance Consideration in Machine Reliability Analysis

We consider the CMS discussed in Section 7.4.1 where there is a PM schedule defined by the organization based on the approach described in Section 7.3, and where *tpr* is the common maintenance interval. Thus, a machine $j$ will undergo PM, a total of $N_j$ times during the planning period $T$:

$$R_j(T) = [R_j(Y_j tpr)]^{N_j} R_j(T - N_j Y_j tpr) \qquad \text{------ (7.27)}$$

Here, we assume that the machine is restored to as-good-as new condition after a PM action is administered every $Y_j\, tpr$ time units (Ebeling, 1997). For Weibull distribution equation (7.27) becomes:

$$R_j(T) = \exp[-N_j\left(\frac{Y_j tpr}{\theta_j}\right)^{\beta_j}]\exp[-\left(\frac{T - N_j Y_j tpr}{\theta_j}\right)^{\beta_j}] \qquad \text{------ (7.28)}$$

Substituting equation (7.28) in system reliability equation of (7.26)

$$R_s(1203) = \exp[-N_1\left(\frac{Y_1 tpr}{\theta_1}\right)^{\beta_1}]\exp[-N_3\left(\frac{Y_3 tpr}{\theta_3}\right)^{\beta_3}]\exp[-N_4\left(\frac{Y_4 tpr}{\theta 4}\right)^{\beta_4}]\times$$
$$\exp[-\left(\frac{T - N_1 Y_1 tpr}{\theta_1}\right)^{\beta_1}]\exp[-\left(\frac{T - N_3 Y_3 tpr}{\theta_3}\right)^{\beta_3}]\exp[-\left(\frac{T - N_4 Y_4 tpr}{\theta_4}\right)^{\beta_4}] \qquad \text{---- (7.29)}$$

which can be simplified as :

$$\ln\frac{1}{R_s(1203)} = N_1\left(\frac{Y_1 tpr}{\theta_1}\right)^{\beta_1} + N_3\left(\frac{Y_3 tpr}{\theta_3}\right)^{\beta_3} + N_4\left(\frac{Y_4 tpr}{\theta_4}\right)^{\beta_4} +$$
$$\left(\frac{T - N_1 Y_1 tpr}{\theta_1}\right)^{\beta_1} + \left(\frac{T - N_3 Y_3 tpr}{\theta_3}\right)^{\beta_3} + \left(\frac{T - N_4 Y_4 tpr}{\theta_4}\right)^{\beta_4} \qquad \text{---------- (7.30)}$$

It may be recalled that, for each machine, the planning period $T$ is divided into a number of **equivalent intervals,** $N_j$ (see equations (7.20) and (7.21)); thus, the term $T\text{-}N_j Y_j\, tpr$ does not exist, and equation (7.30) reduces to:

$$\ln\frac{1}{R_s(1203)} = N_1\left(\frac{Y_1 tpr}{\theta_1}\right)^{\beta_1} + N_3\left(\frac{Y_3 tpr}{\theta_3}\right)^{\beta_3} + N_4\left(\frac{Y_4 tpr}{\theta_4}\right)^{\beta_4} = LIR_{12} \quad \text{--7.31)}$$

In general, for an *(ip)* combination, $LIR_{ip}$ can be represented

$$LIR_{ip} = \sum_{j \in J_{ip}} N_j \left(\frac{Y_j tpr}{\theta_j}\right)^{\beta_j} \quad \text{----------- (7.32)}$$

where $J_{ip}$ is the set machines along the process plan $p$ for processing part type $i$.

Using the expression, $\theta_j = \dfrac{MTBF_j}{\Gamma(1+1/\beta_j)}$ the equation (7.31) can be written as:

$$LIR_{ip} = \sum_{j \in J_{ip}} N_j \left(\frac{Y_j tpr(\Gamma(1+1/\beta_j))}{MTBF_j}\right)^{\beta_j} \quad \text{--------}$$

(7.33)

$LIR_{ip}$ is the inverse of system reliability (in the natural logarithmic scale) corresponding to the machines along a part-processing route *(ip)* when machine failure follows a Weibull distribution and the PM on machine $j$ is conducted at intervals of $Y_j tpr$.. Thus, minimizing $LIR_{ip}$ will increase the system reliability along *(ip)*. This is in fact one of the objectives of the CMS design model discussed in the next section.

### 7.4.3 CMS Design Model

To integrate machine reliability and maintainability in the CMS design process, we consider the CMS design model based on machine reliability to follow Weibull distributions from Section 4.3.

As demonstrated in Section 4.3, the multi-objective CMS design model optimizes system costs in the objective function I and system reliability of machines in the objective function II. To integrate the preventive maintenance planning in the CMS design process we substitute equation (7.33) in the expression for objective function II in the CMS design model.

The modified model may now be presented as follows:

- *Minimize* objective function I:

$$F_1 = VCM + MHC + MNC \quad \text{-------- (7.34)}$$

where, $VCM = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \{CO_{oj}(ip) + CR_{oj}(ip)\} \sum_{c=1}^{C} X_{ojc}(ip)$ -------- (7.34a)

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j \in J_{ipo}} \sum_{\hat{j} \in J_{ip(o+1)}} \sum_{1 \le c, \hat{c} \le C} H_{ijc\hat{j}\hat{c}} Y_{ojc\hat{j}\hat{c}}(ip) \quad \text{-------- (7.34b)}$$

$$MNC = \sum_{j=1}^{m} cp_j \left( 1 - \left[ \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \frac{TO_{oj}(ip) + TR_{oj}(ip)}{A_j(T) b_j} \right] \right) \sum_{c=1}^{C} X_{ojc}(ip) \quad \text{-------- (7.34c)}$$

- Minimize objective function II:

$$F_2 = \sum_{i=1}^{n} \sum_{p=1}^{P(i)} LIR_{ip} \quad \text{----- (7.35)}$$

where, $LIR_{ip} = \sum_{o=1}^{O(ip)} \sum_{j \in J_{ip}} \sum_{c=1}^{C} N_j \left( \frac{Y_j tpr(\Gamma(1 + 1/\beta_j))}{MTBF_j} \right)^{\beta_j} X_{ojc}(ip) \quad \forall i, p$ ------ (7.35a)

using the definitions in equation (7.33)

**Constraints:** Following constraints are defined:

$$\sum_{p=1}^{P(i)} Z(ip) = 1 \quad \forall i \quad \text{--------- (7.36)}$$

$$\sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) = Z(ip) \quad \forall i, p, o \quad \text{-------- (7.37)}$$

$$\sum_{c=1}^{C} M_{jc} \le 1 \quad \forall j \quad \text{------- (7.38)}$$

$$\sum_{j=1}^{m} M_{jc} \le UM \quad \forall c \quad \text{---------- (7.39)}$$

$$\sum_{i=1}^{n} \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} X_{ojc}(ip) \ge M_{jc} \quad \forall j, c \quad \text{---------- (7.40)}$$

$$\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} [\ TO_{oj}(ip) + TR_{oj}(ip)\ ] X_{ojc}(ip) \le b_j M_{jc} A_j(T) \quad \forall j, c \quad \text{--------- - (7.41)}$$

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - 2Y_{ojc\hat{j}\hat{c}}(ip) \ge 0, \ldots\ldots \forall i, p, o \in \{1,2,\ldots\ldots O(ip)-1\}, j \in \{J_{ipo,} \hat{j} \in J_{ip(o+1)}, c, \hat{c}$$

-------------- (7.42)

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - Y_{ojc\hat{j}\hat{c}}(ip) \le 1 \quad \forall i, p, o \in \{1,2,\ldots O(ip)-1\}, j \in J_{ipo}, \hat{j} \in J_{ip(o+1),} c, \hat{c}$$

-------------- (7.43)

To illustrate the applicability of the model, we have solved the 7 machine 12 part cell design problem (Example 1 in section 4.3.3). Table 4.1 gives the part processing information—including part demand—and Tables 4.2 and 7.1 gives the relevant machine and maintenance cost related information for the problem. We first solve the model by taking a hierarchical approach for *optimizing cost only*, and *optimizing reliability only*. After computing the highest achievable reliability from the solution of reliability only optimization, we solve *optimizing cost subject to achievement of the highest reliability*. Table 7.8 compares the model outputs with and without preventive maintenance.

**Table 7.8: Comparison of model outputs for considering and ignoring preventive maintenance (CMS design model based on machine failures to follow Weibull distribution)**

| Comparison Factors | Model outputs | | Comments |
|---|---|---|---|
| | **Preventive maintenance is considered** | **Preventive maintenance is not considered** | |
| *Min* = **Objective function I only** | | | Considerable decrease in the objective function II value when PM is considered. |
| **Objective function I** | **$1,768.45** | **$1,768.45** | |
| Objective function I components | | | |
| VCM | $1,410.33 | $1,410.33 | |
| MHC | $50.00 | $50.00 | |
| MNC | $308.12 | $308.12 | |
| **Objective function II** | **257. 6** | **1110.7** | |
| Cell configuration | Cell 1 =[M1,M2,M3,M4] Cell 2=[M5,M6,M7] | Cell 1 =[M1,M2,M3,M4] Cell 2=[M5,M6,M7] | |
| *Min* = **Objective function II only** | | | 1. Objective function II value improved to a great extent when PM is considered. 2. Changes in other aspects are negligible. |
| **Objective function I** | **$3,401.00** | **$3,320.19** | |
| Objective function I components | | | |
| VCM | $1,827.00 | $1,712.43 | |
| MHC | $750.00 | $650.00 | |
| MNC | $824.00 | $957.76 | |
| **Objective function II** | **197.2** | **$715.4** | |
| Cell configuration | Cell 1=[M4,M7], Cell2 =[M1,M6], Cell 3 =[M3] | Cell 1=[M1,M7], Cell2 [M4,M6], Cell 3 =[M2] | |
| *Min* = **Objective function I,** *s. t.* **Objective function II ≤ Є** | Є=197.2 | Є=715.4 | As expected the model achieved the objective function II according to the target and improved cost factors when preventive maintenance is considered |
| **Objective function I** | **$2,778** | **$2822.90** | |
| Objective function I components | | | |
| VCM | $1768.00 | $1632.92 | |
| MHC | $200.00 | $200.00 | |
| MNC | $810.00 | $989.98 | |
| **Objective function II** | **197.2** | **715.4** | |
| Cell configuration | Cell 1 =[M1,M3,M6,M7] Cell 2=[M4] | Cell 1 =[M1,M2,M4,M6] Cell 2=[M7] | |
| **Total maintenance cost** | **$31,669.00** | **$83,653.00** | Maintenance cost with PM steps is much lower than the cost when PM is ignored. |

When PM is included, the outputs of the CMS design model demonstrate an improvement in the system reliability by a considerable decrease in the objective function II value (the inverse of system reliability in the logarithmic scale). For example, the objective function II value for optimizing costs, optimizing reliability, and optimizing costs subject to the achievement of highest reliability obtained from the solutions are 257.6, 197.2 and 197.2 respectively—against the values 1110.7, 715.4 and 715.4 when PM has been ignored. This improvement in the system reliability is an expected outcome of preventive maintenance planning. Table 7.8 also demonstrates that the inclusion of PM improves the total operations and machine utilization related costs, as well as changes the cell configuration when the model is solved for simultaneous consideration of system reliability and system costs. For example, the model solution for optimizing costs subject to achievement of the highest system reliability (*Min* = objective function I, *s. t.* Objective function II $\leq$ 197.2) reduces the total cost to $2,778.00 from the previous level of $2,822.90 when PM was not been considered. Cell configuration is changed to Cell 1 = [M1,M3,M6,M7], Cell 2 = [M4] as opposed to the previous Cell 1 = [M1,M2,M4,M6] and Cell 2 = [M7]. In addition to this, PM improves overall performance by reducing total maintenance cost. For example, it can be seen in Table 7.8 that by ignoring PM considerations, total maintenance cost comes to $83,653.00, against $31,669.00, when PM is considered.

There is, however, no considerable influence of preventive maintenance on the cell configuration and system costs when the model is solved for only cost (*Min* = Objective function I only) or reliability optimization (*Min* = Objective function II only).

The reliability based PM planning model, centered on the 'effective interval' of individual machines as developed in this chapter for CMS, will aid the user organization to improve resource utilization, and maintenance costs, while at the same time fulfill the desired reliability target. The integration of the PM planning policies in CMS design model will improve the overall performance of the CMS in terms machine reliability.

## CHAPTER 8
## DISCUSSIONS, CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

### 8.1 Discussions and Conclusions

Despite the extensive research that has been conducted in the area of cellular manufacturing systems, only a few studies address the consideration of machine reliability in the design and analysis of CMSs. Another important aspect, which has rarely received attention in the design process, is the simultaneous consideration of system reliability and system cost.

The primary goal of this research was to develop a CMS design methodology that considers machine reliability and system costs simultaneously. A multi-objective, mixed-integer mathematical programming model for designing cellular manufacturing systems has been developed. The model takes into account the manufacturing system costs and reliability of machines along the part-processing routes. The components of system costs are the variable cost of processing the part types, inter-cell material handling costs and machine under-utilization costs. The system reliability of machines along the part processing routes considers the updated reliability status of each individual machine for performing operations on the part types in the design process of CMSs. The approach allocates parts to the processing routes in order to optimize cost and achieve the lowest probability of machine failure while developing the manufacturing cells. Moreover, the approach incorporates provisions for rerouting the parts in the case of machine failure during processing. The potential benefits of the design approach have been demonstrated by the analysis of the numerical examples.

Machine reliability analysis and its integration into the CMS design process is achieved by using the exponential and the Weibull distribution. The exponential distribution approach may make the design more tractable and easy to implement. The model based on this approach requires only the basic machine-maintenance related information, such as MTBF and MTTR, for developing a satisfactory CMS design and improving the overall performance of the system. Since most real world production machines experience increasing failure rate with time, the exponential distribution approach has the practical limitation of not representing the reliability behavior of the system for the aging machines. The Weibull distribution is a versatile approach for machine reliability analysis, and is able to deal with the increasing, decreasing and constant failure rate of machines. This research has developed a CMS design model based on the Weibull distribution that incorporates the increasing failure rate of machines in the design process. The

CMS design models—based on the two most frequently used distributions for machine reliability—have made the approach both suitable and practical for industrial users, and have provided options for selecting the one appropriate for the organization.

The applicability of multi-objective CMS design models has been illustrated by solving numerical examples using the $\epsilon$-constraint approach in Chapter 4. The $\epsilon$-constraint method of solution is easy to understand and analyze, and may be used to balance the reliability and cost requirements of a business.

A performance evaluation model based on the Markovian analysis approach has also been developed, and incorporated in the CMS design model, based on the exponential distribution. This allows the designer or user to emphasize the priority and performance of the individual part-plan combination for each design input according to the system reliability requirements and cost combination. This integration of performance evaluation into the CMS design model provides the user and/or designer with multiple options to select from.

Due to a demanding amount of computational time and resource requirement for large problems, the developed model is not suitable for solving a realistic-size CMS design problem. This led to the development of a heuristic which is able to solve realistic-size problems within a reasonable amount of time and resources. A simulated annealing-based algorithm which is further improved by including GA operators (crossover and mutation) to generate better neighboring solutions has been developed and implemented.

The performance of the heuristic has been evaluated by estimating the GAP% between the heuristic solution and optimal solutions (whenever it was possible to obtain using commercial software), and by estimating the GAP% between the heuristic solution and the LP relaxation solution for large, realistic-size problems. The heuristic performs sufficiently in terms of solution generation time and solution quality for almost all the problem instances attempted in this dissertation. The GAP% comes very close to optimal for reasonable-size problems, while exhibiting a very small variation from the LP relaxation solution for large problems.

The model outputs' sensitivity to changes in key factors has also been investigated to provide a performance perspective for the CMS designer and user organization, so that they can plan to manage the impacts. The effect of the machine reliability related parameter *MTBF* on the model outputs is significant, based on both exponential and Weibull distributions. The impact of another reliability-related parameter *MTTR* is not as high as *MTBF*, but it does affect the output to a reasonable extent in the case of both exponential, and Weibull distribution-based models.

To restrict the deterioration of manufacturing machines and improve the overall performance of the system, this dissertation developed a preventive maintenance planning model for CMSs that combines relevant costs, and machine reliability. The multi-machine preventive maintenance model takes into account the interdependence of CMS machines, resource utilization (in terms of useful resource life), repair cost, and other related costs to decide on preventive maintenance intervals and maintenance scheduling for the individual machines. The dissertation also outlines the development of the CMS design model by integrating preventive maintenance planning policies for system reliability and system costs. Preventive maintenance considerations and the integration of preventive maintenance planning in the design process will make a significant contribution to the overall CMS performance. The solution and analysis—in relation to the maintenance consideration—is illustrated by a few problem instances.

## 8.2 Research Contributions

The research for this work contributes to the area of cellular manufacturing by introducing a multi-objective design model that simultaneously considers the system costs and reliability of machines along part processing routes. The contributions of the research may be summarized as follows:

1. A multi-objective MIP model for designing cellular manufacturing systems has been developed by incorporating the following important factors:
    a. estimation of effective machine capacity by considering machine availability;
    b. selection of part processing routes to achieve the highest system reliability of the machines along the route;
    c. optimization of system costs which consist of part processing cost, inter-cell material handling cost and machine under-utilization cost ;
    d. and inclusion of rerouting options for the parts in case of machine failure.
2. A machine reliability analysis model for the CMS has been developed based on:
    a. the exponential distribution;
    b. and the Weibull distribution.
3. The machine reliability analysis model has been integrated with the basic CMS design model to simultaneously consider system cost and system reliability.
4. A performance evaluation model has been developed by following the Markovian approach, and the performance evaluation in terms of system availability is incorporated in the CMS design model based on the exponential distribution.
5. A heuristic has been developed--based on the basic steps of simulated annealing and solution generation procedure (crossover and mutation) of GA.

6. A reliability-based preventive maintenance planning model for the CMS has been developed to improve the resource utilization and maintenance cost, and to achieve the desired reliability at the same time.
7. A CMS design model is outlined to integrate preventive maintenance planning policies with system reliability and cost and incorporate in the design process for overall improvement of the system's reliability performance.

## 8.3 **Future Research**

The possibilities for future research in this area include:

1. developing a CMS design model that includes any future change in reliability and the dynamic demand of parts;
2. using Genetic Algorithms to solve the model, and comparing the performance of GA-based heuristics with SA-based heuristics;
3. solving real world CMS design problems by integrating preventive maintenance and machine reliability consideration to further explore the applicability of the model;
4. investigation of other reliability distributions in the cell formation and work allocation design; and
5. evaluating the performance of the heuristic in relation to its various factors by developing an appropriate experimental design.
6. developing a cell layout according to the integrated reliability, cost, and maintenance planning based CMS design
7. developing a job scheduling plan to optimize processing times of jobs in a reliability based cell design

## REFERENCES


Abdelmola, A. I. and Taboun, S. M.(1999), Productivity model for cell formation problem: a simulated annealing algorithm, *Computers and Industrial Engineering*, 37(1-2), 327-330

Abernethy, R. B (1996), *The New Weibull Handbook*, (Florida: Robert B. Abernethy, self published)

Agarwal, A., and Sarkis, J. A. (1998), Review and analysis of comparative performance studies of functional and cellular manufacturing layouts, *Computers and Industrial Engineering*, 34(1), 77-89

Akturk, M. S., and Turkan, A. (2000), Cellular manufacturing system design using a holonistic approach, *International Journal of Production Research*, 38(10), 2327-2347

Albadawi, Z., Bashir, H. A., and Chen, M.(2005), A mathematical approach for the formation of manufacturing cells, *Computers and Industrial Engineering*, 48 (1), 3-21

Albino, V., Okogbaa, O.G., and Shell, R. L.(1990), A computerized integrated performance-reliability measure of flexible automated production system, *Computers and Industrial Engineering*, 18(4), 547-558

Albino, V., Carella, G. and Okogbaa, O. G. (1992), Maintenance policies in just-in-time manufacturing lines, *International Journal of Production Research*, 30(2), 369-382

Arzi, Y., Bukchin, J., and Masin, M. (2001), An efficiency frontier approach for the design of cellular manufacturing systems in a lumpy demand environment, European Journal of Operational Research, 134(2), 346-364

Askin, R., and Chieu, K. (1990), A graph partitioning procedure for machine assignment and cell formation in group technology, *International Journal of Production Research*, 28(8), 1555-1572

Askin, R. G., and Estrada, S. (1999), Investigation of cellular manufacturing practices, In S. A. Irani(ed), *Handbook of Cellular Manufacturing Systems* (New York: John Wiley and Sons) pp 25-34

Askin, R. G., Selim, H. M., and Vakharia A. J (1997), A methodology for designing flexible cellular manufacturing systems, *IIE Transactions*, 29(7), 599-610

Asokan, P., Prabhakaran, G., and Kumar, G. S. (2001), Machine-cell grouping in cellular manufacturing systems using non-traditional optimization techniques-a comparative study, *International Journal of Advanced Manufacturing Technology*, 18(2), 140-147

Atmani, A., Lashkari, R. S., and Caron, R. J. (1995), A mathematical programming approach to joint cell formation and operation allocation in cellular manufacturing, *International Journal of Production Research*, 33(1), 1- 15

Ballakur, A., and Steudel, H.( 1987), A within-cell utilization based heuristic for designing cellular manufacturing systems, *International Journal of Production Research*, 25(5), 639-665

Baykasoglu, A., Gindy, N. N. Z., and Cobb, R. C. (2000), Capability based formulation and solution of multiple objective cell formation problems using simulated annealing, *Integrated Manufacturing Systems*, 12(4), 258-274

Berardi, V. L., Zhang, G., and Offodile, F. (1999), A mathematical programming approach to evaluating alternative machine clusters in cellular manufacturing, *International Journal of Production Economics*, 58(3), 253-264

Black, J. T. (1991), *The Design of the Factory with a Future* (New York: McGraw-Hill,)

Boughton, N. J., and Arokiam, I. C. (2000), The application of the cellular manufacturing: a regional small to medium enterprise perspective, Proceedings of the Institution of Mechanical Engineers, Vol 214 (Part B), 751-754

Burbidge, J.L.(1975), *The introduction of Group Technology* ( New York: Wiley)

Cao, D., and Chen, M. (2004), Using penalty function and tabu search to solve cell formation problems with fixed cell cost, *Computers and Operations Research*, 31(1), 21-37.

Caux, C., Bruniaux, R., and Pierreval, H. (2000), Cell formation with alternative process plans and machine capacity constraints: a new combined approach, *International Journal of Production Economics*, 64(1-3), 279-284

Chaharbaghi, K., and Davies, B. L. (1986), The analysis of flexible manufacturing systems for reliability, *The International Journal of Advanced Manufacturing Technology*, 1(4), 79-100

Chandrasekharan, M. P. and Rajagopaln, R. (1986), An ideal seed non-hierarchical clustering algorithm for cellular manufacturing, *International Journal of Production Research*,24(2), 451-464

Chan, H., and Milner, D.(1982), Direct clustering algorithm for group formation in cellular manufacture, *Journal of Manufacturing Systems,* 1(1), 64-76

Chen, M. (1998), A mathematical programming model for system reconfiguration in a dynamic cellular manufacturing environment, *Annals of Operations Research*, 77, 109-128

Cho,I.D., and Parlar, M.(1991), A survey of maintenance models for multi-unit systems, *European Journal of Operational Research*,51, 1-23

Dahel, N.-E., and Smith, S. B. (1993), Designing flexibility into cellular manufacturing systems, *International Journal of Production Research*, 31(4), 933-945

Dai, Y., Zhou, Y.-f., and Jia, Y.Z. (2003), Distribution of time between failures of machining center based on type I censored data, *Reliability Engineering and System Safety*, 79(3), 377-379

Dekker, R., Van Der Schouten, F., Wilderman, R., 1997, A review of multi-component maintenance models with economic dependence, *Mathematical Methods of Operational Research,* 45(3), 411-435

Diallo, M., Perreval, H., and Quillot, A. (2001), Manufacturing cell design with flexible routing capability in presence of unreliable machines. *International Journal of Production Economics*, 74(1-3), 175-182.

Dowsland, K.(1993), Simulated Annealing, In C. Reeves (ed), *Modern Heuristics Techniques for Combinatorial Problems,* (Oxford : Blackwell), pp20-63

Ebeling, C. E. (1997), *An Introduction to Reliability and Maintainability Engineering* (New York: McGraw-Hill)

Faber, Z., and Carter, M. W. (1986), A new graph theory approach for forming machine cell in cellular production systems, In A Kusiak (ed), *FMS: Methods and Studies*( North Holland), 301-315

Falkenauer, E. (1998), *Genetic Algorithm Grouping Problem* (Wiley: New York)

Flynn, B. B., and Jacobs, F. R. A. (1986), A simulation comparison of group technology with traditional job shop manufacturing, *International Journal of Production Research*, 24(5), 1171-1192

Garey, M., and Johnson, D. (1979), *Computers and Intractability: A Guide to Theory of NP-completeness* ( San Francisco, CA :Freeman)

Garret, S. E.(1986), Strategy first: A case in FMS justification, In K. E Stecke and R. Suri (ed), Proceedings of the Second Special Interest Conference on FMS, Ann Arbor, MI, pp17-30

Gindy, N. N. Z., Ratchev, T. M., and Case, K. (1996), Component grouping for cell formation using resource elements, *International Journal of Production Research*, 34(3), 727-752

Groover, M. P.(2001), *Automation, Production Systems and Computer-Integrated Manufacturing*, (NJ: Prentice Hall)

Gupta, S. M., and Kavusturucu, A.(1998), Modeling of finite buffer cellular manufacturing systems with unreliable machines, *International Journal of Industrial Engineering*, 594), 265-277

Hariga, M. A.(1996), A maintenance inspection model for a single machine with general failure distribution, *Microelectronics Reliability*, 36(3), 353-358

Heady, R. B. (1997), Forming minimum-cost machine cells with exceptional parts using zero-one integer programming, *Journal of Manufacturing Systems*, 16(2), 79-90

Heragu, S. S., and Chen, J-S.(1998), Optimal solution of cellular manufacturing system design: benders, decomposition approach, *European Journal of Operational Research*,107(1),175-192

Heragu, S. S.(1994), Group technology and cellular manufacturing, *IEEE Transactions on Systems, Man and Cybernetics,* 24(2),203-215

Ireson,W.G., Coombs-Jr,C.F., and Moss, R.Y.(1996), *Handbook of Reliability Engineering and Management* (New York :The McGraw-Hill Company)

Irani, S. A., Subramonian, S., and Allam, Y. S. (1999), Introduction to cellular manufacturing systems, In S. A Irani(ed), *Handbook of Cellular Manufacturing Systems* (New York :John Wiley and Sons), pp 1-24

Jayaswal, S., and Adil, G. K.(2004), Efficient algorithm for cell formation with sequence data , machine replications and alternative process routings, *International Journal of Production Research*, 42(12), 2419-2433

Jardine, A. K. S. (1973), *Maintenance, Replacement and Reliability*, (Kingsway, London: Pitman Publishing)

Jeon, G., Broering, M., Leep, H. R., Parsei, H. R., and Wong, J. P. (1998a) Part family formation based on alternative routes during machine failure, *Computers and Industrial Engineering*, 35(1-2),73-76

Jeon, G., Leep, H. R., and Parsei H. R.(1998b), Cellular manufacturing system based on new similarity coefficient which considers alternative routes during machine failure, *Computers and Industrial Engineering*, 34 (1), 21-36

Johnson, L. G. (1959), *The Statistical Treatment of Fatigue Experiments*, Research Laboratories, General Motors Corporation, p IVA, 105.

Joines , J. A., King, R. E., and Culbreth, C. T. (1996), A comprehensive review of production–oriented manufacturing cell formation techniques, *International Journal of Flexible Automation and Intelligent Manufacturing*, 3(3/4), 161-120

Kalir, A. and Arzi, Y.(1998), Optimal design of flexible production lines with unreliable machines and infinite buffers, *IIE Transactions*, 30(4) ,391-399

Kaparthi , S., and Suresh, N. C. (1991), A neural network system for shape based classification and coding of rational parts, *International Journal of Production Research*, 29(9), 1771-1784

Kardon, B., and Fredendall, L. D.(2002), Incorporating overall probability of system failure into a preventive maintenance model for a serial system, *Journal of Quality in Maintenance Engineering*, 8(4), 331-345

King, J. R. (1980), Machine component grouping in production flow analysis : an approach using rank order clustering algorithm, *International Journal of Production Research*, 18(2), 213-232

King, J. R., and Nakornchai, V. (1982), Machine-component group formation in group technology: review and extension, *International Journal of Production Research*, 20 (2), 117-133

Kirpatrick, S., Gelatt, C.D. Jr., and Vecchi, M.P.(1983), Optimisation by simulated annealing , *Science,* Vol 220(4598), 671-680

Kizil, M., and Ozbayrak, M. (2004), A tradeoff analysis between the process plan selection and cell formation in cellular manufacturing, *International Journal of Advanced Manufacturing Technology*, 23 (7-8), 501-506

Kuroda, M., and Tomita, T.( 2005), Robust design of a cellular-line production system with unreliable facilities, *Computers and Industrial Engineering*, 48 (3), 537-551

Kusiak, A. (1987), The generalized GT concept, *International Journal of Production Research*, 25(4), 561-569

Kusiak, A. and Chow, W. S.(1987) Efficient solving of GT problem, *Journal of Manufacturing Systems,* 6 , 117-124

Lashkari, R. S., and Kasilingam, R. G.(1990), A Lagrangian relaxation approach to machine allocation in cellular manufacturing systems, *Computers and Industrial Engineering*, 19, 442-446

Lee, S. D., and Chen, Y. L, (1997), A weighted approach for cellular manufacturing design: minimizing intercell movement and balancing workload among duplicated machines. *International Journal of Production Research*, 35 (4), 1125-1146

Li, Y.-C. E., and Shaw, W. H.(1998), Simulation modeling of a dynamic job shop rescheduling with machine availability constraints, *Computers and Industrial Engineering*, 35(1-2),117-120

Lin, C., Madu, C. N., Chien, W., and Kuei, C-H. (1994), Queuing models for optimizing system availability of a flexible manufacturing system, *Journal of Operations Research Society*, 45(10), 1141-1155

Liu, L., and Yuan, X-M.(2001), Throughput, flow times and service level in an unreliable assembly system, *European Journal of Operational Research*, 135 (3),602-615

Logendran, R.(1990), A workload based model for minimizing total intercell and intracell moves in cellular manufacturing, *International Journal of Production Research,* 28(5),913-925

Logendran, R.(1993), Methodology for converting functional manufacturing system into a cellular manufacturing system, *International Journal of Production Economics*, 29(1), 27-41

Logendran, R., and Ramkrishnan, P.(1997) A methodology for simultaneously dealing with machine duplication and part subcontracting in cellular manufacturing systems, *Computers and Operations Research*, 24(2), 97-116

Logendran, R., and Talkington, D. (1997) Analysis of cellular and functional manufacturing system in the presence of machine breakdown, *International Journal of Production Economics*, 53(3), 239-256

Mahesh, O. and Srinivasan, G. (2002) Incremental cell formation considering alternative machines, *International Journal of Production Research*, 40(14), 3291-3310

Malakooti, B., Malakooti, N. R. and Yang, Z.(2004) Integrated group technology, cell formation, process planning with application to emergency room, *International Journal of Production Research*, 42(9), 1769-1786

Mansouri, S. A., Husseini, S. M. M., and Newman, S. T.(2000) A review of modern approaches to multi-criteria cell design, *International Journal of Production Research*, 38(5), 1201-1218

McAuley, J., (1972) Machine grouping for efficient production, *The Production Engineer*, 51, 53-57

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., and Teller, E. (1953), Equation of steady state calculation by fast computing machines, *Journal of Chemical Physics*,21, 1087-1092

Miriyala, K., and Viswanadham, N.(1989), Reliability analysis of flexible manufacturing system, *The International Journal of Flexible Manufacturing Systems*, 2(2),145-162

Moon, C., and Gen, M. (1999), A genetic algorithm based approach for design of independent manufacturing cells, *International Journal of Production Economics*, 60-61 , 421-426

Morris, S. J., and Tersine, R. J. A. (1990), A simulation analysis of factors influencing the attractiveness of group technology cellular layouts, *Management Science*, 36(12), 1567-1578

Nagarajah, C. R., Thompson, W., and McFarlane, C. (1992), Effects of machine reliability and transporter speeds on flow times and production rates in component-process design, *International Journal of production Research*, 30 (2), 267-281

Offodile, O. F., Mehrez, A. and Grazner, J. (1994), Cellular manufacturing: a taxonomic review framework, *Journal of Manufacturing Systems*, 13(3), 196-220

Peker, A., and Kara, Y.(2004), Parameter setting of the Fuzzy ART neural network to part-machine cell formation problem, *International Journal of Production Research*, 42(6), 1257-1278

Plaquin, M-R., and Pierreval, H (2000) Cell Formation using evolutionary algorithms with certain constraints, *International Journal of Production Economics*, 64(1-3), 267-278

Rajamoni,D., Singh, N., and Aneja, Y. P.(1990),Integrated cellular manufacturing system in the presence of alternative process plans, *International Journal of Production Research*, 28(8) 1541-1554

Rajagopalan, R., and Barta, J.(1975), Design of cellular production systems : a graphic-theoretic approach, *International Journal of Production Research,* 13(6), 567-579

Ram, R., and Viswanadham, N. (1992), Performance evolution of cellular flexible manufacturing systems: a decomposition approach, *European Journal of Operations Research*, 57, 287-304

Ram, R., and Viswanadham, N. (1994) Performability of automated manufacturing systems with centralized material handling, *International Journal of Production Research,* 32(8), 1775-1799.

Rao, H. A., and Gu, P. (1993), Design of cellular manufacturing: a neural network approach, *International Journal of Production Research*, 30, 407-424

Rao, A. N., and Bhadhury, B.(2000), Opportunistic maintenance of multi-equipment systems: a case study, *Quality and Reliability Engineering International*, 16(6), 487-500

Rardin, R. L.(1998),*Optimization in Operations Research* (Upper Saddle River,NJ: Prentice Hall)

Rupe, J., and Kuo, W. (2001) Performability of FMS based on stochastic process models, *International Journal of Production Research*, 39(1), 139-155

Savsar, M. (2000) Reliability analysis of flexible manufacturing cell, *Reliability Engineering and System Safety*, 67(2), 147-152

Shafer, S. M., Kern, G. M. and Wei, J. C.(1992) A mathematical programming approach for dealing with exceptional elements in cellular manufacturing, *International Journal of Production Research*, 30(5), 1029-1036

Shafer, S. M., and Charnes, J. M.(1997), Offsetting lower routing flexibility in cellular manufacturing due to machine dedication, *International Journal of Production Research,* 35(2), 551-567

Shankar, K., and Agarwal, A. K. (1997) Models and solution methodologies for the generalized grouping problem in cellular manufacturing, *International Journal of Production Research*, 35 (2), 513-538

Sherwin, D. (1997), A note on block and bad-as-old renewal of components with limited system time horizon , *Journal of Quality Maintenance Engineering*, 3(2), 143-147

Seifoddini, S., and Djassemi, M.(2001), The effect of reliability consideration on the application of quality index, *Computers and Industrial Engineering*, 40(1-2 ),65-77

Selim, H. M, Askin, R. G, and Vakharia, A.(1998), Cell formation in group technology : review, evaluation and directions for future research, *Computers and Industrial Engineering*, 34(1) 3-20

Simeu-Abazi, Z.,Daniel, O., and Descotes-Genon, B. (1997), Analytical method to evaluate dependability of manufacturing systems , *Reliability Engineering and System Safety*, 55(2), 125-130

Singh, N.(1993), Design of cellular manufacturing systems, *European Journal of Operational Research*,69(3),284-291

Sofianopoulou, S.(1999), Manufacturing cells design with alternative process plans and or replicate machines, *International Journal of Production Research*, 37(3), 707-720

Sofianopoulou, S.(1997), Application of simulated annealing to a linear model for the formulation of machine cell in group technology, *International Journal of Production Research*, 35(2), 501-511

Solaimanpur, M., Vrat, P., and Shankar,R. (2004), A multi-objective genetic algorithm approach to the design of cellular manufacturing systems, *International Journal of Production Research*, 42(7), 1419-1441

Spiliopoulos, K., and Sofianopoulou, S.(1998), An optimal tree search method for manufacturing system cell formation problem, *European Journal of Operational Research*, Vol.105, 537-551

Steuer, R. E.(1986), Multiple Criteria Optimization: Theory, Computation and Application (USA: John Wiley & Sons, Inc)

Su, C. T., and Hsu, C. M. (1998) Multi-objective machine-part cell formation through parallel simulated annealing, *International Journal of Production Research*, 36 (8), 2185-2207

Suresh, N. C. and Meridith, J. R.(1994) Coping with the loss of pooling synergy in cellular manufacturing systems, *Management Science*,40(4), 466-483

Sulliman, S. M. A.(2000), A mathematical model for a buffered two stage manufacturing cell with an unreliable transfer device, *International Journal of Production Economics*, 63(1), 69-81

Taha, A. H.,(1992), *Operations Research* (New York: Macmillan Publishing Co. Inc.), pp. 330-331

Talukder, M. S., and Knapp, G. M.(2002), Equipment assignment to multiple overhaul blocks in series system, *Journal of Quality in Maintenance Engineering*, 8(4), 319-330

Tillman, F. A., Hwang, C.-L., and Kuo, W (1980), *Optimization of System Reliability* ( New York: Mercell Dekker, Inc.)

Tsai, C. C., Chu, C-H., and Barta, T. A. (1997), Modeling and analysis of a manufacturing cell formation problem with fuzzy mixed-integer programming, *IEE Transactions*, 29(7), 533-537.

Uddin, M. K., and Shanker, K.(2002), Grouping of parts and machines in presence of alternative processing routes by genetic algorithm, International *Journal of Production Economics*, 76(3),219-228

Vakharia, A. J., and Chang, Y.-L.(1997), Cell formation in group technology: a combinatorial search approach, *International Journal of Production Research*, 35(7), 2025-2043

Vakharia, A. J., Askin, R. J., and Selim, H. M.(1999), Flexibility considerations in cell design, In S. A. Irani(ed), *Handbook of Cellular Manufacturing Systems* (New York: John Wiley and Sons) pp 249-274

Valdez-Flores, C., and Feldman, R.M.(1989), A survey of preventive maintenance models for stochastically deteriorating single unit systems, *Naval Research Logistics*, 36, 419-446

Venugopal, V. and Narendran, T. (1992) Cell formation in manufacturing systems through simulated annealing: an experimental evaluation, *European Journal of Operational Research*, 63 , 409-422

Viswanadham, N., and Ram, R.(1994, April), Composite performance-dependability analysis of cellular manufacturing systems, *IEEE Transactions on Robotics and Automation*, 10(2), 245-258

Wang, K. S., and Wan, E. H. (1993), Reliability consideration of flexible manufacturing system from fuzzy information, *International Journal of Quality and Reliability Management*, 10(7), 44-57

Wang, H.(2002), A survey of maintenance policies of deteriorating systems, Invited review, *European Journal of Operational Research*, 139(3), 469-489

Wemmerlov, U., and Hyer, N. L. (1986), Procedures for the part family machine group identification problem in cellular manufacturing, *Journal of Operations Management* 6(2), 12-147

Wemmerlov, U., and Hyer, N. L. (1989), Cellular manufacturing in the U S industry: a survey of users, *International Journal of Production Research*, 27(9), 1511-1530

Wemmerlov, U. and Johnson, D. J (1997), Cellular manufacturing at 46 user plants : implementations experiences and performance improvements, *International Journal of Production Research,* 35(1), 29-49

Wicks, E. M., and Reasor R. J. (1999), Designing cellular manufacturing systems with dynamic part populations, *IIE Transactions*, 31(1), 11-20

Wilderman, R. E., Dekker, R., and Smit, A.C.J.M.(1997), A dynamic policy for grouping maintenance activities, *European Journal of Operational Research*, 99(3), 530-551

Won, Y.,and Lee, K. C.(2004), Modified p-median approach for efficient GT cell formation, *Computers and Industrial engineering*, 46 (3), 495-510

Wu, N., and Salvendy, G.,(1999), An efficient heuristic for the design of cellular manufacturing systems with multiple identical machines, *International Journal of Production Research*, 37(15), 3519-3540

Xambre, A. R., and Vilarinho, P. M. (2003) A simulated annealing approach for manufacturing cell formation with multiple identical machines, *European Journal of Operational Research*, 151 (2), 434-446

Yazhau, J., Molin, W. andZhixin, J.(1995), Probability failures of machining center failures, *Reliability Engineering and System Safety*, 50(1), 121-125

Yuanidis, P., Styblinski, M.A., Smith, D.R., and Singh, C. (1994), Reliability modeling of flexible manufacturing systems, *Microelectronics and Reliability*, 34(7), 135–139

Yasuda, K., Hu, L.,and Yin, Y. (2005), A grouping genetic algorithm for the multi-objective cell formation problem, *International Journal of Production Research*, 43(4), 829-853

Zakarian, A., and Kusiak, A. (1997, April), Modeling manufacturing dependability, *IEEE Transactions on Robotics and Automation*, 13(2), 161-168

Zhao, C., and WU, Z. (2000), A genetic algorithm for manufacturing cell formation with multiple routes and multiple objectives, *International Journal of Production Research*, 38(2), 385-395

Zolfaghari, S., and Liang, M. (1998) Machine cell/part family formation considering processing times and machine capacities: a simulated annealing approach, *Computers and Industrial Engineering*, 34(4), 813-823

## APPENDIX A.1: BRIEF SUMMARY OF MODELS

| | Description of Model | Solution Procedure | Chapter references |
|---|---|---|---|
| 1. | **System Reliability Model** for the CMS machines associated with a part processing route<br>**Model development** : Analytical approach, considering machine reliability to follow:<br>a) the exponential distribution and<br>b) the Weibull distribution<br>**Purpose** : To develop a machine reliability analysis model and reliability based objectives for CMS design | Optimal solution for CMS design using LINGO 09 | a) Development: **Section 2.3.2**<br>Application: **Section 4.2**<br>( Objective function II)<br>b) Development: **Section 2.4**<br>Application: **Section 4.3.1**<br>( Objective function II) |
| 2. | **System Availability Model** for a part type-process plan combination<br>**Model development** by Markovian approach. The model has three parts.<br>a) Transition probability matrix (TM)<br>b) Steady state probability evaluation<br>c) Evaluation of system availability<br>**Purpose** : Performance evaluation in terms of system availability for part type-process plan assignment | a) Basis data computation by analytical approach,<br>TM by LINGO 09<br>b) Steady state probability by LINGO 09<br>c) System Availability by LINGO 09 | a) Development for TM : **Section 2.4**<br>Application :**Section 4.2.4.1**<br>b) Development for Steady state probability analysis : **Section 2.4**<br>Application : **Section 4.2.4.1**<br>Development and application, **Section 2.4 and 4.2.1** |
| 3. | **Multi-Objective MIP CMS Design Model** using machine reliability consideration to follow the **exponential distribution**<br>**Purpose** : Design of CMS by simultaneous consideration of system reliability and system cost using machine reliability consideration based on the **exponential distribution** | - Є- constraint approach solved using LINGO 09<br>- Solved large size problems using the Heuristic approach | Development and application<br>**Section 4.2** |
| 4. | **Multi-Objective MIP CMS Design Model** using machine reliability consideration to follow the Weibull distribution<br>**Purpose** : Design of CMS by simultaneous consideration of system reliability and system cost using machine reliability consideration based on the **Weibull distribution** | Є- constraint approach<br>Solved using LINGO 09 | Development and application<br>**Section 4.3** |

**APPENDIX A.1: Cont'd**

| | Description of Model | Solution Procedure | Chapter references |
|---|---|---|---|
| 5. | **Maintenance Planning Model for CMS ( NLP )**<br>a) Cost based<br>b) Reliability based<br>c) Combined model to consider both cost and reliability<br>d) Extension of reliability based approach based on effective interval for individual machine to improve resource utilization and improve cost and reliability performance<br>**Purpose** : Preventive Maintenance planning and scheduling | a) Maintenance interval by Golden Section search by MATLAB 6.5. Total cost using LINGO 09<br>b) Reliability based model using LINGO 09<br>c) Combined model by Pre-emptive optimization-using LINGO 09<br>d) Extension of reliability approach using LINGO 09 | Development and application<br>a ) **Section 7.2.1**<br>b) **Section 7.2.2**<br>c) **Section 7.2.3**<br>d) **Section 7.3** |
| 6. | **Integration of PM Planning in Multi-Objective CMS design model**<br>**Purpose** : Integrating PM planning in the CMS design process to consider cost, reliability and maintenance planning for overall improvement of reliability performance and cost in a CMS | $\epsilon$- constraint approach solved using LINGO 09 | Development and application<br>**Section 7.4** |

**APPENDICES A.2, A.3, A.4, A.5 in CD FORMAT**

A.2 LINGO PROGRAM FILES FOR CMS DESIGN
- A.2.1 Design and Performance Model (Exponential Distribution)
- A.2.2 Design Model ( Weibull Distribution, Short Range Beta)
- A.2.3 Design Model ( Weibull Distribution, High Range Beta)
- A.2.4 Design Model ( Weibull Distribution, Long Range Beta)
- A.2.5 Transition Probability Matrix Model
- A.2.6 Steady State Analysis Model

A.3 LINGO PROGRAM FILES PREVENTIVE MAINTENANCE
- A.3.1 Optimum and Maximum Interval Model
- A.3.2 Total Cost Model ( Algorithm Based)
- A.3.3 Total Cost Model ( Cost Based)
- A.3.4 Combined Model
- A.3.5 Cell Design Model with PM Integration

A.4 C++ PROGRAM FILES
- A.4.1 14 Machine 24 Parts Model Solver ( Machine reliability consideration based on exponential distribution)

A.5 MATLAB FILES
- A.5.1 Golden Section Search (Preventive Maintenance Interval)

## VITA AUCTORIS

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